Imperfect Knowledge, Asset Prices Swings and Structural Slumps: A Cointegrated VAR Analysis of their Interdependence

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Imperfect Knowledge, Asset Price Swings and Structural Slumps: A Cointegrated VAR Analysis of their Interdependence

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Abstract

This paper discusses interactions between speculative behavior in the currency markets and aggregate fluctuations in the real economy. It builds on the Structural Slumps theory in Phelps (1994) and the recent theory of Imperfect Knowledge Economics in Frydman and Goldberg (2007). The former emphasizes real interest rates and real exchange rates as potentially important determinants underlying the persistent fluctuations in aggregate activities, and the latter provides the conditions under which speculative behavior in currency markets generates such persistence. The paper argues that by combing the two theories we can shed new light on the two-way interdependence between persistent swings in asset markets and persistent fluctuations in the real economy. In particular, we may improve our understanding of the mechanisms behind the long recurrent spells of high unemployment that continue to mar our economies.

*The views in this paper are strongly influenced by previous and ongoing research with Roman Frydman, Michael Goldberg and Søren Johansen. I am deeply grateful to Roman and Michael for sharing with me their profound insight in international macro and how imperfect knowledge economics can resolve its many empirical puzzles and to my son, Mikael Juselius, for numerous valuable comments on an early version of this paper.
1 Introduction

The aim of this paper is to discuss interactions between speculative behavior in the currency markets and aggregate activity in the real economy inspired by the Structural Slumps theory in Phelps (1994) and the recent theory of Imperfect Knowledge Economics (IKE) in Frydman and Goldberg (2007). The former provides a coherent theoretical framework for how nonmonetary mechanisms can generate unemployment slumps in open economies connected by the world real interest rate and the real exchange rate, whereas the latter gives a rational for why real exchange rates tend to fluctuate persistently around long-run benchmark values and why this is likely to be compensated by similar movements in the real interest differential. To combine the two theories is, therefore, likely to improve our understanding of the two-way interdependence between persistent swings in asset markets and persistent fluctuations in the real economy. In particular, it may shed a new light on the mechanisms behind the long recurrent spells of unemployment that continue to mar our economies.

The main arguments, presented in Section 7, rely heavily on a number of estimated empirical regularities describing the foreign exchange market and wage, price and unemployment dynamics. The findings that (1) the natural rate of the Phillips curve is a function of a nonstationary real interest rate and (2) the ‘profit share’ is a function of a nonstationary real exchange rate, are particularly important for understanding the mechanisms by which the structural slumps and the IKE theories interact. These regularities have been found in several countries’ data by applying the Cointegrated VAR (CVAR) model (Johansen, 1995 and Juselius, 2006). Because the CVAR model offers a precise way of handling unit root nonstationarity and breaks, features which are typical of macroeconomic data, it has been our favoured choice of methodology.

To set the scene, Section 2 discusses exchange rate determination in two models, one based on the Rational Expectations Hypothesis (REH) and the other on the theory of IKE. The discussion focuses on their different implications for basic international parity conditions in terms of time-series
persistence. Section 3 discusses some general principles for how to structure the observed persistence in the data and Section 4 is a brief discussion of how these principles can be used within the CVAR model. Section 5 lists a set of testable hypotheses that can be used to discriminate between REH and IKE based explanations of real exchange rate persistence and Section 6 gives some arguments for why multivariate tests are more reliable than univariate. Section 7 discusses how foreign currency speculation under IKE interacts with a customer market economy where profit shares are adjusting to fluctuations in the real exchange rate and where the natural rate is a function of a nonstationary real long-term interest rate. Section 8 concludes.

2 Expectations formation and real exchange rate persistence

Frydman et al. (2011a) show how the REH versus the IKE assumptions on expectations formation lead to different hypotheses on the persistence of the real exchange rate and the real interest rate differential. Juselius (2011) discuss persistence in terms of $I(0)$, $I(1)$, and $I(2)$, defining processes integrated of order 0, 1 and 2. This section builds heavily on these two papers.

2.1 Rational expectations based models

The REH-based monetary model assumes that PPP holds as an equilibrium condition so that the real exchange rate, $q_t$, is stationary, i.e.

$$q_t = \rho q_{t-1} + \varepsilon_{1,t}$$

where $\rho < 1.0$. The stationarity of the real exchange rate is consistent with UIP as a market clearing mechanism:

$$i_{1,t} - i_{2,t} = \Delta s_{t+1}^e + rp_t$$

where $rp_t$ is a stationary risk premium. Provided (1) and (2) hold, the Fisher parity holds as a stationary condition:

$$i_t = \bar{r} + \Delta p^e$$

where $\bar{r}$ is an average real interest rate. Similarly, under the above conditions the term spread is stationary and the term structure of interest rates is well described by the expectations hypothesis.
Figure 1: The graphs of the dollar/Dmk rate and the relative prices between USA and Germany (upper panel) and the real exchange rate together with the US-German real long-term interest rate differential (lower panel).

2.2 Imperfect Knowledge based models

The theory of IKE (Frydman and Goldberg, 2007) assumes that individuals (bulls and bears) in the foreign currency market recognize their imperfect knowledge about the processes driving outcomes and, therefore, use a multitude of forecasting strategies which they revise over time in a way that cannot be fully prespecified in advance. Under certain conditions on individuals' revision of forecasting strategies and assuming that their forecasting variables are persistent, nominal exchange rates will show a tendency to persistently move away from (and towards) benchmark values. Figure 1, upper panel, illustrates the persistent movements of the nominal dollar/Dmk rate around relative prices as the natural benchmark values. Thus, IKE revisions of forecasts will generate an additional persistence in nominal exchange rates which is different from the persistence implied by REH based models.

Frydman et al. (2011a) showed that under the above conditions, the change in real exchange rate, $\Delta q_t$, can be approximated with the following
model:

\[
\Delta q_t = \zeta_t + \varepsilon_{1,t}
\]  

(4)

where

\[
\zeta_t = \bar{\rho}\zeta_{t-1} + \varepsilon_{2,t},
\]

and \( \zeta_t \) is a drift term measuring the change in the real exchange rate due to a change in individuals’ forecasting strategies, and \( \bar{\rho} \) is an average of \( \rho_t \), \( t = 1, \ldots, T \), such that \( \rho_t \approx 1.0 \) when \( q_t \) is in the neighborhood of long-run benchmark values and \( \rho_t < 1.0 \) when when \( q_t \) is far away from such values. Thus, \( \bar{\rho} \) may vary over different sample periods but generally within a small band close to the unit circle.

As long as the non-constant drift term, \( \zeta_t \), is well approximated with a near \( I(1) \) process, the real exchange rate behaves like a near \( I(2) \) process, i.e. it exhibits pronounced persistence. Figure 1, lower panel illustrates the long swings in the real dollar/Dmk rate. Modelling the real exchange rate as a near \( I(2) \) process is consistent with swings of shorter and longer duration and implies that the length of these swings is not predictable (Frydman and Goldberg, 2007). That the near \( I(2) \) process is a good approximation has been shown empirically in Johansen et al. (2010), thus confirming the theoretically expected results in Frydman and Goldberg (2007, 2011b) and Frydman et al. (2011).

When the real exchange rate is moving away from its benchmark value, the real interest rate differential has to move in a compensating manner to restore equilibrium in the product market (see Frydman and Goldberg, 2007). This implies that the IKE equilibrium relation is a cointegration relation between the real exchange rate, the nominal interest rate differential, and the inflation rate differential:

\[
(p_{1,t} - p_{2,t} - s_{12,t}) = \omega\{(i_{1,t} - i_{2,t}) - (\Delta p_{1,t} - \Delta p_{2,t})\} + e_t
\]  

(5)

where \( e_t \) is a stationary equilibrium error. In (5), the real exchange rate and the nominal interest rate differential are both near \( I(2) \) and cointegrate to near \( I(1) \). Adding the inflation rate differential, being near \( I(1) \), makes the relation stationary. Figure 1 illustrates the close co-movements between the real exchange rate and the real interest rate differential.

Under IKE, the standard UIP needs to be replaced by the Uncertainty Adjusted Uncovered Interest Rate Parity (Frydman and Goldberg, 2007) as
Figure 2: The graphs of the real long-term bond rates together with a 12 months moving average in USA (upper panel) and in Germany (lower panel).

a market clearing mechanism:

\[ i_{1,t} - i_{2,t} = \Delta s^e_{t+1} + u_p_t \]  

(6)

where \( u_p_t = f(p_{1,t} - p_{2,t} - s_{12,t}) \) is an uncertainty premium measuring how far the market has moved away from PPP benchmark values. The nominal interest rate differential and the uncertainty premium, both near \( I(2) \), cointegrate to near \( I(1) \) and the interest rate differential corrected for the uncertainty premium cointegrates with \( \Delta s^e_{t+1} \) to produce a stationary market clearing mechanism.

In an IKE model, nominal interest rate and CPI inflation rate are integrated of different orders and the Fisher parity does not hold as a stationary condition. Figure 2 illustrates the persistence in US and German real long-term interest rates. Similarly, the term spreads are not stationary implying that the term structure of interest rates is not adequately described by the
REH expectations hypothesis. Figure 3 illustrates the pronounced persistence in the US and German short-long interest rate spreads.

3 Persistence as a structuring device

As discussed above, the REH-based models differ from the IKE-based models in a very important aspect: the former imply no persistence in the changes of the real exchange rate whereas the latter are consistent with a marked persistence. To empirically distinguish between the two model classes we need an econometric methodology that can discriminate between different degrees of persistence. The purpose of this section is, therefore, to discuss such a methodology and give the intuition for how it works. A reader not interested in the empirical/econometric methodology can jump to Section 4 without loosing track.
3.1 Time-series persistence

The notion of persistence is typically associated with the strength of the time dependence of a shock to a variable. If the effect of a shock dies out quickly it is transitory and the corresponding variable is considered stationary, whereas if the shock has a lasting effect it is considered permanent and the variable is considered unit root nonstationary. Distinguishing exclusively between transitory (stationary) and persistent (nonstationary) behavior is often too crude for empirical modelling. For example, stationary processes can be divided into highly erratic $I(-1)$ processes and $I(0)$ processes, both of which describe transitory behavior. Nonstationary unit root processes can be generated from shocks which cumulate once, dubbed $I(1)$; or from shocks which cumulate twice, dubbed $I(2)$\(^1\). The latter is particularly important for describing speculative behavior under IKE, whereas the former is generally consistent with REH behavior.

While a classification in $I(0), I(1)$ and $I(2)$ is mathematically unambiguous, it can be more problematic in empirical modelling. This is so because depending on the sample size, the degree of permanence, and the relative noise ratio of $I(1)$ and $I(2)$ components, there are grey zones where data could be said to be near $I(1)$ rather than $I(1)$ or $I(0)$, and near $I(2)$ rather than $I(1)$ or $I(2)$. For example, a random walk process, $x_t = x_{t-1} + \varepsilon_t$, (an $I(1)$ process) and a strongly autoregressive AR(1) process, $x_t = 0.95x_{t-1} + \varepsilon_t$, (mathematically an $I(0)$ process) would often be difficult to distinguish from each other even based on relatively long samples. This is illustrated in Figure 4 where an AR(1) with $\rho = 0.95$ and a random walk are simulated in 200 steps. Both series look similar in terms of persistence. For a short time series, it is even more difficult to discriminate between near unit roots and unit roots. This is illustrated in Figure 5 for a stationary AR(1) process with autoregressive parameter $\rho = 0.80$ and a random walk process simulated in 50 steps. In contrast, an AR(1) process with $\rho = 0.99$ would often be found significantly different from 1.0 in a large sample of, say, 5000 observations even though such a variable would be highly persistent. If we characterize such a process as type $I(0)$, we would give up cointegration as a tool for identifying similar persistency profiles between variables.

Hence, statistical significance alone does not seem to work well as an organizing principle for classifying data into different persistence profiles.

\(^1\)See Johansen (1996) for a mathematically precise definition of the order of integration of stochastic processes.
3.2 Different levels of persistence

Another possibility is to define persistence in terms of the modulus of the characteristic roots of the autoregressive polynomial. For non-explosive models, the roots are defined for the interval (-1, 1) and they can, therefore, be given a convenient interpretation as a measure of the speed of adjustment. For example, consider the simple AR(1) model, \( x_t = \rho_1 x_{t-1} + \varepsilon_t \) or equivalently \( \Delta x_t = -(1 - \rho_1) x_{t-1} + \varepsilon_t \) with \( \rho_1 = 0.9 \). This corresponds roughly to an adjustment coefficient \( \alpha_1 \approx -(1 - \rho_1) = -0.10 \). An adjustment coefficient of -0.10 corresponds to an average adjustment time of \( \ln(2)/(1 - 0.10) \approx 7 \) periods. With annual data this would imply an average adjustment period of 7 years, with quarterly data it would be almost 2 years, with monthly data slightly more than half a year, with weekly data less than 2 months, etc. Whether a characteristic root can be interpreted as evidence of persis-
Figure 5: A simulated AR(1) process with $\rho = 0.8$ (upper panel) and a random walk (lower panel).

tent behavior or not depends, therefore, both on the sample period and the observational frequency.

To illustrate the idea, consider a variable $x_t$ with the autoregressive representation $(1-\varphi_1 L - \cdots - \varphi_p L^p)x_t = \varepsilon_t$ where $\varepsilon_t$ is $Niid$, and define a threshold parameter $\rho^*$ above which the process is considered persistent. The choice of $\rho^*$ is to some extent subject to judgement. With high frequency data its value would generally be closer to the unit circle than with low frequency data. In the context of a specific theory, $\rho^*$ could in some cases be thought of as defining the longest adjustment time for which the policy implications of the model are still useful.

The persistence of $x_t$ could for example by defined as:

- $I(0)$ type when the modulus of the largest root, $\rho_1$, satisfies $\rho_1 < \rho^*$.
- $I(1)$ type when the modulus of the largest root, $\rho_1$, satisfies $\rho^* < \rho_1 \leq 1.0$ and the next root $\rho_2 < \rho^*$. 

\textbf{Figure 6:} The graphs of an AR(1) process with $\rho = 0.95$ (upper panel) and with $\rho = 0.20$ lower panel.

- \textit{I}(2) type when the modulus of the largest root, $\rho_1 = 1.0$, and the next one satisfies $\rho^* < \rho_2 \leq 1.0$.

While the above classification is directly applicable in a univariate model, it needs some modification in a multivariate model like the CVAR. For example, in a univariate model a large characteristic root can be directly associated with the variable in question, $x_{t,t}$, whereas in a $p$-dimensional VAR model of $x'_t = [x_{1,t}, \ldots, x_{p,t}]$, the number of large roots in the characteristic polynomial is a function of the number of exogenous (common) stochastic trends, $p - r$, in the system, where $r$ is the number of cointegration relations, and of number of the stochastic trends which are of first order, $s_1$, or second order, $s_2$, where $s_1 + s_2 = p - r$. Consider for example a five-dimensional VAR model for which three of the characteristic roots are greater than $\rho^*$. This could be consistent with three stochastic trends of first order ($p - r = 3; s_1 = 3$), or, alternatively, with two stochastic trends of first order and one of second order ($p - r = 2; s_1 = 1, s_2 = 1$).
To determine the number of stochastic trends and their division into type $I(1)$ and $I(2)$, the following simple procedure can be used: Start with the unrestricted VAR model ($r = p$) and determine the number, $m^*$, of characteristic roots which are greater than $\rho^*$\(^2\). Next, use the $I(2)$ trace test (Nielsen and Rahbek, 2007) to test the cases $(r, s_1, s_2)$ for which $s_1 + 2s_2 = m^*$. An empirically relevant candidate is found when the trace test is not rejected, all unrestricted characteristic roots $\rho_i < \rho^*$, and the number of restricted unit roots is $m^*$.

### 3.3 Near unit root inference

Another issue to discuss is how the asymptotic $I(2)$ inference is affected when data are near $I(2)$ rather than exactly $I(2)$. It is useful to distinguish between the case when a near unit root is treated as (1) stationary or (2) nonstationary. In the first case, Elliot (1998) showed analytically that the asymptotic distribution is no longer mixed Gaussian and that standard inference can be misleading. This is consistent with the simulation study in Johansen (2006) which found some inference to be very fragile when a near unit root was treated as stationary. For example, up to 5000 observations were needed for the empirical distribution to converge to Students $t$ when the near unit root was 0.998. However, following the rule that the characteristic roots have to be smaller than $\rho^*$ in the preferred model with rank $r^*$ is likely to work as a safeguard against this problem.

In the second case, inference remains mixed Gaussian but some estimators can be biased. However, Corollay 1 in Johansen (1997) can be used to show that inference on $\beta$ and $\alpha$ in the $I(2)$ model is efficient and unbiased also in the near $I(2)$ case\(^3\). Since all results discussed in the subsequent sections have been obtained by cointegration analysis in the $I(2)$ model, the corollary result allows us to attach a fair degree of confidence to our empirical findings. Nonetheless, robustness is always an important issue which needs

\(^2\)Note, however that if a large modulus root corresponds to a complex pair with a significant imaginary part it is not possible to force it to become a unit root on the real line. In this case, it will be considered a stationary, albeit persistent, cyclical component. Also, Nielsen and Nielsen (2009) has shown that if the VAR model is estimated with too many lags (for example adding lags to compensate for a structural break) the number of large, but insignificant, roots will increase. In such a case, the number becomes uninformative.

\(^3\)This is because the second reduced rank condition (which is associated with the $I(2)$ model property) does not affect the asymptotic efficiency of the ML estimator of $\beta$ and $\alpha$. 

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4 Structuring persistence using the CVAR

The CVAR models are inherently consistent with a world where unanticipated shocks cumulate over time to generate stochastic trends which move economic equilibria (the pushing forces) and where the deviations from these equilibria are corrected by means of the dynamics of the adjustment mechanism (the pulling forces). Thus, the CVAR model has a good chance of nesting a multivariate, path-dependent data-generating process and relevant dynamic macroeconomic theories. See Hoover et al. (2008).

In line with the above, Juselius (2006) and Juselius and Franchi (2007) asked the question: Which empirical regularities would we see in the data, if the assumptions of exogenous shocks, equilibrium relations, steady-state behavior, and dynamic adjustment were correct in the theoretical model? The answer was formalized in what was called a theory consistent CVAR scenario, which essentially translates all basic assumptions about the theoretical model’s shock structure, equilibrium relations and steady-state behavior into testable hypotheses on common stochastic trends, cointegration, steady-state values and dynamic adjustment\(^4\). Because of its ability to structure the relevant data into economically meaningful directions \textit{without subjecting them to prior restrictions}, the CVAR can be thought of as providing broadly defined ‘confidence intervals’ within which empirically relevant models should fall.

The CVAR approach starts from an unrestricted VAR model which is essentially just a representation of the covariances of the data. By imposing (testable) reduced rank restrictions on the VAR model, it is formulated as a vector equilibrium-error-correcting model of first order, the \(I(1)\) model, or second order, the \(I(2)\) model. The former is appropriate to describe an economy where growth rates and deviations from equilibria are stationary, the latter where they are unit root nonstationary. See Appendix for a definition of the \(I(1)\) and \(I(2)\) models and an interpretation of their structure.

In the \(I(1)\) model, the deviation from a static equilibrium is assumed to be stationary. For example, REH based models would allow real exchange rates to move away from PPP values albeit in a stationary manner and the \(\alpha\) coefficients in (7) in the Appendix would describe the speed of adjustment back to equilibrium.

\(^4\)For a comprehensive treatment, see Juselius (2006).
In the $I(2)$ model, the deviations from equilibrium values can exhibit a pronounced persistence, implying that the nominal exchange rate, say, can move away from equilibrium values for extended periods of time, provided they are compensated by something else.

5 Testable empirical regularities under REH and IKE

Juselius (2011) used the concept of a CVAR scenario to translate the assumptions on expectations formation and forecasting behavior in an REH contra IKE based model for nominal exchange rate determination into testable hypotheses on the CVAR model, the most important of which are:

1. Under IKE, speculative behavior in the currency markets tends to drive nominal exchange rates away from PPP benchmark values for extended periods of time. These persistent movements have the property of a near $I(2)$ process so that the real exchange rate is near $I(2)$. Under REH, the movements around PPP are stationary, or at most near $I(1)$.

2. Under IKE, the real exchange rate is co-moving with the real interest rate differential. Hence, the latter is empirically near $I(2)$. Under REH, the real interest rate differential is stationary or at most near $I(1)$.

3. Under IKE, the real exchange rate and the real interest rate differential cointegrate to a stationary equilibrium relation. Under REH, they are individually stationary albeit allowed to exhibit some persistence.

4. Under IKE, the nominal interest rate and the inflation rate are not cointegrated so that the Fisher parity does not hold as a stationary condition. Under REH, the Fisher parity is stationary.

5. Under IKE, the term structure of interest rates is driven by two stochastic trends, one typically associated with the short end (monetary policy shocks), the other with the long end (financial market shocks). Thus, the standard expectations hypothesis does not hold and the interest rate spreads are nonstationary but cointegrated. Under REH, the expectations hypothesis implies one stochastic trend and stationary spreads.
These hypotheses (and many more) were tested within a five-dimensional CVAR model based on the principles discussed in Section 3. The test results showed that the IKE-based scenario obtained a remarkable support for every single testable hypothesis, whereas the REH-based scenario was empirically rejected on all counts\(^5\). Detailed results are reported in Frydman et al. (2011) and Juselius (2011)\(^6\).

6 Are near \(I(2)\) results credible?

Many economists would feel uncomfortable treating real exchange rate and interest rate differentials as (near) \(I(2)\), arguing that economic variables or relations cannot be \(I(2)\) as this would have implausible consequences for the economic model. While, in my view, this is confusing economic properties with statistical properties it is, nevertheless, the case that finding economic data to be (near) \(I(2)\) by statistical testing can be interpreted as evidence in favor of an IKE-based model. Therefore, the question by Ricardo Rice, in the discussion of an earlier version of this paper, "How come you find real exchange rates to be near \(I(2)\) when others find them to be near \(I(1)\)" is of considerable interest.

A first answer to the question is that most empirical studies find real exchange rates to be (near) \(I(1)\) because they use univariate testing, whereas I find near \(I(2)\) because I use multivariate testing. For example, based on a CVAR model for US and German prices and the nominal exchange rate, Juselius (2009) explored the consequences of assuming the variables to be \(I(1)\) versus \(I(2)\). In the former case, any choice of rank left one or two near unit roots (of the magnitude 0.99) in the model, rendering any Gaussian based inference completely unreliable. In the latter case, it was possible to account for all large roots in the model, but the estimated relationship between relative prices and nominal exchange rates was not economically plausible. By including nominal interest rates in the empirical model Juselius

\(^5\)For an argument how the empirical difficulties of the REH models discussed here can be traced to their epistemologically flawed micro foundation, see Frydman and Goldberg (2011a, c).

\(^6\)Previously Juselius (1995), Juselius and MacDonald (2004, 2007) reported empirical support for these hypotheses based on data for Danish - German, US -Japan and US - German exchange rates, prices and interest rates in the post Bretton Woods period of currency floats. The theory of imperfect knowledge economics provided the missing theoretical background.
(2011) found a completely plausible relationship between the real exchange rate and the real interest rate differential, but only in the $I(2)$ model. Treating the variables (and the real exchange rate) as $I(1)$ invariably left two large characteristic roots (0.96, 0.96) in the model. But a univariate Dickey-Fuller test would not have detected these large roots due to the small variance of the $I(2)$ component.

A simulation exercise can illustrate why this is the case. I have generated two time series of which the first one, $x_{1,t}$, is a random walk process (consistent with an REH based model for which the real exchange rate is at most $I(1)$), and the second one, $x_{2,t}$, is a random walk augmented with a small, but very persistent, drift term $\zeta_t$ (consistent with an IKE based model). The former is defined as:

$$x_{1,t} - x_{1,t-1} = \varepsilon_t, \quad \varepsilon_t \sim N(0, 1), \quad t = 1, \ldots, 500,$$

and the latter as:

$$x_{2,t} - x_{2,t-1} = \zeta_t + \varepsilon_{1,t}, \quad \varepsilon_{1,t} \sim N(0, 1) \quad t = 1, \ldots, 500$$

$$\zeta_t = 0.95 \zeta_{t-1} + \varepsilon_{2,t}, \quad \varepsilon_{2,t} \sim N(0, 0.15^2)$$

Because the variance of the $\zeta_t$ process is small compared to the variance of $\Delta x_{2,t}$, the fact that $\Delta x_{1,t}$ is $I(0)$ whereas $\Delta x_{2,t}$ is near $I(1)$ can be difficult to detect visually as Figure 7 illustrates. One way of spotting an underlying persistent movement in a noisy series is to plot the series together with a suitable (12 period) moving average as done in Figure 7. It is now obvious that $\Delta x_{2,t}$ has an underlying persistent trend, whereas the moving average trend in $\Delta x_{1,t}$, is much less persistent.

When it comes to unit root testing, the large root associated with $\varepsilon_{2,t}$ in $x_{2,t}$ would not be easily detected based on a univariate test. This is because the estimated residual is the sum of $\varepsilon_{1,t}$ with a large variance and $\zeta_t$ with a much smaller variance. For example, when subjecting simulated versions of the variable, $x_{2,t}$, to a Dickey-Fuller unit root test, the results suggested $I(1)$ or even $I(0)$, whereas $I(2)$ was strongly rejected. Based on multivariate testing the second near unit root was detected both by the $I(2)$ trace tests and by checking the roots of the characteristic polynomial.

\footnote{As a moving average is time-dependent by construction, one should expect some persistence also for the random walk case.}
Figure 7: The graphs of a differenced near I(2) process versus that of a random walk together with a 12 period moving average.

7 Currency speculation and structural slumps

The discussion here is based on many years of systematic investigation by myself and my students of how the persistency of the real exchange rates, real interest rates and the term spreads has influenced aggregate activities in the real economy, in particular how they have affected wage, price and unemployment dynamics. Phelps’ (1994) structural slumps theory was a frequent reference in our early papers, subsequently to be combined with Frydman and Goldberg’s (2007) IKE theory.

This section is a first attempt to put the bits and pieces together into something that eventually may have the potential of becoming a coherent theoretical and empirical framework for understanding persistent fluctua-
tions in the macro economy. While the main results are given in Section 7.4, the logic of the arguments requires a first discussion of the role of a non-stationary long-term interest rate and a nonstationary Fisher parity, what initiates swings and what allows them to be so long-lasting.

7.1 Preliminaries

The structural slumps theory explains how open economies connected by the world real interest rate (set in the global capital market) and by the real exchange rate (determined in a global customers market for tradables) can be hit by long episodes of unemployment. The theory predicts that an exogenous shock to the world level of public debt and/or capital stock will change the world level of interest rates, whereas an exogenous shock to the public debt of an individual open economy increases its interest rate relative to the world interest rate.

The empirical analysis in Johansen et al. (2010), Frydman et al. (2011) and Juselius (2011) finds that shocks to the long-term US bond rate (a proxy for the world interest rate) and to the US-German interest rate differential (measuring relative debt levels between the two countries) are the main exogenous forces in a system comprising US-German prices, nominal exchange rates, and long-term interest rates.

The IKE theory, predicts that the real exchange rate is primarily determined in a speculative market for foreign currency (rather than in a customer market for tradables) and that speculation under some plausible conditions, drives the nominal exchange rate away from long-term PPP values together with a compensating movement in the real interest rate differential.

Assuming that prices of tradable goods are primarily determined in very competitive customer markets, as in Phelps (1994), means that they are not in general much affected by speculation (energy, precious metals and, recently, grain may be exceptions in this respect) and, therefore, do not exhibit persistent swings around benchmark values. If nominal interest rates exhibit persistent swings but consumer price inflation does not, then the real interest rate will also exhibit persistent swings. This might give an incentive for speculation in currency markets. For example, such an incentive would not exist if the Fisher parity is stationary. In this case, an increase in the long-term interest rate would be associated with an expected increase in the inflation rate and speculators would have no specific incentive to invest their long-term capital in such an economy.
7.2 **What is initiating a long swing**

Johansen et al. (2010) and Juselius (2011) find that it was shocks to the interest rate differential and to the level of long-term US interest rate that have been pushing the system. A shock to the former can be interpreted as a shock to the expected change in nominal exchange rate, whereas the latter as a shock shifting the world interest rate level. In this sense, a long swing can be initiated by changes in exchange rate expectations or by pure interest rate shocks. The interdependence between the two makes it difficult to discriminate between them.

For instance, a shock to the long-term interest rate (for example, as a result of a domestic increase in sovereign debt) without a corresponding increase in the inflation rate, is likely to increase the amount of speculative capital moving into the economy. The exchange rate would appreciate, jeopardizing competitiveness in the tradable sector, the trade balance would worsen, and the pressure on the interest rate would increase. Under this scenario, the interest rate is likely to keep increasing as long as the structural imbalances are growing, thus generating persistent movements in real interest rates and real exchange rates. Figure 1 illustrates such historical co-movements.

Thus, persistent shocks to the domestic interest rate relative to the world interest rate are likely to hit the economy as a result of structural imbalances in the domestic economy. In Europe such imbalances have typically been associated with a political reluctance to adequately address painful structural reforms in the labor market. In US, they are typically associated with trade balance problems.

This circle of increasing/decreasing real interest rates and real appreciation/depreciation rates, is empirically manifested in Juselius (2011) as an equilibrium error increasing behavior of the $\delta$ adjustment in (8) in the Appendix. The fact that risk averse individuals will require increasingly large risk premiums for holding the domestic currency as the macroeconomic imbalances grow, will sooner or later cause a reversal in the exchange rate movement (Frydman and Goldberg, 2007). In the empirical analysis this was manifested in the equilibrium error correcting behavior of the $\alpha$ adjustment in (8) in the Appendix.
7.3 Why are swings so long-lasting?

A persistent deviation away from benchmark values, is likely to trigger off a compensating reaction in other sectors of the economy. In the currency market, nonstationary movements in the real exchange rate are compensated by nonstationary movements in the nominal interest rate differential corrected for the inflation rate differential. As long as this interactive (reflexive) process is at work, the deviations from benchmark PPP values can be long-lasting. Such an interactive process between speculative markets and the real economy may explain the pronounced persistence of real exchange that have puzzled economists for such a long time. It also provides a rational for why the I(2) model is a good description of IKE behavior: It is tailor made to describe an economy where persistent deviations from long-run static equilibrium values are compensated by similar persistent movements in other variables. Thus, an IKE economy is still characterized by equilibrating forces but in a dynamic rather than static set-up.

7.4 Persistent fluctuations in the real economy

The tendency of the domestic real interest rate to increase and the real exchange rate to appreciate at the same time is likely to aggravate domestic competitiveness in the tradable sector. In an IKE economy where the nominal exchange rate is determined by speculation, firms cannot in general count on exchange rates to restore competitiveness after a permanent shock to relative costs. Unless firms are prepared to loose market shares, they cannot use constant mark-up pricing as their pricing strategy. To preserve market shares, they would have to adjust productivity or profits rather than increasing their product price. Therefore, in an IKE economy we would expect customer market pricing (Phelps, 1994) to replace constant mark-up pricing. This implies that profits would be squeezed in periods of persistent appreciation and increased during periods of depreciation. Evidence of a nonstationary profit share comoving with the real exchange rate has for instance been found in Juselius (2006).

Next, a customer market firm, facing an increase in the domestic wage cost in excess of the foreign one, would improve labor productivity rather than to increase its price. Labor productivity can be achieved by new technology or by producing the same output with less labor i.e. by laying off the least productive part of the labor force. In the latter case, the increase
in productivity would be achieved at the cost of rising unemployment rate. Therefore, in a customer market economy with the nominal exchange rate determined by speculation, labor productivity and unemployment would rise in periods of real currency appreciation and increasing real interest rates. Evidence of unemployment comoving with trend-adjusted productivity and the real interest rate has been found, among others, in Juselius (2006).

Increasing unemployment generally exerts a downward pressure on nominal/real wage claims and, thus, on wage inflation, $\Delta w$. Thus, wage inflation is negatively associated with unemployment, $u$, in an augmented Phillips Curve relation with a non-stationary natural rate, $u^*$: $\Delta w = -b_1(u - u^*)$, where $u^* = f(r)$ is a function of the real interest rate level, $r$. In Phelps (1994) the latter is a function of domestic government debt and the world real interest rate level. Evidence of a non-stationary natural rate as a function of the long-term real interest rate is found among others in Juselius (2006) and Juselius and Ordonez (2009).

Thus, there seems to be a direct link from financial market behavior causing long persistent swings in real exchange rates and real interest rates to the recurring unemployment slumps discussed in Phelps (1994).

7.5 Further remarks

Frydman and Goldberg (2007) suggest that the uncertainty premium increases with the deviation from the fundamental PPP value. But as discussed above one can also think of the unemployment rate and the profit share as alternative, but related, measures of deviations from benchmark values that eventually will put an end to the long swings movements in nominal exchange rates.

The structural slumps mechanism seems to work well in periods when the major driver underlying the fluctuations in aggregate activity is the long swings in real exchange rates. It is, however, not likely to work well in the aftermath of a fundamental financial crises as the present one as discussed in Koo (2010) and Miller and Stiglitz (2010). This is because when numerous balance sheets in the economy are 'under water', savings will primarily be used for financial consolidation rather than for investment. As the Japanese experience after the collapse of the housing bubble in the nineties showed, not even a zero interest rate will have the intended effect in such a situation.
8 Concluding discussion

Macroeconomic data have a reputation for not being sufficiently informative, thereby justifying the use of ‘mild force’ to make them tell an economically relevant story. Based on my long experience of analyzing macroeconomic data my claim is that macroeconomic data are surprisingly informative, but only if you let them tell the story they want to tell.

So, what do the data tell if they are allowed to speak freely? Some robust findings typical of the last three decades of capital deregulation and globalization can be summarized as follows: First, there is more persistence in the data than standard REH based theories can explain. In particular, basic parity conditions such as purchasing power parity, real interest rates, uncovered interest rate parity, and the term spread seem to exhibit a pronounced persistence untenable with $I(0)$ type stationarity. Second, this persistence seems to originate from complex interactions between speculative financial markets and the real economy that tend to drive prices away from benchmark values.

To summarize, the stories data tell seem consistent with speculative behavior, imperfect knowledge, long swings, and strong reflexivity between the financial and the real economy. Further research along these lines is likely to result in a fruitful synthesis between the theoretical framework of Phelps (1994) and Frydman and Goldberg’s (2007) IKE theory, thereby improving our understanding of the long recurrent spells of high unemployment that continue to mar our economies.

9 References


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8 This includes supervising hundreds and hundreds of seminar papers, BSc, MSc, and PhD theses.


Juselius, K. and Franchi, M. (2007), Taking a DSGE Model to the Data


Nielsen, B. and H.B. Nielsen (2009), The Asymptotic Distribution of the Estimated Characteristic Roots in a Second Order Autoregression, Manuscript under preparation, Economics Department, University of Copenhagen.


**A The I(1) and I(2) model**

**A.1 The I(1) model**

To introduce notation and the idea of structuring the data into pulling and pushing forces, I shall use a simple 3-dimensional VAR model for \( x_t = [p_1, p_2, s_{12}] \), where the three variables describe domestic and foreign prices and the nominal exchange rate. The model is structured around \( p - r \) stochastic trends (the pushing or exogenous forces) and \( r \) cointegration relations (the pulling or equilibrating forces). I shall consider the case \( (r = 1, p - r = 2) \).

The pulling force is formulated as the vector equilibrium error correction model, \( \Delta x_t = \alpha \beta' x_{t-1} + \varepsilon_t \), i.e. as:
where $\beta' x_t$ is an equilibrium error and $\alpha_i$ is an adjustment coefficient describing how the system adjusts back to equilibrium when it has been pushed away. For example, $\beta' x_t = p_{1,t} - p_{2,t} - s_{12,t}$ would describe an economy where purchasing power parity holds as a stationary condition. The $\alpha_i$ coefficients tell us whether it is prices or exchange rates or all three variables that take the adjustment when unanticipated shocks, $\varepsilon_{i,t}$, have pushed the system out of equilibrium.

The pushing forces are analyzed in the moving average form of the CVAR model, describing the cumulated effects of the exogenous shocks, $u_{i,t}$, on the variables:

$$
\begin{bmatrix}
\Delta p_{1,t} \\
\Delta p_{2,t} \\
\Delta s_{12,t}
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix}
\beta' x_{t-1} + \cdots + 
\begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t} \\
\varepsilon_{3,t}
\end{bmatrix}
$$

where $u_{1,t} = \alpha'_{1,1}\varepsilon_t$ and $u_{2,t} = \alpha'_{1,2}\varepsilon_t$ are two autonomous common shocks that have a permanent effect on the system and $\alpha_\perp = [\alpha_{\perp,1}, \alpha_{\perp,2}]$, is a $3 \times 2$ matrix, orthogonal to $\alpha$, defining the two common shocks as linear combination of the VAR residuals $\hat{\varepsilon}_t$ and $\beta_\perp = [\beta_{\perp,1}, \beta_{\perp,2}]$, is a $3 \times 2$ matrix orthogonal to $\beta$ describing the long-run effect of a structural shock to the system.

For example, $\alpha'_{1,1} = [1, -1, 0]$ and $\alpha'_{1,2} = [0, 0, 1]$ would describe an economy where shocks to relative prices and shocks to the nominal exchange rate are the main exogenous driving forces. The case $\beta'_{\perp,1} = [a, a, 0]$ and $\beta'_{\perp,2} = [b, c, b - c]$ would define a stationary real exchange rate:

$$
\begin{bmatrix}
p_{1,t} \\
p_{2,t} \\
s_{12,t}
\end{bmatrix}
= 
\begin{bmatrix}
a & b \\
a & c \\
0 & b - c
\end{bmatrix}
\begin{bmatrix}
\sum_{i=1}^t u_{1,i} \\
\sum_{i=1}^t u_{2,i}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t} \\
\varepsilon_{3,t}
\end{bmatrix}
$$

A.2 The $I(2)$ model

The $I(2)$ model is useful to describe an economy where the persistency in the data is one degree higher than in the $I(1)$ world. To account for this, the $I(2)$ model is formulated in acceleration rates, medium run relations...
between growth rates and dynamic relations. It has a richer but also more complicated structure. The vector \( x_t \) is now integrated of order 2 and the \( p - r \) stochastic trends are divided into \( s_1 \) first order and \( s_2 \) second order stochastic trends, i.e. \( p - r = s_1 + s_2 \). The \( r \) cointegration relations, \( \beta' x_t \), are generally integrated of order 1, i.e. they cointegrate from \( I(2) \) to \( I(1) \) and becomes stationary by adding a linear combination of the growth rates, \( \delta' \Delta x_t \). In addition there are \( s_1 \) linear combinations, \( \beta'_{\perp 1} x_t \sim I(1) \), which can become stationary exclusively by differencing, i.e. \( \beta'_{\perp 1} \Delta x_t \sim I(0) \). Thus, the \( I(2) \) model contains \( p - s_2 \) relations, \( \tau' x_t \), which cointegrate from \( I(2) \) to \( I(1) \), where \( \tau = (\beta, \beta_{\perp 1}) \).

I consider here the case \( (r = 1, s_1 = 1, s_2 = 1) \) implying as before one equilibrium relation and two stochastic trends. The difference is that the equilibrium relation needs to be combined with a growth rate to become stationary and one of the common stochastic trends is an \( I(2) \) trend whereas the other is an \( I(1) \) trend. The former could, for example describe price shocks and the latter exchange rate shocks.

Under this assumption, the vector equilibrium error correcting model for \( I(2) \) data can be formulated as

\[
\begin{bmatrix}
\Delta^2 p_{1,t} \\
\Delta^2 p_{2,t} \\
\Delta^2 s_{12,t}
\end{bmatrix} =
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix}
(\beta' x_{t-1} + \delta' \Delta x_{t-1}) +
\begin{bmatrix}
\zeta_{11} & \zeta_{21} \\
\zeta_{12} & \zeta_{22} \\
\zeta_{13} & \zeta_{23}
\end{bmatrix}
\begin{bmatrix}
\beta' \Delta x_{t-1} \\
\beta'_{\perp 1} \Delta x_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t} \\
\varepsilon_{3,t}
\end{bmatrix}
\tag{8}
\end{equation}

where \( \beta' x_{t-1} + \delta' \Delta x_{t-1} \) describes a deviation from a dynamic equilibrium relation, and \( \beta' \Delta x_{t-1} \) and \( \beta'_{\perp 1} \Delta x_{t-1} \) describe deviations from two medium-run equilibrium relations among growth rates. For example, if \( \beta' x_{t-1} + \delta' \Delta x_{t-1} = (p_{1,t} - p_{2,t} - s_{12,t}) + \delta_1 \Delta p_{1,t} \), then this would describe an economy where deviations from PPP exhibit type \( I(1) \) persistence which is compensated by a similar persistence in country 1 inflation rate.

The common stochastic trends are analyzed in the moving average form of the CVAR model, \( x_t = \beta_{\perp 1} \Sigma \Sigma u_s + B \Sigma u_i + ... + \varepsilon_t \). For the price and

\[ r - s_2 > 0 \], then it is possible to find \( r - s_2 \) relations \( \beta' x \) which are stationary without adding the growth rates.
exchange rate system it can be formulated as:

\[
\begin{bmatrix}
  p_{1,t} \\
  p_{2,t} \\
  s_{12,t}
\end{bmatrix}
= \begin{bmatrix}
  \beta_{12,1} \\
  \beta_{12,2} \\
  \beta_{12,3}
\end{bmatrix}
\begin{bmatrix}
  \sum_{i=1}^{t} \sum_{s=1}^{i} u_{1,s} \\
  \sum_{i=1}^{t} \sum_{s=1}^{i} u_{1,s}
\end{bmatrix}
+ \begin{bmatrix}
  b_{11} & b_{21} \\
  b_{12} & b_{22} \\
  b_{13} & b_{23}
\end{bmatrix}
\begin{bmatrix}
  \sum_{i=1}^{t} u_{1,i} \\
  \sum_{i=1}^{t} u_{2,i}
\end{bmatrix}
+ \ldots
\]

where \( u_{1,t} = \alpha'_{12} \varepsilon_t \) is an autonomous shock that cumulates twice over time, \( u_{2,t} = \alpha'_{11} \varepsilon_t \) is an autonomous shocks that cumulates once over time, \( \alpha_{\perp} = [\alpha_{\perp,1}, \alpha_{\perp,2}] \), is a \( 3 \times 2 \) matrix orthogonal to \( \alpha \), defining the two shocks as linear combination of the VAR residuals \( \varepsilon_t \) and \( \beta_{12} \) is a \( 3 \times 1 \) vector orthogonal to \( \{\beta, \beta_{\perp}\} \) describing the long-run effect of a structural \( I(2) \) shock to the system. If \( u_{1,t} \) is a relative price shock then \( \alpha'_{12} = [1, -1, 0] \) and \( u_{2,t} \) a nominal exchange rate shock, then \( \alpha'_{11} = [0, 0, 1] \). Assuming that only the two prices are affected by the \( I(2) \) trend the system could be described by:

\[
\begin{bmatrix}
  p_{1,t} \\
  p_{2,t} \\
  s_{12,t}
\end{bmatrix}
= \begin{bmatrix}
  1 \\
  1 \\
  0
\end{bmatrix}
\begin{bmatrix}
  \sum_{i=1}^{t} \sum_{s=1}^{i} u_{1,s} \\
  \sum_{i=1}^{t} \sum_{s=1}^{i} u_{1,s}
\end{bmatrix}
+ \begin{bmatrix}
  b_{11} & b_{21} \\
  b_{12} & b_{22} \\
  b_{13} & b_{23}
\end{bmatrix}
\begin{bmatrix}
  \sum_{i=1}^{t} u_{1,i} \\
  \sum_{i=1}^{t} u_{2,i}
\end{bmatrix}
+ \ldots
\]

Thus, prices would be type \( I(2) \), but relative prices and the nominal exchange rate type \( I(1) \). The real exchange rate would generally be \( I(1) \) unless \( b_{13} = b_{11} - b_{12} \) and \( b_{23} = b_{22} - b_{22} \).