A model based approach for benchmarking seasonally adjusted time series

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A MODEL BASED APPROACH FOR BENCHMARKING SEASONALLY ADJUSTED TIME SERIES

Summary: When economic time series are seasonally adjusted directly, a number of the original connections between them are destroyed. However, it is desirable that seasonally adjusted aggregates and underlying variables are aligned perfectly. It is also desirable that seasonally adjusted time series are perfectly aligned to original annual totals. One way to restore the connections is by using a benchmarking model.

The paper presents a multivariate benchmarking model based on Denton’s movement preservation principle. The model consists of an optimisation algorithm under restrictions. The main characteristic of this algorithm is that seasonally adjusted quarter-to-quarter movements are preserved as much as possible while the connection is restored. Simultaneously, all accounting rules applicable are satisfied. The algorithm uses reliability weights in order to deal with quality differences of seasonally adjusted time series.

Benchmarking models are flexible enough to restore the original connection and simultaneously deal with additional requirements. Of course, it is also important to test the applicability of the model in practice and analyse the quality of the results. The model is illustrated with an empirical application on Dutch economic time series of quarterly GDP and its underlying variables. The problems we encountered in practice will be discussed in this paper.

Keywords: Seasonal adjustment, inconsistency, quarterly time series, benchmarking models
1. Introduction

Sub-annual time series of economic variables usually contain seasonal patterns. These patterns may disturb the comparability between periods, months or quarters, within the time series. Users of data, in the private and the public sector, therefore often prefer seasonally adjusted data.

Seasonal adjustment consists in identifying and removing the seasonal pattern from the other components of time series: trend, cycle and the irregular component. When related economic time series are seasonally adjusted, original connections between them and between their annual totals are usually lost. This fact is well known, purely natural and can easily be explained. Therefore, Statistics Netherlands generally publishes these inconsistent time series using the direct method. Nevertheless, even though it is purely natural, the inconsistency is hard to understand and undesired by users.

One easy way to avoid this inconsistency is to seasonally adjust the underlying variables, adjust them to annual totals and to sum them up to the aggregate totals. However, when using this indirect method, seasonally adjusted aggregates might look noisier in comparison with directly seasonally adjusted aggregates, as explained in Laniel and Fyfe (1989). This is a well-known problem in seasonal adjustment.

Another well-known method used for repairing inconsistency is to first seasonally adjust both aggregates and underlying variables. Secondly, the consistency must be restored both between aggregates and underlying variables and between variables and their corresponding annual totals. This process is called benchmarking. A benchmarking model can be used taking into account that the seasonally adjusted aggregates are usually more reliable than their underlying variables.

In Section 2, this paper presents a systematic benchmarking model based on Denton’s movement preservation principle. This model can be used to restore consistency.

The benchmarking model is tested by restoring consistency between the Dutch seasonally adjusted quarterly GDP and its underlying expenditure variables: consumption, gross fixed capital formation, exports, imports and increase in stocks. The results of this scenario are presented in Section 3. Conclusions and recommendations can be found in Section 4.
2. The benchmarking model

Seasonal adjustment consists in identifying and removing the seasonal pattern from the other components of time series: trend, cycle and the irregular component. However, it is not always possible to distinguish the seasonal pattern identified in the underlying variables from the irregular component. When large incidental effects are visible in the underlying variables but not in the aggregate, discrepancies arise between seasonally adjusted time series. If corrections for calendar effects are made separately, discrepancies occur automatically. Because of this phenomenon, natural discrepancies exist between the seasonally adjusted aggregates and the seasonally adjusted underlying variables (contemporaneous inconsistency) and between seasonally adjusted variables and their corresponding annual total (temporal inconsistency).

2.1 Models for benchmarking

A multivariate benchmarking method can be used to restore the contemporaneous and temporal consistency. This method can be based on several different statistical models. Some authors (e.g. Harvey (1990)) describe a state space approach, while others (e.g. Broemeling (1985)) describe models based on Bayesian approaches.

The benchmarking model proposed in this paper is based on the (univariate) method of Denton (1971). The Denton method\(^1\) is based on the “movement preservation principle”, which aims at avoiding step problems. Di Fonzo and Marini (2003) extended the Denton method to perform multivariate benchmarking. The proposed in this section is based on their work. However, some extensions – such as introducing reliability weights – were required. These extensions are explained in more detail in Bikker and Buijtenhek (2006).

2.2 The multivariate Denton method

2.2.1 Outline

This section will present a benchmarking method based on a Denton-type algorithm for optimisation under restrictions. The main characteristic of this algorithm is that quarter-to-quarter movements are preserved as much as possible, while consistencies are restored. The algorithm uses reliability weights in order to deal with quality differences of input data.

2.2.2 Specification of the model

After a process of data collection, integration and seasonal adjustment, this problem starts with a set of inconsistent seasonally adjusted quarterly time series \(x_{ik}\), where \(i\)

\(^1\) The Denton approach will be explained in some detail in the next section.
is a label distinguishing different series, and index \( k = 1, \ldots, N \) enumerates the quarters. The relation between these inconsistent seasonally adjusted quarterly data and the unknown consistent results \( \tilde{x}_{ik} \) can be described as follows

\[
\begin{align*}
    x_{ik} &= \tilde{x}_{ik} + \varepsilon_{ik} \\
\end{align*}
\]

where \( \varepsilon_{ik} \sim (0, \Omega) \) are random disturbances, specified below.

Matrix \( X^* \) can be described as

\[
X^* = \begin{bmatrix}
    x_{11} & \cdots & x_{1N} \\
    \vdots & \ddots & \vdots \\
    x_{M1} & \cdots & x_{MN}
\end{bmatrix}.
\]

(2)

In the following sections we shall represent the inconsistent seasonally adjusted quarterly time series as a single vector \( x = \text{vec}(X^*) \), which contains data for all \( M \) variables during \( N \) quarters. In this way the model in (1) can be reformulated as follows

\[
\begin{align*}
    x &= \tilde{x} + \varepsilon \\
\end{align*}
\]

with \( \varepsilon \sim (0, \Omega) \). Analogously, vector \( y \) contains the corresponding annual totals \( y_{ij} \) where index \( j = 1, \ldots, N/4 \).

To complete the model, matrix \( \Omega \) (the covariance matrix of the error terms) must be specified. Stone et al. (1942) specified matrix \( \Omega \) as the variance matrix \( V_x \) of the data. They assumed that the discrepancies made during the process of seasonal adjustment are not correlated between different series, thus \( E(\varepsilon_{ik} \varepsilon_{it}) = 0 \) for each quarter \( k \) and for each \( s \) and \( t, s \neq t \), in \( M \). This also seems a reasonable assumption in this case. Without corrections for calendar effects, the initial differences between the seasonally adjusted quarters and the original annual totals would be zero. Taking \( \Omega = V_x \) would lead to an appropriate model for smoothing the differences between the aggregates and the underlying variables in each quarter.

However, this problem also concerns temporal inconsistency. Taking \( \Omega = V_x \) for this model would create a discontinuity between the fourth quarter of one year and the first quarter of the next year. The reason for this is that the initial differences between the seasonally adjusted quarters and the original annual total may vary between years because of the correction for calendar effects.

The solution to this problem is called the movement preservation principle, first explained in Denton (1971). Di Fonzo (2002) uses this principle to specify matrix \( \Omega \) as follows

\[
\begin{align*}
    x &= \tilde{x} + \varepsilon \\
\end{align*}
\]

(3)

\( \Theta \) Temporal restrictions must still be taken into account for not disturbing this consistency.
\[
\Omega = \begin{cases} 
(D'D)^{-1} & \text{additive} \\
X'(D'D)^{-1} X & \text{proportional,}
\end{cases}
\]  
\tag{4}

where

\[X = \text{diag}(x)\]  
\tag{5}

and \(D = I_M \otimes D_N\), while the \(N \times N\) matrix \(D_N\) is

\[
D_N = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 \\
-1 & 1 & 0 & \cdots & 0 & 0 \\
0 & -1 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & -1 & 1 & 0 \\
0 & 0 & \cdots & 0 & -1 & 1 
\end{bmatrix}.
\]  
\tag{6}

The choice of \(\Omega\) depends on whether an additive or a proportional model is desired.

Not all seasonally adjusted time series have equal reliability, therefore \(\Omega\) is extended with variation matrix \(V\). Let vector \(v\) contain the coefficients of variation or relative standard errors of \(x_{ik}\), \(v_{ik} = \text{SE}(x_{ik})/x_{ik}\), vectorised in the same way as \(x\). Now, let

\[V = \text{diag}(v),\]  
\tag{7}

so \(X'V'VX\) is the variance matrix \(V_x\). It can be proven that \(\Omega\) can now be written as

\[
\Omega = \begin{cases} 
(D(X'V'VX)^{-1}D)^{-1} & \text{additive} \\
X'(D(V'V)^{-1}D)^{-1} X & \text{proportional,}
\end{cases}
\]  
\tag{8}

2.2.3 Consistency

There are two types of consistency within our system: contemporaneous and temporal. The contemporaneous consistency is based on a set of contemporaneous linear restrictions, which are actually the definitions implied by the accounting framework. If the first \(M-1\) variables are underlying variables which are required to sum up to the final aggregate variable, the linear restriction for each quarter \(k\) would be

\[
\sum_{i=1}^{M-1} \tilde{x}_{ik} - \tilde{x}_{Mk} = 0. \quad \tag{9}
\]

The temporal consistency means that the sum of the quarterly flows adds up to the corresponding annual total. These are also linear restrictions, albeit temporal ones,
as they relate to variables in different quarters. For each year \( j \) and each variable \( i \) the temporal linear restriction would be

\[
\sum_{k=j}^{j+4} \tilde{x}_{ik} = y_{ij}.
\]  

(10)

Contemporaneous and temporal linear restrictions could be treated independently, e.g. in an iterative and hierarchical way\(^3\). It is important to realise that from a statistical point of view the optimal solution can only be found if they are treated simultaneously. Together they form one set of linear restrictions, which the benchmarked quarterly figures must satisfy,

\[
\begin{bmatrix}
C_1 \\
C_2
\end{bmatrix} \tilde{x} = C\tilde{x} = \begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} = b,
\]

(11)

where \( C_1 \) and \( C_2 \) represent the contemporaneous and temporal restrictions, respectively. Vector \( b_1 \) contains either the values of exogenous variables or zeros and vector \( b_2 \) contains the annual totals.

2.2.4 The benchmarking formula

The set of restrictions alone does not lead to an unique solution of the model described in Section 2.2.2. Several degrees of freedom remain, from which a solution must be chosen. The best least squares estimator of the model of 2.2.2 is\(^4\)

\[
(\hat{x} - x)'\Omega^{-1}(\hat{x} - x),
\]

(12)

where the least squares estimator for the benchmarked results \( \tilde{x} \) is represented by the vector \( \hat{x} \). The problem is now formulated as a standard quadratic optimisation problem, where the minimum of the objective function (7) must be found, subject to the constraints \( C\tilde{x} = b \).

Using the Lagrangean and a well-known matrix result and (Magnus and Neudecker (1988)) the solution to this problem is given by

\[
\hat{x} = x + \Omega C'(C\Omega C')^{-1}(b - Cx)
\]

(13)

where \((C\Omega C')^{-1}\) is the Moore-Penrose generalised inverse of \( C\Omega C' \), see also Di Fonzo and Marini (2001). If \( C \) has full rank, the Moore-Penrose generalised inverse can be replaced by the normal inverse.

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\(^3\) For an example see Laniel and Fyfe (1989, pp. 464-465).

\(^4\) See for example Sefton and Weale (1995, pp. 13-15)
3. Benchmarking in practice

In this section, the benchmarking model described in Section 2 is tested with Dutch quarterly economic time series.

3.1 Specifying the problem

Quarterly time series for eight different variables were available for the period 1977 to 2004, for namely

- $Y$: Gross Domestic Product by expenditure (GDP)
- $C$: Private Final Consumption Expenditure incl. NPISH
- $G$: Government Final Consumption Expenditure
- $I_g$: Gross Fixed Capital Formation; Government
- $I_o$: Gross Fixed Capital Formation; Other sectors
- $IS$: Increase in Stocks
- $E$: Exports
- $M$: Imports

For all variables, the series both before and after seasonal adjustment were available, but only in current prices. The annual totals of all variables for 1977 and 2004 together with their means are given in the appendix. This will give an impression of the level of each variable. All original series were directly seasonally adjusted using X12-ARIMA (Census method) after temporarily removing outliers. There was also a correction for calendar effects. The GDP is adjusted for calendar effects. The underlying totals are only corrected if the calendar effects are significant. This means that not all seasonally adjusted quarterly time series are consistent with the annual totals. This consistency must be restored. For current prices, the following restriction may be specified:

$$ Y = C + G + I_g + I_o + IS + E - M $$  \hspace{1cm} (14)

The annual totals fulfil this restriction, but the seasonally adjusted series do not. The model described in Section 2 is used to remove the inconsistencies. Before starting to solve the problem, it might be interesting to look at the size of the inconsistencies to be restored.

Table 1 presents the statistics of these discrepancies. The contemporaneous discrepancies are those between the sum of the seasonally adjusted underlying variables and the seasonally adjusted GDP\(^5\). The temporal discrepancies are the discrepancies between the seasonally adjusted quarterly GDP and the original annual totals. The mean of the GDP over all years is used for the relative discrepancies.

\(^5\) Before annual connection. This means that the disturbances are partly because of the annual disturbances
Table 1. Descriptive statistics of the disturbances of the GDP (in million euro)

<table>
<thead>
<tr>
<th></th>
<th>Contemporaneous discrepancies</th>
<th>Temporal discrepancies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td>Relative</td>
</tr>
<tr>
<td>Mean</td>
<td>-87.6</td>
<td>-0.03%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-1624</td>
<td>-0.60%</td>
</tr>
<tr>
<td>Maximum</td>
<td>1226</td>
<td>0.45%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>452.6</td>
<td>-</td>
</tr>
</tbody>
</table>

The discrepancies are relatively small. To restore consistency, three different scenarios are tested based on different coefficients of variation for the variables. The coefficients of variation are given in Table 2. Note that they are all relative and may be multiplied by a common factor without disturbing the results\(^7\).

In the first scenario, all coefficients are equal. Seasonally adjusted aggregates are often assumed to be better than the seasonally adjusted underlying variables. This information is used in the second scenario by adapting the coefficient of variation of the GDP substantially, which means that nearly all corrections to GDP are made to restore temporal consistency. The variable increase in stocks is often assumed to be less reliable than other variables. This information is used in the third scenario by adapting the coefficient of variation of the variable increase in stocks substantially\(^8\).

Table 2. Coefficient of variation in three different scenarios

<table>
<thead>
<tr>
<th>Variable</th>
<th>Scenario</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>GDP</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>Increase in stocks</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>OTHER</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\(^6\) For comparison, GDP in 2004 was equal to 466 billion (see appendix)

\(^7\) It actually does disturb the variances of the results, but variances of the results are not taken into account.

\(^8\) The variable increase in stocks is a very small variable. The coefficient of variation is a relative measure of reliability. The coefficient of variation must be substantially larger to cause any difference.
3.2 Results

In this sub-section, results from the benchmarking experiments are presented in a graphical way. Each graph contains the directly seasonally adjusted series before and after benchmarking (original and result). Only results for the years 2001 to 2003 are given because the differences are relatively large in these years.

It appeared for all scenarios that the adjustments made on the level of each variable were relatively small because the initial differences were also relatively small. For example, the results of the first scenario for GDP (in current prices) are given in Figure 1. The root mean squared relative differences between the results and the original seasonally adjusted time series is 0.35% in the period 2001-2003. For some quarters the results are below the original seasonally adjusted values, for others they are above the original seasonally adjusted values. From the results presented here, the impression might arise that the benchmarking model yields perfectly satisfactory results.

![GDP Graph](image)

Figure 1. Results of the first scenario expressed in current prices for seasonally adjusted GDP, before and after benchmarking (original and result)

As users of seasonally adjusted data are in fact more interested in the quarterly growth rates than the actual levels, from now on the results of the scenarios are mainly presented accordingly. We focus on four variables: GDP, private consumption, exports and increase in stocks. Other variables show similar behaviour depending on their size.
The resulting quarterly growth rates of the first scenario are given in Figure 2. By looking at the growth rates instead of the level, it appeared that in the first scenario the original seasonally adjusted growth rates of the GDP are considerably disturbed in the results. The reason is that differences in growth rates between the seasonally adjusted GDP and the sum of the underlying variables are large. In the first scenario, all variables have equal coefficient of variation, which means that most corrections are made to the largest variable, GDP.

The resulting growth rates for private consumption and exports in the first scenario are also quite different from their original ones, but the adjustments are smaller. The results for increase in stocks are almost the same as the original seasonally adjusted time series. The reason is that the coefficient of variation of all variables is the same and the correction for calendar effects was probably not significant, which means that the adjustment made to fulfil the contemporaneous restrictions is made on the largest variables.

Figure 2. Results of the first scenario expressed in quarterly growth rates for seasonally adjusted GDP, private consumption and exports and the results of increase in stocks before and after benchmarking (original and result)

9 The variable increase in stocks is usually not presented in growth rates as it is a balance value.
The results for 2001 are noisier than those for 2002 and 2003. The reason is that the initial differences between the quarterly growth rates of the seasonally adjusted GDP and the sum of the seasonally adjusted underlying variables were relatively large. In the fourth quarter of 2001, a positive growth rate has changed into a negative one for GDP.

In the second scenario, the coefficient of variation of GDP has changed. The results are given in Figure 3. They show that the quarterly growth rate of GDP has hardly changed because of the smaller coefficient of variation. However, instead the growth rates of the variables private consumption and exports are considerably disturbed.

In the third scenario, the coefficient of variation of the variable increase in stocks has changed substantially. The results of the third scenario are given in Figure 4. The resulting quarterly growth rates of the GDP are again almost equal to the original ones in the first two years. The results of the quarterly growth rates of private consumption and exports are also satisfying. As expected, almost all corrections are made to the increase in stocks, at least in the first two years.

Figure 3. Results of the second scenario expressed in quarterly growth rates for seasonally adjusted GDP, private consumption and exports and the results of increase in stocks before and after benchmarking (original and result)
For 2003, the variable increase in stocks is almost equal to zero. Even this large coefficient of variation has hardly any impact on the results because it is a relative coefficient. To create contemporaneous consistency, changes are made to exports and private consumption.

Figure 4. Results of the third scenario expressed in quarterly growth rates for seasonally adjusted GDP, private consumption, exports and the results of increase in stocks before and after benchmarking (original and result)

To conclude, regardless of which reliability measures are taken, the differences between the aggregate and the underlying variables have to be eliminated somewhere. Not all initial differences were large and often the results were quite satisfying. However, if the initial differences are too large, benchmarking results may significantly disturb the quarterly growth rates of one or more of the variables involved.

3.3 Discussion

The results of Section 3.2 show that the method of benchmarking seasonally adjusted time series may not provide satisfactory quarterly growth rates if the initial differences are too large. The cause of the relatively large initial differences can be found in the process of seasonal adjustment by identifying the differences between seasonal patterns and irregular components. These differences partly depend on the method of seasonal adjustment, but if seasonal adjustment is done independently for each variable, large initial differences will remain.
The benchmarking model described in Section 2 is flexible, relatively simple and uses reliability weights; it is not unique, there are many different ways to restore consistency after seasonal adjustment. However, the results of Section 3.2 will not be very different because the constraint to restore contemporaneous consistency under large initial differences remains the same.

Using relative reliability weights as in Section 3.1 also introduces some subjectivity in the results. The results are sensitive for large differences in reliability weights, which means that the interpretation of the results is more doubtful.

The results in Section 3.2 are based on current prices. If constant prices were taken into account, it would have been possible to look at quarterly growth rates of volume instead of value. Then the relation between constant and current prices should also be taken into account in the model, which will make it much more complex.

4. Conclusion and recommendations

The benchmarking model presented in this paper is based on Denton’s movement preservation principle, which means that the quarter-to-quarter movement is preserved as much as possible under consistency restrictions. This model is flexible enough for benchmarking seasonally adjusted quarterly time series with different reliabilities.

The model is tested on the Dutch Quarterly GDP and its underlying expenditure variables. Seasonal adjustment is done by Census X12 ARIMA. Obviously, it appeared that the model is flexible enough to correct the seasonally adjusted time series for all disturbances. However, if the original discrepancies are relatively large, the quarterly growth rates can easily be disturbed. In conclusion, the results in this paper show that when initial discrepancies after independently seasonal adjustment are relatively large, restoring consistency leads to undesirable changes to quarterly growth rates.

The origin of the discrepancies can be found in the fact that the seasonally adjusted time series are obtained individually. If seasonal adjustment were done simultaneously for all variables, taking each correction to the aggregate into account for the underlying variables and vice versa, the differences would probably be much smaller. If desired, the differences can still be benchmarked afterwards. However, if the model for seasonal adjustment and the model for benchmarking can be integrated into a simultaneous benchmarking model, the results will probably be better and the interpretation of the results will improve. These will both be topics for further research.
References


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# Appendix

## Table 3. Annual totals for all variables for 1977 and 2004

<table>
<thead>
<tr>
<th></th>
<th>1977</th>
<th>2004</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Domestic Product by expenditure (GDP)</td>
<td>130956</td>
<td>466310</td>
<td>270028</td>
</tr>
<tr>
<td>Private Final Consumption Expenditure incl. NPISH</td>
<td>68818</td>
<td>227687</td>
<td>135531</td>
</tr>
<tr>
<td>Government Final Consumption Expenditure</td>
<td>31769</td>
<td>118003</td>
<td>65259</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation; Government</td>
<td>4853</td>
<td>16716</td>
<td>8740</td>
</tr>
<tr>
<td>Gross Fixed Capital Formation; Other sectors</td>
<td>24552</td>
<td>78699</td>
<td>48866</td>
</tr>
<tr>
<td>Increase in Stocks</td>
<td>404</td>
<td>3</td>
<td>251</td>
</tr>
<tr>
<td>Exports</td>
<td>61051</td>
<td>305195</td>
<td>157053</td>
</tr>
<tr>
<td>Imports</td>
<td>60491</td>
<td>279993</td>
<td>145671</td>
</tr>
</tbody>
</table>