The Global Multi-Country Model (GM): An Estimated DSGE Model for Euro Area Countries


DISCUSSION PAPER 102 | JULY 2019
The Global Multi-Country Model (GM):
An Estimated DSGE Model for Euro Area Countries

Alice Albonico\textsuperscript{a,d}, Ludovic Calès\textsuperscript{a}, Roberta Cardani\textsuperscript{a}, Olga Croitorov\textsuperscript{a}, Fabio Di Dio\textsuperscript{a}, Filippo Ferroni\textsuperscript{a,b,c}, Massimo Giovannini\textsuperscript{a}, Stefan Hohberger\textsuperscript{a}, Beatrice Pataracchia\textsuperscript{a}, Filippo Pericoli\textsuperscript{a}, Philipp Pfeiffer\textsuperscript{e}, Rafal Raciborski\textsuperscript{e}, Marco Ratto\textsuperscript{a}, Werner Roeger\textsuperscript{e}, Filippo Ferroni\textsuperscript{a}, Massimo Giovannini\textsuperscript{a}, Stefan Hohberger\textsuperscript{a}, Beatrice Pataracchia\textsuperscript{a}, Filippo Pericoli\textsuperscript{a}, Philipp Pfeiffer\textsuperscript{e}, Rafal Raciborski\textsuperscript{e}, Marco Ratto\textsuperscript{a}, Werner Roeger\textsuperscript{e}, Lukas Vogel\textsuperscript{e}

Abstract

This paper introduces the Global Multi-country (GM) model, an estimated multi-country Dynamic Stochastic General Equilibrium (DSGE) model of the world economy. We present the model in 3-region configurations for Euro area (EA) countries that include an individual EA Member State, the rest of the EA (REA), and the rest of the world (RoW). We provide and compare estimates of this model structure for the four largest EA countries (Germany, France, Italy, and Spain). The novelty of the paper is the estimation of ex-ante identical country models on the basis of a unified information set, which allows for clean cross-country comparison of parameter estimates and drivers of economic dynamics. The paper also provides an overview of applications of the GM model such as the structural interpretation of business cycle dynamics, the contribution to the European Commission’s economic forecast, the scenario analysis and policy counterfactuals.

JEL Classification: C51, E32, F41, F45.

Keywords: DSGE, Bayesian estimation, EMU, Business cycle, Model fit, Cross-country comparison.

Acknowledgements: The authors would like to thank in particular DG ECFIN Unit A.3 (Economic situation, forecasts, business and consumer surveys) for collecting and providing all the datasets used for estimation as well as the country-desks of Germany, France, Italy and Spain and in particular Anne Jaubertie and João Leal for very fruitful comments and discussions. We also thank the participants of the 13\textsuperscript{th} Dynare Conference (Tokyo, 2017), the 21\textsuperscript{st} International Conference on Macroeconomic Analysis and International Finance (Crete, 2017), the 24\textsuperscript{th} Computing in Economics and Finance (CEF) Conference (Milan, 2018), the 5\textsuperscript{th} International Symposium in Computational Economics and Finance (Paris, 2018), and the internal JRC B.1 seminar for very helpful comments and suggestions. Alice Albonico and Filippo Ferroni contributed to the model development while working at the European Commission.

The closing date for this document was 28 June 2019.

Contact:
\textsuperscript{a} European Commission, Joint Research Centre
\textsuperscript{b} Federal Reserve Bank of Chicago
\textsuperscript{c} Banque de France
\textsuperscript{d} University of Milano-Bicocca
\textsuperscript{e} European Commission, Directorate-General for Economic and Financial Affairs,
lukas.vogel@ec.europa.eu.
# CONTENTS

1. Introduction ........................................................................................................................................ 2

2. The model ........................................................................................................................................... 4
   2.1. EU country households .................................................................................................................. 5
       2.1.1. Ricardian households ................................................................................................................. 5
       2.1.2. Liquidity-constrained households .............................................................................................. 8
       2.1.3. Wage setting ............................................................................................................................ 9
   2.2. EMU country production sector .................................................................................................. 10
       2.2.1. Total output demand .................................................................................................................. 10
       2.2.2. Value added sector .................................................................................................................... 11
       2.2.3. Intermediate goods producers .................................................................................................. 11
   2.3. Trade ............................................................................................................................................. 15
       2.3.1. Exchange rates and terms of trade .............................................................................................. 15
       2.3.2. Import sector ............................................................................................................................. 16
       2.3.3. Export sector ............................................................................................................................. 18
   2.4. Fiscal policy ................................................................................................................................... 18
   2.5. Monetary policy ............................................................................................................................. 20
   2.6. Closing the economy ..................................................................................................................... 20
   2.7. The REA and RoW block .............................................................................................................. 21

3. Model solution and econometric approach ..................................................................................... 24

4. Estimation results ............................................................................................................................... 26
   4.1. Posterior estimates ......................................................................................................................... 27
   4.2. Model fit ....................................................................................................................................... 29
   4.3. Dynamic transmission of shocks .................................................................................................. 31
   4.4. Historical decomposition .............................................................................................................. 37

5. Application ...................................................................................................................................... 45
   5.1. Structural interpretation of business cycle dynamics .................................................................. 45
   5.2. Contributions to the European Commission’s economic forecast ............................................. 45
   5.3. Scenario analysis and policy counterfactuals .............................................................................. 47

6. Conclusion ...................................................................................................................................... 47
1. Introduction

This article presents the European Commission’s Global Multi-country (henceforth, GM) model. The GM model belongs to the class of Dynamic Stochastic General Equilibrium (DSGE) models. It is an estimated structural macroeconomic model that can be used for economic analysis and projections in an open-economy context. The GM model has been developed to be flexible to allow for different country configurations. We introduce the GM model in its configuration designed for EMU-countries (GM3-EMU). The GM3-EMU model configuration consists of three regions: one EMU Member State, the Rest of the Euro Area (REA) and the Rest of the World (RoW). The single EMU country economy features rich dynamics, whereas the REA and RoW economies are defined in a rather stylised form. To date, GM3-EMU has been estimated for the four largest EMU Member State countries (Germany, France, Italy, and Spain) and the respective REA and RoW blocks.

The GM3-EMU structure offers a common platform to compare economic dynamics and shock transmission across EMU countries from different perspectives. Specifically, the GM model has been developed for three main purposes, namely (1) the structural interpretation of business cycle dynamics, (2) contributions to the European Commission’s economic forecast, and (3) scenario analysis and policy counterfactuals. The GM model provides a structural interpretation of macroeconomic developments, i.e. of the historical data, by decomposing the dynamics of observed variables into their driving structural shocks, i.e. by means of historical shock decompositions. Since Autumn 2015, the estimated GM model is being used biannually in the context of the European Commission’s economic forecast to decompose the European Commission’s institutional forecast for EA key variables like real GDP growth, inflation and trade balance similarly to historical shock decompositions. The counterfactuals address the question of how the economy would have evolved if shocks, structural relationships, or policy reactions had differed. Looking forward, the scenarios assess the impact of alternative policies, structural changes, or possible external disturbances on

---

1 The terms EMU (Economic and Monetary Union) and EA (Euro Area) are used interchangeably in the text.

2 The Rest of the Euro Area (REA) is the EA19 minus the individual EMU Member State (Germany, Spain, France, Italy) that is considered separately in the respective model setup.

3 Using the terminology of Blanchard (2018), GM can be classified as a ‘policy model’ or ‘model for policy purposes’ that should provide quantitative insight into the dynamic effects of specific shocks and alternative policies. Fitting the data reasonably well is an important requirement for policy models and can motivate the introduction of elements, e.g. concerning adjustment frictions and the lag structure, that do not obey to the ideal of maximum theoretical purity. ‘Policy models’ need to be able, on the one hand, to capture actual dynamics, and, on the other hand, they need enough theoretical structure to identify shocks, policies, and their effects or transmission.

4 This application is conducted by means of the GM2 configuration (EA+RoW). For more details see Paragraph 5.2.
economic outcomes.

The GM model builds on the estimated version of the QUEST III model (Ratto et al., 2009), from which it inherits most of its structure. The main differences with respect to Ratto et al. (2009) are: (i) the EA is split into two subregions (EMU-country and REA), which are linked by trade and financial linkages plus a common monetary policy for the Euro Area; and (ii) the trade sector incorporates commodity (oil) imports that are used for the production of domestic total output. It seems useful to underline here that, in the last decade, there has been a series of estimated versions of the QUEST III model, designed to address specific research questions at Euro Area (in’t Veld et al., 2011; Kollmann et al., 2013) or EMU-country level (see in’t Veld et al. (2014, 2015) for Spain and Kollmann et al. (2015) for Germany). The QUEST versions have had a larger degree of complexity, e.g. featuring housing and credit-constrained households (borrowing constraint on mortgage loans) and, for Spain, some more detailed financial cross-holdings. Since it is intended to be regularly re-estimated for all different country settings, the GM3-EMU is kept at a lower level of complexity to reduce the computational burden. In the context of this empirical focus, we also show specific modifications to key components of the model (namely labour demand, hybrid Phillips curve, and trade) that can be applied to better fit country-specific patterns in the data, which would be otherwise captured with more difficulty by the plain core specifications. Such modifications are designed to nest the core model specifications.

In terms of model complexity and specification, the GM model lies within the existing literature on medium and large-scale DSGE models developed and used at policy institutions around the world. The numerous publications have helped greatly to increase transparency of model-based analysis and comparability across models. Examples include structural and semi-structural macroeconomic models developed at the ECB or the Eurosystem (see Bokan et al., 2018; Christoffel et al., 2008; Dieppe et al., 2012, 2018; Karadi et al., 2017), the Federal Reserve Board (Erceg et al., 2006), the IMF (Helliwell et al., 1990; Hunt and Laxton, 2004), the OECD (Hervé et al., 2011), or the central bank of New Zealand (Kamber et al., 2016). Additionally, most of the national central banks of the Eurosystem have developed DSGE models tailored to capture country-specific business cycles fluctuations, e.g. NONAME for Belgium (Jeanfils and Burggraeve, 2008), MEDEA (Burriel et al., 2010) and FiMOD (Stähler and Thomas, 2012) for Spain, AINO 2.0 for Finland (Kilponen et al., 2016), or GEAR for Germany (Gadatsch et al., 2016). Related model comparison exercises can be found, e.g., in Wallis (2004) or in more recent studies by Coenen et al. (2012), Taylor and Wieland (2012), and Wieland et al. (2012, 2016).

Even if they belong to the same class of macroeconomic models, however, all of these country-specific structural models differ along some dimensions. More precisely, the inform-
ation set used to discipline the estimations varies substantially from one country model specification to another. Since the information set might change substantially the structural estimates and model conclusions (see Canova et al., 2014), care is needed when comparing estimates proceeding from different data sources. Therefore, there are limits to use the recent empirical literature to perform cross-country comparisons.

The GM3-EMU model contributes to the existing literature by providing a common platform to compare the estimates for, at present, the four largest EA countries. The appeal of the GM3-EMU model is that the core structure used to estimate country-specific dynamics is ex-ante identical. In other words, all country-specific models have the same equations, the same prior parameter distributions and the same shocks. Moreover, when we bridge the EA country-specific models to the data, we use the same information set, i.e. the same time span and an identical selection of observables. These features provide a framework for meaningful cross-country comparisons and for a direct measurement of heterogeneity that can be attributed to either the nature (size and persistence) of the shocks, or the associated transmission mechanisms pinned down by the structural parameters.

The present article is organised to give a detailed overview of the model structure, the estimation results, and applications of the model. Section 2 presents the theoretical specification of the GM model. The model solution and the econometric approach are discussed in Section 3. Section 4 discusses the estimation results. It evaluates the fitting properties of the estimated country models (Germany, France, Italy, and Spain), their ability to replicate key moments in the data, and analyses differences across countries in the internal transmission dynamics. Moreover, the section provides a structural interpretation of the business cycle dynamics in each of the four countries. Section 5 outlines applications of the model in economic analysis and institutional forecasts. Section 6 concludes.

2. The model

The model features three regions: an EMU country, the REA and the RoW. The EMU domestic economy is composed of households, non-financial firms operating either in the domestic market or in the import-export sector, a government and a central bank.

We distinguish between two types of households: Ricardian households are infinitely-lived and have access to financial markets, can smooth their consumption and own the firms; liquidity-constrained households consume their disposable wage and transfer income each period and do not own any financial wealth. Both types of households provide labour services to domestic firms, at the wage set by a labour union with monopoly power.

In the domestic production sector, monopolistically competitive firms produce a variety of differentiated intermediate goods, which are assembled by perfectly competitive firms into
a domestic final output good (value added). In a final step, perfectly competitive firms produce total output by combining value added with energy input.

In the import sector, perfectly competitive firms (import retailers) buy economy-specific goods from the foreign country and assemble them into a final imported good. Final good packagers combine the final imported good with domestic output into final aggregate demand components goods.

The fiscal authority purchases domestic final goods and makes lump-sum transfers to households that are financed by issuing debt and levying distortionary taxes on labour, capital, and consumption, as well as non-distortionary lump-sum taxes. Given the monetary union setting, the European Central Bank (ECB) sets the nominal interest rate following a Taylor rule defined on EA aggregate inflation and the output gap.

The REA and RoW economies are more stylised, featuring a standard three-equation New Keynesian model, consisting of an Euler equation for consumption, a New Keynesian Phillips curve and a Taylor rule. The model is augmented by international trade.

2.1. EMU country households

There is a continuum of households, indexed by \( j \in [0, 1] \), living in each \( k \) region. A share \( \omega_s^k \) of Ricardian households - savers (s) - owns firms and trades assets in the financial market. The remaining share is liquidity-constrained (c) and consumes its entire disposable wage and transfer income each period. Households preferences are defined over consumption and leisure. Additionally, Ricardian’s utility depends on the beginning-of-period financial asset holdings.

2.1.1. Ricardian households

Ricardian preferences are given by the infinite horizon expected life-time utility:

\[
U_s^{j,k} = E_0 \sum_{t=0}^{\infty} (\hat{\beta}_{k,t})^t u_s^{j,k,t}(\cdot),
\]

where \( \hat{\beta}_{k,t} = \beta_k \exp(\varepsilon_{k,t-1}^c) \), \( \beta_k \) is the (non-stochastic) discount factor, \( \varepsilon_{k,t}^c \) captures a shock to the subjective rate of time preference (saving shock).

Ricardians have full access to financial markets, allowing them to accumulate wealth, \( A_{j,k,t} \), which consists of domestic private risk-free bonds, \( B_{j,k,t}^{rf} \), domestic government bonds, \( B_{j,k,t}^G \), one internationally traded bond, \( B_{j,k,t}^W \), and domestic shares, \( P_{k,t}^{S_j,k,t} \):

\[
A_{j,k,t} = B_{j,k,t}^{rf} + B_{j,k,t}^G + \epsilon_{RoW,k,t} B_{j,k,t}^W + P_{k,t}^{S_j,k,t} S_{j,k,t}.
\]
where $P_{S_{k,t}}^t$ is the nominal price of shares at time $t$. Since the international bond is issued in RoW currency, financial wealth depends on the nominal exchange rate $e_{RoW,k,t}$.

The instantaneous utility function of savers, $u^s(.)$, is defined as:

$$u^s_{j,k,t}(C^s_{j,k,t}, N^s_{j,k,t}; \frac{U^A_{j,k,t-1}}{P_{k,t}^{C,vat}}) = \frac{1}{1 - \theta_k} (C^s_{j,k,t} - h_k C^s_{k,t-1})^{1 - \theta_k} - \theta_k \frac{U_{j,k,t-1}^A}{P_{k,t}^{C,vat}}.$$ 

where $C^s_{k,t} = \int_0^1 C^s_{j,k,t} dj$, $h_k$ measures the strength of external habits in consumption and $\omega^N$ the weight of the disutility of labour. $\varepsilon_{k,t}^U$ captures a labour supply shock. The disutility of holding risky financial assets, $U_{j,k,t-1}^A$, takes the following form:

$$U_{j,k,t-1}^A = (\alpha^b_{k} + \varepsilon_{k,t-1}^B) B^G_{j,k,t-1} + (\alpha^{bw}_{k} + \varepsilon_{k,t-1}^{bw}) e_{RoW,k,t} B_{j,k,t-1}^W + \frac{\alpha^{bw}_{k} (e_{RoW,k,t-1} B_{j,k,t-1}^W)^2}{2 \frac{P_{k,t}^Y}{Y_{k,t-1}}} + (\alpha^S_{k} + \varepsilon_{k,t-1}^S) P^S_{k,t-1} S_{j,k,t-1}.$$ 

Internationally traded bonds are subject to transaction costs in form of a function of the average net foreign asset position relative to GDP. The asset specific risk premium depends on an asset specific exogenous shock $\varepsilon^x$, $x \in \{B, S, bw\}$, and an asset specific intercept $\alpha^x$, $x \in \{b_0, S_0, bw_0\}$. Similar to Krishnamurthy and Vissing-Jorgensen (2012) and Fisher (2015), the approach of modelling the disutility of holding risky assets captures the households preferences for safe assets, i.e. the risk-free short term bonds, which generates endogenously a wedge between the return on risky assets and safe bonds.\(^5\) As in Benigno (2009) and Ratto et al. (2009), we assume that only the RoW bond is traded internationally.\(^6\) It follows that households in the Euro Area can invest in both national and foreign assets, while RoW households can only invest in domestic bonds.

The $j^{th}$ Ricardian household faces the following budget constraint:

$$P_{k,t}^{C,vat} C^s_{j,k,t} + A_{j,k,t} = (1 - \tau^N_k) W_{k,t} N^s_{j,k,t} + (1 + r^f_{k,t-1}) B^f_{j,k,t-1} + (1 + r^G_{k,t-1}) B^G_{j,k,t-1} + \frac{P^S_{k,t} + P^Y_{k,t} \Pi^f_{k,t}}{S_{j,k,t-1} + (1 + r^W_{k,t-1}) e_{RoW,k,t} B_{j,k,t-1}^W + T^s_{j,k,t} - t ax^s_{j,k,t}},$$

\(^5\)This modification is along the lines of the money-in-utility approach by Sidrauski (1967), in which model agents derive utility from their holdings of money. In our model, it reflects the costs of holding risky assets relative to risk-free assets. A similar framework is used by Vitek (2014, 2017).

\(^6\)This assumption is consistent with a reduced form of a global bank in RoW lending domestically and abroad. A similar formulation is adopted also by Kollmann et al. (2013).
where \( P_{C,vat}^{C,k,t} \) defines the private consumption deflator\(^7\) in terms of input factors, \( W_{k,t} \) denotes the nominal wage rate, \( N_{s,j,k,t} \) is the employment in hours, \( T_{s,j,k,t} \) are government transfers and \( tax_{s,j,k,t} \) lump-sum taxes paid by savers. Moreover, \( i_{k,t}^{rf} \), \( i_{k,t}^{G} \), and \( i_{k,t}^{W} \) are returns on domestic private risk-free bonds, domestic government bonds, and internationally traded bonds, respectively. As Ricardian households own the firms, they receive nominal profits in form of dividends, \( \Pi_{k,t}^{f} \), that are distributed by differentiated goods producers according to the number of shares held by the households. We define the gross nominal return on shares \( S_{t} \) as:

\[
1 + i_{k,t}^{S} = \frac{P_{k,t}^{S} + P_{k,t}^{Y} \Pi_{k,t}^{f}}{P_{k,t-1}^{S}}.
\]

The Ricardian households maximise the present value of the expected stream of future utility subject to equation (1), by choosing the amount of consumption, \( C_{j,k,t}^{s} \), and next period asset holdings: \( B_{j,k,t}^{rf} \), \( B_{j,k,t}^{G} \), \( S_{j,k,t} \). The maximisation problem results in the following first-order conditions (FOCs):

\[
\lambda_{j,k,t}^{s} = (C_{k,t}^{s} - h_{k} C_{k,t-1}^{s})^{-\theta_{k}}, \tag{2}
\]

\[
1 = \bar{\beta}_{t} E_{t} \left[ \frac{\lambda_{j,k,t+1}^{s}}{\lambda_{j,k,t}^{s}} \left( 1 + i_{k,t}^{rf} \right) \right]. \tag{3}
\]

\[
1 = \bar{\beta}_{t} E_{t} \left[ \frac{\lambda_{j,k,t+1}^{G}}{\lambda_{j,k,t}^{s}} \left( 1 + i_{k,t}^{G} \right) - \left( \varepsilon_{B_{k,t}^{k}} + \alpha_{k}^{B_{0}} \right) \right]. \tag{4}
\]

\[
1 = \bar{\beta}_{t} E_{t} \left[ \frac{\lambda_{j,k,t+1}^{S}}{\lambda_{j,k,t}^{s}} \left( 1 + i_{k,t+1}^{S} \right) - \left( \alpha_{k}^{S_{0}} + \varepsilon_{k,t}^{S_{k,t}} \right) \right]. \tag{5}
\]

The optimality conditions are similar to standard Euler equations, but incorporate asset-specific risk premia similar to Vitek (2014, 2017), which depend on exogenous shocks \( \varepsilon_{B_{k,t}^{k}} \), \( \varepsilon_{k,t}^{S_{k,t}} \). Combining the Euler equation for the risk-free bond (3) with (4) and (5), we obtain the approximated following expressions:

\[
i_{k,t}^{G} = i_{k,t}^{rf} + rprem_{k,t}^{G},
\]

\[
i_{k,t}^{S} = i_{k,t}^{rf} + rprem_{k,t}^{S},
\]

where \( rprem_{k,t}^{G} \) and \( rprem_{k,t}^{S} \) are risk premia on domestic government bonds and domestic

\(^7\)\( P_{C,vat}^{C,k,t} \) is the VAT adjusted private consumption deflator, \( P_{C,vat}^{C,k,t} = (1 + \tau_{C}^{C}) P_{C,k,t}^{C} \), where \( \tau_{C}^{C} \) is the tax rate on consumption (VAT).
shares, respectively. In the absence of explicit international cross-holdings of assets in the current model, these risk premia may capture both the international spillovers (such as comovement of risk premia) that occur via the international financial market channel, and the financial frictions that have contributed to the financial crisis. Analysing, ex-post, the cross-country correlation of the risk-premia shocks on domestic bonds and shares can provide an indication of the international spillover channel.  

Given the monetary union setting, the nominal exchange rate between the $k$-th EMU country and EA is fixed, $e_{EA,k,t} = 1$, implying that $\Delta \ln e_{RoW,EA,t+1} = \Delta \ln e_{RoW,k,t+1}$. We assume that an uncovered interest rate parity condition links the interest rate of the EMU country, $i_{k,t}$, to the EA policy rate set by the European Central Bank (ECB):

$$(1 + i_{k,t}^{RF}) = (1 + i_{EA,t}) - \left( \alpha_{k}^{bw} \frac{e_{RoW,EA,t}B_{k,t}}{P_{k,t}Y_{k,t}} + \varepsilon_{k,t}^{FQ} \right),$$

where $\alpha_{k}^{bw} \frac{e_{RoW,EA,t}B_{k,t}}{P_{k,t}Y_{k,t}}$ captures a debt-dependent country risk premium on net foreign asset holdings as external closure to ensure long-run stability (see Schmitt-Grohe and Uribe, 2003; Adolfson et al., 2008). Following Smets and Wouters (2007) we also introduce an additional risk premium shock, $\varepsilon_{k,t}^{FQ}$ (‘Flight to Safety’), which creates a wedge between the EA policy rate, $i_{EA,t}$, and the return on domestic risk-free assets, $i_{k,t}^{RF}$. Since a positive shock increases the required return on domestic assets and the cost of capital, it reduces current consumption and investment simultaneously and helps explaining the comovement of consumption and investment.

### 2.1.2. Liquidity-constrained households

Liquidity-constrained households have no access to financial markets. Hence, the instantaneous utility function, $u^c(.)$, is:

$$u^c_{j,k,t}(C^c_{j,k,t},N^c_{j,k,t}) = \frac{1}{1 - \theta_k} (C^c_{j,k,t} - h_k C^c_{k,t-1}^{a-k})^{1-\theta_k} - (C^c_{k,t})^{1-\theta_k} \frac{\omega_k^{U} \varepsilon_{k,t}^U}{1 + \theta^N} (N^c_{j,k,t})^{1+\theta^N}.$$ 

In each period, they consume their disposable net income, which consists of labour income and net lump-sum transfers from the government. The budget constraint is described by:

$$(1 + \tau_k^C)P_{k,t}^C C^c_{j,k,t} = (1 - \tau_k^N)W_{k,t}N^c_{j,k,t} + T^c_{j,k,t} - t a x^c_{j,k,t}.$$ 

---

8Observationally, this approach is equivalent to assuming exogenous risk premia as well as endogenous risk premia derived, e.g., in the spirit of Bernanke et al. (1996).
2.1.3. Wage setting

Households are providing differentiated labour services, $N_{j,k,t}$, in a monopolistically competitive market. We assume that there is a labour union that bundles labour hours provided by both types of domestic households into a homogeneous labour service and resells it to intermediate goods producing firms. We assume that Ricardian and liquidity-constrained households’ labour are distributed proportionally to their respective population shares, $\omega_k$. Since both households face the same labour demand schedule, each household works the same number of hours as the average of the economy, $N_{k,t} = N_{c,t} = N_{k,t}$. It follows that the individual union’s choice variable is a common nominal wage rate for both types of households.

The union maximises the discounted future stream of the weighted average of lifetime utility of its members with respect to the wage and subject to the weighted sum of their budget constraints. Nominal rigidity in wage setting is introduced in the form of adjustment costs for changing wages. Additionally, we allow for real wage rigidity as in Blanchard and Galí (2007) and Coenen and Straub (2005), where the slow adjustment of real wages occurs through distortions rather than workers’ preferences. The wage rule is determined by equating the marginal utility of leisure, $U_{N,k,t}$, to the weighted average of the marginal utility of consumption, $\lambda_{k,t}$, times the real wage adjusted for a wage mark-up:

$$\left[ \frac{\mu_k^{w} U_{k,t}^{N} P_{k,t}^{C,vat}}{\lambda_{k,t} P_{k,t}^{Y}} \right]^{1-\gamma^{wr}} \left[ \left( 1 - \tau_{k,t}^{N} \right) \frac{W_{k,t-1}^{Y}}{P_{k,t-1}^{Y}} \right]^{\gamma^{wr}} = \left( 1 - \tau_{k,t}^{N} \right) \frac{W_{k,t}^{Y}}{P_{k,t}^{Y}}$$

$$+ \gamma^{w}_k \left( \frac{W_{k,t}}{W_{k,t-1}^{Y}} - 1 - (1 - sf w_k)(\pi_{k,t-1}^{Y} - \bar{\pi}) - \pi^{w} \right) \frac{W_{k,t}^{Y}}{W_{k,t-1}^{Y} P_{k,t}^{Y}}$$

$$- \gamma^{w}_k \epsilon^{U}_t \left[ \frac{\beta_{k,t}}{\lambda_{k,t} P_{k,t}^{C,vat} N_{k,t+1}^{C,vat}} N_{k,t+1}^{C,vat} \right] \left( \frac{W_{k,t+1}^{Y}}{W_{k,t}^{Y}} - 1 - (1 - sf w_k)(\pi_{k,t}^{Y} - \bar{\pi}) - \pi^{w} \right) \frac{W_{k,t+1}^{Y} W_{k,t}^{Y}}{W_{k,t}^{Y} P_{k,t}^{Y}}$$

where $b^{U}_{k,t}$ is the replacement rate. A similar adjustment is also done on the right hand side:

$$\left[ (1 - \tau_{k,t}^{N}) \frac{W_{k,t-1}^{Y}}{P_{k,t-1}^{Y}} - BEN_{k,t-1} \right]$$

with $BEN_{k,t} = b^{U}_{k,t} \frac{W_{k,t}^{Y}}{P_{k,t}^{Y}}$, where $b^{U}_{k,t}$ is the replacement rate. Since it is only a German-specific labour market shock, we abstract from the inclusion into the general model equations.

As the German government implemented an extensive labour market deregulation in 2003-05 (‘Hartz’ reforms) that included a reduction in unemployment benefits, we capture the effect of the ‘Hartz reforms’ by treating the benefit replacement rate (ratio of unemployment benefit to wage rate) as an autocorrelated exogenous variable. Following the approach by Kollmann et al. (2015), we observe the historical benefit ratio and estimate the labour market reform as an exogenous permanent reduction in the unemployment benefit ratio. Therefore, real unemployment benefits (paid to unemployed workers of the labour force) enters the budget constraints of the households and the government. The wage setting equation on the left hand side is adjusted by $\left[ (1 - \tau_{k,t}^{N}) \frac{W_{k,t-1}^{Y}}{P_{k,t-1}^{Y}} - BEN_{k,t-1} \right]$, with $BEN_{k,t} = b^{U}_{k,t} \frac{W_{k,t}^{Y}}{P_{k,t}^{Y}}$, where $b^{U}_{k,t}$ is the replacement rate. A similar adjustment is also done on the right hand side: $\left[ (1 - \tau_{k,t}^{N}) \frac{W_{k,t-1}^{Y}}{P_{k,t-1}^{Y}} - BEN_{k,t} \right]$. Since it is only a German-specific labour market shock, we abstract from the inclusion into the general model equations.
where $\mu^w_k$ is the gross wage mark-up, $\gamma^w_k$ and $\gamma^{wr}_k$ represent the degree of nominal and real wage rigidity, respectively, $sfw_k$ is the degree of forward-lookingness in the labour supply equation, and $\varepsilon^U_{k,t}$ captures a shock to the wage mark-up (labour supply shock). The marginal utility of leisure is defined as: 

$$U^N_{k,t} = \omega^N_k (C^c_k) \theta^N_k + \theta^N_k (N^c_k - h_k C^c_k - 1)^{-\theta_k}$$

where $s^OIl_k$ is the energy input share and $\sigma^O_k$ is the elasticity of substitution between factors. Each total domestic output firm maximises its expected profits:

$$\max_{Y_{k,t}, Oil_{k,t}} \frac{P^O_{k,t} O_{k,t} - P^Y_{k,t} Y_{k,t} - P^{Oil}_{k,t} Oil_{k,t}}{P^O_{k,t}}$$

subject to the production function (6). The respective first order conditions for the intermediate domestic output and oil are given by:

$$Y_{k,t} = (1 - s^OIl_k) \left( P^Y_{k,t} \frac{P_{k,t}^O}{P_{k,t}^O} \right)^{-\sigma^O_k} O_{k,t},$$

$$Oil_{k,t} = s^OIl_k \left( P^{Oil}_{k,t} \frac{P_{k,t}^O}{P_{k,t}^O} \right)^{-\sigma^O_k} O_{k,t}.$$  

Oil is assumed to be imported from RoW. Hence, the oil price is taken as given:

$$P^{Oil}_{k,t} = e_{Row,k,t} P^{Oil}_{Row,t} + \tau^{Oil} P_t^Y 0,$$

10Note that we do not observe wage dispersion in equilibrium, thus $N^d_{k,t} = N_{k,t}$.  

11Note that $s^OIl_k$ is perturbed by a trend shock to the degree of country openness, as specified below in equation (15).
where $e_{Row,k,t}$ is the exchange rate, measured as price of foreign currency in terms of domestic currency, $\tau^{Out}$ and $P^Y_0$ are the excise duty and the (global) GDP deflator, respectively. The price index of the composite total output is:

$$P^O_{k,t} = \left(1 - s^O_t \right) (P^Y_t)^{\sigma^Y_k - 1} + s^O_t \left( P^O_t \right)^{\sigma^Y_k - 1} \right]^{\frac{1}{1 - \sigma^Y_k}}.$$

### 2.2.2. Value added sector

Value added, $Y_{k,t}$, is produced by perfectly competitive firms by combining a large number of differentiated goods, $Y_{i,k,t}$, produced by monopolistically competitive firms, according to a Dixit and Stiglitz (1977) production technology:

$$Y_{k,t} = \left[ \int_0^1 Y_{i,k,t} \sigma^Y_k \right]^{\frac{1}{1 - \sigma^Y_k}},$$

where $\sigma^Y$ represents the inverse of the steady state gross price mark-up on differentiated goods. The demand for a differentiated good $i$ is then:

$$Y_{i,k,t} = \left( P_{i,k,t} \right)^{-\sigma^Y_k} Y_{k,t},$$

where $P_{i,k,t}$ is the price of intermediate inputs and the corresponding price index is:

$$P^Y_{k,t} = \left[ \int_0^1 (P_{i,k,t})^{-\sigma^Y_k} \right]^{\frac{1}{1 - \sigma^Y_k}}.$$

### 2.2.3. Intermediate goods producers

Each firm $i \in [0, 1]$ produces a variety of the domestic good which is an imperfect substitute for varieties produced by other firms. Given imperfect substitutability, firms are monopolistically competitive in the goods market and face a downward-sloping demand function for goods.

Differentiated goods are produced using total capital, $K^\text{tot}_{i,k,t-1}$, and labour, $N_{i,k,t}$, which are combined in a Cobb-Douglas production function:

$$Y_{i,k,t} = \left[ A^Y_{k,t} \left( N_{i,k,t} - FN_{i,k,t} \right) \right]^{\alpha_k} (CU_{i,k,t} K^\text{tot}_{i,k,t-1})^{1-\alpha_k} - A^Y_{k,t} FC_{i,k},$$

where $\alpha_k$ is the steady-state labour share, $A^Y_{k,t}$ represents the labour-augmenting productivity common to all firms in the differentiated goods sector, $CU_{i,k,t}$ and $FN_{i,k,t}$ are firm-specific
levels of capacity utilisation and labour hoarding, respectively.\textsuperscript{12} FC\textsubscript{i,k} captures fixed costs in production. Total capital is the sum of private installed capital, \( K_{i,k,t} \), and public capital, \( K^G_{i,k,t} \):

\[
K^\text{tot}_{i,k,t} = K_{i,k,t} + K^G_{i,k,t}.
\]

Since total factor productivity (TFP) is not a stationary process, we allow for two types of shocks. They are related to a non stationary process and its autoregressive component:

\[
\log(A^Y_{k,t}) - \log(A^Y_{k,t-1}) = g^\text{AY}_{k,t} + \varepsilon^\text{LAY}_{k,t},
\]

\[
g^\text{AY}_{k,t} = \rho^\text{AY} g^\text{AY}_{k,t-1} + (1 - \rho^\text{AY}) g^{\text{AY}_0} + \varepsilon^\text{AY}_{k,t},
\]

where \( g^\text{AY}_{k,t} \) and \( g^{\text{AY}_0} \) are the time-varying growth and the long-run growth of technology, and \( \varepsilon^\text{LAY}_{k,t} \) is a permanent technological shock.

Monopolistically competitive firms maximise the real value of the firm, \( \frac{p^S_{i,k,t}}{P^Y_{k,t}} S^\text{tot}_{k,t} \), which is the discounted stream of expected future profits, subject to the output demand (7), the technology constraint (8), and the law of motion of capital, \( K_{i,k,t} = I_{i,k,t} + (1 - \delta_k) K_{i,k,t-1} \).\textsuperscript{13}

Their problem can be written as:

\[
\max_{P_i, N_i, I, K, CU, FN} \sum_{s=t}^{\infty} D^S_k \Pi^f_{i,k,t},
\]

where the stochastic discount factor, \( D^S_k \), is:

\[
D^S_k = \frac{1 + r^S_{k,t}}{\Pi^S_{r=t}(1 + r^S_{k,t})}
\]

with \( 1 + r^S_{k,t} = \frac{1+q^S_{i,k,t+1}}{1+q^S_{i,k,t}} \) being the real stock return. The period \( t \) profit of an intermediate goods firm \( i \) is given by:

\[
\Pi^f_{i,k,t} = (1 - \tau_k^K) \left( \frac{P^Y_{i,k,t}}{P^Y_{k,t}} Y_{i,k,t} - \frac{W_{k,t}}{P^Y_{k,t}} N_{i,k,t} \right) + \tau_k^K \delta_k P^I_{k,t} + \frac{P^I_{k,t}}{P^Y_{k,t}} I_{i,k,t} - \text{adj}_{i,k,t},
\]

where \( I_{i,k,t} \) is the physical investment at price \( P^I_{i,k,t} \), \( \tau_k^K \) is the corporate tax and \( \delta_k \) the capital depreciation rate.

\textsuperscript{12}According to Burnside and Eichenbaum (1996), firms prefer not to layoff workers when the demand is temporarily low, because firing workers may be more costly than hoarding them. Additionally, the inclusion of labor hoarding, \( FN_{i,k,t} \), allows to match the observed co-movement between output and working hours.

\textsuperscript{13}We assume that the total number of shares \( S^\text{tot}_{k,t} = 1. \)
Following Rotemberg (1982), firms face quadratic adjustment costs, $adj_{i,k,t}$, measured in terms of production input factors. Specifically, the adjustment costs are associated with the output price, $P^Y_{i,k,t}$, labour input, $N_{i,k,t}$, investment, $I_{i,k,t}$, as well as capacity utilisation variation, $CU_{i,k,t}$, and labour hoarding, $FN_{i,k,t}$:

$$adj^P_{i,k,t} = \sigma^P_k \gamma^P_k \frac{P^Y_{i,k,t}}{Y_{k,t}} \left[ \frac{P^Y_{i,k,t}}{P^Y_{i,k,t-1}} - \exp(\pi) \right]^2,$$

$$adj^N_{i,k,t} = \gamma^N_k \frac{N_{i,k,t}}{Y_{k,t}} \left[ \frac{N_{i,k,t} - FN_{i,k,t}}{N_{i,k,t-1} - FN_{i,k,t-1}} - \exp(g^{pop}) \right]^2,$$

$$adj^I_{i,k,t} = \frac{P^I_{k,t}}{P^Y_{k,t}} \left[ \frac{\gamma^I_{k,1}}{K_{k,t-1}} \left( \frac{I_{i,k,t}}{K_{k,t-1}} - \delta^I_{k,t} \right)^2 + \frac{\gamma^I_{k,2}}{K_{k,t-1}} \left( I_{i,k,t} - I_{i,k,t-1}\exp(g^Y + g^I) \right)^2 \right],$$

$$adj^{CU}_{i,k,t} = \frac{P^I_{k,t}}{P^Y_{k,t}} K_{i,k,t-1} \left[ \frac{\gamma^{CU,1}_k}{(CU_{i,k,t} - 1)} + \frac{\gamma^{CU,2}_k}{2} (CU_{i,k,t} - 1)^2 \right],$$

$$adj^{FN}_{i,k,t} = Y_{t} \left[ \gamma^{FN,2}_k \left( \frac{FN_{i,k,t}}{Actr_{k,t} Pop_{k,t}} - FN \right) + \gamma^{FN,1}_k \left( \frac{FN_{i,k,t}}{Actr_{k,t} Pop_{k,t}} - FN \right)^2 \right],$$

where $\gamma$-s capture the degree of adjustment costs and $\delta^I_{k,t} \neq \delta_k$ is a function of the depreciation rate adjusted for the capital trend in order to have zero adjustment costs on the trend-path.$^{14}$

Given the Lagrange multiplier associated with the technology constraint, $\mu^Y$, the FOCs with respect to labour, labour hoarding, capital, investment, and capacity utilisation are given by:

$$(1 - \tau^K_k) \frac{W_{k,t}}{P^Y_{k,t}} = \alpha_k \left( \mu^y_{k,t} - \varepsilon^{ND}_{k,t} \right) \frac{Y_{k,t}}{N_{i,k,t} - FN_{i,k,t}} - \frac{\partial adj^N_{i,k,t}}{\partial N_{k,t}} + E_t \left[ \frac{1 + \pi^Y_{k,t+1} + \partial adj^N_{i,k,t+1}}{1 + i^s_{k,t+1}} \right],$$

$$(10)$$

$${\mu^y_{k,t} \alpha_k} \frac{Y_{k,t}}{N_{i,k,t} - FN_{i,k,t}} = - \frac{Y_{k,t}}{Actr_{k,t} Pop_{k,t}} \left( \gamma^{FN,1}_k + \gamma^{FN,2}_k \left( \frac{FN_{i,k,t}}{Actr_{k,t} Pop_{k,t}} - FN \right) \right) + \frac{\partial adj^N_{i,k,t}}{\partial FN_{k,t}} - E_t \left[ \frac{1 + \pi^Y_{k,t+1} + \partial adj^N_{i,k,t+1}}{1 + i^s_{k,t+1}} \right],$$

$$(10)$$

$^{14}$We specify $\delta^{I}_{k,t} = exp(g^Y + GAP^{10}) - (1 - \delta_k)$ so that $\frac{I}{K} - \delta^k \neq 0$ along the trend path.
\[ Q_{k,t} = E_t \left[ \frac{1 + \pi_{k,t+1}^Y P_{k,t+1}^I P_{k,t}^Y}{1 + i_{k,t+1}^s P_{k,t+1}^Y P_{k,t}^I} \left( \tau_k^K \delta_k - \frac{\partial \text{adj}_{k,t} \text{CU}_k}{\partial K_{k,t-1}} + Q_{k,t+1}(1 - \delta_k) + (1 - \alpha_k) \mu_{k,t+1}^Y \frac{P_{k,t+1}^Y Y_{k,t+1}}{P_{k,t+1}^I K_{k,t}^{\text{tot}}} \right) \right], \quad (11) \]

\[ Q_{k,t} = \left[ 1 + \gamma_{k,1} \frac{I_{k,t}}{K_{k,t-1}} - \gamma_{k} \frac{I_{k,t} - I_{k,t-1} \exp(g_k^Y + g_{k}^I)}{K_{k,t-1}} \right] - E_t \left[ \frac{1 + \pi_{k,t+1}^Y P_{k,t+1}^I P_{k,t}^Y}{1 + i_{k,t+1}^s P_{k,t+1}^Y P_{k,t}^I} \exp(g_k^Y + g_{k}^I) \gamma_{k,2} \frac{(I_{k,t+1} - I_{k,t} \exp(g_k^Y + g_{k}^I))}{K_{k,t}} \right], \quad (12) \]

\[ \mu_{k,t}^Y (1 - \alpha_k) Y_{k,t} \frac{P_{k,t}^Y}{CU_{k,t} P_{k,t}^I} = K_{k,t-1}^{\text{tot}} \left[ \gamma_{k,1}^{\mu} + \gamma_{k,2}^{\mu} (CU_{k,t} - 1) \right], \quad (13) \]

where \( Q_{k,t} = \mu_{k,t} \frac{P_{k,t}^I}{P_{k,t}^Y} \) represents Tobin’s Q and \( Act_{k,t} \) \( Pop_{k,t} \) is the active labour force of the domestic country. Equations (9) and (10) characterise the optimal level of labour input, taking into account labour hoarding. While (9) equates the marginal cost of labour to its marginal productivity, equation (10) determines the optimal level of labour hoarding at the expense of the loss in the marginal productivity. Equation (11) and (12) define the Tobin’s Q, which is equal to the replacement cost of capital (the relative price of capital). Finally, (13) describes capacity utilisation, where the left-hand side indicates the additional output produced while the right-hand side captures the costs of higher utilisation rate.

Given the Rotemberg set-up and imposing the price symmetry condition, \( P_{i,k,t}^Y = P_{k,t}^Y \), the FOC with respect to \( P_{i,k,t}^Y \) yields the New Keynesian Phillips curve:

\[ \mu_{k,t}^Y \sigma_k \sigma_k^Y = (1 - \tau_k^K) (\sigma_k^Y - 1) + \sigma_k^Y \frac{P_{k,t}^Y}{P_{k,t-1}^Y} \left( \pi_{k,t}^Y - \bar{\pi} \right) \]

\[ - \sigma_k^Y \gamma_k^P \left[ \frac{1 + \pi_{k,t+1}^Y P_{k,t+1}^Y}{1 + i_{k,t+1}^s P_{k,t+1}^Y P_{k,t}^I} \frac{Y_{k,t+1}}{Y_{k,t}} \left( \pi_{k,t+1}^Y - \bar{\pi} \right) \right] + \sigma_k^Y \varepsilon_{k,t}^{\mu}, \]

where \( \varepsilon_{k,t}^{\mu} \) is the inverse of the mark-up shock.

In order to allow firms to be less forward-looking in their price setting, we introduce a backward-looking term \( \pi_{k,t}^* = \rho_k^{\pi} \bar{\pi} + (1 - \rho_k^{\pi})(\pi_{k,t-1}^*), \) where \( \bar{\pi} \) is the steady-state inflation.
The final New Keynesian Phillips curve takes then the following form:

\[ \mu_{k,t} \sigma_k^Y = (1 - \tau_k^K)(\sigma_k^Y - 1) + \sigma_k^Y \gamma_k \frac{P_{k,t}^Y}{P_{k,t-1}^Y} \left( \pi_{k,t}^Y - \bar{\pi} \right) - \sigma_k^Y \gamma_k \frac{1 + \pi_{k,t+1}^Y}{1 + \pi_{k,t+1}^Y} \frac{P_{k,t+1}^Y}{Y_{k,t+1}} \left[ sfp_k \left( \pi_{k,t+1}^Y - \bar{\pi} \right) + (1 - sfp_k)(\pi_{k,t}^* - \bar{\pi}) \right] (14) \]

where \( sfp_k \) is the share of forward-looking price setters.

2.3. Trade

2.3.1. Exchange rates and terms of trade

The nominal effective exchange rate, \( e_{k,t} \), measures the trade weighted average price of foreign currency in terms of domestic currency and is defined as:

\[ e_{k,t} = \prod_l (e_{l,k,t})^{w_{T,l,k,t}}, \]

where \( e_{l,k,t} \) is the bilateral exchange rate between domestic country \( k \) and foreign country \( l \). Similarly, the real effective exchange rate, \( rer_{k,t} \), measures the trade weighted average price of foreign output in terms of domestic output:

\[ rer_{k,t} = \prod_l (rer_{l,k,t})^{w_{T,l,k,t}}, \]

where \( rer_{l,k,t} \) is the bilateral real exchange rate between \( k \) and \( l \). \( w_{T,l,k,t} \) is the trade weight of the foreign trade partner \( l \) in the domestic economy’s external trade and defined as:

\[ w_{T,l,k,t} = \frac{1}{2} \left( \frac{P_{l,k,t}^X X_{l,k,t}}{P_{k,t}^X X_{k,t}} + \frac{P_{l,k,t}^M M_{l,k,t}}{P_{k,t}^M M_{k,t}} \right), \]

where \( X_{l,k,t} \) and \( M_{l,k,t} \) stand for domestic exports to and imports from country \( l \), respectively, and \( P_{l,k,t}^X \) and \( P_{l,k,t}^M \) are the relevant price indices. \( P_{k,t}^M M_{k,t} \) includes oil imports from RoW and is defined as \( P_{k,t}^M M_{k,t} = P_{k,t}^M M_k + P_{k,t}^{oil} OIL_{k,t} \). \( P_{l,k,t}^X \) and \( P_{l,k,t}^M \) are the respective price aggregates and are defined in the next section.

\(^{15}\)When \( \rho^{\pi^*} = 0 \), equation (14) nests the core specification including static expectations. We use and estimate this modified specification for Italy and Spain, as it improves significantly the annual fit of GDP inflation and reduces the previous over-prediction of inflation during the last years of our sample.
The terms of trade, $\text{TOT}_{k,t}$, are the relative price of export over import goods, is defined as:

$$\text{TOT}_{k,t} = \frac{P^X_{k,t}}{P^M_{k,t}}.$$ 

2.3.2. Import sector

Final good packagers (Aggregate import demand)

The final aggregate demand component goods $C_{k,t}$ (private consumption good), $I_{k,t}$ (private investment good), $G_{k,t}$ (government consumption good), $I^G_{k,t}$ (government investment good), as well as $X_{k,t}$ (export good) are produced by perfectly competitive firms by combining domestic output, $O^D_{k,t}$, with imported goods, $M^D_{k,t}$, where $D = \{C, I, G, I^G, X\}$, using the following CES production function:

$$D_{k,t} = A^P_{k,t}\left[\left(1 - u^M_{k,t}s^M_{k,t}\right)^{\frac{1}{\sigma^z_k}}(O^D_{k,t})^{\frac{\sigma^z_k-1}{\sigma^z_k}} + (u^M_{k,t}s^M_{k,t})^{\frac{1}{\sigma^z_k}}(M^D_{k,t})^{\frac{\sigma^z_k-1}{\sigma^z_k}}\right]^{\frac{\sigma^z_k}{\sigma^z_k-1}},$$

where $\sigma^z_k$ is the elasticity of substitution of imports, $A^P_{k,t}$ is a shock to productivity in the sector producing goods, $D$, and $u^M_{k,t}$ is a shock to the share $s^M_{k,t}$ of good-specific import demand components. The shock to the country openness is given by:

$$u^M_{k,t} = \exp(\varepsilon^M_{k,t}) \left[1 + \tau^{\text{MAY}1}_{kk} \varepsilon^{TM}_{kk,t} - \sum_l (1 - \tau^{\text{MAY}1}_{ll}) \varepsilon^{TM}_{ll,t} s^M_{lk} \right].$$

(15)

The shock is partially endogenized and composed of a country-specific shock, $\varepsilon^M_{k,t}$, and a bilateral trend, $\varepsilon^{TM}_{xx,t}$ with $xx = \{kk, ll\}$, which depends on changes in the technology of trading partners. The latter is defined as:

$$\varepsilon^{TM}_{xx,t} = \rho^{TM}_{xx} \varepsilon^{TM}_{xx,t-1} - (1 - \rho^M_{xx}) \left[\tau^{\text{MAY}2}_{xx} \varepsilon^{TM}_{xx,t} Y_{xx,t-1} \exp(g^Y_{xx}) \right],$$

where $\rho^{TM}_{xx}$ captures the persistence of the trade trend, $\tau^{\text{MAY}2}_{xx}$ measures the relative competitiveness of the domestic country, and $\tau^{\text{MAY}1}_{xx}$ captures its impact on the openness. More precisely, an increase in relative productivity lowers the domestic degree of openness (proportionally to $\tau^{\text{MAY}1}_{xx}$) and increases the degree of openness of trading partners towards domestic exports (proportionally to $(1 - \tau^{\text{MAY}1}_{xx})$).\(^{16}\)

From profit maximisation we obtain the following domestic, $O^D$, and foreign, $M^D$, de-

\(^{16}\)The endogenous trend tries to capture the trend in import share and is estimated for Italy and Spain. The core specification without this endogenous component is nested in (15) by setting $\tau^{\text{MAY}2}_{xx} = 0$. Note that the same trend affects similarly the oil import demand.
mand aggregates:

\[ O^{D}_{k,t} = (A^{D}_{k,t})^{\sigma_{k}^{-1}} \left( 1 - u^{M}_{k,t} M^{D}_{k,t} \right) \left( \frac{P^{O}_{k,t}}{P^{D}_{k,t}} \right)^{-\sigma_{k}^{-1}} D_{k,t}, \]

\[ M^{D}_{k,t} = (A^{D}_{k,t})^{\sigma_{k}^{-1}} \left( 1 - u^{M}_{k,t} M^{D}_{k,t} \right) \left( \frac{P^{M}_{k,t}}{P^{D}_{k,t}} \right)^{-\sigma_{k}^{-1}} D_{k,t}, \]

where \( P^{D}_{k,t} \) is the price deflator associated to the demand components:

\[ P^{D}_{k,t} = (A^{D}_{k,t})^{-1} \left[ (1 - u^{M}_{k,t} M^{D}_{k,t}) (P^{O}_{k,t})^{1-\sigma_{k}^{-1}} + u^{M}_{k,t} M^{D}_{k,t} (P^{M}_{k,t})^{1-\sigma_{k}^{-1}} \right]^{\frac{1}{1-\sigma_{k}}} . \]

We define total non-oil imports as:

\[ M_{k,t} = M^{C}_{k,t} + M^{I}_{k,t} + M^{G}_{k,t} + M^{IG}_{k,t} + M^{X}_{k,t}, \]

whereas total imports are:

\[ P^{M}_{k,t} M^{tot}_{k,t} = P^{M}_{k,t} M_{k,t} + P^{O}_{k,t} OIL_{k,t}. \]

**Import retailers (Economy-specific final import demand)**

Final imported goods are produced by perfectly competitive firms combining economy-specific final imports. They maximise the following profit function:

\[ \max_{M_{l,k,t}} P^{M}_{k,t} M_{k,t} - \sum_{l} P^{M}_{l,k,t} M_{l,k,t} \frac{size_{l}}{size_{k}} \]

subject to the following CES production function:

\[ M_{k,t} = \left[ \sum_{l} \left( s^{M}_{l,k,t} u^{M}_{l,k,t} \right) \frac{P^{M}_{l,k,t}}{P^{M}_{k,t}} (M_{l,k,t} \frac{size_{l}}{size_{k}})^{\frac{\sigma_{M}^{F} \sigma_{k}^{M}}{\sigma_{k}^{M} - 1} - 1} \right]^{\frac{\sigma_{M}^{F}}{\sigma_{k}^{M} - 1} - 1}, \]

where \( \sigma_{k}^{M} \) is the price elasticity of demand for country l’s goods and \( \sum_{l} s^{M}_{l,k} = 1 \) are the import shares. The demand for goods from country l is given by:

\[ M_{l,k,t} = s^{M}_{l,k,t} u^{M}_{l,k,t} \left( \frac{P^{M}_{l,k,t}}{P^{M}_{k,t}} \right)^{-\sigma_{k}^{M}} M_{k,t} \frac{size_{k}}{size_{l}}. \]
where $u_{i,k,t}^M$ captures an endogenous bilateral trend component:

$$u_{i,k,t}^M = \frac{1 - (1 - \tau_{i,i}^{MY})\varepsilon_{i,i,t}^M}{1 - \sum_z (1 - \tau_{z,z}^{MY})\varepsilon_{z,z,t}^M s_{z,k}} s_{i,k,t}.$$  \hspace{1cm} (16)

The import prices are:

$$P_{i,k,t}^M = \left[ \sum_l s_{i,k}^M u_{i,k,t}^M (P_{i,k,t}^M)^{1-\sigma_k^M} \right]^{\frac{1}{1-\sigma_k^M}}$$

with $P_{i,k,t}^M$ being the economy-specific import goods prices. Since all products from country $l$ are initially purchased at export price, $P_{l,t}^X$, the economy-specific import goods price can be also expressed as:

$$P_{i,k,t}^M = e_{i,k,t} P_{l,t}^X.$$

2.3.3. Export sector

The exporting firms are competitive and export a good that is a combination of domestic output and import content. Their profit maximisation problem is defined in 2.3.2. The corresponding export price is given by:

$$P_{k,t}^X = \exp(\varepsilon_{k,t}^X) \left[ (1 - u_{k,t}^M s_{k}^M)(P_{k,t}^O)^{1-\sigma_k^O} + u_{k,t}^M s_{k}^M (P_{k,t}^M)^{1-\sigma_k^M} \right]^{\frac{1}{1-\sigma_k^M}},$$

where $\varepsilon_{k,t}^X$ is the export-specific price shock.

2.4. Fiscal policy

The government collects taxes on labour, $\tau_k^N$, capital, $\tau_k^K$, consumption, $\tau_k^C$, excise duties from oil imports, $\tau_k^{Oil}$, and lump-sum taxes, $tax_{k,t}$, and issues one-period bonds, $B_{k,t}^G$, to finance government consumption, $G_{k,t}$, investment, $I_{k,t}^G$, transfers, $T_{k,t}$, and the servicing of the outstanding debt. The government budget constraint is:

$$B_{k,t}^G = (1 + i_{k,t-1})B_{k,t-1}^G - R_{k,t}^G + P_{k,t}^G G_{k,t} + P_{k,t}^I I_{k,t}^G + T_{k,t} + P_{k,t}^Y P_{k,t}^Y,$$

where nominal government revenues, $R_{k,t}^G$, are defined as:

$$R_{k,t}^G = \tau_k^K (P_{k,t}^Y Y_{k,t} - W_{k,t} N_{k,t} - P_{k,t}^I s_{k,t} K_{k,t-1}) + \tau_k^N W_{k,t} N_{k,t} + \tau_k^C P_{k,t}^C C_{k,t}$$

$$+ \tau_{Oil}^O P_{t}^{Oil} Oil_t + tax_{k,t} P_{k,t}^Y Y_{k,t}.$$
For simplicity, all tax rates are assumed to be constant and calibrated according to euro-area historical averages.\textsuperscript{17} To close the government budget constraint, lump sum taxes, $tax_{k,t}$, adjust residually as follows:

$$
tax_{k,t} = \rho_r tax_{k,t-1} + \eta_k^{\text{deflt}} \left( \frac{\Delta B^G_{k,t-1}}{Y_{k,t-1}P_{k,t-1}^Y} - DEFTAR_k \right) + \eta_k^{BT} \left( \frac{B^G_{k,t-1}}{Y_{k,t-1}P^Y_{k,t-1}} - BTA_k \right) + \xi_{k,t}^{\text{tax}},$$

where $DEFTAR_k$ and $BTA_k$ are the targets on government deficit and government debt, respectively, and $\xi_{k,t}^{\text{tax}}$ captures a shock. Hence, the government uses lump-sum taxes as budget closure and increases (decreases) taxes when the level of government debt and the government deficit is above (below) the debt and deficit target. The law of motion of government capital is:

$$K^G_{k,t} = (1 - \delta_k)K^G_{k,t-1} + I^G_{k,t},$$

where $\delta^G_k$ is the depreciation rate of public capital.

The model uses a measure of discretionary fiscal effort (DFE) as defined by the European Commission (2013):

$$DFE_{k,t} = \frac{P^G_{k,t}G_{k,t} + P^I_{k,t}I^G_{k,t} + P^Y_{k,t}T_{k,t}}{Y_{k,t}} - \frac{(\Delta Y_{k,t}^{\text{pot}} - 1)E^G_{k,t-1}}{Y_{k,t}},$$

where $E^G_{k,t}$ is the adjusted nominal expenditure aggregate, and $Y_{k,t}^{\text{pot}}$ is the medium-term nominal potential output.\textsuperscript{18} In order to be consistent with the definition of DFE, which is defined with respect to all primary adjusted government expenditures, we define the aggregate nominal expenditure as:

$$E^G_{k,t} = P^G_{k,t}G_{k,t} + P^I_{k,t}I^G_{k,t} + P^Y_{k,t}T_{k,t}.$$ 

We use the following DFE rules for government consumption, $G_{k,t}$, investment, $I^G_{k,t}$, and

\textsuperscript{17}The presence of constant tax rates is a simplification to streamline the estimation in medium/large-scale DSGE models (see for example Smets and Wouters (2007)). Future work will include fiscal revenue data in the information set for the estimation to better capture country-specific features and for the empirical assessment of automatic fiscal stabilisation and the empirical analysis of various EA-wide fiscal stabilisation mechanisms.

\textsuperscript{18}The adjusted nominal expenditure removes interest payments and non-discretionary unemployment expenditures from total nominal expenditure.
transfers, \( T_{k,t} \):

\[
\frac{\Delta G_{k,t} P_{k,t}^G}{P_{k,t}^G I_{k,t}} = \left( \Delta Y_{k,t}^{pot} \exp(\pi_{k,t}^y) - 1 \right) \frac{G_{k,t-1} P_{k,t-1}^G}{P_{k,t-1}^G I_{k,t-1}} - \alpha_{k,t}^G \left( \frac{G_{k,t-1} P_{k,t-1}^G}{P_{k,t-1}^G I_{k,t-1}} - G \right) + \varepsilon_{k,t}^G,
\]

\[
\frac{\Delta I_{k,t}^T P_{k,t}^T}{P_{k,t}^T Y_{k,t}} = \left( \Delta Y_{k,t}^{pot} \exp(\pi_{k,t}^y) - 1 \right) \frac{I_{k,t-1}^T P_{k,t-1}^G}{P_{k,t-1}^G I_{k,t-1}} - \alpha_{k,t}^T \left( \frac{I_{k,t-1}^T P_{k,t-1}^G}{P_{k,t-1}^G I_{k,t-1}} - I \right) + \varepsilon_{k,t}^T,
\]

where \( \varepsilon_{k,t}^G, \varepsilon_{k,t}^T, \varepsilon_{k,t}^T \) are shocks to government consumption, investment and transfers, respectively. The parameters \( \alpha_{k,t}^G, \alpha_{k,t}^T > 0 \) are policy feedback parameters to ensure long-run stability of the model.

2.5. Monetary policy

Monetary policy is modelled using a Taylor-type rule where the ECB sets the policy rate, \( i_{EA,t} \), in response to the annualised EA-wide inflation gap and the annualised EA output gap (Taylor, 1993).\(^{19}\) The policy rate adjusts sluggishly to deviations of inflation from their respective target level and to the output gap and is subject to a random shock, \( \varepsilon_{EA,t}^i \):

\[
i_{EA,t} - \bar{\pi} = \rho_{EA} (i_{EA,t-1} - \bar{\pi}) + (1 - \rho_{EA}) \left[ \eta_{EA}^{\pi} 0.25 \left( \pi_{EA,t}^C,vat,Q_A - \bar{\pi}_{EA}^C,vat,Q_A \right) \right. \\
+ \left. \eta_{EA}^{y} \log \left( 0.25 \sum_{r=1}^{4} Y_{EA,t-r} \right) - \log \left( 0.25 \sum_{r=1}^{4} Y_{EA,t-r}^{pot} \right) \right] + \varepsilon_{EA,t}^i,
\]

where \( \bar{\pi} = \bar{\pi} + \bar{\pi}_{Y,obs} \) is the steady-state nominal interest rate, equal to the sum of the steady state real interest rate and GDP inflation. Quarterly annualised inflation is defined as:

\[
\pi_{EA,t}^C,vat,Q_A = \log \left( \sum_{r=0}^{3} P_{EA,t-r}^{C,vat} \right) - \log \left( \sum_{r=4}^{7} P_{EA,t-r}^{C,vat} \right).
\]

The policy parameters \( (\rho_i, \eta_i^\pi, \eta_i^y) \) capture interest rate inertia and the response to annualised inflation and output gap, respectively.

2.6. Closing the economy

Market clearing requires that:

\[
Y_{k,t} P_{k,t}^Y + \tau^{Oil}_{k,t} P_{oil,t}^Y = P_{k,t}^C C_{k,t} + P_{k,t}^I I_{k,t} + P_{k,t}^{IG} IG_{k,t} + P_{k,t}^G G_{k,t} + TB_{k,t},
\]

\(^{19}\)We define potential output, \( Y_{k,t}^{pot} \), as the output level that would prevail if labour input equaled steady-state per capita hours worked, capital stock is utilised at full capacity and TFP equaled its trend component.
where the trade balance, $TB_{k,t}$, is defined as the difference between exports and imports:

$$TB_{k,t} = P_k^X X_{k,t} - \sum_l \frac{\text{size}_l}{\text{size}_k} P_{l,k,t}^{M} M_{l,k,t} - P_{RoW,k,t}^{Oil} OIL_{RoW,k,t} e_{RoW,k,t}.$$  

EMU-country exports are the sum of imports from the domestic economy by other countries:

$$X_{k,t} = \sum_l M_{l,k,t},$$

where $M_{l,k,t}$ stands for imports of economy $l$ from EMU country $k$.

Net foreign assets, $B_{k,t}^W$, evolve according to:

$$e_{RoW,k,t} B_{k,t}^W = (1 + i_{t-1}^{bw}) e_{RoW,k,t} B_{k,t-1}^W + TB_{k,t} + ITR_k P_{k,t} Y_{k,t},$$

where $ITR_k$ represents international transfers, which are calibrated to allow a non-zero steady-state of the trade balance.

Finally, net foreign assets of all countries sum to zero:

$$\sum_l NFA_{l,t} \text{size}_l = 0.$$  

2.7. The REA and RoW block

The model of the REA and RoW blocks is simplified in structure. Specifically, REA and RoW consist of a budget constraint for the representative household (Ricardian), demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses only labour as input factor, a New Keynesian Phillips curve, and a Taylor rule. The REA and RoW blocks abstract from capital accumulation. There are shocks to labour productivity, price mark-ups, the subjective discount rate, the relative preference for domestic and imported goods as well as monetary policy shocks. Unless otherwise specified, subindex $k$ corresponds to REA and RoW.

Since RoW is an oil exporter, the resource constraint for the representative household is:

$$P_{RoW,t}^Y Y_{RoW,t} + P_{RoW,t}^{Oil} OIL_{RoW,t} = P_{RoW,t}^C C_{RoW,t} + P_{RoW,t}^X X_{RoW,t} - \sum_l \frac{\text{size}_l}{\text{size}_{RoW}} e_{l,RoW,t} P_{l,t}^X M_{l,RoW,t},$$

where $X_{RoW,t}$ are non-oil exports by RoW.
The resource constraint for the representative household in REA, as an oil importer, is:

\[
P_{REA,t}^Y Y_{REA,t} + \tau^O_{Oil} P_{t}^{Y^0} OIL_{REA,t} = P_{REA,t}^C C_{REA,t}
\]

\[
+ P_{REA,t}^X X_{REA,t} - \sum_l \frac{siz e_t}{siz e_{REA}} c_{l,REA,t} P_{l,t}^X M_{l,REA,t},
\]

where \( \tau^O_{t} P_{t}^{Y^0} OIL_{REA,t} \) captures the excise duty.

Final aggregate demand \( C_{k,t} \) (in the absence of investment and government spending in REA and RoW) is a combination of domestic output, \( Y_{k,t} \) and imported goods, \( M_{k,t} \), using the following CES function:

\[
C_{k,t} = A_{k,t}^P \left[ (1 - u_{k,t}^{MC} s_k^M)^{\frac{1}{\sigma_k}} (Y_{k,t}^C)^{\frac{1}{\sigma_k}} + (u_{k,t}^{MC} s_k^M)^{\frac{1}{\sigma_k}} (M_{k,t}^C)^{\frac{1}{\sigma_k}} \right]^{\frac{\sigma_k}{\sigma_k - 1}},
\]

where \( u_{k,t}^{MC} \) is a shock to input components and \( s_k^M \) the import share. From profit maximisation we obtain the demand for domestic and foreign goods:

\[
Y_{k,t}^C = (A_{k,t}^P)^{\sigma_k - 1} (1 - u_{k,t}^{MC} s_k^M) \left( \frac{P_{k,t}^Y}{P_{k,t}^C} \right)^{-\sigma_k} C_{k,t},
\]

\[
M_{k,t}^C = (A_{k,t}^P)^{\sigma_k - 1} u_{k,t}^{MC} s_k^M \left( \frac{P_{k,t}^M}{P_{k,t}^C} \right)^{-\sigma_k} C_{k,t},
\]

where the consumer price deflator, \( P_{k,t}^C \), is given by:

\[
P_{k,t}^C = \left( 1 - u_{k,t}^{MC} s_k^M \right) (P_{k,t}^Y)^{1-\sigma_k} + u_{k,t}^{MC} s_k^M (P_{k,t}^M)^{1-\sigma_k} \right]^{\frac{1}{1-\sigma_k}}.
\]

The intermediate good producers use labour to manufacture domestic goods (non-oil output) according to a linear production function:

\[
Y_{k,t} = A_{k,t}^Y N_{k,t},
\]

where \( A_{k,t}^Y \) captures a trend in the productivity and \( N_{k,t} = Actr_{k,t} Pop_{k,t} \) is the active population in the economy. Price setting for non-oil output follows a New Keynesian Phillips curve:

\[
\pi_{k,t} - \bar{\pi} = \beta_{k,t} \frac{\lambda_{k,t+1}}{\lambda_{k,t}} [ s f p_k (\pi_{k,t+1} - \bar{\pi}) + (1 - s f p_k) (\pi_k^* - \bar{\pi}) ] + \phi_k^Y \log \frac{Y_{k,t}}{Y_k} + \epsilon_{k,t}^Y,
\]
\[ \lambda_{k,t} = (C_{k,t} - hC_{k,t-1})^{-\theta_k} \]

is the marginal utility of consumption, \( \varepsilon_{Y,t}^k \) is a cost push shock, \( sfp_k \) is the share of forward-looking price setters, and \( \pi_k^* \) measures the weight of backward-looking price setters according to 
\[ \pi_{k,t}^* = \rho_k^* \bar{\pi} + (1 - \rho_k^*)(\pi_{k,t-1}^*). \]

Monetary policy in RoW follows a Taylor-type rule:
\[ i_{RoW,t} - \bar{i} = \rho_{i_{RoW}} (i_{RoW,t-1} - \bar{i}) + (1 - \rho_{i_{RoW}})^2 \left[ \eta_{i_{RoW}} \pi_{RoW,t} - \pi_{CQA,RoW} \right] \]
\[ + \eta_{i_{RoW}}^Y \left[ \log \left( 0.25 \sum_{r=1}^{4} Y_{RoW,t-r} \right) - \log \left( 0.25 \sum_{r=1}^{4} Y_{pot,RoW,t-r} \right) \right] + \varepsilon_{RoW,t}. \]

Oil is considered to be an unstorable exogenous endowment of RoW and it is supplied inelastically:
\[ OIL_{RoW,t} = \sum_l \frac{\text{size}_l}{\text{size}_{RoW}} OIL_{l,RoW,t}, \]

where net oil exporting firms’ revenues in RoW are driven only by its price, \( P_{RoW,t}^{Oil} \), which is assumed to be denominated in RoW currency:
\[ P_{RoW,t}^{Oil} = \frac{P_{Y0}}{\bar{A}^{pot}}. \]

Total nominal exports for REA and RoW are defined as:
\[ P_{k,t}^X X_{k,t} = \sum_l P_{l,k,t}^X M_{l,k,t}, \]

with the bilateral export price being defined as the domestic price subject to a bilateral price shock:
\[ P_{l,k,t}^X = \exp(\varepsilon_{l,k,t}^X) P_{k,t}^Y. \]

We combine the FOCs with respect to international bonds of REA and RoW to obtain the uncovered interest parity (UIP) condition:
\[ E_t \left[ \frac{\varepsilon_{RoW,E_A,t+1}^{E_{RoW,E_A,t}}}{\varepsilon_{RoW,E_A,t}} \right] (1 + i_{RoW,t}) = (1 + i_{E_A,t}) + \varepsilon_{EA,t}^{bw} + \alpha_{EA}^{bw0} + \alpha_{EA}^{bw1} \frac{e_{RoW,E_A,t} D_{E_A,t}^W}{P_{E_A,t} Y_{E_A,t}}, \]

where \( \varepsilon_{EA,t}^{bw} \) captures a bond premium shock between EA and RoW (exchange rate shock), and \( \alpha_{EA}^{bw1} \) is a debt-dependent country risk premium on net foreign asset holdings to ensure long-run stability (Schmitt-Grohe and Uribe, 2003; Adolfson et al., 2008).
3. Model solution and econometric approach

The model is solved by linearising it around its deterministic steady-state. A subset of parameters is calibrated at quarterly frequency to match long-run properties, the remaining parameters are estimated using Bayesian methods.\(^{20}\)

As in Bayesian practice, the likelihood function (evaluated by implementing the Kalman Filter) and the prior distribution of the parameters are combined to calculate the posterior distribution. The posterior Kernel is then simulated numerically using the slice sampler algorithm as proposed by Planas et al. (2015).\(^{21}\)

The estimation uses quarterly and annual data for the period 1999q1 to 2017q2.\(^{22}\) Data for EMU countries and the Euro Area are taken from Eurostat (in particular, from the European System of National Accounts ESA2010), while the Rest of the World series are constructed using the IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.\(^{23}\) The estimated model uses 38 observed series and assumes 39 exogenous shocks.\(^{24}\) On the one hand, the large number of shocks is dictated by the fact that we use a large number of observables for estimation. On the other hand, many shocks are needed to capture key dynamic properties of macroeconomic and financial data (see Kollmann et al., 2015).

The steady-state of the model is calibrated at quarterly frequency to match long-run data properties. Along the deterministic steady-state all real variables (deflated by the GDP deflator) are assumed to grow at a rate of 1.3\% per year (the average growth rate of EA output over the sample period). Prices grow at an EA inflation rate of 2\% per year, adjusted by country-specific average productivities for the demand components (private and public consumption and investment). Population is detrended by the EA average rate of population growth (0.4\% per year). The steady-state ratios of main economic aggregates to GDP are calibrated to match historical ratios for each country over the sample period.

Table 1 provides an overview of the calibrated parameters. The discount factor at quarterly frequency is set to 0.9983 for all countries to match an annual real interest rate of

---

\(^{20}\)We use the Dynare software 4.5 to solve the linearised model and to perform the estimation (see Adjemian et al., 2011).

\(^{21}\)The slice sampler algorithm was introduced by Neal (2003). Planas et al. (2015) reconsider the slices along the major axis of the ellipse to better fit the distribution than any of Euclidean slices. The slice sampler has been shown to be more efficient and offer better mixing properties than the Metropolis-Hastings sampler (Calès et al., 2017).

\(^{22}\)The model is estimated at quarterly frequency, interpolating annual data for the series that are not available at higher frequency.

\(^{23}\)Appendix A provides a detailed description of data sources, definitions and transformations.

\(^{24}\)The list of observables can be found in Appendix B.1. Note that we additionally observe the historical replacement rate in Germany to capture the effect of the ‘Hartz reforms’. 

24
Given the monetary union setting, we calibrate the EA monetary policy parameters according to their estimated values based on a two-region configuration of the GM model (EA-RoW). Therefore, the interest rate persistence is set to 0.845, and the coefficients for the response to the EA inflation gap and the EA output gap are 1.625 and 0.07, respectively.

For simplicity, the share of oil in total output and tax rates are calibrated to EA averages. The labor tax rate is calibrated to close the steady state government budget.

Trade related parameters such as the degree of openness or preferences for imports are calibrated to match the average shares of import content in the demand components as computed by Bussière et al. (2013). The steady-state shares of Ricardian households are calibrated following the survey in Dolls et al. (2012). The debt targets are set to match the average values of the debt-to-GDP ratios over the sample.
Table 1: Selected calibrated structural parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>DE</th>
<th>FR</th>
<th>IT</th>
<th>ES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EA Monetary Policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal interest rate in SS $i_{EA}$</td>
<td>0.0049</td>
<td>0.0051</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI inflation in SS $\phi_{c,vat}^{EA}$</td>
<td>0.0051</td>
<td>0.0049</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate persistence $\rho_{i}^{EA}$</td>
<td>0.8452</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response to inflation $\eta_{i,\phi}^{EA}$</td>
<td>1.6246</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response to GDP $\eta_{i,y}^{EA}$</td>
<td>0.0700</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intertemporal discount factor $\beta$</td>
<td>0.9983</td>
<td>0.9983</td>
<td>0.9983</td>
<td>0.9983</td>
</tr>
<tr>
<td>Share of Ricardian households $\omega^M$</td>
<td>0.6100</td>
<td>0.6600</td>
<td>0.6700</td>
<td>0.6900</td>
</tr>
<tr>
<td>Preference for imports from RoW $s_{RoW}^{EA}$</td>
<td>0.5254</td>
<td>0.4122</td>
<td>0.4935</td>
<td>0.4728</td>
</tr>
<tr>
<td>Preference for imports from REA $s_{REA}^{EA}$</td>
<td>0.4746</td>
<td>0.5878</td>
<td>0.5065</td>
<td>0.5272</td>
</tr>
<tr>
<td>Import share in consumption $s_{M,C}^{EA}$</td>
<td>0.2612</td>
<td>0.2351</td>
<td>0.2050</td>
<td>0.2147</td>
</tr>
<tr>
<td>Import share in investment $s_{M,JG}^{EA}$</td>
<td>0.3608</td>
<td>0.2766</td>
<td>0.2782</td>
<td>0.2864</td>
</tr>
<tr>
<td>Import share in government exp $s_{M,G}^{EA}$</td>
<td>0.0974</td>
<td>0.0876</td>
<td>0.0682</td>
<td>0.0966</td>
</tr>
<tr>
<td>Import share in export $s_{M,X}^{EA}$</td>
<td>0.3100</td>
<td>0.2810</td>
<td>0.2743</td>
<td>0.3101</td>
</tr>
<tr>
<td>Weight of disutility of labour $\omega^N$</td>
<td>1.2771</td>
<td>3.3840</td>
<td>4.2627</td>
<td>4.5919</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cobb-Douglas labour share $\alpha$</td>
<td>0.6500</td>
<td>0.6500</td>
<td>0.6500</td>
<td>0.6500</td>
</tr>
<tr>
<td>Depreciation of private capital stock $\delta$</td>
<td>0.0143</td>
<td>0.0150</td>
<td>0.0136</td>
<td>0.0126</td>
</tr>
<tr>
<td>Depreciation of public capital stock $\delta^G$</td>
<td>0.0143</td>
<td>0.0150</td>
<td>0.0136</td>
<td>0.0126</td>
</tr>
<tr>
<td>Share of oil in total output $\omega^{Oil}$</td>
<td>0.0150</td>
<td>0.0150</td>
<td>0.0150</td>
<td>0.0150</td>
</tr>
<tr>
<td>Linear capacity utilisation adj. costs $\gamma^{u,1}$</td>
<td>0.0166</td>
<td>0.0161</td>
<td>0.0145</td>
<td>0.0148</td>
</tr>
<tr>
<td><strong>Fiscal policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption tax $\tau^C$</td>
<td>0.2000</td>
<td>0.2000</td>
<td>0.2000</td>
<td>0.2000</td>
</tr>
<tr>
<td>Corporate profit tax $\tau^k$</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.3000</td>
</tr>
<tr>
<td>Labour tax $\tau^N$</td>
<td>0.3942</td>
<td>0.5277</td>
<td>0.4488</td>
<td>0.3539</td>
</tr>
<tr>
<td>Deficit target $\text{def}T$</td>
<td>0.0210</td>
<td>0.0234</td>
<td>0.0366</td>
<td>0.0190</td>
</tr>
<tr>
<td>Debt target $\bar{BG}$</td>
<td>2.5521</td>
<td>2.8418</td>
<td>4.4551</td>
<td>2.3131</td>
</tr>
<tr>
<td><strong>Steady state ratio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private consumption share in SS $C/Y$</td>
<td>0.5662</td>
<td>0.5490</td>
<td>0.6018</td>
<td>0.5888</td>
</tr>
<tr>
<td>Private investment share in SS $I/Y$</td>
<td>0.1723</td>
<td>0.1898</td>
<td>0.1721</td>
<td>0.2046</td>
</tr>
<tr>
<td>Gov’t consumption share in SS $G/Y$</td>
<td>0.1877</td>
<td>0.2313</td>
<td>0.1923</td>
<td>0.1845</td>
</tr>
<tr>
<td>Gov’t investment share in SS $IG/Y$</td>
<td>0.0214</td>
<td>0.0387</td>
<td>0.0273</td>
<td>0.0361</td>
</tr>
<tr>
<td>Transfers share in SS $T/Y$</td>
<td>0.1664</td>
<td>0.1839</td>
<td>0.1777</td>
<td>0.1357</td>
</tr>
<tr>
<td><strong>Others</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size of the country (% of world) $\text{size}$</td>
<td>4.6563</td>
<td>3.5673</td>
<td>2.8992</td>
<td>1.8633</td>
</tr>
<tr>
<td>Trend of total factor productivity $\gamma^{AY0}$</td>
<td>0.0029</td>
<td>0.0017</td>
<td>0.0027</td>
<td>0.0010</td>
</tr>
</tbody>
</table>

4. Estimation results

In this section, we discuss the key aspects of fitting observed data and the associated modelling and estimation approaches. We present the posterior estimates of key model parameters, the ability of the model to fit the data and impulse response functions. We also evaluate the drivers of business cycle fluctuations in each country by analysing the historical decomposition of real GDP growth and the trade balance-to-GDP ratio.

As usual in empirical work, there are critical areas that may require modelling extensions
in the attempt to improve the model fit. The general approach is that modifications should
nest the core model specification in such a way that they are only used when the overall
fit improves significantly. Against this background, we have identified two features of the
model for which we apply additional specifications. First, there is a tendency to over-predict
inflation. To address this issue, we have used a modified formulation of the hybrid Phillips
curve as discussed in equation (14). Second, we observe very large import demand shocks,
which are often related to trends in the import share. One possible way to reduce the size
of these shocks is to interpret part of them as deriving from a gain/loss in competitiveness
associated to diverging trends in productivity, which has been implemented in equations (15)
and (16). These modifications have been selected for Italy and Spain.

4.1. Posterior estimates

The posterior estimates (with HPD intervals) of key model parameters are reported in
Table 2. The estimated habit persistence is relatively high in Italy, which implies a slow
adjustment of consumption to changes in income. Risk aversion coefficients are similar for
all countries and range between 1.38 - 1.51. The inverse of the labour supply elasticity
is relatively high in Germany (2.98) compared to the other countries. The import price
elasticity coefficient in Italy is lower (1.13) than in the other countries (1.27 - 1.38). Since
we use the modified Phillips curve (equation 14) for Italy and Spain, the estimated share of
forward-looking price setters is significantly lower in Italy (0.36) and Spain (0.74) compared
to Germany and France. Price and nominal wage adjustment costs are higher in France (36
and 4.07, respectively) and rather low in Italy and Spain. Real wage rigidity is high for
all countries. Employment adjustment costs vary significantly among the countries. The
labour market rigidity is linked to the two adjustment cost parameters in labour demand
and labour hoarding. The former appears to be relatively rigid in France (108) compared to
Spain (6.4) and Italy (38). The latter features similar levels in Germany (1.58), Italy (1.62)
and Spain (1.57), with a somewhat lower level in France (1.24). Also the estimates related
to the share of forward looking price setters vary extensively across the countries. This
parameter pinpoints how much firms take into account future information in their decisions,
namely how firms weigh the forward looking components (relative to the backward looking
components) in setting prices. It may capture several specific features of the economy,
including different kinds of goods market rigidities and indexation schemes. The different
estimates across countries are thus likely to reflect differences in the economic setup and
institutional rules in which firms operate. Capacity utilisation adjustment costs are similar
across the four countries, whereas Italy (19) and Spain (22) face lower investment adjustment
costs compared to Germany (31) and France (34).
Table 2: Prior and posterior distribution of key estimated model parameters.

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr</td>
<td>Mean</td>
<td>St.Dev</td>
<td>DE</td>
<td>FR</td>
<td>IT</td>
</tr>
<tr>
<td>Consumption habit persistence</td>
<td>h</td>
<td>B</td>
<td>0.5</td>
<td>0.73</td>
<td>0.78</td>
<td>0.81</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>θ</td>
<td>G</td>
<td>1.5</td>
<td>1.41</td>
<td>1.38</td>
<td>1.42</td>
</tr>
<tr>
<td>Inverse Frisch elasticity of labor supply</td>
<td>θ^N</td>
<td>G</td>
<td>2.5</td>
<td>2.98</td>
<td>2.04</td>
<td>2.07</td>
</tr>
<tr>
<td>Import price elasticity</td>
<td>σ^z</td>
<td>G</td>
<td>2</td>
<td>1.30</td>
<td>1.38</td>
<td>1.13</td>
</tr>
<tr>
<td>Oil price elasticity</td>
<td>σ^o</td>
<td>B</td>
<td>0.5</td>
<td>0.10</td>
<td>0.10</td>
<td>0.26</td>
</tr>
<tr>
<td>Share of forward-looking price setters</td>
<td>sfp</td>
<td>B</td>
<td>0.5</td>
<td>0.98</td>
<td>0.99</td>
<td>0.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nominal and real frictions</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Price adjustment cost</td>
<td>γ^P</td>
<td>G</td>
<td>60</td>
<td>20</td>
<td>36</td>
<td>11</td>
</tr>
<tr>
<td>Nominal wage adjustment cost</td>
<td>γ^w</td>
<td>G</td>
<td>5</td>
<td>3.43</td>
<td>4.07</td>
<td>2.69</td>
</tr>
<tr>
<td>Real wage rigidity</td>
<td>γ^wr</td>
<td>B</td>
<td>0.5</td>
<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>Employment adjustment cost</td>
<td>γ^N</td>
<td>G</td>
<td>60</td>
<td>54</td>
<td>108</td>
<td>38</td>
</tr>
<tr>
<td>Labour hoarding adjustment cost</td>
<td>γ^FN</td>
<td>G</td>
<td>2</td>
<td>1.58</td>
<td>1.24</td>
<td>1.62</td>
</tr>
<tr>
<td>Capacity utilisation adjustment cost</td>
<td>γ^CU</td>
<td>G</td>
<td>0.003</td>
<td>0.004</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>γ^I</td>
<td>G</td>
<td>60</td>
<td>31</td>
<td>34</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiscal policy</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lump sum taxes persistence</td>
<td>ρ^T</td>
<td>B</td>
<td>0.5</td>
<td>0.86</td>
<td>0.96</td>
<td>0.88</td>
</tr>
<tr>
<td>Lump sum taxes response to deficit</td>
<td>ρ^DEF</td>
<td>B</td>
<td>0.03</td>
<td>0.022</td>
<td>0.023</td>
<td>0.032</td>
</tr>
<tr>
<td>Lump sum taxes response to debt</td>
<td>ρ^B</td>
<td>B</td>
<td>0.02</td>
<td>0.0032</td>
<td>0.0022</td>
<td>0.0037</td>
</tr>
</tbody>
</table>

Notes: Cols. (1)-(2) list model parameters. Cols. (3)-(4) indicate the prior distribution function (B: Beta distribution; G: Gamma distribution). Identical priors are assumed across countries. Cols. (5)-(8) show the mode and the HPD intervals of the posterior distributions of DE, FR, IT, and ES parameters, respectively.

It is worth mentioning that high employment adjustment costs translate into less hiring and firing (in France this value is relatively high), due e.g. to stricter employment protection, whereas higher labour hoarding adjustment costs imply less flexible use of employed labour. A combination of high employment adjustment costs and relatively low labour hoarding adjustment costs (as in France) implies elevated persistence in employment (less hiring and/or firing) together with a more flexible use of officially employed labour.

The fiscal feedback rule on lump-sum taxes exhibits relatively high persistence for France (0.96) and Spain (0.95), implying a more drawn-out response to debt and deficit levels. The estimated responses of taxes to deficit and debt targets are in the same order of magnitude across countries. The posterior estimates of key model innovations can be found in Ap-
4.2. Model fit

In order to evaluate the capability of the model to fit the data, Table 3 compares sample and model-implied moments for a subset of key statistics. In particular, we focus on volatilities and persistence of real GDP, consumption, investment, employment, the trade balance-to-GDP ratio and the GDP deflator as well as the cross-correlation of GDP with its main components. We use first differences for the trade balance-to-GDP ratio and quarter-on-quarter growth rates for all other variables. The estimated models tend to overestimate the volatility of real variables. However, the relative magnitudes seem to be preserved, e.g. \( \text{std}(GC)/\text{std}(GY) \). Of particular note is the high volatility of investment, which is in line with the data patterns. Most of the correlations between GDP growth and its components are fairly well captured. More precisely, all country models replicate well the correlation of consumption, investment and employment with output. In our model the trade balance is positively correlated with output, but matches the data pattern only for Germany. Moreover, our estimated models are able to replicate both negative (for Germany and Italy) and positive (for France and Spain) correlations between GDP inflation and GDP growth. First-order autocorrelations are particularly well seized in Spain, whereas the other countries show a more differentiated picture.

The last two columns in Table 3 report the \( R^2 \) of the 1-year and 2-year ahead forecast. We define the \( R^2 \) as follows:

\[
R^2 = 1 - \frac{e_j'y_j}{y_j'y_j},
\]

where \( y_j = [y_{1,j}, \ldots, y_{T,j}]' \) is the country-specific \( j \)-th time series in deviation from the model-implied steady-state and \( e_j = [e_{1,j}, \ldots, e_{T,j}]' \) is the associated \( k \)-step ahead forecast error obtained from the Kalman filter recursions. The definition implies that \( R^2 \) has an upper bound located at 1 and is unbounded from below. This means that in the perfect case where the model generates no forecast error, the \( R^2 \) is one and it declines monotonically as the forecast error increases. Since the volatility of the forecast error can be larger than the volatility of the observed time series, the \( R^2 \) can be negative. In that case, a constant forecast centered on the sample mean would do a better job since its \( R^2 \) coincides with zero.

The graphical representation of the \( k \)-step ahead forecast, i.e. the 1-year and 2-year ahead forecast at each point in time, can be found in Figures B.1 - B.4 in Appendix B.3.

The 1-year ahead \( R^2 \) is mostly positive for all analysed countries, indicating that the model forecast errors are not very large. Even the 2-year ahead forecast provides a relatively good fit, especially for IT and ES. A different picture arises for Germany and France, for
Table 3: Theoretical moments and model fit.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Std(%)</th>
<th>AR(1)</th>
<th>Corr (x, GY)</th>
<th>r2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP growth (GY)</td>
<td>0.84</td>
<td>1.25</td>
<td>0.41</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption growth (GC)</td>
<td>0.58</td>
<td>1.21</td>
<td>-0.24</td>
<td>0.53</td>
</tr>
<tr>
<td>std(GC)/std(GY)</td>
<td>0.69</td>
<td>0.97</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Investment growth</td>
<td>4.22</td>
<td>4.85</td>
<td>-0.02</td>
<td>0.20</td>
</tr>
<tr>
<td>GDP deflator</td>
<td>0.35</td>
<td>0.69</td>
<td>0.44</td>
<td>0.70</td>
</tr>
<tr>
<td>Hours growth</td>
<td>0.54</td>
<td>0.64</td>
<td>0.28</td>
<td>0.20</td>
</tr>
<tr>
<td>Δ Trade balance to GDP</td>
<td>0.67</td>
<td>0.90</td>
<td>-0.07</td>
<td>0.01</td>
</tr>
</tbody>
</table>

| France                        |        |       |              |       |
| GDP growth (GY)               | 0.48   | 0.94  | 0.59         | 0.08  |
| Consumption growth (GC)       | 0.45   | 0.82  | 0.18         | 0.62  |
| std(GC)/std(GY)               | 0.95   | 0.87  | -            | -     |
| Investment growth             | 2.77   | 3.17  | 0.16         | 0.27  |
| GDP deflator                  | 0.29   | 0.49  | 0.70         | 0.83  |
| Hours growth                  | 0.39   | 0.52  | 0.61         | 0.15  |
| Δ Trade balance to GDP        | 0.39   | 0.66  | -0.20        | 0.01  |

| Italy                         |        |       |              |       |
| GDP growth (GY)               | 0.74   | 1.30  | 0.68         | 0.06  |
| Consumption growth (GC)       | 0.58   | 0.85  | 0.67         | 0.57  |
| std(GC)/std(GY)               | 0.78   | 0.66  | -            | -     |
| Investment growth             | 4.12   | 5.20  | 0.05         | 0.25  |
| GDP deflator                  | 0.54   | 0.76  | -0.24        | 0.33  |
| Hours growth                  | 0.57   | 0.71  | 0.31         | 0.20  |
| Δ Trade balance to GDP        | 0.41   | 0.81  | 0.20         | -0.01 |

| Spain                         |        |       |              |       |
| GDP growth (GY)               | 0.70   | 1.43  | 0.91         | 0.14  |
| Consumption growth (GC)       | 0.88   | 1.43  | 0.62         | 0.48  |
| std(GC)/std(GY)               | 1.25   | 1.00  | -            | -     |
| Investment growth             | 2.91   | 3.85  | 0.33         | 0.42  |
| GDP deflator                  | 0.47   | 0.60  | 0.75         | 0.69  |
| Hours growth                  | 1.33   | 1.24  | 0.27         | 0.31  |
| Δ Trade balance to GDP        | 0.63   | 0.95  | 0.20         | -0.02 |

* Note: The r2 is reported for the absolute nominal trade balance.

which the estimated model delivers a poor (in-sample) forecast accuracy particularly for consumption.

An additional way to assess the fit of the model is to compare the estimates of endogenous variables with their observable counterparts. For example, capacity utilisation is an endogenous variable defined in the GM3-EMU model and it is treated as a latent variable, endogenously determined by the firm’s decision rules. As a consequence, the Kalman filter allows us to retrieve a model-consistent estimate of capacity utilisation over the business

---

25The difficulty in fitting of consumption is common in the empirical DSGE model literature. The consumption decision is essentially driven by the Euler equation, which may not properly capture the empirical behaviour of households. For example, the model may benefit from introducing some form of credit/wealth contraints. In our case, the consumption fit is particularly difficult for Germany and France, while for Spain and Italy results are more satisfactory. Similar issues in fitting consumption in the case of Germany are reported in Kollmann et al. (2015).
cycles. While capacity utilisation is not directly measurable in national account statistics, we use a ‘model-free’ or reduced-form proxy that has been constructed to compare the model-based and model-free estimates of capacity utilisation.\textsuperscript{26} Figure 1 plots the time series of capacity utilisation implied by the reduced form proxy and the GM implied one computed via the Kalman filter.

As the differences are minimal and the two measures coincide, it gives additional credit to the plausibility of the estimated structural models to replicate key features of EA Member State business cycles. It is useful to underline that this match has been improved for France, Italy and Spain by the introduction of labour hoarding.

4.3. Dynamic transmission of shocks

Figures 2 - 7 show the dynamic responses of the main variables to domestic supply (TFP), domestic demand (private saving and government spending), EA monetary policy and foreign demand shocks as well as an exchange rate shock to the euro currency. All figures report the response of a temporary shock of 1\% except for TFP, where it is a temporary shock to the growth rate (i.e. permanent to the level). In all cases we report expansionary shocks. Each panel shows, for the four countries, the dynamic response of the following endogenous

\textsuperscript{26}For details on the construction of the capacity utilisation series, see Havik et al. (2014).
variables: real GDP, private consumption, private investment, total hours worked, real wages, real interest rate, GDP inflation, real effective exchange rate, and the trade balance-to-GDP ratio. Real variables are presented as percentage deviation from their steady-state. GDP inflation and the trade balance-to-GDP ratio are expressed in percentage-point deviations from steady-state. Real interest rates are shown in annualised percentage-point deviations from their steady-state.

Figure 2: Permanent positive TFP shock.

Permanent positive TFP shock.

A one-time increase in the level of TFP is shown in (Figure 2). Following a positive technology innovation, as expected, output, consumption and investment rise permanently. The higher productivity brings about a fall of firm marginal costs. Since domestic demand adjusts very sluggishly, to accomodate the higher productivity, less capital is needed for Germany, France and Spain in the short term. This is why investment is shown to reduce on impact for these countries. Besides, the increase in the real interest rate and higher investment adjustment costs for these countries further dampen the immediate response of investment. Employment temporarily decreases in Spain, but reacts positively in Germany, France and Italy, where labour demand adjustment costs are much more elevated. The exchange rate depreciates and the trade balance improves temporarily due to substitution of imports by domestic demand. However, on impact, the relatively slow adjustment in prices
and increased demand induce a negative trade balance in Italy.

**Negative private saving shock (positive shock to consumption demand).**

A negative shock to the saving rate, which is modeled as a persistent increase in the subjective rate of time preference of households, boosts domestic consumption with a concomitant increase in domestic output and prices (Figure 3). The shock triggers a rise in the policy rate and an increase in the real interest rate in the medium term, leading to a decline in investment. The trade balance deteriorates on impact due to a combination of higher import (domestic demand expansion) and lower export demand (real exchange rate appreciation). Figure 3 also shows that due to low investment adjustment costs in Italy the positive shock to domestic consumption has particularly negative consequences on investment.

It is worth noting that that the estimated persistence of the shock is high, inducing a very prolonged propagation mechanism for this shock.

Additionally, lower price and labour market frictions (labour hoarding and wage stickiness) lead to a more persistent decrease of real wages in Italy compared to the other countries.\(^\text{27}\)

**Government expenditure shock.**

\(^{27}\)The dynamic response of real wages in Italy is also influenced by the modified formulation of the hybrid Phillips curve (see equation 14), which we use for better fitting GDP inflation in Italy.
An increase in government expenditure raises domestic output and crowds out consumption and investment in the medium term. Upward pressure on prices leads to a real exchange rate appreciation and a deterioration of the trade balance. The fiscal multiplier is close to one on impact and similar in size across the four countries. While consumption in Germany is negative on impact, we can see some crowding-in of consumption in the other countries, particularly in Spain. Furthermore, the lower estimated labour market frictions in Spain lead to a more pronounced positive effect on employment and real wages.

**EA monetary policy shock.**

An expansionary monetary policy (lowering the annualised interest rate by 1pp) implies an increase in aggregate demand components (Figure 5). Investment raises substantially due to a decline in real interest rates. Higher domestic demand induces firms to increase labour demand which results in higher employment. The real exchange rate depreciates due to a strong initial depreciation of the euro. The gain in competitiveness improves the trade balance-to-GDP ratio.

**Negative shock to the RoW savings rate (positive shock to foreign demand).**

Figure 6 presents dynamic responses to a positive foreign demand shock, namely a negative shock to RoW savings. Analogously to domestic saving shocks, the negative RoW savings shock is modeled by a decline in the subjective discount rate. The shock increases RoW demand and activity in combination with a real effective depreciation in the four coun-
Figure 5: EA monetary policy shock.

Figure 6: Positive foreign demand shock.
tries, leading to trade balance improvements. The rise in policy and real interest rates in response to higher output and inflation in the EA dampens consumption and investment demand. Due to a lower estimated share of forward-looking price setters in Italy, GDP inflation increases less on impact, which boosts domestic consumption and activity compared to the other countries.

Positive shock to preferences for international bond (Euro depreciation).

Figure 7 presents dynamic responses to a preference shock for international bonds, which mimics a euro depreciation. The gain in competitiveness increases the trade balance via a rise in exports and a decline in import demand. Consequently, domestic GDP and employment increase. The real interest rate is negative on impact but increases due to the monetary response to inflation, which is primarily caused by a deterioration of the terms of trade. Lower domestic import demand (higher foreign prices) decreases consumption on impact. Investment decreases on impact due to capital outflows (preference shock towards foreign assets). Subsequently, both consumption and investment return to equilibrium following a path determined by the intertemporal substitution implied by the real interest rate.
4.4. Historical decomposition

This subsection highlights the estimated contribution of different shocks to historical time series in the period 2008-2016\(^{28}\). Figures 8-15 show the historical decomposition of the four countries for two macroeconomic variables, namely the year-on-year growth rate of real GDP and the trade balance-to-GDP ratio. In each subplot, the continuous black line shows historical time series, from which sample averages have been subtracted. The vertical black bars show the contribution of different groups of exogenous shocks (see below) to the historical data, while stacked light bars show the contribution of the remaining shocks. Bars above the horizontal axis (steady-state) represent positive shock contributions, while bars below the horizontal axis show negative shock contributions. The sum of all shock contributions equals the historical data.

We plot the contributions of the following (groups of) exogenous variables originating in the respective domestic country: (1) shocks to the Total Factor Productivity (TFP); (2) labour and goods market adjustment as captured by wage and price mark-up shocks; (3) oil price shock; (4) domestic demand shocks, i.e., changes in consumption and investment demand that are not explained by fundamentals such as household income, interest rates and return expectations on physical capital and financial assets; (5) international bond premium shock; (6) shocks to world demand and international trade, which include foreign demand and supply shocks, and deviations of trade volumes and prices from the estimated export and import demand and pricing equations; (7) monetary policy shocks that capture deviations of short-term interest rates from the estimated policy rule. The remaining shocks and the effect of initial conditions are summarised as ‘others’. The groups (1) to (3) act mainly on the economy’s supply side, whereas (4) to (7) predominantly affect demand for goods and services in the short and medium term. The model-based decompositions presented below identify the importance of each of these groups of shocks.

**Shock decomposition of real GDP growth.**

A large share of real GDP growth fluctuations in all four countries during 2008-2016 are attributed to domestic demand shocks (in particular those driving investment demand and ‘flight to safety’), whereas the role of supply shocks is much smaller.

\(^{28}\)Since the discussion in this Section focuses on the post-crisis adjustments, we show a restricted time frame, for clarity of visualization. Full sample shock decompositions are reported in Appendix B.5.
Figure 8: Germany

Figure 9: France
The growth slowdown during the financial crisis (2008-2009) is largely associated with a fall in domestic demand basically driven, in particular, by an increase in the investment risk premium. Labour and price mark-up shocks, capturing sluggish price and wage adjustments in the context of contracting economic activity, also exerted a negative contribution on growth, especially in Germany. For Germany, France and Italy this was accompanied by large
adverse trade shocks, while in Spain consumption (positive saving shock or ‘deleveraging’) plays a larger role within the ‘domestic demand’ group. In Italy the bond premium shocks (capital outflow) exacerbated the negative effects on GDP growth more than in the other countries. In contrast, expansionary monetary policy had a noticeable stabilising effect on domestic GDP growth in all four countries during the 2008-09 financial crisis. It is worth noticing that the monetary policy shock is related to a standard monetary rule, while non-conventional measures in place from 2008-09 to recent years are not captured by this shock. The effect of non-standard measures is captured by savings, investment risk premia and bond premia shocks.

In 2010, the crisis was followed by a relatively rapid partial recovery due to domestic (fall in investment risk premia) and foreign demand shocks. The main drivers during this period were relatively homogeneous across the four countries. Specifically, in Germany, France, Italy a decrease in investment risk premia, a positive consumption shock and a positive contribution from trade were the main factors accounting for the recovery, while in Spain persistent negative demand shocks are observed. Also the positive contribution from the sovereign bond market (bond premium shock) significantly contributed to the recovery of all countries.

The post crisis slump in France, Italy and Spain was mainly driven by domestic demand: namely an increase in investment risk premia and negative consumption shocks (positive saving shock). The fiscal austerity due to the sovereign debt and banking crisis made the strongest negative contribution in Spain.

The main drivers of above-trend GDP growth in Germany during the most recent years have been the fall in oil prices, positive trade shocks as well as the depreciation of the euro (explained in the model by an increase in the risk premium on euro-denominated bonds). The recovery in Spain and Italy in recent years has been driven by negative mark-up shocks, a recovering domestic demand (consumption and the weaning off related to the flight to safety shock, i.e. a reduction in the intra-euro risk premium compared to the crisis years).29

Our estimates suggest that EA monetary policy shocks had a relatively moderate effect on GDP growth.30 Since we do not impose a zero lower bound on the nominal interest rate

---

29Our results for Italy are in line with the historical decomposition by Acocella et al. (2018), in which also preference shocks played a positive role to favor the recovery. For Spain, a similar key role of demand shocks is found in in’t Veld et al. (2014, 2015) as well as in Aspachs-Bracons and Rabanal (2010), where housing demand and technology shocks are found to be the main drivers of the Spanish pre-crisis housing boom.

30The moderate impact of monetary policy shocks on real GDP growth is in line with the study by Rafiq and Mallick (2008), which analyses the effects of monetary policy shocks on output in Germany, France and Italy. They conclude that monetary policy innovations play only a modest role in explaining fluctuations in output for these countries, thus making the problem of a one-size-fits-all policy in a currency union less worrying.

40
as a constraint on monetary policy, the negative contributions to GDP growth during 2013 and 2015 originate from a lower model-implied policy rate compared to the observed policy rate which is at the zero bound. Hence, the gap is closed by positive (tightening) monetary policy shocks. It has to be stressed that ‘monetary policy’ only refers to the Taylor rule shock and excludes non-conventional measures that are rather be part of receding investment risk premia, declining savings rates, and exchange rate depreciation shocks in the logic of the model.

It is interesting to notice that the GM model attributes the subdued levels of the Italian output growth over the full sample to a sequence of persistent negative TFP shocks which act as a persistent drag to the economy. TFP shocks are a reduced form representation for whatever is left out from combining capital and labour inputs and their intensity in utilisation. Therefore, one can think of total factor productivity as bundling together intangible assets (i.e. unobservable or difficult to measure quantities) such as technological innovation and/or input misallocations. In light of this, the decomposition of the Italian output growth offers a narrative which is coherent with other studies that, by exploiting the cross-sectional variation, explain the Italian low productivity in terms of limited ICT investment and penetration (see Hassan and Ottaviano, 2013; Pellegrino and Zingales, 2017).

Shock decomposition of the trade balance-to-GDP ratio.

The steady-states of the trade balance-to-GDP ratios for the four countries are set to the mean of the observed country-specific time series. Therefore, the trade balance steady-state in Germany is around 5% of GDP, in France around -1%, in Italy close to 0% and in Spain around -2% of GDP.
During the global 2008-09 crisis, Germany’s trade balance declined because of the simultaneous contraction of RoW real activity and global trade, although Germany’s trade balance surplus has been accumulated since the beginning of the 2000s. Beside the traditionally high saving rate in Germany (captured within the ‘domestic demand’ group), the increase in global trade and RoW demand, the depreciation of the euro (explained in the
model by an increase in the risk premium on euro-denominated bonds) and the decline in oil prices have led to an even more pronounced increase of Germany’s trade balance surplus in recent years.\footnote{A closer analysis of the trade balance evolution for Germany can be found in Kollmann et al. (2015).}

While France has shown a gradual and persistent trade balance deterioration since the beginning of the sample, Italy and Spain have experienced a rapid trade balance reversal since 2011-2012. France has suffered more than the other three EMU countries considered from negative trade shocks after the financial crisis. Domestic demand, notably an increase in the risk premium on investment that in the model lowers import demand, the euro depreciation as well as an increased demand from RoW have contributed to a stabilisation of the trade balance in France. Among domestic demand shocks, a positive contribution of the flight to safety shock suggests relatively persistent capital outflows to the rest of the euro area.

Italy and Spain show a similar pattern in terms of main drivers during the most recent years. Both countries have suffered from negative trade shocks and low foreign demand. The after-crisis trade balance improvements were mainly driven by domestic demand (an increase in savings and investment risk premia) and the depreciation of the euro.

In the post-crisis period, the ‘flight to safety’ contribution to the trade balance-to-GDP ratio has been considerably positive across the countries, especially in Italy and Spain. The shock captures a reduction of the intra-euro risk premium and suggests capital outflows from the domestic country, improving the trade balance.

Summarising the key patterns of the historical decomposition across the countries, the model suggests that:

(1) The GDP growth slowdown during the 2008-2009 financial crisis was largely due to an increase in investment risk premia and negative shocks to foreign demand and trade. The positive contributions of stabilising fiscal and monetary policy during the financial crisis is visible across countries.

(2) The partial recovery in the aftermath of the crisis was due to a fall in investment risk premia and a recovery of world trade and demand, particularly for Germany.

(3) During most recent years, the main drivers of GDP growth have been a normalisation of consumption after a period of post-crisis deleveraging, the fall in oil prices, positive trade development and the euro depreciation. Fiscal shocks have contributed considerably to GDP growth in Spain.

(4) The trade balance development in Germany differs substantially from those in the other countries. Overall, the improvement of the trade balance-to-GDP ratios after the financial crisis are mainly driven by increasing private savings (lower consumption
demand), an increase in investment risk premia, the depreciation of the euro, and a
recovery of world demand and trade. Weak foreign demand from REA has weighted
negatively on the trade balance of the countries since the Great Recession.

5. Applications

The GM model has been developed for three main purposes, namely (1) the structural
interpretation of business cycle dynamics, (2) contributions to the European Commission’s
economic forecast, and (3) scenario analysis and policy counterfactuals.

5.1. Structural interpretation of business cycle dynamics

The GM model provides a structural interpretation of macroeconomic developments,
i.e. of the historical data, by decomposing the dynamics of observed variables into their
driving structural shocks. Examples for individual EA Member States include the shock
decompositions for real GDP growth and the trade balance in Germany, France, Italy, and
Spain presented in subsection 4.4 above. Kollmann et al. (2016) have used the first estimated
three-region version of the GM model (Euro Area, US, and Rest of the World) to analyse
the drivers of the post-crisis slump in the Euro Area and the US on the basis of shock
decompositions for real GDP growth, inflation dynamics, and net trade, while Giovannini
et al. (2018) have adopted the same approach and country configuration more recently to
investigate post-crisis EA and US external rebalancing. Estimated versions of the European
Commission’s QUEST model have been used in the same spirit to investigate the (recent)
economic history of EA Member States, including the boom-bust cycle in Spain (in’t Veld
et al., 2014, 2015) and the large and persistent current account surplus of the German
economy (Kollmann et al., 2015). In addition, the estimated model projects paths for a
number of variables that are not directly observed (‘latent’), but can be of interest to the
policy maker, such as the output gap, or equilibrium interest and exchange rates. The shock
decompositions for GDP, inflation, consumption, investment, the trade balance, and other
variables provide an interpretation of historical economic developments in the EA and EA
Member States. Besides the narrative value, forward-looking policy makers may benefit from
this type of analysis when the latter points to ongoing macroeconomic imbalances or informs
about the sustainability of internal and external adjustment.

5.2. Contributions to the European Commission’s economic forecast

Since Autumn 2015, the estimated GM model in its two-region version with the EA and
the RoW has been used biannually in the context of the European Commission’s economic
forecast. More precisely, the estimated GM model is used to decompose the European
Commission’s institutional forecast for EA real GDP growth. As a first step, we append the forecasts over the forecasting horizon to the historical database. We do that for all the variables for which the European Commission produces a forecast. EA forecasts are obtained by aggregating the forecasts for individual EA Member States. The set of variables typically includes real GDP, inflation, demand components, employment, fiscal variables, and the external technical assumptions on interest rates, the euro exchange rate, global demand, and commodity prices. The model is then ‘inverted’ (Issing, 2004) to derive the out-of-sample innovations needed to align the model predictions based on estimated (on historical data) equations and historical shocks with the European Commission’s forecast. These model-consistent innovations are then analysed to provide an economic interpretation of the forecast in the context of the GM model and to discuss the forecast’s internal consistency, e.g. with respect to the dynamics of output versus inflation, the forecasts for demand and external trade, or the projected paths of fiscal variables and GDP growth.

The advantage of using a detailed structural model, such as the GM model, to decompose historical time series as well as forecasts derives from the model’s ability to exploit the richness of the information set. In particular, the detailed model allows an identification of the driving forces on the basis of restrictions that they impose across variables and over time. The size and sign of the various domestic and foreign demand and supply shocks is determined by the ability of these shocks to fit not only GDP, but also the other observed variables of the model, such as consumption, investment, international trade, and employment, and the co-movement between them.

The decomposition of the EA real GDP growth forecast with the GM model is included in the forecast publication, e.g. in European Commission (2017). The multitude of shocks is grouped into several categories to facilitate the presentation. More precisely, the grouping may distinguish between domestic productivity shocks, goods and labour market adjustment as captured by price and wage mark-up shocks, commodity price shocks, monetary and exchange rate shocks (deviations of short-term interest rates from the estimated monetary policy rule and shocks moving the exchange rate independently of the monetary policy stance), domestic demand shocks, and shocks to foreign demand, supply, and international trade.

Corresponding forecast decompositions are also available for the four largest EA Member States for which country blocks have been estimated to date as presented in this paper. Decompositions at country level provide a consistency check for country desks, in charge of

---

32Based on the forecast data and the implied model-consistent shocks, the model also projects paths for those (over the forecast horizon ‘unobserved’ or ‘latent’) model variables which have no forecast entry.
5.3. Scenario analysis and policy counterfactuals

Policy analysis produces a constant need for counterfactual scenarios that economists in policy institutions are confronted with. The counterfactuals address the question of how the economy would have evolved if shocks, structural relationships, or policy reactions had differed. Looking forward, the scenarios assess the impact of alternative policies, structural changes, or possible external disturbances on economic outcomes. Many of these questions are not only of qualitative, but of quantitative nature, i.e. the scenarios need to provide a quantitative assessment of the effects.

Estimated DSGE models provide a suitable platform for counterfactual or scenario analysis. Firstly, the basic structure of DSGE models derives from the objectives of households, firms and governments, their respective budget, resource, and technology constraints, and related market clearing conditions, which are all taken as given. However, the private sector’s (firms and households) decisions do not take the government policy actions as invariant, and do react to different policy regimes. As a consequence, the comparison of alternative policy scenarios is conceptually and internally consistent. To be useful for policy analysis though, DSGE models should be data-based. If sufficiently detailed for the problem at hand, DSGE models provide enough structure to identify the parameters and/or shocks that need to be modified to obtain a particular scenario or counterfactual. By fitting the data patterns reasonably well, DSGE models can therefore provide plausible quantitative results (Coenen et al., 2017).

The GM model’s country blocks are used for scenario or counterfactual analysis. In the context of the forecast (see the subsection 5.2 above), the model can be used to assess the macroeconomic impact of particular external assumptions by means of conditional forecasts (see also Christoffel et al., 2008; Issing, 2004). Similarly, the impact of particular changes to fiscal policy on economic activity (fiscal multipliers) can be assessed on the basis of simulations; this is exemplified by the IRFs displayed for a set of standard shocks in the subsection 4.3 above.

6. Conclusion

This paper presents the European Commission’s Global Multi-country (GM) model, an estimated structural macroeconomic model. The GM model is used for the structural interpretation of macroeconomic dynamics and institutional forecasts, and for (counterfactual) scenario analysis.
The GM model is designed to be flexible for alternative country configurations. This paper presents a version of the GM model suitable for the analysis of EMU Member States (GM3-EMU), which has been estimated for the four largest economies (Germany, France, Italy, and Spain). Across countries, model specification, data sample, observed variables, prior parameter distributions, and shocks are ex-ante identical. Hence, cross-country heterogeneity is purely data-driven and is expressed ex-post by differences in the estimated parameter values and shock processes. This enables us to analyse and compare business cycle properties and heterogeneity in the transmission of fundamental shocks and policy interventions.

The paper describes the detailed theoretical model specification and the estimation results. We analyse the fit of the model for the four selected countries, highlight the transmission mechanisms of shocks and explain their contributions to the observed behaviour over time of real GDP growth and the trade balance-to-GDP ratio.

Further steps in the development of the GM model have been made to aggregate the estimated EMU countries into a GM6 model (Germany, France, Italy, Spain, Rest of the Euro Area, and Rest of the World), which shall be particularly relevant for analysing and assessing cross-country spillovers.

Future work on the GM model project will aim at improving the empirical fit in these dimensions and may include avenues such as an improved characterisation of trade shocks and of the distinction between short-term versus long-term trade elasticities, the inclusion of good-specific import shares, the possibility of pricing to market, an improved modelling of labour supply and, possibly, an inclusion of (downward) nominal wage rigidity, and a more structural interpretation of unconventional monetary policy.
References


53


Appendix A. Data source and transformations

We use quarterly and annual data for the period 1999q1 to 2017q2. Data for EMU countries and the Euro Area aggregate (EA19) are taken from Eurostat (in particular, from the European System of National Account ESA2010). Bilateral trade flows are based on trade shares from the GTAP trade matrices for trade in goods and services. The Rest of the World (RoW) data are annual data and are constructed using IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

Series for GDP and prices in the RoW start in 1999 and are constructed on the basis of data for the following 58 countries: Albania, Algeria, Argentina, Armenia, Australia, Azerbaijan, Belarus, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Egypt, Georgia, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Israel, Japan, Jordan, Korea, Lebanon, Libya, FYR Macedonia, Malaysia, Mexico, Moldova, Montenegro, Morocco, New Zealand, Nigeria, Norway, Philippines, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Syria, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, and Venezuela.

When not available, quarterly-frequency data are obtained by interpolating annual data. We seasonally adjust the following time series using the TRAMO-SEATS package developed by Gómez and Maravall (1996): nominal public investments (for EA19, Germany, France, Italy, and Spain), nominal social benefits other than transfers in kind (for EA19, Germany, France, Italy, and Spain), government interest expenditure (for EA19, Germany, France, Italy, and Spain), compensation of employees (for Germany, France, Italy, and Spain), general government net lending (for Italy and Spain), employees (for EA19, Germany, France, Italy, and Spain).

Table B.1 lists the observed time series. GDP deflators and relative prices of aggregates are computed as the ratios of current price value to chained indexed volume. The trend component of total factor productivity is computed using the DMM package developed by Fiorentini et al. (2012). The obtained series at quarterly frequency is then used to estimate the potential output. In Germany, we additionally observe the historical unemployment benefit ratio (constructed as the ratio of unemployment benefits to the wage rate).

We make a few transformations to the raw investment series. In particular, we compute the deflator of public investments based on annual data and then obtain its quarterly frequency counterpart through interpolation. This series together with nominal public investments is then used to compute real quarterly public investments. In order to assure consistency between nominal GDP and the sum of the nominal components of aggregate demand, we impute change in inventories to the series of investments.
Appendix B. Estimation results

Appendix B.1. List of observables

<table>
<thead>
<tr>
<th>EMU countries</th>
<th>Euro Area*</th>
<th>Rest of the World</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP (nom. and real)</td>
<td>GDP (nom. and real)</td>
<td>GDP (nom. and real)</td>
</tr>
<tr>
<td>TFP trend</td>
<td>GDP trend</td>
<td>GDP trend</td>
</tr>
<tr>
<td>Hours worked</td>
<td>Effective exchange rate (nom.)</td>
<td>Oil price</td>
</tr>
<tr>
<td>Wages (nom.)</td>
<td>Interest rate (nom.)</td>
<td>Interest rate (nom.)</td>
</tr>
<tr>
<td>Imports (nom. and real)</td>
<td>Imports (nom. and real)</td>
<td>Population</td>
</tr>
<tr>
<td>Exports (nom. and real)</td>
<td>Exports (nom. and real)</td>
<td></td>
</tr>
<tr>
<td>Government consumption (nom. and real)</td>
<td>Population</td>
<td></td>
</tr>
<tr>
<td>Government investment (nom. and real)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private consumption (nom. and real)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total investment (nom. and real)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government transfers (nom.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government interest payments (nom.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government debt (nom.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Active population rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Note: We observe EA aggregate variables and compute area-wide observation equations within the model, by aggregating EMU country and Rest of Euro area variables.

Table B.1: List of observables.
## Appendix B.2. Estimated key model innovations

Table B.2: Prior and posterior distributions of key model innovations.

<table>
<thead>
<tr>
<th>Autocorrelations of forcing variables</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr</td>
<td>Mean</td>
</tr>
<tr>
<td>Subjective discount factor $\rho^{UC}$</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>Investment risk premium $\rho^{S}$</td>
<td>B</td>
<td>0.85</td>
</tr>
<tr>
<td>Domestic price mark-up $\rho^{MUY}$</td>
<td>B</td>
<td>0.05</td>
</tr>
<tr>
<td>Flight to safety $\rho^{FQ}$</td>
<td>B</td>
<td>0.85</td>
</tr>
<tr>
<td>Trade share $\rho^{M}$</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>International bond preferences $\rho^{BW}$</td>
<td>B</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard deviations (%) of innovations to forcing variables</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr</td>
<td>Mean</td>
</tr>
<tr>
<td>Subjective discount factor $\varepsilon^{UC}$</td>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>Investment risk premium $\varepsilon^{S}$</td>
<td>G</td>
<td>0.1</td>
</tr>
<tr>
<td>Domestic price mark-up $\varepsilon^{MUY}$</td>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>Flight to safety $\varepsilon^{FQ}$</td>
<td>G</td>
<td>0.8</td>
</tr>
<tr>
<td>Trade share $\varepsilon^{M}$</td>
<td>G</td>
<td>0.4</td>
</tr>
<tr>
<td>International bond preferences $\varepsilon^{BW}$</td>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>Monetary policy $\varepsilon^{i}$</td>
<td>G</td>
<td>0.4</td>
</tr>
<tr>
<td>Government consumption $\varepsilon^{G}$</td>
<td>G</td>
<td>0.4</td>
</tr>
<tr>
<td>Gov transfers $\varepsilon^{T}$</td>
<td>G</td>
<td>0.4</td>
</tr>
<tr>
<td>Permanent TFP $\varepsilon^{LAY}$</td>
<td>G</td>
<td>0.4</td>
</tr>
<tr>
<td>Labor supply $\varepsilon^{U}$</td>
<td>G</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Notes: Cols. (1)-(2) list model innovations. Cols. (3)-(4) indicate the prior distribution function (B: Beta distribution; G: Gamma distribution). Identical priors are assumed across countries. Cols. (5)-(8) show the mode and the HPD intervals of the posterior distributions of DE, FR, IT, and ES key innovations, respectively.
Appendix B.3. Annual Fit

Figures B.1 - B.4 show the unconditional 1- and 2-year ahead forecast of selected observed variables for the four EMU countries. The solid blue line depicts the observed annual time series, the red solid line shows the unconditional model-implied 1- and 2-year ahead prediction at each point (year) in time. The dashed slim green and blue lines connect the 1- and 2-year predictions, respectively.

This graphical representation of the 1- and 2-year ahead forecast error, discussed in section 4.2, suggest that our estimated models deliver a relatively good (in-sample) forecast accuracy. For example, looking closer to the huge drop in real GDP growth during the global financial crisis for the four estimated countries, the models 2-year ahead predictions in 2008 forecast a further decrease in GDP growth in 2009 before it forecasts a recovery in 2010. We are also able to fit fairly well nominal and real export and import growth across countries. However, we face some difficulties in delivering a well-performing (in-sample) forecast accuracy, e.g., for consumption growth and GDP inflation in Germany and France.

Figure B.1: Annual fit in Germany.

Note: The solid black line depicts the observed annual series, the red solid line shows the unconditional 1- and 2-year ahead prediction.
Figure B.2: Annual fit in France.

Note: The solid black line depicts the observed annual series, the red solid line shows the unconditional 1- and 2-year ahead prediction.

Figure B.3: Annual fit in Italy.

Note: The solid black line depicts the observed annual series, the red solid line shows the unconditional 1- and 2-year ahead prediction.
Note: The solid black line depicts the observed annual series, the red solid line shows the unconditional 1- and 2-year ahead prediction.
Appendix B.4. Cross correlation

Figures B.5 - B.8 depict the lead-lag structure of real GDP growth with its main components (consumption, investment, employment, and the trade balance) and GDP inflation for the four EMU countries. We use first differences for the trade balance-to-GDP ratio and quarter-on-quarter growth rates for all other variables. It compares the model-generated cross correlations (black) (auto-correlation for GDP growth) with the ones of the observed data (blue). The horizontal dashed red lines represent the 95% confidence bounds.

In the figures, lag refers to the timing of the second argument of the couple, where GDP is always the first. For example, looking at the subplot of consumption growth in B.5, it provides information on the cross-correlation of consumption growth, ranging from $t - 2$ to $t + 2$, on contemporaneous GDP growth at time $t$: when lag is positive, consumption leads GDP by lag periods; when lag is negative, consumption lags GDP by lag periods. Therefore, the cross-correlation of consumption growth in $t + 2$ on GDP growth in $t$ can also be interpreted as the cross-correlation of GDP growth in $t - 2$ on consumption growth in $t$.

The figures suggest that most of the correlations between GDP growth and its components are fairly well captured. More precisely, all country models replicate the contemporaneous correlation of consumption, investment and employment with output. In our model the trade balance is positively correlated with output, but matches the data pattern only for Germany. Moreover, all estimated models generate a negative contemporaneous correlation between GDP inflation and GDP growth, which matches the data only in Germany and Italy. Persistency patterns are particularly well seized in Spain.
Figure B.5: Lead-Lag structure of output growth with its main component and GDP inflation for Germany.

Figure B.6: Lead-Lag structure of output growth with its main component and GDP inflation for France.
Figure B.7: Lead-Lag structure of output growth with its main component and GDP inflation for Italy.

Figure B.8: Lead-Lag structure of output growth with its main component and GDP inflation for Spain.
Appendix B.5. Full sample historical decomposition

Shock decomposition of real GDP growth.

Figure B.9: Real GDP growth - Germany

Figure B.10: Real GDP growth - France
Figure B.11: Real GDP growth - Italy

Figure B.12: Real GDP growth - Spain
Shock decomposition of the trade balance-to-GDP ratio.

Figure B.13: Trade balance-to-GDP ratio - Germany

Figure B.14: Trade balance-to-GDP ratio - France
Figure B.15: Trade balance-to-GDP ratio - Italy

Figure B.16: Trade balance-to-GDP ratio - Spain
European Economy Discussion Papers can be accessed and downloaded free of charge from the following address:

Titles published before July 2015 under the Economic Papers series can be accessed and downloaded free of charge from:
GETTING IN TOUCH WITH THE EU

In person
All over the European Union there are hundreds of Europe Direct Information Centres. You can find the address of the centre nearest you at: http://europa.eu/contact.

On the phone or by e-mail
Europe Direct is a service that answers your questions about the European Union. You can contact this service:
- by freephone: 00 800 6 7 8 9 10 11 (certain operators may charge for these calls),
- at the following standard number: +32 22999696 or
- by electronic mail via: http://europa.eu/contact.

FINDING INFORMATION ABOUT THE EU

Online
Information about the European Union in all the official languages of the EU is available on the Europa website at: http://europa.eu.

EU Publications
You can download or order free and priced EU publications from EU Bookshop at: http://publications.europa.eu/bookshop. Multiple copies of free publications may be obtained by contacting Europe Direct or your local information centre (see http://europa.eu/contact).

EU law and related documents
For access to legal information from the EU, including all EU law since 1951 in all the official language versions, go to EUR-Lex at: http://eur-lex.europa.eu.

Open data from the EU
The EU Open Data Portal (http://data.europa.eu/euodp/en/data) provides access to datasets from the EU. Data can be downloaded and reused for free, both for commercial and non-commercial purposes.