How to monitor and forecast public deficit every month. The Case of France.

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Monitor and Forecast State deficit.
Use of intra-annual (monthly) information to obtain annual model.
Pure time series approach.
We use aggregation techniques for ARIMA models.
Annual predictions may be updated when new monthly information is released.
Our predictions are very accurate and improve, by far, the official ones.
Why is intra-annual important?

- The Stability and Growth Pact (SGP): The budgetary surveillance mechanisms (article 104, SGP) foresee that the Commission and the Council have to take decisions based on budgetary developments
  - Stability programmes are based on annual predictions

- This increases the need for reliable and frequently updated forecasts for government finances
  - Fiscal policy surveillance of the Council of Ministers
  - Coordination of fiscal policies timely
Why State deficit?

General government deficit is divided in:

- Local authorities: No intra annual and no volatile
- Social Security: No intra annual. Deficit is not very big.
  - Health and unemployment: They could be modelled
  - Pensions: Easy to forecast
- State deficit: Monthly. It is the most important

<table>
<thead>
<tr>
<th>%GDP</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
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<td>General</td>
<td>-1.4</td>
<td>-1.5</td>
<td>-3.2</td>
<td>-4.1</td>
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<tr>
<td>State</td>
<td>-2.5</td>
<td>-2.3</td>
<td>-3.8</td>
<td>-3.9</td>
</tr>
<tr>
<td>Local authorities</td>
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<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Social Security</td>
<td>0.5</td>
<td>0.3</td>
<td>-0.3</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

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Why State deficit?

More evidence:

Evolutions of the State and general government deficits
State deficit: Decomposition

State deficit is the balance between revenues and expenditures. In turn they are the sum of several components:

- **Revenues:**
  - Taxes
  - Other fiscal and non fiscal

- **Expenditures:**
  - Wages and pensions
  - Debt interest payments
  - Military expenditures
  - Other
State deficit: Decomposition

Graphically

Revenues
- 40% VAT
- 9% Other fiscal revenues
- 10% Non fiscal revenues
- 21% Income tax
- 11% Corporate tax
- 9% Tax on oil products

Expenditures
- 34% Wages and pensions
- 14% Debt interest payments
- 11% Military expenditures
- 5% Civilian capital expenditures
- 9% Economic interventions
- 12% Social interventions
- 6% Functioning expenditures
- 9% Other state expenditures
Our approach

- Model, via ARIMA, the monthly components. Sample from January 1995 to December 2003.
- Use econometric techniques of temporal aggregation to infer the annual model from the monthly information.
- Forecast one-step ahead year for all the components.
- Sum the forecasts to get a forecast of the State deficit.
- Update the models and the predictions as new intra annual information is available.
Take a look to graph side of data

Revenues

Corporate tax

Income tax

Tax on oil products

Other fiscal revenues

Non fiscal revenues

VAT

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Take a look to graph side of data

Revenues

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Expenditures

- Wages and pensions
- Debt interest payments
- Military expenditures
- Civilian capital expenditures
- Operating expenditures
- Social interventions
- Other interventions
- Economic interventions

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Take a look to graph side of data

Expenditures

- Wages and pensions
- Debt interest payments
- Military expenditures
- Civilian capital expenditures
- Functioning expenditures
- Social interventions
- Other interventions
- Economic interventions

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We use TRAMO (ARIMA + outliers). General results:

- All series have an unit root in seasonality
- 12 over 14 have seasonal MA component
- 2 over 14 do not have outliers (proxy of discretionary measures)
Temporal Aggregation

Next, we aggregate the monthly observations into annual frequency:

\[ y_T^* = \sum_{j=0}^{11} y_{t-j} = \sum_{j=0}^{11} L^j y_t \]

where \( y_T^* \) is a NON-OVERLAPPING sequence of annual observations.
Temporal Aggregation

We have estimated the monthly model:

\[ \text{ARIMA} (p, d, q) \times (P, D, Q)_{12} \]

\[ \Phi(L^{12}) \phi(L) \Delta^d \Delta^D y_t = \Theta(L^{12}) \theta(L) \varepsilon_t, \]

And we are now interested in the annual model:

\[ \text{ARIMA} (p', d', q') \]

\[ \alpha(B) \Delta^{d'} y^*_T = \eta(B) \varepsilon^*_T, \]

where \( B = L^{12} \) because of the non overlapping.
Temporal Aggregation

The estimated parameters of the annual model are a function of the aggregated observations

Is there a way to link both models? Is there a way to infer the parameters of the annual model from the parameters of the monthly model?

Is there a way to incorporate all the monthly information into the annual model?
The answer is yes. Intuition:

**Monthly Data**

ARIMA(θ)

Data $y_i$ → $\hat{\theta}(y)$

**Annual Data**

ARIMA(α)

Data $y^*_T = \sum_{j=0}^{11} y_{t-j}$ → $\hat{\alpha}(y^*) = \hat{\alpha}(y)$

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Temporal Aggregation

\[ ARIMA(p, d, q) \times (P, D, Q)_{12} \]
\[ \Phi (L^{12}) \phi (L) \Delta^d \Delta^D y_t = \Theta (L^{12}) \theta (L) \varepsilon_t \]

\[ \downarrow \]

\[ ARIMA(p', d', q') \]
\[ \alpha (B) \Delta^{d'} y^*_T = \eta (B) \varepsilon^*_T \]

There are three things to be solved

- The orders \( p', d', q' \)
- The AR parameters in \( \alpha (B) \)
- The MA parameters in \( \eta (B) \)
Temporal Aggregation

The polynomial orders:

\[ p' = p + P \]
\[ d' = d + D \]
\[ q' = \left\lfloor 12^{-1} \left( 11 (p + d + 1) + q + 12Q \right) \right\rfloor , \]

Remember:

- Monthly model: \( ARIMA (p, d, q) \times (P, D, Q)_{12} \)
- Annual model: \( ARIMA (p', d', q') \)
Temporal Aggregation

The AR parameters: Example $AR(1)$

$$y_t = \phi y_{t-1} + \epsilon_t \rightarrow (1 - \phi L)y_t = \epsilon_t$$

\[\downarrow\]

$$d = 0, D = 0, p = 1, P = 0, q = 0, Q = 0$$

\[\downarrow\]

$$d' = 0, p' = 1, q' = 1$$

$$y_T^* = \alpha y_{T-1}^* + \eta \varepsilon_{T-1}^* + \varepsilon_T^* \Rightarrow \alpha?$$

We multiply the monthly model by:

$$T(L) = \frac{1 - \phi L^{12}}{1 - \phi} \sum_{j=0}^{11} L^j$$
Temporal Aggregation

\[ T(L) = \frac{1 - \phi^{12} L^{12}}{1 - \phi L} \sum_{j=0}^{11} L^j \]

And we get easily the AR parameter:

\[ (1 - \phi L)T(L)y_t = (1 - \phi^{12} L^{12}) \sum_{j=0}^{11} y_{t-j} = (1 - \alpha B)y_T^* \]

\[ \Rightarrow \alpha = \phi^{12} \]
Temporal Aggregation

The MA parameters: Example $ARIMA(0, 0, 0) \times (0, 1, 1)$

$$(1 - L^{12})y_t = (1 + \Theta L^{12})\varepsilon_t$$

\[\downarrow\]

$d = 0, D = 1, p = 0, P = 0, q = 0, Q = 1$

\[\downarrow\]

$d' = 1, p' = 0, q' = 1$

$$(1 - B)y^*_T = (1 - \eta B)\varepsilon^*_T \Rightarrow \eta, \sigma^2_{\varepsilon^*}$$

We multiply the model by:

$$T(L) = \sum_{j=0}^{11} L^j$$

which gives

$$(1 - L^{12}) \sum_{j=0}^{11} y_{t-j} = (1 + \Theta L^{12}) \sum_{j=0}^{11} \varepsilon_{t-j}$$
Temporal Aggregation

\[
(1 - L^{12}) \sum_{j=0}^{11} y_{t-j} = (1 + \Theta L^{12}) \sum_{j=0}^{11} \varepsilon_{t-j}
\]

\[
(1 - B)y^*_T = (1 - \eta B)\varepsilon^*_T
\]

We equate the MA variance-covariance matrices

\[
\begin{align*}
12 \left(1 + \Theta^2\right) \sigma^2_\varepsilon &= (1 + \eta^2) \sigma^2_{\varepsilon^*} \\
\text{Variance monthly} &\quad \text{Variance annual}
\end{align*}
\]

\[
12\Theta \sigma^2_\varepsilon = \eta \sigma^2_{\varepsilon^*}
\]

Covariance order 1 monthly \quad Covariance order 1 annual
In general: Multiply the model by the polynomials

\[
T(L) = \prod_{i=1}^{p} \left[ \frac{1 - \delta^k_i L^k}{1 - \delta_i L} \right] \left[ \frac{1 - L^k}{1 - L} \right]^{d_k-1} \sum_{j=0}^{\infty} L^j
\]

\[
A(L) = \prod_{j=1}^{P} \left[ \frac{1 - (\tau_j L)^{ks^*}}{1 - (\tau_j L)^s} \right] \left[ \frac{1 - L^{ks^*}}{1 - L^s} \right]^D
\]

where \(\delta_i, i = 1, \ldots, p\) are the inverted roots of the polynomial \(\phi(L)\), \(\tau_j, j = 1, \ldots, P\) are the inverted roots of the polynomial \(\Phi(L^{12})\) and \(s^* = s/k\).
Temporal Aggregation

- The model is multiplied by these two polynomials.
- The powers of the resulting polynomials are divisible by the aggregation frequency (12 in our case).
- That is, the only non-zero coefficients in

\[ T(L)A(L)\Phi\left(L^{12}\right)\phi(L)\Delta^d\Delta^Dy_t \]

should be those of \( L \) divisible by the lower frequency (12 in our case).

And the orders of the annual polynomials are given by:

\[
\begin{align*}
p' &= p, P' = P \text{ or } p' = p + P \\
q' &= \left[ k^{-1}(p + d + 1)(k - 1) + (P + D)s^*k + (Q - P - D)s + q \right] \\
d' &= d, D' = D \text{ or } d' = d + D
\end{align*}
\]
Annual Forecasting

- We aggregate the model and we forecast one year.
- We start with information up to January 2002.
- Then EVERY MONTH the models, and the predictions, are UPDATED.
- We do this updating for the whole 2002 and 2003.
- We compare our forecasts with:
  - Monthly forecasts
  - French Official forecasts
Annual Forecasting 2002

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Annual Forecasting 2003

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Conclusions 1/2

- Methodology to forecast annual variables exploiting intra annual information.
- Based on simple ARIMA models.
- Forecasts are updated as new information is released.
- Paramount importance for the Eurozone policy makers.
- Effective surveillance method.
- Applied to French State deficit successfully.
This methodology has a great number of advantages for EU macroeconomic data:

- It exists the cross-section equivalent. Important applications:
  - Inflation components
  - From country numbers to EU numbers (aggregate 25 countries)
  - National Accounting

- But we may also disaggregate
  - Annual GDP into monthly GDP?
  - National into regional numbers?
  - Annual ESA95 accounting into quarterly?