Analysing the permanent and cyclical components of GDP of the euro-area countries in a global context: The role of cross-sectional dependence

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Luxembourg: Publications Office of the European Union, 2016

ISBN: 978-92-79-60960-2
ISSN: 2315-0807
doi: 10.2785/754309
Cat. No: KS-TC-16-014-EN-N

Abstract

This paper focus on an analysis of the GVAR model across euro-area countries when detrending. The GVAR model accommodates cross-country as well as cross-variable dependencies among the euro-area countries. We focus on the role of cross-sectional dependence in the production of trend and cycle estimates of the Euro-area countries by comparing the GVAR trend and cyclical components extracted for individual countries with the estimates produced using a restricted GVAR in which cross-sectional dependencies are set to zero. By implementing an analysis in real-time and across countries, we see how accommodating dependencies across countries in the detrending model performs compared to models restricting such dependencies to zero; and we see how changing levels of cross-country dependence affect real-time trend-cycle estimates.

Acknowledgements: Paper accepted at the 32nd CIRET conference in Hangzhou (October 2014). The paper has benefitted from the outcomes of the multi-annual PEEIs project financed by Eurostat.

JEL classification code: C22, E17

Keywords: GVAR, cross-sectional dependence, cyclical components, de-trending

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1 Introduction

Model-based parametric methods are often used to decompose macroeconomic time series into trend and cyclical components. These decomposition methods are usually based on (vector) autoregressive moving average model or unobserved component models. De-trending models of this type are often in an univariate setting. If there exists any possible interlinkages among economies and also variables within an economy/country, such interlinkages are not captured in the trend and cycle estimates. Traditionally, trends and cycle components are estimated for each country separately, albeit often using multivariate (country-specific) VAR or UC models. However, given the existence of globalisation or indeed Europeanisation one may expect better trend and cycle estimates to be obtained by incorporating such dependencies into the de-trending models. Therefore, multivariate models that allow for non-zero cross-country and cross-variable dependencies provide a better means to de-trend Euro-area countries' macroeconomic time series. Multivariate de-trending models also nest univariate de-trending models, as the latter can be obtained by applying appropriate restriction to the former.

However, the estimation of multivariate models that involves datasets with high dimension is usually infeasible, certainly when classical estimation methods are employed. Different approximations have been proposed to resolve this "curse of dimensionality". Among these approaches, the Global VAR (GVAR) model of Pesaran et al. (2004) has been increasingly prominent in the recent literature. The GVAR allows for interdependencies among a large number of countries and disaggregate variables. The GVAR model consists of a system of country-specific VARX* models. Cross-sectional dependencies across countries and variables are captured via both non-zero parameters (of foreign variables and its lags; and also a common global factor) and non-zero cross-country, cross-variable covariances. Estimation is conducted by first estimating country-specific models, then stacking up these models via a link (weight) matrix to solve for the GVAR parameters and covariances. This technology can accommodate a large dimensional dataset; therefore it provides a feasible estimation method and an efficient way to study cross-sectional dependence. In the case of zero cross-sectional dependence, the GVAR reduces to a stack of country-specific VAR models, thus the multivariate model nests a univariate setting naturally. For a discussion of the relationship between the disaggregate model and the multivariate global model see Lui and Mitchell (2012)

In their paper, Dees et al. (2009) propose a new method to de-trend using the GVAR. Unlike multivariate modelling using a VAR, the GVAR offers a means of de-trending even when the dimension of VAR is high. It uses the multivariate Beveridge-Nelson decomposition, but conditions on a wider range of (disaggregate) information when forming the long-run forecasts which lie behind the BN trend. They show how the trend and cycle estimates can be obtained from a GVEC representation of the GVAR, and thus allow for the estimation of the two components to capture unit root and co-integration behaviour in the global economy. The GVAR estimates are consistent with the estimates obtained from dynamic stochastic general equilibrium (DSGE) model. While at the same time, it resolves the problems of weak instruments and the biasedness of estimates due to misspecification of steady state caused by the existence of stochastic trends and cointegration, commonly encountered in DSGE de-trending methods. GVAR also nests both the aggregate and disaggregate models naturally through its multivariate setting. As discussed in Pesaran et al. (2004), GVAR captures cross-sectional dependence among countries and variables in three different and related ways: (1) contemporaneous dependence of domestic (country-specific) indicator variables on foreign variables and its lags is non-zero; (2) dependence of country-specific variables on common global exogenous variables is non-zero; (3) contemporaneous dependence of shocks among countries is non-zero. While (1) and (2) assume on non-zero parameters, (3) assumes non-zero off-diagonal element of the GVAR covariance matrix i.e. non-zero covariances between countries and variables. Therefore, by imposing restriction on the parameters and on the covariance matrix, GVAR can be reduced to a stack of country-specific VAR models.

While the evidence of globalisation and Europeanisation provides an empirical argument in favour of considering cross-sectional dependence when de-trending, this evidence also suggests possible convergence in the trend and cyclical components of individual countries' macroeconomic time series. Evidence from the recent recession suggests not only an increased level of cross-sectional dependence when there is an economic downturn, but the contagion of the recession among countries also implies convergence among individual countries' economic growth rates. There is, therefore, a linkage between cross-sectional dependence and convergence. An increased level of cross-sectional dependence implies convergence among countries. In fact, modelling cross-sectional dependence among countries can be viewed as a way of modelling convergence.

In this paper, we focus on an analysis of the GVAR model across Euro-area countries when de-trending. It will draw out the role of cross-sectional dependencies when de-trending in particular individual countries' GDP. In the application we will consider, in real-time, the impact of temporal changes in cross-sectional dependence on trend and cycle estimates for individual countries. We will also consider the cases when cross-sectional dependence among countries is set to zero. We will look into how the trend and cycle estimates of individual countries relate to each other in both cases.

We will also examine the correlations between the GDP cycles among the Euro-area countries when cross-country dependencies are captured in the de-trending model; and also the correlations between their cycles with that of the Euro-area as a whole when cross-country dependences is switched on and off in the de-trending model. The findings of such analysis should provide a basis of future investigation on whether the cyclical component of GDP among countries are moving together in real-time especially during the recession when cross-sectional dependencies among countries are high. This, in turn, could provide a starting point for further studies on the relationship between changing level of interdependencies and convergence over time.

The plan of this paper is as follow. In section 2, we review the global model. We discuss how the global model is derived from the country-level VARX* models. Following Dees et al. (2009), we explain how the trend and cycle components can be estimated from a global vector error correction (GVEC) form of the GVAR, that captures the pure unit root and co-integrating relationships among the economies. We then discuss the restricted form of GVAR when cross-sectional dependencies are set to zero. Section 3 presents the empirical results. Section 4 concludes.

2 The Global VAR Model – A review

In this section, we first present the GVAR model of Pesaran et al. (2004), Dees et al. (2007) and Pesaran et al. (2009). We then follow Dees et al. (2009) and consider the estimation of the permanent component (trend) and the transitory component (cycle) of a time series using a GVAR.

Let $\mathbf{x}_{it}$ be a k_i–dimensional vector containing the domestic variable of country i, and $_{it}$ denotes a k_i – dimensional vector containing the foreign counterparts of the variables containing in $\mathbf{x}_{it}$. So k_i and k^* are the number of variables in $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$, respectively. For simplicity, we further assume $k_i = k^*_i$. Assuming all variables in $\mathbf{x}_{it}$ and $\mathbf{x}^*_{it}$ are quarterly, consider the following (unconditional) VAR with 4 lags in the logarithm of $\mathbf{x}_{it}$ and $\mathbf{x}^*_{it}$.

$$\begin{aligned} \mathbf{x}_{it} &= c_{0i} + c_{1i}t + \mathbf{\Psi}_{i1}x_{it-1} + \mathbf{\Psi}_{i2}x_{it-2} + \mathbf{\Psi}_{i3}x_{it-3} + \mathbf{\Psi}_{i4}x_{it-4} + \\ &\quad \mathbf{Y}_{i1}\, x^*_{it-1} + \mathbf{Y}_{i2}\, x^*_{it-2} + \mathbf{Y}_{i3}\, x^*_{it-3} + \mathbf{Y}_{i4}\, x^*_{it-4} + u_{it} \end{aligned} \tag{1}$$

$$\begin{aligned} \mathbf{x}^*_{it} &= c^*_i + c^*_{1i}t + \boldsymbol{F}_{i1}x_{it-1} + \boldsymbol{F}_{i2}x_{it-2} + \boldsymbol{F}_{i3}x_{it-3} + \boldsymbol{F}_{i4}x_{it-4} + \\ &\quad \mathbf{\Theta}_{i1}\, x^*_{it-1} + \mathbf{\Theta}_{i2}\, x^*_{it-2} + \mathbf{\Theta}_{i3}\, x^*_{it-3} + \mathbf{\Theta}_{i4}\, x^*_{it-4} + u^*_{it} \end{aligned} \tag{2}$$

$\mathbf{x}^*_{it-p}$ and $\mathbf{x}^*_{it-p}$ with p = 1, 2, 3, 4 denotes the lagged domestic and foreign variables, **t** is a linear time trend. We also define $\mathbf{u}_{it} = (u_{it}^{y}, u_{it}^{ip}, u_{it}^{bal})$ and $\mathbf{u}^*_{it} = (u_{it}^{y}, u_{it}^{ip}, u_{it}^{bal})$. The $k_i \times k_i$ covariance matrix of the errors in the above VAR(4,4), Σ, as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma^{(11)} & \Sigma^{(12)} \\ \Sigma^{(21)} & \Sigma^{(22)} \end{bmatrix} \tag{3}$$

where $\Sigma_{(11)}$ contains the variances and covariances of the domestic variables, and $\Sigma_{(22)}$ contains the variances and covariances of the foreign variables. $\Sigma_{(12)} = \Sigma_{(21)}$ contains the covariances between the domestic and foreign variables. We estimate these country-level VAR(4,4) for each i = 1,..., N country. Then we solve for the (conditional) country-level VARX*(4,4), using the fact that the errors in u_{it} follow a multivariate normal distribution, the conditional mean of the error in the domestic equations on the errors in the foreign equations, $E\,(\mathbf{u}_{it}|\mathbf{u}^*_{it})$, can be written as

$$E\,(\mathbf{u}_{it}|\mathbf{u}^*_{it}) = E\,(\mathbf{u}_{it}) + \Sigma_{(12)}\Sigma^{-1}_{(22)}(\mathbf{u}^*_{it} - E\,(\mathbf{u}^*_{it})) = \Sigma_{(12)}\,\Sigma^{-1}_{(22)}\,\mathbf{u}^*_{it} = \Xi\,\mathbf{u}^*_{it}$$

where $\Xi = \Sigma_{(12)}\,\Sigma^{-1}_{(22)}$ is simply a ${}^{k_i}/_2 \times {}^{k_i}/_2$ matrix of the products of the rows and columns in $\Sigma_{(12)}$. Writing the conditional equation of the error in (1) in the form

$$\mathbf{u}_{it} = E\,(\mathbf{u}_{it}|\mathbf{u}^*_{it}) + \boldsymbol{\nu}_{it} = \Xi\mathbf{u}^*_{it} + \boldsymbol{\nu}_{it} \tag{4}$$

Following the substitution of (4) into (1), and let $\mathbf{a}_{i0} = (\mathbf{c}_{0i} - \Xi\,\mathbf{c}^*_{0i})$, $\mathbf{a}_{i1} = (\mathbf{c}_{1i} - \Xi\,\mathbf{c}^*_{1i})$, $\boldsymbol{\Lambda}_{i0} = \Xi$, $\boldsymbol{\Phi}_{ip} = (\boldsymbol{\Psi}_{ip} - \Xi\,\boldsymbol{F}_{ip})$, and $\boldsymbol{\Lambda}_{ip} = (\boldsymbol{\Upsilon}_{ip} - \Xi\,\boldsymbol{\Theta}_{ip})$, with p = 1, 2, 3, 4. Thus, the above 6-variable VAR(4) can be written as the following 3-variable VARX*(4,4)

$$\begin{aligned} \mathbf{x}_{it} \quad = \quad & \mathbf{a}_{i0} + \mathbf{a}_{i1}\mathbf{t} + \boldsymbol{\Phi}_{i1}\mathbf{x}_{it-1} + \boldsymbol{\Phi}_{i2}\mathbf{x}_{it-2} + \boldsymbol{\Phi}_{i3}\mathbf{x}_{it-3} + \boldsymbol{\Phi}_{i4}\mathbf{x}_{it-4} + \\ & \boldsymbol{\Lambda}_{i0}\mathbf{x}^*_{i1} + \boldsymbol{\Lambda}_{i1}\mathbf{x}^*_{it-1} + \boldsymbol{\Lambda}_{i2}\mathbf{x}^*_{it-2} + \boldsymbol{\Lambda}_{i3}\mathbf{x}^*_{it-3} + \boldsymbol{\Lambda}_{i4}\mathbf{x}^*_{it-4} + \boldsymbol{\nu}_{it} \end{aligned} \tag{5}$$

The foreign variables, $\mathbf{x}^*_{it}$ all of domestic variables vary by country i and are defined as the weighted averages of

$$x^*_{it} = \sum_{j=1}^{N} w_{ij}\, x_{jt}; \; w_{ii} = 0 \tag{6}$$

with the predetermined weights w_{ij} defined as the GDP weights in our analysis. That is, the size of country j's GDP relative to the sum of GDP in the other $N - 1$ (excluding country i) countries. We further let $z_{it} = (x'_{it}, x^{*'}_{it})'$, rewriting (5) as

$$\mathbf{A}_i\mathbf{z}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}\mathbf{t} + \mathbf{B}_{i1}\mathbf{z}_{it-1} + \mathbf{B}_{i2}\mathbf{z}_{it-2} + \mathbf{B}_{i2}\mathbf{z}_{it-2} + \mathbf{B}_{i3}\mathbf{z}_{it-3} + \mathbf{B}_{i4}\mathbf{z}_{it-4} + \boldsymbol{\nu}_{it} \tag{7}$$

where $\mathbf{A}_i = (\mathbf{I}_k, -\boldsymbol{\Lambda}_{i0})$, and $\mathbf{B}_{i1} = (\boldsymbol{\Phi}_{i1}, \boldsymbol{\Lambda}_{i1})$, $\mathbf{B}_{i2} = (\boldsymbol{\Phi}_{i2}, \boldsymbol{\Lambda}_{i2})$, $\mathbf{B}_{i3} = (\boldsymbol{\Phi}_{i3}, \boldsymbol{\Lambda}_{i3})$, $\mathbf{B}_{i4} = (\boldsymbol{\Phi}_{i4}, \boldsymbol{\Lambda}_{i4})$,. Since the domestic variables can be written as

$$\mathbf{z}_{it} = \mathbf{W}_i\mathbf{x}_t \tag{8}$$

where $\mathbf{W}_i$ is the weight matrix, and $\mathbf{x}_t$ is the global vector i.e. $x_t = (x'_{1t}, x'_{2t}, \ldots, x'_{Nt})$. Using (7) and (8), we have

$$\mathbf{A}_i\mathbf{W}_i\mathbf{x}_t = \mathbf{a}_{i0} + \mathbf{a}_{i1}\mathbf{t} + \mathbf{B}_{i1}\mathbf{W}_i\mathbf{x}_{t-1} + \mathbf{B}_{i2}\mathbf{W}_i\mathbf{x}_{t-2} + \mathbf{B}_{i3}\mathbf{W}_i\mathbf{x}_{t-3} + \mathbf{B}_{i4}\mathbf{W}_i\mathbf{x}_{t-4} + \boldsymbol{\nu}_{it}$$

The GVAR model is thus derived by stacking up all N country-specific VARX* models to yield

$$\mathbf{Gx}_t = \mathbf{a}_0 + \mathbf{a}_1\mathbf{t} + \mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{H}_2\mathbf{x}_{t-2} + \mathbf{H}_3\mathbf{x}_{t-3} + \mathbf{H}_4\mathbf{x}_{t-4} + \boldsymbol{\epsilon}_t \tag{9}$$

$$\mathbf{x}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{G}^{-1}\mathbf{H}_2\,\mathbf{x}_{t-2} + \mathbf{G}^{-1}\mathbf{H}_3\,\mathbf{x}_{t-3} + \mathbf{G}^{-1}\mathbf{H}_4\,\mathbf{x}_{t-4} + \mathbf{G}^{-1}\boldsymbol{\epsilon}_t \tag{10}$$

where

$$\boldsymbol{G} = \begin{bmatrix} A_0 W_0 \\ \vdots \\ A_N W_N \end{bmatrix}, \qquad \boldsymbol{H}_1 = \begin{bmatrix} B_{01} W_0 \\ \vdots \\ B_{N1} W_N \end{bmatrix}, \qquad \boldsymbol{H}_2 = \begin{bmatrix} B_{02} W_0 \\ \vdots \\ B_{N2} W_N \end{bmatrix}, \tag{11}$$

$$\boldsymbol{H}_3 = \begin{bmatrix} B_{03} W_0 \\ \vdots \\ B_{N3} W_N \end{bmatrix}, \qquad \boldsymbol{H}_4 = \begin{bmatrix} B_{04} W_0 \\ \vdots \\ B_{N4} W_N \end{bmatrix}, \qquad \boldsymbol{\varepsilon}_t = \begin{bmatrix} v_{0t} \\ \vdots \\ v_{Nt} \end{bmatrix}, \tag{12}$$

The GVAR in (10) captures cross-country and cross-variable dependencies through the off-diagonal elements in its parameter matrices and the covariance matrix of its error. By imposing zero restrictions on the off-diagonal elements on the parameter and the covariance matrix, the global model can be reduced to a stack of country-specific VARs. For a discussion of the relationship between disaggregate models and multivariate global model, see for example, Lui & Mitchell (2012).

2.1 The vector error correction form of the global model

GVAR as in (10) can be written in an error correction form i.e. a GVEC. To obtain the GVEC form of (10), we write

$$\mathbf{x}_t - \mathbf{G}^{-1}\mathbf{H}_1\mathbf{x}_{t-1} - \mathbf{G}^{-1}\mathbf{H}_2\mathbf{x}_{t-2} - \mathbf{G}^{-1}\mathbf{H}_3\mathbf{x}_{t-3} - \mathbf{G}^{-1}\mathbf{H}_4\mathbf{x}_{t-4} = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{a}_1\mathbf{t} + \mathbf{G}^{-1}\boldsymbol{\epsilon}_t$$

$$(\mathbf{I} - \mathbf{G}^{-1}\mathbf{H}_1 L - \mathbf{G}^{-1}\mathbf{H}_2 L^2 - \mathbf{G}^{-1}\mathbf{H}_3 L^3 - \mathbf{G}^{-1}\mathbf{H}_4 L^4)\,\mathrm{x}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{a}_1\mathbf{t} + \mathbf{G}^{-1}\boldsymbol{\epsilon}_t \tag{13}$$

define $\boldsymbol{\rho} = \mathbf{G}^{-1}\mathbf{H}_1 + \mathbf{G}^{-1}\mathbf{H}_2 + \mathbf{G}^{-1}\mathbf{H}_3 + \mathbf{G}^{-1}\mathbf{H}_4$; and $\xi_s = -[\mathbf{G}^{-1}\mathbf{H}_{s+1} + \mathbf{G}^{-1}\mathbf{H}_{s+2} + \ldots + \mathbf{G}^{-1}\mathbf{H}_p]$, for $s = 1, 2, \ldots, p-1$. For any values of $\mathbf{G}^{-1}\mathbf{H}_1$, $\mathbf{G}^{-1}\mathbf{H}_2$, $\mathbf{G}^{-1}\mathbf{H}_3, \ldots, \mathbf{G}^{-1}\mathbf{H}_p$, the polynomial in L of (13) is equivalent to

$$(\mathbf{I} - \boldsymbol{\rho}L) - (\xi_1 L + \xi_2 L^2 + \xi_3 L^3)(\mathbf{I} - L) \tag{14}$$

So

$$(\mathbf{I} - \boldsymbol{\rho}L)\,\mathrm{x}_t - (\xi_1 L + \xi_2 L^2 + \xi_3 L^3)(\mathbf{I} - L)\,\mathrm{x}_t = \mathbf{G}^{-1}\mathbf{a}_0 - \mathbf{G}^{-1}\mathbf{a}_1\mathbf{t} - \mathbf{G}^{-1}\boldsymbol{\epsilon}_t$$

$$(\mathrm{x}_t - \boldsymbol{\rho}\mathrm{x}_{t-1}) - (\xi_1\Delta\mathrm{x}_{t-1} + \xi_2\Delta\mathrm{x}_{t-2} + \xi_3\Delta\mathrm{x}_{t-3}) = \mathbf{G}^{-1}\mathbf{a}_0 - \mathbf{G}^{-1}\mathbf{a}_1\mathbf{t} - \mathbf{G}^{-1}\boldsymbol{\epsilon}_t \tag{15}$$

and

$$\mathrm{x}_t = \mathbf{G}^{-1}\mathbf{a}_0 + \mathbf{G}^{-1}\mathbf{a}_1\mathbf{t} + \xi_1\Delta\mathrm{x}_{t-1} + \xi_2\Delta\mathrm{x}_{t-2} + \xi_3\Delta\mathrm{x}_{t-3} + \boldsymbol{\rho}\mathrm{x}_{t-1} + \mathbf{G}^{-1}\boldsymbol{\epsilon}_t \tag{16}$$

substitute $\boldsymbol{\xi}_i$ and $\boldsymbol{\rho}$ back into the equation, and subtract x_{t-1} from both sides of the equation gives

$$\begin{aligned}\mathrm{x}_t - \mathrm{x}_{t-1} = {} & \mathbf{G}^{-1}\mathbf{a}_0 - \mathbf{G}^{-1}\mathbf{a}_1\mathbf{t} + \mathbf{G}^{-1}[-\mathbf{H}_2 - \mathbf{H}_3 - \mathbf{H}_4]\,\Delta\mathrm{x}_{t-1} + \\ & \mathbf{G}^{-1}[-\mathbf{H}_3 - \mathbf{H}_4]\,\Delta\mathrm{x}_{t-2} + \mathbf{G}^{-1}[-\mathbf{H}_4]\,\Delta\mathrm{x}_{t-3}\,(\boldsymbol{\rho} - \mathbf{I})\,\mathrm{x}_{t-1} + \mathbf{G}^{-1}\boldsymbol{\epsilon}_t\end{aligned}$$

where $(\boldsymbol{\rho} - \mathbf{I}) = -(-\mathbf{I} - \mathbf{G}^{-1}\mathbf{H}_1 - \mathbf{G}^{-1}\mathbf{H}_2 - \mathbf{G}^{-1}\mathbf{H}_3 - \mathbf{G}^{-1}\mathbf{H}_4) = -\mathbf{G}^{-1}\mathbf{H}(\mathrm{I})$. And $-\mathbf{G}^{-1}\mathbf{H}(\mathrm{I}) = -\mathbf{G}^{-1}\tilde{\alpha}\tilde{\beta}'$, where $\tilde{\alpha}$ is the block diagonal matrix of the global loading coefficients and $\tilde{\beta}$ is the cointegrating matrix. So

$$G\Delta xt = \mathbf{a}_0 + \mathbf{a}_1\mathbf{t} + [\mathbf{H}_2 - \mathbf{H}_3 - \mathbf{H}_4]\,\Delta x_{t-1} + [-\mathbf{H}_3 - \mathbf{H}_4]\,\Delta x_{t-2} + [-\mathbf{H}_4]\,\Delta x_{t-3} - \tilde{\alpha}\tilde{\beta}'\,\Delta x_{t-1} + \boldsymbol{\epsilon}_t$$

Let $\Gamma_1 = -\mathbf{H}_2 - \mathbf{H}_3 - \mathbf{H}_4$; $\Gamma_2 = -\mathbf{H}_3 - \mathbf{H}_4$; $\Gamma_3 = -\mathbf{H}_4$. The above equation can be written as

$$G\Delta x_t = a_0 + a_1\mathbf{t} + \Gamma_1\Delta x_{t-1} + \Gamma_2\Delta x_{t-2} + \Gamma_3\Delta x_{t-3} - \tilde{\alpha}\tilde{\beta}'\,x_{t-1} + \boldsymbol{\epsilon}_t \quad (17)$$

$$G\Delta x_t = a_0 + a_1\mathbf{t} - \tilde{\alpha}\tilde{\beta}'\,x_{t-1} + \sum_{i=1}^{p-1}\Gamma_i\,\Delta x_{t-i} + \boldsymbol{\epsilon}_t; \quad p=4 \quad (18)$$

(17) is the error correction form of the GVAR. To ensure no quadratic trends in the variables in the global model, the trend coefficients, a_1, must be restricted to satisfy the condition

$$a_1 = \tilde{\alpha}\tilde{\beta}'\,\boldsymbol{\gamma}$$

where $\tilde{\alpha}\tilde{\beta}' = (G - H_1 - H_2 - H_3 - H_4)$. $\tilde{\alpha}$ is a k × r block-diagonal matrix of the global loading coefficients, with diagonal elements α_j and $r = \sum_{i=1}^{N} r_i$ and r_i is the cointegrating rank of country i. $\tilde{\beta}$ is the k × r cointegrating matrix. $\boldsymbol{\gamma}$ is a k × 1 vector of fixed constants.

2.2 Estimating the trend and cycle components using the GVAR

Before we derive the permanent and the transitory components using GVAR, we first recall the transitory component of country *i*'s GDP, y^C_{it} , is the deviation of GDP from its steady state. i.e. $y^C_{it} = y_{it} - y^P_{it}$, where y^P_{it} denotes the permanent component of GDP. We can see the permanent and transitory component can be obtained from the decomposition of the variables in the above GVAR, that is

$$\mathbf{x}_t = \mathbf{x}^P_t + \mathbf{x}^C_t$$

$$\mathbf{x}_t = (\mathbf{x}^P_{dt} + \mathbf{x}^P_{st}) + \mathbf{x}^C_t$$

where $\mathbf{x}^P_{dt}$ and $\mathbf{x}^P_{st}$ are the permanent-deterministic component and the permanent-stochastic components of $\mathbf{x}^P_t$. The permanent deterministic component, $\mathbf{x}^P_{dt} = \boldsymbol{\mu} + \mathbf{g}_t$, where μ and g are k × 1 vectors of fixed constants, while t is the deterministic time trend. The permanent stochastic component, $\mathbf{x}^P_{st}$, is uniquely defined as the “long-horizon forecast”, i.e.

$$x^P_{st} = \lim_{h\to\infty} E_t\,(x_{t+h} - x^P_{d,t+h})$$

Dees et al. (2009) point out that $\mathbf{x}^P_{st}$ is identically equal to zero if $\mathbf{x}_t$ is trend stationary, but it will be equal to x_t if x_t has a pure unit root or is non-cointegrated. GVAR captures the unit root and co-integration properties of the global economy, and at the same time, allows for the permanent and transitory components to be conditional on the cross-country and cross-variable dependence.

We now return to the GVAR decomposition, and first derive a VAR representation of the global error correction form. Rewrite (17) as

$$\mathbf{G}x_t - \mathbf{G}x_{t\text{-}1} = a_0 + \tilde{\alpha}\tilde{\beta}'\gamma\mathbf{t} + \Gamma_1\,\mathbf{x}_{t\text{-}1} + (\Gamma_2 - \Gamma_1)\,\mathbf{x}_{t\text{-}2} + (\Gamma_3 - \Gamma_2)\,\mathbf{x}_{t\text{-}3} - \Gamma_3\mathbf{x}_{t\text{-}4} - \tilde{\alpha}\tilde{\beta}'\,\mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t$$

$$\mathbf{G}x_t = a_0 + \tilde{\alpha}\tilde{\beta}'\gamma\mathbf{t} + (\mathbf{G} + \Gamma_1 - \tilde{\alpha}\tilde{\beta}')\,\mathbf{x}_{t\text{-}1} + (\Gamma_2 - \Gamma_1)\,\mathbf{x}_{t\text{-}2} + (\Gamma_3 - \Gamma_2)\,\mathbf{x}_{t\text{-}3} - \Gamma_3\mathbf{x}_{t\text{-}4} + \boldsymbol{\epsilon}_t$$

$$\mathbf{x}_t = \mathbf{G}^{-1}a_0 + G^{-1}\tilde{\alpha}\tilde{\beta}'\,\gamma\mathbf{t} + \mathbf{G}^{-1}\,(\mathbf{G} + \Gamma_1 - \tilde{\alpha}\tilde{\beta}')\,\mathbf{x}_{t\text{-}1} + \mathbf{G}^{-1}\,(\Gamma_2 - \Gamma_1)\,x_{t\text{-}2} + \quad (22)$$

$$\mathbf{G}^{-1}\,(\Gamma_3 - \Gamma_2)\,\mathbf{x}_{t\text{-}3} - \mathbf{G}^{-1}\Gamma_3\mathbf{x}_{t\text{-}4} + \mathbf{G}^{-1}\boldsymbol{\epsilon}_t \quad (23)$$

or define $b_0 = G^{-1}a_0$; $b_1 = G^{-1}\tilde{\alpha}\widetilde{\beta}'\gamma$; $\Omega_1 = G^{-1}$ $(\mathbf{G} + \Gamma_1 - \tilde{\alpha}\widetilde{\beta}')$; $\Omega_2 = G^{-1}(\Gamma_2 - \Gamma_1)$; $\Omega_3 = G^{-1}(\Gamma_3 - \Gamma_2)$; $\Omega_4 = - G^{-1}\Gamma_3$ and $\boldsymbol{\varepsilon}_t = G^{-1}\boldsymbol{\epsilon}_t$. 22 can be written as

$$x_t = b_0 + b_1\mathbf{t} + \Omega_1 x_{t-1} + \Omega_2 x_{t-2} + \Omega_3 x_{t-3} + \Omega_4 x_{t-4} + \boldsymbol{\varepsilon}_t \quad (24)$$

If we rewrite (24) in terms of (20),

$$x_t = \mathbf{x}^P{}_{dt} + \underbrace{\mathbf{C}(1)\, s_{\varepsilon_t}}_{\mathbf{x}^P{}_{st}} + \underbrace{\mathbf{C}^*(L)\, \boldsymbol{\varepsilon}_t}_{\mathbf{x}^C{}_t}$$

$$x_t = \boldsymbol{\mu} + \mathbf{gt} + \mathbf{C}(1)\, s_{\varepsilon_t} + \mathbf{C}^*(L)\, \boldsymbol{\varepsilon}_t$$

Define

$s_{\varepsilon_t} = \sum_{j=1}^{t} \varepsilon_j$; $C(1) = \sum_{j=0}^{\infty} C_j$; and

$\mathbf{C}_j = \mathbf{C}_{j-1}\,\Omega_1 + \mathbf{C}_{j-2}\,\Omega_2 + \mathbf{C}_{j-3}\,\Omega_3 + \mathbf{C}_{j-4}\,\Omega_4$, for $j = 2, 3, \ldots$;

$\mathbf{C}_0 = I_k$; $C_1 = -(\mathbf{I}_k - \Omega_1)$; $C_j = 0$ for $j < 0$;

$\mathbf{C}^*_j = \mathbf{C}^*_{j-1} + \mathbf{C}_j$, for $j = 1, 2, \ldots$, with $\mathbf{C}^*_0 = \mathbf{C}_0 - C(1)$

The permanent stochastic trend is given by

$$\mathbf{x}^P{}_{st} = C(1) \sum_{j=1}^{t} \varepsilon_j \quad (25)$$

and this is equivalent to the long-horizon forecast in (21), which is also the multivariate version of the Beveridge-Nelson stochastic trend. $\mathbf{x}^P{}_{st}$ can then be computed from the GVAR as (25) is simply a moving average representation involving the errors. So $\widehat{x^P_{st}} = C(1) \sum_{j=1}^{t} \hat{\varepsilon}_j$. In our case when the GVAR does not include a time trend, the cyclical component, $\mathbf{x}^C{}_t$ is given by

$$\mathbf{x}_t = (\mathbf{x}^P{}_{dt} + \mathbf{x}^P{}_{st}) + \mathbf{x}^C{}_t$$

Let $v_t = (x_t - \mathbf{x}^P{}_{st})$. So the estimated $\hat{v}_t$ is therefore

$$\hat{v}_t = \hat{\boldsymbol{\mu}} + \hat{\boldsymbol{g}}t + \widehat{\boldsymbol{x}^C_t}$$

$\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{g}}$ can be estimated by OLS for each country and each variable by regressing the elements of $v_{i,l,t}$ on $\mu_{i,l,t}$ and $g_{i,l,t}$, for country i and variable l in the country's k_i vector of variables. That is,

$$\widehat{v_{i,l,t}} = \widehat{\mu_{i,l,t}} + \widehat{g_{i,l,t}}t + \widehat{\eta_{i,l,t}}$$

The cyclical component, $\widehat{\boldsymbol{x}^C_t}$, is thus the stack of these OLS errors.

Recall the global vector $x_t = (x'_{1t}, x'_{2t}, \ldots, x'_{Nt})'$ contains the domestics variables of all N countries, as do the trend and cyclical components of $\mathbf{x}_t$. Our interest is to use the GVAR to derive trend and cyclical components for individual countries within the Euro-area. However, the aggregate EA trend and cycle can actually be computed by aggregating up the country-specific GVAR trend and cycle components. The country- specific, as well as the aggregate trend and cycle estimates thus accommodate the cross country dependencies among the N countries.

2.3 A restricted GVAR

We have seen in the previous section how GVAR accommodate cross-sectional dependence through non-zero parameters and covariances. In fact when appropriate restrictions are applied, the models can be reduced to a univariate setting. Consider the case when crosssectional dependence is assumed to be zero, this implies parameters on the foreign variable in the country-specific models in (5) to be zero. In this case, equation (5) will be reduced to

$$\mathbf{x}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}\mathbf{t} + \mathbf{\Phi}_{i1}\mathbf{x}_{it-1} + \mathbf{\Phi}_{i2}\mathbf{x}_{it-2} + \mathbf{\Phi}_{i3}\mathbf{x}_{it-3} + \mathbf{\Phi}_{i4}\mathbf{x}_{it-4} + \nu_{it}$$

Zero cross-country, cross-variable covariances implies the off-diagonal elements of the error ν_{it} to be zero. That is, *Cov* $(\nu_{it}, \nu_{jt}) = 0$. This is the same as estimating equation (1) with no reference to (2) as $\Sigma_{(12)} = \Sigma_{(21)} = 0$ in (3). The GVAR with zero cross-sectional dependence can therefore a stack of equation (1).

$$\mathbf{x}_t = \mathbf{J}_0 + \mathbf{J}_1\mathbf{t} + \mathbf{D}_1\mathbf{x}_{t-1} + \mathbf{D}_2\mathbf{x}_{t-2} + \mathbf{D}_3\mathbf{x}_{t-3} + \mathbf{D}_4\mathbf{x}_{t-4} + \boldsymbol{E}_t \quad (26)$$

Where the covariance matrix of $\boldsymbol{E}_t$ has non-zero diagonal element but zero off-diagonal elements. J 'sand **D**'s are parameter matrices. (26) is therefore a stack of country-specific unconditional VAR models. Each model can then be estimated separately for each country. The trend and cyclical components can be computed separately for each country's unconditional VAR model. If one's interest is on aggregate EA trend and cycle estimates, they can still be obtained by aggregating up the disaggregate country-level trend and cycle estimates. In fact, if cross-sectional dependence plays a role in de-trending country-level GDP, we would expect both the country-level estimates and the aggregate EA estimates from disaggregate VAR to be different from that of GVAR.

3 Empirical Results

3.1 Analysing the Permanent and cyclical components of GDP of the euro-area countries

To examine how changing levels of cross-sectional dependencies among countries over time impacts on trend and cycle estimates, we conduct trend and cycle decomposition in real-time for twelves euro-area countries (So N = 12), the EA12. They are Germany, France, Italy, Spain, Netherlands, Austria, Belgium, Portugal, Finland, Greece, Ireland and Luxembourg. To ensure any difference in the resulting trend and cycle estimates from the GVAR and the restricted GVAR are solely due to cross-sectional dependencies among countries and indicator variables, we employ the same information set for both de-trending models. The information set contains, individual countries' hard data, namely GDP and IP, obtained from the EuroInd database. As well as individual countries' soft data namely the Economic Sentiment Indicator (ESI) published by the DG-ECFIN of the European Commission, except for Ireland we use the Consumer Confidence Indicator as the Irish ESI composite is not available. The ESI combines various information from qualitative business tendency surveys, including expectation questions, into a single confidence indicator.

The first real-time vintages of the hard data we took for the hard data are for the release of 2004Q1 quarterly estimates, while the last vintage we took are for the release of the 2012Q4 quarterly estimates. First releases of the hard data are used. There are therefore, a total of 36 quarterly real-time trend and cycle estimates for each model employed. We consider the logarithm of GDP, IP and survey data at levels. Since GDP are published quarterly while IP and survey data are both published monthly, we compute the quarterly aggregates for the monthly variables as the mean of their log level of 3 months in a quarter. So the detrending models are quarterly models. To avoid the problem of small T in estimation, especially in the global model while the number of variables in total is large, we backcast all real-time data triangle to start from 1985Q1 (1985m1 for IP).

3.2 The role of cross-sectional dependencies in estimating the trend and cycle components for euro-area countries

Our empirical analysis will focus on the role of cross-sectional dependencies in estimating trend and cycle estimates for individual countries in the euro-area. We will look at the impact of changing levels of cross-sectional dependence among countries on their trend and cycle estimates. In particular, we will investigate how trend and cycle estimates have changed over time in two scenarios: (1) when cross-sectional dependencies are accommodated; (2) when cross-sectional dependence is assumed to be zero. In other words, we will consider a GVAR and restricted GVAR in which cross-sectional dependencies are restricted to zero. Conducting de-trending in real-time and over time, would allow one to draw inference of the impact of changing level of cross-sectional dependence on the trend and cycle estimates of the euro-area countries. We also present evidence on the correlations of the cycles among euro-area countries, as well as the correlation of the country-level cycles with the euro-area cycle as a whole. We will show why it matters to capture interdependencies in the trend and cycle decomposition; as well as in the examination of correlations of the cyclical components of GDP between countries and the euro-area.

3.3 Real-time GDP trend and cycle estimates for euro-area countries

We produce real-time trend and cycle estimates using the GVAR and the restricted GVAR, explained in the previous subsections, as the de-trending models. We tried with different lag orders for both models. A multivariate Beveridge-Nelson decomposition is employed. It means the GVAR trend and cycle estimates condition on a larger information set as compare to traditional univariate and multivariate application of the Beveridge-Nelson trend and cycle decomposition.

We present results of trend and cycle estimates of both GVAR and restricted GVAR with 2 lags (data starts from 1985Q3 as two quarterly data points are lost) and 4 lags (data starts from 1986Q1 as four quarterly data points are lost). There are in total 96 plots of all estimated real-time trend and cycle series of individual countries. They are provided to the Eurostat in an Appendix folder namely ”Appendix figures” which contains the PDF files of all the 96 plots of the estimated series. Figure 1a to 1l in the ”Appendix figures” folder present the real-time cycle of the EA12 countries estimated using a GVAR(4). Figure 2a to 2l present the real-time cycles using a GVAR(2). While Figure 5a to 5l and Figure 6a to 6l plot the estimated trends from the GVAR(4) and GVAR(2), respectively. Figure 3a to 3l plot the real-time cycles of the EA12 countries estimated using restricted GVAR(4), while Figure 4a to 4l plot the real-time cycles from restricted GVAR(2) models. Whereas Figure 7a to 7l and Figure 8a to 8l plot the real-time trends of the restricted GVAR(4) and restricted GVAR(2), respectively.

We take a sample of the plots from the Appendix to present them here. The samples we have taken are GVAR and Restricted GVAR trend and cycles of Austria, Germany and Spain. These plots are shown in Figure 1 to 12.

Cycles from GVAR(4) appears to be more volatile than that of GVAR(2) post-2009Q1. It can be seen from the graphs that although the cycle estimates fluctuates around zero, they fluctuate within a wider range. It can also be seen that while both models produce smooth trends pre-2009Q1, the trend estimates from GVAR(4) are more noisy post-2009Q1. In fact, GVAR(2) trends appears to be relatively smoother than GVAR(4) trends in general, except the trend estimates for the first two quarters of 2009 appear to be noisy. Perhaps it is not surprising as it was the recession period. Besides, cycle estimates from both models appear to produce similar patterns pre-2009 in general, but not from 2009

Figure 1: Real-time Austrian cycles from GVAR(4)

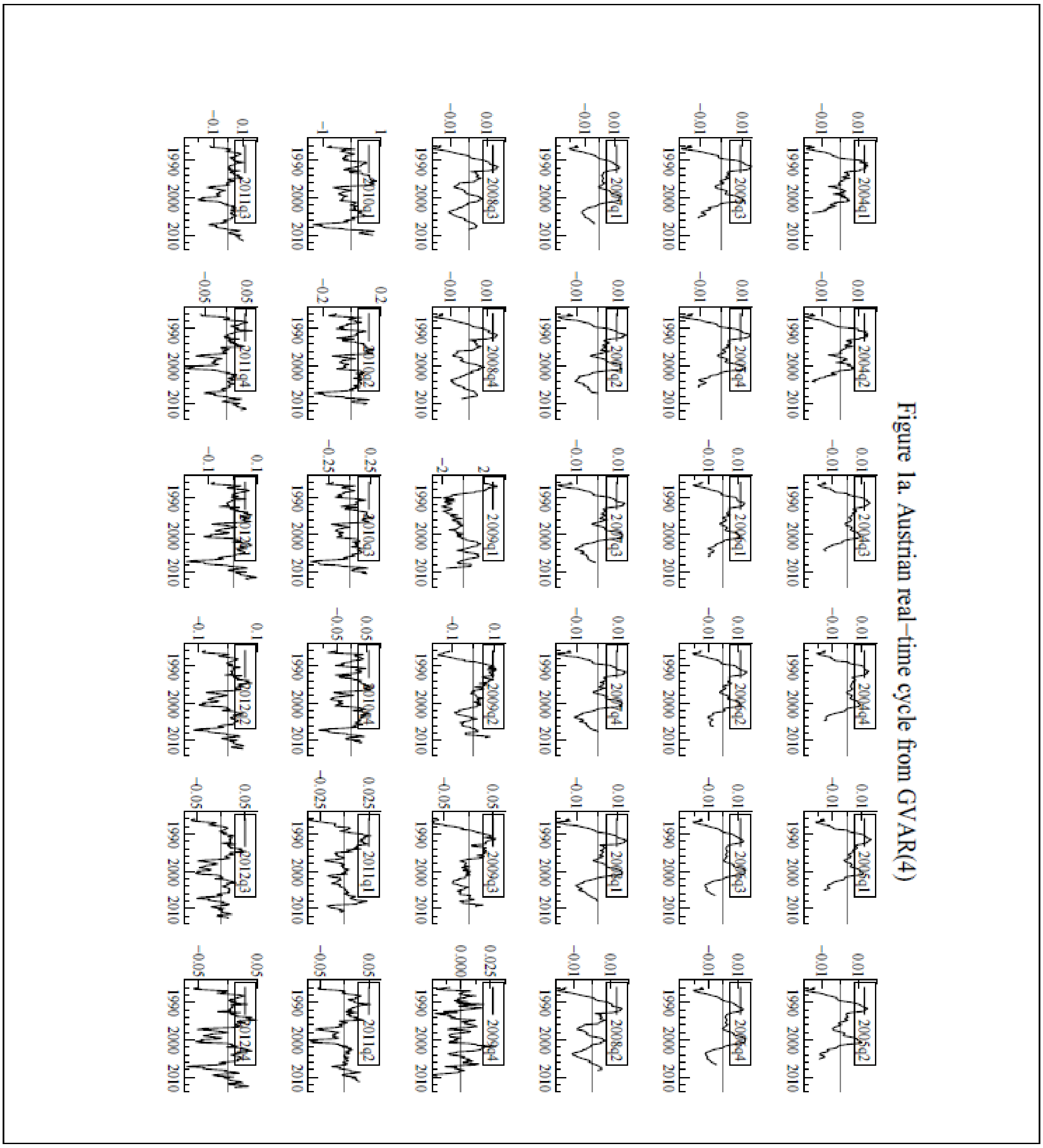


Figure 2: Real-time Austrian trend from GVAR(4)

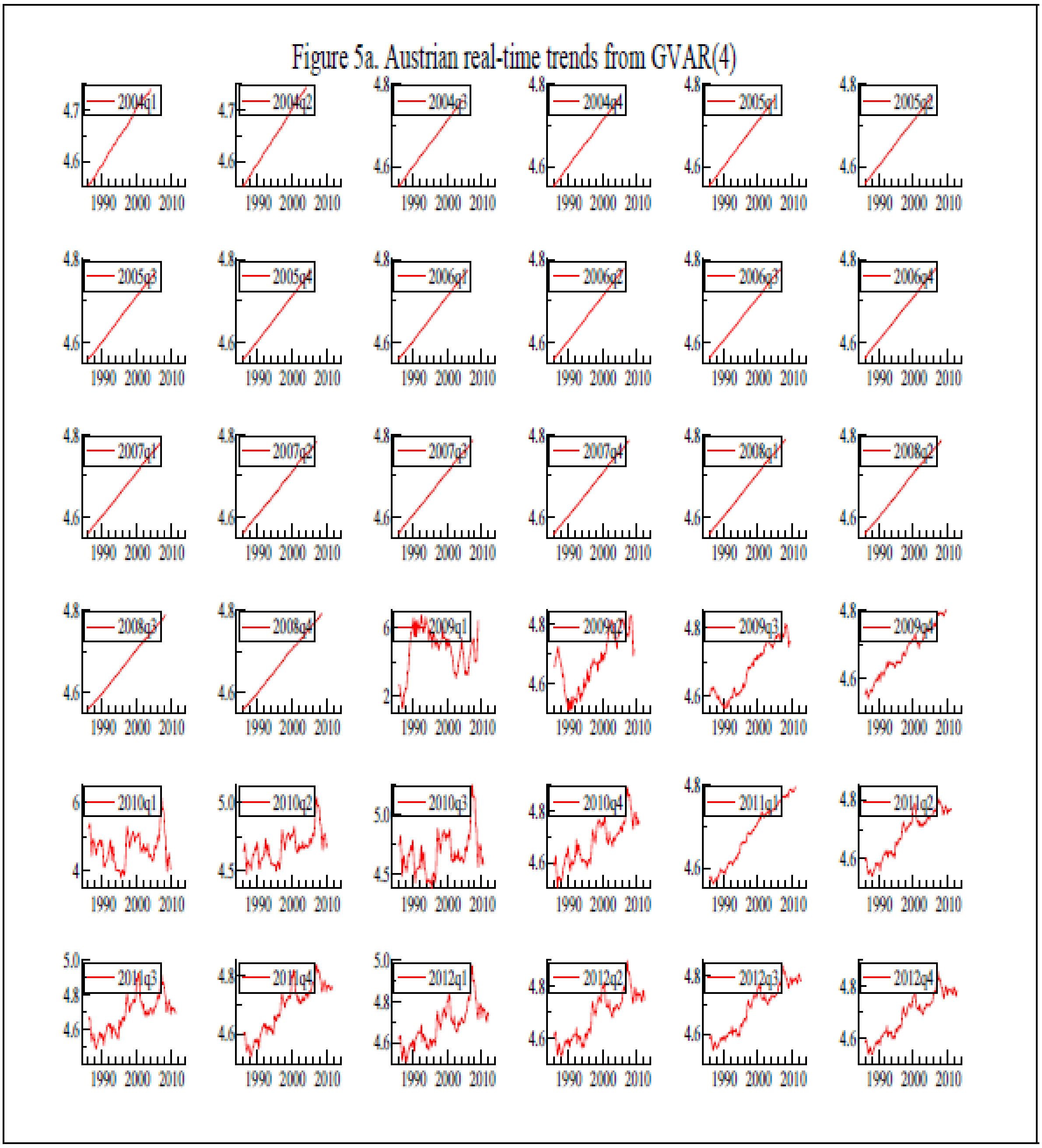


Figure 3: Real-time German cycles from GVAR(4)

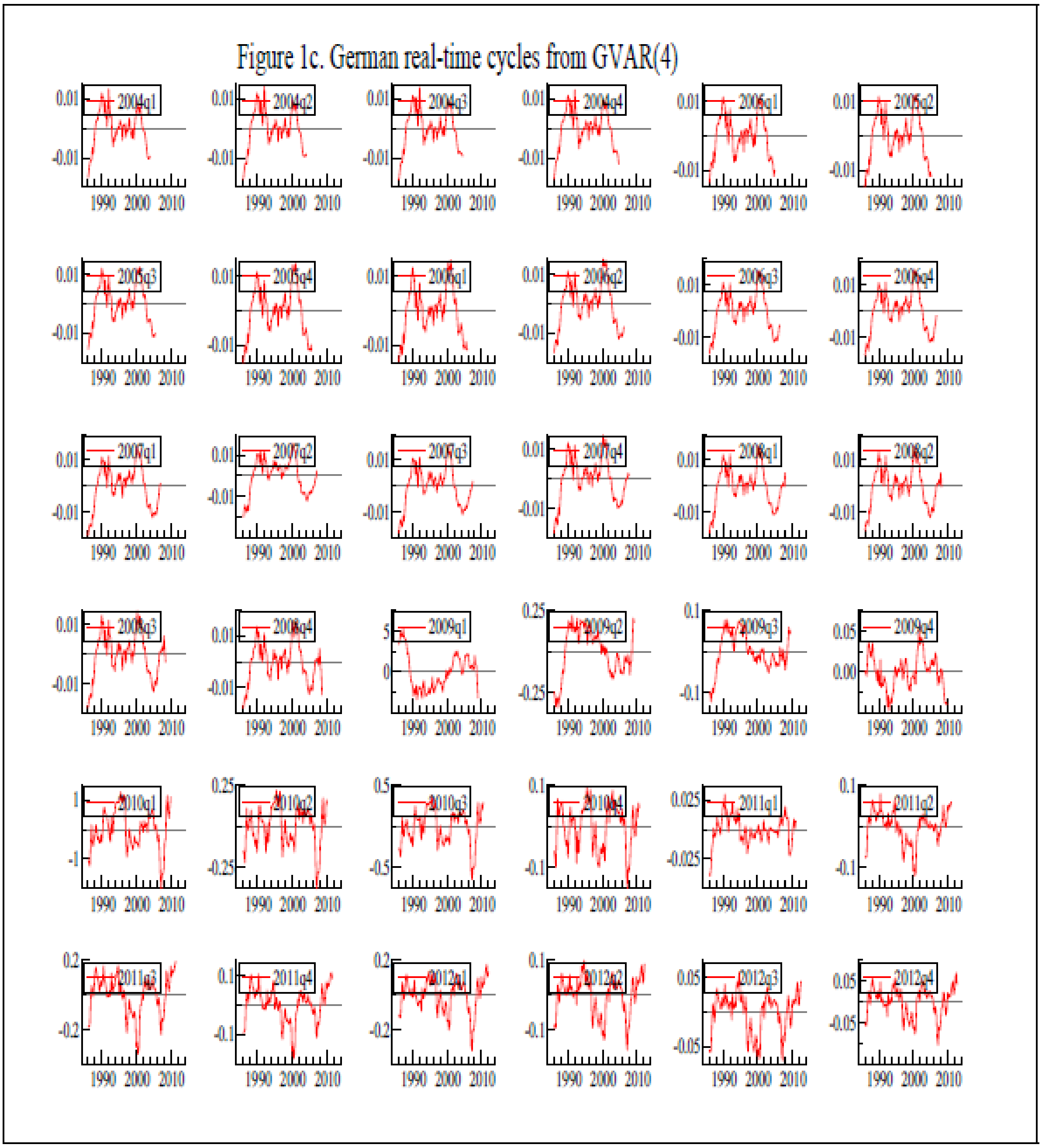


Figure 4: Real-time German trend from GVAR(4)

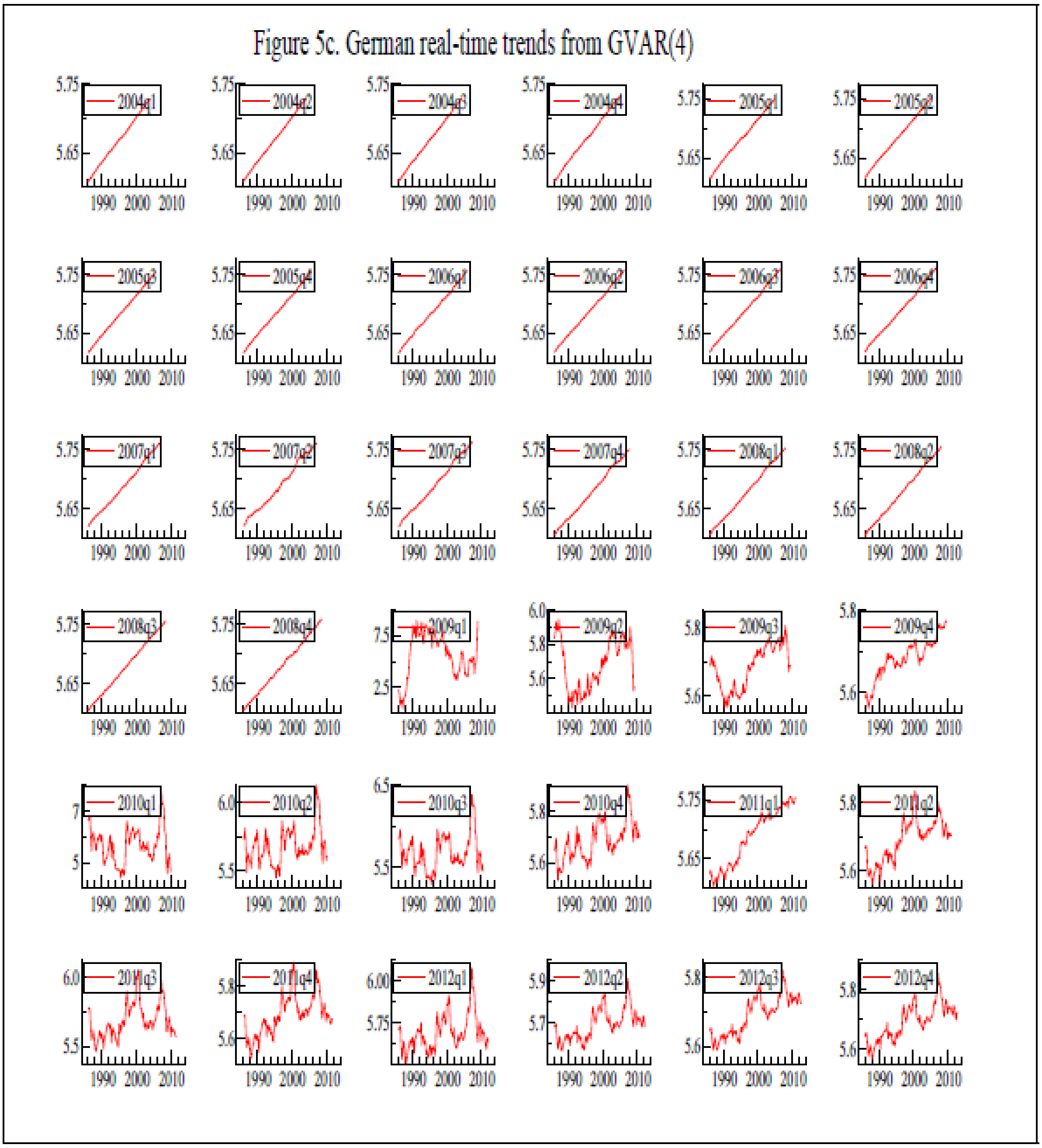


Figure 5: Real-time Spanish cycles from GVAR(4)

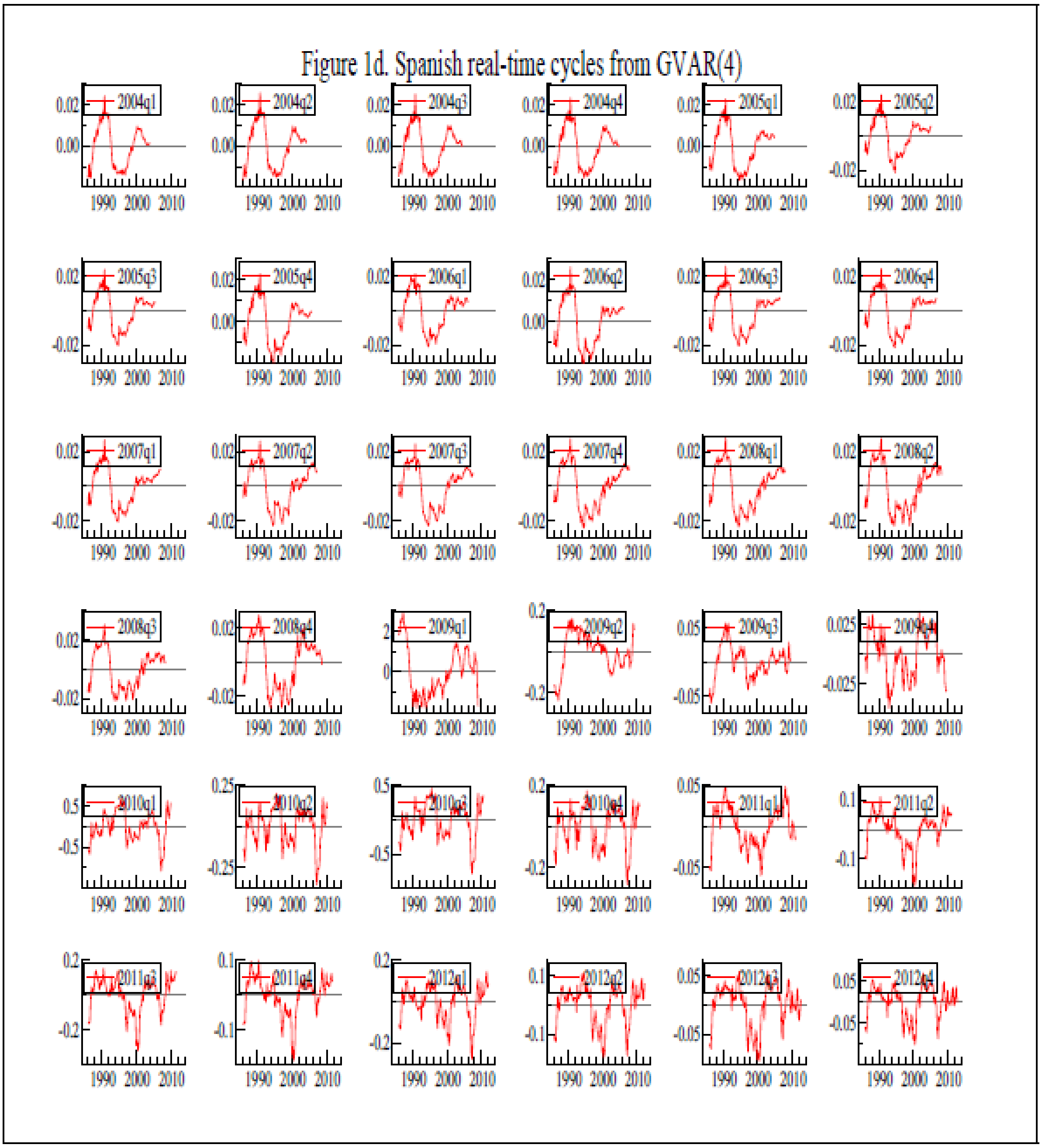


Figure 6: Real-time Spanish trend from GVAR(4)

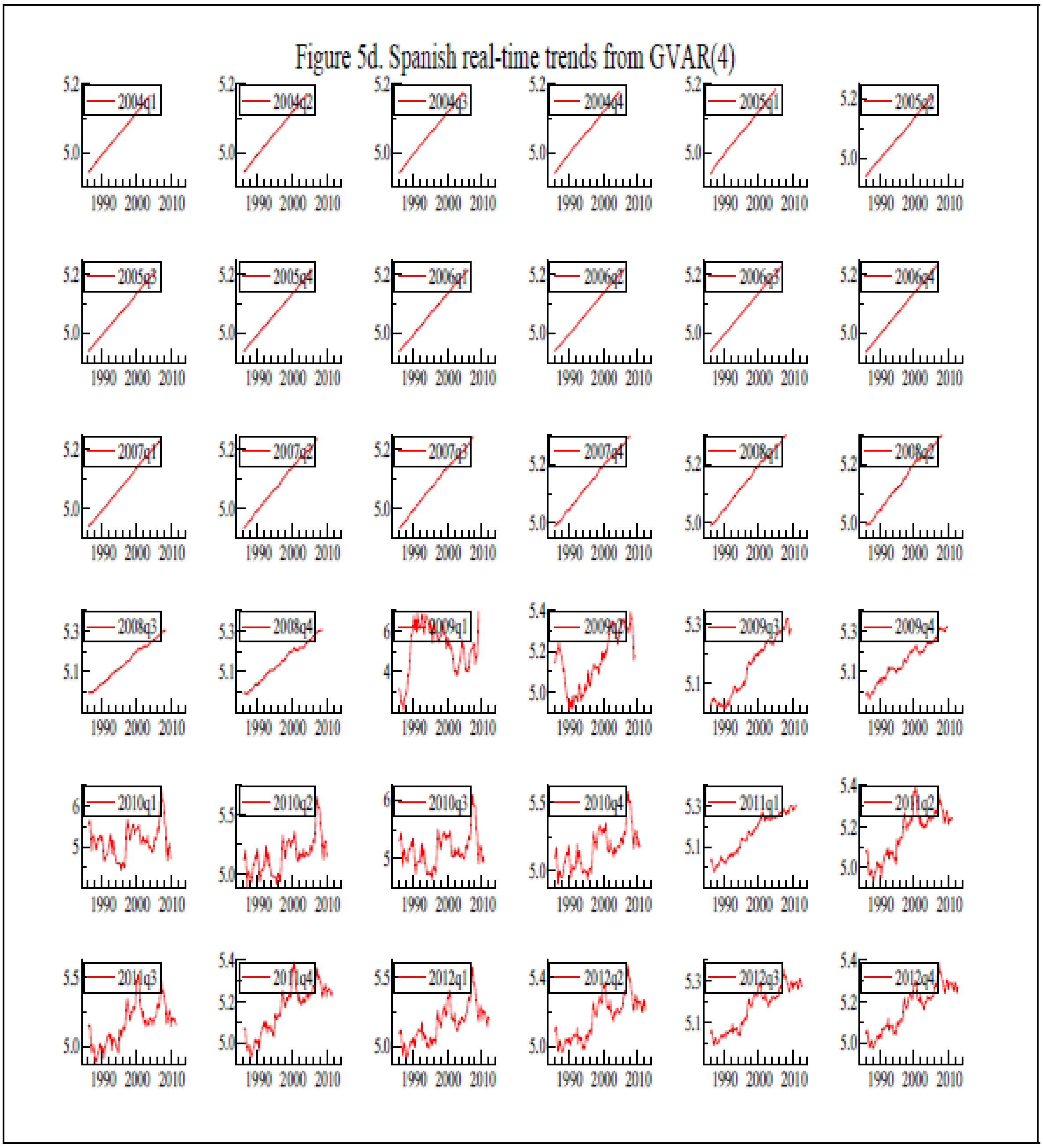


Figure 7: Real-time Austrian cycles from VAR(4)

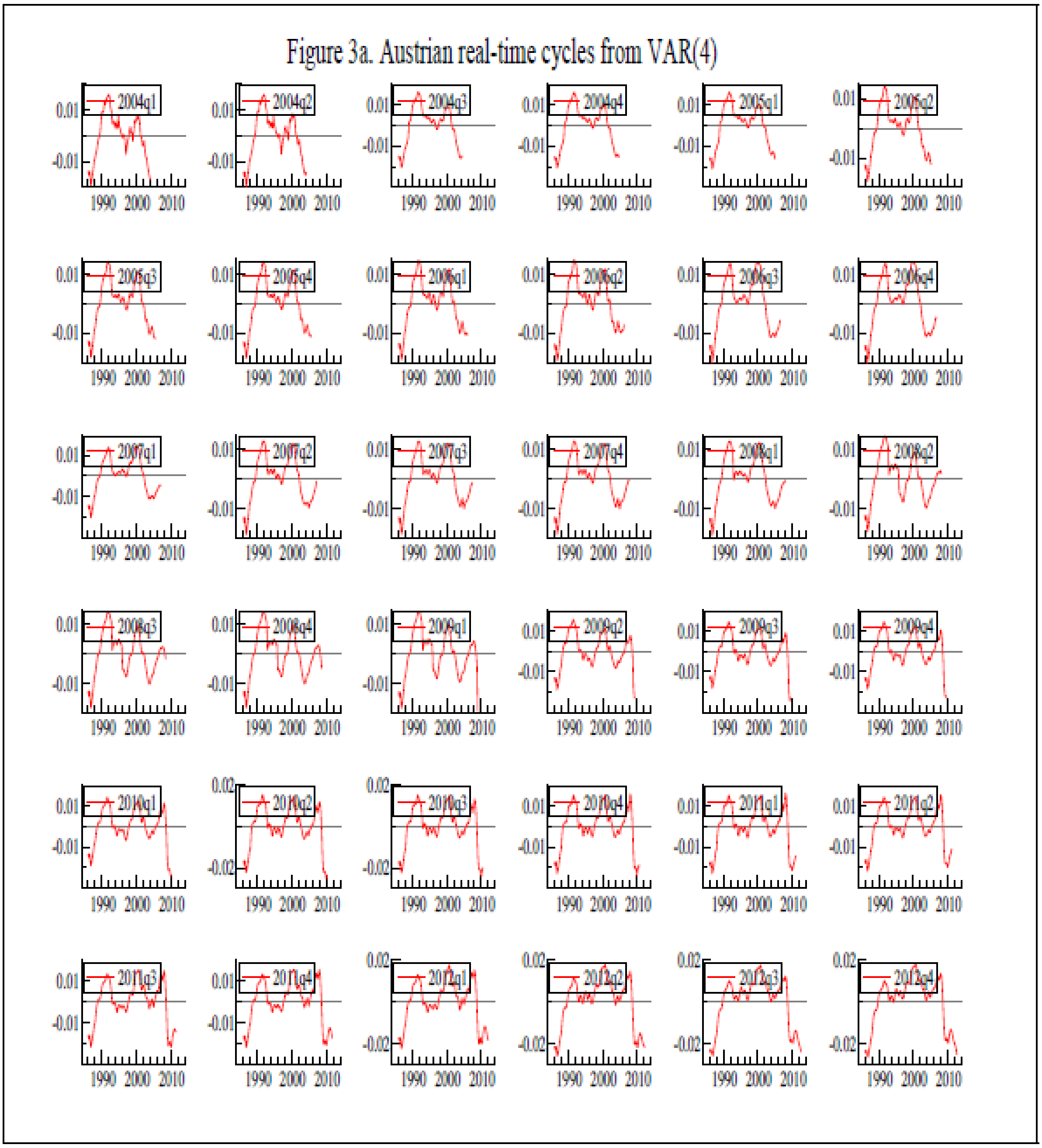


Figure 8: Real-time Austrian trend from VAR(4)

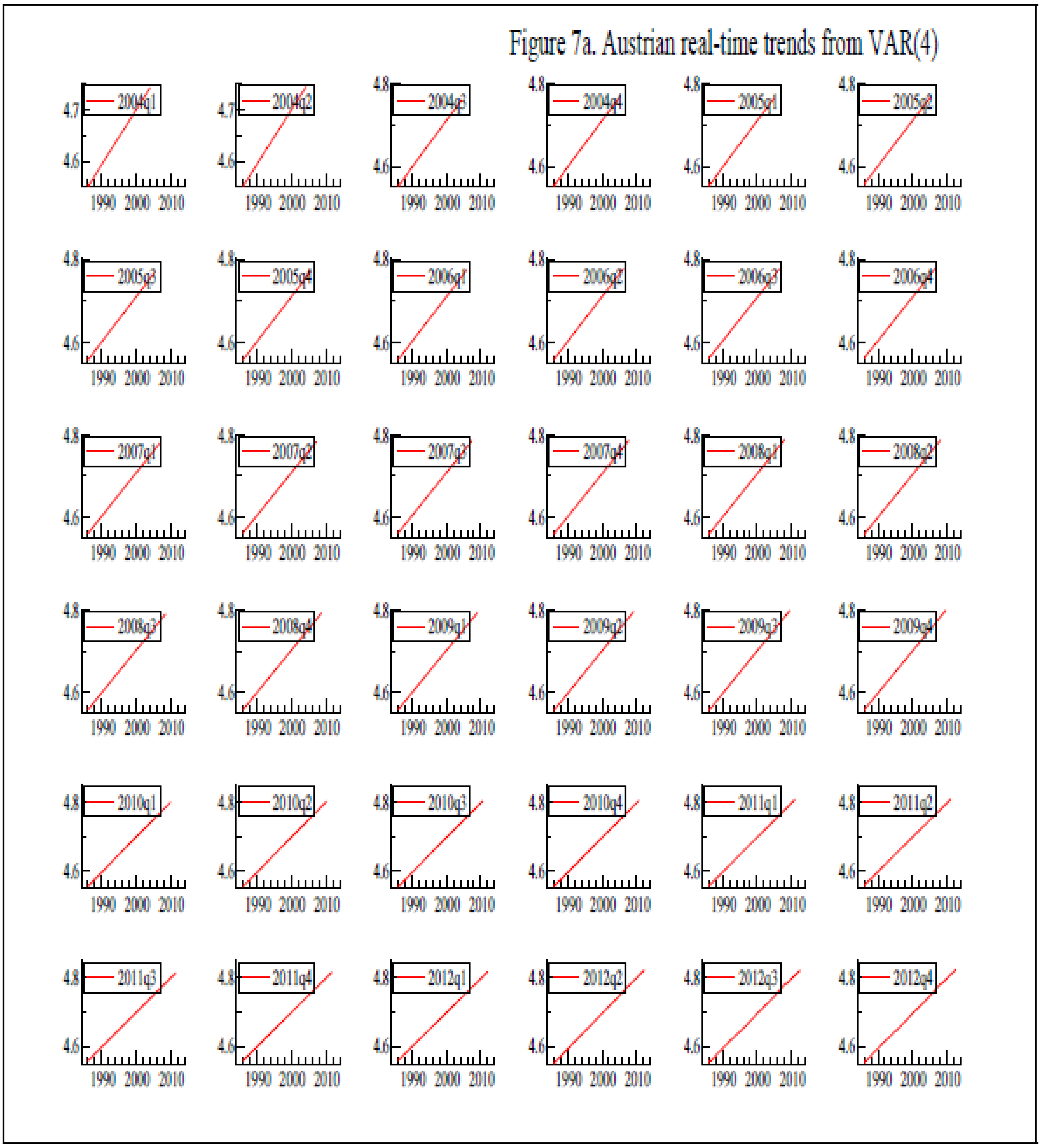


Figure 9: Real-time German cycles from VAR(4)

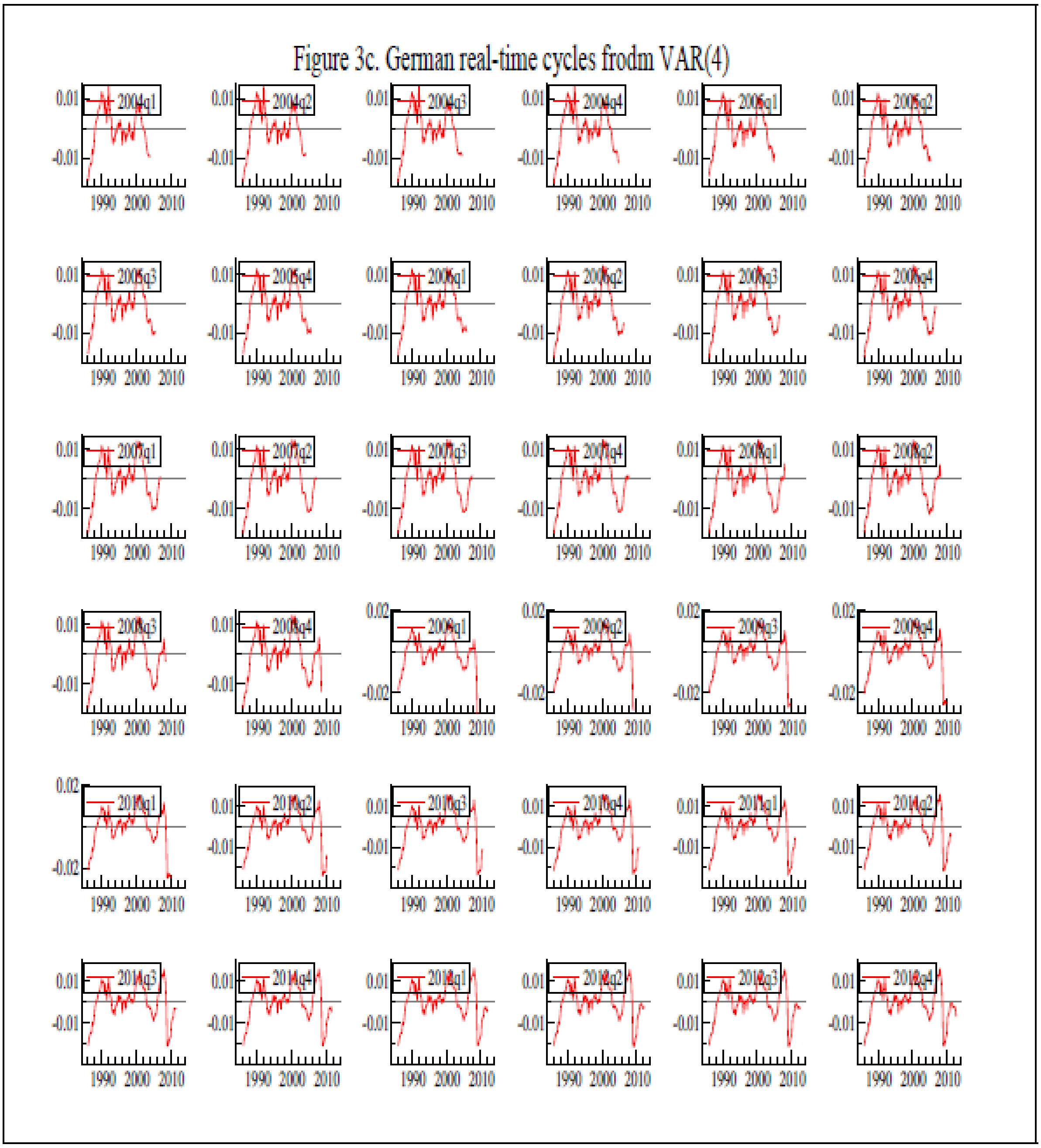


Figure 10: Real-time German trend from VAR(4)

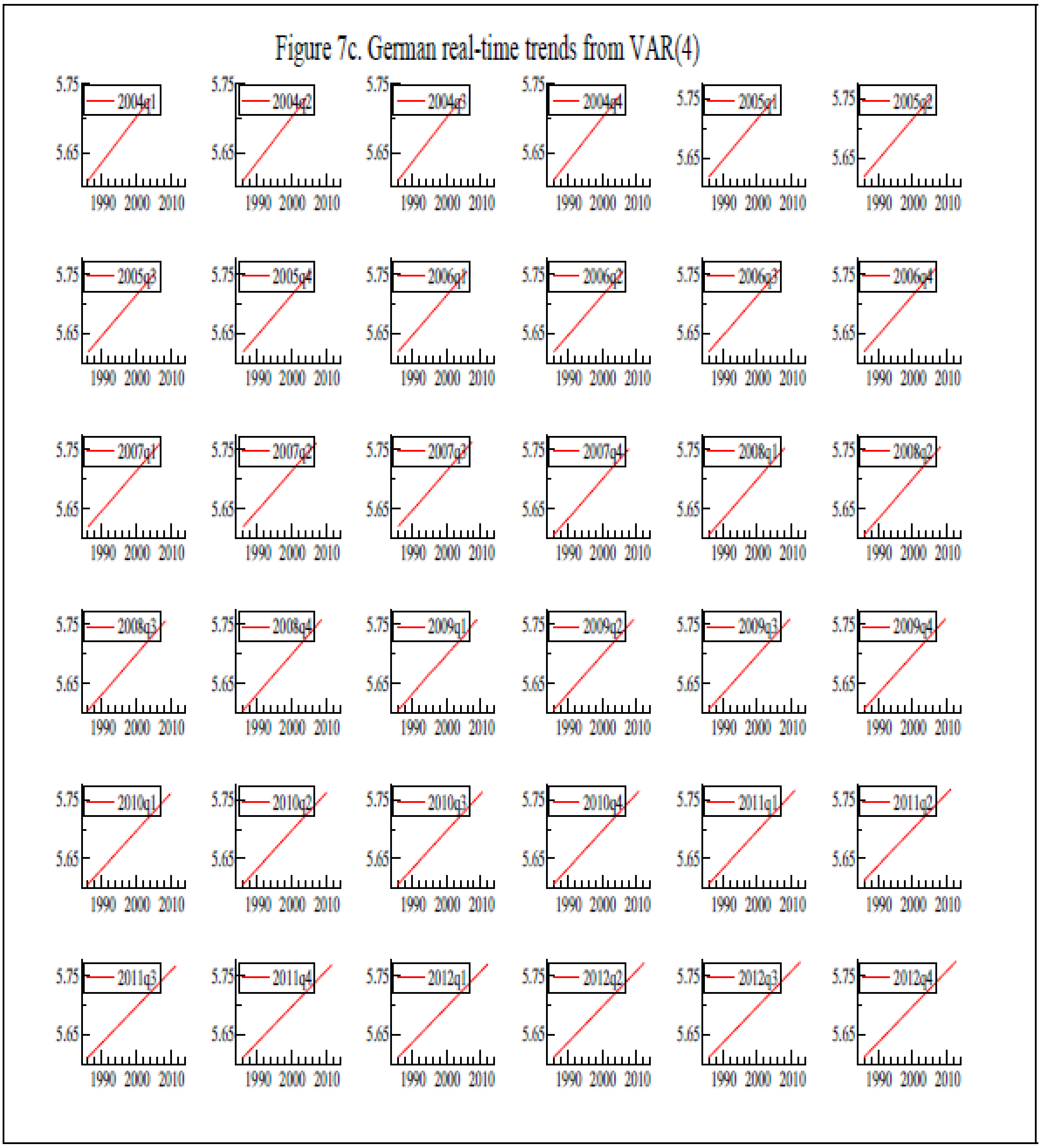


Figure 11: Real-time Spanish cycles from VAR(4)

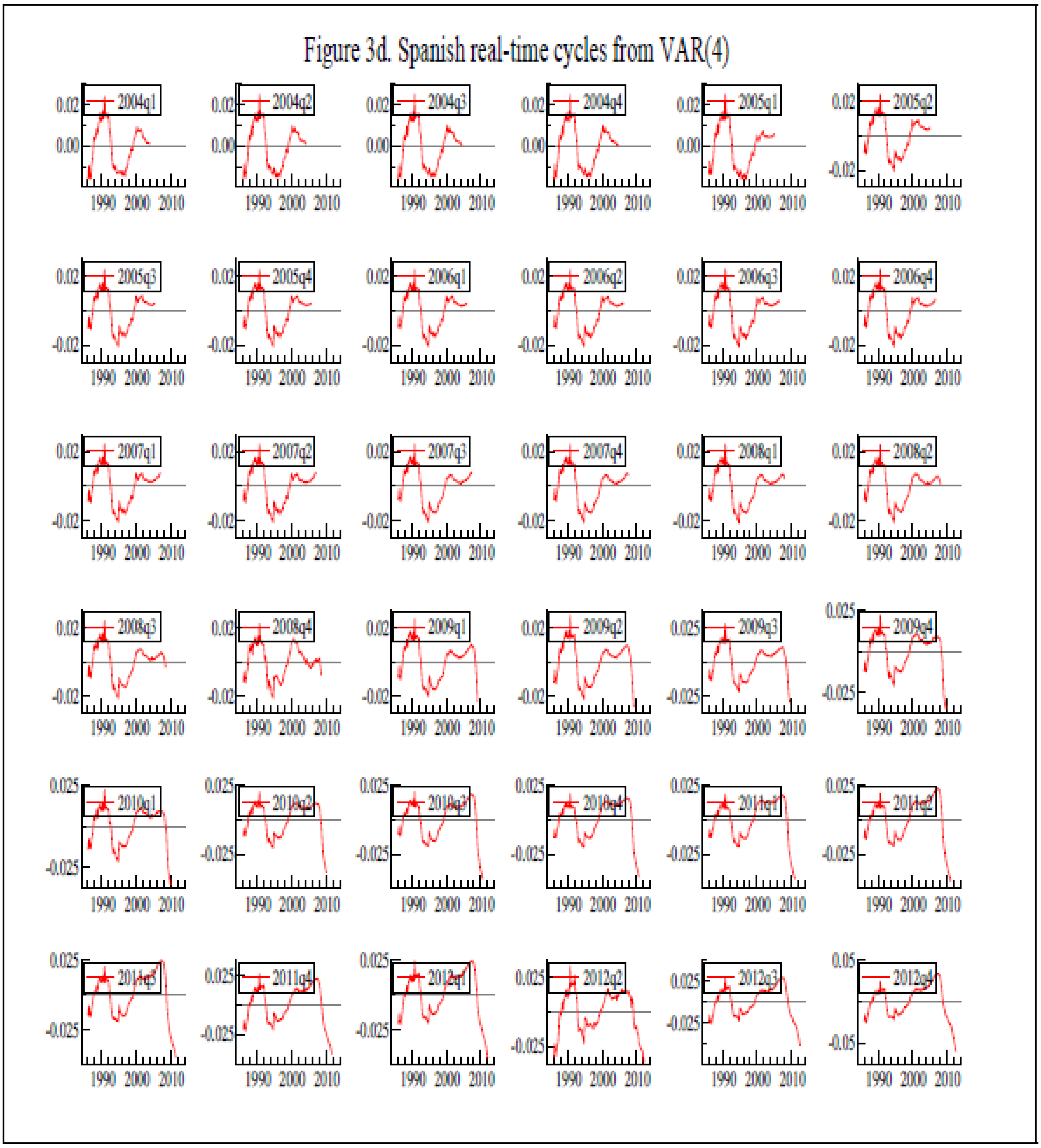
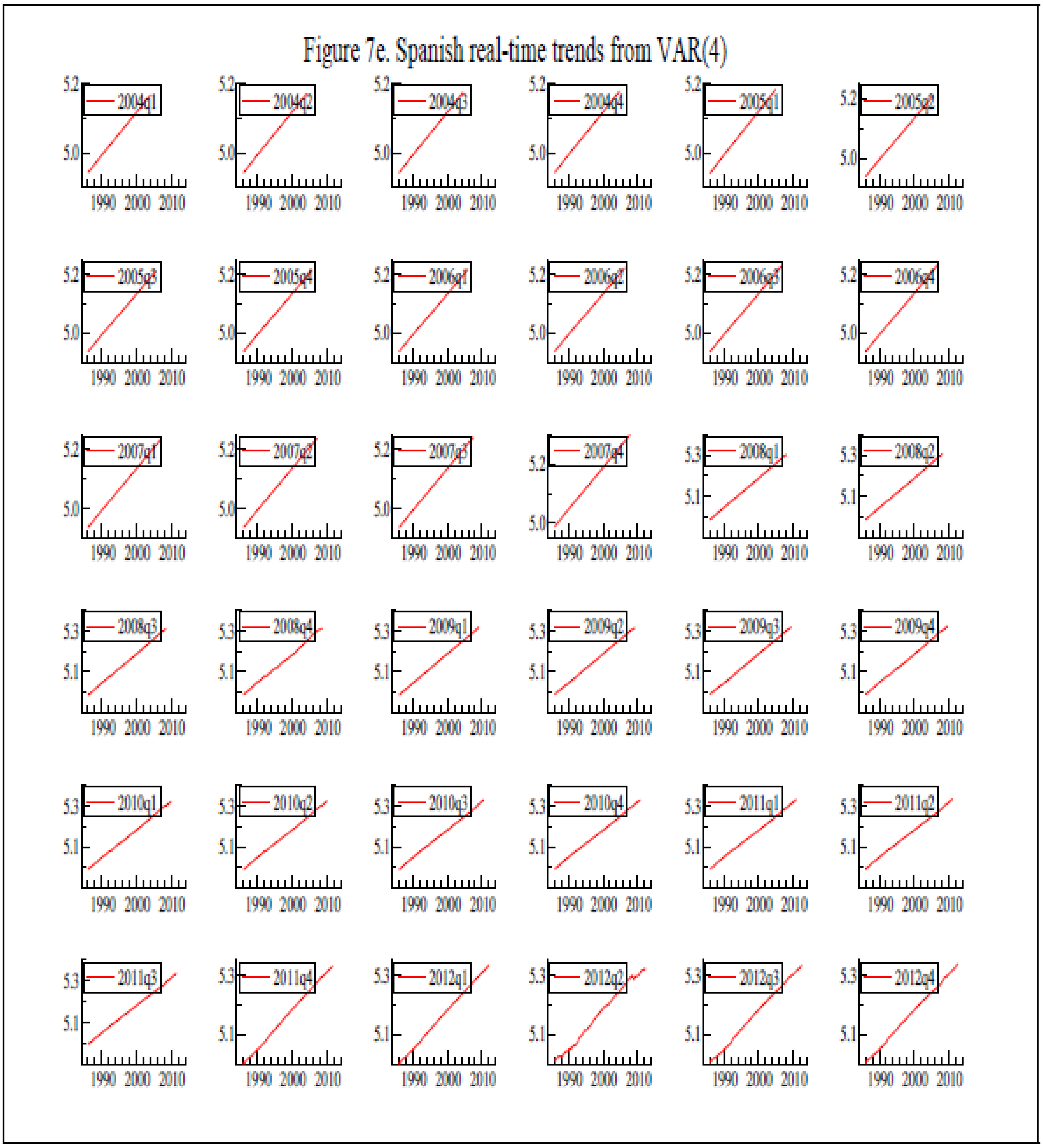


Figure 12: Real-time Spanish trend from VAR(4)



onwards. In general, VAR trends and cycles show similar patterns across different vintages and between the two lag orders. The effect of crisis on the estimates as shown in GVAR estimates are not often observed.

Restricted GVAR trends always appear much smoother than GVAR trends. This is true even for post-2009 vintages (except noisy Irish trends for 2008Q4 and 2009Q1 and noisy Portuguese trends post-2009, that can be seen from the plots in the Appendix folder). Noisy trend estimates as shown in GVAR for the first 2 quarters of 2009 are not observed in the trends from restricted GVAR model in general.

Cycle estimates from GVAR look different across time as compared to the restricted GVAR in which no cross-country dependencies are captured, with the estimates produced using the vintages from the recessions being quite peculiar. This is perhaps not surprising as we should expect unusual cyclical movements and trend movements from series during the crisis. But such finding may lead to one consider the use of more advanced techniques in capturing abrupt changes across time (i.e. breaks) in detrending methods, so as to see whether smoother trend and cycle estimates can be produced. By using a multivariate global model like the kind of GVAR, we have captured cross-sectional variations (across countries, across variables), but changes across time in the form of structural breaks are still not captured.

In fact insignificant cross-country and cross-variable dependencies should lead to GVAR produces very similar trend and cycle estimates to those from a restricted GVAR. It is because in the extreme cases when cross-sectional dependencies are zero, GVAR reduces to a restricted GVAR (the former nested the latter). However, the results from our countrylevel analysis show that the estimates from the two models are very different. This is an evidence to show that the level of cross-sectional dependencies do play an important role on the trend and cycle estimates. In fact our work from last year on flash estimation have found the evidence of increased level of cross-sectional dependencies during the recession. So perhaps one should expect more volatile estimates even during the crisis period unless more abrupt changes across time (i.e. breaks) are captured by the detrending model.

We also examine the correlations between the real-time cyclical estimates of the euro-area countries. We present an analysis using the estimates from a GVAR(4). Table 1 shows the correlations between the EA12 countries's real-time estimates. The reported correlation coefficients are computed using the real-time cyclical series estimated for the

Table 1: Correlations of real-time GVAR(4) cycle estimates between EA 12 countries (2004q1 to 2012q4)

	DE	FR	IT	ES	NL	AT	BE	PT	FI	GR	IE	LU
DE	1	0.984	0.996	0.993	0.996	0.995	0.995	0.904	0.98	0.498	0.825	0.998
FR		1	0.968	0.983	0.968	0.996	0.99	0.823	0.937	0.634	0.907	0.985
IT			1	0.991	0.999	0.983	0.988	0.921	0.992	0.429	0.781	0.994
ES				1	0.993	0.991	0.996	0.884	0.97	0.525	0.843	0.997
NL					1	0.985	0.99	0.928	0.988	0.444	0.788	0.996
AT						1	0.996	0.865	0.959	0.579	0.875	0.995
BE							1	0.878	0.962	0.558	0.864	0.998
PT								1	0.936	0.18	0.563	0.9
FI									1	0.33	0.707	0.975
GR										1	0.896	0.519
IE											1	0.839
LU												1

period 2004q1 to 2012q4. We have found an average correlation of 0.8655 for the estimates over all countries. It can be seen that the correlations among the real-time estimates of the euro-area countries are in general quite high, except for Greece for which its real-time cyclical estimates in general have a relatively low correlations with the other countries. High correlation among the cyclical estimates among countries implies a more correlated movement of the output gap towards the same direction. It may be considered as an indication of convergence. The fact that Greece has a lower correlations with the other euro-area countries in general may in fact implies Greece is “moving away” from the rest of the euro-area. In fact we will see in the later part of this paper on examining the correlations between the euro-area cycle and the member countries’ cycles that real-time Greek cycles does appear to have a lower correlation with the EA cycle overall, especially prior to the onset of the crisis.

We also compute the correlations between these real-time cycle estimates with the cycle estimates from the “final” vintage (release for 2012q4). Table 2 reports these correlations. We note that the real-time estimates for 2009Q1 is somewhat peculiar. As it is not surprising from what is shown in the Figures of individual cycle series of the GVAR(4). If we remove the estimate of this quarter and look into the correlation, we found an average correlation of 0.144 (while if this quarter is included, the correlation goes down to -0.028). In fact, we have seen from the figures the trend and cycle estimates post-2009 from the

Table 2: Correlations between GVAR(4) real-time cycle estimates and final estimates (2004Q1 to 2012Q4)

	Entire period	**excluding 2009q1**
DE	-0.013	0.21
FR	-0.078	-0.003
IT	-0.078	0.135
ES	-0.171	0.096
NL	-0.108	0.245
AT	-0.08	0.12
BE	-0.027	0.258
PT	-0.084	0.219
FI	0.128	-0.204
GR	0.194	0.249
IE	0.056	0.21
LU	-0.078	0.197
Average	-0.028	0.144

GVAR are very different from those prior to then. We should not be surprised that the real-time estimates and the final estimates do not have high correlations in general.

3.4 Correlations between real-time GDP cycles of the euro-area aggregate and euro-area countries

As a first step to examine whether the cyclical components of the GDP of the euro-area countries are moving together with that of the euro-area (EA) as a whole, we compute the correlations of the real-time estimated cycle series between the EA aggregate and the EA12 countries, using the estimates from a GVAR and restricted GVAR (i.e. VAR) with 4 lags. The real-time EA trend and cycle series are computed by aggregating up the estimated real-time country-level trend and cycle series from both models. In theory, restricted GVAR can be viewed as a disaggregate approach in trend and cycle decomposition while the GVAR is a multivariate approach. The former is nested within the latter. In other words, if the level of interdependencies is insignificant, we should expect the trend and cycle estimates from both approaches to be very similar, and being the same in the extreme case of zero interdependencies. Therefore, the low level of interdependencies should lead to the correlations between the EA cycles and the country-level cycles to be very similar regardless of what model is used. And in fact, if the GDP cycles of the countries are not correlated with the EA cycle over time, the values of the correlation coefficients should be low whether the de-trending model captures interdependencies or not.

We compare the findings between both de-trending models to see if accommodating interdependencies in de-trending models shows any differences in the indication of co-movement between the EA aggregate GDP cycle and the EA countries' GDP cycles. We also examine the magnitudes of these correlations over time, to see if there appear any change in correlation over time and in real-time, as evidence of increase level of correlations between the EA cycle and the EA countries' cycles may indeed form the basis of further investigation into convergence among GDP cycles, which can be conducted in future empirical studies.

Figure 13 and 14 plots the correlations between EA cycles and cycles from individual EA12 countries in real-time. It can be seen that prior to 2009, the level of correlations between the EA and country-level GVAR cycles are usually lower. The correlations between the EA and country-level GVAR cycles even lies below that of the VAR cycles for Germany, France, Spain, and Ireland. While for some countries, the correlations with the EA cycles computed using both de-trending models follow similar patterns pre-2009 especially for Italy and Belgium, a bit less so for Netherlands, Portugal, Greece and Luxembourg.

It can also be seen that from almost all of the plots the correlations between EA cycles and EA countries cycles, computed from the GVAR, show significant increase in value since the start of 2009. Except for Portugal, the correlation of its cycle with that of the EA peaks in 2009 but dropped significantly to a negative value at the beginning of 2010 and went back to high value since 2011. The values of correlations between EA and the countries' cycles computed using GVAR always appear larger than that computed using a restricted VAR in general since 2009 and remain high since then for all countries, with occasional variations. Moreover, the patterns of fluctuations between country-level GVAR cycles and the EA GVAR cycles are pretty consistent post-2009 for almost all countries, except for Portugal, Greece and Finland. A lot of variations in the level of correlation between the EA GVAR cycle and the Finnish cycles since 2009 with the value fluctuates between -0.5 to 0.99, which is somewhat different from other countries.

This findings indicates that since the start of the recession, the cyclical components of individual countries' GDP has shown significant co-movements towards the same direction with the EA as a whole. The values of these correlations are in general very high and in the post-2009 periods for most countries, mostly above 0.9. Perhaps such findings are not surprising as it is in line with the argument of financial contagion during the recession.

The patterns of correlations between EA and the countries' cycles obtained from a VAR do not appear to be consistent across countries in general. This is different from what we have seen from the patterns of correlations between EA and the GVAR cycles for most countries. Although correlations with the EA cycles computed from the VAR also shows an increase since 2009, the size of such increase is less pronounced than that reflected in the correlations computed from the EA and countries' GVAR cycles. In fact it is known that the level of interdependencies among countries' output has increased during the recession, this is supported by empirical evidence of financial contagion and also by the findings from our research in the previous year.

We also break down the entire sample period into three sub-periods to look at how the average correlations between EA cycle and EA countries' cycles differ across time. The results are reported in table 3 and 4. First of all, GDP cycles of big countries are usually found to be more highly correlated with the EA cycles, this is true regardless whether cross-sectional (country) dependencies are captured in the de-trending model.

In the case of GVAR, average correlations between EA and country-level cycles shows an increase over the 3 sub-periods, except for Spain, Netherlands and Belgium their correlations with the EA has gone down moving from the period 2004q1-2007q2 to 2007q3-2009q4. But this is consistent with what is shown in the plots in Figure 13 and 14. The correlation with the EA cycle is the lowest for Greece for the first 2 sub-periods but it does show a significant increase in value for the period 2010q1-2014q4. Interestingly, the correlation with the EA cycle in the case of VAR shows a different story, with a clear decline in average correlations for almost all countries moving from the period 2004q1-2007q2 to 2007q3-2009q4, except for Belgium, Portugal and Luxembourg. However, the correlations with the EA VAR cycles do in general show an increase when moving to the period of 2010q1-2012q4. The average correlation with EA cycle during the period 2010q1-2012q4 is overall higher in the case of GVAR than in the case of VAR, which is consistent from the graphical illustrations. The values of the average correlations with the EA cycle for the entire sample period are higher in almost all countries in the case of GVAR than in the case of VAR.

Table 3: Average correlations between GVAR(4) cycles of EA and EA12 countries

DE	2004q1-2007q2 0.82	2007q3-2009q4	2010q1-2012q4	Entire sample period
FR	0.923	0.952	0.985	0.952
IT	0.865	0.876	0.992	0.91
ES	0.786	0.75	0.981	0.841
NL	0.638	0.608	0.965	0.739
AT	0.583	0.706	0.983	0.75
BE	0.87	0.584	0.958	0.82
PT	0.713	0.699	0.522	0.645
FI	0.46	0.582	0.25	0.424
GR	0.243	0.372	0.966	0.52
IE	0.483	0.662	0.982	0.699
LU	0.49	0.628	0.88	0.658

Table 4: Average correlations between VAR(4) cycles of EA and EA12 countries

DE	2004q1-2007q2 0.841	2007q3-2009q4	2010q1-2012q4	Entire sample period
FR	0.944	0.895	0.962	0.936
IT	0.869	0.805	0.894	0.86
ES	0.842	0.791	0.895	0.846
NL	0.628	0.524	0.787	0.652
AT	0.58	0.662	0.881	0.703
BE	0.867	0.456	0.663	0.685
PT	0.726	0.59	0.476	0.605
FI	0.488	0.456	0.478	0.476
GR	0.277	0.23	0.38	0.298
IE	0.579	0.492	0.707	0.597
LU	0.486	0.369	0.348	0.408

Figure 13: Correlations between EA cycles and cycles of Germany, France, Italy, Spain, Netherlands and Austria

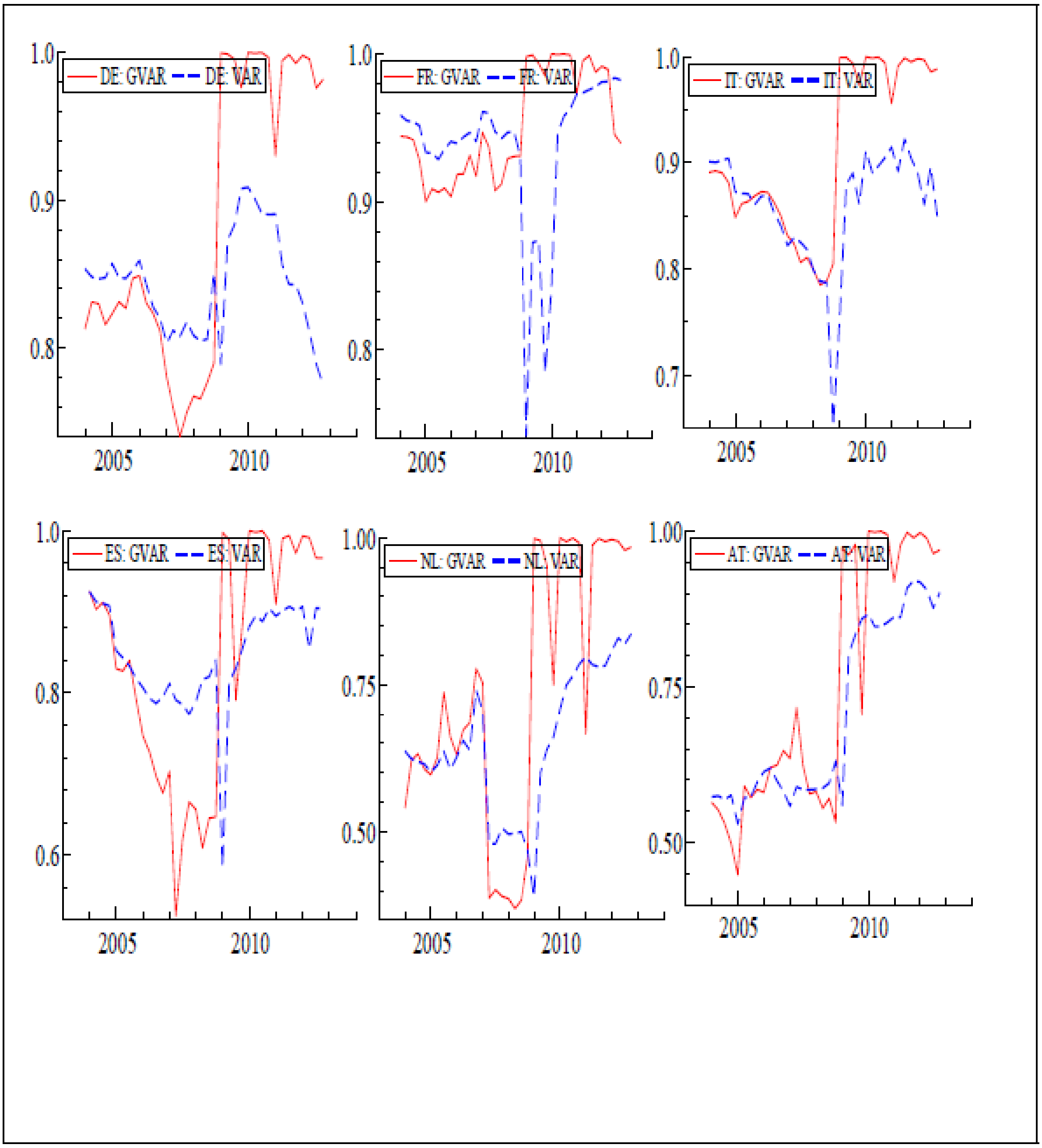
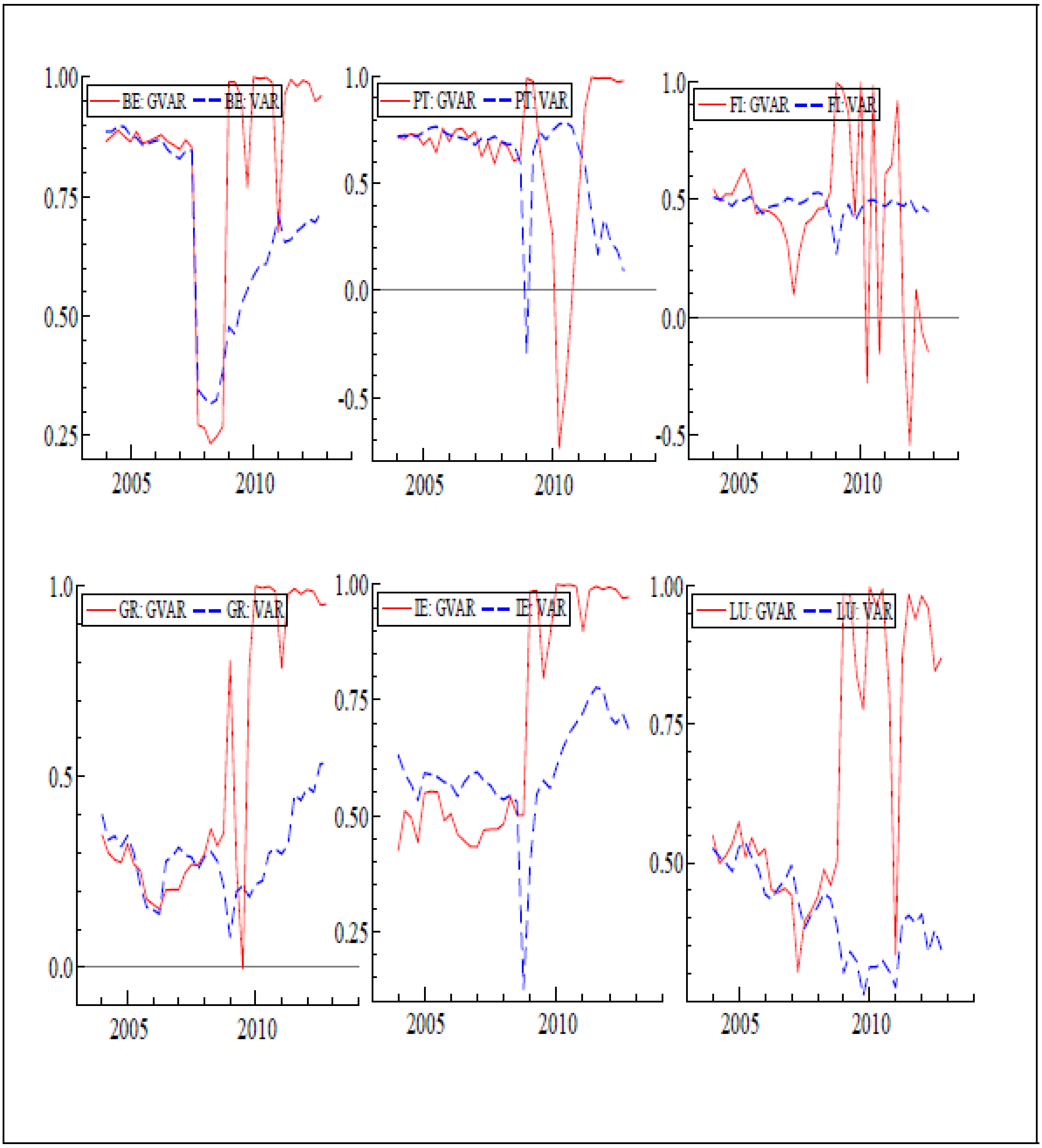


Figure 14: Correlations between EA cycles and cycles of Belgium, Portugal, Finland, Greece, Ireland and Luxembourg



To sum up, the correlations with the EA cycles are less pronounced in the case of VAR than in GVAR. This implies that when significant interdependencies are not captured in the de-trending method, the picture of co-movement of the cyclical estimates (or convergence) cannot be clearly reflected. In fact if the level of interdependencies are insignificant, we should expect the pattern of these correlations computed from the GVAR cycles to be consistent with the VAR cycles. The fact that they are different means these interdependencies are significant and capturing these interdependencies does matter. It once again confirmed that it is statistically important to capture the changing level of cross-country dependencies when de-trending, and in the examination of co-movements of the country-level and EA cycles over time. Moreover, the significant increase in correlations with the EA GVAR cycles post-2009 also raises the question of whether there exists only just one EA cycles that applies to almost all countries during the recession, or whether such findings support the argument of convergence of individual countries' cycle towards to the EA cycle.

4 Conclusions

This paper uses a global model that captures cross-country and cross-variable dependencies as a means of de-trending euro-area GDP into trend and cycle components. The model uses multivariate Beveridge Nelson decomposition to compute the estimates. By conducting the decomposition exercise in real-time and by comparing the trend and cycle estimates from the GVAR with those computed using a restricted GVAR in which all cross-country, cross-variable dependencies are set to zero. We can draw inference of the impact of changing level of cross-sectional dependencies on the trend and cycle estimates of the euro-area countries, as well as to see how such dependencies play a role in decomposing country-level GDP into trend and cycle estimates.

We have shown that the Global VAR model can be used to simultaneously de-trend European countries' GDP and thereby accommodate the interdependencies that exist between these European countries. Since a multivariate Beveridge Nelson decomposition is employed, it implies the GVAR trend and cycle estimates are produced by conditioning on a larger information set than in traditional univariate and multivariate application of the Beveridge-Nelson trend and cycle decomposition.

We have found the estimates from a GVAR for individual countries to be very different from that of a restricted GVAR. In particular, when cross-sectional dependencies are completely switched off, the trend and cycle estimates from such model do not pick up the abnormality of the crisis as much as the GVAR. This is true in general for all countries. We have also found more obvious evidence of changing patterns of cycle estimates across different vintages. Both findings confirms a changing level of cross-sectional dependencies over time impacts on trend and cycle estimation, and such dependencies should be captured when detrending. More volatile estimates during the crisis may lead to further investigation on whether allowing detrending model to capture breaks could help to produce smoother estimates.

The out-of-sample simulations reveal that the real-time cyclical estimates from the GVAR do not correlate well with the "final" estimates. This is to be expected and it can be explained by the magnitude of the recent recession. But since the GVAR estimates capture the dependencies between countries, despite their unreliability in real-time, they are to be preferred to more stable estimates which incorrectly ignore the manifest cross-sectional correlations between countries.

We have shown that it is important statistically to capture the changing cross-sectional (country) dependencies. This explains the fact that we find big differences between the trend-cycle estimates from a GVAR and those from a restricted GVAR (VAR) model that ignores these dependencies. The GVAR trend estimates are found to be much noisier, but smoothness is not necessarily a good thing. Our results show that statistically it is important to allow for the dependencies between countries. The recent recession had a marked effect on the trend and cycle estimates from the GVAR and introduced considerable volatility.

Since the recession involved an increase in cross-sectional dependence, it is important to capture these cross-sectional dependencies via a GVAR.

Correlations with the EA cycles are less pronounced in the case of VAR than in GVAR implies that when significant interdependencies are not captured in the de-trending method, the picture of co-movement of the cyclical estimates (or convergence) cannot be clearly reflected. It once again confirmed that it is statistically important to capture the changing level of cross-country dependencies also for the examination of co-movements of the country-level and EA cycles over time. The significant increase in correlations with the EA GVAR cycles post-2009 also raise the question of whether there exists only just one EA cycles that applies to almost all countries during the recession, or whether such findings support the argument of convergence of individual countries' cycle towards to the EA cycle.

Evidence of Europeanisation provides an empirical argument in favour of considering cross-sectional dependence when de-trending, this evidence also suggests possible convergence in the trend and cyclical components of individual countries' macroeconomic time series. Evidence from the recent recession suggests not only an increased level of cross-sectional dependence; the contagion of the recession among countries also implies convergence among individual countries' economic growth rates. An increased level of cross-sectional dependence may imply convergence. If modelling cross-sectional dependence can be viewed as an alternative way of modelling convergence, then the GVAR model should provide a tool for drawing inference about convergence among countries.

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KC-TC-16-014-EN-N

Analysing the permanent and cyclical components of GDP of the euro-area countries in a global context: The role of cross-sectional dependence

SILVIA LUI, GIAN LUIGI MAZZI AND JAMES MITCHELL

This paper focus on an analysis of the GVAR model across euro-area countries when detrending. The GVAR model accommodates cross-country as well as cross-variable dependencies among the euro-area countries. We focus on the role of cross-sectional dependence in the production of trend and cycle estimates of the euro-area countries by comparing the GVAR trend and cyclical components extracted for individual countries with the estimates produced using a restricted GVAR in which cross-sectional dependencies are set to zero.

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ISBN 978-92-79-60960-2