HANDBOOK FOR INNOMATH

TRAINING COURSE FOR TEACHERS FACILITATORS:

SUPPORTING MATHEMATICALLY GIFTED STUDENTS

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INNOMATH Training Course for teacher facilitators: Supporting Mathematically Gifted Students

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Introduction

This handbook, supporting the "**INNOMATH Training Course for teacher facilitators: Supporting Mathematically Gifted Students**", is a collection of all the Learning plans of the Modules, developed for the purpose of supporting mathematically gifted students. These Learning plans are designed for INNOMATH subjects which can be implemented inside and outside school enviroment.

The main target groups of this Handbook are teachers and future facilitators. Furthermore, students of age 10-18, with aptitude for Mathematics will benefit from the implementation of these learning plans of Modules. This Handbook can play an important role in the innovative educational transformation.

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INNOMATH: Innovative enriching education processes for Mathematically Gifted Students in Europe

INNOMATH Training Course for teacher facilitators: Supporting Mathematically Gifted Students

Module 1 : Access and use of INNOMATH project findings, MID Day model and Supporting Talented students with advanced maths

Note: This Module does not have a Learning Plan. This Module focuses on providing information in regards to the Mathematics in Industry Day (MID-Day) methodology to support mathematically gifted students.

A. Guidelines for a MID day

You should firstly identify gifted students, convinced an industry that their lateral thinking capacities might be worth a try and motivated academics and teachers to back you up. When you achieve these then you have a plan to design a MID-day (Mathematics meets Industry Day). This Module acts as a guide to plan and conduct a successful MID-day event.

B. Definition and Glossary

Partner: In these guidelines, while talking about partners, we mean an industry ready to invest a day or two talking with academics, teachers and students in order to define and address an industrial problem.

Industrial problem: A suitable issue brought forward by the industrial partner, that is understandable by students, teachers and academics. It has to be adapted on both sides, address some useful issues on the side of the industry and be challenging, yet feasible on the side of the students. The state of the problem is the result of a common brainstorming between the industrial partner, the academics and the teachers, prior to the MID-Day. We provide a list of possible problems in section E below.

MID-Day: An actual Math and Industry Day, when students tackle a problem set up by an industrial partner and adapted by the academics and teachers. It might be more an ideation, the refinement of an idea, than actually the full solution of a problem.

C. Methodology, Requirements and guidelines for schoolteachers

The responsibilities of teachers are to select and prep-up students in a way that will make them fit for a fruitful MID-Day. On the MID-Day, teachers have to facilitate students' work.

You have first selected students, whether using a written test or based on preselection by parents' association, grades at school, Olympiad or math clubs selection… You have obtained formal agreements from parents and schools for the participation of everyone.

After your first discussion with the industrial partner and the academics, you have decided on the type of MID-day to conduct. You will find below a few proposals. You will have to set up a scratch course based on a lesson plan that we provide, to level up the students to the adequate degree. This implies as well adjusting roles according to levels: some more advanced or mature students can act as coaches or tutors to younger ones, as *guide on the side* rather than *sage on the stage*.

Facilitation during the MID-Day involves propping up intimidated students, supporting self-confidence, reassuring them that their knowledge is not insufficient to start thinking about the problem and easing communication, making sure that no one feels insecure or imposes one's view, that every voice is heard, that every student feels needed and valued in a way or another. Division of the problem in sub-tasks has to be planned ahead for this purpose.

You will then have to report on how things went, observing regulation and self-regulation, how each student reacted, what triggers or withers his/her motivation, how he/she belonged, what was proposed and created.

After the MID-Day, pay a special attention on how the material raised during the MID-Day can be further used in everyday math class, relating it with institutional knowledge. Further look for opportunities for students to share their work with others in fairs, competitions, forums, clubs or setting up an exhibition in your school. This will help recruiting students for the year to come. Do not forget to mention that participation in a MID-Day can be valued by students in their CV when applying for grants. This event will spring and nurture their professional network!

D. Methodology, Requirements and guidelines for universities supporting MID days

University partners should build the team of students that will work on the industrial problem. That means working with students but as well training students in managing work as a team. In collaboration with the industrial partners and teachers, they have to shape the problem in a way that it can be worked upon by students.

Your main responsibility is to set up, organise and manage the MID-Day and the preparatory meetings before that. A typical interaction with teachers and students will have three phases:

In a 1st meeting: You will meet in an informal way students and their teachers, motivate them, build a team around original, creative and fun math material. You can reach out for participants in math club, science fair, math day, math competitions, inter-schools' meetings and so on.

In a 2nd meeting, you will level up selected students into building the group as a team. That means that you will teach them the tools to work as a team (see below).

The 3rd meeting will consist in the MID-Day *per se*. You will have to select facilities convenient and large enough for the meeting, whether at the university or at the industrial partner's site, have coffee breaks and lunch at hand and so on. A hybrid version, with online teleconferences might be in order if no possible physical meeting is feasible.

You will have to facilitate communication and team's work, asking the right questions so that everybody feels adequate and productive. Be careful not to look for answers by yourself without relying on students. Make sure students are relevant and the focus of the MID-Day. You are not here as a teacher that teaches students, they should be seen as trainee fellow researchers.[1]

Basic tools for working as a team

- Make divergences clear. While keeping a secure environment, reach a fruitful debate: do not vote, do not compromise, do not select at random, avoid averaging and bargaining but strive for unanimity.
- Listen and involve: Do not let anyone impose their standpoint but argue, do not dismiss but propose rebuttals.
- Do not personalise debates, do not take things personally. Look for solutions.
- Accept solutions only if it is understood and shared by everyone.
- Giving a light training in these themes, both for students and teachers, increases the probability to get original solutions and a productive team.

E. Examples of MID days

- Discuss with your industrial partner. Optimization problems can be found in many businesses: Whether the industrial partner is cutting glass, numerically controlling robots, managing a storage house or arranging a supply chain, they surely have a basic open optimization problem at hand that needs to be clarified if not solved.

- A **Statistical Research Survey** will interest your industrial partner, they might want to know better the needs of their clients or employees, and a market or human resources research project is relatively easy to set up, fun and creative to design and can lead to unexpected results when seen from an unusual point of view: *"how to transform this bleak open-space in the covid context?"*. A lesson plan in statistics and survey methodology is in order before actually getting down to business.

- A **Scientific Trail** on the industrial site is a great way for the industry to promote their work during class visits, and for the students to become problem posers and look at their surroundings with a scientific eye: "*how large is this tank? how long will it take to fill it up?*" The lesson plan should concentrate on modelling and measurement methodology,

[1] Hall, Jay (1971) Decisions, Decisions, Decisions, Psychology Today, November, 51-54, 84-88.

discovering for example that your own body is a pretty precise tool for everyday quantities estimations.

- If you do not have direct contact with an industrial partner, do not forget the **educational industry**, and have the students set up pedagogical games for you: let them invent new ways to teach by gamifying some of your content. Learn from them! The lesson plan should teach neural basis of cognition, gamification, determining aims and sub-aims, defining challenges and levels, ways to collaborate, helping the students understand their own cognitive process better. And you never understand a concept better than having to explain it, especially to a computer!

Once the main theme is set, you should plan the lesson path to get the students from where they stand to the required competencies regarding the tasks at hand. That is: you have to conduct the lesson in an introduction to the project. This first day at school should be relaxed and joyful, students working in teams and getting to know one another.

Once the problem is refined with the partner, you should send it to students in advance with a possibility to react, interact, with clarifying questions.

During the big day of the MID-Day, students visit the industrial site or at least discuss with representatives of the industry and work on their project together during the morning.

Here is a possible schedule for the MID-Day:

- 9h-9:30: Welcome, small talk ice breaking

- 9:30-10:30: Team building with a group activity like « lost on the moon » or similar. https://netmind.net/play-the-moon-landing-exercise/

- 10:30-10:45 Break

- 10:45-11:45 Discussion in sub-groups, regarding the problem of the MID-Day, comprehension of the issue, first tracks of thoughts…

- 11:45-12:15 First synthesis on possible strategies addressing the issues, setting crews on specific tasks.

- 12:15-13:45 Lunch break

- 13:45-15:45 Work in sub-groups on specific strategies and tasks.

- 15:45-16 Break

- 16-16:30 Preparation of the synthesis and its presentation by the students

- 16:30-17:15 Presentation in front of the industrial partners, discussion

F. Evaluation results, Dos and Donts

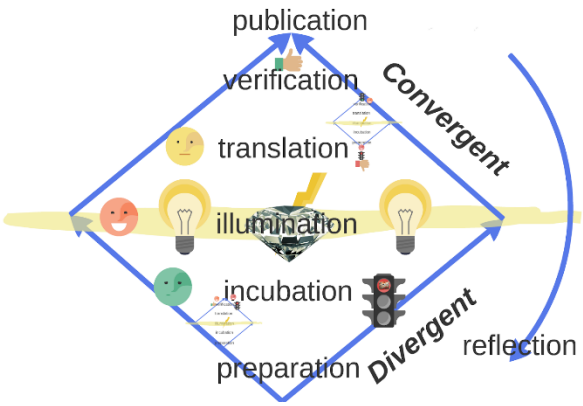
During the MID-Day

Beware that every student belongs to a small group, at most 8 students, formed in the preparatory activities, for example with older students supervising the group as a coach, making

everybody feel needed and belonging. One has to pay a special attention to insecure students by asking suitable questions, highlighting and developing the special abilities of each student, increasing self-confidence. The industrial problem is thus analysed with every perspective in mind, so that it is possible to divide partial tasks among all team members, fostering the collaboration of all team members.

The evaluation should focus on these aspects: that every student felt taken into account as an individual thinker and creator. Of course, not all ideas can lead to a successful product, maybe no production will result from the MID-Day and it should not be felt as a failure, but the potentialities and the creativity of ideas and productions should be put forward. If you need numbers, or want to assign individual trophies, you can objectify the evaluation of the creativity of the process of ideation that took place along the four dimensions described by Guilford: Fluidity, Flexibility, Elaboration and Originality:

- Fluidity is the capacity to produce a lot, to workout many examples.
- Flexibility is the ability to change a strategy when you are stuck, serendipity and adaptation are clearly the keywords here.
- Elaboration digs in depth a question, looks for the fine-grained details, is exhaustive in its search.
- Originality is the type of creativity you recognize when you see it: you have never seen anything like this, it is new and provocative.



Since we are talking about ideation, be aware that we are not in the students' head, to be fully creative, an idea has to go through the whole cycle of creativity with the two moments of divergent and convergent phases: from preparation to incubation, then illumination (the Aha moment), but this idea has to get out of the student's head and translated into words, sketches and so on, in order to be verified, and then published, proposed to colleagues to see what they think about it, whether they validate it. And in the end this idea can be reflected upon and integrated into a greater cycle of ideas in order to address the problem at stake.

One should not mistake creativity with only divergent thinking: a phase of brainstorming is acceptable, but at some point, it has to depart from daydreaming and converge to an actual workable solution, and there is a need for as much creativity in this convergent phase. Likewise, the Aha moment of illumination is not to be overvalued, poetry and industry can mix only to a point, what seems like an original idea has to translate to actual schemes. These subtle points have to be taken into consideration by the team in order to foster a productive mindset among students.

After the MID-Day:

All results will not show straight away during the MID-Day. Students will come back home with lots of observations, ideas and plans. In the last phase, this work can be reworked and presented again to the industrial partner. It is important to organize a way for them to exchange with others.

This can be further used, for school fairs, forums, schools twinning, integration weeks, competitions and so on in order to show what has been done.

G. Why Industry should get involved, benefits for industry

This point of the interest of the industrial partner has to be cleared beforehand but it is good to revisit it at the end of the MID-Day and go through the checklist: Have all parties benefited from the MID-Day? Has lateral thinking really unblocked a creativity situation or simply has the industry been able to show its business to outsiders? The benefits really are different depending on the type of MID-Day that was conducted. To go through the generic examples, we gave:

- Has the **Statistical Research Survey** been able to make evident elements that went unseen before, or was it simply a way to communicate, whether internally or with potential customers? Allowing customers or better employees to express their views, to see that they are taken into account is already something that can be valued.
- Is the **Scientific Trail** fun and informative? Can it be used with potential customers, with classes? Does it open the eyes of participants on the challenges that the industry addresses? In the case of a touristic visit of a site, does the partner discover the site with a new eye? Will it serve its purpose of scientific tourism?

As for the **educational industry**, have students setup interesting, informative and fun pedagogical games? Did they learn something out of designing the pedagogical game?

You can find examples of implementing MID-day events in various Countries in the O3: Electronic Guidebook of Methods and Tools for teacher facilitators, published in the INNOMATH website (https://innomath.eu/outputs/)

You can also see examples of MID-days in the link below:
https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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Module 2 – Learning Plan: Inquiry Based Learning

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

This module focuses on the approach of inquiry learning. Students are invited to create facts and data, to organise it and infer from it generalization, rules and principles. It stems from preliminary data collection to induce the process. The need for data organization is primordial and should not be dropped from a metaphorical helicopter, because most of the used formulae are supposed to be known but they are actually not available to be used. Teachers, as guide on the side rather than sage on the stage, guide the group of learners to form relevant questions and to shape conjectures using critical thinking. Helping students voice their thoughts, hear one another and appreciate each other's ideas is important in the process.

This approach can be used at basic level to deeply root important concepts that otherwise *fall from the sky* as a *deus ex machina* that actually don't make much sense. This is why we present as examples, at a basic level, the context of constructing the meaning of the formula of the volume being linearly dependent on any independent side lengths or rather, the *cube* of the size of an object. At a more advanced level, we investigate continued fractions of square roots of integers through origami. But both modules culminate in the same final activity: the discovery of the volume of a regular tetrahedron.

Derivation of volume growth with respect to enlargement

Students are guided to discover that, depending on the dimension, enlarging an object has a nonlinear effect on its measure (area or volume). Constructing paper models of different objects of different sizes, students gather data and arrive at the conclusion that the enlargement factor is used multiplicatively three times for the volume. Their findings is put to the use for the discovery of a formula for the volume of a regular tetrahedron, through an origami 3D puzzle.

Learning Outcomes: With the completion of this module the trainees should be able to
1. understand that, under enlargement, the volume of an object depends on the cube of the enlargement factor;
2. construct paper origami models of regular tetrahedra and octahedra;
3. find a formula for the volume of a regular tetrahedron.

Content and Resources:

1. Simple volume growth
    - worksheet page 1
    - nets of simple volumes.
2. Volume growth of any shape
    - worksheet page 2
3. The volume of the tetrahedron
    - worksheet page 3
    - geogebra book

Methodology and approaches for the module training presentation:

The methodology of the module is based on the approach of inquiry learning as a process of active construction of knowledge.
The presentation gives an overview of the topic, target group, content-related prerequisite, step sequence of the module and its objectives, materials and respective processing time.

Instruments/ Tools/ Supporting Material/ Resources to be used:

- worksheet (each of the three steps comprises one page)
- worksheet (suggested solution)
- geogebra book
- sheets of paper
- a bucket of sand, semolina, lentils, chick peas or any other flowing material
- plastic balloons, water bucket, camera

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity Number and broad Description: 0 Introductory	
Development	In the introduction, ideas around the notions of physical quantities such as length, area, volume are gathered, together with their units and meaning of cm, cm^2, cm^3. Formula for the volume of a cuboid is retrieved, especially its linear dependency with regard to each dimension: 1x2, 2x2, 2x2x2.
Materials	12 squares, 8 or better 27 dice.
Resources	Presentation (ppt)
Estimated Time	5 min
Environment/Room Setting	frontal space/ general introduction/ demonstration
Trainees' role	activate previous knowledge, express oneself, gather ideas

Development activities with integrated practical activities (hands-on activity)

Activity Number and broad Description: 1 Simple volume growth	
Development	Students constructs different simple volumes (cuboid, cone, pyramid, cylinder) at a given unit scale and its double. They count how many times the smaller one fills the larger one with sand (or semolina). They conjecture the multiplication by a factor 8 of the volume when doubling the size. For fast students: an enlargement factor 3/2 or another one.
Materials	- worksheet page 1 - strong paper (cardboard), scissors and tape, sand or semolina, buckets.
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills
Activity Number and broad Description: 2 Volume growth of any shape	
Development	Students try out their hypothesis on other shapes, especially the sphere. A graph of its volume with respect to its size can summarize the data.
Materials	- worksheet page 2 - digital camera (smartphone, webcam) or projection screen - inflatable balloons, water, buckets, measuring jug
Resources	Presentation (ppt)
Estimated Time	approx. 30 min
Environment/Room Setting	own workspace/ individual work phase, need of a sink, a basinet or outside
Trainees' role	active role active construction of knowledge and skills

Evaluation of Learning Outcomes

Activity Number and broad Description: 3 The volume of the tetrahedron	
Development	Students construct an origami model of tetrahedra and octahedra. Solving a little 3D puzzle for doubling the tetrahedron with these pieces, they infer that a certain cube, of size $\sqrt{2}$ a, contains 24 tetrahedra of size a. This allows to infer that the volume of the tetrahedron is $(\sqrt{2}\ a)^3/24$.
Materials	- worksheet page 3, tetrahedron and octahedron folding instructions - geogebra book - paper
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ collective work phase
Trainees' role	active role active construction of knowledge and skills

Reflection and Closure activity:

In order to make the learning success of this module visible, the students should share their individual insights gained through this learning module in their own words in a padlet. This collaborative method is best suited because all participants of this module can access it at the same time and read and discuss each other's contributions. In doing so, the padlet offers the possibility to prepare these individual reflections in written form as text, auditorily as a voice message, visually as a graphic or audio-visually as a video. These contributions can also be networked so that connections become clear. The teacher can take up these presented contributions in a joint final discussion and correct or deepen them if necessary.

Source:

IREM de Lyon

Continued fraction of square roots of integers

Students are guided to discover that looking at a positive real number as the ratio of a rectangle allows to experimentally get a rational approximation for it. They investigate the A4 paper format and discover that its ratio is √2 and, with the help of folding paper, compute its continued fraction approximation. Its diagonal is in a ratio of √3. Likewise, we construct a paper of ratio √3 and infer its continued fraction expansion. Understanding geometrically square roots of integers, we set the scene for their continued fraction expansion computations. Their findings is put to the use for the discovery of a formula for the volume of a regular tetrahedron, through an origami 3D puzzle.

Learning Outcomes: With the completion of this module the trainees should be able to
1. understand that the A4 paper is of ratio √2 and why;
2. compute the continued fraction expansions of √2, √3 and maybe √n;
3. construct paper origami models of regular tetrahedra and octahedra;
4. find a formula for the volume of a regular tetrahedron.

Content and Resources:

1. Simple volume growth
    - worksheet page 1
    - nets of simple volumes.
2. Volume growth of any shape
    - worksheet page 2
3. The volume of the tetrahedron
    - worksheet page 3
    - geogebra book

Methodology and approaches for the module training presentation:

The methodology of the module is based on the approach of inquiry learning as a process of active construction of knowledge.

The presentation gives an overview of the topic, target group, content-related prerequisite, step sequence of the module and its objectives, materials and respective processing time.

Instruments/ Tools/ Supporting Material/ Resources to be used:

- worksheet (each of the three steps comprises one page)
- worksheet (suggested solution)
- geogebra book
- sheets of paper
- a bucket of sand, semolina, lentils, chickpeas or any other flowing material
- plastic balloons, water bucket, camera

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity Number and broad Description: 0 Introductory	
Development	In the introduction, ideas around the notions of physical quantities such as length, area, volume are gathered, together with their units and meaning of cm, cm^2, cm^3. Formula for the volume of a cuboid is retrieved, especially its linear dependency with regard to each dimension: 1x2, 2x2, 2x2x2.
Materials	12 squares, 8 or better 27 dice.
Resources	Presentation (ppt)
Estimated Time	5 min
Environment/Room Setting	frontal space/ general introduction/ demonstration
Trainees' role	activate previous knowledge, express oneself, gather ideas

Development activities with integrated practical activities (hands-on activity)

Activity Number and broad Description: 1 Simple volume growth	
Development	Students constructs different simple volumes (cuboid, cone, pyramid, cylinder) at a given unit scale and its double. They count how many times the smaller one fills the larger one with sand (or semolina). They conjecture the multiplication by a factor 8 of the volume when doubling the size. For fast students: an enlargement factor 3/2 or another one.
Materials	- worksheet page 1 - strong paper (cardboard), scissors and tape, sand, buckets.
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills
Activity Number and broad Description: 2 Volume growth of any shape	
Development	Students try out their hypothesis on other shapes, especially the sphere. A graph of its volume with respect to its size can summarize the data.

Materials	- worksheet page 2 - digital camera (smartphone, webcam) or projection screen - inflatable balloons, water, buckets
Resources	Presentation (ppt)
Estimated Time	approx. 30 min
Environment/Room Setting	own workspace/ individual work phase, need of a sink, a basinet or outside
Trainees' role	active role active construction of knowledge and skills

Evaluation of Learning Outcomes

Activity Number and broad Description: 3 The volume of the tetrahedron	
Development	Students construct an origami model of tetrahedra and octahedra. Solving a little 3D puzzle for doubling the tetrahedron with these pieces, they infer that a certain cube, of size √2 a, contains 24 tetrahedra of size a. This allows to infer that the volume of the tetrahedron is $(\sqrt{2}\,a)^3/24$.
Materials	- worksheet page 3 - geogebra book - paper
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ collective work phase
Trainees' role	active role active construction of knowledge and skills

Reflection and Closure activity:

In order to make the learning success of this module visible, the students should share their individual insights gained through this learning module in their own words in a padlet. This collaborative method is best suited because all participants of this module can access it at the same time and read and discuss each other's contributions. In doing so, the padlet offers the possibility to prepare these individual reflections in written form as text, auditorily as a voice message, visually as a graphic or audio-visually as a video. These contributions can also be networked so that connections become clear. The teacher can take up these presented contributions in a joint final discussion and correct or deepen them if necessary.

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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Module 3 – Learning Plan: Discovery Based Learning - Derivation and modeling of constrained growth processes with the natural exponential function

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

This module focuses on the approach of (self-)discovery learning. In contrast to teacher-centred, rather passive learning, the focus is on the students' own activities. After a short description of the tasks, the students should solve them independently and work with the materials provided. The teacher deliberately takes a step back and gives room to the momentum of the solution process. Through this, networked knowledge can emerge through the active construction of knowledge. This approach pursues the goal that learning is a subjective, active-discovering and constructive process.

The discovery of special exponential functions is used as an exemplary context. Starting from known exponential functions, the students analyse relations between the functions and their derivative with Geogebra. They make assumptions about a special exponential function and finally derive Euler's number e using the differential quotient. Then the students repeat (known) simple derivation rules in the new context of exponential functions and learn the chain rule. They use these to form the derivative of functions at different levels of difficulty. Finally, students analyse graphical representations and derive the equations for constrained and logistic growth functions. They work through several exercises (also in factual relations) and give function equations, determine growth rates and learn about properties of growth concepts.

Learning Outcomes: With the completion of this module the trainees will be able to

1. derive exponential functions qualitatively, name Euler's number e as a special basis of an exponential function and derive it as a limit value of a sequence.
2. derive exponential functions using various derivation rules.
3. describe equations for constrained and logistic growth functions and explain their properties (in factual contexts) also computationally.

Content and Resources:

Derivation und modelling of constrained growth processes with the natural exponential function

1. Derivation of the (natural) exponential function
    - worksheet page 1
    - geogebra file 1
    - youtube videos
      Derivation and summary of Euler's number e:
      https://www.youtube.com/watch?v=m2MIpDrF7Es
      Excursus on further properties and connections of Euler's number e:
      https://www.youtube.com/watch?v=AuA2EAgAegE&t=219s

2. Derivation rules for exponential functions
    - worksheet page 2
    - youtube video
      Proof of the chain rule:
      https://www.youtube.com/watch?v=m0LZX19DyyI

3. Modelling constrained and logistic growth
    - worksheet page 3
    - geogebra file 2
    - calculator
    - if available, but not necessary: computer algebra system (CAS)

Methodology and approaches for the module training presentation:

The methodology of the module is based on the approach of self-discovery learning as a process of active construction of knowledge.
The presentation gives an overview of the topic, target group, content-related prerequisite, step sequence of the module and its objectives, materials and respective processing time.

Instruments/ Tools/ Supporting Material/ Resources to be used:

- worksheet (each of the three steps comprises one page)
- worksheet (suggested solution)
- geogebra file 1
- geogebra file 2
- video: Derivation and summary of Euler's number e:
  https://www.youtube.com/watch?v=m2MIpDrF7Es
- video: Excursus on further properties and connections of Euler's number e:
  https://www.youtube.com/watch?v=AuA2EAgAegE&t=219s
- video: Proof of the chain rule:

https://www.youtube.com/watch?v=m0LZX19DyyI

- presentation (PPT)
- padlet: https://kant-gymnasium.padlet.org/okaufmann/jmjw07stw5mli2uy

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity Number and broad Description: 0 Introductory	
Development	The introduction is done through a short demonstration of exponential functions and their derivatives using geogebra as well as visual impressions of key words and associated graphics. In doing so, the students should generate a reality reference as well as motivating internal mathematical questions and activate their previous knowledge of exponential functions and the concept of derivatives.
Materials	Geogebra file 1, graphics, keywords
Resources	Presentation (ppt)
Estimated Time	5 min
Environment/Room Setting	frontal space/ general introduction/ demonstration
Trainees' role	passive role activate previous knowledge

Development activities with integrated practical activities (hands-on activity)

Activity Number and broad Description: 1 Derivation of the (natural) exponential function	
Development	The students discover the relationship between known exponential functions and their derivatives using Geogebra. They make assumptions about a special exponential function and finally derive Euler's number e using the differential quotient.
Materials	- worksheet page 1 - geogebra file 1 - youtube videos *Derivation and summary of Euler's number e:* https://www.youtube.com/watch?v=m2MIpDrF7Es *Excursus on further properties and connections of Euler's number e:* https://www.youtube.com/watch?v=AuA2EAgAegE&t=219s
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills

Activity Number and broad Description: 2 Derivation rules for exponential functions	
Development	The students repeat (known) simple derivation rules in the new context of exponential functions and learn the chain rule. They use these to form the derivative of functions of varying degrees of difficulty.
Materials	- worksheet page 2 - youtube video *Proof of the chain rule:* https://www.youtube.com/watch?v=m0LZX19DyyI
Resources	Presentation (ppt)
Estimated Time	approx. 30 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills

Evaluation of Learning Outcomes

Activity Number and broad Description: 3 Modelling constrained and logistic growth	
Development	The students analyse graphical representations and derive the equations for constrained and logistic growth functions. They work on several exercises (also in factual contexts) and give function equations, determine growth rates and learn about properties of the growth concepts.
Materials	- worksheet page 3 - geogebra file 2 - calculator - if available, but not necessary: computer algebra system (CAS)
Resources	Presentation (ppt)
Estimated Time	approx. 75 min
Environment/Room Setting	own workspace/ individual work phase
Trainees' role	active role active construction of knowledge and skills

Reflection and Closure activity:

In order to make the learning success of this module visible, the students should share their individual insights gained through this learning module in their own words in a padlet. This collaborative method is best suited because all participants of this module can access it at the same time and read and discuss each other's contributions. In doing so, the padlet offers the possibility to prepare these individual reflections in written form as text, auditorily as a voice message, visually as a graphic or audio-visually as a video. These contributions can also be networked so that connections become clear. The teacher can take up these presented contributions in a joint final discussion and correct or deepen them if necessary. Here you can find the link to the prepared padlet:

https://kant-gymnasium.padlet.org/okaufmann/jmjw07stw5mli2uy

Sources:

- KIRA Deutsches Zentrum für Lehrerbildung Mathematik (2021): *Entdeckendes Lernen im Mathematikunterricht.* In: https://kira.dzlm.de/lernen-wie-kinder-denken/entdeckendes-lernen-im-mathematikunterricht
- Brandt, D. et al. (2016): *Lambacher Schweizer Mathematik. Oberstufe mit CAS-Einsatz*. 1st edition. Ernst Klett Verlag. Stuttgart, p. 224 – 233.

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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Module 4 – Learning Plan: Problem Solving Methods

Introduction and Broad Description of the Context and Goal of the area/ topic addressed: In this module we will deal with problem solving. We will try out and experience different problem-solving strategies using examples and shed light on the theoretical background of problem solving.

Learning Outcomes: With the completion of this module the trainees will be able to ……

1. Understand the importance of problem solving in mathematics education
2. Have a collection of problems of different difficulty levels and different topics to practice problem solving with students
3. Know various problem-solving strategies and can apply them purposefully and provide specific guidance to students about them

Content and Resources (providing information on the various constituents/ dimensions of the topic under consideration): Examples are presented from most grade levels and many school-relevant thematic areas. These will be made available in a beamer presentation. A beamer presentation with solutions and supplemental information is also available for the instructor.

Methodology and approaches for the module training presentation: The participants can experience problem solving themselves with many examples and thus experience the positive effects of successful problem solving. In addition, the strategies used in the process are reflected upon and thus made accessible for other problems and, in particular, the repertoire of problem-solving strategies is expanded. This encourages and further empowers teachers to integrate and train problem solving even more intensively in the classroom.

Instruments/ Tools/ Supporting Material/ Resources to be used: ……

- Module4-innomath-problem-solving-beamer.pdf (presentation)
- Module4-innomath-problem-solving-handout.pdf
- Module4-innomath-problem-solving.pdf (only for lecturer)
- Module4-Worksheet-Examples-5-10.pdf
- Module4-Handout-Polya-Questions.pdf
- Module4-Worksheet-Making-Task-Accessible.pdf
- Module4-Making-Task-Accessible.ggb (GeoGebra-file)

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities

Activity Number and broad Description: Problem Solving: General Positions, Definition	
Development	Introduction and entry
Materials	Lecture slides 1 to 8
Resources	Module4-innomath-problem-solving-beamer.pdf
Estimated Time	10 min
Environment/Room Setting	Lecture, all participants together
Trainees' role	Listening, taking notes

Development activities

Activity Number and broad Description: Problem Solving in elementary school: Examples	
Development	Getting first ideas of low level problem solving
Materials	Lecutre slides 9 to 16
Resources	Module4-innomath-problem-solving-beamer.pdf
Estimated Time	10 min
Environment/Room Setting	Discussion, all participants together
Trainees' role	Discussion

Practicing Activities (hands-on activity)

Activity Number and broad Description: Examples	
Development	Solving Problems – Examples
Materials	Module4-Worksheet-Examples-5-10.pdf and Lecture slides 17 to 35
Resources	Module4-innomath-problem-solving-beamer.pdf, Module4-Worksheet-Examples-5-10.pdf
Estimated Time	100 min
Environment/Room Setting	Switch between small groups and discussions with all participants
Trainees' role	Work in small groups and contribute to discussion with all participants

Development activities

Activity Number and broad Description: Problem Solving – Polya-Questions and Cognitive Structures	
Development	Getting to know a helpful tool for teachers
Materials	Lecture slides 36 to 41, Module4-Handout-Polya-Questions.pdf
Resources	Module4-innomath-problem-solving-beamer.pdf, Module4-Handout-Polya-Questions.pdf
Estimated Time	5 min
Environment/Room Setting	Lecture, all participants together
Trainees' role	Paying attention

Reflective activities

Activity Number and broad Description: Problem Solving – Reflection on Heuristic Strategies	
Development	Reflecting on heuristic strategies used
Materials	Lecture slides 42 to 47
Resources	Module4-innomath-problem-solving-beamer.pdf
Estimated Time	15 min
Environment/Room Setting	Discussion, all participants together
Trainees' role	Discussion

Development activities

Activity Number and broad Description: Problem Solving – General Heuristic Structures	
Development	Informations on some more general heuristic structures
Materials	Lecture slides 48 to 71
Resources	Module4-innomath-problem-solving-beamer.pdf
Estimated Time	15 min
Environment/Room Setting	Lecture, all participants together
Trainees' role	Paying attention

Evaluation of Learning Outcomes

Activity Number and broad Description:	
Development	Making tasks accessible and promoting creativity
Materials	Lecture slides 72 to 82, Module4-Worksheet-Making-Tasks-Accessible.pdf, GeoGebra: Module4-Making-Task-Accessible.ggb
Resources	Module4-innomath-problem-solving-beamer.pdf, Module4-Worksheet-Making-Tasks-Accessible.pdf, Module4-Making-Task-Accessible.ggb
Estimate Time	25 min
Environment/Room Setting	Lecture and working in small groups
Trainees' role	Paying attention during short lecture part, working in small groups

Reflection and Closure activity:

Finally, the participants will work in small groups to develop suggestions on how to make a specific problem accessible to students. For this the participants will need at least one computer with GeoGebra for each group. This will then be discussed in plenary.

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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Module 5 – Learning Plan: Project Based Learning

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

Project Based Learning (PBL) and is a teaching methodsin which students learn by actively engaging in real-world and personally meaningful projects. In Project Based Learning teachers make learning come alive for students.
This module explains what this two methos and why is so important for the students. At the same time, the module focuses on how does PBL differ from “doing a project”, seven essential project design elements and seven project based teaching practises. The objective of this module is the teachers to feel confident and ready to use Project based Learning at schools.

Learning Outcomes: With the completion of this module the trainees will be able to

1. Understand the definitions and the meanings of the Project Based Learning.
2. Why to involve Project Based Learning at schools.
3. The impact on students.
4. Understand how to approach this method.

Content and Resources (providing information on the various constituents/ dimensions of the topic under consideration):

The training includes a presentation on PowerPoint and some examples of Project based Learning ready for the teachers to use and a video where a school applied this method.

Methodology and approaches for the module training presentation:

The methodology for Project Based Learning is divided into three parts. The first part is an overview of the PBL. The definitions and the importance of PBL are given to the traineers. In the second part , trainees will see how does PBL differ from “doing a project”, seven essential project

design elements and seven project based teaching practises. In the third part, some PBL ideas and a video are provided to the trainees ready to use at schools

Instruments/ Tools/ Supporting Material/ Resources to be used: ……

1. PowerPoint Presentation on the Project based Learning and Project Work Methodology.
2. A quick quiz on the traineers in order to assess if they understand how does PBL differ from “doing a project”.
3. Presentation of some PBL ideas that teachers can use in their lessons.
4. A video shows how a school applied the PBL.

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity 1 - Introduction	
Development	At the beginning of this module the trainees will start with the definition of PBL.
Materials	PowerPoint presentation
Resources	Lecture slides
Estimated Time	5 min
Environment/Room Setting	Lecture room or ZOOM
Trainees’ role	Listening, taking notes

Development activities

Activity 2 – Impact on students	
Development	Here the trainees will find out how PBL impacts on school students and what the students develop from PBL.
Materials	PowerPoint presentation,
Resources	Lecture slides-ppt presentation
Estimated Time	10 min
Environment/Room Setting	Lecture room or ZOOM
Trainees’ role	Listening, taking notes, Answer the quiz

Activity 3 – Approach to PBL	
Development	Here the trainees will discover how to approach PBL at schools. The trainees will discover seven essential Project design elements and seven project based teaching practises for PBL.
Materials	PowerPoint presentation
Resources	Lecture slides-ppt presentation
Estimated Time	35 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Listening, taking notes, Thinking

Practicing Activities (hands-on activity)

Activity 4 - How does PBL differ from "doing a project"?	
Development	Here the trainees will discover the differences between PBL and "doing a project".
Materials	PowerPoint presentation
Resources	Lecture slides-ppt presentation, Quick quiz
Estimated Time	10 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Listening, taking notes, Thinking, Answer the quiz

Activity 5 – Examples of PBL	
Development	Here the trainees will see real examples for how to use PBL at schools.
Materials	PowerPoint presentation and video
Resources	Lecture slides-ppt presentation
Estimated Time	25 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Listening, taking notes, Thinking

Evaluation of Learning Outcomes

Activity 6- Changing ideas	
Development	Discussion between trainees and trainers.
Materials	PowerPoint presentation
Resources	
Estimate Time	5 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Discussing, Changing ideas

Reflection and Closure activity:

The trainees will work in small groups to develop suggestions on how to implement Project Based Learning at schools.

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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Module 6 – Learning Plan: Presentation and Communication Skills

Introduction and Broad Description of the Context and Goal of the area/ topic addressed: The aim of the training is to practice the skills of making presentations and public speaking. During the training, issues related to image building, communication with the audience, presentation techniques, constructing presentations, and contact with recipients will be discussed. Part of the training will also be devoted to the practical creation of presentations, so that each participant receives feedback on their strengths and weaknesses during the presentation.

Learning Outcomes: With the completion of this module the trainees will be able to ……

1. Prepare a professional presentation in accordance with the rules
2. Adjust the course of your speech to the needs of the audience
3. Communicate effectively (verbally and non-verbally)
4. Skillfully control the course of the presentation and control the group
5. Build a professional image of yourself and appropriate relationships with participants
6. Convince others of your opinion and argue effectively

Content and Resources (providing information on the various constituents/ dimensions of the topic under consideration):
The training includes multimedia presentations on issues related to creating shows and communication with listeners and other group members. Additionally, there are materials for individual and group exercises.

Methodology and approaches for the module training presentation: The training is divided into three parts. In the first stage, participants will watch films and presentations that aim to introduce and develop the knowledge and skills of creating multimedia materials and their presentation, with an emphasis on issues related to communication in the group. The trainer, using the MS Power Point program, in practice, will explain the rules regarding the basics of

creating a presentation and will also indicate some useful elements and inspiring ideas. The second part consists of workshops that practice the practical skills necessary to work with a group of talented students. In the third part, participants will be able to evaluate the activities and try to answer where and how they will be able to apply the acquired skills.

Instruments/ Tools/ Supporting Material/ Resources to be used: ……

(list of file, web links, videos, PPT…. use file names inseting the Module number)
Module 6 Intro presentation.ppt
Module 6 Presentation and communication skills 1.ppt
Module 6 Presentation 2.ppt
Module 6 communication styles and types of behavior.pdf
Module 6 communication styles and types of behavior - appendix.pdf
Module 6 You listen.pdf
Module 6 You listen – appendix.pdf
Module 6 Say it differently.pdf
Module 6 Say it differently – appendix.pdf
Module 6 Peer exercise.pdf
Module 6 Peer exercise – appendix.pdf
Module 6: https://www.youtube.com/watch?v=-FOCpMAww28
Module6:
https://www.ted.com/talks/julian_treasure_how_to_speak_so_that_people_want_to_listen?language=pl#t-19718
Module 6 Survey: https://forms.gle/iohhGTJANsUSHM3g8

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity Number and broad Description: 1. In this part, we will tell you how important in teaching and working with a talented student is careful preparation of a presentation, proper choice of means of communication and body language. We will show examples of good and unsuccessful practices.	
Development	knowledge and skills for creating presentations and giving presentations
Materials	
Resources	Module 6 Intro presentation.ppt *(ppt in preparation)*, Module 6: https://www.youtube.com/watch?v=-FOCpMAww28
Estimated Time	10 min
Environment/Room Setting	Lecture, all participants together
Trainees' role	Listening, taking notes

Development activities

Activity Number and broad Description: 2. In this part, we will tell you how to create an interesting and professional presentation from the visual side.	
Development	knowledge and skills for creating presentations
Materials	Lecture slides – ppt presentation, MS Power Point 2019
Resources	Module 6 Presentation and communication skills 1.ppt
Estimated Time	60 min
Environment/Room Setting	Lecture, all participants together
Trainees' role	Listening, taking notes

Development activities

Activity Number and broad Description: 3. In this section, we'll tell you how to give a great presentation.	
Development	knowledge and skills for giving presentations
Materials	Lecture slides – ppt presentation,
Resources	Module 6 Presentation and communication skills 2.ppt, Module 6: https://www.ted.com/talks/julian_treasure_how_to_speak_so_that_people_want_to_listen?language=pl#t-19718
Estimated Time	50 min
Environment /Room Setting	Lecture, all participants together
Trainees' role	Listening, taking notes

Practicing Activities (hands-on activity)

Activity Number and broad Description: Communication type test	
Development	getting to know your own style of communication
Materials	handouts, pens, sheets of paper
Resources	Module 6 communication styles and types of behavior.pdf, Module 6 communication styles and types of behavior - appendix.pdf
Estimated Time	15 min
Environment/Room Setting	independent work, work in small groups
Trainees' role	individual exercise

Activity Number and broad Description: You listen.	
Development	
Materials	handouts, pens, sheets of paper
Resources	Module 6 You listen.pdf, Module 6 You listen – appendix.pdf
Estimated Time	15 min
Environment/Room Setting	individual work, work in a small group
Trainees' role	performing exercises, group work, discussion

Activity Number and broad Description: Say it differently	
Development	the ability to use the language going to senses, in understandable and the correct way
Materials	handouts, pens, sheets of paper
Resources	Module 6 Say it differently.pdf, Module 6 Say it differently – appendix.pdf
Estimated Time	15 min
Environment/Room Setting	individual work, presentation in a group
Trainees' role	performing exercises, group work, discussion

Activity Number and broad Description: Peer exercise	
Development	ability to build statements, control presentation time, voice work
Materials	handouts, pens, sheets of paper, stopwatch
Resources	Module 6 Peer exercise.pdf, Module 6 Peer exercise – appendix.pdf
Estimated Time	15 min
Environment/Room Setting	individual work, presentation in a group
Trainees' role	individual exercise, preparation and performance of the speech, discussion

Evaluation of Learning Outcomes

Activity Number and broad Description: Evaluation	
Development	assessment of activity and involvement, assessment of the usefulness of exercises
Materials	Google Forms online survey
Resources	Module 6 Survey: https://forms.gle/iohhGTJANsUSHM3g8
Estimate Time	10 min
Environment/Room Setting	individual work
Trainees' role	filling out the survey

Reflection and Closure activity:

Participants will have the opportunity to exchange comments and ideas that came to their mind during the exercises. They will also be able to share examples from their backgrounds.

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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Module 7 – Learning Plan: Raising the entrepreneurial mindset in school students through learning activities

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

Nowadays, entrepreneurship is a much-discussed word especially for young people. This module explains the meaning of entrepreneurship and why entrepreneurship is so important for school students. At the same time, the module focuses on some Learning and Creativity plans which the teachers can use in order to involve the entrepreneurship factor in their lesson activities. The objective of this module is the teachers to know how to raise the entrepreneurial mindset of their school students through learning activities and at the end of this module the teachers should feel confident and ready to configure their lesson plans in order to involve the entrepreneurship factor in them.

Learning Outcomes: With the completion of this module the trainees will be able to:

1. Understand the meaning of the entrepreneurship
2. Why involve school students with Entrepreneurship
3. Understand how to create a Learning & Creativity Plan which involves the Entrepreneurship factor.
4. Understand how Virtual Business Entrepreneurship works

Content and Resources (providing information on the various constituents/ dimensions of the topic under consideration):

The training includes a presentation on PowerPoint and two examples of Learning & Creativity Plans ready for the teachers to use.

Methodology and approaches for the module training presentation:

The methodology is divided into three parts. The first part is an overview of the topic how to raise entrepreneurial mindset to school students through activities. In this part , definitions and reasons for the importance of entrepreneurship are given. In the second part , trainees will see actual Learning & Creativity plans that can be used throughout school periods. In the third part, the trainees will understand what is a virtual Business and how it works at schools.

Instruments/ Tools/ Supporting Material/ Resources to be used: ……

1. PowerPoint Presentation on the Entrepreneurship and Virtual Business Entrepreneurship
2. Presentation of two Learning & Creativity plans
3. Presentation of some activities that teachers can do in their lessons.

Pedagogical/Learning Sequencing and Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity 1 - Introduction	
Development	At the beginning of this module the trainees will discover how entrepreneurship increased over the time. (a graph is given). Then important definitions are given.
Materials	PowerPoint presentation
Resources	Lecture slides
Estimated Time	15 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Listening, taking notes

Development activities

Activity 2 – Importance of Entrepreneurship	
Development	Here the trainees will discover why entrepreneurship is so important for school students and the impact on students.
Materials	PowerPoint presentation
Resources	Lecture slides-ppt presentation
Estimated Time	25 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Listening, taking notes, Thinking

Activity 3 – Virtual Business Entrepreneurship	
Development	Here the trainees will discover how Virtual Business Entrepreneurship works at school and the impact on school students.
Materials	PowerPoint presentation
Resources	Lecture slides-ppt presentation
Estimated Time	35 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Listening, taking notes, Thinking

Practicing Activities (hands-on activity)

Activity 4 - Learning & Creativity plans	
Development	Here 2 learning & activity plans will be given to trainees in order to learn how to include the entrepreneurship idea to schools.
Materials	PowerPoint presentation
Resources	Lecture slides-ppt presentation
Estimated Time	60 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Listening, taking notes, Thinking

Evaluation of Learning Outcomes

Activity 5- Changing ideas	
Development	Discussion between trainees and trainers.
Materials	PowerPoint presentation
Resources	
Estimate Time	30 min
Environment/Room Setting	Lecture room or ZOOM
Trainees' role	Discussing, Changing ideas

Reflection and Closure activity:

The trainees will work in small groups to develop suggestions on how to implement entrepreneurship at school. They can create their own Learning & creativity plans.

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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Module 8 – Learning Plan: Cloud Computing and Cloud Education Leadership

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

In the context of promoting skills and competencies for learning facilitators, supporting mathematically gifted students, it is essential that they should be able to:

- Work in a contemporary environment that is shaped by the developments of the 4th Industrial Revolution,
- Work in an environment that promotes the achievement of the goals of mathematics
- Connect the work of the students to the real world and particularly the industry
- Provide to the students the means for developing competencies for creativity and innovation but also for the development of positive attitudes towards mathematics.

As a result of this a facilitator should be competent in some of the tools of the new era. Such a tool is Cloud Computing and Cloud Education Leadership.

An approach for achieving this goal is to provide the means to the trainees for taking the advantages that are the outcomes of the L-Cloud project. In this module we are aiming at introducing and presenting to them the main components of this project.

Learning Outcomes: With the completion of this module the trainees will be able to:

1. Identify the need to endeavor in areas that will provide the background for an educational leader with skills and competencies relating to the technologies that are setting up the framework of working in education 4.0
2. Refer to what about is the project L-Cloud and the concepts involved in it.
3. Consider the concept of education in the context of L-Cloud
4. Specify the goals and objectives of L-Cloud

5. Refer and know of a framework of competencies in the Educational Environment of Cloud Computing.
6. Know the content of the training course for adaptive educational leaders and
7. Refer to the process of Certification of the adaptive educational leaders in the context of L-Cloud
8. Assess the extent of usefulness of the various competencies that an educational leader should develop in the context of

Content and Resources (providing information on the various constituents/ dimensions of the topic under consideration):

The project L-Cloud: https://www.l-cloud.eu/en_US/

Methodology and approaches for the module training presentation:

Collaborative learning: brainstorming, debates, co-design and planning
Constructionism: inquiry based and project-based learning
Developing case studies and worksheets, answering questionnaires and quizes
Investigating-researching using the web
Maieutic: Socratic method of questioning

Instruments/ Tools/ Supporting Material/ Resources to be used:

(list of file, web links, videos, PPT…. use file names etc.)
The L-Cloud platform: https://www.l-cloud.eu/en_US/
The School on the Cloud Platform: https://www.schoolonthecloud.net/#:~:text=By%20encouraging%20collaboration%20and%20knowledge,countries%2C%20distributed%20widely%20across%20Europe

Pedagogical/Learning Sequencing and Activities Plan: ……..

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc.)

Activity 1: Introduction and reflection on how the developments in the 4th Industrial Revolution leads to the need of promoting in the educational forum the ideas of Cloud Computing and Cloud Education Leadership	
Development	Refer to education in connection with the idustrial revolutions Write some answers on the following Questions What are some aspects that characterize the 21st century in the context of the economy, the social conditions, the realms of meaning, the educational forum and so on? To what extent do these aspects influence the work of a learning facilitator? What is the conclusion from these observations for a learning facilitator? Does the term adaptable educational leader make sense to you?

	Reflection and discussion on these answers Refer to the relation of education with the developmnts in the digital era and in particular to Cloud Computing
Materials	ppt
Resources	
Estimated Time	30 min
Environment/Room Setting	In the case of a class: Circular arrangement in order to facilitate discussion In the case of online presentation: Provisions for chatting
Trainees' role	Discussion and reflection

Development activities

Activity 2: Description of the L-Cloud project and the basic concepts involved in it	
Development	Presentation and explanation of the various concepts involved in L-Cloud
Materials	ppt, laptops
Resources	The L-Cloud webpage: https://www.l-cloud.eu/en_US/ The School on the Cloud webpage: https://www.schoolonthecloud.net/
Estimated Time	30 min
Environment/Room Setting	In the case of a class: Circular arrangement in order to facilitate discussion In the case of online presentation: Provisions for chatting
Trainees' role	Discussion and reflection

Activity 3: Presentation of the framework of competencies expected from an educational Leader in the context of a cloud education environment	
Development	Reflection on the needs for a framework of competencies for an educational leader in the cloud educational environment Presentation and reflection on this framework
Materials	ppt, laptops, internet
Resources	The outcomes of the L-Cloud project
Estimated Time	30 min
Environment/Room Setting	In the case of a class: Circular arrangement in order to facilitate discussion In the case of online presentation: Provisions for chatting
Trainees' role	Discussion and reflection

Activity 4: Description of the course and the certification process leading to a certified cloud education leader	
Development	Presentation and explanation of the course and the certification process of L-Cloud
Materials	ppt, laptops, internet
Resources	The outcomes of the L-Cloud project
Estimated Time	30 min
Environment/Room Setting	In the case of a class: Circular arrangement in order to facilitate discussion In the case of online presentation: Provisions for chatting
Trainees' role	Discussion and reflection

Practicing Activities (hands-on activity)

Activity 5 : Reflection on ideas of L-Cloud that can provide resources for the facilitators of training mathematically gifted students and discussion on how L-Cloud could become a catalyst for the achievement of the goals of the INNOMATH project.	
Development	Presentation, Reflection and discussion on the usefulness of the ideas of L-Cloud for facilitators in the training of gifted children in the context of the INNOMATH project
Materials	ppt, laptops, internet
Resources	The outcomes of the L-Cloud project
Estimated Time	30 min
Environment/Room Setting	In the case of a class: Circular arrangement in order to facilitate discussion In the case of online presentation: Provisions for chatting
Trainees' role	Discussion and reflection

Evaluation of Learning Outcomes

Activity 6: Questionnaire on the extend of support and usefulness of the L-Cloud framework of competencies to the learning facilitators in the INNOMATH project	
Development	Presenting and requesting answers by the participants to a Questionnaire relating to the competencies of an educational leader and its usefulness in the context of the INNOMATH approach
Materials	Questionnaire in the APPENDIX
Resources	ppt, laptops, internet
Estimate Time	The outcomes of the L-Cloud project
Environment/Room Setting	30 min
Trainees' role	In the case of a class: Circular arrangement in order to facilitate discussion In the case of online presentation: Provisions for chatting
	Discussion and reflection

Reflection and Closure activity:

QUESTIONNAIRE

Assessing and commenting on the extent of usefulness of the ideas of L-Cloud in the INNOMATH context

Complete the following questionnaire by assessing the degree of contribution of each competency in the strengthening of a teacher's competencies, aiming to support mathematically gifted students. Mark with: 4 (Excellent), 3 (Good) 2(satisfactory), 1(little). Furthermore add your comments on the advantages, practicality and difficulties you would expect in applying these competencies in an INNOMATH class

AREAS	COMPETENCES	MARK 1,2,3,4	COMMENTS
1. Communication, Collaboration and Participation	1.1. Knowledge of the foundations of communication as applied to Cloud Education Environments **(CEE)**.		
	1.2. Ability for communication in CEE.		
	1.3. Ability to establish a shared vision on CEE in the educational organization.		
	1.4. Capacity to build and consolidate communities of interest related to CEE.		
	1.5. Negotiation skills (social and political interactions) with multiple educational stakeholders, actors and contexts, and decision making in cloud education.		
	1.6. Ability to manage personal emotions.		
	1.7. Disposition to team building and active participation in CEE.		
2. Innovation, Creativity and Creation	2.1. Knowledge of the foundations of creativity as applied to CEE		
	2.2. Ability to lead cloud education innovations in parallel to the pedagogical project and the infrastructure of the centre		
	2.3. Ability to creatively use CEE in different educational contexts.		
	2.4. Disposition to investigate about current research, innovations and best practices in the field of cloud education.		
	2.5. Disposition to express creative ideas, experiences and emotions about CEE.		
3. Professional Development	3.1. Understanding, construction and continuous reflexion on educational leaders' professional digital identity in CEE.		
	3.2. Ability to actively participate in educational research and practitioner networks in CEE.		
	3.3. Ability to critically assess your own practice as leaders and develop their understanding of effective and sustainable leadership.		
	3.4. Disposition to participate in cloud education Professional Development programmes (CPD).		

	3.5. Disposition to promote reflexive practice and professional development focused on engagement, responsibility, teaching, learning and leadership, and keeping abreast of change.		
4. Ethics and professional responsibility	4.1. Knowledge on the effective and ethical use of the different types of CEE (public, private and hybrid) and their services, tools and functionalities (SaaS, PaaS and IaaS).		
	4.2. Knowledge on legal issues related to safety, data protection, privacy and a responsible use of CEE.		
	4.3. Ability to solve complex problems in CEE.		
	4.4. Disposition to Identifying and removing barriers to create/maintain a cloud education infrastructure.		
	4.5. Disposition to motivating, encouraging, trusting and valuing colleagues to create and use cloud education in their contexts		
	4.6. Disposition to social and global awareness and responsibility in relation to CEE.		
	4.7. Disposition to promote and build an ethical digital identity in cloud education.		
5. Intercultural relationships and internalization	5.1. Knowledge on international and global aspects of CEE.		
	5.2. Ability to build and maintain intercultural relationships with partner, stakeholders and the educational community as a whole through CEE.		
	5.3. Disposition to promote international mobility, entrepreneurship, training and cooperation on cloud education.		
	5.4. Disposition to respect and become aware of the diversity of learners' cultures and identify common values.		
	5.5. Commitment to foster inclusion, cross-cultural skills and equal opportunity in CEE.		
6. Pedagogical and Organizational	6.1. Pedagogical content knowledge in relation to different subjects, contents and structure in CEE.		
	6.2. Knowledge on contextual, institutional, organizational aspects of educational policies		
	6.3. Knowledge on class management, assessment and feedback processes in CEE.		
	6.4. Skills on using, developing, creating and managing CEE, including applications, devices, and networks		
	6.5. Skills on selecting, creating, organizing, sharing and publishing educational content according to different CEE.		
	6.6. Skills to identify students' learning needs, and learning progress in the cloud.		
	6.7. Disposition to accept responsibilities to planning and implementing CEE.		

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/

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INNOMATH: Innovative enriching education processes for Mathematically Gifted Students in Europe

INNOMATH Training Course for teacher facilitators: Supporting Mathematically Gifted Students

Module 9 – Learning Plan Evolution of Education 1.0 to 4.0 , Future Schools

Introduction and Broad Description of the Context and Goal of the area/ topic addressed:

The education systems implemented in most countries today are characterised by the elements of Education 2.0, while very few countries are pushing for reforms defined by Education 3.0. The features of the development stages of Education from Education 1.0 to Education 4.0 are more or less clear. But talking about Education 4.0 with today's technologies may not be so accurate. We do not know what the technologies in 15-20 years will be and how these will affect the learning processes. However, we do know that today's technologies are not fully utilized in the current learning processes in school education systems. One of the objectives of this course is to try to answer the question: is this an evolution or a revolution? Some results of EU funded projects, like L-Cloud: Development of tomorrow's Cloud Education Leaders will be discussed, the STEAME project and other.

Learning Outcomes: With the completion of this module the trainees will be able to

1. Understand the evolution differences of Education 1.0, 2.0, 3.0 and 4.0
2. Understand the need to be adaptable to change
3. Understand and develop creative thinking for future schools

Methodology and approaches for the module training presentation:

1. Presentations of definitions
2. Presentation of Reasoning for change and highlights from reports on evolution of learning
3. Presentation of tools, methods and new approaches in learning
4. Discussion sessions and debates

Instruments/ Tools/ Supporting Material/ Resources to be used: ……

(list of file, web links, videos, PPT…. use file names inseting the Module number)

1. PPT on Evolution of Education from EDU 1.0 to EDU 4.0
2. Presentation of related videos of modern schools as a case study for discussion
3. Comparative Tables and studies
4. Adaptable Cloud Education Leaders related reports
5. Guidelines for STEAME Activities and Schools
6. Related Websites supporting learning of the future such as www.steame.eu, https://www.schooleducationgateway.eu/en/pub/index.htm

Pedagogical/Learning Sequencing and Activities Plan: ……..

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Activity 1 – Comparative thinking:	
Development	Compare past, present and consider what has changed over the last 100 years.
Materials	PPP, pictures, presentation by trainees of their school learning environment
Resources	Slides, Pictures
Estimated Time	30 min
Environment/Room Setting	U-Shape or ZOOM
Trainees' role	React to questions raised by the trainer

Development activities

Activity 2 – Needs for change:	
Development	What has to change? What teachers in Europe say?
Materials	EU reports for schools and learning needs Survey reports by the STEAME project and other
Resources	Web reports, pictures, videos
Estimated Time	30 min
Environment/Room Setting	U-Shape or ZOOM
Trainees' role	Reading, discuss, imagine, create

Activity 3 – What experts say?:	
Development	Think Tank suggestions
Materials	Publications by experts, reports by authorities
Resources	Publications, Executive Summaries, Web reports, pictures, videos
Estimated Time	30 min
Environment/Room Setting	U-Shape or ZOOM
Trainees' role	Considering, Evaluating, Estimating, imagine, realise, discuss

Practicing Activities (hands-on activity)

Activity 4 – Design the future with critical evaluation:	
Development	Exercise to write a set of pedagogical changes, learning environments and methods, infrastructure changes that could be done if the teacher had the power to do
Materials	Small self stick papers. Teachers are asked to write what they will stop in red papers, what they will keep in yellow papers and what they will change or introduce in green papers
Resources	Boards in the room so teachers can stick their ideas
Estimated Time	20 min to prepare the papers, 5 min to stick them on the board, 15 min to peer review and evaluation for what colleagues uploaded .
Environment/Room Setting	U-Shape with boards available ZOOM with sending ideas via chat or use of online breakout groups
Trainees' role	Moderate the activity and coordinate the discussion at the end

Evaluation of Learning Outcomes

Activity 5 – Conclusion of Activity 4:	
Development	Discuss with trainer and prioritize.
Materials	Live Board
Resources	Prioritize listing for the three categories Stop-Keep-Introduce (SKI)
Estimate Time	30 min
Environment/Room Setting	U-Shape with boards available ZOOM with sending ideas via chat
Trainees' role	Moderate the priority listing

Reflection and Closure activity: Do we have a plan of change?

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Module 10 – Cooperative Learning for teachers and students

Target Group: Students from 11^{h} grade through to 12th grade (between 16 and 18 years old)

Goal/ Content/ Description: Systems of linear equations are useful for modelling certain scientific problems, but also for solving them using numerical methods. Examples: systems of nonlinear equations, function approximation, equations with partial derivatives, optimization problems, etc. Finding the solutions of a higher-order system (or the inverse of a higher-order matrix) can be a difficult task in practice because of:

1. The large number of arithmetic operations needed;
2. Errors that propagate in long sequences of operations.

Solving systems of equations is required in most linear algebra problems, problems which will introduce the students to the simplex algorithm used in a large number of industrial applications where it is necessary to solve problems with thousands of restrictions and variables.

Objectives:

- Computing the inverse of a matrix using two methods;
- Solving a system of linear algebraic equations using two methods.

Materials/ Tools: interactive whiteboard, computer, linear algebra computer applications, powerpoint.

Resources used by the teacher: Interactive whiteboard, computer, linear algebra computer applications, powerpoint, https://matrixcalc.org/ro/slu.html.

Resources for the student: Interactive whiteboard, computer, linear algebra computer applications, powerpoint, https://matrixcalc.org/ro/slu.html.

Approaches/ Methodology:

There exist many methods for solving systems of equations. Among the methods learned up to this point are: substitution method, Cramer's rule (for systems with the same number of equations and unknowns and which have a unique solution), Gaussian elimination. Today we will solve systems of equations which have the same number of equations and unknowns, square coefficient matrices, and non-zero determinants (systems with unique solutions) using the substitution lemma. Cramer's rule, while simple in essence, is not practical when the number of equations is greater than 3. The same thing can be said when finding the inverse matrix, process which has a high computation time due to the large number of operations.

As a first step, students are asked to find the solutions of a system of equations using the substitution lemma in parallel with the methods learned according to the current curriculum.

As a second step (which will be part of a future lesson), students will learn to use the simplex algorithm, then use this newly acquired knowledge to solve problems.

Activities Plan:

Introductory activities (creation of interest, reference to real value issues, relation to background experiences etc)

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
2 weeks earlier than the classroom consideration	Substitution lemma	Provide a document with written instructions
1 week earlier than the classroom consideration	Computing the inverse of a matrix using the substitution lemma	Provide a document with written instructions

Development activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
20 min	**Solving systems of equations using the substitution lemma.** Let us consider the following system of equations: (S) $\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$	

Let $A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}$ be the coefficient matrix of the system (S) with:

$a_j = \begin{pmatrix} a_{1j} \\ \vdots \\ a_{nj} \end{pmatrix}, j = \overline{1,n}$ being the columns of the matrix A, so $A = \{a_1, \dots, a_n\}$,

$b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$ being the column of the right-hand-sides of the equations (the free terms) of (S),

and $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ being the unknowns of the system.

Using these notations the system (S) can be written in matrix form as: $A \cdot x = b$.

We will associate the following table to the system:

Base	a_1	a_2	...	a_k	...	a_n	b
e_1	a_{11}	a_{12}	...	a_{1k}	...	a_{1n}	b_1
⋮	⋮	⋮		⋮		⋮	⋮
e_l	a_{l1}	a_{l2}	...	a_{lk}	...	a_{ln}	b_l
⋮	⋮	⋮		⋮		⋮	⋮
e_n	a_{n1}	a_{n2}	...	a_{nk}	...	a_{nn}	b_n

Where $e_l = \left(\underbrace{0, \dots, 0}_{l-1}, 1, 0, \dots, 0\right) \in R^n, \{e_1, e_2, \dots, e_n\}$ – canonical (standard) basis in R^n.

Up next we will replace the basis vectors, one by one, like so: e_1 with the vector a_1, e_2 with the vector a_2, and so on until we replace all the vectors e_i with a_i, $i = \overline{1,n}$.

After the first change of base we will get the table of the components in the new base:

Base	e_1	a_2	...	a_k	...	a_n	b
a_1	1	a'_{12}	...	a'_{1k}	...	a'_{1n}	b'_1
e_2	0	a'_{22}	...	a'_{2k}	...	a'_{2n}	b'_2
⋮	⋮	⋮		⋮		⋮	⋮

	a_k | 0 | a'_{l2} | … | a'_{lk} | … | a'_{kn} | b'_l ⋮ | ⋮ | ⋮ | | ⋮ | | ⋮ | ⋮ e_n | 0 | a'_{n2} | … | a'_{np} | … | a'_{nn} | b'_n Where: $a'_{l1} = \begin{cases} 0, & l \neq 1 \\ 1, & l = 1 \end{cases}$ $a'_{lk} = \begin{cases} \frac{a_{11} \cdot a_{lk} - a_{1k} \cdot a_{l1}}{a_{11}}, l = \overline{2,n}, k = \overline{2,n} \\ \frac{a_{1k}}{a_{11}}, 1 = l \end{cases}$ $b'_l = \begin{cases} \frac{b_l \cdot a_{11} - a_{l1} \cdot b_1}{a_{11}}, 1 \neq l \\ \frac{b_1}{a_{11}}, l = 1 \end{cases}$ After all the vectors e_i from the base have been replaced with the vectors a_i, the b column will represent the solution of the system.	

Practicing Activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
15 min	**Example:** Solve the following system using the substitution lemma: $\begin{cases} 2x + y + 3z = 4 \\ x - 2y + z = 4 \\ -x + 3y - 2z = -6 \end{cases}$. The associated coefficient matrix is $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 1 \\ -1 & 3 & -2 \end{pmatrix}$, The column of the right-hand-sides is b=$\begin{pmatrix} 4 \\ 4 \\ -6 \end{pmatrix}$.	

The associated table is:

Base	a_1	a_2	a_3	b
$\leftarrow e_1$	2	1	3	4
e_2	1	-2	1	4
e_3	-1	3	-2	-6

After the first base change we get:

Base	a_1	a_2	a_3	b
a_1	1	$1/2$	$3/2$	2
$\leftarrow e_2$	0	$-5/2$	$-1/2$	2
e_3	0	7/2	-1/2	-4

After the second base change we get:

Base	a_1	a_2	a_3	b
a_1	1	0	7/5	12/5
a_2	0	-1	$1/5$	-4/5
$\leftarrow e_3$	0	0	-6/5	-6/5

After the last base change, we get:

Base	a_1	a_2	a_3	b
a_1	1	0	0	1
a_2	0	1	0	-1
a_3	0	0	1	1

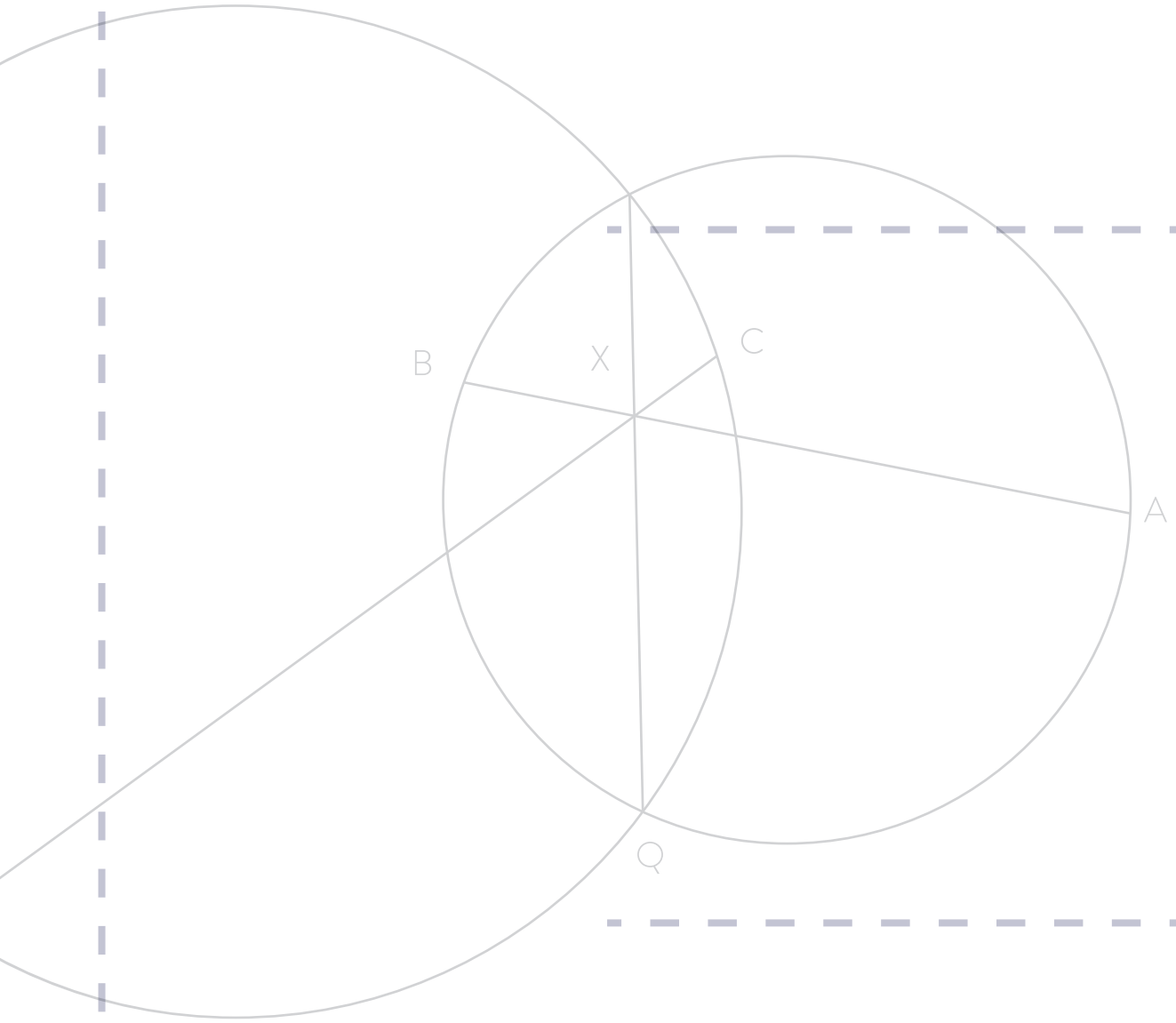
The solution of the system is X=$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

Assessment activities

Time When / length	Description of the activity	Instructions/ Hints/ Support/ Comments
30 min	The students will be divided into two groups and each group will receive a worksheet with the same system of	

	equations. The first group will solve the system using Cramer's rule and the second group will use the substitution lemma. The students will have 20 minutes at their disposal, after which a representative from each group will show the class how they have solved the problem and what answer they have obtained. The students will analyze the two methods and draw conclusions. **Worksheet** Find the real solutions of the following system of equations: $$\begin{cases} 2x_1 - x_2 + x_3 - x_4 = 0 \\ 3x_1 + 2x_2 - x_4 = 2 \\ 2x_1 - 2x_2 - x_3 = 3 \\ x_1 + x_2 + x_3 + 3x_4 = 3 \end{cases}$$ • Group 1 will solve using Cramer's rule • Group 2 will solve using the substitution lemma Expected answer: $x = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$ Can you list other methods for solving this system? (at least three examples)	

More material (presentation, video and other) on this Module can be found here: https://innomath.eu/innomath-training-course-for-teacher-facilitators/



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