Subsidising the next generation infrastructures. Consumer-side or Supply-side?

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Abstract:
The constant increase of households' bandwidth consumption reveals the need of an ultra broadband infrastructure. Furthermore, it is widely accepted that such infrastructure will improve economic growth and employment. However, the cost of such a roll out is high and the households' rate of adoption is uncertain. Therefore operators hesitate to invest massively. In such a context, public intervention could help rollout.

But what exactly is the most efficient form of intervention and to what degree is each form most appropriate. This paper studies, more specifically, subsidy strategies: Subsidizing the demand by a contribution to each household's subscription fee for a predetermined amount of time (a refund, a tax cut) or subsidizing the infrastructure by means of a contribution to operators' infrastructure costs? In this paper, we explain that subsidizing the demand is more efficient, in welfare terms, than infrastructure subsidies as long as the consumers' demand for ultra broadband remains elastic enough and that the decrease in costs is dynamic enough to allow private operators to extend the roll out of the infrastructure fast enough without subsidies.

1 Introduction

Public policies to promote broadband are quite diversified. There is a wide range of alternatives and many papers have studied their effects from either a theoretical or an empirical point of view. (Leighton 2001), (Gillett et al. 2004), as well as (Cava-Ferreruela & Alabau-Munoz 2006) have drawn up a taxonomy of public strategies according to the level of public authorities involvement; from the lightest ones, where public authorities settle to create the appropriate conditions for market development, to the heaviest ones where they invest directly in publicly owned network infrastructures,(Jeanjean 2010). These strategies include actions from both the supply side and demand side.

The light-intervention strategy actions on the supply side usually consist of regulatory rules in order to foster competition or to reduce barriers to entry (unbundling policies) or to reform ordinances that affect road and building construction codes, technologies standards or cable pulling. The soft-intervention strategies on the demand side consists of educating the population in the use of new technologies, the promotion of broadband applications and access for public institutions.

Subsidies are a type of medium-intervention strategy. They generally have been used in geographic areas where broadband services are not available because there is a lack of infrastructure. The high fixed-costs of the infrastructure deter the investments in scarcely populated area where revenues are too low.

Several analysis have highlighted the positive effect of both suppy-side and demand-side promotion, (Cava-Ferreruela & Alabau-Munoz 2006) and (Atkinson et al. 2008) Empirical evidence, (Goolsbee 2003), (Wallsten 2006), has shown, that, in unserved areas, supply-side subsidies had a positive impact on broadband penetration. (Goolsbee 2003) has compared the supply-side subsidies in unserved areas with demand-side subsidies in already served areas in order to improve the broadband penetration rate in the USA. He found that demand-side subsidies had a better impact on penetration than supply-side subsidies but were less cost effective. The low cost effectiveness of demand-side subsidies arises from the fact that the action is limited to already served zones. However, if we consider the investment incentive for telcos in unserved areas induced by the increase of consumers' willingness to pay, the action can become cost effective. This paper studies the conditions in which such subsidies are cost effective and when they are more cost effective than supply-side subsidies, in sections 3 and 5.

Indeed, operators invest in areas that are profitable. As dense areas are more profitable than rural ones, dense areas will be served first. If we consider geographic density as a continuum, there is a point where operators stop investing because it is no longer profitable (Valletti et al. 2002), (G\ötz 2009). This break-even point depends on the one hand, on the infrastructure costs, which tend to decrease over time thanks to learning and demand effects, allowing less dense areas to become profitable over time, and on the other hand, it also depends on the rate of network expansion, use development and overall network effects . Because consumer subsidies encourage households to spend more on the services proposed by operators, the business plans of the latter are improved; subsidies given to incite the public demand allow areas that would not have been served at one point of time to become profitable sooner than they would have been without subsidies.

This stimulates operators to invest earlier in those areas. Consumer subsidies serve as a catalyst for investments which accelerate the infrastructure rollout. However, the effectiveness of consumer subsidies depends on consumers' demand for ultra broadband. The more elastic it is, the more consumers react affirmatively to subsidies and the more operators are stimulated to invest earlier.

But subsidies should not last forever; they just need to last long enough to be an incentive for operators. That is to say that they should last at least as long as the duration of the roll out without subsidies would have lasted. In such a case, operators are certain that their anticipated investments will be profitable in the long run.

Subsidizing infrastructure boils down to a decrease in infrastructure costs for the investors which, just like consumer subsidies, allows an anticipation of investment in the areas that otherwise would not have been served at that time. In this case, however, subsidies are sunk costs, they are spent. Their effect is expected to last over the duration of the investment lifetime and it is not possible to recover them once they have been spent. Conversely, the duration of consumer subsidies does not necessarily need to last so long. They should not last any longer than the time to be incentive enough for operators. If this time is shorter than the infrastructure lifetime, consumer subsidies will prove more efficient over time. If, however it is longer, then infrastructure subsidies will prove to be more efficient.

Dense areas would tend to be served before rural ones. The impact of an investment on consumer demand is higher in dense areas, and a slight decrease in infrastructure costs allows a greater coverage increase, therefore the time that consumer subsidies should last, to remain a sufficient incentive, is shorter. The minimum efficient duration of consumer subsidies increases with infrastructure costs and likewise decreases with the density of the population. That is why consumer subsidies are more efficient at the beginning of the roll out in dense areas. Conversely, in rural areas, the duration of consumer subsidies will have to be longer and perhaps indefinite. In which case the infrastructure subsidies will be more efficient in the long run.

This paper compares the welfare provided by the two forms of subsidies and concludes that if the time required to subsidize the demand is shorter than the infrastructure lifetime, then consumer subsidies are more efficient than infrastructure subsidies.

The paper takes into account only the direct effects explained above, but not the indirect effects such as network externalities or innovation spillovers, which improve the case for consumer subsidies.

This paper consists of seven sections. The first is the introduction, the second is a basic model for pricing and coverage constraints, the third is the detailed mechanism of consumer subsidies, the fourth is the detailed mechanism of infrastructure subsidies, the fifth is a comparison of the two types of subsidies and the conditions where each is more efficient. The sixth is a numerical application with the case of an average-size country as an example, with an appraisal of the infrastructure costs and population spread. The seventh is the conclusion with some recommendations and relevant policy implications. Both policy instruments, consumer-side and supply-side subsidies have been employed for broadband market (Götz 2009) when the market was already matured, these experiments often resulted in preferring the supply side. (Leighton 2001) advised against resorting to such subsidies but that was in 2001 and in a context of an already built network “The wires over which broadband service can be transmitted are already in place-owned by telephone, cable, and even electricity providers”, since then, the context has changed a lot. In the present context, (Atkinson et al. 2008), however, recommend consumer-subsidies through an exemption of broadband access from federal, state and local taxes in order to encourage the growth of consumer demand. However, this paper shows that in the case of an emerging market, to consumer side can be very relevant.

2 The basic model

The firms are in competition for the ultra broadband services provided by the NGA infrastructure.

The coverage of an area with the NGA infrastructure consists of delivering an ultra broadband outlet in each household, but the service installation of the outlet and the terminal supply are not included.

The infrastructure cost *f* depends on the density of the population *d*.

The cost of the outlet c_{outlet} is the infrastructure cost for each household covered by the infrastructure. Let us assume that *n* is the total number of households covered and *S* the surface of the area.

$$d = \frac{n}{S}$$

$$c_{outlet} = \frac{f(d)}{n} = \frac{f(d)}{d\,S}$$

Let us also assume that, c_{outlet} is strictly convex and decreases in *d*. The denser the area, the less expensive it is to connect each outlet. (If $f(d)$ is constant, then c_{outlet} is convex and decreases in d. In fact, as certain empirical study show it, see chapter 6, $f(d)$ is not completely constant, but its variations are sufficiently weak so that c_{outlet} remains convex and still decreases in *d*.)

A firm decides to invest in an area if its profit is higher than its fixed costs. According to the density of the population there can be three cases:
1) No firm decides to invest. The area is not covered.
2) Only one firm decides to invest and has the monopoly
3) More than one firm decides to invest and a competitive market is created. For simplification purposes, we will limit our study to the case of a duopoly.
We will consider a two-stage game: first, the firms choose the areas they plan to invest according to the density of the population.
Second, the firms compete in price (Bertrand)

2.1 The demand

The utility function that we have chosen to adopt is quadratic and strictly concave (Singh & Vives 1984), (Götz 2009), (Valletti et al. 2002)

$$U(q_1, q_2) = a\,(q_1 + q_2) - \frac{(q_1^2 + 2\sigma\, q_1 q_2 + q_2^2)}{2}$$

q_1 and q_2 are respectively the probability that a household will subscribe to firm 1 or firm 2's ultrabroadband services. We assume that each household can not choose to subscribe to both firms' services so $q_1 + q_2 \leq 1$. a and σ are positive coefficients: a is the maximum willingness to pay for ultrabroadband services and σ is the coefficient of product differentiation $\sigma \in [0,1]$. When $\sigma = 1$ then the services of the two firms are complete substitutes and when $\sigma = 0$ then they are completely independent.

We assume here that both firms propose services of an equivalent level of quality for consumers. Consumers have the same willingness to pay a for the services of the two firms.

The representative consumer aims to maximize

$$U(q_1, q_2) - q_1 p_1 - q_2 p_2$$

p_1 and p_2 are the prices of ultrabroadband services provided respectively by firm 1 and firm 2

This gives rise to a linear demand structure, inverse demands are represented by:

$$p_1 = a - q_1 - \sigma q_2$$
$$p_2 = a - q_2 - \sigma q_1$$

We can write direct demand as follows:

$$q_1(p_1, p_2) = \frac{a(1-\sigma) - p_1 + \sigma p_2}{1-\sigma^2}$$
$$q_2(p_1, p_2) = \frac{a(1-\sigma) - p_2 + \sigma p_1}{1-\sigma^2}$$

The quantities of subscriptions y_1 and y_2 respectively for firm 1 and firm 2 in the area are given by $y_1 = q_1 n$ and $y_2 = q_2 n$

Let us assume that the two firms have the same marginal costs *c*. Therefore the firms' profits are respectively:

$$\pi_1(p_1, p_2) = n\frac{a(1-\sigma) - p_1 + \sigma p_2}{1-\sigma^2}(p_1 - c)$$
$$\pi_2(p_1, p_2) = n\frac{a(1-\sigma) - p_2 + \sigma p_1}{1-\sigma^2}(p_2 - c)$$

As the firms are symmetric, the non cooperative Nash equilibrium leads them to set the same price

$$p_1 = p_2 = p = \frac{a(1-\sigma) + c}{2-\sigma}$$

And then $q_1 = q_2 = q = \dfrac{(a-c)}{(2-\sigma)(1+\sigma)}$ with $q_1 + q_2 \leq 1$. This gives rise to the following condition: $a \leq c + \dfrac{(2-\sigma)(1+\sigma)}{2}$

and the quantities are:

$$y_1 = y_2 = y = q\,n = \frac{(a-c)\,d\,S}{(2-\sigma)(1+\sigma)}$$

2.2 Decision to invest:

Firms' profit depends on both the density *d* and on σ. It can be rewritten accordingly: $\pi_1(d,\sigma) = \pi_2(d,\sigma) = \pi(d,\sigma)$

$$\pi(d,\sigma) = (p(\sigma)-c)\,q(d,\sigma)\,n = \frac{(a-c)(1-\sigma)}{(2-\sigma)}q(d,\sigma)\,n$$

Firms invest in the area if their profit is higher than the cost price of infrastructure. Cost price of infrastructure is the fixed costs impacted by an amortization coefficient which is for a fully amortizing payment $\tau = \frac{\rho}{1-(1+\rho)^{-T}}$ when ρ is the depreciation rate and T the infrastructure life time.
We have chosen the fully amortizing payment in order to have a constant coefficient τ independent of time.

The coverage is at a maximum under monopoly, (Götz 2009)

$\pi(d,\sigma) \geq f(d)\tau$ or

$$(p-c)q \geq \frac{f(d)\tau}{n} = c_{outlet}(d)\tau$$

This leads to a condition on $c_{outlet}(d)$

$$c_{outlet}(d)\tau \leq \left(\frac{a-c}{2-\sigma}\right)^2\left(\frac{1-\sigma}{1+\sigma}\right)$$

Let us assume that the minimum density the firm can cover is $\underline{d}$. $\underline{d}$ is such that

$$c_{outlet}(\underline{d})\tau = \left(\frac{a-c}{2-\sigma}\right)^2\left(\frac{1-\sigma}{1+\sigma}\right)$$

c_{outlet} decreases in d and $\left(\frac{a-c}{2-\sigma}\right)^2\left(\frac{1-\sigma}{1+\sigma}\right)$ decreases in σ, therefore, $\underline{d}(\sigma)$ increases in σ. For simplification purposes, we will note $\underline{d}(\sigma) = \underline{d}_\sigma$. $\underline{d}_\sigma$ is at a minimum when $\sigma = 0$, in the case of a monopoly, $c_{outlet}(\underline{d}_0) = \frac{(a-c)^2}{4\tau}$ and it is at a maximum when $\sigma = 1$, $c_{outlet}(\underline{d}_1) = 0$

The coverage is at a maximum under monopoly, (Götz 2009)

Let us assume that the density of a country ranges from $\overline{d}$ the highest to $\underline{\underline{d}}$ the lowest. $\overline{d} \geq \underline{d}_\sigma \geq \underline{d}_0 \geq \underline{\underline{d}}$

Market structure depends on d:
In the areas where $\underline{d}_0 > d > \underline{\underline{d}}$, there is no investment, the density is too low to make the investment profitable, and so there is no infrastructure. In the areas where $\underline{d}_\sigma > d > \underline{d}_0$, a monopoly is the only structure that can be profitable. In the areas

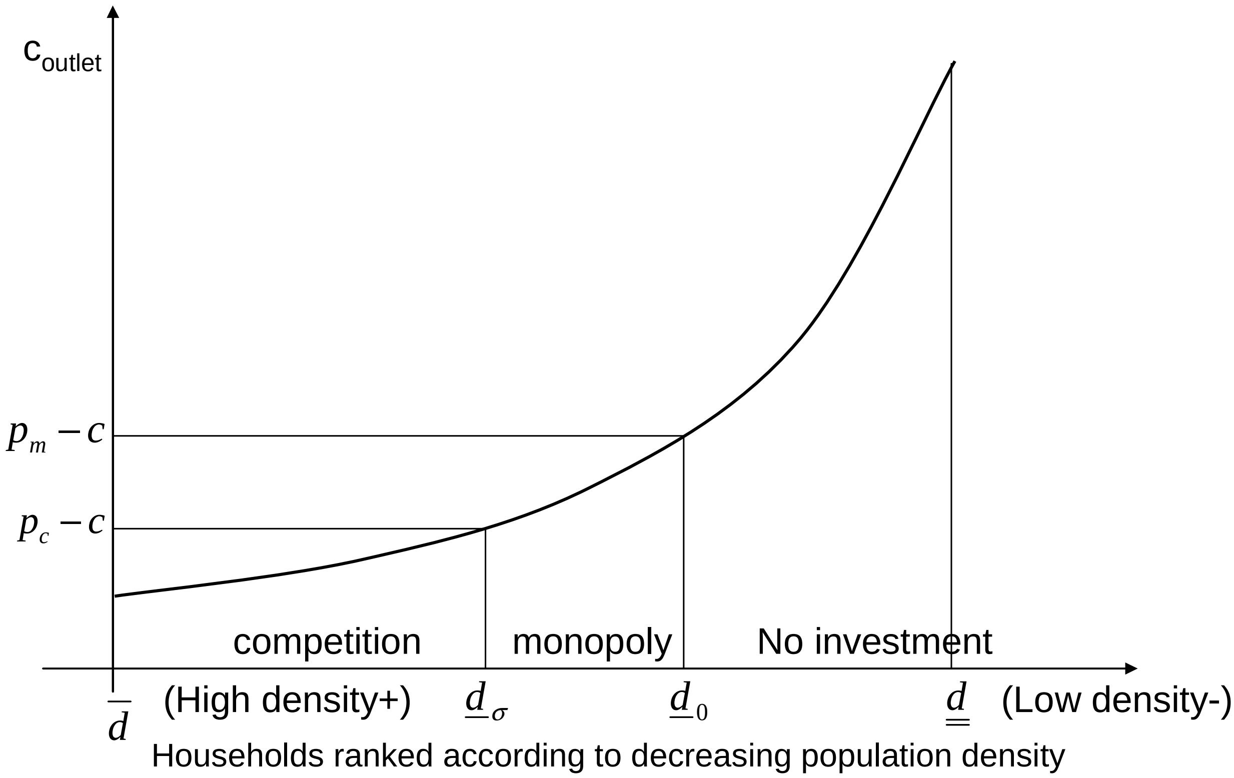
where $\overline{d} \geq d > \underline{d}_{\sigma}$, competition is possible, the density of the population is high enough to make the investments profitable for several firms.

The willingness to pay a helps investment whereas marginal cost(s) *c* deters it.
The graph below (fig.1) illustrates the different areas in a country that are covered by a competition market structure or by a monopoly according to the density of the population. As we assume the coverage starts from the densest areas to the least dense, the households are ranked according to their density, from highest to lowest.

In the case of a monopoly: $p_m - c = \dfrac{a-c}{2}$

In the case of competition, with $\sigma \neq 0$ $p_c - c = \dfrac{(a-c)(1-\sigma)}{2-\sigma} < \dfrac{a-c}{2}$

p_m and p_c are respectively the monopoly and the competitors' prices.



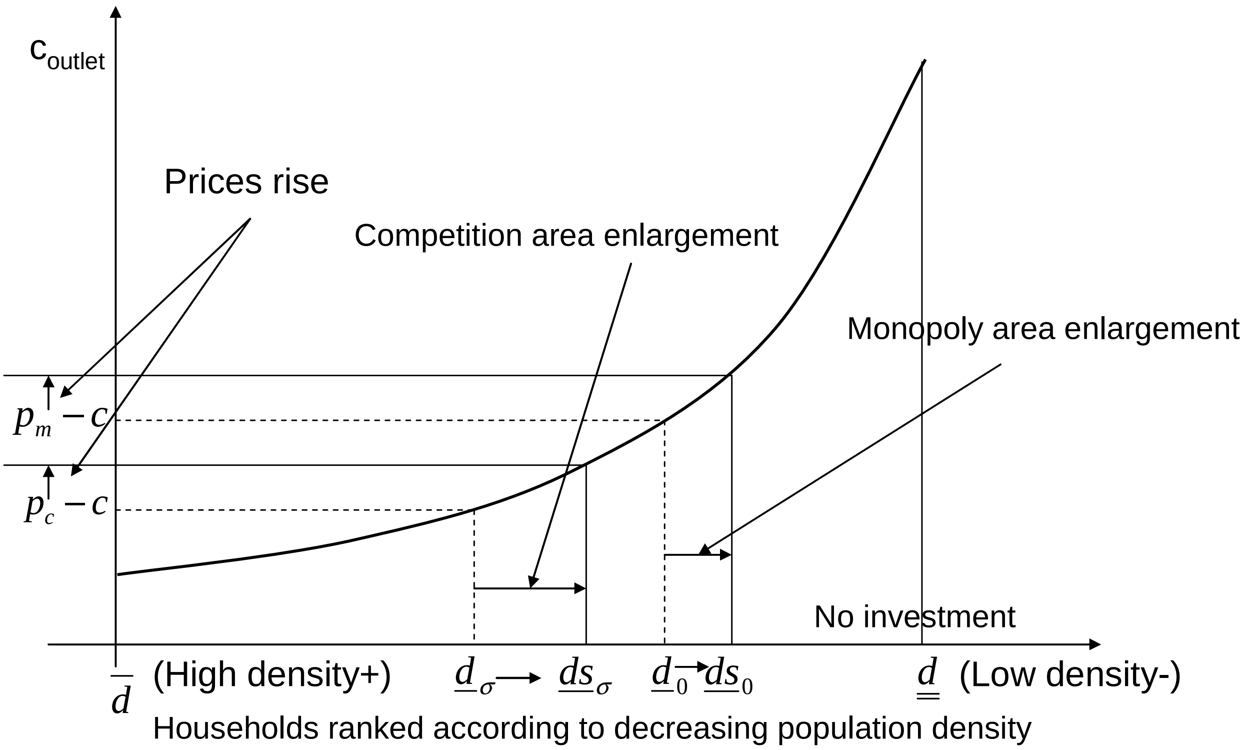
(fig.1)

3 Consumer subsidies

Consumer subsidies s consist of complementing the consumer's increased willingness to pay *a* for a limited amount of time *t* (i.e. via refunds or a tax cut). The complemented willingness to pay become $a_s = a + s$.

c_{outlet} increases with a, the willingness to pay. Therefore $\underline{d}_{\sigma}$ decreases in a. The growth of the willingness to pay reduces the minimum density $\underline{d}_{\sigma}$ and $\underline{d}_0$, and at the same time, both monopoly and competition prices increase, which therefore allows operators to invest in a larger area.

Because c_{outlet} is convex, the competition area grows faster than the monopoly area . The graph below (fig.2) illustrates the growth of the areas covered by a monopolistic or a competitive market structure.



(fig.2)

How does the penetration rate evolve when the willingness to pay increases?
Two factors have a conclusive impact on the efficiency of the consumer subsidies efficiency: The improvement of the investment conditions over time (infrastructure costs decrease and/or ultrabroadband adoption increases), and price elasticity.

3.1 Improvement of the investment conditions over time:

Apart from subsidies, investment conditions improve over time. Infrastructure costs and operating costs as well as the household adoption rate for ultrabroadband evolve and tend to improve due to technical progress and services developments: the skills development of technical staff and the bandwidth needs of households increase.

The drawback of encouraging consumers is that there is always a time limit to the funding. When consumer encouragement comes to an end, the consumer's willingness to pay is abruptly deterred, and, if the market situation has not been evolving , it will revert back to its previous level. Firms will only take into account the consumer stimulation in their investment decision if the subsidies last long enough to allow time for investment conditions to improve substantially. In such a case, when the consumer

stimulation abruptly comes to a halt, willingness to pay is nonetheless deterred; however, its level will be sufficient to cover the investments. The duration of the consumer subsidies is crucial. It has to be clearly announced ahead of time because firms need this information to plan their investment roadmap.
The longer the duration last, the more efficient it is, however, the subsidies will be more costly. Therefore, it is crucial to determine the most cost effective duration of time dependent on the improvement of the investment conditions. If the duration is too short, there will not be enough incentive to allow firms to invest. If it is too long, the incentive will be sufficient however the amount of the subsidies will be prohibitive. In fact, the investment conditions will improve faster when the optimal duration is shorter and the amount of subsidies is lower.

Let us assume that the cost of an outlet decreases over time. $c_{outlet}(t,d)$ is the cost of an outlet at time *t* while the density is d, as a result of learning by doing effect and of technical progress.
Let us assume that the costs decrease regularly over time by a ratio of $\delta \in [0,1]$ such that $c_{outlet}(t,d) = c_{outlet}(0,d)(1-\delta)^t$
Let us assume that the consumer subsidies begin at t = 0. The minimum density area is covered by the monopoly with the density $\underline{d}_0$ which confirms:

$$c_{outlet}(0,\underline{d}_0) = \frac{(a-c)^2}{4\tau}$$

Consumer subsidies increase the maximum willingness to pay from a to a_s, the minimum density area covered decreases from $\underline{d}_0$ to $\underline{ds}_0$ and $\underline{ds}_0$ confirms:

$$c_{outlet}(0,\underline{ds}_0) = \frac{(a_s-c)^2}{4\tau}$$

The duration of the consumer subsidies must last long enough to allow the cost of an outlet the time to decrease enough to reach the density $\underline{ds}_0$. The minimum duration $\underline{t}$ must confirm:

$$c_{outlet}(\underline{t},\underline{ds}_0) = c_{outlet}(0,\underline{d}_0)$$

Which means:

$$\frac{(a_s-c)^2}{4\tau}(1-\delta)^{\underline{t}} = \frac{(a-c)^2}{4\tau}$$

And so,

$$\underline{t} = \frac{\ln\left(\frac{a-c}{a_s-c}\right)^2}{\ln(1-\delta)}$$

The minimum duration $\underline{t}$ increases in a_s and decreases in δ. The faster the costs decrease, the shorter the minimum duration will be. We can even write $\lim_{\delta \to 1} \underline{t} = 0$; whereas, if costs do not decrease $\lim_{\delta \to 0} \underline{t} = +\infty$ the minimum duration will last forever.

3.2 Cost of consumer subsidies

The total cost of the subsidies depends on the duration and the number of households impacted.
The number of households impacted depends on the distribution of the population in a country.
Let us assume that the distribution is defined by the density contingent on the surface area of the country $d(S)$ (fig.3 page 10). We assume the coverage of the country begins with the high density areas and ends with the low density areas. If S = 0 represents the highest density area and if $S = S_{country}$, the total country surface, represents the lowest density area.
The population covered at S is:

$$dn = d(S)\,dS$$

The maximum density area is $\overline{d}$, then $d(0) = \overline{d}$ and the minimum density covered is covered by the monopoly $\underline{d}_0$ for a covered area $\overline{S}_m$ then $d(\overline{S_m}) = \underline{d}_0$
Therefore the number of households covered is:

$$n = \int_0^{\overline{S_m}} d(S)\,dS$$

The number of subscribers depends on the limit between the monopoly area and the competition area. This limit is the density $\underline{d}_\sigma$ for a covered area $\overline{S}_c$ and $d(\overline{S_c}) = \underline{d}_\sigma$

$$y = q_c \int_0^{\overline{S_c}} d(S)\,dS + q_m \int_{\underline{S_c}}^{\overline{S_m}} d(S)\,dS$$

q_c and q_m represent respectively the adoption rates in competition and monopoly areas

$$q_c = \frac{2(a-c)}{(2-\sigma)(1+\sigma)} \text{ and } q_m = \frac{a-c}{2}$$

The consumer subsidies increase the adoption rate both for competition and for monopoly. The adoption rate becomes:

$$q_{cs} = \frac{2(a_s-c)}{(2-\sigma)(1+\sigma)} \text{ and } q_{ms} = \frac{a_s-c}{2} \text{ with the subsidies}$$

Let us assume the consumer subsidies are offered only to the newcomers for the duration t_s, we suppose that all the newcomers take advantage of the consumer subsidies.
The number of newcomers created by the consumer subsidies is y_s

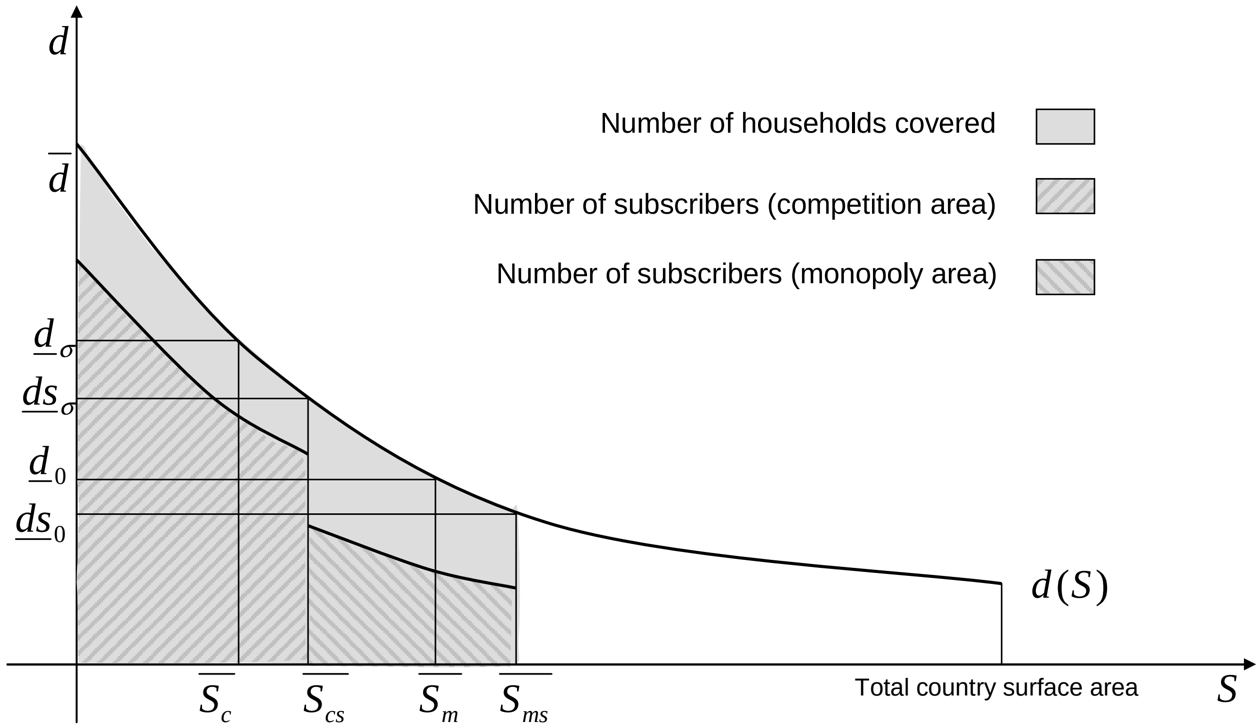
$$y_s = q_{cs}\int_0^{\overline{S_{cs}}} d(S)\,dS + q_{ms}\int_{\overline{S_{cs}}}^{\overline{S_{ms}}} d(S)\,dS - q_c\int_0^{\overline{S_c}} d(S)\,dS - q_m\int_{\overline{S_c}}^{\overline{S_m}} d(S)\,dS$$

$\overline{S}_c$ and $\overline{S}_m$ represent the areas covered, respectively, in the competition and monopoly areas without subsidies; and $\overline{S}_{cs}$, $\overline{S}_{ms}$ the areas covered with subsidies (fig.4 p11).

$$q_m = \frac{a-c}{2}\,;\, q_{ms} = \frac{a_s - c}{2}\,;\, q_c = \frac{2(a-c)}{(2-\sigma)(1+\sigma)}\,;\, q_{cs} = \frac{2(a_s - c)}{(2-\sigma)(1+\sigma)}$$

$\overline{S}_{cs}$, $\overline{S}_{ms}$, q_{ms} and q_{cs} increase in a_s , therefore y_s increases in a_s too.

The graph below (fig.3) illustrates a country's density distribution.



(fig.3)

The cost of demand subsidy for public authorities is $C_s = y_s\,(a_s - a)\,t_s$.

When $t_s < \underline{t}$, operators limit their investments to the areas with a density higher than $\underline{d}_{\lim} > \underline{d}_0$

$\underline{d}_{\lim}$ is such that

$$c_{outlet}(\underline{d}_{\lim}) = \frac{(a_{\lim} - c)^2}{4\tau}$$

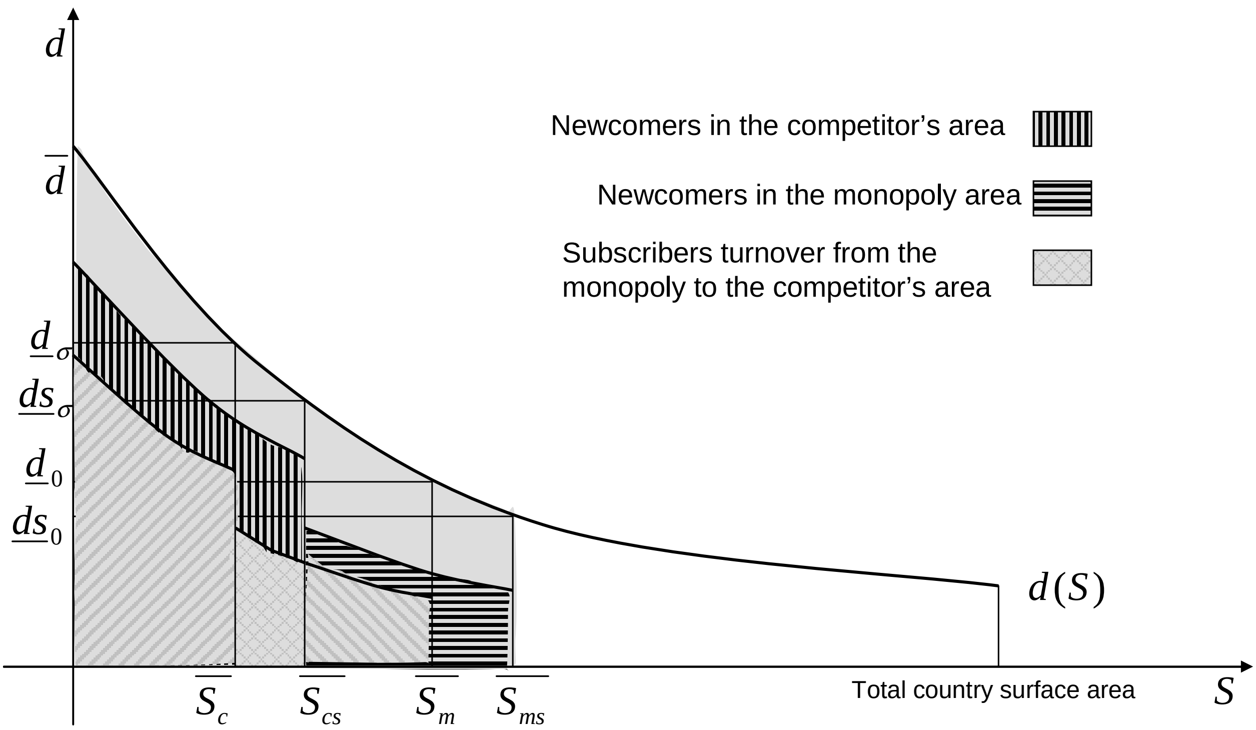
This density $\underline{d}_{\lim}$ corresponds to the value of willingness to pay $a_{\lim} < a_s$ such that

$$a_{\lim} = (1-\delta)^{-\frac{t_s}{2}}(a-c) + c$$

This leads to a less extensive coverage and a lower number of subscribers $y_{\lim}$ than that which could be expected with the subsidies. $y_{\lim} < y_s$.
This duration is not optimal because the level of subscribers $y_{\lim}$ could have been reached with a lower amount of subsidies, one which corresponded to the subscribers' willingness to pay $a_{\lim}$.

While $t_s > \underline{t}$ the duration is enough to encourage companies to invest and reach the number of newcomers y_s ; however, all time beyond $\underline{t}$ will be useless because it will not allow companies to exceed y_s .
Therefore, the optimal duration for the consumer subsidies is $t_s = \underline{t}$

The graph below illustrates the distribution of the newcomers (fig.4)



(fig.4)

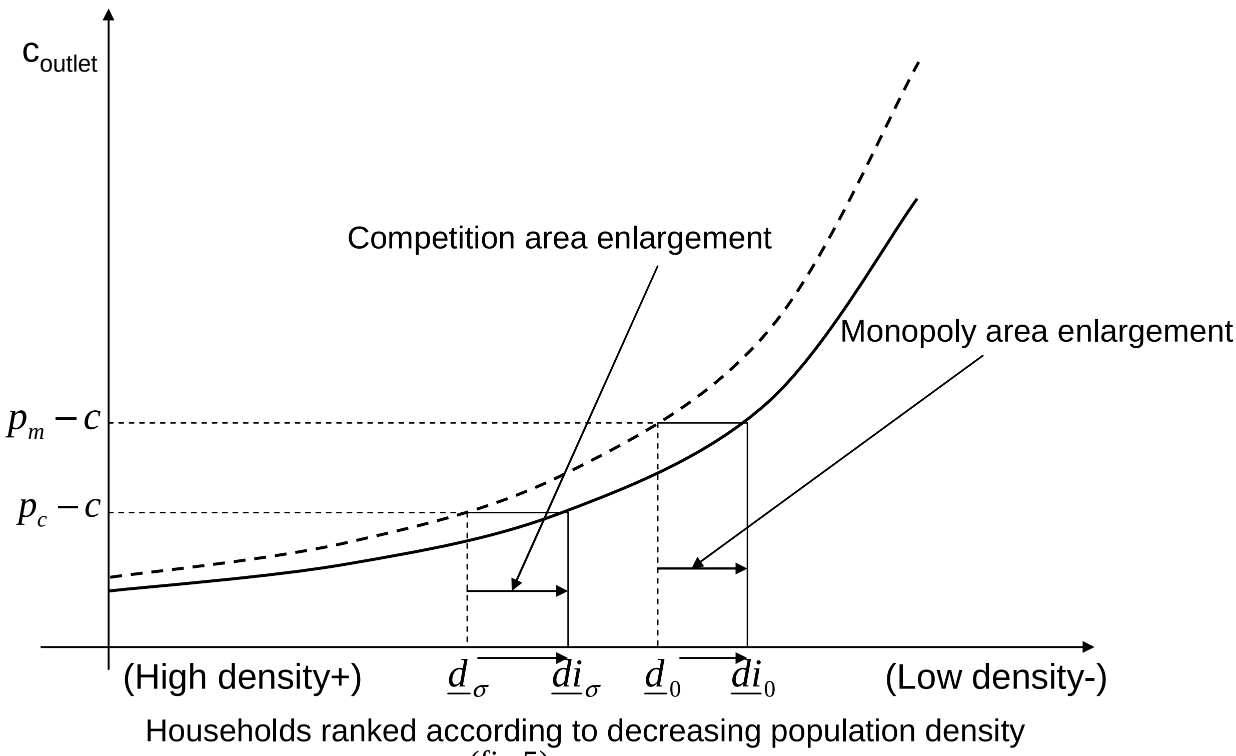
4 Infrastructure subsidies

Infrastructure subsidies refer to the proportion α of infrastructure costs granted in aid to an operator in order to improve its coverage.
Subsidies lower the costs that companies have to pay and encourage them to increase their coverage (fig.5).
Costs are shared between public authorities and companies. Cost of an outlet in a *d* density area becomes $c_{outlet}(d)(1-\alpha)$ for a company and $c_{outlet}(d)\alpha$ for public authorities.
Therefore $\underline{di}_0$, the lowest population density that a company can cover with a subsidy of proportion α, is located in the monopoly area.

$\underline{di}_0$ is such that $c_{outlet}(\underline{di}_0)(1-\alpha) = c_{outlet}(\underline{d}_0)$

The competitor's area is also extended from $\underline{d}_\sigma$ to $\underline{di}_\sigma$ such that:

$c_{outlet}(\underline{di}_\sigma)(1-\alpha) = c_{outlet}(\underline{d}_\sigma)$



(fig.5)

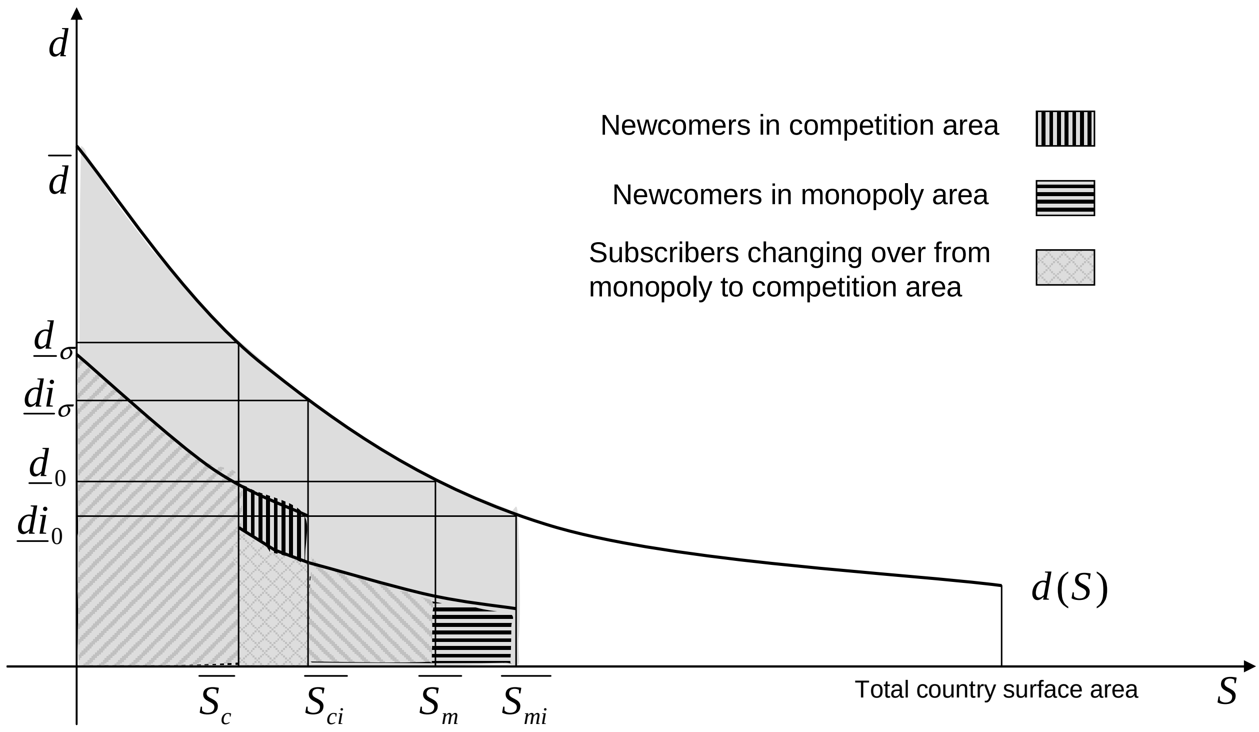
The subsidies allow the enlargement both of the competition and the monopoly area. The number of newcomers created by the infrastructure subsidies is

$$y_i = (q_c - q_m)\int_{\overline{S_c}}^{\overline{S_{ci}}} d(S)\,dS + q_m \int_{\overline{S_m}}^{\overline{S_{mi}}} d(S)\,dS$$

$\overline{S}_{ci}$, $\overline{S}_{mi}$ are the areas covered with infrastructure subsidies (fig.6).

The cost of infrastructure subsidies is the sum of all households covered thanks to the subsidies:

$$C_i = \left(\int_{\overline{S_c}}^{\overline{S_{ci}}} c_{outlet}\left(d(S)\right) d(S)\,dS + \int_{\overline{S_m}}^{\overline{S_{mi}}} c_{outlet}\left(d(S)\right) d(S)\,dS \right)\alpha$$



(fig.6)

We can notice that the number of subscribers reached for an equal density is higher with the consumer subsidies because they increase the adoption rate q both for competition and for monopoly area (comparison between fig.4 and fig.6).
There is a great difference between consumer and infrastructure subsidies for the authorities that grant them. While the former have a limited duration, the latter are tied up for the entire infrastructure lifetime T which is usually quite long, about 20 or 30 years. This means that when the duration of consumer subsidies t_s is shorter than T then they will be borne less time than infrastructure subsidies.

5 Comparison

Infrastructure subsidies are spent over a long period of time and consumer subsidies are spent little by little. The amount of the former includes the amortization of capital costs while the amount of the latter does not.

5.1 Coverage

We will compare both types of subsidies: When the consumer and infrastructure subsidies cover the same surface area of the country with the same population density.
$\underline{di}_0 = \underline{ds}_0$
In such a case: $c_{outlet}(\underline{di}_0)(1-\alpha) = c_{outlet}(\underline{ds}_0)(1-\alpha) = c_{outlet}(\underline{d}_0)$
And therefore we can deduce that there is a relation between the infrastructure subsidizing rate α and the increase of willingness to pay a_s:

$$\alpha = 1 - \left(\frac{a-c}{a_s - c} \right)^2$$

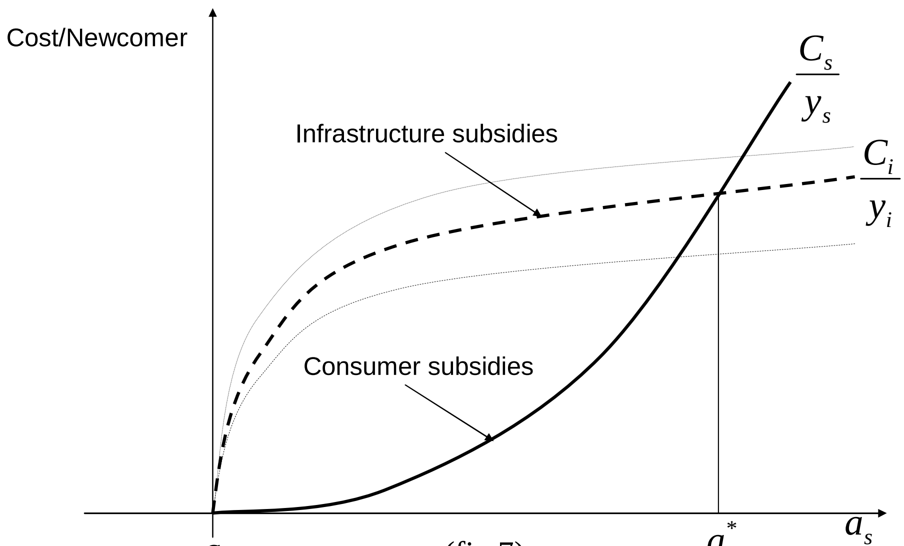
α increases in a_s. If $a_s = a$, $\alpha = 0$ and $\lim_{a_s \to \infty} \alpha = 1$

Let us notice that if $\underline{di}_0 = \underline{ds}_0$ then $\underline{di}_\sigma = \underline{ds}_\sigma$, i.e, the size of both zones (the competitor and the monopoly) remains equivalent irrespective of the type of subsidy. (see proof in the annexes)
However, the number of subscribers is higher with consumer subsidies than with infrastructure subsidies because the adoption rate is better.

5.2 Cost per newcomer

In this part, we will compare the cost of the subsidies per newcomer, in the two types of subsidies.

For high values of a_s, we can write $\frac{C_s}{y_s} > \frac{C_i}{y_i}$ and for low values of a_s, we can write $\frac{C_s}{y_s} < \frac{C_i}{y_i}$ as we can notice in (fig.7) (see proof in the annexes)



(fig.7)

The cost of consumer subsidies increases in two ways, on the one hand, with the amount of the subsidy and on the other hand because the necessary duration increases too. That is the reason why the consumer subsidy curve is convex. The cost of the infrastructure subsidies increases quickly for low values of a_s because α increases quickly and then it slows down for higher values of a_s because the growth of α slows down. That is why the infrastructure subsidy curve is concave.
Therefore where the paths of the two curves cross, there is a cross value of $a_s = a_s^*$.
So for the values of a_s lower than the cross value, $a_s < a_s^*$, the cost per newcomer is

lower with consumer subsidies; for the values of a_s higher than the crossing value, $a_s > a_s^*$, the cost per newcomer is lower with infrastructure subsidies.
The amount of consumer subsidies should not go beyond the service price, this sets a limit on the level of a_s.

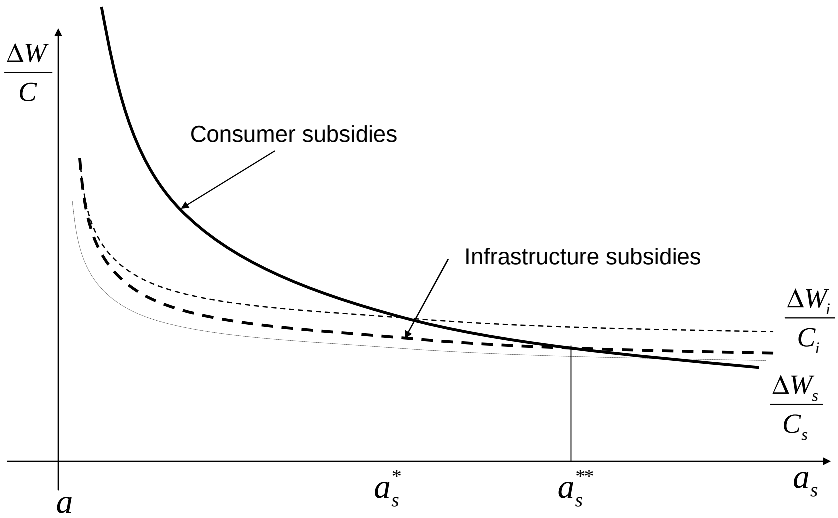
$a_s - a \leq p$; $a_s \leq (2-\sigma)\, a + c$

If this limit is under the cross value, $(2-\sigma)\, a + c < a_s^*$ consumer subsidies remain the most efficient means for the coverage.
We can observe (fig.4) that consumer subsidies lead to an increase of newcomers both in the newly covered area and in the formerly covered area. We must not forget that even if the newcomers in the formerly covered area do not contribute to enlarge the coverage, they truly contribute to improve consumer surplus and welfare. That's why the results concerning consumer surplus and welfare improve consumer subsidy advantages.

5.3 Consumer surplus and Welfare

In the previous paragraph we compared the cost per newcomer regarding consumer and infrastructure subsidies. We will now compare the increase of Consumer Surplus and Welfare resulting from the subsidies for the same level of coverage.

There is a value of $a_s = a_s^{**}$ beyond which it is advantageous to opt for infrastructure subsidies and below which it is advantageous to opt for consumer subsidies (fig.8). But this value of a_s^{**} is higher than the one for the cost per newcomer, $a_s^{**} > a_s^*$ because consumer subsidies generate newcomers not only in the newly covered areas, but also in the formerly covered areas, and these newcomers improve Consumer Surplus and Welfare.
If $a_s < a_s^{**}$ then $\frac{\Delta W_s}{C_s} > \frac{\Delta W_i}{C_i}$ and if $a_s > a_s^{**}$ then $\frac{\Delta W_s}{C_s} < \frac{\Delta W_i}{C_i}$ (see proof in the annexes)



(fig.8)

5.4 Dynamic of the model

The purpose of the subsidies is to accelerate the NGN coverage of an area (region, country). The dynamic of the coverage in this model is the decrease in costs which turn non profitable areas into profitable ones and therefore encourage operators to invest. Subsidies act like a catalyst which improves profitability, stimulates and eventually anticipates private investment in an area. It is also possible to determine the amount of time it would have taken for the area to be covered without any subsidies. ($\underline{t}$ for consumer subsidies).

Depending on the amount of time the public authorities hope to save when they anticipate the amount of time it will take to cover the area, they will choose the amount of the subsidies a_s. If $a_s > a_s^{**}$ then they should opt for infrastructure subsidies and if $a_s < a_s^{**}$ then they should opt for consumer subsidies to be the most cost effective regarding Welfare.

However, each year, market conditions change. The cost of an outlet for a given area decreases, and marginal costs may decrease as well. This leads to an increase of the ratio Welfare/cost both for consumer and infrastructure subsidies, but this effect favours infrastructure subsidies even more. Therefore, a_s^{**} decreases and may even fall below a_s, the value of the chosen subsidies. In the beginning phases of coverage, when $a_s < a_s^{**}$, what may be most apt to happen, because costs are so high, it may seem wiser to choose consumer subsidies. However, they could become less efficient than infrastructure subsidies at any time if a_s^{**} becomes too low.

In order to formulate a judicious opinion on the respective efficiency of the two subsidy models, a realistic example is essential.

6 Example: a numerical case of an average country

Imagine a country with 10 million households. The infrastructure costs and the distribution of the population given in chart 1 below, come from the Idate study (Pouillot et al. 2009).

The population is distributed into seven zones with different densities as follows:

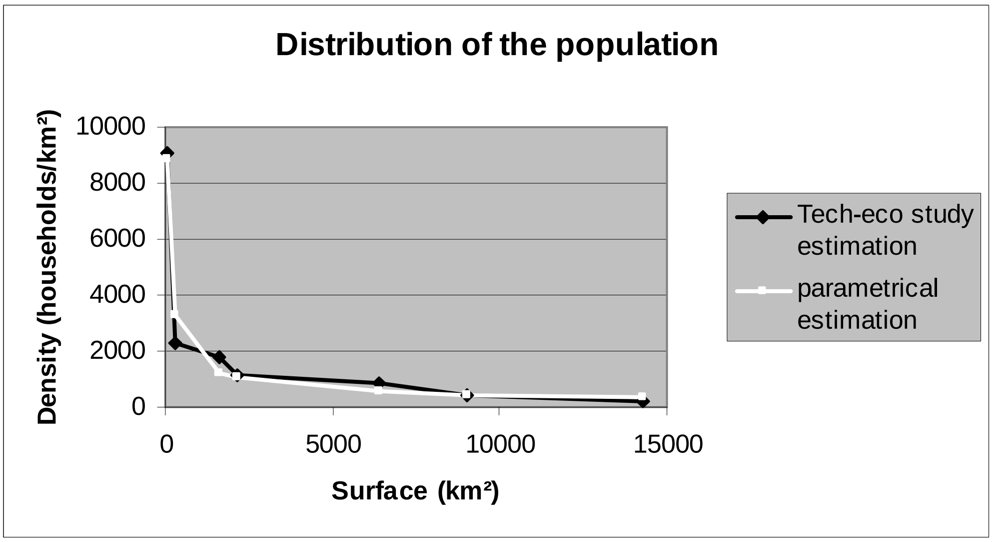
Type of zone	Highly concentrated urban	Wide area concentrated urban	High rise suburban	Medium density urban	Residential suburban	Concentrated rural	Wide area rural	Total country
Percentage of the population	5%	5%	25%	5%	40%	10%	10%	100%
average number of people per household	2.2	2.2	2.8	2.2	2.8	2.5	2.5	2.65
Number of inhabitants (million)	1.31	1.31	6.57	1.31	10.52	2.63	2.63	26.29
Number of households million)	0.6	0.6	2.35	0.6	3.76	1.05	1.05	10.00
Average population density (per km²)	20 000	5 000	5 000	2 500	2 500	1 000	500	3 775
Average households density (per km²)	9 091	2 273	1 786	1 136	893	400	200	1 565
Surface (km²)	66	329	1 643	2 169	6 376	9 005	14 264	14 264

(chart.1)

We can formulate the relation between the density and the surface. This relation can be formulated as a function in the form of: $d(S) = A_1 S^{-B_1}$

Where A_1 and B_1 are constant positive parameters.

The graph below (fig.9) illustrates this relation and its parametrical estimation.



(fig.9)

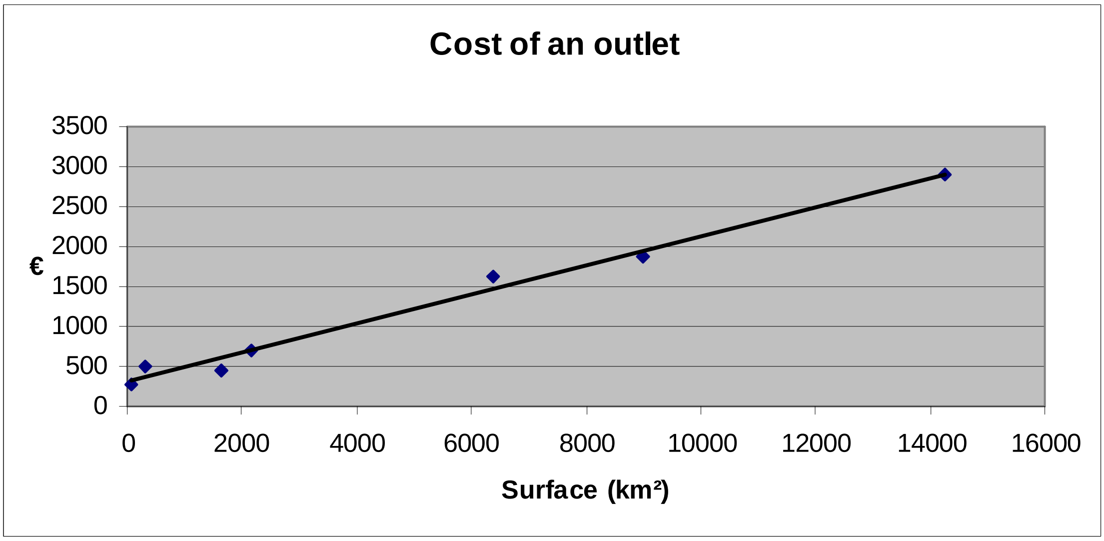
We have chosen $A_1 = 115078$ and $B_1 = 0.6119$ in the figure above for the parametrical estimation. The coefficient of determination $R^2 = 0.9132$ demonstrates that the parametrical model is relevant.

The cost of an outlet using the GPON 32 technology as referred to in the Idate study in chart.2. (the details of the chart are explained in the annexe)

Type of zone	Highly concentrated urban	Wide area concentrated urban	High rise suburban	Medium density urban	Residential suburban	Concentrated rural	Wide area rural
cost of outlet (€)	274	502	450	701	1 635	1 875	2 895
Service installation and terminal supply (€)	450	450	450	450	450	450	450

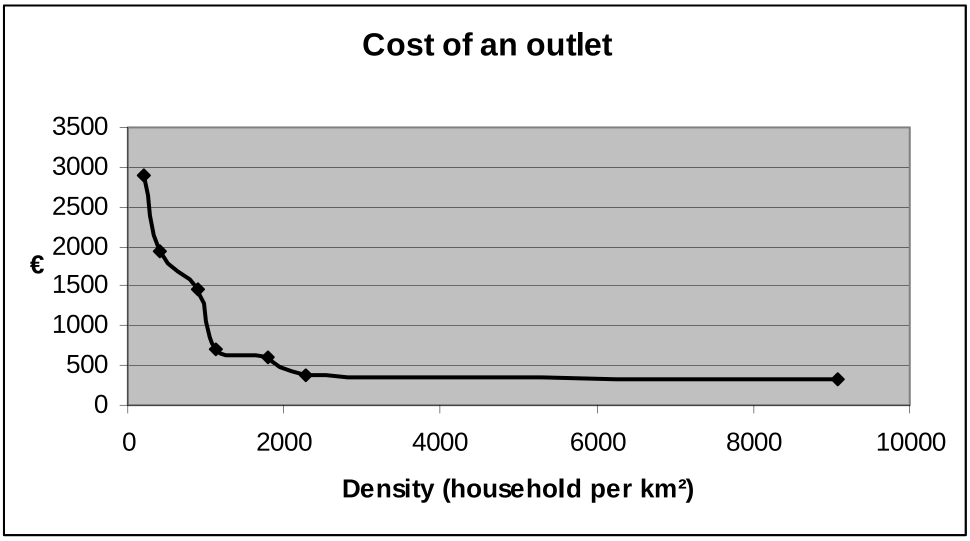
(chart.2)

We can formulate the relation between the cost and the surface area covered. This relation can be formulated as a linear function: $c_{outlet}(S) = A_2 S + B_2$. (fig.10)



(fig.10)

We have chosen $A_2 = 0.182$ and $B_2 = 310.25$ for the parametrical estimation, the coefficient of determination $R^2 = 0.9863$ shows that the parametrical model is relevant.
We check that $c_{outlet}(d)$ is convex and decreases in d (fig.11) like it is indicated in chapter 2.



(fig.11)

We will represent the evolution of the coverage (covered surface) over time in different cases.
In each case, we will compare, on the one hand, the amount of the subsidies generated by consumer subsidies and, on the other hand, by infrastructure subsidies for the same covered surface covered.

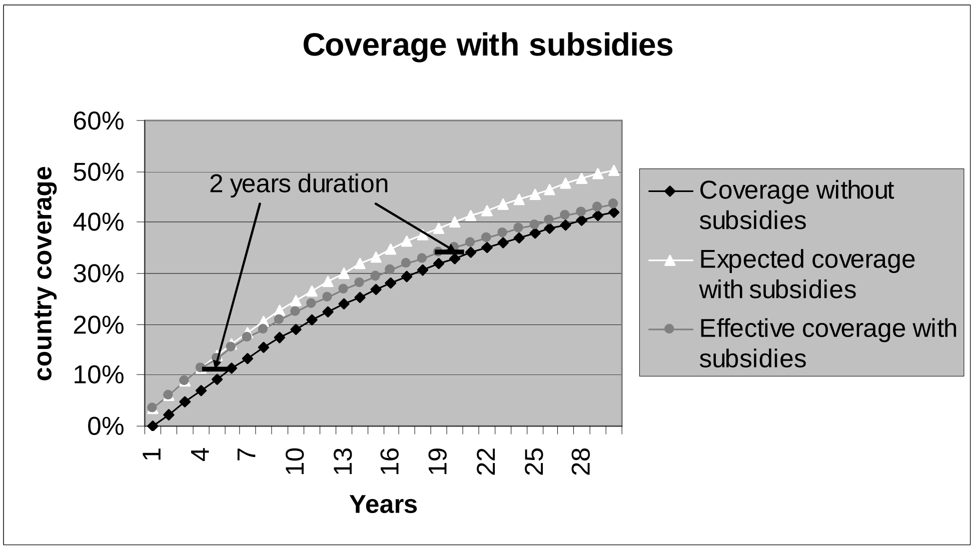
The infrastructure costs decrease by a ratio $\delta = 1\%$
Let us assume the marginal cost is composed of c_0, a portion which is constant, and $c_1(t) = c_1(0)(1-\theta)^t$, a portion which decreases over time, $c = c_0 + c_1$.

The constant portion corresponds to the acquisition and administrative costs that we will suppose are constant and the one which decreases over time due to technical progress θ corresponds to service installation and supply of the terminal.
As an example: $c_0 = 12$ € $c_1 = 13(1-\theta)^t$ and $\theta = 10\%$
Therefore at $t = 0; c = 25$ €
The coefficient $\tau = 0.176$
The substitutability coefficient $\sigma = 0{,}5$ in the competition area.
The details concerning the origin of the values of the parameters can be found in the annexe.

Let us assume that the covered surface area is nil at $t = 0$, therefore the cost of an outlet is $c_{outlet} = B_2$ and therefore the willingness to pay: $a = \sqrt{4B_2\tau} + c = 39.90$ €. This value of a will generate $p_m = 32.45$ € corresponding to the price the monopoly will set.

At the end of the period, the ultra broadband will cover less than 50 % of the total surface area and about 60% of the population.

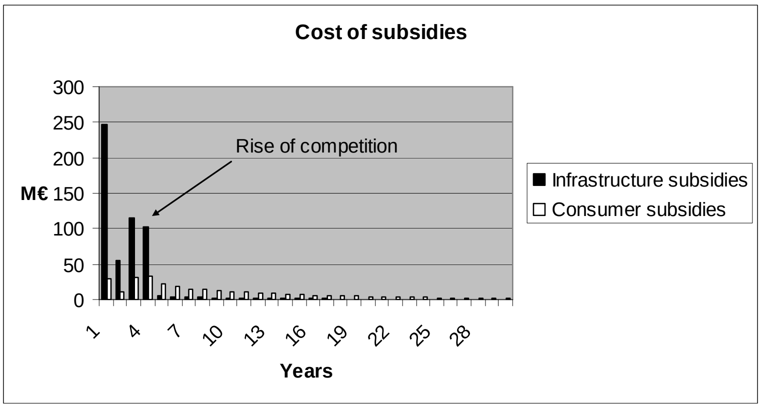
Scenario 1 : slow coverage $t_s = 2$; the amount of subsidies $a_s - a = 2$ € per newcomer. The graph below illustrates the coverage in scenario 1 (fig.12)



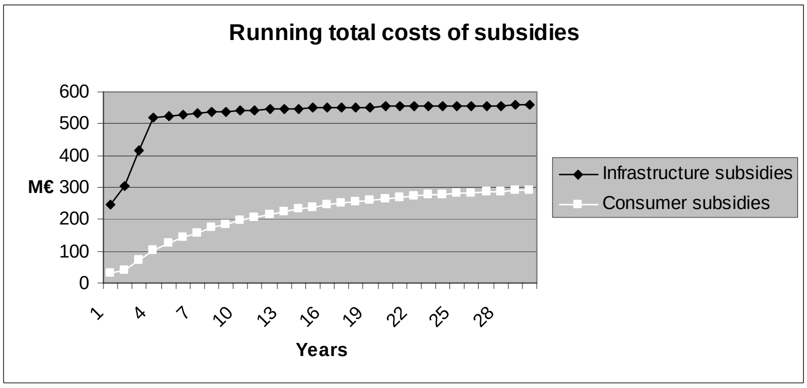
(fig.12)

In the first four years of the roll out, a 2-year period of subsidies, t_s, is long enough, but becomes too short in the fifth one.
From this moment on, the effective coverage, (gray curve), with the subsidy of 2€/month per newcomer, falls below the expected coverage (white curve). The duration becomes lower than optimal: $t_s < \underline{t}$. The operators will invest less than expected because the coverage without subsidies (black curve) will take $\underline{t}$ which is longer the 2-year period of subsidies, to reach the same coverage. Therefore if the operators had invested as expected, with the subsidies cancelled after only two years, this would have led to a fall in the demand as it is explained in paragraph 3.3.

The cost of the subsidies is clearly lower for consumer subsidies as we can notice in the following graphs (fig.13) and (fig.14):



(fig.13)

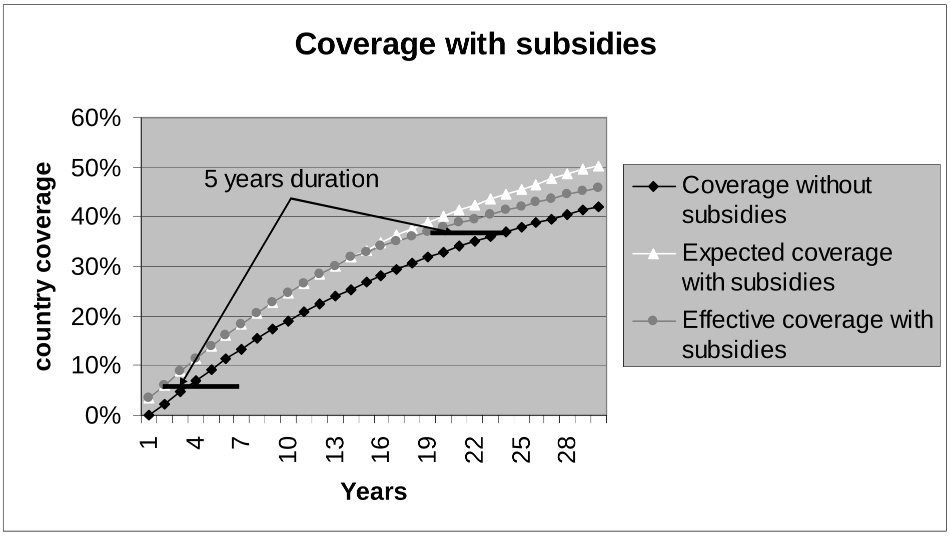


(fig.14)

However, it is apparent that from the fifth year onwards, the cost of consumer subsidies increases faster than infrastructure subsidies. From that time on, the actual coverage slows down, therefore infrastructure subsidies decrease heavily. In contrast, consumer subsidies remain high but their efficiency is reduced by the insufficient duration.

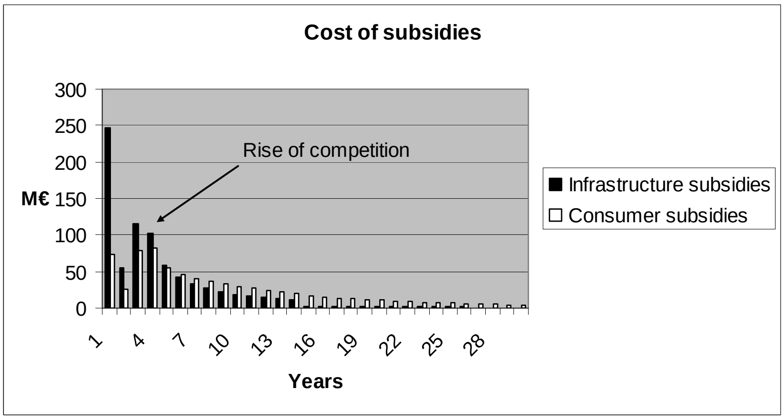
As of the fifth year, the infrastructure subsidies appear to be cheaper, or else it will be necessary to increase the duration.

Scenario 2 : a longer duration: $t_s = 5$; the amount of subsidies: $a_s - a = 2$ € per newcomer. The graph below illustrates the coverage in scenario 2 (fig.15)

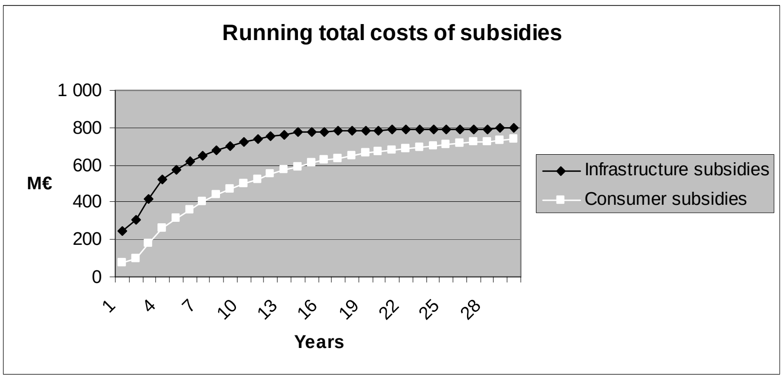


(fig.15)

In this scenario, an increase in the duration of the subsidies allows the maintenance of the the expected coverage for 15 years. However, the cost of consumer subsidies will increase dramatically. In particular, during the first years, the duration is too long $t_s > \underline{t}$ and therefore consumer subsidy costs are too expensive for the results attained. The ideal duration for the subsidies is $t_s = \underline{t}$, but because $\underline{t}$ increases over time, and it is difficult to constantly change t_s, the duration. However it could be a good idea to readjust it regularly.
The subsidy costs are illustrated in the following two graphs (fig.16) and (fig.17):



(fig.16)

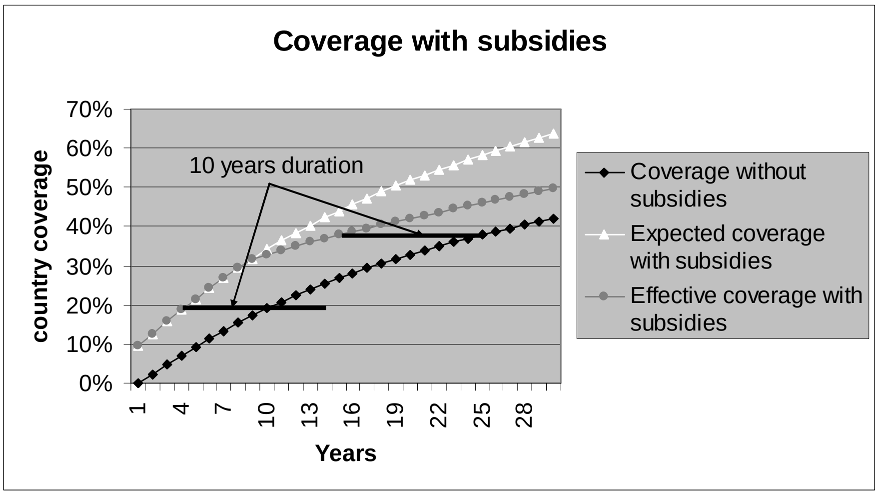


(fig.17)

In the first five years, in spite of the long duration, consumer subsidies remain clearly cheaper than infrastructure subsidies. The total cost of subsidies is higher than in scenario 1.

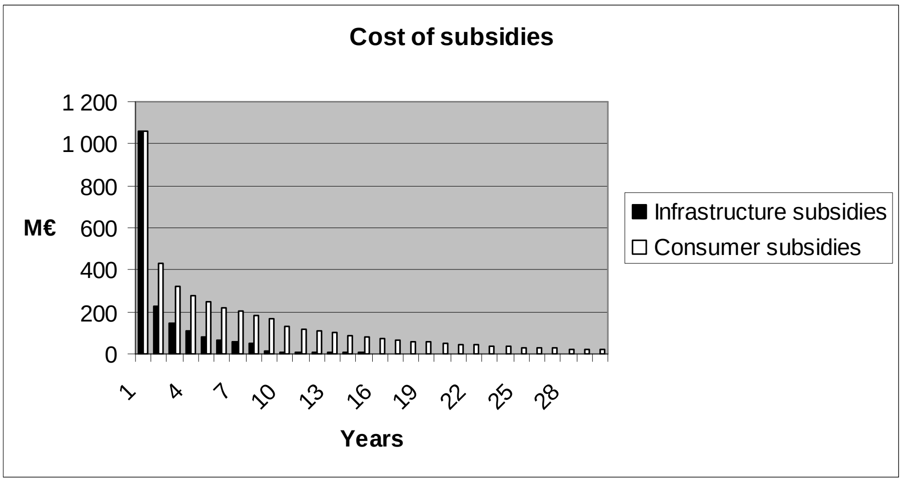
Scenario 3 : quick and expensive coverage $t_s = 10$; the amount of subsidies $a_s - a = 5€$ per newcomer.

The graph below illustrates the coverage in scenario 3 (fig.18)

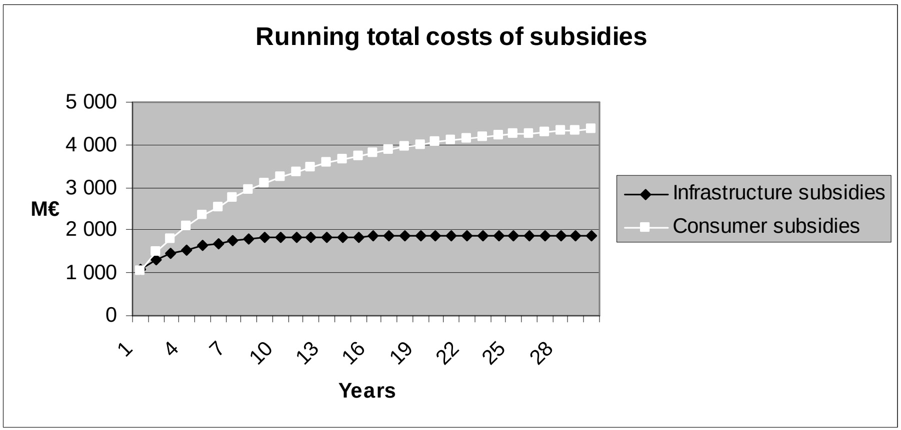


(fig.18)

In the first year of the subsidies, the coverage reaches about 1600 km² more than 10 % of the country's surface area . This represents 1.5 million households for infrastructure subsidies and about 2 million for consumer subsidies.
The 10-year period of subsidies is long but the subsidy amount, 5€/month, is also high and eventually it is not sufficient after the ninth year.
The subsidy costs are illustrated in the two following two graphs (fig.19) and (fig.20):



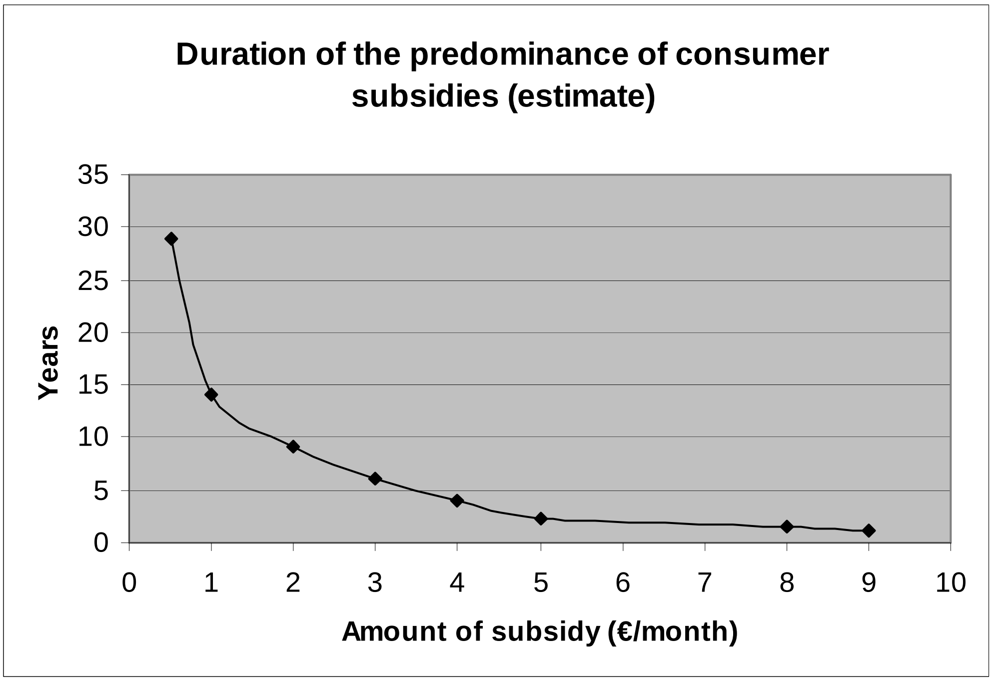
(fig.19)



(fig.20)

As noticed in paragraph 5.2, the cost is higher for consumer subsidies due to the high level of subsidies. The cost is very high, more then 1 billion € for the first year and about 5 billion € for consumer subsidies and "only" 2 billion € for infrastructure subsidies!
As demonstrated in fig.19, infrastructure subsidies are better fitted for quick and expensive roll outs rather than consumer subsidies.
The quicker and more expensive the coverage is, the more adapted infrastructure subsidies will be.
Generally, at the beginning of the coverage period, the consumer subsidies are better fitted, and remain better fitted for a certain length of time. The higher the amount the subsidy is, the shorter the length of time will be.

In the following scenario, the duration of the subsidy has been adapted on a yearly basis, $t_s = \underline{t}$, in order to determine the optimal length of time that consumer subsidies are best fitted. The following results are obtained (fig.21):



(fig.21)

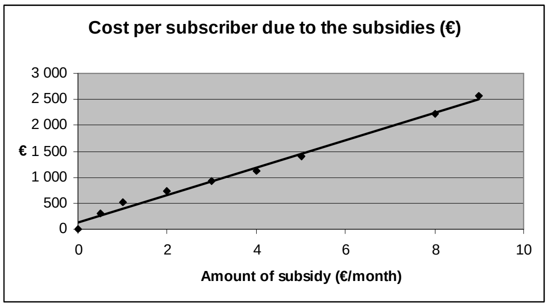
Above 9€/month, the predominance of consumer subsidies lasts less than a year.

The chart below estimates: the total cost, the number of subscribers and the cost per subscriber, for a 30-year roll out, using different values for the amount of subsidy. Coverage starts with consumer subsidies during the time it is the most efficient and finishes with infrastructure subsidies (chart.3).

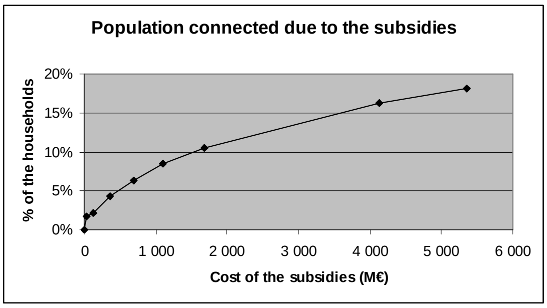
Amount of subsidy (€/month)	Subscribers (000)	Subsribers due to subsidies (000)	Total cost (M€)	Cost/subsriber due to subsidies (€)
0	5 378	0	0	0
0,5	5 551	173	38	220
1	5 596	218	128	587
2	5 810	432	369	854
3	6 018	640	690	1 078
4	6 222	844	1 094	1 296
5	6 422	1 044	1 683	1 612
8	7 002	1 624	4 137	2 547
9	7 190	1 812	5 353	2 954

(chart.3)

When newcomers who subscribe only because of the incentive subsidies granted, the subsidy costs per subscriber increase. As the costs per subscriber increase, the proportion of households covered increases less and less quickly in relation to the total cost of the subsidies, as the figures below illustrate (fig.22) and (fig.23).



(fig.22)



(fig.23)

The above are generated in relation to the decreasing ratio of the costs δ for the infrastructure and θ for the marginal costs. The predominance of the consumer subsidies increases in δ and θ, therefore the total costs of the subsidies and the cost per subscriber due to the subsidies decreases in δ and θ.

7 Conclusion

The consumer subsidies are all the more relevant when the speed of the coverage without subsidies is dynamic. Consumer subsidies are cheaper than infrastructure subsidies when the coverage without subsidies is dynamic enough and the example above shows that this should be the case at the beginning of the roll out.
Demand-side subsidies, from a dynamic point of view, do not affect only the consumers located in the already covered areas, but they also encourage the

investment in non served areas because the threshold of profitability becomes easier to reach.
As soon as the investment incentive is high enough, i.e, the roll out without subsidies is fast enough to allow the duration of the subsidies to remain short, consumer subsidies are truely cost effective.
When the speed of the roll out without subsidies tends to slow down over time as it reaches less dense areas. Therefore, the duration of the consumer subsidies needs to increase over time to remain optimal. This, in turn, increases the cost of the subsidies and deters investment in unserved areas. In such cases, subsidies affect almost exclusively the already covered zones. This is consistent with Goolsbee [3] who considers that demand-side subsidies affect only the already covered areas. More generally, the more the roll out without subsidies is dynamic, the more the dynamic point of view is relevant and, contrarily, when it is not fast enough, the static point of view becomes more relevant. In the case of a new infrastructure roll out, like Next Generation Infrastructure which leads to pull fibre cables, the dynamic effect may be significant especially in the beginning of the roll out. Notice that in the example in section 6, the values of δ and θ chosen are quite low. Actually, the roll out of the infrastructure improves the operators' knowhow and efficiency, (learning by doing). This paper has not enquired about the effect on how the consumer subsidies stimulate innovation and the deployment of new usages. This impact should be positive and should improve consumer subsidies. Most of the aforementioned authors have noticed that people's predisposition and skills for using new technologies had a positive effect on Broadband penetration and connection speed. The reverse should be true, because application and content providers should be encouraged to provide more applications and more contents when a greater portion of the population is connected. This, in turn, would also improve people's familiarity with new technologies. There is no clear evidence of this phenomenon yet, but I think it is a good strategy for future research.
The relevant policy implications that authorities should take into account are: Firstly, the estimation of the rhythm of infrastructure rollout without subsidies. Secondly, the choice of the duration of the subsidies. Thirdly, the selection of areas where consumer subsidies are the most efficient way for the duration they have chosen. Third, the announcement of the subsidy rules (direct subsidies, taxes decreases, etc), specifying in particular, the duration of the measure. Operators have to know this duration to plan the areas where they intend to invest. Fourthly, regularly adjusting the duration so that it remains at the optimal level and forecasts the approximate moment in time when the infrastructure subsidies would be the best way for the remainder of the roll out.

8 Annexes

8.1 Proof of 5.1

Indeed, if $\underline{di}_0 = \underline{ds}_0$ then $\alpha = 1 - \left(\frac{a-c}{a_s - c}\right)^2$ then

$\left(\frac{a-c}{2-\sigma}\right)^2 \left(\frac{1-\sigma}{1+\sigma}\right) = \left(\frac{a_s-c}{2-\sigma}\right)^2 \left(\frac{1-\sigma}{1+\sigma}\right)(1-\alpha)$ and so

$c_{outlet}(\underline{d}_\sigma) = c_{outlet}(\underline{ds}_\sigma)(1-\alpha)$ and yet $c_{outlet}(\underline{d}_\sigma) \equiv c_{outlet}(\underline{di}_\sigma)(1-\alpha)$, see figure 5, therefore $c_{outlet}(\underline{di}_\sigma) = c_{outlet}(\underline{ds}_\sigma)$ and thus $\underline{di}_\sigma = \underline{ds}_\sigma$

8.2 Proof of 5.2

For the consumer subsidies we choose the optimal value for the duration $t_s = \underline{t}$.
The cost per newcomers is:

$\frac{C_s}{y_s} = (a_s - a)\underline{t}$ for consumer subsidies and

$$\frac{C_i}{y_i} = \frac{\left(\int_{S_c}^{\overline{S_{ci}}} c_{outlet}\left(d(S)\right) d(S)\, dS + \int_{S_m}^{\overline{S_{mi}}} c_{outlet}\left(d(S)\right) d(S)\, dS \right)\alpha}{(q_c - q_m)\int_{S_c}^{\overline{S_{ci}}} d(S)\, dS + q_m \int_{S_m}^{\overline{S_{mi}}} d(S)\, dS}$$ for infrastructure subsidies

We can insert $\frac{C_i}{y_i}$ between two values which are easier to work with:

$\frac{c_{outlet}(\underline{d}_\sigma)\,\alpha}{\max(q_m, q_c - q_m)} < \frac{C_i}{y_i} < \frac{c_{outlet}(\underline{\underline{d}})\,\alpha}{\min(q_m, q_c - q_m)}$ When α is low, $\frac{C_i}{y_i}$ tends to come closer to the left side's term, whereas when α increases $\frac{C_i}{y_i}$ tends to come closer to the right side.

$$\frac{c_{outlet}(\underline{d}_\sigma)}{\max(q_m, q_c - q_m)}\left(1 - \left(\frac{a-c}{a_s - c}\right)^2\right) < \frac{C_i}{y_i} < \frac{c_{outlet}(\underline{\underline{d}})}{\min(q_m, q_c - q_m)}\left(1 - \left(\frac{a-c}{a_s - c}\right)^2\right)$$

Let us rewrite $\frac{C_s}{y_s} = \frac{(a_s - a)\ln\left(\frac{a-c}{a_s-c}\right)^2}{\ln(1-\delta)}$

8.3 Proof of 5.3

Let us divide the newcomers into two groups: Newcomers in the competition area and newcomers in the monopoly area.
Newcomers generated by consumer subsidies:

$$y_s = \underbrace{(q_{cs} - q_c)\int_0^{\overline{S_c}} d(S)\, dS + (q_{cs} - q_m)\int_{\overline{S_c}}^{\overline{S_{cs}}} d(S)\, dS}_{\textit{competition area}} + \underbrace{(q_{ms} - q_m)\int_{\overline{S_{cs}}}^{\overline{S_m}} d(S)\, dS + q_{ms}\int_{\overline{S_m}}^{\overline{S_{ms}}} d(S)\, dS}_{\textit{monopoly area}}$$

Newcomers generated by infrastructure subsidies:

$$y_i = \underbrace{(q_c - q_m)\int_{S_c}^{\overline{S_{cs}}} d(S)\,dS}_{competition\ area} + \underbrace{q_m \int_{S_m}^{\overline{S_{ms}}} d(S)\,dS}_{monopoly\ area}$$

We notice that if $\overline{S_{ms}} = \overline{S_{mi}}$ then $\overline{S_{cs}} = \overline{S_{ci}}$ because $c_{outlet}(\underline{di}_\sigma) = c_{outlet}(\underline{ds}_\sigma)$

It is evident that $y_s > y_i$

Let us assume $yc_s = \underbrace{(q_{cs} - q_c)\int_0^{\overline{S_c}} d(S)\,dS + (q_{cs} - q_m)\int_{S_c}^{\overline{S_{cs}}} d(S)\,dS}_{competition\ area}$;

$$ym_s = \underbrace{(q_{ms} - q_m)\int_{S_{cs}}^{\overline{S_m}} d(S)\,dS + q_{ms}\int_{S_m}^{\overline{S_{ms}}} d(S)\,dS}_{monopoly\ area}$$

$yc_i = \underbrace{(q_c - q_m)\int_0^{\overline{S_{cs}}} d(S)\,dS}_{competition\ area}$ and $ym_i = \underbrace{q_m \int_{S_m}^{\overline{S_{ms}}} d(S)\,dS}_{monopoly\ area}$

It is obvious that $y_s > y_i$; $yc_s > yc_i$ and $ym_s > ym_i$

The increase of Consumer Surplus generated by consumer subsidies is:

$$\Delta CS_s = (a_s - p_c)\,yc_s + (a_s - p_m)\,ym_s = (a_s - c)\left(\frac{yc_s}{2-\sigma} + \frac{ym_s}{2}\right)$$

The consumer surplus with infrastructure subsidies:

$$\Delta CS_i = (a - p_c)\,yc_i + (a - p_m)\,ym_i = (a - c)\left(\frac{yc_i}{2-\sigma} + \frac{ym_i}{2}\right)$$

The variation of the profit for consumer subsidies:

$$\Delta\pi_s = (p_c - c)\,yc_s + (p_m - c)\,ym_s = (a_s - c)\left(\frac{(1+\sigma)yc_s}{2-\sigma} + \frac{ym_s}{2}\right)$$

The variation of the profit for infrastructure subsidies:

$$\Delta\pi_i = (p_c - c)\,yc_i + (p_m - c)\,ym_i = (a - c)\left(\frac{(1+\sigma)yc_i}{2-\sigma} + \frac{ym_i}{2}\right)$$

The variation of welfare for consumer subsidies:

$$\Delta W_s = \Delta CS_s + \Delta\pi_s = (a_s - c)(yc_s + ym_s) = (a_s - c)\,y_s$$

The variation of welfare for infrastructure subsidies:

$$\Delta W_i = \Delta CS_i + \Delta\pi_i = (a - c)(yc_i + ym_i) = (a - c)\,y_i$$

It is obvious that $\Delta CS_s > \Delta CS_i$, $\Delta\pi_s > \Delta\pi_i$ and $\Delta W_s > \Delta W_i$

To compare infrastructure and consumer subsidies, we will now calculate the ratio variations of Welfare/Cost. This ratio, like the cost per newcomer, illustrates that there is a value of $a_s = a_s^{**}$ beyond which it is advantageous to opt for infrastructure subsidies and below which it is advantageous to opt for consumer subsidies (fig.8).

But this value of a_s^{**} is higher than the one for the cost per newcomer, $a_s^{**} > a_s^{*}$ because consumer subsidies generate newcomers not only in the newly covered areas but also in the formerly covered areas, and these newcomers improve Consumer Surplus and Welfare.

$$\frac{\Delta W_s}{C_s} = \frac{(a_s - c)}{(a_s - a)\underline{t}}$$

$\frac{\Delta W_i}{C_i} = \frac{(a-c)\,y_i}{C_i}$ and we can insert $\frac{\Delta W_i}{C_i}$ between two values which are easier to work with.

$$\frac{(a-c)\max(q_m, q_c - q_m)}{c_{outlet}(d_\sigma)\,\alpha} > \frac{\Delta W_i}{C_i} > \frac{(a-c)\min(q_m, q_c - q_m)}{c_{outlet}(\underline{\underline{d}})\,\alpha}$$

8.4 Values of chart 2

The Idate's study gives the average investment per subscriber according to the penetration/coverage rate by zone (chart.4). This allows us to calculate the fixed costs of the infrastructure in GPON 32 for each zone and the connection costs (service installation and terminal supply).

Investment per subscriber (€)	Zone						
Penetration/coverage rate	Highly concentrated urban	Wide area concentrated urban	High rise suburban	Medium density urban	Residential suburban	Concentrated rural	Wide area rural
10%	3200	5500	-	7450	-	-	-
20%	1800	3000		3950	-	-	-
30%	1360	2100	1950	2800	5900	6700	10100
50%	1000	1450	-	1850	-	-	-
100%	725	950	-	1150	-	-	-

(chart.4)

The investment includes the connection cost c_c and the cost of an outlet c_{outlet}, accordingly:

$I = c_c + \frac{c_{outlet}}{\lambda}$ where λ is the penetration/coverage rate.

The connection cost can be deduced by the different values of investment for different penetration/coverage rates:

$$c_c = \frac{I_i \lambda_i - I_j \lambda_j}{(\lambda_i - \lambda_j)}$$

where I_i and I_j are the investment according to the penetration/coverage rate respectively λ_i and λ_j

The value of c_c remains quite the same irrespective of the values of λ_i and λ_j.

$c_c = 450$ €

We can deduce the values of the cost of an outlet for each zone:

$c_{outlet} = (I - c_c)\,\lambda$

And we obtain the results indicated in the chart 2.

8.5 *Values of the parameters of paragraph 7*

8.5.1 The value of τ

τ is the sum of the amortization coefficient $\tau_1 = \frac{\rho}{1-(1+\rho)^{-T}}$ and the rate of maintenance.

The Idate's study explains that technical operations costs, including maintenance fees, were measured as a percentage of the investment in the corresponding equipment. This percentage varies depending on the technology deployed, it is estimated at $m = 7\%$ of the investments per year using GPON. That is the reason why m is added to τ_1 which is also a multiplier coefficient of the investment.

If $\rho = 10\%$ and $T = 30\, years$ then $\tau = \tau_1 + m = 0{,}176$

8.5.2 Marginal costs

The marginal costs include a fixed part comprised of acquisition costs, management costs and business expenses as well as a variable part, the connection costs which decreases over time.

Let us assume for the example of paragraph 7 that the connection costs decrease by $tp = 10\%$ per year, representing the technical progress rate.

The annual connection cost price is affected by an amortization coefficient τ_2, in exactly the same as for infrastructure costs

$$\tau_2 = \frac{\rho}{1-(1+\rho)^{-T_2}}$$

T_2 is the lifetime of the household terminal (OLT) which is estimated $T_2 = 5.25 years$

Therefore, the cost price of the connection is $c_c(\tau_2 + m)$

Acquisition costs are also affected by an amortization coefficient $\tau_3 = \frac{\rho}{1-(1+\rho)^{-T_3}}$

T_3 is the lifetime of a subscription which is estimated $T_3 = 3 years$

The acquisition costs are estimated at $c_a = 80$ € per subscriber.

Management and business costs are estimated at $c_m = 60$ € per year and per subscriber

$c_b = 60$ € per year and per subscriber

We can write the marginal costs per month:

$$c = \frac{1}{12}\left(\underbrace{c_c(\tau_2 + m)}_{\text{variable part}} + \underbrace{c_a\tau_3 + c_m + c_b}_{\text{fixed part}}\right)$$

The variation of the marginal costs from year *t* to year *t*+1 is:

$$c(t+1) = c(t) - \frac{c_c(t)(\tau_2 + m)\,tp}{12}$$

When $t = 0$, $c(0) = 25$ € and $c(t) = 12 + 13(1-tp)^t$

References:

Atkinson, R.D., Correa, D.K. & Hedlund, J.A., Explaining international broadband leadership. *The information Technology and Innovation Foundation*

Cava-Ferreruela, I. & Alabau-Munoz, A., 2006. Broadband policy assessment: A cross-national empirical analysis. *Telecommunications Policy*, 30(8-9), 445–463.

Götz, G., 2009. Competition, regulation, and broadband access to the internet. *MAGKS Papers on Economics*.

Gillett, S.E., Lehr, W.H. & Osorio, C., 2004. Local government broadband initiatives. *Telecommunications Policy*, 28(7), 537–558.

Goolsbee, A., 2003. Subsidies, the value of broadband, and the importance of fixed costs. *Broadband: should we regulate high-speed Internet access*.

Jeanjean, F., Social Efficiency of the Investments of Local Authorities in the Access and Backhaul Networks. *(May 28, 2010). Available at SSRN: http://ssrn.com/abstract=1617333*

Leighton, W.A., 2001. Broadband deployment and the digital divide. *Policy Analysis*, 410.

Singh, N. & Vives, X., 1984. Price and quantity competition in a differentiated duopoly. *The RAND Journal of Economics*, 15(4), 546–554.

Valletti, T.M., Hoernig, S. & Barros, P.P., 2002. Universal service and entry: The role of uniform pricing and coverage constraints. *Journal of Regulatory Economics*, 21(2), 169–190.

Wallsten, S., Broadband and unbundling regulations in OECD countries. AEI-Brookings Joint Center Working Paper No. 06-16. Available at SSRN: http://ssrn.com/abstract=906865

Pouillot, D., Montagne, R. & Hain, T., 2009. FTTx business models- Key variables to achieving profitability. *Idate* M81508-February 2009