Benford testing procedure validation

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JRC I.3 – Text and Data Mining Unit
Statistics and Information Technologies for Anti-Fraud, Security and Trade

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in terms of size, power and rate of false signals on a real environment.
Objective: study and assess the properties of the Benford testing procedures described in


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In the article, the proposed procedure is shown to work properly in simulated markets.

However, no data were available for a huge validation experiment.
Description of the proposed procedure:

Trader X: 

\( N \) transactions on 
\( M \) distinct products

\( N_1 \) transactions on
Product 1

\( N_2 \) transactions on
Product 2

…

\( N_M \) transactions on
Product \( M \)
Description of the proposed procedure:

Trader X: 
N transactions on 
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\[ \begin{align*}
N_1 & \text{ transactions on Product 1} \\
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& \vdots \\
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\end{align*} \]

\[ \chi^2 \text{ on the first digit of the N statistical values} \]
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**Pseudo-Trader X:**
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Exact p-value
Simulated markets were used to assess the properties of the procedure:

\[ K \text{ simulated traders} \begin{cases} \eta K \text{ fraudulent} \\ (1-\eta)K \text{ honest} \end{cases} \]

Application of the testing procedure on every simulated trader

- **Size**
- **Rate of false signals**
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- Different fractions of fraudulent traders \( \eta \).
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The dataset provided included all the trades involving products sensible to under-evaluation strategies in the period between 2011 and 2018.
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Our main focus!
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1) **Traders with at least 50 transactions**

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2) A clear distinction between **honest** and **fraudulent** traders
   
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   Not easy!

   It is difficult to give a reliable definition of an honest trader
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<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Count</th>
</tr>
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<tbody>
<tr>
<td>Transactions considered</td>
<td>all</td>
<td>9,729,946</td>
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### Checked companies considered

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### Definition of honest trader

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**Trader A**

100 transactions

99 not-checked

1 checked

*Regular*
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<th>100 transactions</th>
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<tbody>
<tr>
<td>Regular</td>
<td></td>
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### Trader B

<table>
<thead>
<tr>
<th>100 transactions</th>
<th>35 not-checked</th>
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<tr>
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<td></td>
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- 100 transactions
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- 65 checked
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- 100 transactions
- 99 not-checked
- 1 checked
- 1 under-eval.

**Trader D**
- 100 transactions
- 35 not-checked
- 65 checked
- 50 under-eval.
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Results will then consider different definitions of the honest traders set.
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Improve the reliability of honest traders definition by considering only the traders with a significant fraction (40%) of transactions checked.
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Size: 8.00%
Rate of False Signals: 18.18%
Power: 75.00%
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Rate of False Signals: 18.18%
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Nice results but too few traders...
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Focus only on the 55,710 checked records. In this way, it is possible to exactly discriminate honest and fraudulent traders, since all transactions have been checked.
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Good results in general.
Low power, but also low fraud intensity (median: 12.5%)
Still high size and high rate of false signals...
Further investigations are required...
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Range = \log_{10}(Max(V)) - \log_{10}(Min(V))
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Therefore we decided to further filter the traders...
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**Rate of False Signals:** 25.00%

**Power:** 15.38%
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Size very close to the nominal level (1%)
Improved rate of false signals
Lower power, but also lower fraud intensity (median: 3.9%)
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- A refined simulation scheme taking into account similarities and dependences of traders behavior will be object of future studies.