# Teacher Training for Chess and Mathematics

A (chess) game-based approach to problem solving

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*IO3*

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# 1. Introduction

The pattern of work in the digital-robotic future will be transformed. Citizens will need to navigate through new economic models, systems, and processes. Knowledge and expertise will be less important than the ability to adapt continually to new conditions. Those with the ability to master a domain will be the most sought after. Multi-careered workers will need to think critically, solve problems, recognize systems of cause and effect as well as have the confidence to communicate. Pressing ecological and economic challenges require people to collaborate, communicate and solve problems.

Furthermore, international educationists have argued that children could get a lot more out of their school experience. The harsh focus on mathematics and English has failed to inspire the majority of children. The continual testing desolates what should be a fruitful period of their lives. The obsession with league tables and international PISA rankings distorts policy. Fortunately, there are countervailing currents from practitioners in those areas which have been squeezed out of the core curriculum: the arts, music, languages, and sport. For example, the International Baccalaureate still requires its students to undertake 'Creativity, Activity, Service'.

Traditional classroom learning often falls short of equipping students with the knowledge they need to thrive in this changing world. A board game that trains the mind in structured thinking, enhances memory and facilitates social interaction could be an ideal enriching tool in this new environment. Chess fits the bill well: it can be used to develop a wide spectrum of personal and social skills, such as visualization, creativity, focus, cooperation and good manners, amongst many others.

Schools could also do a lot more with games. Children derive intrinsic satisfaction and meaning from playing games. Many pupils' brains have already been rewired since toddlerhood by constant exposure to digital devices. Playing games requires solving a series of problems, which reinforces their willingness to solve problems. What more fertile ground could there be to embed some instructive material? Computer games may not be so amenable to school instruction, but there is a game which fits the purpose.

Chess is a classic board game notable for its intellectual variety and depth. Empirical studies show that children who play this abstract strategy game improve their thinking, planning, and problem-solving capabilities, and also their overall academic performance. Hence, over the last decade the international 'scholastic chess' movement has emerged.

Chess has evolved over a millennium and a half to match society's requirements. What we typically associate with chess – chequered red boards, long-range pieces and clocks – are historic artefacts. Chess is now seen not just as a competitive board game: rather it is a domain in which the interaction of pieces, moves, and rules, with subsets and variants, give rise to intricate problem-solving challenges.

The foregoing insights have given rise to a search for exercises that lend themselves well to classroom instruction. For those who know how to play chess, miniature endgame studies, checkmates, and proof games are particularly instructive. Ideas can be expressed in the purest form in scenarios with very few pieces or moves. For those who do not play chess, the most accessible approach is through mini-games comprising subsets of chess armies, where even a simple objective, for example reaching the other side, can generate a strategically rich game. Strategy in this context means that one's best move is dependent upon the assumed response by the opponent.

The purpose of the CHAMPS (Chess and Mathematics in Primary School) project is to develop a new category of 'chess-maths' exercises in which mathematical games and puzzles are represented in a chess format. The objective is to insert logic and mathematics into situations where children are most receptive. As far as the children are concerned, they already play on the chessboard with pieces, so attempting some questions from a different perspective does not seem like crossing into the feared 'maths lesson' territory. Conversely, some maths lessons may be enriched by such exercises without the need to be a chess player.

The quest has been to find 50 instructive problems for children to solve using the chessboard and the pieces but – and here is the innovative part – these are not chess problems. The children are exploring with enjoyment the mathematical characteristics of a constrained domain - an 8 x 8 grid with pieces that have a variety of moves and with a variety of winning conditions. This is a pedagogical approach which touches on the fields of arithmetic, geometry, combinatorics, graph theory, and game theory, and gives rise to fascinating problems involving *inter alia* symmetry, polarity, tiling, and binary multiplication. We have tapped into a rich vein which combines games, mathematics and children having fun.

A crucial requirement is to ensure that the material can be understood not only by the children but also by their teachers. Teachers are naturally uncomfortable dealing with those activities in which the children may outperform them. We address this by providing a comprehensive teachers' guide which sets out the solution methods and answers for each of the 50 exercises.

This project was funded through ErasmusPlus from the European Union. The partners on the CHAMPS project were the Slovak Chess Federation (co-ordinator), Chess in Schools and Communities (UK), Ludus (Portugal), University of Girona (Spain) and Veľká Ida Primary School, Slovakia. The following individuals also contributed to the project: Carlos Santos, Carme Saurina Canals, Josep Serra, Mark Szavin, Stefan Löffler, Alessandro Dominici, Malcolm Pein, Chris Fegan, Zdenek Gregor, Eva Repkova, Vladimir Szucs, Viera Kebluskova, Niki Vrbova and Viera Harastova. The project manager was Stefan Marsina.

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*March 2019*

# 2. Mathematics Curriculum

The primary school topics corresponding to the 50 Exercises book are as follows:

1. Co-ordinates, positions, movements, pencil
2. Parity, Visualisation, pencil
3. Patterns, sorting, order, sequences, pencil
4. Parity, symmetry
5. Parity, symmetry
6. Arithmetic, symbols, equations, pencil
7. Geometry, spatial notions, enumeration, intersection
8. Enumeration, pigeonhole principle, maximum/minimum
9. Arithmetic, trial and error, input/output
10. Enumeration, symmetry
11. Geometry, straight lines, slope, ruler
12. Geometry, straight lines, slope, ruler, pencil
13. Working backwards from the target
14. Parity, pencil
15. Enumeration
16. Symmetry
17. Symmetry, Elimination
18. Shapes, symmetry
19. Enumeration, Venn diagrams, Trial and Error
20. Enumeration, parity
21. Disjoint properties, union, symmetry
22. Logic
23. Straight lines, slopes, trial and error
24. Enumeration, spatial notions
25. Enumeration, spatial notions
26. Logic, information, tree diagrams
27. Symmetry, working backwards from the target
28. Shapes, parity
29. Trial and Error, symmetry, multiples, divide to conquer
30. Enumeration, trial and error
31. Geometry, distance measures, peer learning
32. Symmetry, multiples
33. Symmetry, multiples, powers
34. Symmetry, multiples, tree diagrams, working backwards
35. Symmetry, representation
36. Symmetry, parity, working backwards
37. Enumeration, Pascal's triangle
38. Enumeration, Pascal's triangle
39. Enumeration, Pascal's triangle
40. Decomposition of shapes, trial and error
41. Decomposition of shapes, enumeration, trial and error, area measures, square numbers
42. Enumeration, trial and error
43. Enumeration, trial and error
44. Enumeration, shapes, arithmetic, organising information in tables
45. Logic, organising information in tables
46. Enumeration, trial and error, Symmetry
47. Enumeration, proportion, organising information in tables
48. Exponential Growth, geometric sequence
49. Enumeration, shapes, organising information in tables
50. Enumeration, symmetry, angles

## 3. Training to Teach Chess and Mathematics

The objective of this document is to outline the design principles for a Scholastic Chess Instructor Certificate suitable for those wishing to:

- teach chess for mathematics enrichment and/or
- use chess as an educational tool primarily in the teaching of mathematics.

Ideally, this Certificate would be recognized in all EU countries, and thus would provide a highly desirable qualification amongst teachers who seek continuing professional development.

Adopting this approach, we have gathered teacher materials from around the world relating to chess, mathematics, and problem solving. The material was critically reviewed by educational experts on the CHAMPS team.

**How much chess knowledge is required?**

There is a minimal requirement for knowledge of chess to tackle chess and mathematics problems:

- chessboard essentials: files, ranks, and naming the squares
- how pieces move and capture
- concepts of 'controlling a square' and 'attacking a piece'
- exchange value of the pieces (for the arithmetic exercises)
- (for some exercises) check and checkmate

**Competencies developed**

Many mathematical competencies can be developed with Chess and Mathematics. The general competencies are:

- analytical reasoning
- strategic thinking
- pattern recognition
- spatial awareness

In addition, the following competencies are targeted specifically:

- parity as a problem-solving aid
- understanding rotational and line symmetry
- applying the rules of logic
- enumeration and systematic counting
- collection and use of information in tables

# 4. Chess and Mathematics

Classical Chess and Mathematics exercises are conventionally classified as falling into the area of Recreational Mathematics. They span across many mathematical fields, for example:

- Logic: the study of reasoning
- Geometry: properties of space, and shape and size of objects
- Combinatorics: counting and listing elements in a finite structure
- Graph theory: the study of graphs (set of nodes connected by edges)
- Game Theory: the science of strategy and decision making

Geometry is the field that is most closely linked to the primary curricula, so the question arises: are there any compelling reasons why we should use Chess and Mathematics problems in primary schools? We are persuaded that there are several important reasons:

1) Children who are familiar with chess are also comfortable working in chess themes.

2) A great number of mathematical problems can be presented on the chessboard with or without chess pieces.

3) Game-based learning, including mathematical games, is an area of growing importance given how well children relate to playing games.

4) Very little chess knowledge is required for Chess and Mathematics, so hopefully a lot of teachers would welcome these exercises in their classroom.

## 4.1. Problem Solving in Chess and Mathematics

Many national curricula state that mathematics is about problem solving. It is imperative that we place emphasis on this aspect of using Chess and Mathematics in the primary classroom. It is essential to instil an exploratory mindset in all learners and enhance the core competencies of mental strategies such as visualisation and pattern recognition.

There are generally two different approaches to mathematical development: instrumental understanding and relational understanding. Instrumental understanding is having a mathematical rule and being able to use it. Relational understanding is about using the mathematical rule, and also knowing why it works. Our goal is to develop a relational understanding of mathematical concepts and thoughtful presentation of Chess and Mathematics can facilitate this.

## 4.2. Simplifying Methods

Chess is a rich 'low-threshold, high-ceiling'[1] game that can be played and enjoyed on many different levels. Everyone can have fun and find meaning in it regardless of their experience and quality of play. This is in contrast with 'Chess and Mathematics' problems which can be inherently difficult and are generally not meant for the primary classroom. On the other hand, suitably selected and exercises provide a rich reservoir of questions that can be explored from many different angles.

The educator's challenge is to take suitable high-ceiling Chess and Mathematics problems and tailor them into low-threshold activities accessible for a younger audience. The most straightforward way to simplify a problem is to part-solve it. As Pólya put it, `If you can't solve a problem, then there is an easier problem you can solve: find it.´

This can be applied to almost any question, for example:

- No need to juggle with five dominating queens – instead reveal the position of four queens and give learners the task of placing the fifth queen on the board.
- Only play out the last few moves in a game. For example, in the Sliding Rooks game slide the rooks together in ranks 1-6 and only play in ranks 7 and 8.
- Play the game against the student with perfect technique thus revealing the winning strategy.

Asking leading questions and providing more information are also useful simplifying methods. Introductory questions that gently tap into the problem provide a good starting point. They raise students' interest and may boost their confidence and willingness to tackle the problem in more detail.

[1] The term originally from Seymour Papert (1980) has been popularised by the NRich mathematics outreach project from the University of Cambridge.

## 4.3. POLYA PROBLEM SOLVING STRATEGY

George Pólya was a famous Hungarian mathematician and teacher (1887-1985). He wrote extensively about problem solving, namely in his renowned book *How to Solve it*. Pólya's six-step problem solving strategy is applicable to mathematics as well to other disciplines. Pupils can benefit from learning and using these strategems:

1. Understand the problem. You must have a clearly defined initial situation and a desired goal situation.

2. Determine a plan of action. What resources will you use, how will you use them and in what order? What strategies will you apply?

3. Anticipate undesirable outcomes arising from carrying out your plan of action. Return to step 1 or 2 if large problems would be created as a result of your action.

4. Carry out your plan of action in a reflective, thoughtful manner.

5. Check to see if your desired goal has been achieved. If the problem has not been solved, return to step 1 or 2. The reflective thinking you practised in step 4 has increased your expertise, and you may come up with a more suitable plan of action. At this point you may also decide to stop working on this problem.

6. Analyse the results you have achieved. Reflect on what you have learnt by solving this problem. Your expertise may come useful in future problems!

## 4.4. Concrete-Pictorial-Abstract method

We have followed the widely approved Concrete-Pictorial-Abstract method (CPA) in the 50 Exercises. This method is also central to the Singapore method for teaching mathematics which has proved successful in international comparisons.

**Concrete Phase**

Every abstract concept is first introduced using physical materials such as the chess board, chess pieces, coloured or numbered discs, shapes for tiling the board and many others. Introducing concepts in a concrete and tangible way allows children to explore the problem with some free play, clarify ambiguous points, get closer to the solution and communicate all these ideas with their peers.

We have given careful consideration to the Concrete phase in the design of IO1. Suggestions are given whether to undertake this activity alone, in pairs or in smaller groups. Many problems begin with a teacher-led discussion prior to the Concrete phase.

**Pictorial Phase**

The Pictorial phase is the next step in the investigation. It mostly takes the form of working with pencil and paper on printouts of empty boards or chess positions. A few exercises start directly with the Pictorial phase. The Rook's Tour is an example in which the Concrete phase is omitted as it would provide little added value and is likely to confuse learners since tracking the rook's movement is difficult on the physical chessboard. Filling out partially completed tables is another example for the Pictorial phase such as in 'King's random walk and 'How many rectangles on a chessboard'.

**Abstract Phase**

The Abstract phase is often reserved for differentiation and takes the form of an extension exercise. Abstract mathematical reasoning is mostly beyond the grasp of primary school children. Therefore, full proofs to problems are not given if they are considered too challenging for the target age.

# 5. Training Course Design

Having critically reviewed the international material, the CHAMPS team formulated a trial one-day training course which was run under the auspices of the European Chess Union in London in December 2018. There were 14 participants of which 11 were from Europe and 3 were from the USA. The majority of the participants were primary school teachers, and the rest comprised chess coaches and organisers of chess education. It was a full-day course led by two instructors, Rita Atkins and John Foley. The participants completed a test at the end of the course.

**Course Timetable**

10:00 Course start

Introductions and group discussion
Starter problems
Tailoring problems for a young audience
Maths minigames

13.00 Lunch

Visualising mathematics
Logic puzzles
Problem solving
More challenging exercises
A look at advanced topics

16.00 Test

17:30 Course End

The feedback from this course helped us to refine further the pedagogical approach. The following section summarises the practical insights gained from running the trial training course.

## 5.1. PRACTICAL CONSIDERATIONS

**Course duration**

The amount of material contained within the 50 Exercises book and the detail of the Solution Methods require a two-day training course for teachers. A preceding training day to learn chess is advisable for those teachers who have never encountered the game.

**Explain the connections between chess and mathematics**

Teachers found it helpful early on to explore the connections between chess and mathematics in some depth. In tutor-led discussions, we listed the mathematical concepts inherent in chess. We discussed whether chess could help develop mathematical skills. An important topic, discussed in groups, was if and how mathematical training improves the understanding of chess.

**Presenting the exercises**

It is important to be able to present each of the exercises in the correct way. Always start simply and then gradually add layers of complexity. Teachers should not hesitate to start with even more basic material than is mentioned in the exercise. Ensure that the class understands the basic elements before releasing them to start solving.

**Teacher knowledge**

Teachers need to fully understand each of the classroom exercises in order to be able to present them effectively. This means that the teachers should know the basic rudiments of playing chess. This will maintain their credibility with the children and improve response rates.

**Age level**

Each exercise has an indicated age level at which pupils may be expected to try to solve the exercise, taking into account the mathematical topic and level of difficulty. If the topic is developed carefully, then children may be able to solve problems for a higher age group.

**Not about chess**

The course is not about the mathematics of playing chess. It does not cover the analytical side of chess: material balance, zugzwang, king geometry etc. For example, it is irrelevant whether the queen is worth 9 points or 10 points except for its convenience in the arithmetic exercises.

**Scope Limits of Course**

The scope of the course is limited to the training of educational practitioners. The course does not present the outcomes of scientific investigations into chess as a pedagogical tool in the teaching of mathematics. The course focus is manifested in the selected 50 exercises which represent the most recommended activities from respondents across Europe. The course is not part of a research project and so it does not delve into how chess helps develop mathematical skills nor how mathematical training can improve the understanding of chess.

**Problem solving**

We discussed Problem Solving Methods at great length. Useful conclusions were drawn, many of which are reflected in the Solution Methods sections of the 50 Exercises. Here is the list of the most relevant problem-solving methods we have found for Chess and Mathematics:

- look for a starting point
- trial and error
- list all possible outcomes
- eliminate impossible outcomes
- work backward from the solution
- put information into a table
- follow the rule

**Concrete v abstract**

Course attendees prefer concrete activities which require only a limited amount of prior knowledge. Problems that required a high level of abstraction, such as those involving mathematical proof, were not so well received. We decided to omit or truncate a few challenging topics such as the knight's tour, tiling with tetrominoes, tiling with equivalent shapes and Napier's binary arithmetic. These topics would be more suited to a secondary school treatment. Hands-on problems where the participants are provided with materials (paper, pencil, ruler, counters, etc) generated more engagement than those exercises which were abstract or thought experiments.

## 5.2. GAMIFICATION

An important observation from the Trial Training Course was that everybody enjoyed playing the games on the chessboard. In order to carry on with the course we typically had to interrupt their games as everyone wanted to continue playing. Heated discussions accompanied the games and a lot of peer-learning took place. Given that adults enjoyed the Chess and Mathematics games so much, we were confident that children would be equally receptive to its charms, therefore we decided to gamify as many problems as possible.

**Rendering exercises into games**

Games based on Chess and Mathematics proved very popular and so many of them were retained within the 50 Exercises book. We also made it a priority to gamify as many exercises as possible. Almost every exercise can be gamified with a little bit of imagination.

Games have a specific role in children’s lives: it is their main type of activity. Participation in a game is associated with positive feelings of joy, excitement and relaxation. A game seems to be a natural and fun way to obtain knowledge and acquire new mental processes. Certain educational theories, such as Waldorf schools, consider game playing as their main teaching method. The stimulating nature of play, the way it increases the engagement of pupils and how it spontaneously makes them use different abilities could be the reasons for Chess and Mathematics to gain a firm foothold in primary schools.

Education has many goals and there is a huge amount of research about game-based learning. Some key points in favour of game-based education versus traditional methods:

- Intrinsic motivation: learners being engaged because they love to play.
- Transfer of learning from game-playing environments to other environments.
- Learning some general strategies for problem-solving.
- Games provide an excellent environment for computational thinking.
- High level of engagement, because the learner wants to win.
- Games provide an environment in which one can interact with other people and develop certain social skills.
- Learners can think critically and carefully and mostly outside of the box!

## 5.3. Testing Knowledge

At the end of the Trial Training Course a test was administered in-course using eight multiple-choice and three open-ended questions. The questions closely mirrored the contents of the course. While the multiple-choice questions posed little difficulty to the participants, many people found the open-ended questions rather challenging. The mathematical aspects of questions nine and ten troubled a few participants, and others with limited fluency in the English language struggled with the essay-style question eleven. Even though the test was by no means straight forward, we were delighted that everybody has passed!

The test that was administered at the end of the course proved slightly too difficult for primary school teachers. We appreciate that course attendees have had to climb a steep learning curve. The test was more suitable for teachers of children who at the high ability range in primary school or are aged 11 and above. Modifications, such as tailoring of the open-ended questions would make it more suitable as a standardised test for primary school teachers.

Participants of the Trial Training Course have given us valuable feedback about the content and delivery of the course. Participants generally enjoyed the course and promised to use the material and techniques to enrich their mathematics lessons or enliven their chess coaching. The pilot testing was immensely useful in the finalisation of the 50 Exercises. Most exercises were retained, some were simplified, and a few were omitted. The final set of exercises has been refined continuously to ensure that they are usable by teachers and instructive to children. Each teacher will have their own way of presenting the material.

The Test and Marking scheme are enclosed in the Appendix.

# 6. Conclusion

The CHAMPS project provided vital insights into the types of Chess and Mathematics exercise to suit the interests and aptitudes of primary school teachers whilst providing children with instructive mathematical problems and solution methods. We are confident that the recommended range of exercises included within the 50 Exercises book has been extensively tested in the classroom and the training room. The quality of the material is sufficiently high to justify inclusion in any primary school mathematics curriculum. The 50 Exercises form a suitable basis for a training course.

# APPENDIX

**Chess and Mathematics Trial Training Course Test**

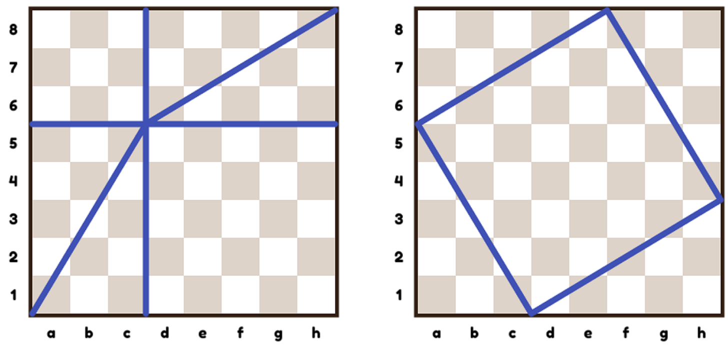
Time: 40 minutes **Name:**

For Questions 1-8 circle the correct answer. Only one answer is correct.

1. Give a reason why mathematics problems on the chessboard are great teaching tools.
   A) These problems relate closely to the primary mathematics syllabus.
   B) Children who are familiar with chess feel comfortable working in chess themes.
   C) In order to facilitate mathematical learning it is essential that children learn how to play chess.
   D) So that children learn about graph theory.

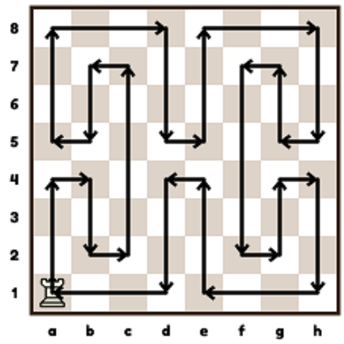
2. What chess knowledge is NOT required for solving mathematics problems on the chessboard?
   A) Moves of the pieces
   B) Stalemate
   C) The concept of controlling a square
   D) How pieces capture

3. Which famous mathematical theorem is illustrated on these diagrams?
   A) Pythagoras Theorem
   B) The sum of the first n odd numbers equals $n^2$
   C) Fermat's last theorem
   D) Prime number theorem



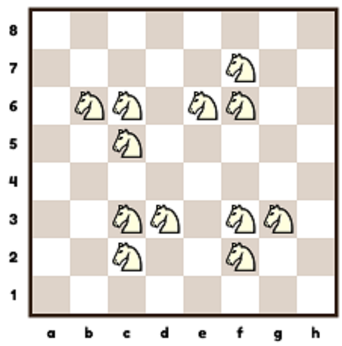

4. Which is the ONLY Simplifying Trick
   A) Trial and Error
   B) Look for a starting point
   C) Work backward from the solution
   D) Part-solve

**5.** Which statement about the diagram is TRUE?



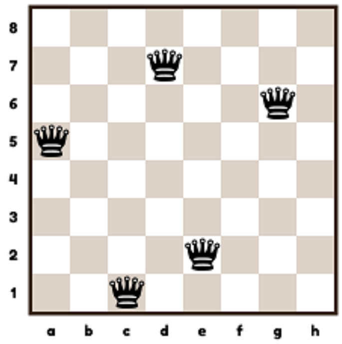

A) This is a Closed Tour of the rook that takes 28 moves.
B) This is an Open Tour of the rook that takes 28 moves.
C) This is a Closed Tour of the rook that takes the minimum number of moves.
D) This is an Open Tour of the rook in which there are equal number of horizontal moves and vertical moves.

**6.** Twelve knights can dominate the chessboard. On the diagram one of the knights was put on the wrong square. Which one is it and where should it go so that the knights attack every free square of the chessboard?



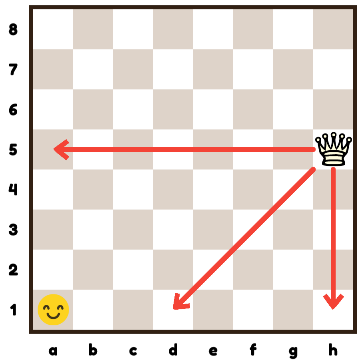

A) The knight from f7 must go to g6
B) The knight from f2 must go to f4
C) The knight from d3 must go to b3
D) The knight from c6 must go to a6

7. Eight independent queens can be placed on the chessboard. Place the missing three queens so that no queen can attack any other.



A) b3, f8 and h4
B) b8, f4 and h3
C) b8, f4 and h1
D) b3, f4 and h8

8. We have encountered Wythoff's game: a 'Wythoff's queen' is a chess queen constrained to move only south, west or southwest. Starting with a single queen on h5, players take turns to move. The first player to reach a1 is the winner.

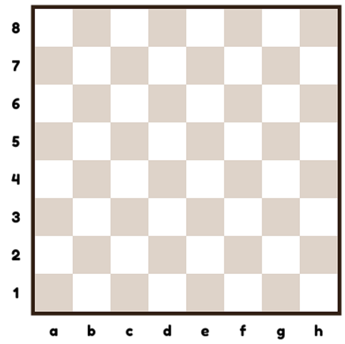
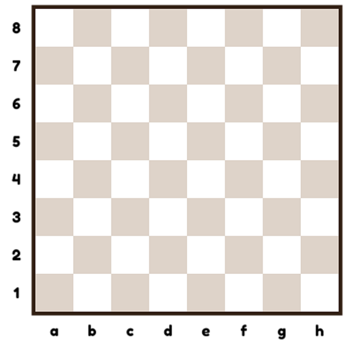
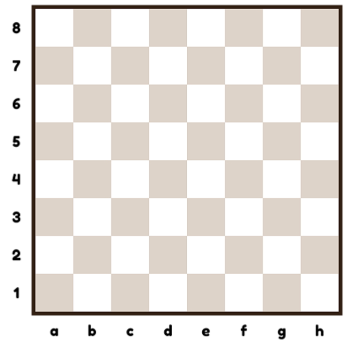


The queen can move any number of squares in the direction of the arrows.

Which statement about the winning strategy is TRUE?

A) The player must land the queen on one of three safe squares to win
B) The first player has the winning strategy
C) The queen starting on g5 would not change the outcome of the game (assuming optimal play)
D) If a player moves the queen to b3 or c2, he or she can win

**9.** There are many ways to tile the chessboard with 16 squares. Find three distinct solutions. Solutions in which the same set of squares is rearranged counts as a single one.

**10.** This is a game we have discussed:
The first player puts a knight on the board. The second player moves the knight to another square. They alternate to move the knight to any square where it has not been before. The last player who can make a move is the winner.

We have drawn a grid on the chessboard as shown on the diagram. The key was to make a move inside the region defined by this grid every time the opponent moves the knight into that region.

A) The first player begins the game by putting the knight on a square of the chessboard. Who has the winning strategy, the first or the second player?

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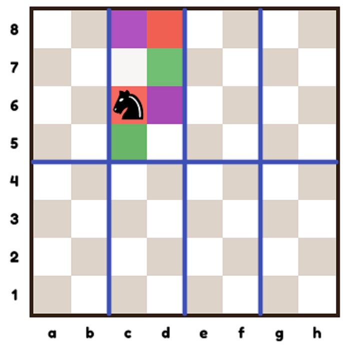
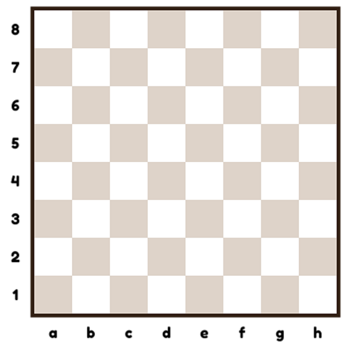
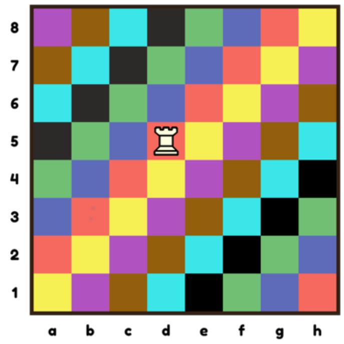



B) We now play this game with a king:
The first player puts a king on the board. The second player moves the king to another square. They alternate to move the king to any square where it has not been before. The last player who can make a move is the winner.
Find the winning strategy for this game. Draw a grid on the chessboard to illustrate your answer. Explain your strategy below.

……………………………………………………………………………………

**11.** Here is a well-known colouring problem:

We wish to colour every square of the chessboard. A rook is placed on a square of the chessboard that is painted to a particular colour. In every move the rook can make it must land on a square that has a *different* colour from the square where the rook moved from. The rook can start from any square. What is the minimum number of colours you need to colour the chessboard with?

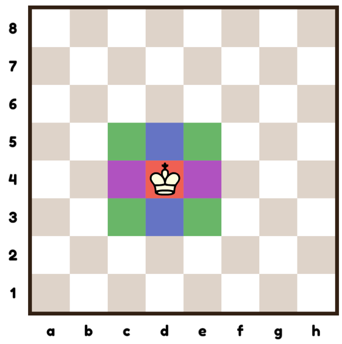


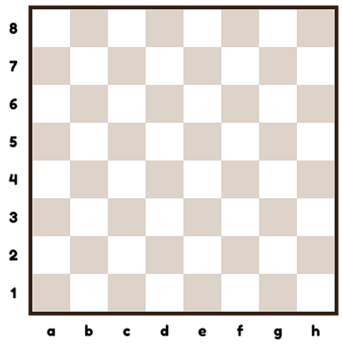
Answer: eight colours suffice as the diagram shows.

The same problem can be set for other pieces as well.

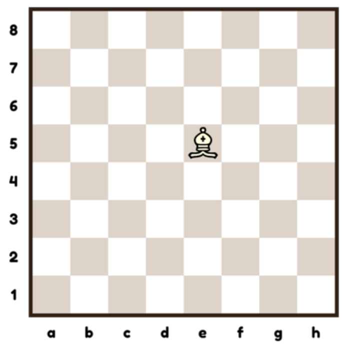
A) What is the minimum number of colours you need for the knight?

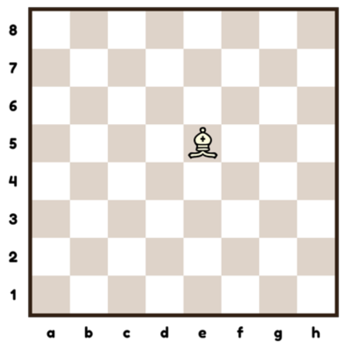
………………………………………………………………………………………





B) Solve the problem for the bishop. Either colour the squares or use numbers to represent colours in your answer.





C) Your task is to teach the colouring problem for the king to a class of ten-year-old pupils. Explain how you would introduce the problem and what teaching aids you would use. Mention any simplifying tricks and solving tricks you might employ in your delivery of the material. Give as much detail as necessary. Below are two diagrams you may wish to use.

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**Chess and Mathematics Trial Training Course**

**Marking Scheme**

The total number of marks is 36. The pass mark is 18 or more.
Each multiple-choice question is awarded 2 marks. Only one answer is correct.

**1.** B

**2.** B

**3.** A

**4.** D

**5.** A

**6.** B

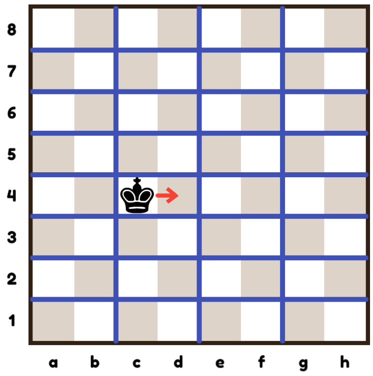
**7.** A

**8.** D

**9.** There are eight distinct solutions. Any three are rewarded with 2 marks each.

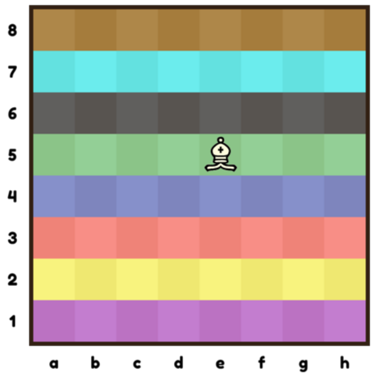
**10.** A) The second player wins (2 marks)

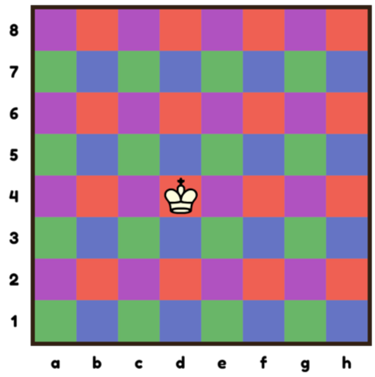
B) Make a 2x1 grid such as the one shown on the diagram (2 marks). The second player has a winning strategy. (1 mark). He must move inside the region defined by this grid every time the opponent moves the knight into that region. (1 mark)




**11.** A) Two colours (1 mark).

B) Eight colours suffice. Colour either every rank or every file in a different colour. (2 marks)




C) The correct colouring solution that is shown on the diagram is awarded with 2 marks. Maximum 3 more marks can be given for suitable points of introduction, simplifying and problem-solving methods. Some examples:

1. Provide a good starting point.
2. Let children use coloured counters to fill the board.
3. Ask them to use no more than four different colours.
4. Talk about the resulting colour pattern.
5. Work on printouts of empty boards and coloured pencils.