Explicit Unconditionally Stable Dissipative Integration Algorithms for Real-time Hybrid Simulations of Complex Structural Systems

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\[ \mathbf{M} \ddot{\mathbf{X}}_{i+1} + \mathbf{C} \dot{\mathbf{X}}_{i+1} + (\mathbf{R}^a_{i+1} + \mathbf{R}^e_{i+1}) = \mathbf{F}_{i+1} \]

Numerical integration

\[ \mathbf{X}_{i+1} \text{ and } \dot{\mathbf{X}}_{i+1} \]

Analytical substructure

Experimental substructure

KR-α

RTHS

Conclusions
In RTHS using explicit algorithms, generally the mass and initial stiffness proportional damping (PD) models are used to model inherent damping in the system:

\[ C = a_0 M + a_1 K_I \]

- Known to produce unrealistically large damping forces and inaccurate results when structure undergoes inelastic deformations (A).

Alternatively nonproportional damping (NPD) can be used:

\[ C = a_0 M + a_1 K_I^* \]

- Produces accurate results in nonlinear dynamic analysis using implicit algorithms.
- Produces erroneous results in nonlinear dynamic analysis using explicit algorithms (e.g., CR) with realistic time step size.
  - Member forces become contaminated with participation of spurious higher modes.
  - The problem becomes worse by experimental error in RTHS, including the effects of actuator delay compensation algorithms which amplify high frequency signals.

Numerical damping can be used to circumvent the above problem.

Explicit KR-α Method

- Unconditional stability, 2nd order accuracy, controllable numerical energy dissipation

Velocity update: \[ \dot{X}_{i+1} = \dot{X}_i + \Delta t\alpha_1 \ddot{X}_i \]

Displacement update: \[ X_{i+1} = X_i + \Delta t\dot{X}_i + \Delta t^2\alpha_2 \ddot{X}_i \]

Weighted equations of motion: \[ M\ddot{X}_{i+1} + C\dot{X}_{i+1-\alpha_f} + R_{i+1-\alpha_f} = F_{i+1-\alpha_f} \]

where,

\[ \hat{X}_{i+1} = (1 - \alpha_3)\dot{X}_{i+1} + \alpha_3 \ddot{X}_i \]

\[ \dot{X}_{i+1-\alpha_f} = (1 - \alpha_f)\dot{X}_{i+1} + \alpha_f \dot{X}_i \]

\[ X_{i+1-\alpha_f} = (1 - \alpha_f)X_{i+1} + \alpha_f X_i \]

\[ F_{i+1-\alpha_f} = (1 - \alpha_f)F_{i+1} + \alpha_f F_i \]

Initial acceleration: \[ M\ddot{X}_0 = [F_0 - C\dot{X}_0 - R_0] \]

Integration Parameters

- Parameter controlling numerical energy dissipation
  - $\rho_\infty = \text{spectral radius when } \Omega = \omega_n\Delta t \to \infty$
    - varies in the range $0 \leq \rho_\infty \leq 1$
  - $\rho_\infty = 1$: No numerical energy dissipation
    - Algorithm identical to the CR algorithm
  - $\rho_\infty = 0$: Asymptotic annihilation

- Integration parameters ($\alpha_1$, $\alpha_2$, and $\alpha_3$) are determined using the KR-$\alpha$ family of algorithms

- Scalar integration parameters:
  - $\alpha_m = \frac{1}{\rho_\infty + 1}$
  - $\alpha_f = \frac{\rho_\infty}{\rho_\infty + 1}$
  - $\gamma = \frac{1}{2} \alpha_m + \alpha_f$
  - $\beta = \frac{1}{4} \left( \frac{\gamma}{\gamma + 1} \right)^2$

- Matrix integration parameters:
  - $\alpha_1 = [M + \gamma \Delta t C + \beta \Delta t^2 K]^{-1} M$
  - $\alpha_2 = \left( \frac{1}{2} + \gamma \right) \alpha_1$
  - $\alpha_3 = [M + \gamma \Delta t C + \beta \Delta t^2 K]^{-1} [\alpha_m M + \alpha_f \gamma \Delta t C + \alpha_f \beta \Delta t^2 K]$
Numerical Characteristics

\[ \Omega = \omega_n \Delta t \]

Spectral radius

Lower modes of interest (typ.)

Spurious higher modes (typ.)

\[ \rho_\infty = 1 \]
\[ \rho_\infty = 0.75 \]
\[ \rho_\infty = 0.50 \]
\[ \rho_\infty = 0.25 \]
\[ \rho_\infty = 0.0 \]

\[ \xi = -0.05 \]
\[ \xi = 0 \]
\[ \xi = 0.05 \]

\[ \xi = 0 \]
\[ \xi = 0.05 \]
Implementation for RTHS

Definitions:

\[ A = \Delta t \alpha_1 [M - M \alpha_3]^{-1} \]
\[ B = \frac{1}{\Delta t} M \alpha_3 \alpha_1^{-1} \]
\[ D = \frac{1}{\Delta t} \alpha_1^{-1} \]

Set \( i = 0 \)

Excitation forces: \( F_{i+1} - \alpha_f \)

Responses: \( X_i, \dot{X}_i, \ddot{X}_i, \) and \( R_i \)

Set \( i = i + 1 \)

Optional calculation:

\[ \ddot{X}_{i+1} = D \ddot{X}_{i+1} \]

Extrapolation Effects – small

\( \delta t = \frac{1}{1024} \) sec. small

\( X_{i+1} = \dot{X}_i + \ddot{X}_i \)

\[ X_{i+1} = X_i + \Delta t \dot{X}_i + \left( \frac{1}{2} + \gamma \right) \Delta t \ddot{X}_i \]

\[ D_{i+1}^{C(0)} = X_i^e + \frac{j}{n} (X_{i+1}^e - X_i^e) \]

Experimental Substructure

Set \( j = j + 1 \)

\[ R_{i+1}^{m(j)} \]

\[ R_i = R_{i+1}^{m(n-1)} + K^e \left[ X_i^e - D_{i+1}^{C(n-1)} \right] + C^e \left[ \dddot{X}_i^e - V_i^{C(n-1)} \right] \]

Analytical Substructure

\[ R_{i+1}^a \]

\[ F_i = B_i \]

Prototype Building

- 3-story, 6-bay by 6-bay office building located in Southern California
- Seismic design category D
- Moment resisting frame (MRF) with RBS beam-to-column connections; damped braced frame (DBF), gravity system

Seismic tributary area for one MRF and DBF

Plane View of 3-Story Prototype Building

Elevations of 3-Story Prototype Building

Dong, B. “Seismic performance evaluation of steel structures with nonlinear viscous dampers using real-time large-scale hybrid simulation”, PhD Dissertation, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA 2015. (in preparation)
Prototype and Test Structure

- MRFs designed to satisfy ASCE7 code strength requirement
- Story drift controlled by nonlinear elastomeric dampers installed in DBFs
- DBFs designed to remain elastic under design basis earthquake (DBE) ground motion
- Test structures derived by scaling down the prototype by a factor of 0.6

Substructures for RTHS

Time discretized weighted equation of motion (KR–α Method):

\[ M\ddot{X}_{i+1} + C\dot{X}_{i+1-\alpha_f} + (R^a_{i+1-\alpha_f} + R^e_{i+1-\alpha_f}) = F_{i+1-\alpha_f} \]
Analytical Substructure

- **FE model developed in HybridFEM** (Karavasilis et. al., 2012)
- **Columns and beams**
  - displacement-based nonlinear beam-column fiber elements and elastic beam-column elements
- **MRF panel zone**
  - nonlinear panel-zone elements
- **Nonproportional damping (NPD) model**
- **Gravity system**
  - lean-on-column using elastic elements with second order \( P - \Delta \) effects
- **247 DOFs and 74 elements**

RTHS: Ground motion and time step

- **Ground motion**
  - B-WSM180 component of the 1987 Superstition Hills, California earthquake recorded at the Westmoreland Fire Station
  - Scaled to two hazard levels
    - Design basis earthquake (DBE)*: Scale factor = 1.51
    - Maximum considered earthquake (MCE)*: Scale factor = 2.26

- **Time step**
  - \( \Delta = \frac{4}{1024} \) sec, the smallest time step within which the numerical computation can be finished in real-time

*Note:  DBE has 475 year return period (10% probability of exceedance in 50 years)
MCE has 2475 year return period (2% probability of exceedance in 50 years)
Servo hydraulic actuator control

- Dynamics of combined servo-hydraulic systems and experimental substructure causes delay and change in amplitude in actuator displacements – requires compensation

- Adaptive time series compensation (ATS) with measured specimen feedback $x^m$ used – enables compliance and dynamics of test setup to be compensated for:

$$u^{c(j)}_k = a^{(j)}_{0k} x^{t(j)}_k + a^{(j)}_{1k} \dot{x}^{t(j)}_k + a^{(j)}_{2k} \ddot{x}^{t(j)}_k$$

  - $u^{c(j)}_k$ is the compensated actuator command displacement at the $j^{th}$ substep of the $k^{th}$ time step
  - $x^{t(j)}_k$, $\dot{x}^{t(j)}_k$, and $\ddot{x}^{t(j)}_k$ are target displacement, velocity, and acceleration, respectively, at the $j^{th}$ substep of the $k^{th}$ time step
  - Coefficients $a^{(j)}_{0k}$, $a^{(j)}_{1k}$, $a^{(j)}_{2k}$ are calculated using measured specimen displacement $x^m$ and its first and second derivatives of the previous window (typically of 1 sec. width), and the least squares method.
  - Ceiling and floor limit values for coefficients used to avoid overcompensation leading to instability.

Large-scale RTHS on Structure with Nonlinear Viscous Dampers: Procedure

- Compensator coefficients:
  - Initial values for coefficients are based on mean values from low level BLWN response

Dong, B. “Seismic performance evaluation of steel structures with nonlinear viscous dampers using real-time large-scale hybrid simulation”, PhD Dissertation, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA 2015 (in preparation)
Actuator control: Typical MCE level test & \( \rho_\infty = 0.75 \)

<table>
<thead>
<tr>
<th>Error indices</th>
<th>Floor-1</th>
<th>Floor-2</th>
<th>Floor-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. amp. error (%)</td>
<td>0.27</td>
<td>0.46</td>
<td>0.91</td>
</tr>
<tr>
<td>NEE (%)</td>
<td>0.04</td>
<td>0.50</td>
<td>0.58</td>
</tr>
<tr>
<td>NRMSE (%)</td>
<td>0.29</td>
<td>0.14</td>
<td>0.13</td>
</tr>
</tbody>
</table>

\[
A_k^{(j)} \approx \frac{1}{a_0} \left( \frac{a_{i_k}^{(j)}}{a_{0_k}} \right)
\]

\[
\tau_k^{(j)} \approx \frac{a_{i_k}^{(j)}}{a_{0_k}}
\]

Max. amp. error = \( \frac{\max |x^t| - \max |x^m|}{\max |x^t|} \)

\[
N_{EE} = \left[ \frac{\sum_{i=1}^n x_{i}^t \cdot x_{i}^t - \sum_{i=1}^n x_{i}^m \cdot x_{i}^m}{\sum_{i=1}^n x_{i}^m \cdot x_{i}^m} \right] : \text{ sensitive to amplitude error (Bursi, et al.)}
\]

\[
N_{RMSE} = \frac{\sqrt{n} \sum_{i=1}^n (x_{i}^t - x_{i}^m)^2}{\max(x^m) - \min(x^m)} : \text{ sensitive to period/ phase error (Bursi, et al.)}
\]

Synchronization subspace plots
MCE level RTHS using $\rho_\infty = 1.0$

Input ground excitation

Floor response

Real-time hybrid simulation using explicit unconditionally stable parametrically dissipative KR-\(\alpha\) method

Ground excitation: B-WSM180 component, 1987 Superstition Hills, Westmoreland Fire Station
Hazard level: Maximum considered earthquake (MCE)
Algorithmic parameter: $\rho_\infty = 1.0$

Freq. $\approx f_{Nyq} = \frac{1}{2\Delta t}$

High frequency oscillations in member forces

- Under nonlinear structural behavior, pulses are introduced in the acceleration at the Nyquist frequency \( \left( = \frac{1}{2\Delta t} \right) \) when the state of the structure changes occur within the time step.
- These pulses excite spurious higher modes present in the system which primarily contribute to the member forces.
- The problem becomes worst by the noise introduced through the measured restoring forces and the actuator delay compensation which can amplify high frequency noise.

- How can we remove them?
  - Reduce the time step: Not always possible due to the computation time required for each time step.
  - Introduce controllable numerical damping.
MCE level RTHS using $\rho_\infty = 0.75$

Hysteretic response: analytical substructure

(a) first-story MRF column base, north end
(b) first-story MRF column base, south end
(c) center of roof RBS, south end
Hysteretic damper response: MCE level test
Summary and Conclusions

- Reviewed formulation and numerical characteristics of the explicit unconditionally stable parametrically dissipative KR-\(\alpha\) method
- Proposed an efficient implementation for real-time hybrid simulation using the KR-\(\alpha\) method
- Experimentally demonstrated the significance of numerical energy dissipation in eliminating participation of spurious higher modes through large scale real-time hybrid simulations
- Controllable numerical energy dissipation in the KR-\(\alpha\) method is shown to be effective for conducting RTHS
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  - 5-Year grant, commencing on Jan. 1, 2016
  - replaces NEES
- Shared-use experimental facility with large-scale multi-directional hybrid simulation testing capabilities for multi-hazards:
  - Earthquake, Wind
  - Soil-structure interaction Effects
  - Advanced instrumentation
• More information:
  http://www.nees.lehigh.edu/
  https://www.youtube.com/watch?v=YWYaQE-Cf98
  http://www1.lehigh.edu/news/lehigh-wins-5m-natural-hazards-engineering-research

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• Lehigh NHERI Researchers’ Workshop
  • 1-day workshop at Lehigh on Nov. 9, 2015
  • Agenda
    • NHERI@Lehigh Equipment Facility capabilities
    • Basics of RTHS through lectures and hands-on demonstrations
    • How NHERI@Lehigh Equipment Facility capabilities can enhance your research
    • Information for preparing research proposals which utilize the NHERI@Lehigh Experimental Facility
  • Visit www.atlss.lehigh.edu for information postings
Thank you