Reliability Assessment of Real-time Hybrid Simulation in Presence of Actuator Tracking Error

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Outline of Presentation

• RTHS Background
• Probabilistic Analysis
• Implementation
  o Use of Tracking Indicator
  o Use of Frequency Domain Analysis
• Summary and Conclusion
• Acknowledgement
Servo-Hydraulic Actuators

- Apply desired responses to experimental substructures in a real-time manner;
- Measure the restoring forces of the experimental substructures and feed back to the integration algorithm;

Critical to maintain the boundary conditions between substructures!

Courtesy of Lehigh RTMD
Actuator Delay in Predefined Test

Maximum tracking error 16.90 mm (35% of command maximum)!

Command Maximum: 50 mm
Frequency Content: 0 ~ 5 Hz
Effect of Actuator Tracking Error

Comparison of structural responses for a linear SDOF Structure with 2 Hz natural frequency and 1 msec delay
Questions to be answered?

How will the tracking errors affect the accuracy of simulated structure response from RTHS?

How will researchers assess the accuracy of simulated response in replicating the true structural response when the latter is not available?
Probabilistic Analysis

\[ m \ddot{x}(t) + c \dot{x}(t) + k x(t) = -m \ddot{x}_g(t) \]

Exact response \( X_{\text{ext}} \)

\[ m \ddot{x}(t) + c \dot{x}(t) + k x(t - \tau) = -m \ddot{x}_g(t) \]

Delay response \( X_{\text{del}} \)

\[ \text{MAX} = \frac{\max(|x_{\text{ext}} - x_{\text{del}}|)}{\max(|x_{\text{ext}}|)} \times 100\% \]

or other variables such as RMS and NRMS, etc
Reliability Assessment

With actuator delay in RTHS, probability of simulated response having MAX error larger than 10%

Critical delays based on 10% MAX error
Effect of Nonlinear Behavior

Critical delay distribution for different ductility demands for an SDOF
Effect of Stiffness Degradation

- Slight stiffness degradation
- Significant stiffness degradation

Critical delay for same ductility demand and different stiffness degradation

Legend:
- No
- Slight
- Moderate
- Significant

Graphs show normalized frequency against critical delay for different stiffness degradation levels.
Different Natural Frequencies

Distribution of critical delay for SDOF structures with significant stiffness degradation for different natural frequencies
Effect of Strength Degradation

Critical delay for same ductility demand and different strength degradation
Probabilistic Assessment

With known delay in RTHS, probability of simulated response having MAX error larger than 10% can be adjusted to account for nonlinear behavior and stiffness degradation.

10% MAX error critical delay distribution can be adjusted to account for nonlinear behavior and stiffness degradation.
Implementation using TI

Tracking Indicator (Mercan and Ricles 2010)

Positive TI
Application of Proposed Approach

Perform real-time hybrid simulation

![Comparison Graphs]

- d^c
- d^m

Compare

![Probability of Exceedance Graph]

![T1 Graphs]

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Implementation using TI

Analytical Substructure Properties:
- structural mass: $m = 503.4$ ton;
- natural frequency: $f_n = 0.77$ Hz;
- viscous damping ratio: $\zeta = 0.02$;

Analytical Substructure modeled using Bouc-Wen model [Wen 1980]

Implementation using TI

Test 1: IC with $\alpha_{es}=15$  
Test 2: AIC with $\alpha_{es}=15$  
Test 3: AIC with $\alpha_{es}=30$
Implementation using Freq. Anal.

Input $I(t)$
Output $O(t)$

Input $I'(t)$
Output $O'(t)$

$F[I'(t)]$
$F[O'(t)]$

Window
Technique

$FEI = \sum_{j=1}^{p} \left| \mathcal{F}[I'(t)]_j \right|^2 \cdot f_i$

Amplitude
$A$

$FEI = \frac{\sum_{j=1}^{p} \left| \mathcal{F}[I'(t)]_j \right|^2 \cdot f_i}{\sum_{i=1}^{p} \left| \mathcal{F}[I'(t)]_i \right|^2}$

Time Delay
$d$

Phase
Diff. $\Phi$

$feq = \frac{\sum_{i=1}^{p} \left| \mathcal{F}[I'(t)]_i \right|^2 \cdot f_i}{\sum_{i=1}^{p} \left| \mathcal{F}[I'(t)]_i \right|^2}$

$\phi = d = \arctan \left[ \frac{\text{Im}(FEI)}{\text{Re}(FEI)} \right]$
Implementation using Freq. Anal.

Integrated Time and Frequency Domain Analysis for Probabilistic Assessment

- Real-Time Hybrid Simulation → Critical Delay Distribution (NL)
- Freq. Domain Analysis → Equivalent time delay
Implementation using Freq. Anal.

\[
\text{d(t)} = \text{Analytical Substructure} + \text{Experimental Substructure}
\]

<table>
<thead>
<tr>
<th>Test</th>
<th>(\alpha_{es})</th>
<th>A</th>
<th>(\phi) (rad)</th>
<th>f (Hz)</th>
<th>d (msec.)</th>
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<tr>
<td>1</td>
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<td>1.002</td>
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Summary and Conclusion

• Actuator tracking error could lead to error in RTHS deviating from actual structural responses under earthquakes.

• Probabilistic approach is more appropriate for reliability assessment of RTHS results due to the fact that actual response is not known before or even after the experiments.

• With proper adjustment, critical delay distribution from linear structure analysis can account for nonlinear behavior with stiffness and strength degradation.

• Probabilistic analysis could be implemented through both tracking indicator and frequency domain analysis.
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Thanks for your attention!

Questions?