ISSN 1977-0375

eurostat
Methodologies and working papers

Work session on demographic projections

Bucharest, 10–12 October 2007

2007 edition

EUROPEAN COMMISSION

Luxembourg: Office for Official Publications of the European Communities, 2007

ISBN 978-92-79-04759-6
ISSN 1977-0375
Cat. No. KS-RA-07-021-EN-N

Theme: Population and social conditions
Collection: Methodologies and working papers

FOREWORD

Since 1989, Eurostat and the United Nations Economic Commission for Europe (UNECE) have jointly organised Work Sessions on Demographic projections. These sessions are part of the Work Programme of the Conference of European Statisticians. The aim is to continue and intensify international co-operation in the field of demographic projections.

While the focus has always been on methodological innovations, the last two Work Sessions have also addressed concrete policy questions and provided a high level forum for discussion among producers and users of population projections.

There is a pressing need to address many issues related to the "demographic characteristics" of our society. Contemporary society is undergoing demographic changes, characterised by low fertility and increasing longevity leading to an ageing population. Moreover, further demographic changes, ethno-cultural diversity, changing patterns in partnership behaviours and household formation confront our society with complex challenges. Demographic projections provide the means to look into the different development paths of our society in the future.

The Work Session in Bucharest was attended by about 80 participants from 35 countries from all over the world, international organisations, universities and leading demographic institutes. It was hosted by the National Statistical Institute of Romania and received scientific support from the International Institute for Applied Systems Analysis (IIASA), the Vienna Institute of Demography (VID) and the Netherlands Interdisciplinary Demographic Institute (NIDI). The two keynote lectures were delivered by (i) Henri BOGAERT, Commissioner for the Belgian Federal Planning Bureau, on 'Long Term Population Projections in Europe; How they influence policies and accelerate reforms', and (ii) by David REHER, Professor at Universidad Computense de Madrid, on ' Towards long term population decline: Views at a critical juncture of world population history'.

The papers of this Work Session, the oral presentations and the discussions were of high quality and relevance. They addressed a very large spectrum of methodological and policy issues covering fertility, mortality, household and migration projections, as well as projections of specific sub-population groups, and their importance for policy formulation. The round table discussion allowed a dynamic exchange of ideas and was devoted to the question 'Uncertainty of future population trends in Europe and outside: Is it a question for demographers, decision-policy makers or both?'

Eurostat and the United Nations Economic Commission for Europe plan to continue to organise such Work Sessions at regular intervals in the future, within the framework of the work programme of the Conference of European Statisticians. The aim is to ensure the ongoing review of developments in the area of demographic projections and provide a forum for discussions by scientists and users. The next session has been tentatively proposed to take place in Nicosia, Cyprus in June 2009.

Michel Glaude
Director

ACKNOWLEDGEMENTS

The Work Session was organised jointly by the Statistical Office of the European Communities – Eurostat and the United Nations Economic Commission for Europe (UNECE) Statistical Division. It was financially supported by Eurostat, and hosted by the Romanian National Institute of Statistics that kindly provided all necessary means for a successful meeting. We would like to thank the members of the Scientific Committee and the Organising Committee for their appreciated contribution to the success of the Work Session.

The views expressed in the current publication are purely those of the authors and may not in any circumstances be regarded as stating an official position of the European Commission.

Members of the Scientific Committee

Prof. Dr Graziella Caselli – University of Rome “La Sapienza”

Prof. Dr Vasile Ghetau – University of Bucharest

Prof. Dr Nico Keilman – University of Oslo

Prof. Dr Wolfgang Lutz – International Institute for Applied Systems Analysis

Prof. Dr Frans Willekens – Netherlands Interdisciplinary Demographic Institute

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Dr Paolo Valente – UNECE Statistical Division

STATISTICAL COMMISSION
and ECONOMIC COMMISSION FOR EUROPE

STATISTICAL OFFICE OF THE
EUROPEAN COMMUNITIES (EUROSTAT)

CONFERENCE OF EUROPEAN STATISTICIANS

Joint Eurostat-UNECE Work Session on Demographic Projections

Bucharest (Romania), 10-12 October 2007

AGENDA AND TIMETABLE

The meeting will start on Wednesday, 10 October 2007, at 10:00 a.m.
at the Conference room of National Institute of Statistics of Romania,
16 Libertatii Avenue, sector 5, Bucharest, Romania

The meeting is hosted by the National Institute of Statistics of Romania

SUMMARY OF AGENDA ITEMS FOR THE MEETING:

1. Opening of the meeting and welcoming remarks
2. Adoption of the agenda and designation of officers
3. Key – note lectures
4. Fertility
5. Mortality
6. Population projections
7. Household projections
8. Specific projection issues
9. Round table discussion
10. Future work
11. Adoption of the report

TIMETABLE

Time	Item	Session/Activity
		WEDNESDAY, 10 October 2007
9:30-10:00	Registration of participants	
10:00-10:50	1.	**Opening of the meeting** **Welcoming remarks by:** - Vergil Voineagu, National Institute of Statistics, Romania - Eugen Nicolaescu, Minister of Health, Romania - Michel Glaude, Eurostat - Paolo Valente, UNECE
10:50-11:00	2.	**Adoption of the agenda and designation of officers**
11:00-12:30	3.	**Key note lectures**
11:00-11:45	3.1	♦Towards long-term population decline. Views at a critical juncture of world population history. ***David Reher*** – *University of Madrid*
11:45 –12:30	3.2	♦Long Term Population Projections in Europe: How they influence policies and accelerate reforms. ***Henri Bogaert*** – *Federal planning bureau, Belgium*
12:30-14:00	Lunch break	
14:00-15:30	4.	**SESSION 1: FERTILITY – Chair: Wolfgang Lutz**
14:00-14:20	4.1	♦ Projections of age-specific fertility rates through an agent-based model of social interaction ***Belinda Aparicio Diaz, Thomas Fent, Alexia Prskawetz*** – *Vienna Institute of Demography (Austrian Academy of Sciences)* ***Laura Bernardi*** – *Max Planck Institute for Demography*
14:20-14:40	4.2	♦Trends in partnership behaviours in Japan from the cohort perspective ***Miho Iwasawa, Ryuichi Kaneko*** – *National Institute of Population and Social Security Research, Japan*
14:40-15:00		♦ A meta-analysis of fertility trends by social status ***Vegard Skirbekk*** – *International Institute for Applied Systems Analysis (IIASA)*
15:00-15:20	4.3	♦ Age profiles estimation for family and fertility events based on micro data: the MAPLE (Method for Age Profile Longitudinal Estimation) ***Roberto Impicciatore*** - *University of Milan & "Carlo F. Dondena" Centre for Research on Social Dynamics* ***Francesco C. Billari*** – *Bocconi University & "Carlo F. Dondena" Centre for Research on Social Dynamics*
15:20-15:50		Questions & Discussion
15:50-16:05	Coffee break	

Time	Item	Session/Activity
16:05–16:45	5.	**SESSION 2: MORTALITY – Chair: Nico Keilman**
16:05–16:25	5.1	♦ An approach to improve the consistency of mortality projections obtained by the Lee–Carter method **Dalkhat Ediev** – Vienna Institute of Demography (Austrian Academy of Sciences)
16:25–16:45		Questions & Discussion
From 17:00 onwards	**ORGANISED SOCIAL EVENT**	
THURSDAY, 11 October 2007		
9:15 – 10:45	5.	**SESSION 2: MORTALITY (cont.) – Chair: Nico Keilman**
9:15 – 9:35	5.2	♦Mortality and longevity projections for the oldest–old in Portugal ***Edviges Coelho, Maria Graça Magalhães*** – *Statistics Portugal, Portugal* ***Jorge Miguel Bravo*** – *University of Évora*
9:35 – 9:55	5.3	♦Mortality rates in population projections: a stochastic approach to inference **Therese Karlsson, Gustaf Strandell** – Statistics Sweden
9:55 – 10:15	5.4	♦Life expectancy adjustments in the Norwegian pension reform **Helge Brunborg** – Statistics Norway
10:15–10:55		Questions & Discussion
10:55–11:10	Coffee break	
11:10 –12:30	6.	**SESSION 3: POPULATION PROJECTIONS – Chair: Graziella Caselli**
11:10 –11:30	6.1	♦A new technique for stochastic population projections ***Salvatore Bertino, Eugenio Sonnino*** – *Università di Roma "La Sapienza"*
11:30 –11:50	6.2	♦Population Prospects from the Lowest Fertility with the Longest Life: The New Official Population Projections for Japan and their Life Course Approaches ***Ryuichi Kaneko*** – *National Institute of Population and Social Security Research, Japan*
	6.3	♦Supporting paper (not presented): Replicating the official population projection for Sweden using a time series approach ***Gustaf Strandell***– *Statistics Sweden*
11:50 – 12:30		Questions & Discussion
12:30 –14:00	Lunch break	

Time	Item	Session/Activity
		THURSDAY, 11 October 2007
14:00 –15:30	6.	**SESSION 3: POPULATION PROJECTIONS (cont.) – Chair: Graziella Caselli**
14:00 –14:20	6.4	♦Population forecast and the impact on the long–term growth potential ***Ion Ghizdeanu*** – *National Commission for Economic Forecasting, Romania*
14:20 –14:40	6.5	♦ A tool for projecting age patterns based on a standard age schedule and assumptions on relative risks using linear splines: TOPALS ***Joop de Beer, Nicole van der Gaag, Frans Willekens*** – *Netherlands Interdisciplinary Demographic Institute(NIDI)*
14:40 –15:00	6.6	♦Population Forecasting via Microsimulation: the software design of the MicMac Project ***Jutta Gampe, Sabine Zinn*** – *Max Planck Institute for Demographic Research* ***Frans Willekens, Nicole van den Gaag*** – *Netherlands Interdisciplinary Demographic Institute(NIDI)*
15:00–15:30		Questions & Discussion
15:30–15:45	Coffee break	
15:45 –17:15	7.	**SESSION 4: HOUSEHOLD PROJECTIONS – Chair: Vasile Ghetau**
15:45 –16:05	7.1	♦On future household structure ***Nico Keilman*** – *Department of Economics, University of Oslo* ***Juha Alho*** – *University of Joensuu*
16:05–16:25	7.2	♦Towards a dynamic model for household projections for The Netherlands ***Coen van Duin*** – *Statistics Netherlands*
16:25–16:45	7.3	♦ Probabilistic household projections based on an extension of headship rates method with application to the case of Russia ***Sergei Scherbov*** – *Vienna Institute of Demography (Austrian Academy of Sciences)* ***Dalkhat Ediev*** – *International Institute for Applied Systems Analysis (IIASA)*
16:45–17:15		Questions & Discussion

Time	Item	Session/Activity
FRIDAY, 12 October 2007		
9:15–10:45	8.	**SESSION 5: SPECIFIC PROJECTION ISSUES – Chair: Frans Willekens**
9:15 – 9:35	8.1	♦Bayesian Model Selection in Forecasting International Migration: Simple Time Series Models and Their Extensions ***Jakub Bijak*** *– Central European Forum for Migration and Population Research*
9:35- 9:55	8.2	♦Conception of Spatial Units Appropriate for Regional Population Forecasts ***Branislav Bleha*** *– Comenius University in Bratislava*
9:55 –10:15	8.3	♦Labour force projection at territorial level in Romania ***Marcela Postelnicu*** *– National Statistical Institute of Romania*
10:15-10:45		Questions & Discussion
10:45-11:00	Coffee break	
11:00-12:00	8.	SESSION 5: SPECIFIC PROJECTION ISSUES (cont.) – Chair: Frans Willekens
11:00–11:20	8.4	♦Projecting ethno–cultural diversity of the Canadian population using a microsimulation approach ***Eric Caron Malenfant, Alain Bélanger, Laurent Martel, René Gélinas*** *– Statistics Canada*
11:20–11:40	8.5	♦New Times, Old Beliefs: Projecting the Future Size of Religions in Austria, Canada and Switzerland ***Anne Goujon, Katrin Fliegenschnee*** *– Vienna Institute of Demography (Austrian Academy of Sciences)* ***Vegard Skirbekk*** *– International Institute for Applied Systems Analysis (IIASA)*
11:40–12:00		Questions & Discussion
12:00-14:00	Lunch break	
14:00-15:30	9.	**ROUND TABLE DISCUSSION - Chair: Frans Willekens** ♦ Uncertainty of future population trends in Europe and outside: Is it a question for demographers, decision – policy makers or both? Graziella Caselli – University of Rome "La Sapienza" Vasile Ghetau – University of Bucharest Michel Glaude – Eurostat Nico Keilman – University of Oslo Wolfgang Lutz – International Institute for Applied Systems Analysis Silvia Pisica – National Statistical Institute of Romania Marcela Postelnicu – National Statistical Institute of Romania
15:30-15:45	Coffee break	
15:45-16:10	10.	Future work ♦ Proposal on future work
16:10-16:30	11.	Adoption of the report
16:30		CLOSING OF THE MEETING

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Key note lectures

Session 1: Fertility

Session 2: Mortality

Session 3: Population projections

Key note lectures

TOWARDS LONG TERM POPULATION DECLINE: VIEWS AT A CRITICAL JUNCTURE OF WORLD POPULATION HISTORY

David S. Reher
*Universidad Complutense de Madrid**

Note:

This is a shortened and revised version of the paper that was published originally as: Reher, David S. "Towards long-term population decline: a discussion of relevant issues," ***European Journal of Population*** *23 (2007): 189-207. The present paper was deliverd as a keynote lecture at the Joint Eurostat – UNECE Work Session on Demographic Projections, Bucharest, 10-12 October 2007.*

1. Towards long-term population decline

There are indications that a large part of the world is about to commence a prolonged period of population decline. This will bring to a close three centuries of unfettered and extremely rapid population growth, itself a unique experience in human history. For a number of decades during the second half of the twentieth century, world population growth rates surpassed 1.75 percent per year, exceeding 2 percent between 1970 and 1975, and were considerably higher in many world regions. Not only is this period of growth ending, there are also real perspectives for prolonged population decline in many of the world's regions during the twenty-first century. There can be little doubt that this process has started in Europe and in other developed nations. It may just be getting under way in many of the lesser developed countries of the world as well. Only in the least developed regions of the world is it still a matter of serious doubt, though there too population growth rates have declined substantially in recent years.

The mechanics of decline can be traced to a prolonged reduction in fertility nearly everywhere in the world. In many of the developed regions of the world, fertility began to fall over a century ago. Since then, this slide has been unchecked, with the brief interlude of the baby boom of the 1950s and 1960s. In other parts of the world, fertility decline started much later (1960s-1980s), though the pace of decline has been far faster than it was in the developed world. The result of this is that inter-regional disparities in fertility at the beginning of the twenty-first century are far smaller than they were only 50 years ago. In large parts of the world, below-replacement fertility has been common for some time now, and in others there is a good chance that fertility, at present just above replacement levels, may be headed in the same direction. This process will be stimulated by the a decline in the number of women of reproductive age, itself the result of earlier declines in the number of births. The process of declining numbers of births began in Europe some decades ago, and is already under way in a number of developing countries.

The very idea of decline and of population shortage is largely foreign to our society, mostly because for several centuries there has been no experience of shrinking population at a societal level. Even in developed regions, where the process is well-advanced, the idea of population decline and its implications is having difficulty being assimilated by large sectors of society (Caldwell and Schindlmayr, 2003: 257). In most of the developing countries, the problems of population *abundance* continue to dominate scientific, social and political agendas.

Despite these signs of incipient decline, world population is likely to continue increasing over the coming decades, reaching a total of perhaps 8-9 billion persons by 2050 (current levels are 6.4 billion). By mid-century, however, the structural changes discussed here will be well on their way to turning growth into decline for the entire world.

2. Population decline and the demographic transition

It is our contention here that the persistent extremely low fertility in developed countries cannot be satisfactorily attributed to economic stress, unemployment, public policies or lack thereof, or to passing trends such as the postponement of reproduction, though all may contribute. Extremely low fertility has been around for too long for it to portend anything other than major long-term social change. It gives every indication of having become a structural aspect of the developed world.

There is reason to believe that the low fertility in European populations is the outcome of the demographic transition that started well over a century ago. This hallmark event of human history unleashed powerful forces of social change, leading to modernization in many parts of the world. Much as the demographic transition theory argues, the transition itself may have been triggered or at least been accompanied by more general societal changes. The process itself of reproductive change, however, tended to generate social and economic synergies of its own. The links between the demographic transition and social change can be seen in age structures, migration patterns, the distribution of family labor, education and the quality of children, and adult health. All of these were powerful agents of change in themselves and have done much to accelerate patterns of economic growth and social and political modernization during the twentieth century in Europe, America and in areas of East Asia.

In order to understand this process more thoroughly, it is helpful to look briefly at how the demographic transition contributed to social change generally, and especially to how it contributed to the transformation in the role of women in society. This is the key issue, one that is present in all historic demographic transitions and likely in the more recent ones as well. Women were the central figures for the initial demographic transition in Europe. They were the ones who contributed most to reproductive outcomes and were likely also the ones initiating fertility control within marriage. They also held the key to the health improvements of their children, especially before the aftermath of World War II when medicine and public health assumed greater relative importance. The demographic transition was, in its very origins, a key event in the empowerment of women. It also initiated a series of social, political and cultural changes affecting their role that mark social change during the twentieth century.

By implication, the demographic transition led to greater reproductive efficiency: reaching the desired family size took less time and less individual effort than ever before, though it may well have cost considerably more. Ronald Lee has estimated that women went from spending 70 percent of their adult lives bearing and rearing children before the demographic transition, to spending only about 14 percent of it in more recent times. It meant a massive liberation of women's time, minimizing their 'wasted investments' on children who eventually died.

At first mortality declined faster than fertility and, despite diminishing numbers of children ever-born, completed family size tended to increase. Eventually, however, completed family size decreased; a process that implied important ideational changes because it meant that people –women- were aiming at –and achieving- smaller families. It led to an emphasis on children of 'quality': surviving children began to receive more parental attention. This included increasing investments in education, for boys as well as for girls, in both public and private spheres. By implication, the economic costs related to childbearing and childrearing also increased.

Ultimately, this process of increasing reproductive efficiency with its ideational and economic implications can be seen as a prerequisite for the entry of women into the labor force. The increases in the labor force participation rates of woman have their own set of economic, social, and cultural causes. One of them, however, was the revolution in reproductive efficiency and the way it affected women and families: it made labor force participation possible in terms of time, helped create the economic need to do so and paved the way for the increases in education necessary to make this sort of activity a part of the life expectations for women. Taking a job and keeping a job after marriage became standard fare for the great majority of women in these countries.

All of this has led to a substantial rearrangement of women's position in society and, by implication, that of men as well, providing an overall reduction in the gender differentiation of public and private life. This is one of the most important social changes of the entire twentieth century, one whose implications should not be underestimated. Women are now as highly educated as men, have activity rates that are every bit as high, and make an important contribution to family economies. All of this has led to sharply declining fertility coupled with profound changes in certain dimensions of family forms, the meaning of the family, and family life generally.

Having children no longer has the type of overriding importance that it once did for women (and for men) only half a century ago. Historically, by implication a successful life for a young woman meant having children and a family. With some exceptions, if you didn't have a family you were not successful in life, either in the eyes of society or in your own. In situations such as these, sacrifices were made to be successful, no matter what the cost. Today, having a family is still an important part of success for most women in the developed world, but it has a much lower priority than it did before. In a somewhat arbitrary way, we might say that in the past having children and a family was 80 percent of what could be considered success in life, and now it is closer to 30 percent. As this happens, the opportunity costs for reproductive success necessarily become higher and people are more willing to negotiate, especially when circumstances are not ideal.

It is not difficult to see how problems can abound in these sorts of situations. They can include problems with a person's career expectations, with finding the right partner, with the housing or the job market, with the willingness of men to share fully in home and family responsibilities, with gender equity, with the reality of having to lower one's expected living standards in order to have a child, or with the difficulties inherent in raising children in modern societies where there is little social, private or public support for families raising children. The importance of these factors may vary across societies, thus helping to explain existing differences in fertility. Even so, these concerns are common to young couples everywhere in the developed world and figure mightily in their reproductive decisions. Having a family is an expensive, long-term investment. Since it is no longer an overwhelming priority for women (or for men), as it once was, they are much more willing to negotiate these expectations.

For men, being successful in life tended to be based mainly on professional success, more than on having a family. It was women who made families function and held them together, not men. This is why these changes we have described in women's values and expectations have had such a profound effect on reproduction and the family. The persistently low fertility over the past half century in much of the developed world is impossible to explain without this sort of ideational change. Should the current levels of fertility in developed societies continue to be linked to the role of women in society, by implication, then, below-replacement fertility will be with us for a long time to come.

Contrary to what had been expected in classical demographic transition literature, fertility did not decline to replacement levels. Replacement fertility proved to be only a road sign en route to significantly lower levels of reproduction and, eventually, to falling numbers of births. This occurred in different parts of Europe some time between the mid 1960s and the early 1980s. The demographic result of this is very clear as the process of aging and eventually of population decline accelerated and became common fare for most of these societies.

3. Perspectives for the developing world

What about the rest of the world? There continues to be a general feeling that for the most part fertility will remain above replacement and so the prospects for the coming years point to a slow down in growth, but not to a reversal of growth. Is this supposition reasonable? The existence of a hypothetical 'floor' to fertility decline was a widespread belief in Europe during much of the twentieth century, and the upturn in fertility in the postwar years seemed to confirm this belief. Yet historical reality showed how unrealistic that expectation was.

Is it realistic for much of the developing world? The relative lack of economic development, low levels of education and strong family cultures suggest that it may well be. Yet there are also signs to the contrary. At present, fertility is already below replacement in nearly 60 of the world's nations, and many of them are not developed countries. We can understand the processes at work better if we look at the demographic transition in these nations. Very generally, its specifically demographic characteristics can be summed up in the following points:

1) Fertility decline began for most of the world's populations some time between 1955 and 1980; and mortality decline began much earlier.

2) The long lag between mortality decline and fertility decline led to accelerating rates of population growth that have only recently begun to slow.

3) The pace of decline, both of fertility and of mortality, has been far faster than in the historic demographic transitions; possibly twice as fast or more. This disparity in pace is due to a large extent to the technological context within which these transitions took place.

4) Population aging in these countries is also proceeding at a far more rapid pace than it ever did in the historic transitions. In many countries, the number of births has already begun to shrink and promises to continue to do so in the future.

5) Completed family size is now declining rapidly as the reduction in fertility outpaces improvements in mortality. This is exactly the opposite of the process taking place during the central decades of the twentieth century when over the course of a generation completed family size nearly doubled with respect to pre-transitional levels due to rapid mortality decline coupled with high and sometimes rising levels of fertility.

The role of the developed West in this process is central to the way the demographic transition took place in much of the rest of the world. The initial decreases in mortality were, to a large extent, more the product of the application of Western technologies with respect to health than they were to social development or maternal education as occurred in Europe. With respect to fertility, the role of family planning and efficient contraception, both strongly promoted by the developed world, were important for the onset and for the pace of decline (Demeny and McNicoll, 2006a: 12-39). These countries were both passive and eager recipients of European technology and ideals, with the consequence, at least in demographic terms, that there was an extended delay between mortality decline and the onset of fertility decline and, once started, the pace of fertility decline was extremely fast.

Will fertility stay above replacement and will the number of births continue to increase or at least remain at levels near where they are at present? Should current trends continue, many developing countries will have below replacement fertility in the very near future. This pace of reduction of fertility would seem nearly unstoppable, unless a baby boom takes hold in these regions, as it did for historic transitions during the 1950s and 1960s. The baby boom delayed the onset of below-replacement fertility by nearly 20 years in the developed world, though it did not stop it. Fertility decline became a two stage process: an original decline, followed by a pause or even an increase in fertility during the baby boom, followed in turn by another period of intense decline. In the developing world, that same baby boom may have been a factor leading to the persistence of high fertility despite rapidly declining mortality.

Will there be a second baby boom in the developing world? Our guess is that there will not, mostly because the conditions of the baby boom were world-wide at the time and appear unlikely to be repeated, especially in light of the existence of efficient contraceptive technologies. For this reason, there is a chance that many developing regions will pass straight from the first period of fertility decline into the second one with little or no pause.

A more pertinent question, however, is whether the role of women in society is also being dramatically altered. Increasing reproductive efficiency, so vital for Europe's social and economic transformation, is unquestionably affecting women the world over. Will this lead to increasing investments in the quality of children? We believe it will, especially as completed family size continues to decrease. It is unquestionable that the role of women in society is changing, though there continue to be enormous disparities. In some areas, there are already indications that woman no longer see their future as simply getting married and having children. Women's education has been increasing dramatically the world over. Despite problems in estimating female labor force participation in different societies over time, it too appears to be on the increase.

One of the characteristics of historic transition processes is that they commenced in a wide variety of contexts, though in the end the effects tended to converge everywhere. This also appears to be true in much of the developing world: social, economic and cultural disparities amid demographic similarities. Everywhere the value of children and the costs of raising them will increase, and so will the pressures on women to take advantage of new-found time available to them to generate further income for their families. There may be disparities in timing, but the process appears to be widespread.

The implications of the demographic transition in much of the developing world are becoming clear. The process of aging will be far more rapid and more intense than it was in historic populations. In this way, similar changes will take place in as little as half the time that it took in European nations. Throughout the developing world, aging and its attendant economic and social challenges will become an acute social issue relatively soon after it becomes a central concern for societies in the developed world. The intensity of change will leave these nations with but a brief window of the opportunity for modernization within which to take full advantage of the "demographic dividend" derived from their own transitions (Bloom, et. al, 2003).

Labor shortages will be one aspect of the issue of aging. In some countries, this shortage of working age population is easy to predict because numbers of births have already been declining for several years. We believe that it is only a matter of time (perhaps 2-3 decades) before they begin to affect many or most societies in the developing world. The availability of surplus labor (potential migrants) to compensate the dearth of labor in the developed world may eventually be called into question, as the sending countries begin to suffer labor strictures of their own.

Should present trends continue, ultimately they will lead to significant decreases in the population of reproductive age. At that stage, the negative population momentum so visible in many developed regions of the world will make itself felt elsewhere as well. Long-term population decline will set in, only three to four decades after it commences in the developed regions of the world. The gap between the onset of fertility decline and the onset of population decline, which spanned more than a century in historic Europe, promises to be far shorter in the developing world.

4. Some conjecture regarding the future

The trend towards population decline has been building for many years now. In some areas where this process is further advanced population will decline by as much as 20 percent in the next 50 years. Should present trends in fertility continue, decline by the end of the century will be much greater. Since this upcoming period of decline will hinge on low fertility, populations will tend to be loaded with elderly persons, and children and working age populations will be shrinking.

From the standpoint of natural resources and the environment, over the long run this will be good news indeed. Eventually the twenty-first century will be seen as one in which the excesses of the nineteenth and twentieth centuries were corrected. This is not to say that over the coming decades there may not be vast struggles for control of certain shrinking natural resources. In the long run, human demand has had a dramatic effect on the environmental balance throughout the world, and the upcoming period first of slow growth and then of decline, will be a powerful correction for this process.

As for the society of the future, expectations are not nearly so optimistic. Severely skewed age structures, an unavoidable by-product of the process underway, will have important consequences for all aspects of social welfare that depend on the redistribution of resources.

It is important to remember here that the present state of affairs in the developed world with declining numbers of births has been reached despite ever-greater numbers of women of childbearing age. In most of the developed world in the very near future, the number of women of childbearing age will begin a process of reduction that even in optimistic circumstances is likely to last for many decades. In other words, we are entering into a world of below-replacement fertility and shrinking numbers of women of childbearing age. This means that the pace of reduction of births should begin to accelerate, even in the face of moderately rising fertility.

Economists are well-acquainted with the issue of aging and grapple with potential solutions ranging from later retirement to increasing women's labor force participation, large-scale immigration, or reducing pensions and dismantling what is left of the welfare state. While certain doubts exist as to the economic expediency of many of these mechanisms or whether or not they will bring with them unwanted side effects, especially in the case of international migration, it is unquestionable that they represent a safety valve for rapidly aging societies. If current trends persist, however, over the long run none of them may prove to be more than partial remedies.

International migration itself, the focus of much current attention and concern, is unlikely to represent more than a temporary and rather inadequate solution for skewed age structures and population decline for two reasons. 1) Fertility among migrants, while initially higher than among the native populations, very quickly tends to decline to levels holding in the host society. 2) More important, perhaps, is the fact that many sending regions will be experiencing labor shortages of their own within two or three decades. It is unquestionable that these countries currently have abundant supplies of surplus labor that can be funneled fairly directly to receiving countries, normally developed ones, suffering from labor shortages. This situation, however, cannot be sustained indefinitely because of the dramatic fertility decline taken place among those sending countries. This is not to say that the developed world of low fertility will not continue to be able to attract immigrants. It will thanks to higher wages and living standards, though this process may become far more conflictive than it is at present where problems basically only really affect the social integration and adaptation of migrants in the host societies. The point here is that in a few short decades there is a good chance that labor shortages will become a problem affecting most of the world and not just one of the developed nations.

A shortage of labor and the abundance of tax revenues it must generate that are increasingly diverted from productive assets towards more pressing social needs will eventually be harmful for living standards and welfare systems. Economic growth itself may also be adversely affected by general population decline, as demand for goods and services and levels of investment shrink accordingly. The relative importance for living standards and economic growth of these two issues (age structures and population decline) is not entirely clear. Even so, it is difficult to argue with the idea that together their effects will far surpass either of them taken alone.

More than any other, the key issue here is the number of children born into society. With a moderately balanced age structure, all the challenges posed by increasing longevity can be met successfully, at least at a societal level. If age structures are severely skewed, however, it is much more difficult to be optimistic about the future. As Massimo Livi Bacci (2001) said, children are not just a matter of personal consumption and preference, but also one of social investment. It is difficult to argue with this sort of reasoning.

The key issue is just how this bottom line –having children- can be met. Many influential authors suggest that public policy can make a difference. Indeed it can, but just how much of a difference can it make? In Northern European societies, where aggressive pronatal public policies are in effect, fertility is also considerably below replacement and has been so for more than three decades now. Can policy convince women (couples) to have children? Recent experience in Europe suggests that policy alone cannot be successful.

This leads us to the pivotal issue of how to reconcile the commonweal and self-interest, at least in terms of reproduction. Self-interest, as Adam Smith reminded us, has always been a key part of human life, past and present. How were the two of them brought into line in the past? The historical record is filled with examples of how slow population growth was guaranteed by means of economic limitations to population growth within a context of high pressure demographic regimes and close-knit cultural structures.

All of this changed with the demographic transition. Living standards rose and compensating for high mortality ceased to be an important part of people's reproductive strategies. More important, perhaps, from a cultural standpoint, self-interest was no longer bound by such strict norms. The ability of the family and traditional culture to govern reproductive decisions lost much of its traditional relevance with modernization. As a result, fertility decisions became conscious and individual, more influenced by social networks and by secular consumer society than by tradition. In so doing, the developmental idealism defined by Thornton became a guiding principle of modern life and an instrument itself of social change.

In addition, by the mid-twentieth-century the revolution in contraceptive technologies enabled women to control their reproductive outcomes with considerable precision. Unexpected by all, the great historical achievement of increasing reproductive efficiency –the centerpiece of the demographic transition- turned into dangerously low fertility. This process began many years ago in developed societies and appears to be well under way in the developing world. Ultimately, sustainable human reproduction may not be easily compatible with liberal economies that reward careers for women outside the home immersed in an increasingly pervasive consumer society with considerable amounts of individual economic insecurity.

Has the genie really been let inadvertently out of the bottle? Having children is ultimately an expression of confidence in the future; in the security of the life you can expect your children to be able to lead. At one level, this sort of confidence is subject to economic and political constraints. At another, deeper one, it is related to social and cultural stability. There is an immense cultural change under way in much of the West and it is related, at least in part, to the role of women in society. It is also related, of course, to the triumph of secularization, individualism and consumer society, long considered hallmarks of modernization processes. Despite what can be very legitimately viewed as the achievements of recent history, it is also true that this is a time of insecurity for both men and women as to their roles in society, the nature of their gendered relationships and the future. It is also a time of deepening concern about the sustainability of society as we know it. We are witnessing the demise of the ideological foundations upon which society has been built for the past two centuries. Times of flux are not times that are conducive to optimism about the future.

In Europe and in other world regions we do not know the ending date of the process underway, but it may well not be soon in coming. When it does, it will be at significantly lower population levels than those existing today or perhaps at any time during the twentieth century. It is unclear just how these adjustment mechanisms will come about or how effective they might be. In any case, the second half of the 21st century may be the b eginning of a long downward spiral of world population.

From our vantage point at the turn of the millennium, we can envisage a great trend change with potentially enormous consequences. In that respect, we are fortunate indeed, at least from a scientific and historical standpoint. For our children, and especially our grandchildren, persistent population decline –and possibly lower living standards- will likely be the only reality they will ever experience and the times of runaway population growth so prevalent in the nineteenth and twentieth centuries will be but a distant memory of the past.

Are other scenarios possible? Yes they are, but, at least at this stage, they are less likely than the one I have described. Some of these scenarios may be more benign (a return to replacement fertility everywhere aided by policies and changes in values), others may imply a complete turnaround in our attitude towards the family (the advent, for example, of certain technological innovations rendering personal reproductive decisions irrelevant), while others may be much less benign, implying aggressive public policies, social and political conflict, and the progressive abandonment of the social, economic and political achievements of the past two centuries. Even though the future is not really ours to know, demographers have an important role to play in bringing such crucial issues to the front and stimulating much-needed debate.

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FPB – Economic analyses and forecast - Belgium

LONG-TERM POPULATION PROJECTIONS IN EUROPE: HOW THEY INFLUENCE POLICIES AND ACCELERATE REFORMS

October 2007

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Abstract

The long-term demographic projections have progressively raised concerns about the consequences of ageing population. To better understand those changes and measure their size, projections of social expenditure have been built and progressively refined. Confronted with a large budgetary cost of ageing in the long run, the Government's alternative is: solve the problem when it comes up or try to anticipate the negative results and prevent them. Three ways are to be considered that are not mutually incompatible: reforming the social system in order to reduce the cost for the present and future generations, increasing the tax or contribution receipts by pushing up employment rates and the trend growth of GDP and saving now in the public sector to cover the increase of the future expenditure. The paper shows that, since the end of the nineties, a broad movement of reforms has taken place in the EU which involves this three-pronged strategy.

Jel Classification – H55, J18, J26

Keywords

Pension reforms, pension expenditure, pension projections, sustainability, demographic projections

Acknowledgements

This paper has been prepared for the Joint Eurostat-UNECE Work Session on Demographic Projections in Bucharest, 10-12 October 2007. The author thanks Micheline Lambrecht and Jean-Maurice Frère from the Belgian Federal Planning Bureau for their comments and contributions. He wishes also thank all the members of the Ageing Working Group attached to the EU Economic Policy Committee: the material used for this paper is extensively based on the reports of the group.

Executive Summary

1. During the eighties and the nineties, the long-term demographic projections have demonstrated progressively the important changes that are observed and will continue to be observed in the future population structure: life expectancy is persistently increasing, low fertility rates weigh on the growth of the young population and the baby boom generation is progressively reaching the age of retirement: three characteristics which have been associated with the ageing of population.

2. The long-term demographic projections have progressively raised concerns in various policy domains. The first domain is the long-term financial sustainability of branches of the social protection, where expenditure is age-related to the age of the population. The ageing of population essentially changes the dynamics of receipts and expenditure of the pension system, which is in most of the EU countries based on pay-as-you-go systems. Health care and long-term care are two other types of expenditure that are influenced by the increasing average age of the population. To better understand those changes and measure their size, projections of social expenditure have been built and progressively refined. This had a first consequence: the reform process that was characterized by successive steps of relative increase of welfare of elderly people from the fifties to the eighties has been stopped or managed cautiously, and in some cases, even reversed during the nineties. Other types of social expenditure that have a close relationship with the ageing of population have been considered: unemployment, family allowances, education, etc. Systematic projection exercises have been developed by international organizations, especially the EU. Those projections, while still very imperfect, have had a very powerful impact on policies. One of the reasons is the following: the projections were conducted not only by academic researchers but also by international organizations. Among the latter, the projections conducted by the EU, which have been very influential because they were conducted on a multi-country basis with the same methodologies and consistent demographic and macroeconomic assumptions. Accordingly, the results were comparable. In most countries, the long-term increase in age-related expenditure was contained in a relatively narrow range. In the few countries where the increase was much higher or lower than the average, the reasons were easily identifiable. The second domain of concern is social policy itself. How to achieve a successful social security system when it is considered as unfinished and requiring more resources in order to reduce the risk of poverty and to warrant an acceptable level of protection of the population in general and which is at the same time considered as financially unsustainable in the long run? This has induced a large production of studies and academic literature about the reform of the social security systems. The third domain of concern is economic growth itself. During the last decade, the European growth performance has been generally very disappointing: growth in productivity has especially been very low and has stopped the catching up process of the European standard of living compared to the US. If this is to carry on and is combined with a declining population of working age, growth in the next decades will be very low and converge towards a halving of what it has been during the last two centuries. The consequences of such a change in the dynamism of a society are still to be investigated and have not yet received sufficient attention from economists and sociologists. This is one of the reasons why at the EU level such a great emphasis is being put on enhancing potential growth through the Lisbon agenda.

3. Confronted with a large budgetary cost of ageing in the long run, the government's alternative is: solve the problem when it comes up or try to anticipate the negative results and prevent them. The first solution would imply, in a pay-as-you-go system, that the cost of ageing would be covered by a progressive proportional increase of the tax pressure. There are two major drawbacks to this solution: first, in a globalized world, tax competition is a major constraint which limits the possibilities of increasing the cost of labour or of capital, second, the first solution is considered to be against an intergenerational equity principle since the cost is raised by one generation and paid by the next one. The alternative is to try to solve the problem now or in the medium term. Three ways are to be considered that are not mutually incompatible: reform the social system in order to reduce the costs for the present and future generations, increase the tax or contribution receipts by pushing up the trend growth of GDP and saving now in the public sector to cover the increase of the future expenditure. Those different ways of reducing the problem of ageing for the government can be combined. The weight associated to each of them will depend on the room for manoeuvre in the long run peculiar to each country. Four variables are of particular interest to figure out the dimensions of the general problem: the general tax pressure, the generosity of the social security system, the employment rate and the public debt. Moderating the increase of pension and health care systems is always difficult when it implies a reduction of the benefit ratio in the future. It is even more difficult when the benefit ratio is relatively low and the system less generous. In countries where the social system is less generous, one should see that the effort would be put on the other dimensions: employment or S2. Accordingly, the variety of situations among the EU countries has led the Commission and the Council to recommend a three-pronged strategy to be tailored for each country.

4. Social reforms have been directed towards increasing the employment rate and reducing the benefit ratio. A number of countries have reformed their legal pension systems. Few of them have undertaken a radical reform. Most of them have changed the system by increasing the age eligibility conditions, changing the indexation of pension from wage indexation to price indexation and changing the replacement ratio for future pensioners by increasing the number of contribution years. In the recent past, from 2000 to 2007, many countries have introduced some changes in their pension system. This is clearly seen in the progress made by the employment rates of older workers. This shows up very clearly in the projection of the benefit ratios. In many countries, those reforms substantially improved the view that we can have on the risk of long-term unsustainability of their public finance.

5. Some countries have also increased the sustainability of their public finance by accumulating savings. That has been explicitly the case when a reserve fund was created and funded by the government. Implicitly, the same process has been put in place by pursuing a fiscal policy aiming at reaching and maintaining a surplus conducing to a rapid decrease in public debt. A generalization of such a policy is now under consideration for every EU country in the framework of the Stability and Growth Pact (SGP). Sustainability is now at the centre of the debate at the "Medium Term Objective" (MTO) of the budget balance.

6. Since 2005, when the Council revised the SGP, sustainability has become a major issue. A monitoring process is now being followed involving the long-term projection and the assessment of sustainability. The long-term projection is based on a commonly agreed methodology conducted every three or four years by the member states and the Commission and the assessment of sustainability based on commonly agreed quantitative and qualitative indicators but conducted exclusively by the Commission. One can say that this process, actually launched in 2000, has been a success among the many other processes decided by the Council.

1. Introduction

I would like to thank Eurostat for inviting me to this conference on long-term demographic projections. Those projections have now become an essential starting point of the work done by the European Ageing Working Group attached to the Economic Policy Committee that I have the honour to chair since 2000. As this work is already getting to be well known, I shall not present it systematically. Rather, the question that I would like to ask today is the following: Let us assume that there is no available demographic projection or that we are completely myopic about the demographic future of our countries, what would this change to our daily life? Put in another way: what did the long-term demographic projections change in the social, economic and financial policies?

Since the last decade, when long-term demographic projections have started to highlight the future ageing of population, a large number of reforms have been undertaken. I would like to show you that these important reforms depend on figures derived from the very long-term, and that if we only had at our disposal the figures of today, the present policies would have been very different in many cases. In the meanwhile, it does not mean that policies have fully responded to the challenges posed by the demographic projections. Many countries are still far from an ideal and sustainable position in terms of social, financial and economic goals in a long-term perspective.

I would like to develop the discussion along the following way: first, I shall present the main public policy challenges that have emerged from the demographic projections; secondly, I shall show what is until now the response in terms of general strategy; thirdly, I would like to show you what has changed in social policies and, fourthly, how the long-term is influencing present fiscal policy.

2. Raising concerns emanating from population projections

Demographic developments have always been central to economic growth and welfare. In the sixties and the seventies, the international conferences on demography were motivated primarily by the strong population growth in the developing countries. The first UN Conference on Population was held in 1974 in Bucharest, it was followed by those held in Mexico in 1984 and Cairo in 1994. A UN Division on Population has been created in order to work out statistics, studies and forecasts.

During the eighties and the nineties, the long-term demographic projections have demonstrated progressively the important changes that are observed and will continue to be observed in the future population structure: life expectancy is persistently increasing, low fertility rates weigh on the growth of the young population and the baby boom generation is progressively reaching the age of retirement: three characteristics which have been associated with the ageing of population.

UN conference was already devoted to ageing in Vienna in 1969 and replicated in Madrid in 1999.

Beside the UN and national statistical institutes, Eurostat has been one of the leading international organizations which have very early[1] provided, almost every five years, long-term demographic projections alarming about the potential impacts of ageing (see table 1 to have the main demographic indicators from the last Eurostat projection).

Table 1 Main demographic indicators from the last Eurostat projection

	EU15		EU10	
	2004	2050	2004	2050
Fertility rate	1,5	1,6	1,2	1,6
Life expectancy at birth - men	76,4	82,1	70,1	78,7
Life expectancy at birth - women	82,2	87	78,2	84,1
Net migration flows (thousands)	1347	778	-3	101
Net migration flows (as % of population)	0,4	0,2	0	0,1

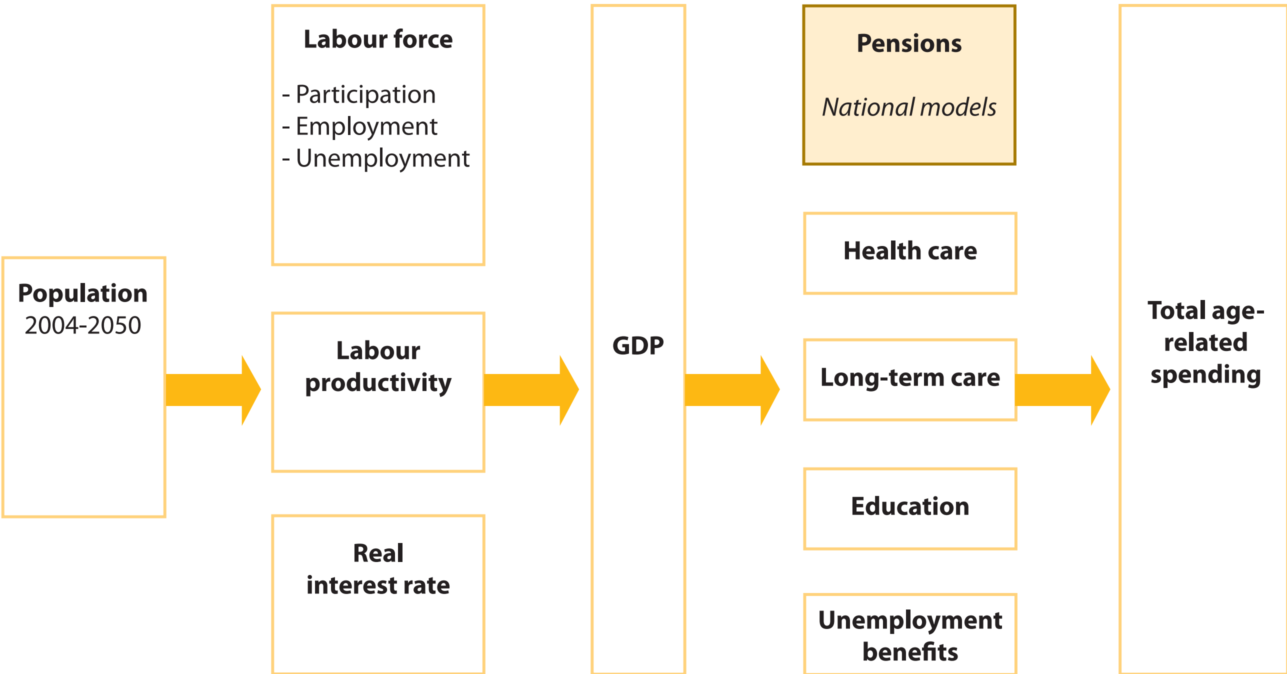
Source: European Economy (2006).

Concerns were raised in various domains of policy. The first domain is the long-term financial sustainability of the branches of social protection, where the expenditure is age-related to the age of the population. The ageing of population essentially changes the dynamics of receipts and expenditure of the pension system, which is in most of our countries based on pay-as-you-go systems. Health care and long-term care are two other types of expenditure that are influenced by the increasing average age of the population. To better understand those changes and measure their size, projections of social expenditure have been built and progressively refined. Those projections have first been developed in academic research projects. One of the first attempts to build such a projection at the international level was made by the IMF in the early nineties. Systematic projection exercises have been developed by academic research projects and by international organizations, especially in the EU. Other types of social expenditure that have a close relationship with the ageing of population have been considered: unemployment, family allowances, education, etc. In 1999, at the EU level, the EPC established the Working Group on Ageing Population with a mandate to project age-related public expenditures until 2050 on the basis of a Eurostat demographic projection. They were published in 2001 for 15 EU member states. In 2003, an attempt was made to update the projections and taking on board the new acceding countries. In 2006, a complete redrafting of the methodology and the use of more accurate and comparable data has led to a new set of projections for the 25 EU countries.

The methodology that has been developed by the AWG proceeds in several steps. The first step is the demographic projection achieved by Eurostat. Starting from the population, the labour force has been projected following the so-called cohort approach. An assumption of a slight decline in the structural unemployment contributes to the projection of employment by age and gender. We then have followed a "production function" approach to project productivity and wage development as well as GDP.

[1] 6 sets of projections have been provided (projections beginning with the years: 80, 85, 90, 95, 2004).

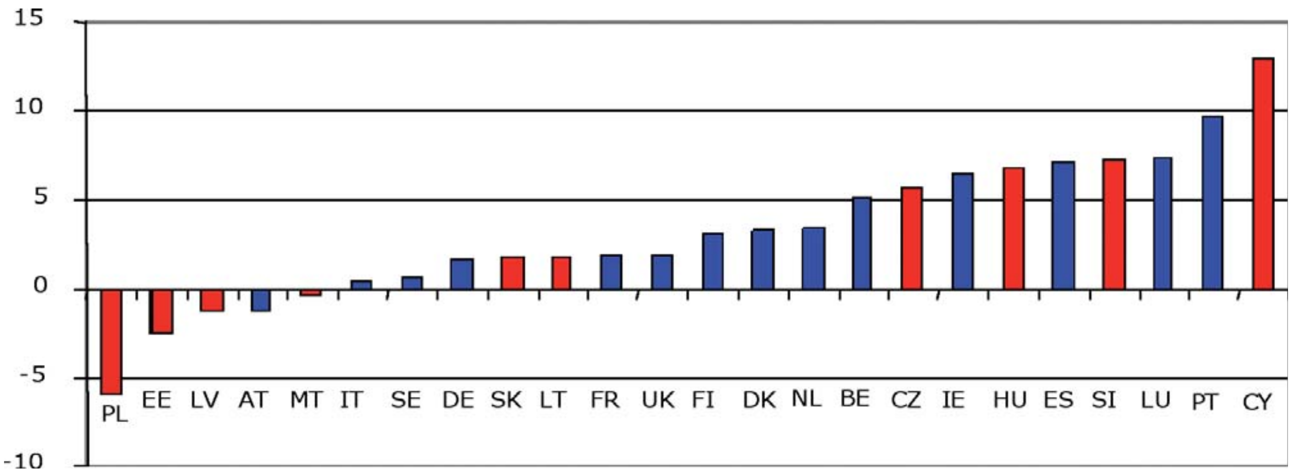
Figure 1 Overview of the 2005/2006 projection exercice



On the basis of these common assumptions, national models have been run to provide the pension projections, whilst the other age-related expenditure was computed by the Commission services with a methodology commonly discussed and agreed by the Commission and the member states.

The results of the pension projection are given in the following chart. The blue bars identify the EU 15 countries and the red ones the 10 new EU member states. The results show large differences between countries. Seven of them, mainly Eastern European countries, but also Italy and Sweden, having introduced radical reforms, and particularly having introduced either a switch to private pensions or a notional defined contribution system (NDC), have a negative or small increase of pension expenditure in percent of GDP. Eight countries show an increase which is smaller than 5 percent and 9 countries higher than 5 percent of GDP.

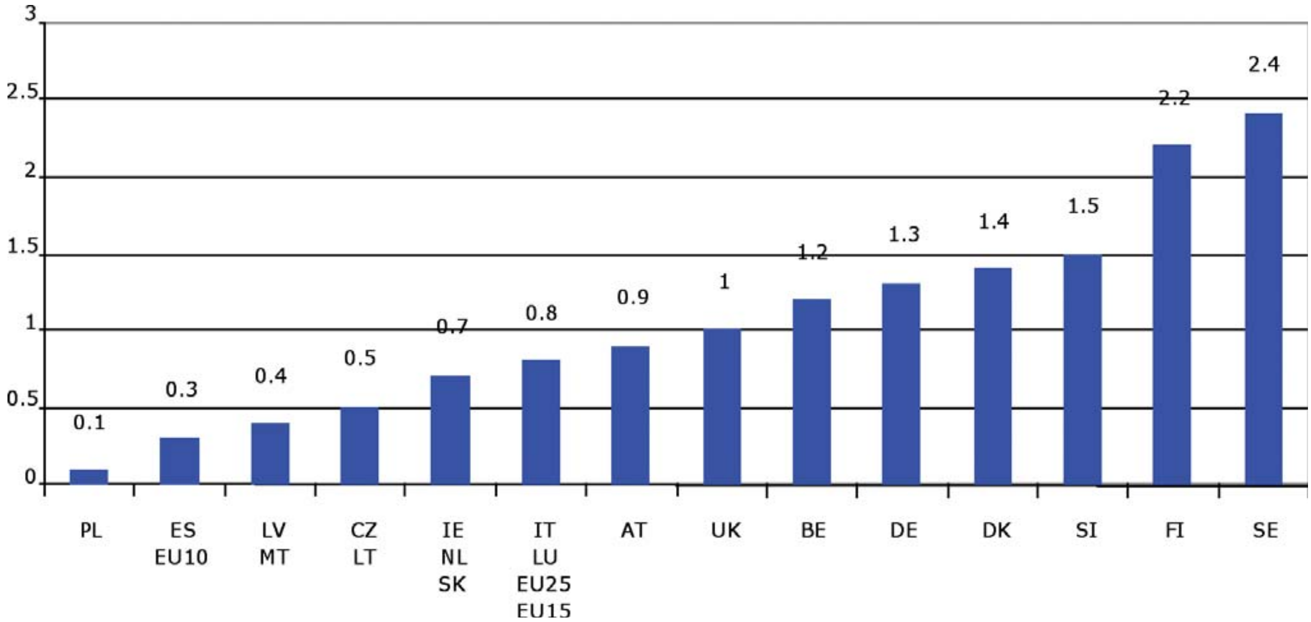
Figure 2 Projected changes in public pension expenditure between 2004 and 2050 *(in percent of GDP)*



Source: European Economy (2006).

The other important increase in age-related expenditure is health care. Health care spending is mainly driven by the average health status of the population, economic development, technologies and medical progress, etc. The ageing of population tends to decrease the average health status since a large share of the population requires more care. There is a debate in literature on the extent to which, as life expectancy increases, the health status or morbidity of the population may change. That is the reason why different hypotheses on the evolution of healthy life expectancy have been envisaged. Several scenarios have been constructed. The figures presented in the chart below attempt to isolate the pure effects of an ageing population on health care spending assuming that the age-related spending per capita on health care in the base year (2004) remains constant over time. It assumes de facto that the gains in life expectancy up to 2050 are assumed to be spent in bad health.

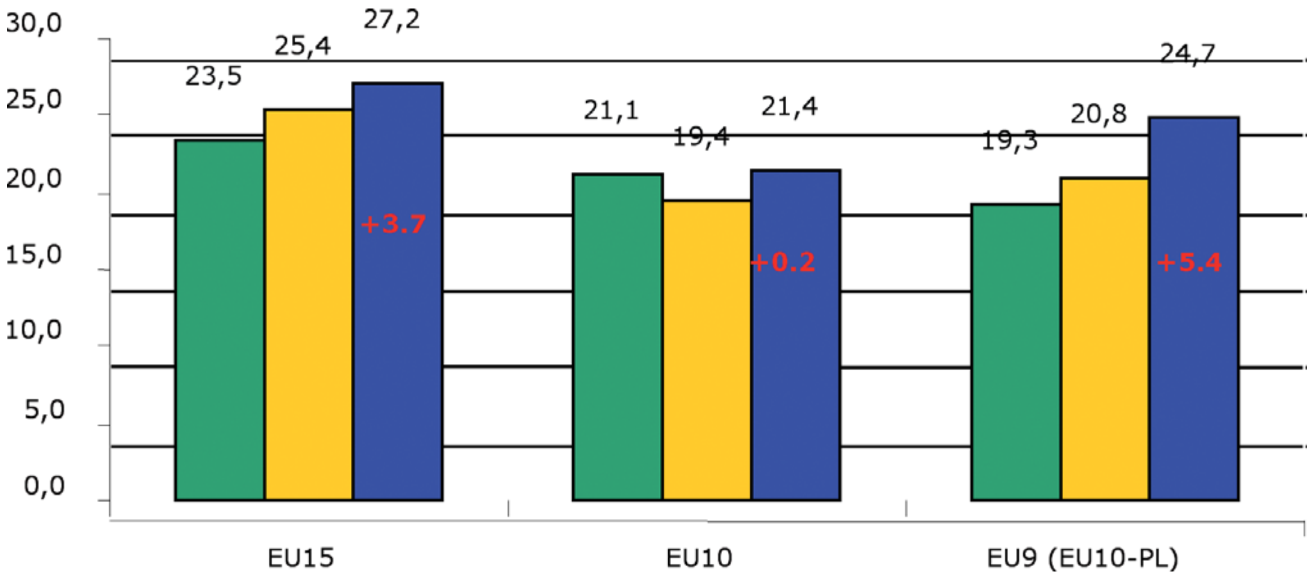
Figure 3 Change in health expenditure 2004-2050
(In percent of GDP)



Source: European Economy (2006).

The changes in the overall age-related expenditure that was considered by the AWG, i.e.: pensions, health and long term care, unemployment and education, are then added and .expressed in terms of GDP. The following figure illustrates their evolution from 2004 to 2030 and 2050.

Figure 4 Projected age-related expenditure between 2004 and 2050 *(in percent of GDP)*



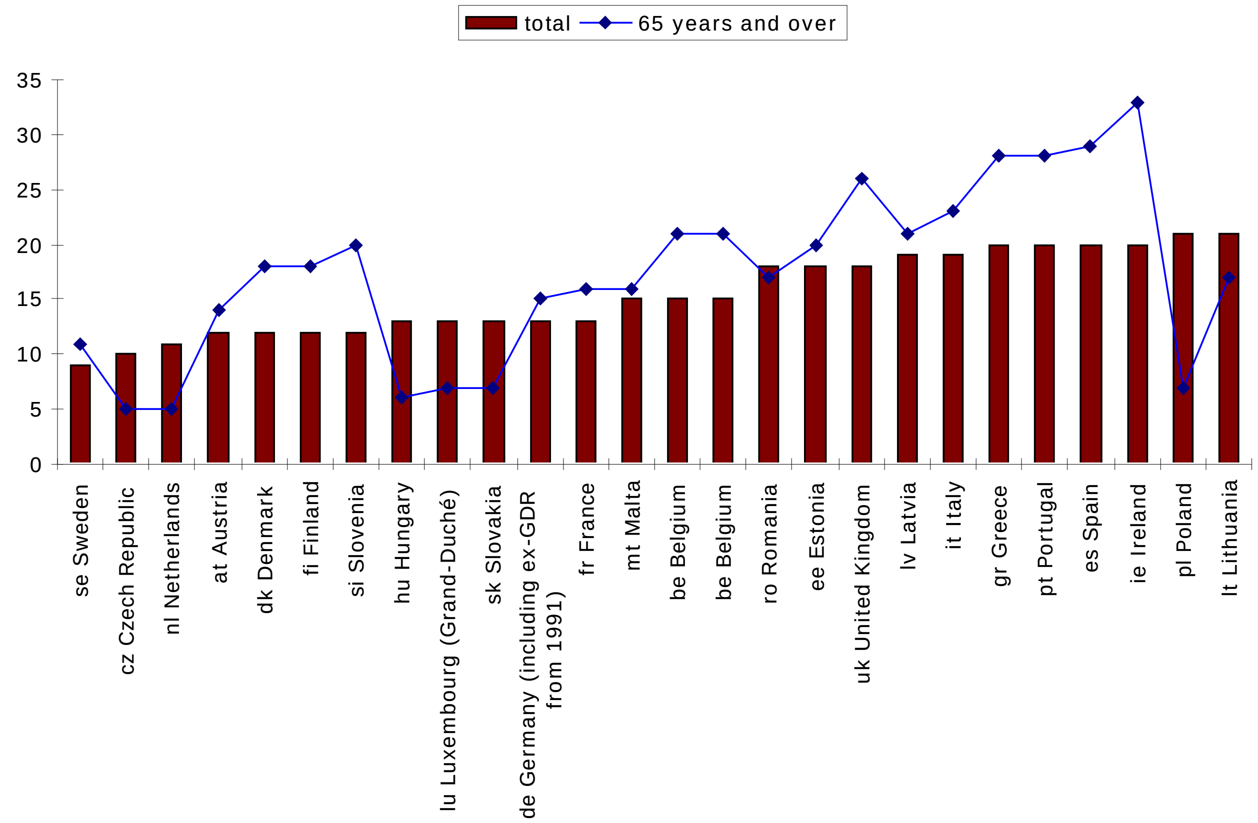
Source: European Economy (2006).

In general, the reduction in of the cost of unemployment and education is unable to compensate the projected changes in pension and health care spending. For the EU 15, the total increase would represent 3.7 percent of GDP, while, in the new member states, it would represent only 0.2 percent. Excluding Poland, where a broad and radical reform of the pension system was introduced, this increase climbs to 5.4 percent of GDP.

From the beginning, the publication of long-term age-related expenditure projections had a first consequence: the reform process, which was characterized by successive steps of relative increase of welfare for elderly people from the fifties to the eighties, has been managed more cautiously and even, in some cases, reversed during the nineties. The projections, while still very imperfect, have had a very powerful impact on policies. One of the reasons is the following: the projections were conducted not only by academic researchers but also by international organizations. Among the latter, the projections conducted by the EU, which have been very influential because they were conducted on a multi-country basis with the same methodologies and consistent general demographic and macroeconomic assumptions. Accordingly, the results were comparable. In most of the countries, the long-term increase in age-related expenditure was contained in a relatively narrow range. In the few countries where the increase was much higher or lower than the average, the reasons were easily identifiable.

The second domain of concern is social policy itself. How to achieve a successful social security system when it is considered as unfinished and requiring more resources in order to reduce the risk of poverty and to warrant an acceptable level of protection for the population in general and whilst it is considered at the same time as financially unsustainable in the long run? This has induced a large production of studies and academic literature about the reform of the social security systems. The risk of poverty is still high, and sometimes very high, especially among the oldest people. It mainly reflects past accruals and ongoing indexation of pensions. If unsustainable pension systems put the public finance at risk, equally inadequate pensions generate a demand for social policies to prevent situations of poverty. In this respect, the Social Protection Committee of the EU underlines that minimum income provision schemes for older people have an essential role in alleviating or reducing the risk of poverty amongst the elderly. Whether those systems contribute to this objective depends on the initial level of these provisions and the indexation rules applied to them. As it can be seen on the following graph, these minimum provisions are still inadequate in many countries.

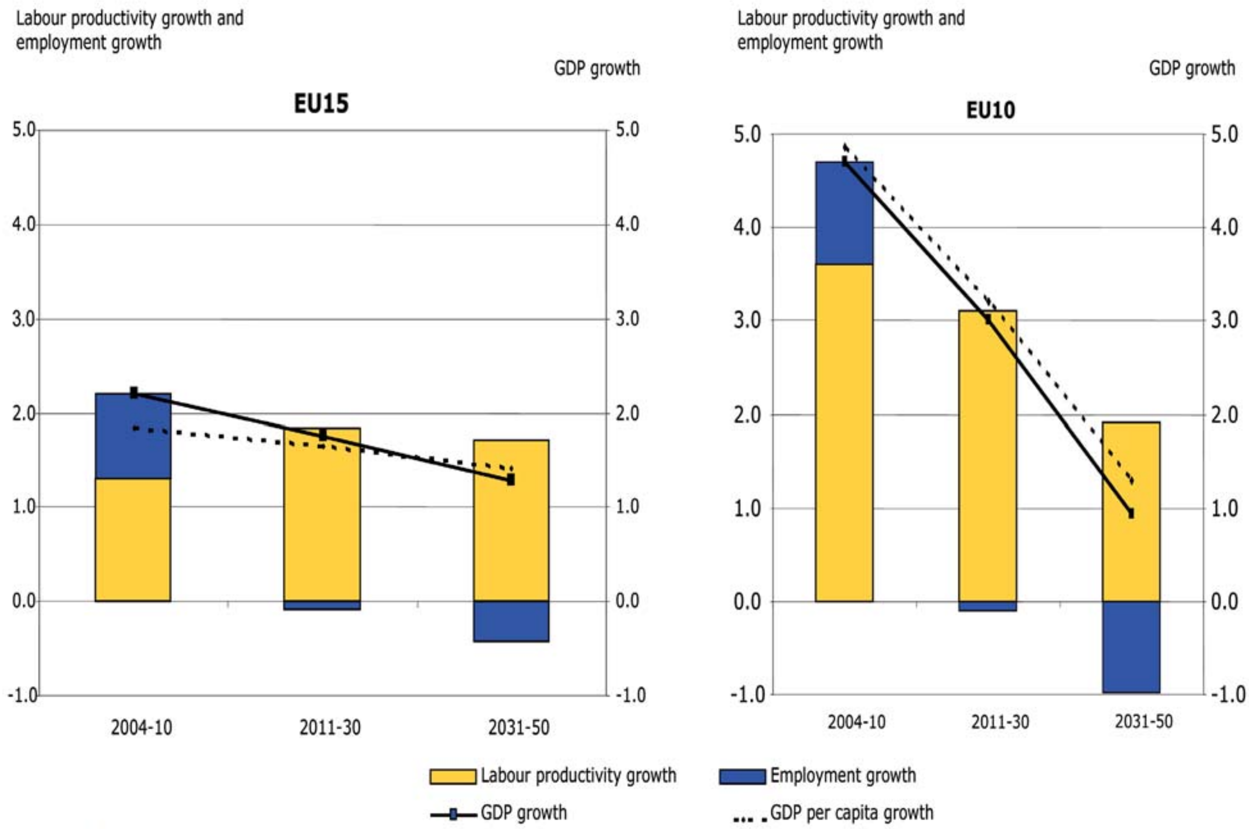
Figure 5 At risk of poverty rate in 2005 for the whole population and for the population 65 and over *(poverty threshold: 60% of median income after social transfers)*



Source: Eurostat.

The third domain of concern is economic growth itself. During the last decade, in European growth performance has been generally very disappointing: growth in productivity has especially been very low and has stopped the catching up process of the European standard of living compared to the US. If this is to carry on and is combined with a declining population of working age, growth will be very low in the next decades and converge towards a halving of what it has been during the last two centuries. The consequences of such a change in the dynamism of a society are still to be investigated and have not yet received sufficient attention from economists and sociologists. This is one of the reasons why at the EU level such a great emphasis is being put on enhancing potential growth through the Lisbon agenda.

Figure 6 Projected potential growth rates and their determinants *(in percent)*

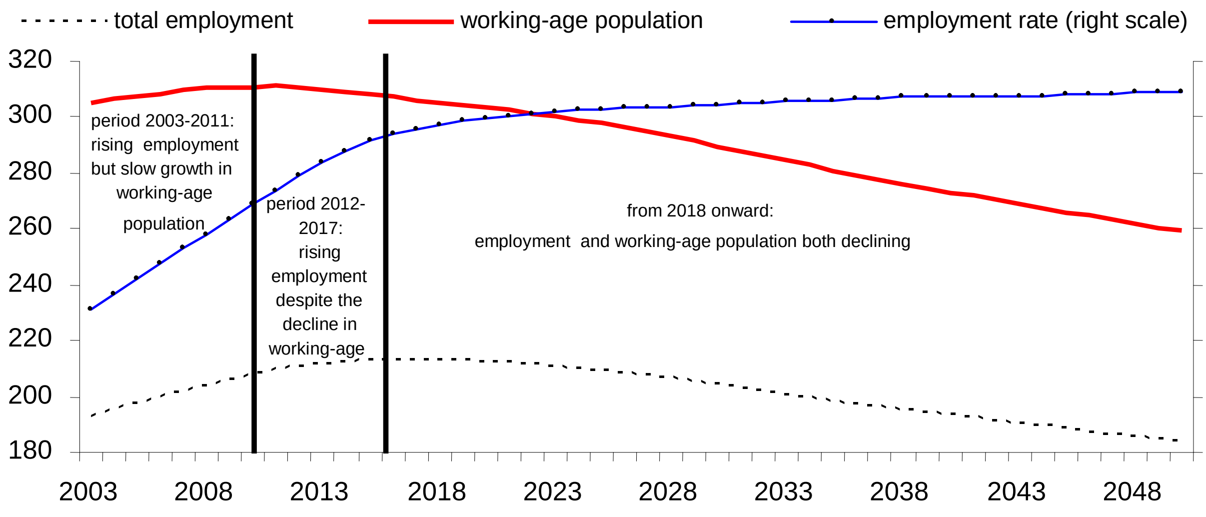


Source: European Economy (2006).

In a no policy change scenario, for the EU-15, the annual average potential GDP growth rate will fall from 2.2% in the period 2004-2010 to 1.8% in the period 2011-2030 and to 1.3% between 2031 and 2050. An even steeper decline is foreseen in the EU-10, from 4.3% in the period 2004-2010 to 3% in the period 2011-2030 and to 0.9% between 2031 and 2050.

These impressive reductions are mainly the result of the decline in the projected employment, which is itself driven by the decline of the working age population despite the rise of the overall employment rate. The employment rate is projected to rise from 63% in 2003 to 70% in 2020, the Lisbon employment rate target, mainly due to higher female employment rates and the increase of the older workers' participation rate. The following graph is particularly illustrative.

Figure 7 Projected working-age population and total employment *(EU25; in millions)*



Source: European Economy (2006).

In the latest and coming years, we do not see any impact of ageing on the labour market, on the contrary. Employment is growing substantially, while unemployment does not decline. This is due to a huge increase in participation rate and a still growing working-age population. Between 2011 and 2017, despite the decline in working-age population employment continues to rise because the participation is still growing rapidly among females and older workers. During this period, we nevertheless expect some shortages on the labour market that can result in increasing wage pressures. These pressures on the labour market will be permanent after 2017 when working–age population declines and participation rates are at maturity. Accordingly employment declines continuously and weighs on output growth.

Concerns are, in consequence, put on macroeconomic policies. The tightness of the labour market after 2011 can potentially produce inflation pressures which would lead to a restrictive monetary policy, rising interest rates and decreases in investment and in potential growth[2]. Moreover, low economic growth means small margins of manoeuvre for future governments which has always proven to be a very difficult situation for a government, especially if it has to maintain a strictly balanced budget or even a surplus.

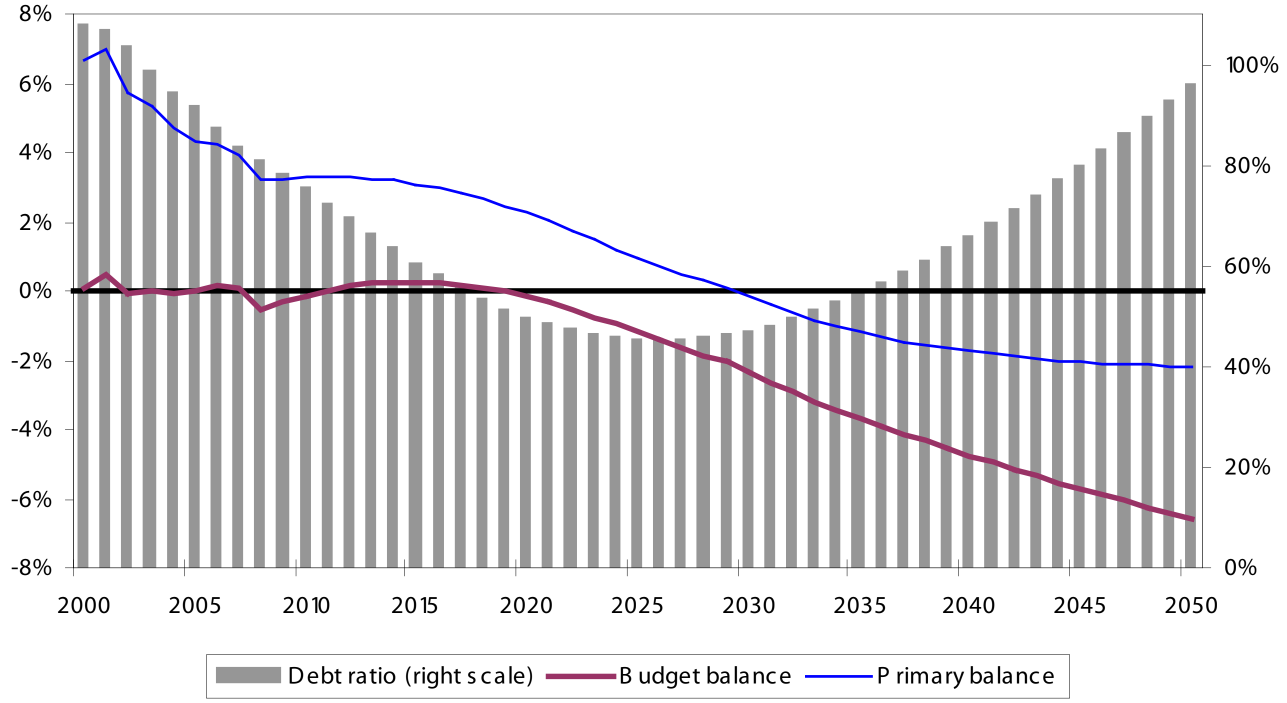
3. From diagnosis to prevention

Confronted with a large budgetary cost of ageing in the long run, the Government's alternative is: solve the problem when it comes up or try to anticipate the negative results and prevent them. The first solution would imply, in a pay-as-you-go system, that the cost of ageing would be covered by a proportional increase of the tax pressure. There are two major drawbacks: first, in a global and open environment, tax competition is a major constraint which limits the possibilities of increasing the cost of labour or capital; then, this first solution is considered to be against an intergenerational equity principle since the cost is raised by one generation and paid by the next one.

This choice can be illustrated by a "no policy change scenario" projection for any European country. I have chosen to show, on the Belgian case, that the political problem is far from being clear cut.

[2] Such a scenario is described for the euro area in the last world medium term projection issued by the Belgian Fe-deral Planning Bureau: "The NIME Outlook for the World Economy 2007-2013".

Figure 8 Long-term fiscal framework in Belgium in a "no policy change scenario" *(in percent of GDP)*



Source: FPB Perspectives économiques 2007-2012.

The Belgian government has succeeded to reduce substantially the deficit during the last decade and to maintain a balanced budget during the present decade. A "no policy change" scenario would imply that the reduction of the debt ratio, driven by the increase of the denominator - GDP - would entail a reduction of the interest burden in percent of GDP, which will compensate for the increase of the age-related expenditure during several years. The consequence will be that a balanced budget can easily be maintained until 2020. Later, this compensation will no longer happen. Age-related expenditure will grow faster and the reduction of the interest burden will be small since the debt itself would reach very low levels. The result is that the deficit increases and is fuelled by the snowball effect of the debt and interests payments. Politically, with regards to the short and medium term, this scenario is such that it will be very difficult for a government to change anything in its policy. The only reason to reform the social system or to build up a structural surplus in order to frontload the long-term shock is: intergenerational equity and tax competition.

The alternative is to try to solve the problem now or in the medium term. Three ways are to be considered that are not mutually incompatible: (i) reforming the social system in order to reduce the cost for the present and future generation, (ii) increasing the tax or contribution receipts not by increasing rates but by pushing up the trend growth of the tax base, i.e. GDP, and (iii) saving now in the public sector to cover the increase of the future expenditure. These different ways of reducing the problem of ageing for the government can be combined. The weight associated to each of them will depend on the room for manoeuvre in the long run that are peculiar to each country. Four variables are of particular interest to figure out the dimensions of the general problem: the general tax pressure, the generosity of the social security system, the employment rate and the public debt (the so-called S2 indicator). First, if the tax pressure is already high, tax competition will make that in the future a government will not be able to use this instrument to finance the age-related expenditure. On the contrary, competitiveness and wage wedge induced structural unemployment will push policies to reduce tax rates. Secondly, moderating the increase of pension and health care systems can be part of a solution. But, this is always difficult as it implies a reduction of the benefit ratio in the future. It is even more difficult when the benefit ratio is relatively low and the system less generous. Thirdly, in countries where the social system is less generous, one should see that the effort is being put on other strategies: reforming the system in order to increase employment of older worker and postpone the take-up of pension. This strategy will at the same time increase GDP and reduce pension spending[3]. Fourthly, a government confronted with a situation where tax pressure is high, pensions are not generous, and employment rate is also high can decide to somehow accumulate savings now by building up budget surplus and accelerate the reduction of public debt. Accordingly, the variety of situations among the EU countries has led the Commission and the Council to recommend a three-pronged strategy being tailored for each country, i.e. reducing debt at a fast pace, raising employment and productivity, and reforming pension and health and long-term care systems.

4. Inducing social reforms

Pension reforms have primarily aimed at increasing the employment rate of older workers and reducing the benefit ratio, i.e. the average pension divided by the average wage. A number of countries have reformed their public pension systems. Few of them have undertaken a radical reform. Most of them have changed the system by increasing the age eligibility conditions, changing the indexation of pension from wage indexation to price indexation and changing the replacement ratio for future pensioners by increasing the number of contribution years. In the recent past, from the end of the nineties to 2007, many countries have introduced some changes in their pension system. In its last report, the AWG has recorded the pension reforms that were introduced among the EU-25. A summary can be found in the following table.

[3] Especially if the reforms aim at rising the age eligibility conditions for benefiting from a pension instead of introducing financial incentives (bonus) for those who take up their pension later on.

Table 2 Recent pension reforms in EU25 (as recorded by the AWG in 2005[4])

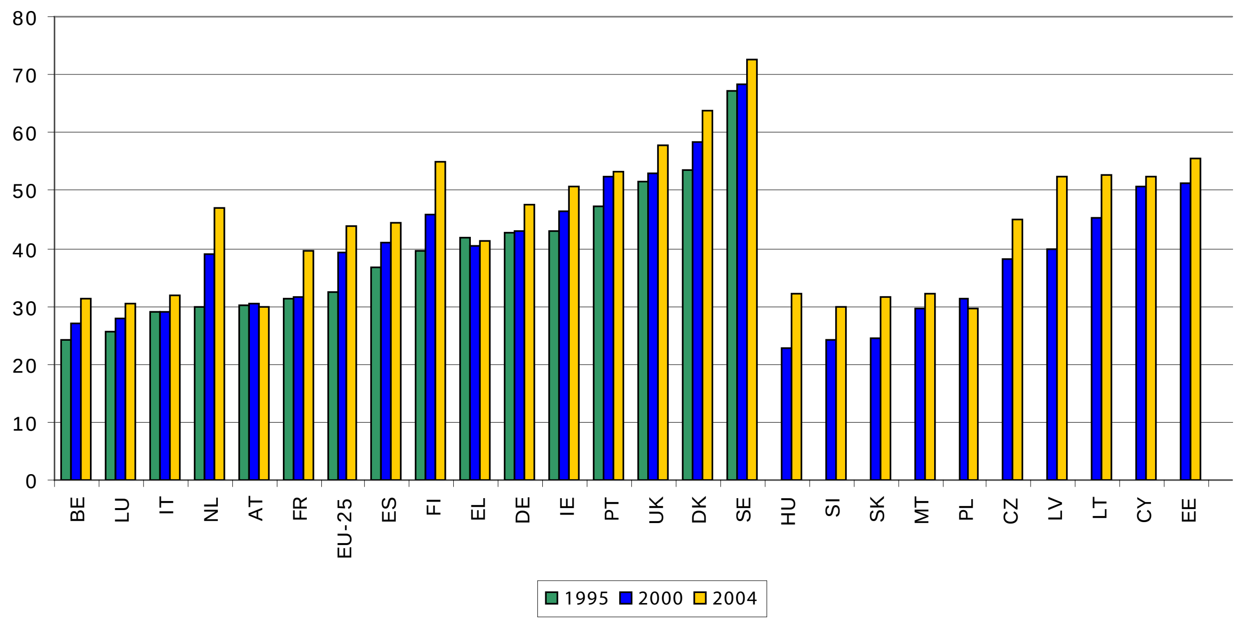
Country	Year	Retirement age or length of contribution period	Pension indexation	Attractive-ness of early retirement	Disability	Develop-ment of funded schemes	Pension formula	Reserve fund
BE	2003	Y				Y		Y
DK	2003,2004			Y	Y			
DE	1992-2001,2002,2004	Y	Y (sustainability factor)	Y	Y	Y		
EL								
ES	2002-2005			Y			Y	
FR	2004	Y	Y (to prices)	Y				
IE	1999,2000,2003					Y		Y
IT	2004	Y					NDC	
LU								
NL	2006			Y				
AT	2003,2004	Y	Y (to prices)	Y				
PT	2002,2005	Y		Y			Y	
FI	2003-2005			Y	Y		Y(life time)	
SE	1998			Y		Y	NDC	
UK	2002-2003	Y						
CY								
CZ	2003							
EE	2001	Y				Switch to private schemes		
HU	1997	Y				Y		
LT	1995,2004	Y				Switch to private schemes		
LV	1996	Y				Switch to private schemes	NDC	
MT								
PL	1999					Switch to private schemes	NDC	
SK	2004	Y				Switch to private schemes		
SL	2000	Y				Y	Y	
Number of countries	21	13	3	9	3	11	8 of which 4 NDC	2
Number of reforms from 2000 to 2005	+/- 26							
Number of reforms recorded	+/- 32							

4 in: European Economy (2006).

According to the table, the reform process has been impressive! The number of reforms recorded is more or less 32. Among the EU-25, 21 countries have introduced one or more reforms. Several countries have even introduced radical reforms: four have implemented notional defined contribution (NDC) systems: Italy, Latvia, Poland and Sweden, according to which the actuarial accumulation of contributions is converted in an annuity at the time of retirement. The system remains, nevertheless, a pay-as-you-go system: no funding in individual accounts are implemented in the NDC system. One of the salient features of the NDC system is the progressive change of the generosity of the pension in function of the increases in life expectancy. In order to safeguard the actuarial rationale of the system a pensioner in an older age group has either to work longer or to benefit from a lower pension than someone in the former age group. Other countries have introduced this kind of mechanism as well, without the actuarial philosophy (e.g. FR). A radical shift towards private pensions has been made by EE and LV, while a part of the social pension scheme has been switched to private schemes in Lithuania, Hungary, Poland, Slovakia and Sweden.

Apart from these, the majority of the countries have not radically changed their system, but parametric changes have been introduced. The table shows that in most of the countries, a particular emphasis has been put on increasing the employment rate of older worker. This has been introduced by rising the eligibility age condition to early retirement (notably by progressively aligning the retirement age of females to that of males) or by diminishing the advantages given to early retirement or by abolishing the unemployment pathway to early retirement. The EPC/AWG report[5] records recently enacted reforms having potential effects on older worker participation in 17 countries: BE, DE, SP, FR, IT, AU, FI, SW, UK, CZ, EE, HU, LI, LV, PL, SK, SL. The following graphs show the evolution of the participation of older workers since 1990 and the estimated impact of pension reforms on average exit age from the labour force which will continue to influence the participation in the future.

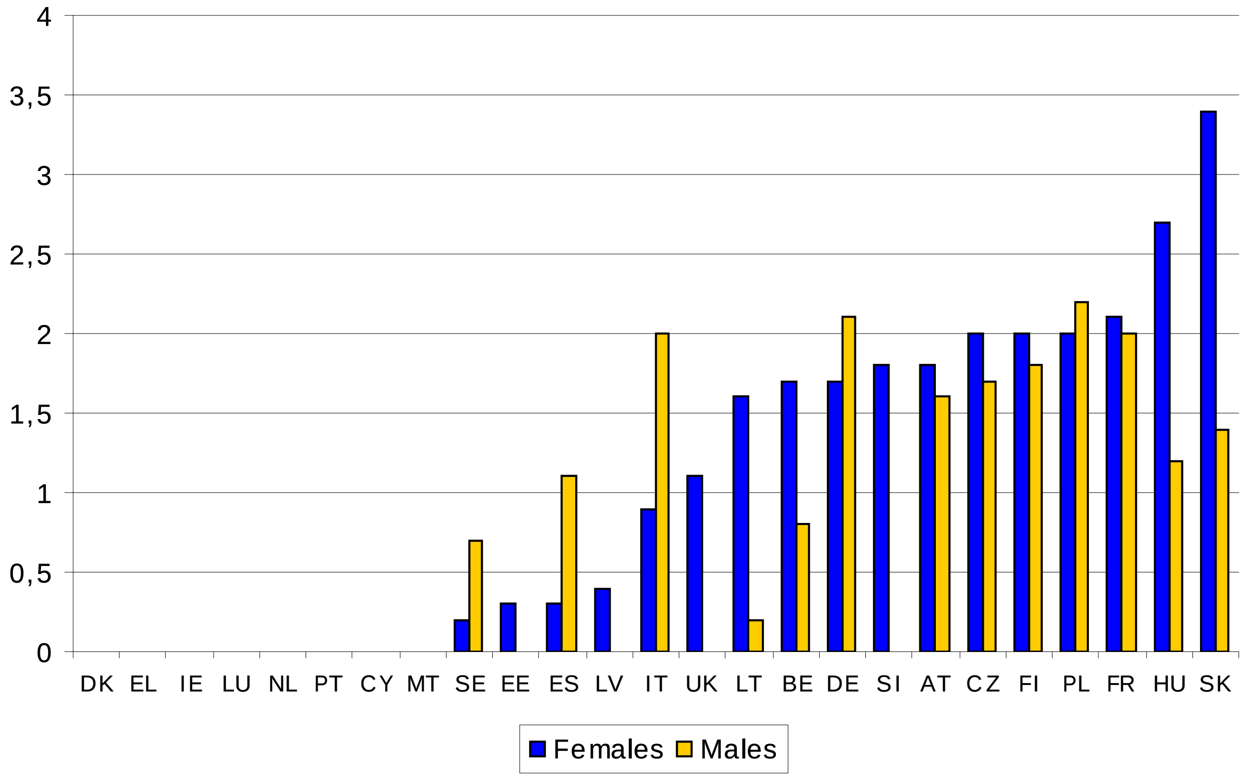
Figure 9 Historical participation rates: older workers aged 55 to 64 *(in percent)*



Source: European Economy (2005).

[5] See European Economy, Special Report 4/2005, pp. 51-56.

Figure 10 Impact of pension reforms on average exit age from the labour force
(number of years)

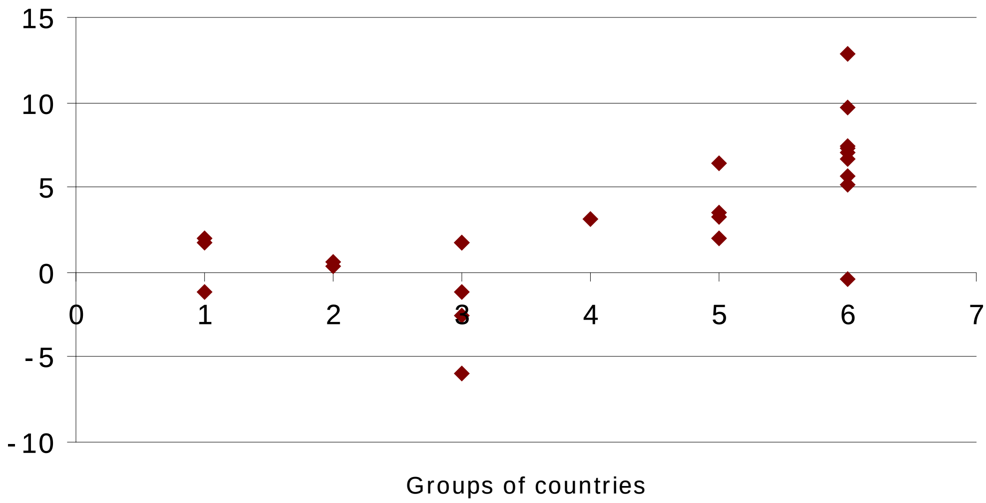


Source: European Economy (2005).

In a number of countries, the indexation formula of pensions has been changed when indexation was related to the average wage. Now, in most of the countries, pensions are indexed to prices plus a factor, this factor being, anyway, less than indexation to wages. There are two exceptions: countries where pensions are simply a flat rate (DK and NL) where pensions are legally indexed to wages, and countries where pensions are determined as a notional defined contribution system. After the last reform, Germany is a country which applies consistently the philosophy behind the pay-as-you-go system: pensions are primarily indexed to wages but the index is adjusted by the change in the contribution rate and the change in the pensioners/employees ratio (called the sustainability factor)

The first consequence of these reforms is the lower (than previously foreseen in the 2001 report) change of the pension to GDP ratio between 2004 and 2050. As it can be seen on the following graph, countries which have implemented reforms that switch the indexation of pensions to prices, switch to private pensions or introduced in the pension formulation a life expectancy factor like in NDC systems show up a lower future increase in the costs of pensions.

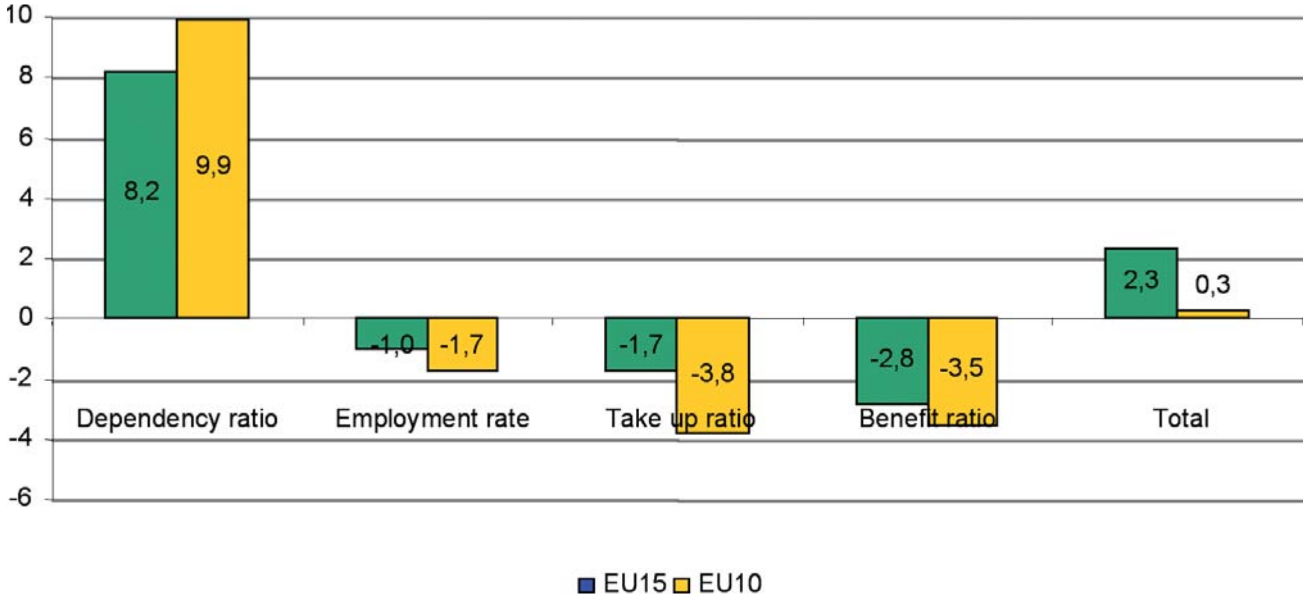
Figure 11 Projected changes in pension expenditures 2004-2050 by group of reforming countries *(in percent of GDP)*



Figures on horizontal axis represent groups of countries: 1): countries having introduced large reforms: 1=shift in indexation and broad reform of defined benefit systems (DE, FR, AT), 2=shift towards NDC system (SW, IT), 3=shift to private pensions or NDC + private; 2) (LT, LV, EE, PL) other countries: 4=important change in the pension formula (FI), 5=flat rate system (DK, IE, NL, IK), 6= defined benefit systems.

Another consequence of these reforms will be a substantial decrease in the benefit ratio over time in many countries. In its 2005 report the AWG has tried to disentangle the increase in the costs of pensions until 2050. The pension to GDP ratio can be expressed as the product of the dependency ratio, the employment ratio, the take up of pension ratio and the benefit ratio. In the following graph, this decomposition clearly shows that the rise in the old age dependency ratio is the dominant factor pushing up public spending, while employment rate, eligibility rate and relative benefit level will offset part of the demographic pressure (70% in EU-15). The benefit ratio, which is not the replacement ratio, but the ratio of the average pension relative to output per worker, is so impressive in some countries (DE, FR, IT, AT, PT, SW, EE, LV, MT, PL, SK) that it will inevitably lead, if it is not accompanied by private complements, to future pressure to policy changes and reforms in the reversed direction. This is especially true, in my view, for the indexation mechanism where, if there is only an indexation to prices (SP, FR, IT, AU, PL), someone who stays in the pension system during several decades, and who has no complementary private pension, will suffer a huge lost in its relative standard of living.

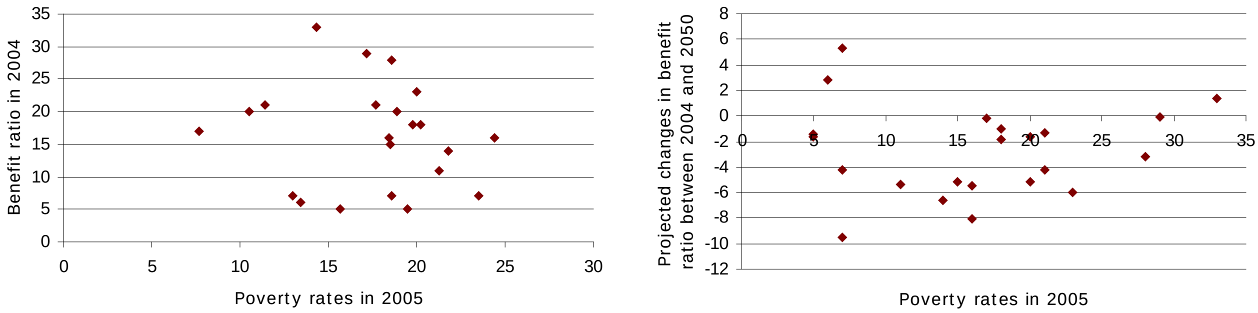
Figure 12 Decomposition of the change between 2005 and 2050 in all public pensions relative to GDP[6] *(in percent of GDP)*



Source: European Economy (2006).

Despite the fact, as it can be seen on the following chart, that there is no correlation between the level of the benefit ratio and the poverty rate, large reductions in the benefit ratio can finally have an important impact on poverty in the future. A warning about the social sustainability of the projected change in the benefit ratio is all the more important as there is no correlation either between the projected change of the benefit ratio and the poverty rate.

Figure 13 **Poverty rates and benefit ratios** *(in percent)*

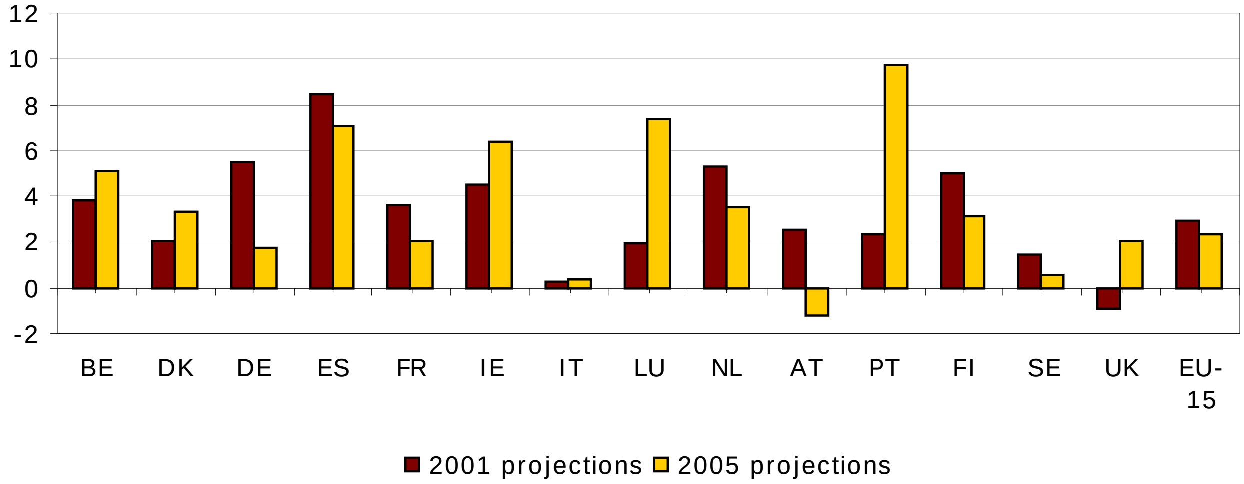


Source: European Economy (2006) and Eurostat.

In many countries, the pension reforms have changed radically the view that we can have on the risk of long-term unsustainability of their public finance. The comparison between the results of the 2005 and 2001 projections is presented in the following graph. The differences are not only due to the introduction of reforms since 2000, but are also the result of changes in the data used, in the demographic projections and, in some cases, in the modelling techniques. Anyway, the AWG report suggest that the smaller projected increase in public pension spending can be largely attributed to major pension reforms undertaken since 2001, in particular in DE, FR, AT and FI. Reforms undertaken in other countries have probably affected the projected evolution of pension expenditure, but their effects are more difficult to disentangle.

[6] Pension to GDP ratio can be expressed as the product of four factors: the dependency ratio (population over 65/population 15-64), the inverse of the employment ratio (employment/population 15-64), the take-up ratio (number of pensioners/population over 65), and the benefit ratio (average pension/GDP per worker or averedge wage rate).

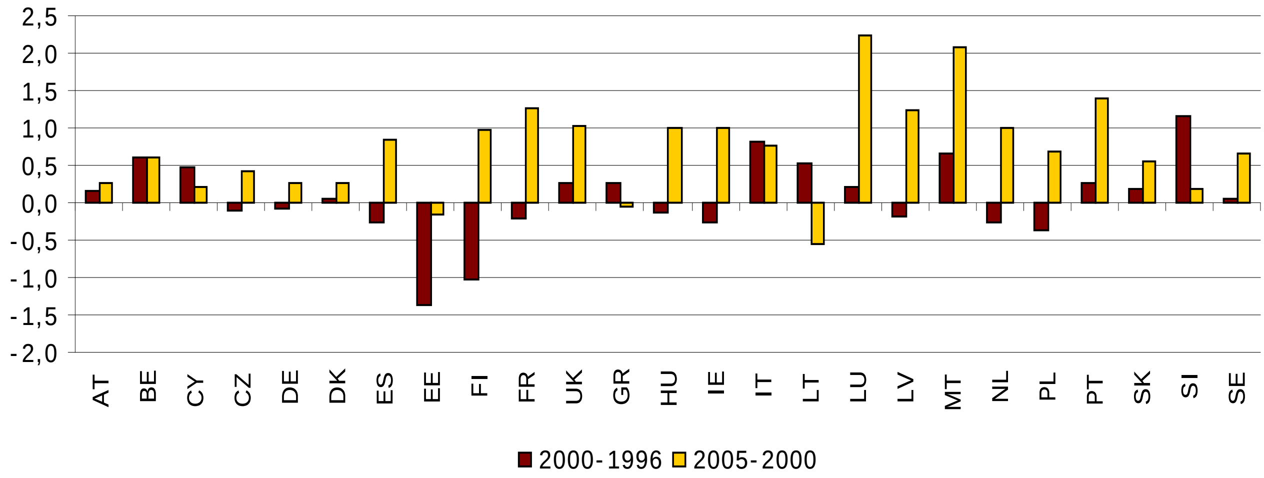
Figure 14 Projected change between 2004/2005 and 2050 of gross public pension expenditure as a hare of GDP



Source: European Economy (2006).

If pension reforms have been introduced successfully in most of the EU countries in order to moderate their increase, this is not the case when we consider health expenditure. The next figure shows the large increase in total health expenditure that was observed between 1996 and 2000 and between 2000 and 2005. In general, we can see that the increase is general and important. But we can also see that financial sustainability warnings, far from having induced more moderate changes, were not able to curb the trends. This is quite understandable in the new member states were the health care systems have to catch-up the western standards, but important increases have also accelerated in countries like SP, FR, FIN, UK, LUX, PT while in the countries like BE, IT or SW the increase continues to be significant.

Figure 15 Total health expenditure in percent of GDP: change between 1995 and 2000 and between 2000 and 2005

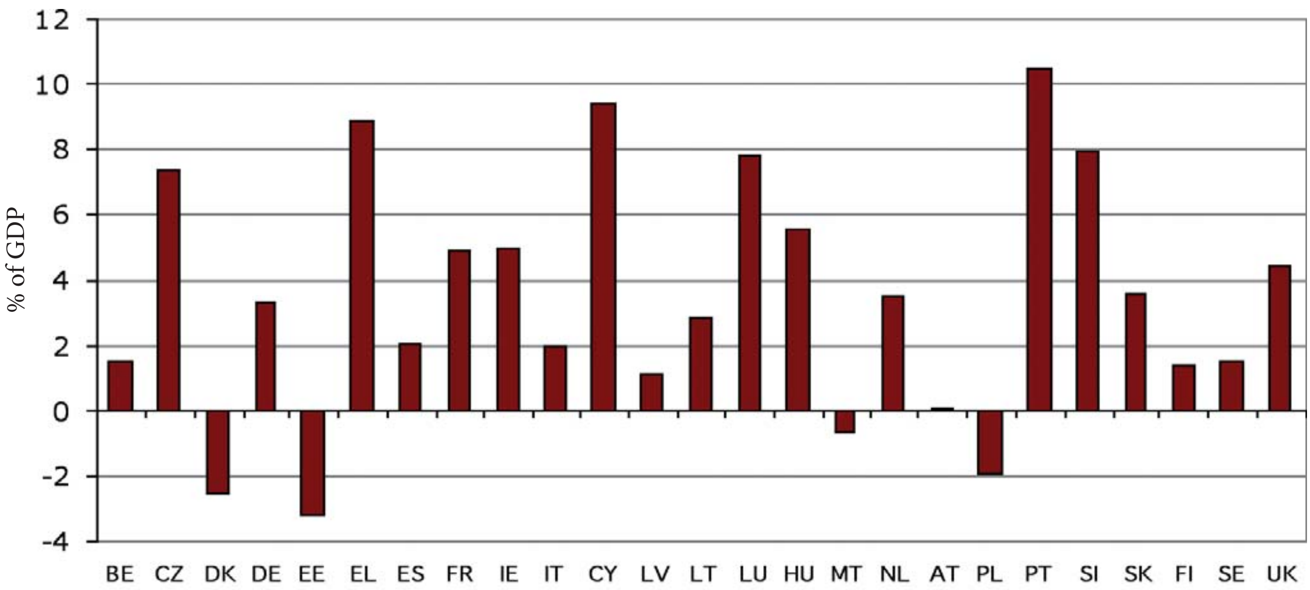


Source: WHO Website – National health accounts.

5. Influencing fiscal policy

Some countries have also increased the sustainability of their public finance by accumulating savings. This has been explicitly the case when a reserve fund was created and funded by the government. Implicitly, the same process has been followed by countries that pursue a fiscal policy aiming at reaching and maintaining a budget surplus leading to a rapid decrease of public debt. A generalization of such a policy is now under consideration for every EU country within the framework of the Stability and Growth Pact (SGP). Sustainability is now at the centre of the discussion about the level of the MTO: the Medium Term Objective is the level of the budget balance that should be maintained on average during the business cycle. According to the revised SGP, the MTO should be established at -1% of GDP. This allows the budget balance to fluctuate around this value without going beyond the 3% deficit ceiling fixed in the Maastricht Treaty. For countries where the debt ratio is higher than the 60% ceiling, a lower deficit or a small surplus has been recommended by the Council. Now, the long-term impact of ageing on social expenditure represents also an implicit liability which requires a more restrictive fiscal policy. This is the reason why an indicator has been constructed in order to know what the permanent improvement of the budget balance should be if the sum of discounted future deficits resulting from the impact of ageing is to be financed. This was called the S2 indicator or the sustainability gap and is shown in the following graph. It shows that for most of the countries, the required improvement of the structural budget balance is rather large.

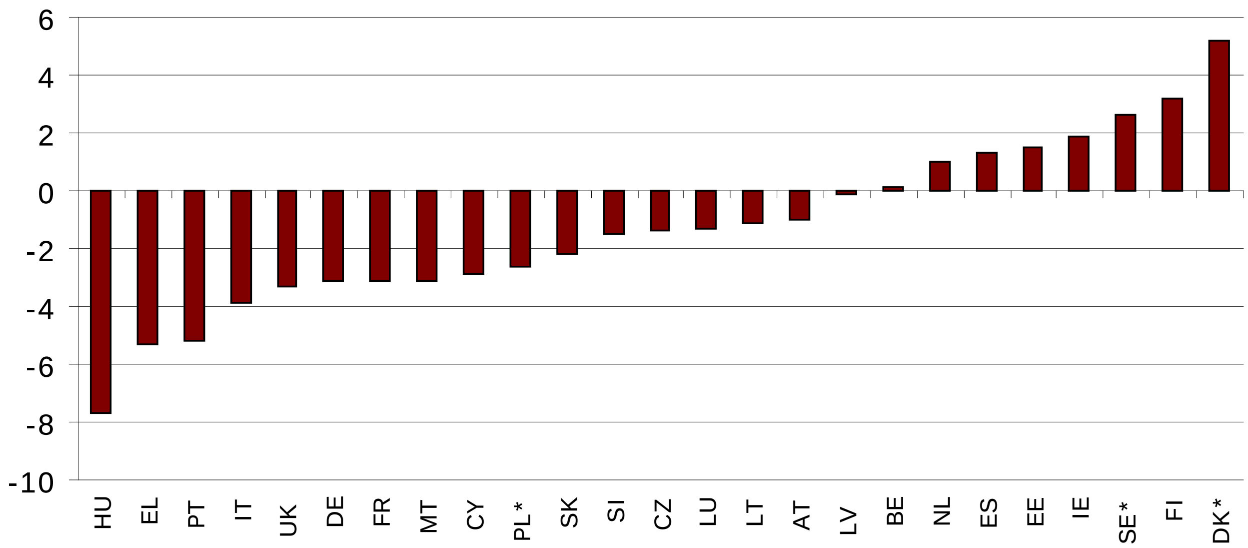
Figure 16 The S2 indicator in the Sustainability Report
(In percent of GDP)



Source: Commission services.

The sustainability gap has been internalized in fiscal policy in several countries. The following graph shows the structural balance reported for 2006 in the Stability and Growth Programmes of the member states. In most of the countries the deficit is still large and beyond the -1% recommended as the MTO structural balance. This is the case for 15 out of 25 countries: CZ, DE, EL, FR, IT, CY, LT, LU, HU, MT, PL, PT, SL, SK and UK. Eight countries: (BE, NL, DK, EE, ES, IE, FI and SE) are close to balance or have a surplus the purpose of which is directly linked to the long-term sustainability objective and, indeed, these countries have no sustainability gap.

Figure 17 Government structural balances in 2006
(in percent of GDP)



* For countries (DK, PL, SE), the balances are given including revenue to the funded part of the mandatory pension scheme.

Source: Commission services.

The risk of unsustainability remains significant in many countries. When the Council revised the SGP in 2005, this has been a major concern and long-term financial sustainability has been a central issue of the revised pact. A monitoring process is now followed involving: (1) the long-term projection (based on a commonly agreed methodology) conducted every three or four years by the MS and the Commission (The Projection Report by the AWG), (2) the assessment of sustainability based on the projection and on commonly agreed quantitative and qualitative indicators and conducted exclusively by the Commission (The Sustainability Report), (3) a yearly updated sustainability assessment taking into account new reforms and new developments in a country fiscal policy. Specific country recommendations of reforms or changes of fiscal policy will be then issued by the Council.

Figure 18 European Sustainability Assessment Process

As shown in this paper, among the many processes decided by the Council, one can say that this process, actually launched in 2000, has been a success. There is, however, a large variation in the degree of risks to the sustainability of public finance that the EU countries are still facing. Five countries are assessed to be at high risk, ten at medium risk and eight at low risk in the Commission's assessment and the Council's opinion. For several countries, particularly the large ones, the risk is no more linked to the long-term cost of ageing but to the short term disequilibrium in the budget. Taking this into account, and mainly due to the reforms, the assessment has largely improved over time.

Figure 19 Overall classification of risks to the sustainability of public finance in the 2006/2007 updates of stability and convergence programmes

Risk category	Country
Low	DK, EE, LV, LT, NL, PL, FI, SE
Medium	BE, DE, ES, FR, IE, IT, LU, MT, SK,UK
High	EL, CY, HU, PT, SI

6. Conclusions

We have seen that the long-term demographic projections have very important negative consequences on the sustainability of public finance, on employment and growth. Since the end of the nineties, many countries reacted to that prospective situation. The EU also launched several processes in order to encourage member states to take preventive action against the negative consequences of prospected demographic developments. The general strategy was to reform the social system, to promote employment especially among the elderly and to fund the cost of the demographic shock through a rapid reduction of the public debt ratio. Those three kinds of policies could be clearly observed during the last 10 years among the EU member states.

The reform process of the pension system is particularly impressive in several countries, notably those who have switched towards a private system or a system which relates the pension to the life expectancy of the cohort, like the NDC systems, and those who have switched the indexation system towards prices instead of wages. Generally, the earnings-related systems where this kind of reform was not introduced show a rather large increase in the pension to GDP ratio over time. At present, there is no significant relation between the benefit ratio and the poverty rate of the elderly, but the projected decrease of the benefit ratio questions social sustainability of some of the reforms. Particular attention should be given the minimum pension and the indexation of this minimum, which must be compatible with the evolution of the average wage. Most of the countries have introduced reforms to increase the effective age of retirement from the labour force. The impact on the evolution of the pension to GDP ratio is smaller than the reforms of the pensions mentioned above, often because the increases are obtained by higher pension at a later age, but the impact on employment and economic growth is substantial. As for the other expenditure, especially health care, the same reform process has not been observed yet.

The employment rate of the elderly is already improving very fast, which proves that the reform process is again showing up again. Nevertheless, the AWG projections show that the average employment rate can improve more and more rapidly over the next 50 years. Reforming the systems and the labour market in order to retire later is still a challenge in many countries. This challenge is the key to a sustainable and high economic growth in the coming decades.

Last but not least, we see that in some countries fiscal policy is heavily influenced by the consequences of the long-term demographic projections. The intention is to frontload a share of the impact by raising funds for a reserve fund or by reducing the debt ratio. This is clearly the case for eight countries where the structural budget balance is positive or larger than the deficit of one percent which would be required if there were no demographic chock. More should be done in the future, as recommended by the Commission. Nevertheless, a trade off exists between reforms having a possible negative and unsustainable social impact and the restrictive fiscal policy that can be suffered in the short run from the adjustment of the structural balance. Though, we have seen that almost eight countries have radically changed their fiscal policy, merely to cope with the information given by the demographic projections.

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Session 1: Fertility

Chair: Wolfgang Lutz

PROJECTIONS OF AGE-SPECIFIC FERTILITY RATES THROUGH AN AGENT-BASED MODEL OF SOCIAL INTERACTION

September 21, 2007

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Abstract

In demographic projections future fertility rates are usually predicted using time series extrapolations or expert-opinion-based expectations. Both approaches have their foundation at the macro-level which is motivated by the fact that variables like total fertility rates or age specific fertility rates are measured at the macro-level. The cause of fertility, however, is at the micro-level where individual fertility decisions are taken. Hence, a theory-based approach must be attached to the micro-level and generate macro level observations as the result of individual behaviour. This is often referred to as the micro-macro link.

We suggest the use of agent-based models (ABMs) to predict future fertility rates. Empirical studies indicate that the transition to parenthood is influenced by an individual's peer group. ABMs allow us to model individual fertility decisions considering the influence of peer groups and the influence of the society as a whole. We build a one-sex model and provide agents with four different characteristics. Based on theses characteristics agents endogenously form their network. Network members then may influence the agents' transition to higher parity levels. Our numerical simulations indicate that accounting for social interactions can explain the shift of fertility rates in Austria over the period 1991 to 2001. We apply our model to forecast age-specific fertility rates up to 2021.

1. Introduction

Human behavior, including childbearing behavior, is performed by socialized actors deeply rooted in a web of social relationships like those created by kinship, love, power, friendship, competition, or interest. Within one's social circle of relationships individuals may exchange information about possibilities and consequences of specific childbearing choices, learn about other persons' preferences, form expectations on their future choices, feel induced to conform to others' norms about family-related behavior, and modify their interpretation of a specific behavior.

Interpersonal interactions among these relatively small groups of individuals produce social effects observable in macro patterns of behavior and demographic research on union and family formation has concentrated on the latter. Empirical evidence increasingly suggests social interaction as an important determinant of demographic behavior. Diffusion processes are currently an integral part of the literature on fertility decline (Knodel and van de Walle 1979, Watkins 1987, Cleland and Wilson 1987, Mason 1992, Pollak and Watkins 1993, Palloni 1998). While most research is carried out in developing countries some contagion models have been applied to union behavior in the European context (Nazio and Blossfeld 2003). Diffusion approaches build on the idea that social networks of kin, peers and institutions, as markets and legal and the administrative system, are potential communication channels for ideas and behavior (Granovetter 1985, Rogers 1995).

In fertility explanations social interaction are relevant both at the micro and at the macro level. Individual and population fertility are interdependent because the aggregation of individual fertility behavior produces externalities (like the erosion of norms, pressure to conform, path dependency of the information exchange). Kohler (2001) efficiently summarizes the features of this micro-macro link: a) social interaction can alter the distribution of knowledge in the population and affect reproductive decisions under uncertainty by conveying information on the consequences of low fertility or on the dynamics of social change, b) it may establish a collective behavior among community members and initiate a fertility change when other factors would instead inhibit it, c) it may induce an endogenous transformation of social institutions and social norms.

Montgomery and Casterline (1996, 1998) separate the concept of diffusion into the two components social learning and social influence. Social learning takes place interpersonally when other individuals provide information that shape a persons' subjective beliefs while social influence denominates the effects of interpersonal interactions that are intrinsically "social" such as to avoid conflict within social groups. The latter are expressed in an individuals' preferences and in her information sets as well. Kohler et al. (2000) empirically investigate the impact of family planning programs taking into consideration the influence of social interactions on the diffusion of knowledge, attitudes, and behaviors related to family planning. They distinguish between the direct and indirect effect of family planning programs on an individuals' probability to apply family planning. According to their estimates the contribution of social interactions amounts to 43% of the total program effects. Moreover, intensified social interactions may either increase or decrease the total effect.

The analysis of social mechanisms like social learning and social influence plays an increasingly relevant role in demographic explanations of observed family formation patterns also in contemporary Europe, like in the hypothesis formulated by Kohler et al. (2002) on the emergence of lowest-low fertility. However, the increasing inclusion of social interaction in the demographic theoretical framework matches with a relatively unrealistic model of social learning and social influence mechanisms (Chattoe 2003).

This lack of precision seems to constitute a general problem in the development of demographic behavior theory. Specifically, there is a certain agreement that demography suffers from a poor level of precision in the theoretical construction, a statistical modeling that is not or insufficiently theory-driven, and the non – or hard – observability of important concepts and indicators involved in the theory (Burch 1996, de Brujin 1999). Partially this is due to the inadequateness of the demographers' methodological toolbox to answer demographic relevant questions. The very recent inclusion of agent-based simulations and systematic and comparative in-depth investigations offer new possibilities to develop cognitive valid behavioral theories and to speculate on the consequences of alterative micro-macro feedbacks in order to explain demographic patterns (Billari and Prskawetz, 2003, Billari et al. 2003, Billari et al., 2006).

Modelling individual fertility decisions requires to include the decision mechanism at the microlevel, the society at the macrolevel, the interactions between the micro- and the macro-level, and the interactions among individuals within their peer groups. We need a sound microfoundation of individual behaviour and a mechanism taking into account social interaction between individuals and describing the interaction between the individuals and the environment in terms of socioeconomic conditions. Agent based models, and to some extent also microsimulations, have the potential to include these three attributes.

Agent-based models allow for a straightforward modeling of social interactions among agents and of interactions between the micro and macro level. Agent-based models enable us not only to consider past developments but also to build scenarios for alternative structural frameworks. Unlike the socially isolated actors in a typical microsimulation model, the agents in an ABM interact independently. The key features characterising ABMs are that agents are autonomous, interdependent, follow simple rules, and they are adaptive and backward-looking (Macy and Willer, 2002). Consequently, ABMs allow for a higher degree of complexity than for instance microsimulations. While the focus is on the aggregate level (age-specific fertility in Austria), our model is based on the micro level and explains how aggregate level properties emerge from the behavior of the agents on the micro level. Thus, aggregate outcomes are rooted in the interaction of agents.

In this paper we introduce an agent-based model to study social interaction and in particular endogenous network formation and its implication for the transition to parenthood. In the first place such a model allows us to test whether transitions in age-specific fertility experienced in the past can be explained by social interactions. Secondly, and the focus of the current paper, we can use this model to project age-specific fertility rates. Section 2 is devoted to the implementation of the model. The data we use to calibrate our model are discussed in section 3. Simulation results and fertility projections based on these simulation results are presented in section 4. Finally, section 5 concludes our findings.

2. Model Implementation[1]

We set up a one-sex model that allows us to simulate the different life cycle stages of females. Each individual agent has an identity number *id*, four characteristics, and a social network that includes friends, siblings and the agent's mother[2]. The agent's characteristics are age x, education e, intended education *ie*, and parity p. We set the lower and upper age of reproduction to be equal at 15 and 49 years respectively and the maximum age of our agents as 95 years. Though agents older than 49 cannot give birth in our model, they still may influence other agents. Education is an influential factor for social network formation and size (Moore, 1990) and thus becomes our second characteristic. We distinguish three stages of education: primary and lower secondary, upper secondary, and tertiary. Since education does not effect an agent's network on the day of graduation but already during training, we further include the intended education as an important characteristic of the agent[3]. Based on these three characteristics – age, education and intended education – an adult agent chooses on average s members for her social network. These members influence the agent's decision of childbearing, i.e. her parity, that constitutes the fourth characteristic of the agent. The agent's desire to give birth, that is to increase parity, is weakened or intensified by the influence of the social network *snw*.

2.1 Initial population

We initialize the simulation with N individuals and base our simulations on Austrian data, as defined in section 3. The Austrian age distribution for females constitutes the initial age distribution. The level of education of individuals aged 15 or older is assigned according to the Austrian age-specific educational distribution for females. On the basis of the assigned age and educational level, each agent is assigned her parity according to the Austrian age- and education-specific parity distribution of females.

Since most people finish their education before they turn 30, we assume that the educational distribution at age 30 in the base year determines the intended education at earlier ages[4].

We do not allow an intended education *ie* lower than the already achieved education e. Therefore, agents with $e = 2$ are assigned intended education $ie = 2$ or 3 and agents with $e = 3$ are assigned $ie = 3$. Agents younger than 15 are not assigned an intended education and for all individuals above the age of 28 the intended education ie is set equal to the actual education[5]. Moreover, individuals at the educational level 1 and older than 20 are also assigned their actual education 1 as their intended education since transition between level 1 and 2 practically happens solely until the age of 20. For agents with parity greater or equal to one an age at first birth a is assigned according to the education-specific distribution of age at first births (cf. section 3). Since the behavior of women in training for education level e is more comparable with the behavior of those who already achieved the level e, we assign the age at first birth a according to the agents' intended education *ie*. Once all initial agents have got assigned their individual characteristics, adult agents create their social network by choosing relevant others based on the three characteristics: age, education, and intended education.

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1 See Aparicio Diaz et al. 2007 for further details.

2 The agent's mother and siblings are not known for the initial population.

3 The argument to include intended education in addition to attained education is based on the anticipatory analysis in life-course research discussed in Hoem and Kreyenfeld (2006).

4 Of course, some individuals finish secondary or tertiary education later than age 30. Therefore, it seems to be desirable to look at the educational distribution, for instance, at the age of 40 or 50 to be sure not to loose any individual obtaining a higher level of education during her life course. However, applying the educational distribution of older cohorts would result in a bias toward lower levels of education since higher education was not that common for older cohorts this holds in particular for females.

5 Although there are some cases of individuals who advance to higher levels of education above that age limit, the period data on which we base the empirical estimations do not lead to strictly positive transition rates for that age group.

2.2 Simulation steps

During each simulation step, each agent ages by one year and dies off at the age of 95. Individuals younger than 15 are considered as children without education. At age 15 an individual becomes an adult with education level one and an intended education assigned on the basis of the education distribution of the population aged 30. Further she builds her own social network which includes friends chosen according to the procedure described below. Agents born during the simulation already have a social network consisting of their mother and siblings[6]. Though children do not exhibit their own social network of friends, they can nevertheless be part of one. When an agent turns 50, we assume that childbearing ceases. However agents older than 50 may still influence other adults of childbearing age.

In the course of the simulation an adult agent may change her educational level. The age-specific educational transition rate is based on empirically observed transition rates *etra* for Austria (see section 3). From empirical data we know that agents with a higher intended education are more likely to increase their level of education, likewise are non-mothers. To achieve this we scale the empirical education transmission rate *etra* by *a* multiplier $w(c)$ where c is a vector of characteristics $c = (x,e,ie,p)$. We assume that every agent may increase her educational level but postulate that those who have not yet attained their intended education are subject to a higher transition rate and consider that women with a higher parity have a lower transition rate to higher education – in particular there is a pronounced difference between mothers and non-mothers.

2.3 Endogenous social network

We apply a model of endogenous social network formation. Regarding Granovetter (1973, 1983) the network ties shape an individual's opportunities as well as her norms and values. A vast bunch of literature studying the topology of social networks has emerged recently (Watts and Strogatz, 1998, Barabasi, 1999, Newman, 2003). However, these models do not provide any theory explaining how such complex network structures may emerge (Flache and Macy, 2006). Therefore, we use a searchable network topology originating from Watts et al. (2002) who introduced a network structure taking into account the fact that individuals partition the social world in more than one way. They applied this network to explain the process of delivering messages to a target person. In the sequel we will use a similar network structure for the diffusion of childbearing behavior. As mentioned in the introduction, our model should take into consideration that links in a social network may be based on any individual characteristic like age, kinship, love, power, friendship, professional occupation, geography, and so on. Thus, we have agents living in a multidimensional space, where each dimension represents one characteristic.

The agents within such a searchable network exhibit network ties and individual characteristics. For our purpose we consider the characteristics age, education, and intended education to create a social network *snw*. Watts' approach envisions that individuals organize the society hierarchically into a series of layers, where the top layer represents the whole population which is split according to the agents' characteristics into smaller subsets of individuals which are likewise split into more specific subgroups. The social groups that are formed through this hierarchic division depend on the branching ratio b and the group size g of the lowest hierarchic level.

The similarity among any two individuals, d_{ij}, is given by the height of their lowest common ancestor level in this hierarchy. If two individuals i and j belong to the same group we define their similarity d_{ij} equal to one, if they belong to different groups which are directly connected, their similarity becomes $d_{ij} = 2$ and so on. The probability of acquaintance (i.e. the probability of a link) between two individuals with a distance d is given by

$$pr(d) = c\ exp(-\alpha d), \tag{1}$$

with α being an adjustable parameter and c being a constant required for normalization. Thus, even two individuals belonging to the same group are not necessarily connected. However, if the parameter α, determining the agent's level of homophily, is assigned high values, the chance of a connection between individuals in the same group becomes high. To build up the social network an agent chooses a distance d according to the above probability distribution (1) and then picks

[6] Through the inclusion of the mother as a peer, we attain the effect that the number of siblings influences the agent's fertility, in addition to the parity of the siblings themselves.

a friend uniformly among all individuals with distance d[7]. This procedure is repeated until an average number of friends, s, is found. The mean network size is an exogenous parameter. The actual number of friends for an agent is log-normally distributed.

Since individuals belong to three groups (by age, education, and intended education) the procedure described in the previous paragraph is repeated for each of these characteristics. Since we postulate that the characteristics are independent people belonging to the same group in one dimension may be far away from each other in another dimension. However, if there is a link established in one dimension due to the random process described above, the agent considers the chosen agent to be a member of her peer group. Since networks are known to be unstable over time (Wellman 1997), we assume that each adult may exchange one or more members of her social network.

2.4 Social influence and parity transition

An adult agent (aged between 15 and 49) may give birth to a child. The decision to change her parity status is influenced by her social network (see for instance Bernardi 2003, 2007). The propensity to have a first child increases with the share of parents within the agent's social network. Similarly the propensity to higher order births increases with the share of parents of higher order parity. To ensure that the social influence modeled at the individual level is "anchored" at the social influence we observe at the macro level, we postulate that the social influence vanishes if the parity distribution of an agent's network coincides with the parity distribution at the macro-level. Formally, the social influence *si* for an agent of parity p is modeled as a function of the difference between the share of mothers at parity $\tilde{p} > p$ within the social network, *rop*, and in the whole population, *ROP*. The social influence positively affects the age- and parity-specific birth probabilities *bpr* of Austria (see section 3).

To determine the social influence *si*, we first define the relevant share of network members *rop(p)* whose parity exceeds the agent's parity p.

$$rop(p) = \frac{\#\{j : p_j > p \; AND \; j \in snw\}}{\#\{j : p_j \geq p \; AND \; j \in snw\}}, \tag{2}$$

where p_j denotes the current parity of agent j who is a member of agent i 's social network *snw* and $\#\{j : p_j > p \; AND \; j \in snw\}$ denotes the number of network members with parity greater p. Note, that for higher order births we ignore (in the numerator of equation (2)) those agents within the peer network who are at parity $\tilde{p} < p$[8].

Likewise, we compute the share of adult agents with parity greater p, *ROP(p)*, on the aggregate level,

$$ROP(p) = \frac{\#\{j : p_j > p\}}{\#\{j : p_j \geq p\}}.$$

The difference between *ROP* on the aggregate level and *rop* on the individual level determines the social influence on an agents age- and parity-specific birth probability $bpr(x,p)$. We model social influence as an s-shaped function with slope β,

$$si(p) = 0.1 * \frac{\exp(\beta * (rop(p) - ROP(p)))}{1 + \exp(\beta * (rop(p) - ROP(p)))} + 0.95. \tag{3}$$

The parameter β gives the intensity of the social influence when the share of network members of a specific parity diverges from the one on the aggregate level. Choosing $\beta = 0$ results in a social influence of 1 in any case, which means that the influence of the social network is completely ignored. Lyngstad and Prskawetz (2006) point to a weaker influence for second birth, thus we reduce the social influence for higher order births si $(p>0)$ by decreasing β to a fifth of its original value.

7 Technically this procedure is implemented in the way that the agent draws a random number in the interval (0,1) and the random number then determines the specific value of d as determined by the probability distribution (1).

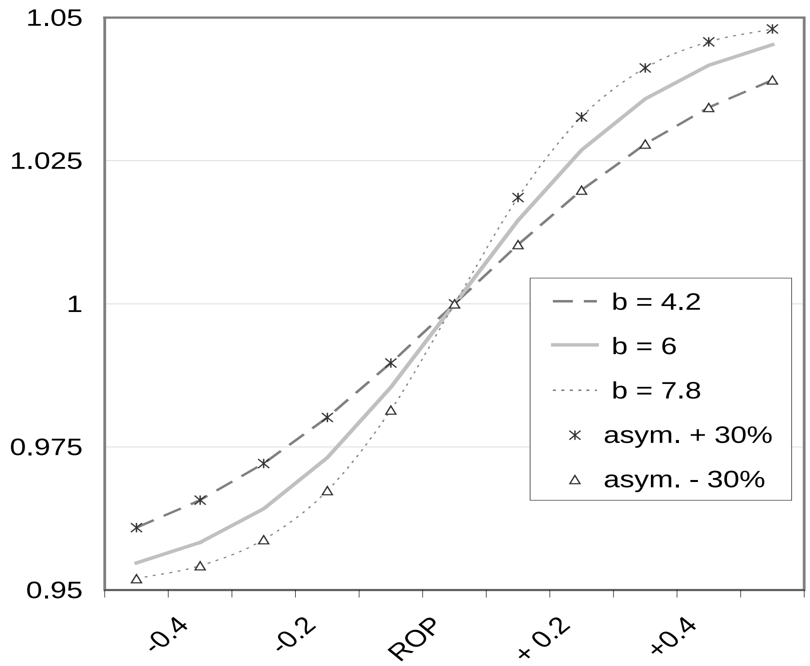
8 Bernardi et al. 2007 found that women who already have children do not refer to childless peers concerning former fertility decisions.

The value *si* is multiplied with the empirical age- and parity-specific birth probability at time *t*, $\overline{bpt}_t(x, p)$, to take the social influence into account. Thus, an agent *i* at age *x* is assigned a probability of birth,

$$bpt_i(x, p) = \overline{bpr}_t(x, p)si(p). \tag{4}$$

The multiplier given in (3) ensures that the birth probability *bpr*(*x*,*p*) of an agent *i* facing a value of *rop* within her social network which is equal to *ROP* on the aggregate level is not being distorted. Put differently, when the social influence at the individual/micro level is equal to the social influence at the macro level we assume that the social influence vanishes (i.e. it is equal to one). In case that the micro level share *rop(p)* differs from the macro level share *ROP(p)*, the social influence is assigned a value in the range (0.95, 1.05) assuming that positive and negative deviations are symmetric. To also allow for an asymmetric social influence, but retaining the condition of *si* = 1 if *ROP(p)* = *rop(p)*, we introduce the asymmetry through the slope β. We postulate an asymmetry that strengthens the positive and weakens the negative social influence. More precisely, a social influence function with slope $\beta = 6$ and an asymmetry of +30%, as shown in Figure 1, actually leads to an influence function with slope $\beta = 6 - 1.8 = 4.2$ for negative influence, thus for agents with a lower *rop(p)* at the individual level than *ROP(p)* at the macro level, and a slope $\beta = 6 + 1.8 = 7.8$ for positive influence. In this way we achieve that the asymmetric social influence modeled at the individual level is again "anchored" at the social influence we observe at the macro level.

Figure 1 Symmetric and asymmetric social influence function



Interacting macrolevel data on age- and education-specific fertility rates with microlevel data on socio-economic characteristics and social influence within the agents' peer groups allows us to model the agents individual propensity of childbearing. The structure of our modelling approach captures an individual's situation and her exposure to social norms and social pressure. Individual changes on the micro level result in a modified probability to give birth at the macro level. Thus, the birth probabilities at *t*+1 become

$$\overline{bpr}_{t+1}(x, p) = \overline{bpr}_t(x, p)\overline{si}_t(x, p) \tag{5}$$

where $\overline{si}$ *(x,p)* is the average of the social influence values *si* of all agents at age *x* and parity *p*. These updated probabilities to give birth enter equation (4) for the next time step.

Transition to parenthood: After transition to parenthood an agent increases her parity by one. Since we work with a one-sex model we refer to the Austrian sex ratio at birth *srb* (see section 3) as a multiplier for the number of new agents. Hence only the female babies are created as new agents. Then they age each simulation step until arriving at adulthood (at age 15) when they choose their friends for the social network. During childhood an agent's network only consists of the agent's mother and siblings, to whom the new agent is also added as a network member.

3. Data

Age Distribution: For the initial population we alternatively use the age-distribution of Austrian females in 1991 or 2001[9].

Distribution by Age and Education: We assign the level of education according to the agents' age. Agents younger than 15 receive education 0, while all other agents may get a primary/lower secondary, upper secondary, or tertiary education according to the age-specific educational distribution of Austrian females in 1991 or 2001[10.] We distinguish (for adult agents) three stages of education, whereas the Austrian data we use as input distinguish 6 to 8 stages. We therefore merged these groups as follows: (i) primary/lower secondary education encompasses basic schooling (up to 9 years) and lower secondary education (including apprenticeships and normally between 10 and 12 years of schooling), (ii) upper secondary education which encompasses the Austrian gymnasium and its equivalents, such as corresponding non-academic vocational training at a similar level and (iii) tertiary education (including postgraduate studies, the training of primary school and gymnasium teachers, art academies, and so on).

Distribution by Age, Education, and Parity: Based on the Austrian distribution by age, education and parity of 1991 or 2001[11], we assign a corresponding parity for the initial agents.

Parity-specific Birth Probability by Age: The birth probabilities we apply in our simulations derive from computations by Tomas Sobotka on the basis of data provided by Statistik Austria. We apply the corresponding data from 1991 and 2001.

Educational Transition Rate by Age: The age-specific transition rates for educational groups are based on period measures. We alternatively start from the age and educational structure of the population in 1991, or 2001 and denote $F(x,e)$ the number of agents at age x and with educational level e. For each age group we build the share of females having primary or lower secondary, upper secondary and tertiary education:

$$f(x,e) = \frac{F(x,e)}{\sum_e F(x,e)} .$$

By working with shares instead of absolute values we control for different cohort size. We then presume that the age and educational structure of the population stays constant over time and build the age-specific transition rates as follows:

$$t(x,e) = \frac{f(x+1,e+1) - f(x,e+1)}{f(x,e)} ,$$

where $t(x,e)$ indicates the transition rate at age x from the educational level e to level e+1 in the next time step.

Age at First Birth by Education: We use data on age at first birth taking into account the mothers' level of education from the census 1991 or 2001. Since these data are only provided for five year age groups we interpolate the data with piecewise cubic hermite polynomials.

9 Source: Statistik Austria (2005a), Table 8.7.

10 Sources: Statistik Austria (1994), Table 14, Statistik Austria (2004), Table 15.

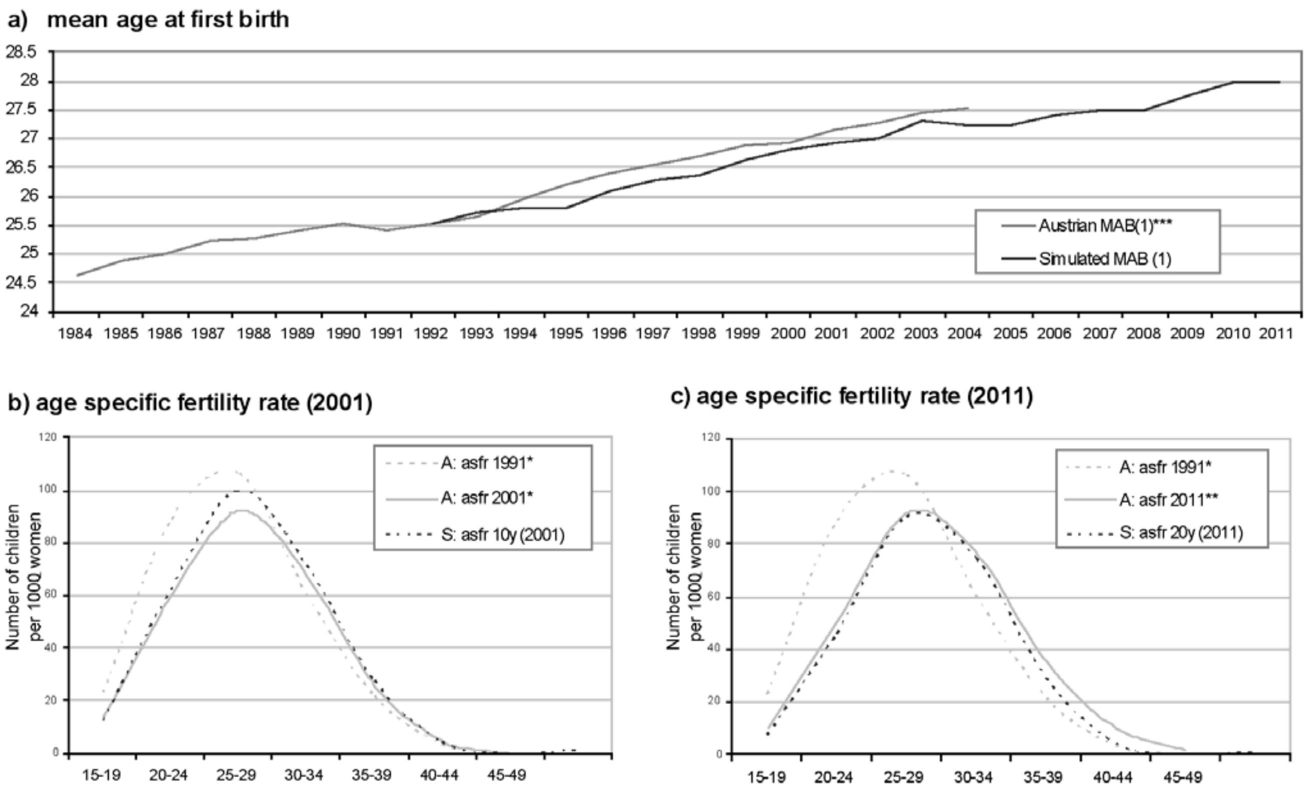
11 Sources: Statistik Austria (1996), Table 48, Statistik Austria (2005c), Table 47
Analogous as for the distribution by age and education we pool the eight educational groups to 3 groups.

Sex Ratio at Birth: Since we do not include male agents to our model, we need the sex ratio at birth to calculate the number of new agents per simulation step. We again use Austrian data[12] of the particular base year for this purpose.

4. Simulation results

In this section we discuss the results from simulations of the agent-based model introduced in the previous sections. We set the population size equal to $N = 6000$ and present the average over 200 simulation runs. In a first experiment we initialize the model with Austrian data from 1991 and run our simulation for 20 years up to 2011. A thorough sensitivity analysis indicates, that we obtain the best fit to actual data by postulating an asymmetry in the functional form of the social influence for the 1990s. Thus we add an asymmetry of 30% (see Figure 1) during this decade and an asymmetry of 60% from 2000 onwards. The social influence on fertility behavior is therefore amplified if *rop>ROP* and dampened for *rop<ROP*. Model parameters are assigned the following values: We set the group size of the hierarchy equal to 5 individuals ($g = 5$), the branching ratio b equal to 2 and postulate an average size of the network of 10 peers ($s = 10$, Fliegenschnee, personal communication, 2006). Results for this experiment – both historical developments (from 1991 to 2001) and some first projections (to 2011) – are summarized in Figure 2. The mean age at first birth (Figure 2a) depicts an increasing trend and validates the promising performance of our proposed model though the empirically observed line is slightly above the simulated one as caused by a bend in the early nineties which is not replicated by our model.

Figure 2 Simulation results for simulating 20 years starting from 1991



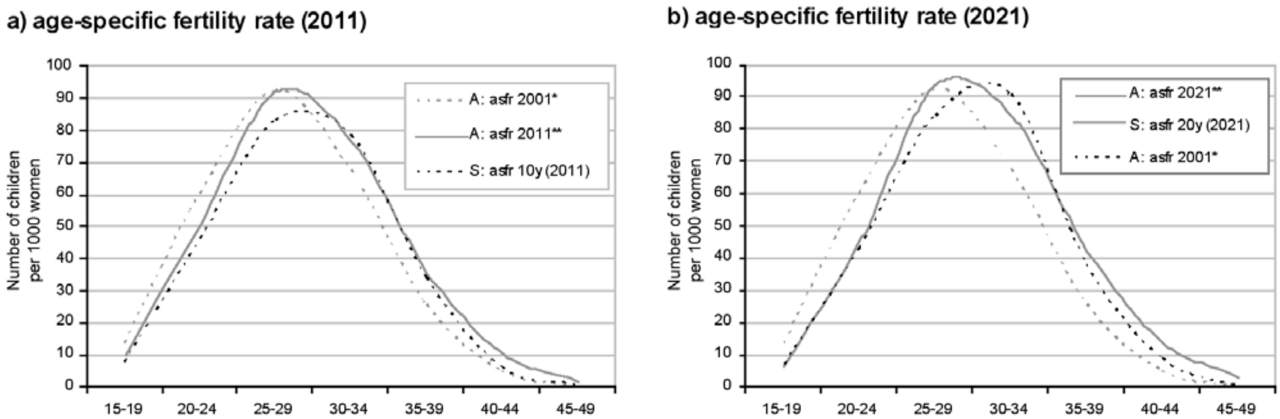
After the first 10 years of the simulations, the probability of first birth comes close to the empirically observed curve in 2001 (Figure 2b). Further simulations for another 10 years yields the first birth probabilities in 2011 (Figure 2c). Since similar data are not available from Statistik Austria we compare the 2011 time series of first birth probabilities with the corresponding (last obtainable) empirical data from 2005. The evolution of age-specific fertility rates (empirically observed and projected ones by Statistik Austria for 2011 as well as simulated ones) is presented in Figure 2a and Figure

[12] Statistik Austria (2005b), Table 2.26.

2b. Simulated fertility rates for 2001 (Figure 2a) overestimate the empirically observed rates (particularly for ages between 25 to 29). This difference can be explained by the extremely low fertility rate in Austria during 2001 as caused by a change in family policies (introduction of new child benefits in the following year). In 2001 the total fertility rate reached a low of 1.33 as compared to 1.36 in 2000 and 1.39 in 2002. The latter Figures are closer to the simulated fertility rate of 1.39 for the year 2001.

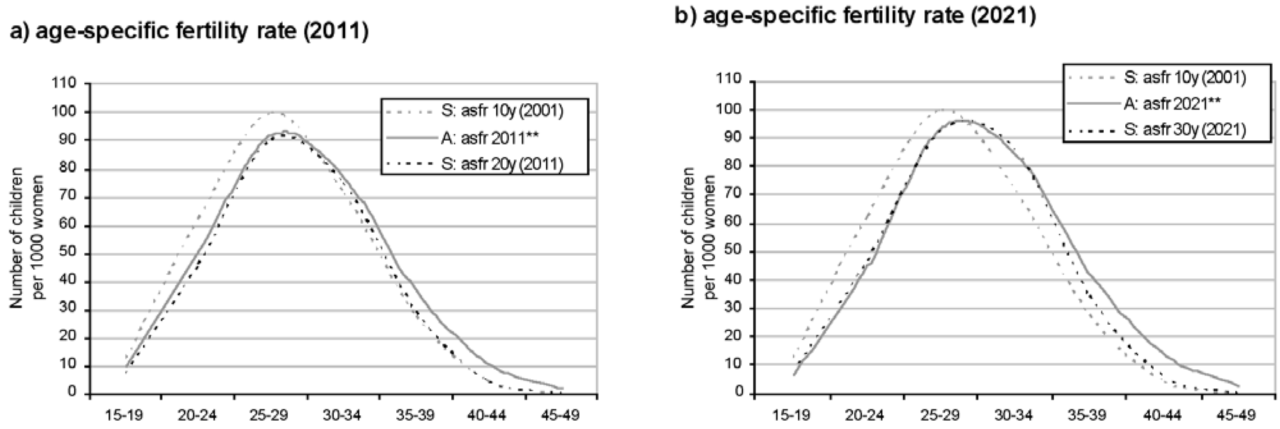
So far we have demonstrated that our model is capable to reproduce changes in the timing of fertility that occurred during the last decades. Next we apply our model to project future trends of fertility and compare our projections to the age-specific fertility assumptions applied by Statistik Austria for their recent population projection (Hanika, 2006). While population forecasts are usually based on time series extrapolation of recent fertility trends combined with some expert knowledge, our approach has a theoretical foundation. We use a causal model to explain trends in timing of fertility rather than continuing existing trends. Sanderson (1998) argues that combining forecasts from such models with standard forecasts results in more accurate predictions if the forecast errors of the two different approaches are not highly correlated.

Figure 3 Simulation results for simulating 20 years starting from 2001



Starting from the year 2001, we forecast fertility rates to 2021. We retain the model parameters as in previous simulations and postulate an increase in the asymmetry of the social influence from 30% prior to 2010 to 90% from 2010 onwards. Figure 3 compares simulated age-specific fertility rates and those assumed by Statistic Austria for 2011 and 2021 respectively. The simulated rates for 2011 (Figure 3a) are considerably lower compared to the assumptions by Statistic Austria. This underestimation of fertility rates in our simulations is mainly caused by the exceptionally low fertility rates in Austria in 2001, which is the base year of this simulation. The relatively low birth probabilities of 2001, especially for the age group 25 to 29, are passed on through the whole simulation implying also the quite low fertility rate in 2021 (see Figure 3b).

Figure 4 Simulation results for 30 years starting from 1991



As we consider the year 2001 not to be an appropriate base year – due to its exceptionally low fertility rates – we project fertility rates for the years 2011 and 2021 again using 1991 instead of 2001 as the base year. The results are depicted in Figure 4. The shape of the age-specific fertility rate as projected by our simulations for 2011 and 2021 is rather similar as the corresponding rates postulated by Statistik Austria with one exception. Fertility rates at higher ages (above age 40) are projected to be lower in our simulations as compared to the assumptions underlying the projections by Statitik Austria. The reason for this difference is the assumption of Statistik Austria that currently low fertility at younger ages (postponement) will be partially offset by a higher fertility rate at higher ages (recuperation) in the future. Yet, there is no empirical evidence indicating to what extent recuperation indeed will take place and, consequently, our model solely based on age- and parity-specific fertility rates of the past and mechanisms of social networks and social learning is not designed to capture expert opinions.

Figure 5 depicts age specific fertility rates in 2021 and Figure 6 illustrates the time trend of total fertiliy rate from 1991 to 2021. Both graphs are based on 542 simulations with a population size of N = 12000 agents initialized with data from 1991. Again a comparison with the projection of Statistik Austria is provided. In addition to Figures 2, 3, and 4, plotting the mean values of series of simulations, Figures 5 and 6 show the median value and the confidence intervals comprising 50%, 75%, and 95% of all simulations. Therefore, these graphs do not provide just one projection but also confidence intervals indicating the bandwith of uncertainty. Comparing the median value (labeled 0.500 in the legend) in Figure 5 with the standard projection again reveals a rather strong correspondence for young ages but a slight deviation beginning at the age of 35 and becoming more pronounced for the 40+ age groups. Within the age interval from 40 to 42 the line indicating the standard projection roughly coincides with the 75% confidence interval but for age groups above 42 it departures even further.

Figure 5 Age-specific fertility rate (births per 1000 women) in 2021

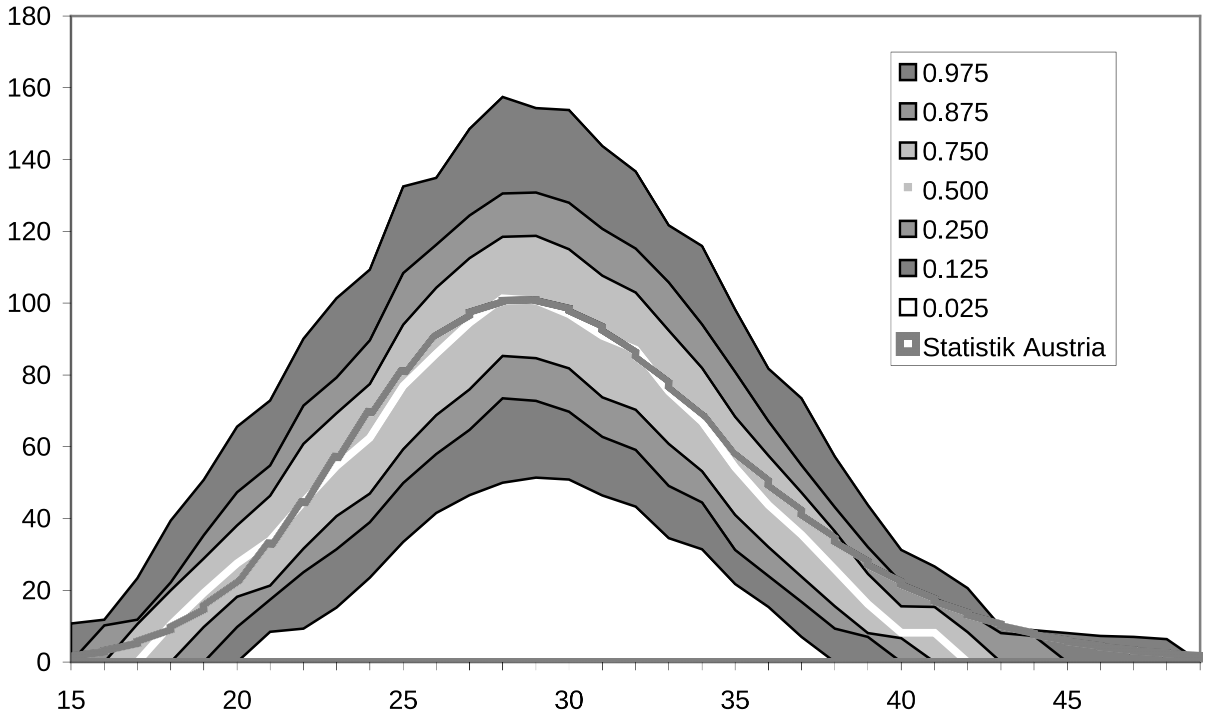
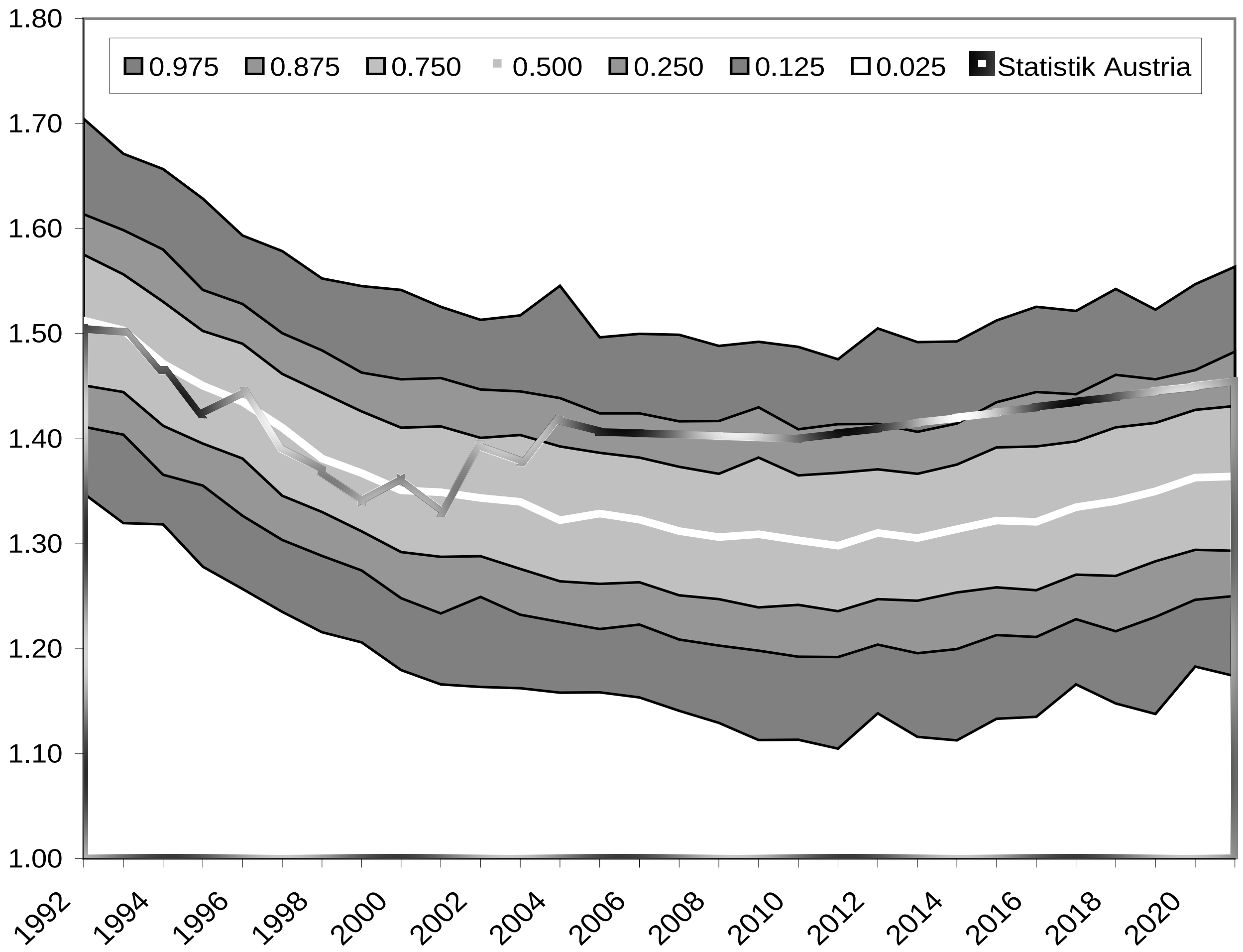


Figure 6 exhibits an apparent agreement for the time horizon up to 2002 but a pronounced departure starting in 2003. This is again due to the assumption of recuperation which is based on expert knowledge but cannot and should not be captured within a causal model of fertility. Nevertheless, the deviation in Figure 6 stays within the 75% confidence interval for most of the time.

Figure 6 Total fertility rate 1991 - 2021



5. Conclusions

As recently shown by various authors (Kohler et al. 2002, Bernardi 2003) social learning and social influence play an increasing role in demographic explanations of observed family formation patterns also in contemporary Europe. The increasing inclusion of social interaction in the demographic theoretical framework however matches with a relatively unrealistic model of the mechanisms underlying those social interactions.

We propose to apply the methodology of agent-based models (ABMs) to study the role of social interaction for explaining observed demographic patterns. Such models allow "thought experiments that explore plausible mechanisms that may underlie observed patterns" (Macy and Willer 2002, p.147). Different to micro or macro simulations ABMS provide a theoretical bridge between the micro and macro level. The dynamic bottom up approach of ABMs – to explain global patterns by simple local interactions – is particularly useful when aiming to explain trends in fertility timing and quantum over the last decades.

Billari et al. (2007) developed a model taking into account the impact of social interactions within an individuals' peer group on her decision to get married. In this paper we present an ABM including endogenous formation of social networks.

This model has the ability to capture the impact of social interaction – via peer groups – on the transition to parenthood. Calibrating our model to Austrian data we show that our model captures the observed changes in the timing and quantum of fertility over the last three decades to a high degree. Hence, one might argue that social interactions and their influence on childbearing decisions may be one driving force explaining recent fertility transitions.

Moreover, this model can also be used to project future trends of age- and education-specific fertility patterns. Thus, we apply our model to forecasts age-specific fertility rates in Austria for the next two decades. Different to common practice in population forecasts that are usually based extrapolations of past fertility trends combined with expert opinions, the agent based approach has a theoretical foundation. Our approach explains trends in timing and quantum of fertility by social interactions within an endogenous social network rather than continuing existing trends. The underlying network topology is based on a sound sociological foundation. Sanderson (1998) argues that combining model-based forecasts including knowledge of the socio-economic determinants of population change with standard forecasts results in more accurate predictions if the forecast errors of the two different approaches are not highly correlated. Moreover, since an agent based model is per se probabilistic, this approach can also be used for probabilistic projections.

The next step is to apply our model to different European countries and test its validity. Within the framework of our ABM we can experiment with alternative mechanisms that may underly the timing and quantum of fertility in different social environments. The exploration of plausible mechanisms that underlie observed patterns is the main challenge demographers are confronted with in order to propose efficient explanations of past trends and provide reliable projections of future demographic developments. To demonstrate the feasibility of such an approach–by applying it to the topic of the transition to parenthood–is the main aim of the current paper.

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Joint Eurostat/UNECE Work Session on Demographic Projections
Organized in cooperation with the National Institute of Statistics of Romania (INSSE)

TRENDS IN PARTNERSHIP BEHAVIOURS IN JAPAN FROM THE COHORT PERSPECTIVE

Bucharest, 10-12 October 2007

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1. Introduction

Assumptions about the future age-specific fertility rates required for population projections can be obtained using the cohort fertility method. With this method, we predict the average completed family size of younger cohorts based on the actual birth process of preceding cohorts. Since childbearing behaviour is affected by family formation and dissolution, it is essential to consider these processes in the construction and assessment of future fertility assumptions. Results in this paper are based on the preliminary analyses for producing official population projections for Japan conducted by the National Institute of Population and Social Security Research (NIPSSR 2007, Kaneko 2007).

In this paper, we describe patterns of partnership formation and dissolution from a birth cohort perspective. Recognizing that declining exposure to marriage may place a strong structural restriction on childbearing, we then examine the extent to which these behavioural changes contribute to fertility decline, by cohort. In addition to marriage, divorce, widowhood, and remarriage may also be significant factors for fertility.

However, in most developed countries, the link between marriage and fertility has been weakening. We also look into the trends in new patterns of family formation: cohabitation, non-marital fertility, and marriage preceded by pregnancy. We provide cohort indices for cohorts born from 1935 to 1990, incorporating some estimation for cohorts born after 1956.

2. Data

We use Vital Statistics data for calculation of fertility, marriage, and divorce rates. We also use Japanese National Fertility Surveys, conducted by NIPSSR every five years, for estimating average family size by marital status.

Since roughly 10% of marriages and 30% of divorces are not registered in the year in which they occur (Ishikawa 1995), we estimated the number of marriage and divorces in the year they actually occurred using ratios of delayed to on-time registration obtained from the observed data.

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Due to the recent increase in non-Japanese residents and the relatively poor data on them, trends in indices among all residents in Japan including non-Japanese could be unstable and difficult to understand. We therefore focus on events in the lives of Japanese female residents and look into indices for those people.

3. Trends in marriage

We will focus on the relationship between fertility trends and changes in marriage, divorce, widowhood and remarriage. In Table 1 shown in a UN report, low-fertility regions are classified according to four characteristics of nuptiality: age at first marriage, prevalence of marriage in the prime reproductive ages, prevalence of cohabitation, and prevalence of union dissolution. Characteristics of Eastern Asia and Southern Europe, such as high age at first marriage, and low prevalence of both marriage and cohabitation, seem to have the most negative impact on fertility. However, it has been suggested that in Japan the occurrence of divorce is actually higher than expected. Therefore, Japan might be moving into a more unfavorable situation for childbearing than ever before. We will return to the quantifiable change in divorce later.

Postponement of family formation, which has been widely witnessed in most industrial countries since the second half of the 20th century (Billari 2005), can be also seen in Japan. The mean ages at first marriage and first childbearing have risen dramatically since the late 1970s, and total fertility rates and total first marriage rates (the sum of age-specific first marriage rates for ages 15 to 49) continue declining (Figure 1, Figure 2).

To obtain data on cohort age-specific first marriage rates for those who have not reached the end of their reproductive ages, we applied a generalized log-gamma distribution model presented by Kaneko (2003) to the actual values for the cohort age-specific first marriage rates. Because future trends are highly uncertain, we work with three sets of assumptions (medium, high, and low).

The mean age at first marriage and the proportion of those never married has risen for cohorts born after 1950. We show the cumulative first marriage rates relative to those of the 1950 birth cohort, using the scheme provided by Frejka and Calot (2001). For recent birth cohorts, a decline from the base cohort observed in the 20s will not be entirely made up in the 30s (Figure 3, Figure 4).

4. Trends in Divorce and Marital Status Composition

Divorce rates in Japan have risen since the 1960s, and the total divorce rate (sum of age-specific divorce rates for ages between 15 and 49) is beyond 0.25 in the 2000s (Figure 5). We tried to calculate the cohort index on divorce experience: that is, the proportion of those who experience divorce at least once among women with marriage experience, by age and birth cohort. For future values, we made three assumptions. Medium values were produced with the assumption that the average trends in the past three years will continue. The lowest combination of rates over the last 10 years was used for the high assumption, and the highest combination over the last 10 years was used for the low assumption.

According to the medium assumption (synthetic cohort projection), 36 percent of first-married women in the 1990 birth cohort will eventually experience divorce by age 50 (Figure 6). This is consistent with the results on the proportion of divorce experience by marriage duration and marriage cohort calculated by Raymo et al. (2005), projecting that at least one third of marriages in 2002 may end in divorce within 20 years. Figure 7 shows that the occurrence of divorce in Japan more closely resembles Central Europe rather than Southern Europe.

Based on these trends in divorce experience and the future trends in the proportion of women never married by the age of 50 and the occurrence of remarriage estimated from National Fertility Survey data, we obtained the actual and assumed composition of marital status of women at age 50 by birth cohort from 1930 to 1990. We can see that first-marriage couples were in the majority up to the 1950s birth cohort, but due to the increase in women without marriage experience and divorced women, the proportion of first-marriage couples has been decreasing, and will eventually be around 50 percent (Figure 8).

While divorce rates have been increasing, widowhood has been declining due to the decline in male mortality rates. Widowhood is expected to be increasingly rare for recent birth cohorts.

5. Impact on cohort fertility

What impact does the change in partnership behaviour - declining marriage rates and increasing divorce rates – have on fertility? We measured the contribution of both factors using models for cohort TFR.

We use a mathematical model in which cohort completed fertility consist of its segments from the factors, i.e., marriage, divorce, and couple's reproductive behavior with in marriage. With the model, the cohort cumulative fertility rate at age 50 (CTFR) is expressed as;

$$\begin{aligned} CTFR &= (1-\gamma)\cdot CEB\cdot\delta \\ &= (1-\gamma)\cdot\left(CEB^{*}(afm)\cdot\kappa\right)\delta . \end{aligned}$$

Here, γ is the proportion of never-married women at age 50 (one minus cumulative first marriage rate), the CEB is the average completed number of children of women in the first-marriage couples, and δ is the coefficient of the divorce and widowhood effects. As the second line of the equation indicates, the CEB can be broken down into the expected cumulative number of births (CEB^*) that is dependent of the age pattern of first marriage (denoted as *afm*) and κ, a coefficient that represents deviation of marital reproductive behavior from the expected pattern derived from the previous standard cohorts.

Change in γ and the age pattern of first marriage (*afm*) reflect behavioural change in first marriage, change in δ indicates behavioural change in divorce, and change in κ reflects changes in couples' reproductive behaviour after marriage.

By calculating the counterfactual CTFR with unchanged coefficients, we can see the contribution of each behavioral change on the CTFR compared with the medium assumptions for the projection.

CEB^* requires a standard pattern for the completed number of children, by age at first marriage. We obtained this from the average pattern of 1932 - 1965 birth cohorts using the 7th through the 13th Japanese National Fertility Surveys (Figure 9).

Figure 10 shows the three simulated CTFRs. The first line is based on the assumption of no change in γ, *afm*, κ, or δ, since the 1955 birth cohort, while the second curve provides the results where only γ and *afm*, (i.e., marriage behavior), have changed. The third curve uses the medium assumption with changes in all coefficients. Changes in marital behaviour explain 73 percent of the difference in family size for the cohorts born in 1950 and 1990.

The value of δ was set in the following manner. Using data from the NFS, we calculated the completed number of children of women with marriage experience by four marital status categories: first-marriage couples (*ff*), couples with a first-married wife and a remarried husband (*fr*), couples with a remarried wife (*r.*), and divorced/widowed women (*dw*) (Figure 11). We can obtain the indices for each marital status category relative to the average number of children for first-marriage couples (*R..*). In the previous section, we generated the predicted composition of female marital status at age 50 (*P..*) by birth cohort. δ is the weighted mean of *R..* with *P..* as weights, as defined as below.

$$\delta = \left\{P_{ff} + P_{fr}R_{fr} + P_{r\cdot}R_{r\cdot} + P_{dw}R_{dw}\right\}/(1-\gamma) .$$

δ is represented as the function of the proportion of those who experience divorce by age 50 (Figure 12).

Cumulative divorce rates among first-married women have been increasing across cohorts (Figure 6). Based on a synthetic cohort projection, 36 percent of first-married women will eventually experience divorce by age 50 in the 1990 birth cohort. Therefore, the valve of δ in this cohort is 0.925.

Using the variable δ, we can obtain a CTFR without the divorce, widowhood and remarriage effects or one with δ held constant since the 1955 birth cohort (Figure 13).

However, since widowhood has been declining and some proportion of divorced women get remarried, the net contribution of the change in divorce, widowhood and remarriage to cohort fertility decline between 1950 and 1990 is about 3%.

6. New patterns of family formation: Cohabitation, nonmarital childbearing, and marriage preceded by pregnancy

Do these changes in partnership formation and dissolution mean the emergence of new patterns of family formation? Here we show some aspects of change regarding new patterns of family formation recently observed in Japan.

In the context of the second demographic transition, novel patterns of family formation, such as cohabitation and extramarital childbearing, were once considered to be related to fertility decline to below replacement level. However, very low levels of these behaviours are now commonly found among the lowest-low fertility countries. The visibility of cohabitation remains low in Japan but is clearly becoming an increasingly common part of the union formation process (Figure 14, Figure 15). There is, however, substantial variation around the median duration of 15 months for the most recent cohabitating unions, with one-fifth of the cohabitations lasting under six months and roughly one-third lasting two years or more. Experience of cohabitation is associated with a rapid transition to parenthood through premarital pregnancy (especially among those at the lower end of the educational distribution), but cohabitation experience delays the first birth beyond age 25 for women with higher education.

The fertility of unmarried women is still very low in Japan, but has increased slightly since the 1990s. The projected lifetime unmarried fertility rate of the 1990 cohort is nearly .03 children; two percent of their cohort TFR. Compared with the pattern of age-specific nonmarital fertility rates in 1990, the pattern in 2005 shows a dramatic increase for women under age 25 (Figure 16). This change might be in common with some Anglo-Saxon countries such as the UK or the US, where the birth rates among unwed young mothers contribute to relatively high fertility. When we consider the impact of these changes on future fertility, we need to examine whether these people remain as single mothers or move into marital relationships and continue to reproduce as members of the latter group.

Another notable behavioural change is an increase in marriages preceded by pregnancy (MPP). Based on extrapolative projections, the proportion of MPP to first-married women will rise to over 20% in the 1990 cohort, while it was less than 5% among cohorts born prior to 1950. The increase in childbearing of unmarried women is concentrated among teenagers, and the MPP is concentrated among women in their early 20s. These behaviours seem to be related to the trends in use of contraceptives and unintended pregnancy.

7. Conclusion

Fertility assumptions for the latest population projections for Japan based on the 2005 census suggests the extremely low level of fertility– in 2030 and after, the medium variant TFR for Japanese women is assumed to be 1.20. These prospects were led by drastic changes in the patterns of family formation and dissolution. Among the 1990 birth cohort, the mean age at first marriage is 28.2, the proportion of never-married women at age 50 grows to 23.5%, and 36% of first-married women will eventually experience divorce.

Counterfactual CTFRs with variant patterns of family formation and dissolution have shown that over 70% of the CTFR decline is attributed to a decline in marriage rates. The contribution of increase in divorce rate on CTFR's reduction would be 3% in the 1990 birth cohort according to calculation of the counterfactual value if divorce behaviour remained unchanged since the 1955 birth cohort.

Developed countries with relatively high fertility rates show relatively high levels of unmarried couples cohabiting and childbearing at young ages. The visibility of cohabitation and childbearing of unmarried couples is still low in Japan, but among cohorts born in the 1980s and later, these new patterns of family formation have been increasing. Since these changes could lead to a rise in fertility rates for women in their 20s in the near future, we need to pay attention to these trends.

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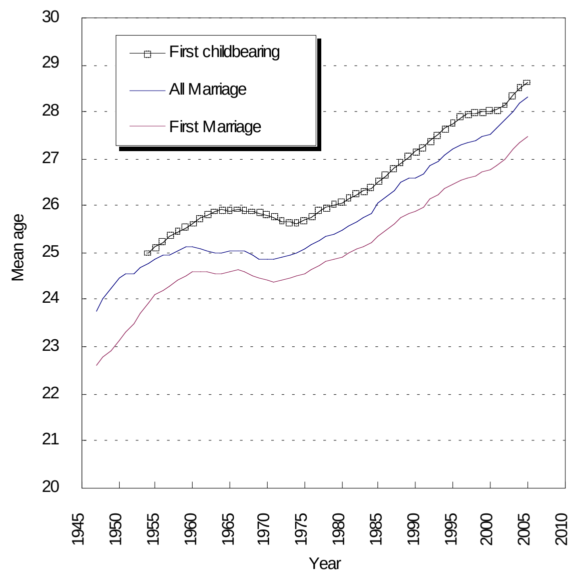
Table 1 Low-fertility regions by selected partnership characteristics

Age at first marriage	Prevalence of			Region
	Marriage	Cohabitation	Divorce	
Low	High	High	Low	
			High	
		Low	Low	Eastern Europe (bu, pl, ro yu)
			High	Eastern Europe (cz,hu,it,md,ru)
	Low	High	Low	
			High	
		Low	Low	
			High	
High	High	High	Low	
			High	
		Low	Low	
			High	
	Low	High	Low	
			High	Nothern Europe, Western Europe, Nothern America, Australia/New Zealand
		Low	Low	Eastern Asia, Southern Europe
			High	Japan?

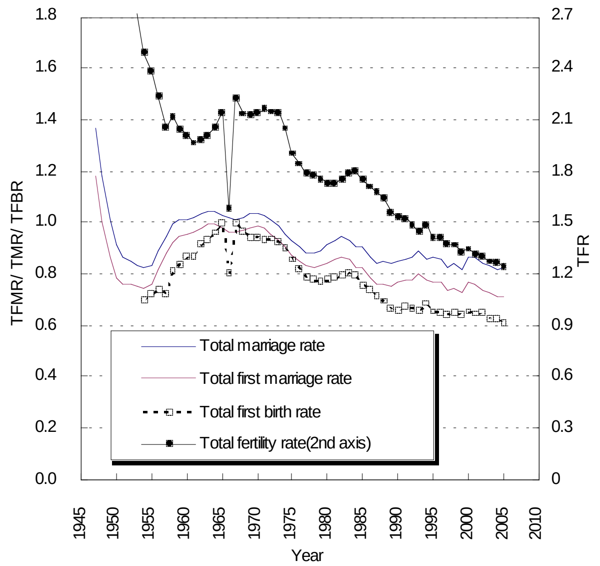
Source: UNPD (2003) (rearranged by the authors).

Figure 1 Mean age at first marriage, all marriage, and first childbearing among Japanese women



Data: Vital Statistics in Japan (own calculations based on age-specific marriage rates and fertility rates of Japanese women).

Figure 2 Total fertility rates, total (first) marriage rates, and total first birth rates among Japanese women



Data: Vital Statistics in Japan (own calculations based on age-specific marriage rates and fertility rates of Japanese women).

Figure 3 Cumulative age-specific first marriage rates: Actual values and assumptions birth cohort 1935-1990

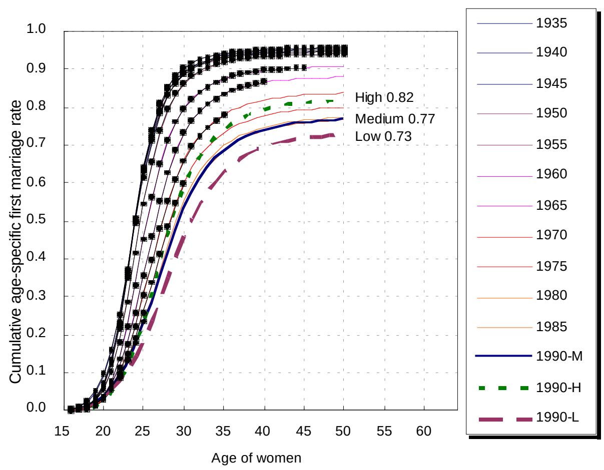


Figure 4 Differences in cumulative cohort first marriage rates between base and subsequent cohort

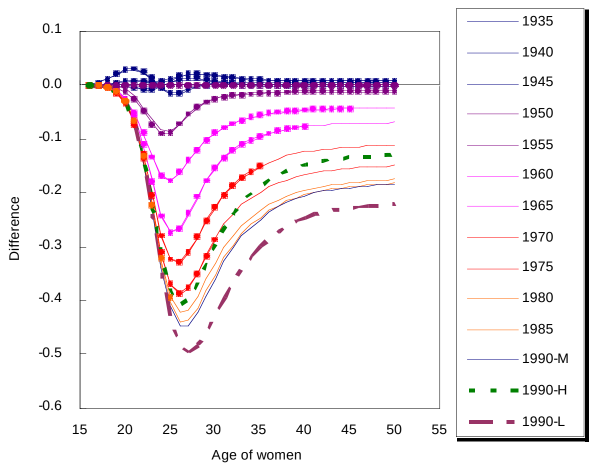
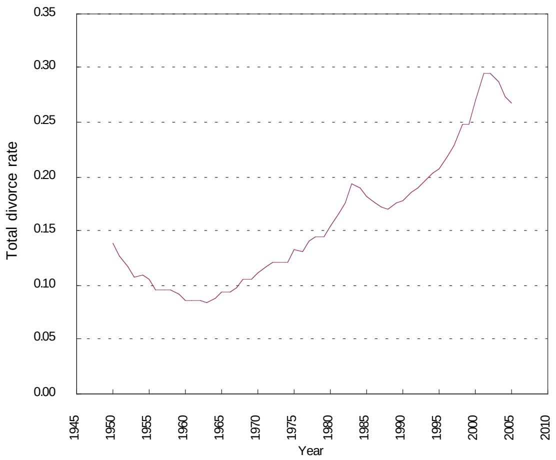


Figure 5 Total divorce rates among Japanese Women



Data: Vital Statistics in Japan (own calculations based on age-specific divorce rates of Japanese women).

Figure 6 Cumulative divorce rates among women who have been married at least once: Actual values and assumptions, birth cohort 1935-1990

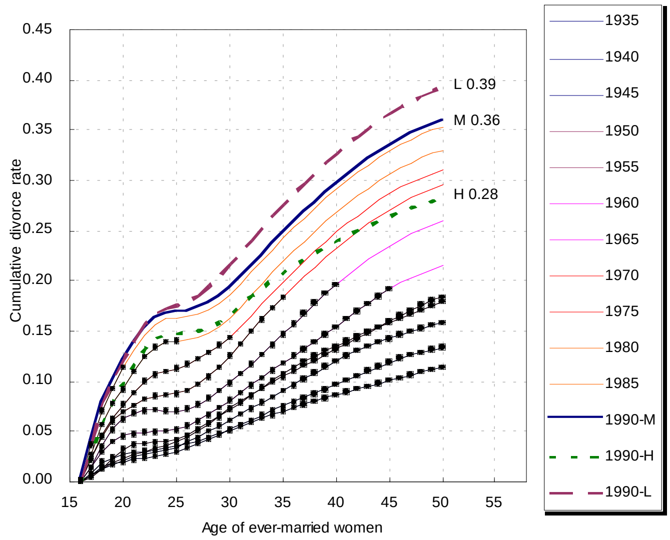
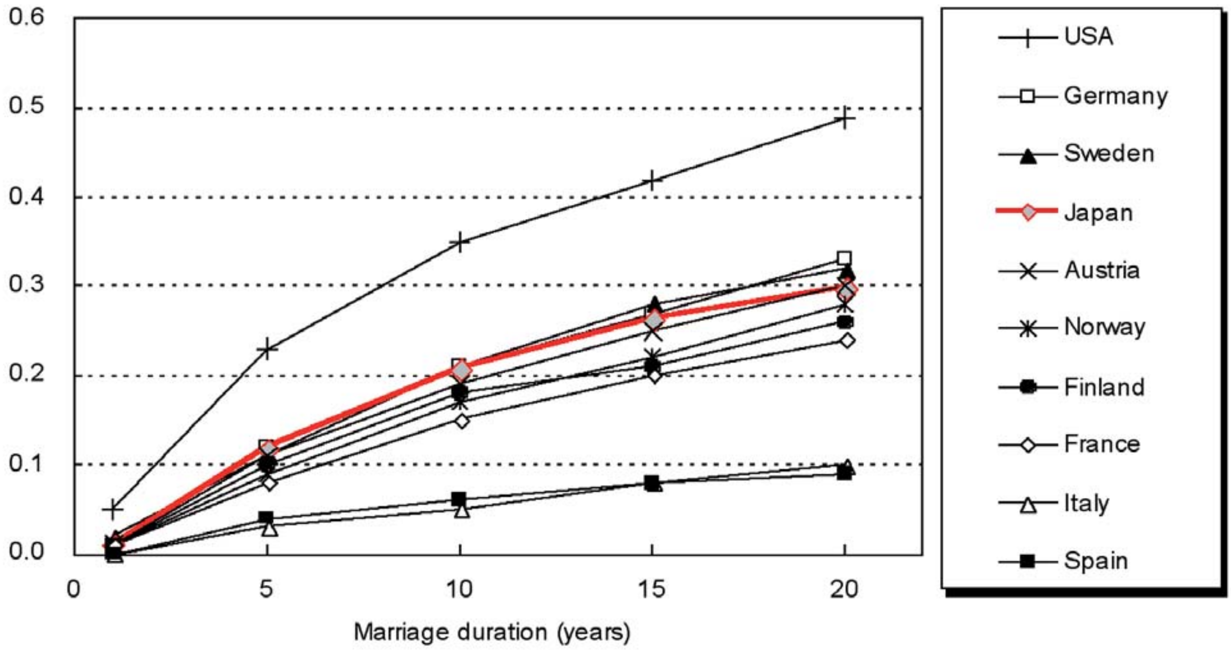


Figure 7 Cumulative probality of marital dissolution by marriage duration (Life table estimates). Selected countries



Source: For Japan, Raymo, Iwasawa, and Bumpass (2005, 2006). For others, Andersson and Philipov (2001)

Figure 8 Marital status of women at age 50: Actual values and medium assumptions, birth cohort 1930-1990

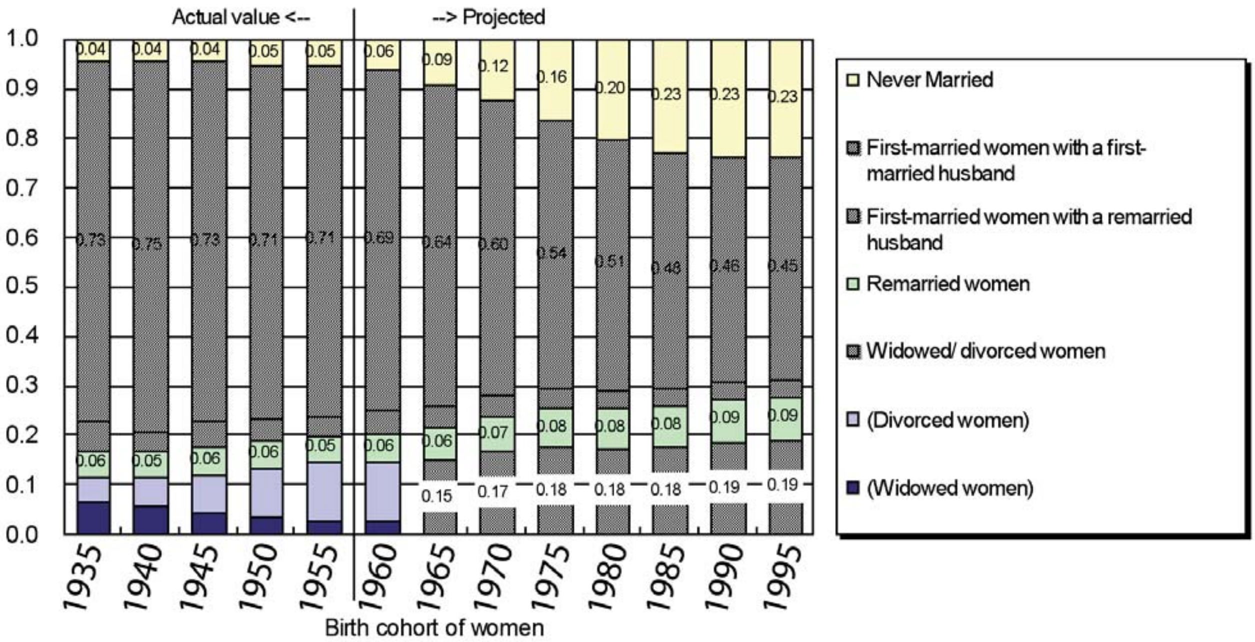
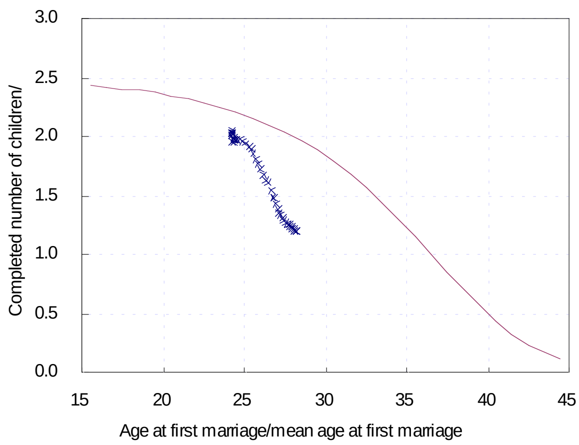


Figure 9 Completed number of children of first married couples by wife's age at first marriage and CTFRs by mean age at first marriage



Data: Women aged 40-49 in the 7th -13th Japanese National Fertility Survey (NIPSSR).

Figure 10 Simulated results with respect to marital behavior and medium assumptions for cohort total fertility, rates, birth cohort 1935-1990

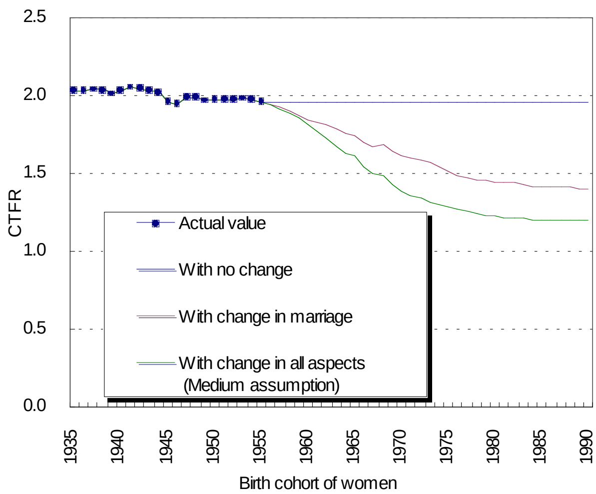
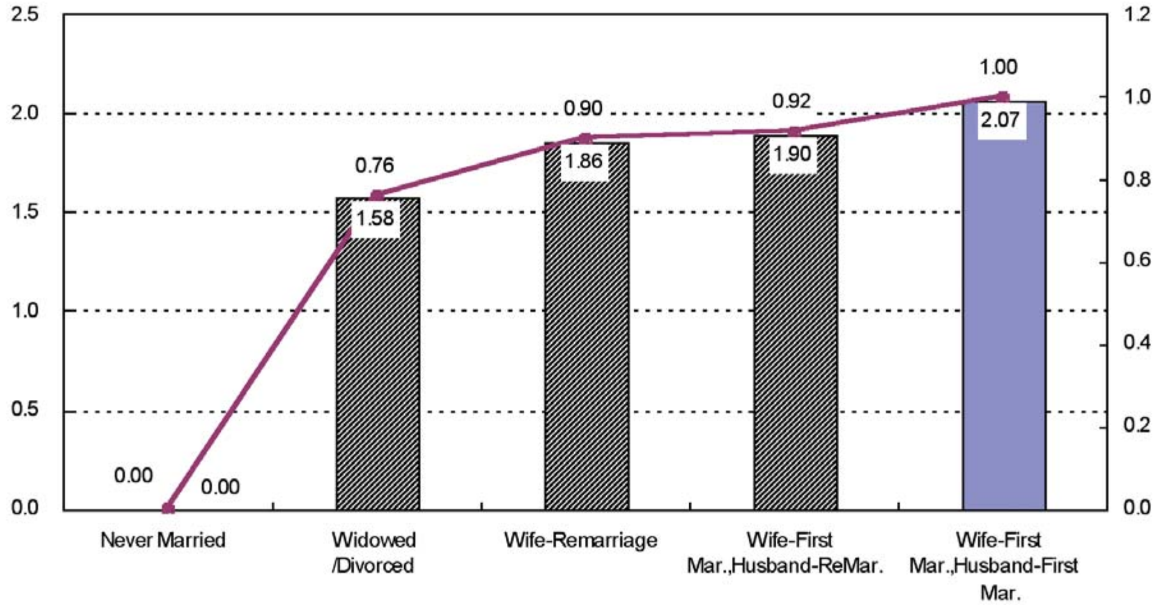


Figure 11 Completed number of children by marital status and relative ratio to first marriage couples



Data: Women aged 40-49 in the 13th Japanese National Fertility Survey (2005).

Figure 12 Association between the proportion of women with divorce experience at age 50 and the...

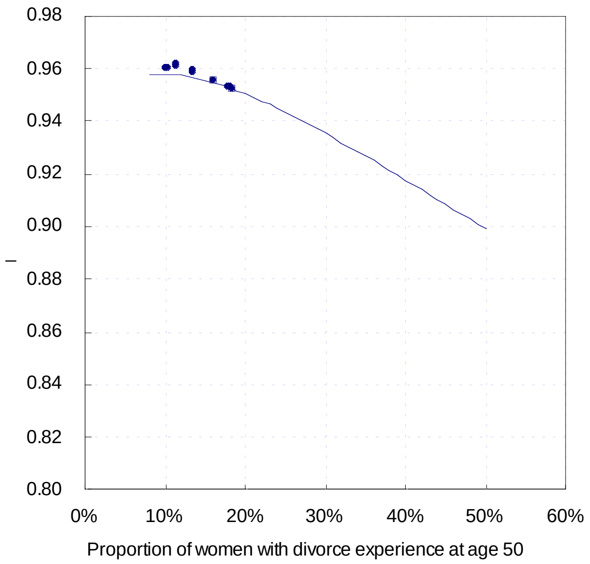


Figure 13 Simulated results with respect to divorce behavior and medium assumptions for cohort total fertility, rates by birth cohort of women

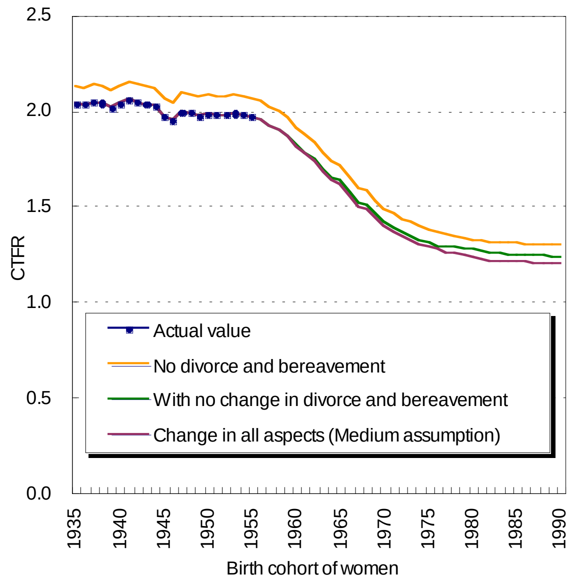
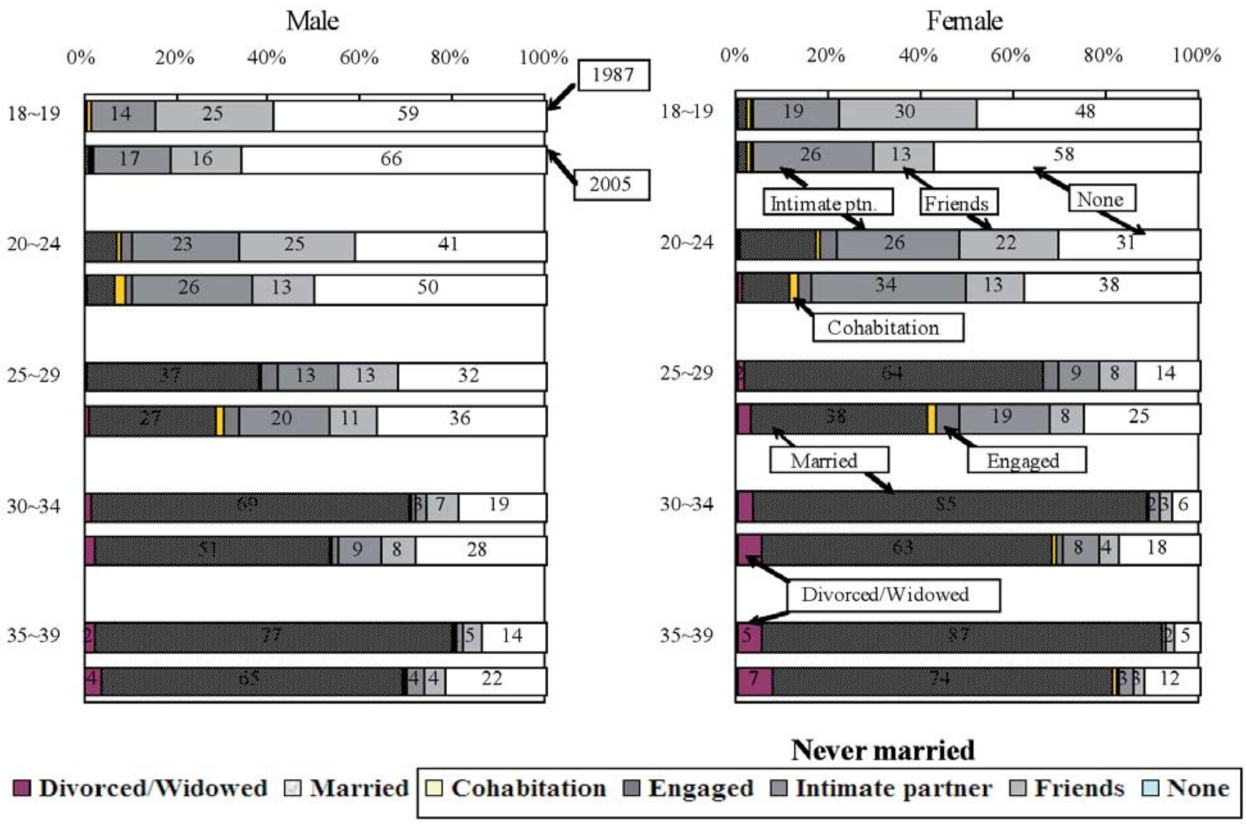


Figure 14 Partnership status by age group in 1987 and 2005



Data: Japanese National Fertility Survey in 1987, 2005 (NIPSSR)

Figure 15 Proportion of women who have ever cohabited: 1998 and 2004

Source: Own calculations from the 24th National Survey on Family Planning (1998) and the 1st SPFG(2004) conducted by the Population Problems Research Council, the Mainichi Newspapers (Iwasawa 2005).

Figure 16 Age-specific non-marital fertility rates: 1990 and 2005

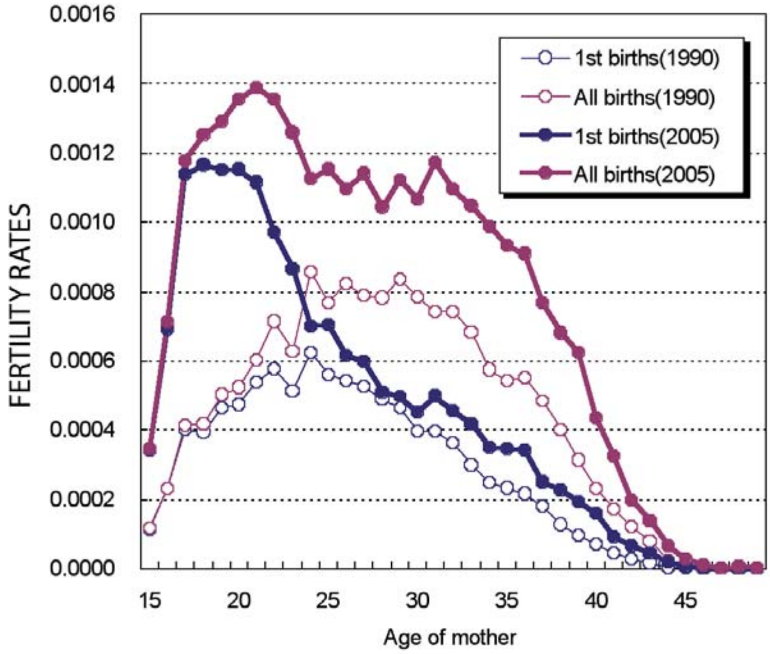


Table 2 Cohort indices based on medium variant assumptions for future fertility trends

Cohort Index (Japanese Women)		Women's birth cohort										
		1955	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005
Proportion never-married		5.8%	9.3%	12.0%	16.2%	20.0%	22.6%	23.3%	23.5%	23.6%	23.6%	23.6%
Mean age at first marriage		24.9	25.7	26.5	27.0	27.5	27.9	28.1	28.2	28.2	28.3	28.3
CTFR		1.96	1.81	1.61	1.39	1.28	1.23	1.21	1.20	1.20	1.20	1.20
Completed number of children of first married couples		2.16	2.06	1.93	1.84	1.78	1.74	1.71	1.70	1.69	1.69	1.69
Distribution	Childless	12.7%	17.5%	22.7%	30.0%	34.3%	36.4%	37.4%	37.4%	37.4%	37.5%	37.5%
	One	11.8%	13.8%	16.9%	19.0%	18.9%	18.3%	18.1%	18.2%	18.2%	18.2%	18.2%
	Two	47.1%	43.5%	40.8%	36.0%	33.9%	33.4%	33.1%	33.1%	33.1%	33.1%	33.2%
	Theree	23.4%	20.5%	15.8%	11.8%	10.2%	9.5%	9.4%	9.4%	9.4%	9.4%	9.4%
	Four or more	5.0%	4.7%	3.9%	3.3%	2.7%	2.3%	2.1%	1.9%	1.8%	1.8%	1.8%
Mean age at childbearing	All	28.2	28.7	29.3	29.7	30.0	30.2	30.3	30.3	30.3	30.3	30.3
	1st	26.3	27.0	27.8	28.4	28.7	29.0	29.1	29.1	29.1	29.1	29.1
	2nd	28.8	29.4	30.1	30.5	30.9	31.0	31.1	31.1	31.2	31.2	31.2
	3rd	31.3	31.6	32.0	32.3	32.6	32.7	32.9	33.0	33.1	33.1	33.1
	4th or more	33.7	34.0	34.3	34.4	34.5	34.6	34.7	34.7	34.7	34.8	34.8

Source: NIPSSR, Population Projection for Japan: 2006-2055 (2007).

AGE PROFILES ESTIMATION FOR FAMILY AND FERTILITY EVENTS BASED ON MICRO DATA: THE MAPLE (METHOD FOR AGE PROFILE LONGITUDINAL ESTIMATION).

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Abstract

Focusing on fertility behaviors and transitions between various living arrangements, in this paper we deal with the methodological background of the age profiles estimation based on longitudinal survey data. The method, that starts with the specification of the required micro-level data, includes a data processing routine that leads to the estimation of regression models. These models permits, for each transition, to evaluate the smoothed age profile and the relative risks given by time-fixed and time-varying covariates. The age profile for the relevant transitions in the field of families and fertility, will represent data to be included as input in the MicMac model[1].

The major advantage of the method developed here is flexibility: it can be applied to every setting where micro-level data on transitions are available from a large-scale representative survey (e.g., Fertility and Family Survey; Generations and Gender) and for different kind of transitions. Moreover, based on regression model the method permits to evaluate confidence intervals and to test hypotheses. The whole method is written in R software and it could be easily recalled as a R function in order to be applied to real data.

Acknowledgments

The data for this analysis have been provided by ISTAT (Italian National Institute of Statistics) and Centraal Bureau voor de Statistiek, The Netherlands.

[1] More information on MicMac is available on the website www.micmac-projections.org.

1. Introduction

The new methodology for population forecasting that is being developed within the MicMac project does not only take into account macro demographic changes but also life course trajectories (see, e.g., Willekens, 2005; van der Gaag et al., 2006). In this framework, the life course is viewed as a sequence of states and events; each event marks a transition from one state to another state. The study of a single transition is based on the estimation of a transition rate (from the original state to the destination state). From the literature on living arrangements and fertility (see, e.g., Billari et al., 2005) we know that transition rates vary with age. Indeed, such variation with age has been traditionally exploited in demographic forecasting.

The present paper proposes a general method for the calculation of age profiles for the main transitions experienced by individuals, as far as living arrangement and fertility behaviours are concerned. We call this method MAPLE (Methof for Age Profile Longitudinal Estimation). Our analytical strategy starts from micro data, on the assumption that such data have to be collected allowing for a longitudinal (at least in a retrospective sense) reconstruction of the life course. This means that our method requires data that allow to reconstruct the biography of the individual and, for this purpose, dates of the most important events must be collected. Longitudinal data can be obtained both from retrospective surveys and from panel surveys. Here, we only refer to the former case. *Retrospective surveys* concerning family and fertility behaviours are rather available for most of European countries (e.g., from the Fertility and Family Survey project, from the Generations and Gender Project and from other data collection ventures based on National Statistical Offices). Retrospective surveys permit to collect a wide range of information relating to the past experience of individuals with limited costs. On the other hand, the attention is limited uniquely to the survivors since we do not have information about deaths nor about emigrated individuals. However, this feature is not necessarily a disadvantage in methodological terms since we consistently reduce the number of events that drives the individual out of the observed sample over time, simplifying the calculation of transition rates. In the literature on demographic microsimulation, biographic information collected in retrospective surveys has often been used (see, e.g., Wachter et al., 1998).

The method can be segmented in four steps. The first step consists in the specification of the required micro-data for the evaluation of the age profiles for all the relevant transitions relating to living arrangement and fertility (see de Beer et al., 2006). In Section 2 we indicate all the information that the starting data files should contain in order to apply the method of calculation. The second step relates to data preparation and includes the computation of ages at various events and the definition of the window of observation that we will consider for each individual. In the third step we specify a transition-specific data matrix: within the window of ages, the events experienced and the time spent in a specific state contribute to the calculation of events and time of exposure for all the individuals in the sample. Starting from a data file where each record is a set of dates and status variables relating to the i-th individual, we obtain a matrix for each transition where the single row refers to a specific age x and containing the number of events and time of exposure. Second and third step are discussed in Section 3. In the last step, presented in Section 4, we analyze how an observed set of events and time of exposure can be modelled by a smoother function, obtaining age profiles. In particular, we develop GAM (Generalized Additive Model) that permits to evaluate the smoothed age profile as transition rate baseline and, at the same time, to estimate the effects of a vector of covariates as multiplicative changes from the baseline. Finally, in Section 5 and 6 the method is applied to Italy and the Netherlands. As an example, we show the results relating to the transition "never married-first marriage".

Figure 1 Organization of the method

First step

Data requirement (specification of the required micro-data)

Second step

Data preparation (computation of ages; definition of the observation window)

Third step

Transition-specific data matrix

Fourth step

GAM models and relative risks

The data used in this report are the following:

- the ISTAT survey called "Famiglia e soggetti sociali" (FFS-IT) conducted in Italy at the end of 2003
- the Fertility and Family survey for Netherlands (FFS-NL) conducted between February and May of 2003.

The characteristics of the information contained in these datasets require some specific adaptation to the general indications given in Section 2. Besides, some transitions could not be considered because of missing information in the data set (for example, in both surveys there are no questions concerning the return in the parental home after a temporary exit) or because some transitions are very rare, thus implying a too small number of cases for the method to be correctly applied (see Impicciatore and Billari, 2007 for details). All the passages of the method are developed in R, a very suitable statistical free and open source software.

2. Input data requirements

This section describes the data required for the calculation of age profiles on the transitions between various living arrangements and concerning fertility. Considering micro level data originated from a retrospective survey, in the starting data file we need the individual complete biography concerning family and fertility behavior. More specifically, we assume to have information on the date of birth, the date of the interview and the date of each event experienced within the range considered. All dates are expected to be available on a monthly time scale, i.e. to be expressed in calendar month (format MM: 1 to 12) and year (format YYYY).

Moreover, for each specific transition, we need a *status* variable at the interview that is, for a generic transition *TR*:

- 0 if the individual has never experienced *TR* at the time of the interview
- *1* if the individual experienced *TR* before the interview
- *9* if the case is not applicable, i.e. the individual has never been at risk to experience *TR*.

As an example, we consider the transition *TR*="first child→second child". The status variable is 1 if the second child was born before the interview; is 0 if the individual has only one child; is 9 if the individual is still childless at the interview.

2.1 Marital status

The *state space* for marital status is composed by four states:

- never married
- married
- divorced
- widowed

These categories imply that we shall not consider explicitly the order of marriage. That is, the behavior of persons in their second marriage cannot be distinguished from persons in their first marriage. The qualitative shape of the transition matrix is as follows:

From \ to	Never married	Married	Divorced	Widowed
Never married	O	TR1		
Married		O	TR2	TR3
Divorced		TR4	O	
Widowed		TR5		O

"TRX"   transition between categories
"empty space" impossible event
"O"   non-event

All these transitions are experienced when one of these events (marriage, divorce, death of spouse) occurs. In the starting data file we need dates for these events (if any) and 5 status variables at the interview (we do not consider third or higher marriage):

- date of first marriage
- date of second marriage
- date of divorce
- date of death of spouse
- status for TR1 (first marriage)
- status for TR2 (divorce)
- status for TR3 (death of spouse)
- status for TR4 (second marriage after a divorce)
- status for TR5 (second marriage after death of spouse)

2.2 Living arrangement

The choice of possible states for the living arrangements strongly depends on the characteristics of the country examined. As a general rule (see de Beer et al., 2006), we could consider the following states: living with parents, living without a partner, living with a partner, living with other persons, living in an institution. However, some of these states and/or transitions are relatively rare in most of the country. This is, for example, the case of Italy, where the distinction between living alone (without a partner), with other persons, and in institution cannot be adequately studied. In order to have an adequate number of cases and to give general rules we prefer to aggregate some states. In detail, we consider: living with parents, living with a partner, living alone or with other people. The qualitative shape of the transition matrix is:

From \ to	at parental home	with a partner	alone or with other persons
at parental home	O	TR6	TR7
with a partner	TR8	O	TR9
alone or with other persons	TR10	TR11	O

"TRX" transition between categories
"empty space" impossible event
"O" non-event

We need all dates of the events that cause these transitions and a status variable for each transition:

- date of exit from parental home
- date of (first, second, third) return into parental home
- date of the beginning of (first, second, third) union
- date of the end of (first, second, third) union
- status for TR6 (exit from parental home with a partner)
- status for TR7 (exit from parental home alone or with other persons)
- status for TR8 (return into parental home after a union)
- status for TR9 ("alone or with other persons" after a union")
- status for TR10 (return into parental home after "alone or with other persons")
- status for TR11 (union after "alone or with other persons")

Given that all the events considered here are repeatable, we need to underline that these transitions could be experienced twice or more by the same individual (for example, one can experience T6, then TR9, then TR10 and T6 again). In this case, we should consider more than one record for each individual nut it is possible only if we at our disposal a very detailed biography. Very often, this is not the case since it is very difficult to rely on the complete set of information mentioned above. As a consequence, we are forced to totally exclude some transitions and limit the others only at the first experience (for example, we can consider only the first exit from parental home).

2.3 Fertility (number of children ever born)

Women and men are distinguished by the number of children ever born. The possible states are then childless, 1 child, 2 children, 3 children, 4 or more children. The qualitative shape of the transition matrix is:

From \ to	childless	1 child	2 children	3 children	4+ children
Childless	O	TR12			
1 child		O	TR13		
2 children			O	TR14	
3 children				O	TR15
4+ children					O

"TRX" transition between categories
"empty space" impossible event
"O" non-event

Transition such as 0 → 2, 1→3, etc. caused by multiple births are not taken into account. A childless woman who has a twin birth simply experiences the transition 0→1 and 1→2 at the same date.

The *i*-th status variable accounts for the *i*-th child even born at the time of the interview (is 1 if the woman gave birth to at least *i* children, 0 if the woman gave birth to only *i-1* children, 9 if the woman has less than *i*-1 children). Evidently, for TR12 the status variable can only take value 0 or 1. Then, we need:

- date at first birth
- date at second birth
- date at third birth
- date at fourth birth
- status for TR12 (first child)
- status for TR13 (second child)
- status for TR14 (third child)
- status for TR15 (fourth child)

2.4 Children in the household

Differently from the previous section, now we consider the presence of children in the parental home, thus not simply the births. Given that in many applications, a dichotomy is sufficient (see de Beer *et al*, 2006), we consider two states: with and without children. The qualitative shape of the transition matrix is:

From \ to	Without children	With children
Without children	O	TR16
With children	TR17	O

"TRX" transition between categories
"empty space" impossible event
"O" non-event

The events that cause transition TR16 are "birth of the first child" or "entry into a relationship with a partner who has one or more children". Transition TR17 is experienced when the last child exits from parental home (or dies) or following a divorce or separation causing children living at ex-partner's.

The status variables are dichotomous variables assuming value 0 when no event that causing transitions is experienced before the interview, and 1 otherwise.

Briefly, we need:

- date of entry into the status "with children"
- date of exit from the status "without children"
- status for TR16 (without children→ with children)
- status for TR17 (with children → without children)

2.5 Structure of the initial data set

The initial dataset should contain all the dates and variables specified in the previous sections. Then, the structure of records should be as follow:

ID	Weight	m.birth	y.birth	m.inter	y.inter	m.event1	y.event1	m.event2	y.event2	….
Identification number	weight	Month of birth	Year of birth	Month of interview	Year of interview	Month of event 1	Year of event 1	Month of event 2	Year of event 2	….

….	TR1	TR2	…	TR17	Sex	Edu
….	Status for transition TR1	Status for transition TR2	…	Status for transition TR17	Sex	Level of educational attainment

3. Data preparation

Given the initial dataset, the first step towards the estimation of age profiles is the calculation of exact ages in which each event occurred. Then, we must define a *window of observation* for our data. Our final aim is the estimation of the transition rates that will be used as input in the models developed within MicMac for population forecasting. Nevertheless, we easily understand that in order to forecast, or "project", individual behaviors in the future, we need to start from recently observed behaviors and, in case, manipulate the outcomes of such observations. Since we are working with retrospective data, we have information on the whole biography of individuals interviewed at all ages, including events (and transitions) experienced many years ago. This is why we limit our observation to the more recent past, excluding all the events that happened before a certain limit. The choice of the window of observation is evidently subjective. The third step is the computation of a new status variable, that we can call *censor* and that tells us if the individual must be considered in the analysis, if she/he has experienced the specific transition in the fixed window of observation or if it is left censored.

Through ages and status variable, we have all we need for the effective calculation of relative risks. Depending on the type of observational plan, transition rates can be distinguished in *period rates*, based on period-age observations (calendar year in which an event occurs and the age at the time of event), *cohort rates* based on cohort-age observations (cohort to which a person belongs and the age in calendar years) and *period-cohort rates* based on period-cohort observation (calendar year in which the event occurs and the cohort to which the person belongs). Since we are using retrospective data, we use cohort rates for which the observation period extends over two calendar years and the age is in completed years. The last step is the determination of a smoother function for the observed age profile.

3.1 Transformation of dates into ages

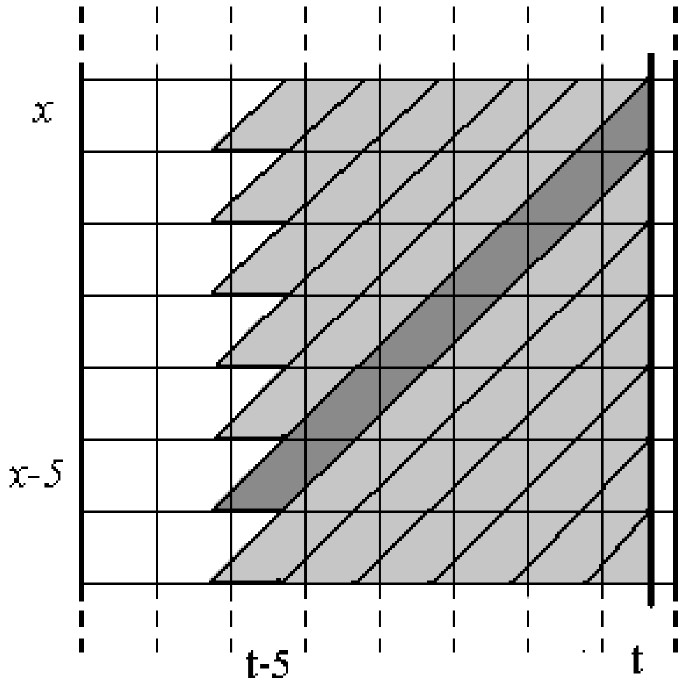
If we have, for each individual, the date of a specific event in the format month (MM: 1 to 12) – calendar year (YYYY), we can calculate the exact age at any event with a two-digits precision. First of all, we have to transform a date from the format month-year in a decimal expression considering the number of years and fractions of years since 1 January 1900. Then, the exact age at a specific event is given by the difference between this decimal expressions and the decimal expression of the date of birth. For example, if first marriage is experienced on November 1993 (decimal expression=93.88) and the individual was born on May 1965 (decimal expression=65.38), the exact age at first marriage is 38.50 years.

3.2 The window of observation

All the information on the events in the living arrangement and fertility fields, are collected through surveys based on interviews to respondents aged at least 15 or 18 years at the interview. This is consistent with the dynamics of such behaviors in contemporary Europe (see, e.g., Billari et al., 2005). From now on, we consider all the individuals aged at least 18 years at the interview.

We have already stressed the necessity to focus our attention only on the more recent past. A plausible period could be the last five years before the interview. Then, we can consider all the events experienced within the age interval [x-5, x], where x is the exact age at the interview. However, as we have already underlined, the type of data used requires transition rates calculated as cohort rates (based on cohort-age observations) and then it seems reasonable to consider the beginning of the window of observation at the (*x*-5)-*th* birthday. For example, if an individual were interviewed on November 2003 at the exact age 40.35, the window of observation starts at the 35th birthday and ends at 40.35 years. Fig. 1 shows graphical representation of windows of observation in the Lexis diagram.

Figure 2 The window of observation in the Lexis diagram. Individuals are interviewed in a precise point in time during year t



3.3 Episode limits and censoring

From this point onward, the procedure is transition-specific and it has to be repeated for each transition. Once we know the window of observation, we can define the limits of the episode that must be included in the window. The episode is the time interval spent in the initial state (i.e. when the individual is at risk to experience the transition) and it is delimited by a starting age and a final age.

The starting age could be the age at the beginning of the window of observation (*agesw*) or a higher age, in case the individual enters the initial states after *agesw*. For example, in the transition TR1 (never married → married), the starting age is always the beginning of the window of observation, whereas for TR2 (married → divorced) the maximum between age at marriage (if any) and *agesw* (age at the beginning of the window of observation).

The age at the end of the episode is the minimum among the following ages:

- the age at the interview, if the case is censored at the interview (*ageint*);
- the age at the considered event, if the individual experienced the transition (*ageev*);
- the age at any other event that causes the exit from the observation (*agecens*).

The next step is the computation of a new status variable called *censor*, which indicates if the individual has to be included in the analysis, if it is censored or not and, in the case, the kind of censoring (interview or other events). More in detail, for a generic transition TR*X*, *censor* assumes the following values:

0 censored at the interview

1 the event is experienced in the window of observation

2 the individual experienced another event in the window of observation that causes the exit from observation. Since we consider retrospective data, we do not have deaths and migrations. However, other competing events could cause a censoring. For example, in the transition TR2, death of spouse causes the exit from observation.

9 the individual cannot be considered in the analysis. This situation emerges when the transition (or an event that causes the exit from observation) is experienced before *agesw*. This means that in the period considered, the case cannot contribute to the time of exposure.

3.4 Transition-specific data matrix

In order to estimate transition rates, the number of events and time of exposure have to be measured. For every transition, we consider single years of age from 0 to 100+. Considering a generic transition TR*X* from a state *A* to a state *B*, for the *j-th* individual we have a window of observation included between $x_j^{N\_WIN}$ and $x_j^{FIN\_WIN}$, and an episode that starts at the age x_j^{in} and ends in x_j^{fin}.

In general, we have that

$$\underbrace{x_j^{IN\_WIN} \leq \overbrace{x_j^{in} \leq x_j^{fin}}^{episode} \leq x_j^{FIN\_WIN}}_{Window\ of\ observation}$$

The *transition rate* r_x at age x is the ratio between the number of events E_x experienced at age x and the amount of time spent in the initial states (time of exposure PY_x) by individuals at the same age. Therefore, the next step is the computation of E_x *and* PY_x for every age x. Considering N as the number of individuals, we have:

$$E_x = \sum_{j=1}^{N} E_{j,x}$$

$$PY_x = \sum_{j=1}^{N} PY_{j,x}$$

where, for the j-th individual:

$$E_{j,x} = \begin{cases} 1 & \text{if } x_j^{in} < x < x_j^{fin} \text{ and the transition is experienced at the exact age x} \\ 0 & \text{otherwise} \end{cases}$$

$$PY_{j,x} = \begin{cases} 1 & \text{if } x_j^{in} < x < x_j^{fin} \text{ and neither transition nor exit from observation} \\ & \text{are experienced at the age x} \\ \delta_{j,x} & \text{if } x_j^{in} < x < x_j^{fin} \text{ and transition or exit from observation is experienced} \\ & \text{at the age x} \\ 0 & \text{if } x < x_j^{in} \ or \ x > x_j^{fin} \end{cases}$$

where $\delta_{j,x}$ is the fraction of year spent in the initial state at the exact age in which the individual experiences the transition or the exit from observation. For example, let us suppose that in the transition (married → divorced) the j-th individual has an episode that start at the 40th birthday and ends at the exact age 42.31 with a divorce. At age 41 his contributions for the event is 0 and his contribution for the time of exposure is 1; for the age 42 he contributes 1 event and 0.31 years to the time of exposure.

We can include individual post-stratification weights w_j in the computation. The formulas become:

$$E_x = \sum_{j=1}^{N} E_{j,x} \cdot w_j$$

$$PY_x = \sum_{j=1}^{N} PY_{j,x} \cdot w_j$$

It is also possible to take into account some covariates by computing events and time of exposure separately for any combination of two or more categorical variables. In order to do so, we need to select sub-samples (defined for each combination of levels) to which apply the previous calculations. For example, we can consider two *timed-fixed* covariates, in the sense that their values remain fixed for the whole window of observation, sex and level of education, coded as follow:

Sex: 1 Men; 2 Women.
Level of educational attainment: 1 primary education and less (ISCED 01); 2 lower secondary education (ISCED 2); 3 upper secondary education (ISCED 3-4); 4 tertiary (ISCED 5A-6, 5B).

In table 1, we show a segment of a transition-specific data matrix that takes into account sex and education. For the age x we have 8 rows, one for each combination of levels for the covariates sex and education. We have a column with the number of cases (CASES) relating to a specific row (combination of age x and the specific level of covariates); unweighted (EVENT and EXPOS) and weighted (EVENTW and EXPOSW) events and time of exposure.

Table 1 Transition-specific data matrix

ID	AGE	CASES	EVENT	EVENTW	EXPOS	EXPOSW	SEX	EDU
[110,]	27	46	3	2.7286948	41.83	45.99903	1	1
[111,]	27	669	41	44.4088040	596.41	622.97497	1	2
[112,]	27	783	32	34.2193542	693.11	715.09001	1	3
[113,]	27	275	6	4.9138153	252.44	269.64722	1	4
[114,]	27	21	3	3.6084344	19.27	16.23176	2	1
[115,]	27	324	36	37.1002991	280.89	262.49605	2	2
[116,]	27	607	47	46.0331352	528.09	516.12050	2	3
[117,]	27	347	31	35.4466726	306.03	312.34037	2	4
[118,]	28	45	6	7.0276129	37.83	43.06423	1	1
[119,]	28	658	46	52.2252426	579.47	602.05693	1	2
[120,]	28	717	53	58.0949265	615.20	653.72342	1	3
[121,]	28	272	7	5.3103960	250.33	265.40648	1	4
[122,]	28	23	2	1.1703283	19.18	16.29679	2	1
[123,]	28	300	34	33.5822526	258.96	245.48345	2	2
[124,]	28	528	53	54.3586481	444.91	447.92291	2	3
[125,]	28	320	17	23.9958731	285.22	291.48234	2	4

4. GAM models and transition rates

If we consider the transition rate for a specific event as the dependent variable, we should model it as a function of age and a set of covariates. However, age profiles for a specific transition should never be considered as a linear function. Smoothing or graduating rates, or more specifically the age profile of rates, has been a traditional issue in various disciplines, including demography and actuarial science. Traditional approaches based on polynomials have been criticized in the literature since a long time, with authors proposing the use of spline functions as a solution (see, e.g., McNeil et al., 1977); recent developments include Smith et al., (2004) and, on age-specific fertility rates, Schmertmann (2003).

For our purpose, suitable solutions are the so-called *Additive Models* (Hastie & Tibshirani, 1990; Chambers & Hastie, 1992; Hastie *et al*, 2001) that are a generalization of linear model where the dependent variable Y can be modeled as a sum of non-linear (smoother) functions.

The model structure is

$$g(\mu) = \beta_0 + f(age) + \sum_k \beta_k X_k \qquad (1)$$

where

$\mu = E(Y)$, $g(.)$ is the link function and Y is the response variable (distributed as a exponential distribution); X_k is a generic covariate and β_k the corresponding parameter; β_0 is the intercept and $f(age)$ is the smooth function of age.

Since transition rates at age x for a specific event is given by the ratio between number of events (*Events*) and the time of exposure (*Exp.time*), considering natural logarithm as link function, for each i-th row of data matrix[2] we can write:

$$\ln\left(\frac{Events_i}{Exp.time_i}\right) = \beta_0 + f(age_i) + \sum_k \beta_k X_{ki} + \varepsilon_i$$

where ε_i is the random error term. Then,

$$\ln(Events_i) = \ln(Exp.time_i) + \beta_0 + f(age_i) + \sum_k \beta_k X_{ki} + \varepsilon_i$$

or, considering the expected value

$$\ln(E[Events]) = offset[\ln(Exp.time)] + \beta_0 + f(age) + \sum_k \beta_k X_{ki} \qquad (2)$$

where $Events \sim Poisson$.

It is important that the term ln(*Exp.time*) has no coefficient to be estimated.

2 We remember that each row of the data matrix is given by a specific combination of age x and the levels of categorical covariates.

The smooth function *f* is a *piecewise cubic spline*, a curve made up of sections of cubic polynomials joined together so that they are continuous in value, as well as first and second derivatives. The points at which the sections join are known as the *knots* of the spline, that are placed at quantiles of the distribution of unique *x* values. The number of knots defines the *degree of smoothness* (i.e. number of knots + 2). In order to avoid the choice of this parameter, that is essentially arbitrary, the degree of smoothness of *f* is estimated by Generalized Cross Validation[3] (Wood, 2006). The *mgcv* package contains a GAM implementation in which the degree of smoothness of model terms is estimated as part of fitting (see Wood, 2006). Calling the fitted values of this model (estimated number of events), the transition rate will be estimated as

$$\hat{r} = \frac{\hat{y}}{Exp.time} \tag{4}$$

We estimate relative risks for each transition separately for men and women. The covariates in the models are:

1. level of education at the interview (considered constant throughout the window of observation)
2. be married (Yes/No) (*time-varying*)
3. having children in the household (Yes/No) (*time-varying*)

A time-varying variable is obtained by splitting the episode at the point where the event occurs (see Blossfeld and Rohwer, 2002). Each sub-episode will be characterized by a unique value of the variable. For example, let's consider the transition to the first child and an episode that starts at 22 and ends at 25.83 years of age. If the individual married at 23.24 years, then the episode is splitted into the two sub-episodes (22; 23.24) and (23.24; 25.83). The covariate "be married" has value "No" in the first sub-episode and "Yes" in the second.

The variable "having children in the household" requires a particular procedure: we must account for the entry of the first child in the household and for the exit of the last child. Therefore, if needed, we will split our subinterval twice.

We must underline three crucial aspects emerging from this approach:

a. In our dataset, *Events* are calculated starting from individual weighted information. As a consequence, number of events and time of exposure are not integers. Since the *Poisson* distribution is defined only for integers, we need to round the number of weighted events. Empirical analyses (here not shown) suggest that this approximation appears acceptable.

b. The effect of covariates should be considered as differences from the grand mean, i.e. from the mean risk for the whole sample[4]. Therefore, we use the "deviation coding" system that permits to compare the mean of the dependent variable for a given level to the overall mean of the dependent variable. If we consider, for example, the categorical covariate *Education* with 4 levels (primary school, lower secondary school, upper secondary school, tertiary school), the deviation coding is accomplished by assigning value "1" to level 1 for the first comparison (because level 1 is the level to be compared to all others), to level 2 for the second comparison (because level 2 is to be compared to all others), and to level 3 for the third comparison (because level 3 is to be compared to all others). The value "-1" is assigned to level 4 for all three comparisons (because it is the level that is never compared to the other levels). The value "0" is assigned to all other levels (See table 2).

3 The way to control smoothness by altering the basis dimension, is to keep the it fixed at a size a little larger than it is believed could reasonably be necessary, but to control the model's smoothness by adding a "wiggliness" penalty to the least squares fitting objective (penalized regression spline). (Wood, 2006).

4 We need this feature because in the development of the MicMac project we will use the effect of covariates estimated with micro data to the baseline age profile based on macro data.

Table 2 Deviation coding for level of education

Level of education	Dummy 1 (Primary vs. mean)	Dummy 2 Low. sec. vs mean	Dummy 3 Upp. sec vs mean
Primary	1	0	0
Lower secondary	0	1	0
Upper secondary	0	0	1
Tertiary	-1	-1	-1

The contrast estimate gives the proportional effect to be applied to the baseline risk. Given that the expected values of the dummies specified in such a way are always zero[5], we can obtain the baseline transition rate as:

$$baseline_i = e^{\beta_0 + f(age_i)}$$

c. The use of GAM models allows to include covariates in the equation and to evaluate their proportional effect on the smoothed age. Therefore, the estimated coefficients express multiplicative changes to be applied at the baseline age profile in order to evaluate the estimated risk for each year of age.

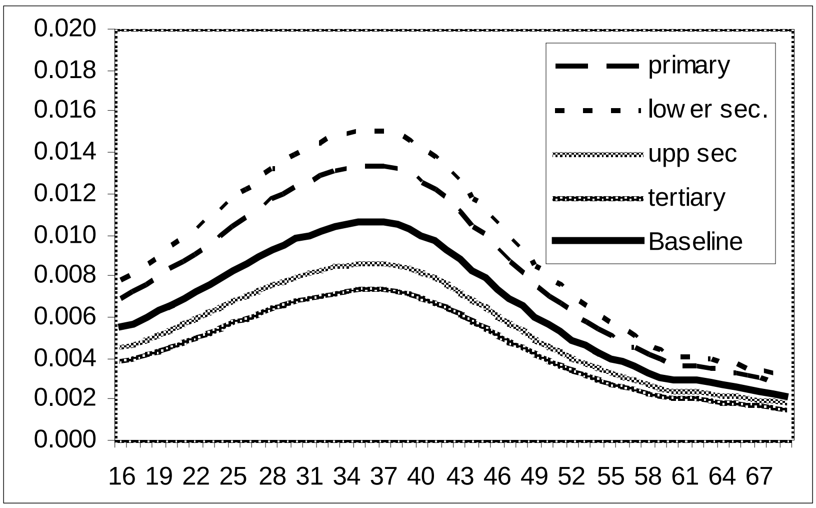
   The simplest way to do so is to consider a vertical shift throughout the whole range of age. For example, in fig. 3 it is shown the multiplicative effect of the level of education on an unspecified transition.

   However, very often the effect of a covariate shows a combination of vertical and horizontal shifts. In order to take into account this feature, a solution could be the estimation of the vertical shift for different specific sub-interval of age. In our analysis we split the range of age into 3 sub-intervals at two specific knots. The knots are fixed automatically at the 33rd and at the 67th percentiles (i.e. at the ages x_1 and x_2 at which, respectively, the 33% and the 67% of all the events are experienced before these ages).

The resulting model takes into account the baseline transition rate and the interaction between covariates and the age sub-intervals. The covariate deviation coding is adapted to this new feature: we evaluate the effect of each level of categorical variable within each subinterval.

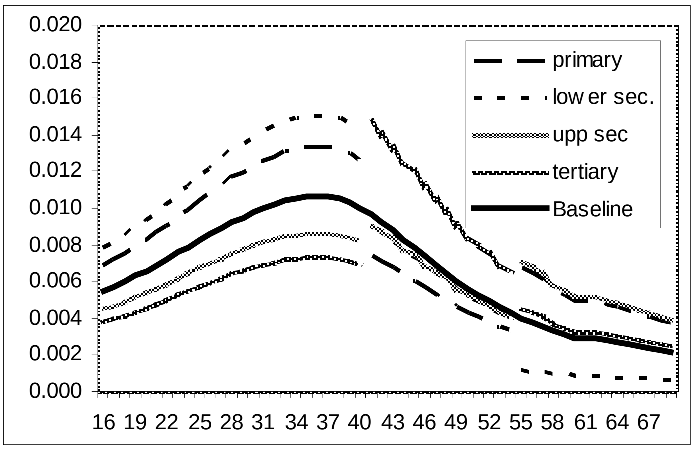
In fig. 4 we can see an example of the effects of education on transition TR2 (married-divorced) obtained by dividing the age range into three subintervals.

Figure 3 Multiplicative effects of covariates estimated with additive model



[5] More precisely, the expected values are zero if the number of cases is (approximately) the same for each levels. In our analysis this condition is satisfied given the structure of our data-matrix (equal number of rows for each combination of levels of covariates).

Figure 4 Proportional effects of education on the transition TR2 (married→divorced)



Level of education. Relative risks			
	15-40	41-54	55-90
primary	1.259	0.765	1.702
lower sec.	1.419	0.928	0.295
upp sec	0.812	0.936	1.774
tertiary	0.689	1.505	1.122

Finally, we can test the statistical significance of an additional covariate in the model by dropping it and noting the change in the deviance. The fitted models are compared using an analysis of deviance table. The tests are usually approximated, unless the models are un-penalized (Wood, 2006). We will only consider covariates that significantly increase the fit of the model when we add them into the equation.

5. Application examples in Italy and Netherlands

5.1 Age profiles of family and fertility events in Italy

Data for the Italian case come from the multipurpose survey called "Famiglia e soggetti sociali (FSS)". Carried out at the end of 2003, these data contain wide retrospective information on life course trajectories and the transition to adulthood, including data on the history of marital unions, cohabitations (followed by a marriage or not) and marital disruption, for a large sample of the resident population. The longitudinal nature of the survey makes it possible to update the collected information and to follow the same individual over time.

However, in this dataset we find limitations in the data available that requires some adaptation.

a. We do not have the date of birth of the respondent, but only the age (in completed years) at the interview (i.e. 15 November 2003): an individual with the age x (say, 48) has an exact age that is in the interval (48.00, 48.99) and the date of birth is included between 16 November 1954 and 15 November 1955. This means that, even if we have dates in month and year for every event, we cannot define the exact age at which these events were experienced. Since we do not have further information, we can avoid this indetermination adding a random month of birth extracted from a discrete uniform distribution $U(1,12)$.

b. For some events, only the calendar year has been asked. In particular, we do not have information on the month for the following events:

- exit from parental home
- divorce
- death of spouse
- exit from parental home (or death) of the last child

If the individual experienced the event, we can consider a random month for each event extracted from a discrete uniform distribution $U(1,12)$. A remark is needed: in this case we add a second approximation to the one already introduced in point a. Since the available information concerns the respondent's age in completed years at the interview and the calendar year in which the divorce occurred, the exact age at this event is included in a two-years range. For example, an individual aged 48 at the interview who divorced during 1995, could have experienced this event at an exact age included in the interval (39.12, 41.11), i.e. at 39, 40 or 41. For the month of exit from parental home we can use the additional information given by the month of marriage: if the individual experienced both exit from home and first marriage and if the two events occurred in the same calendar year, we can assume that the two events occurred simultaneously. Therefore, the month of exit from home corresponds to the month of first marriage.

It is not possible to analyze all the transitions proposed in section 2 because of lack of data or very few events (rare transition). In particular we cannot consider:

- TR8, TR9, TR10: no information are available for the events that cause these transitions;
- TR11: we have information on the event that causes transition (entry into a union) but not on the event that causes exit from observation (return parental home);

Transition-specific age profiles are estimated separately for men and women. Moreover, we considered the effect of educational level and the following time-varying covariates:

- married (Yes/No), included in TR12, TR13, TR14, and TR15
- with children in the household (Yes/No), included in TR1,TR2, TR4 and TR5.

In table 4, we have *p*-values associated with the Null Hypothesis that a specific covariate does not increase the fit of the model. In the columns marked with (1) the significance Chi-square test related to the comparison between the model without covariate and the model with education is shown; in columns marked with (2) and (3) the model with education is compared with the model adding, respectively, "married or not" and "with/without children in the household". Results obtained for women suggest to include education everywhere excepting TR4, TR5 and TR15, where the difference from the base model (without covariates) is not significant (at 95% level). For men, education could be excluded in TR2, TR3, TR4, TR5 and TR15. Time-varying covariate "married" could be excluded in TR14 and TR15 for both sex whereas "with children" should be included in TR2, TR4 and TR5 for women and TR1 and TR2 for men.

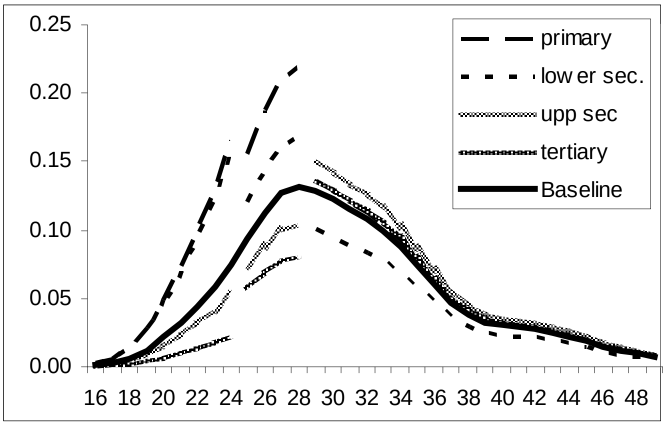
Table 4 Analysis of Deviance. Significance Chi-square test (p-value) comparing model without covariate and model with education (Column 1); model with education and model with "married or not" (column 2); model with education and model with "with/without children in the household" (column 3) by sex and transition

	Women			Men		
	education	married	with children	education	married	with children
	(1)	(2)	(3)	(1)	(2)	(3)
TR1 never married -> married	0.000	-	0.066	0.000	-	0.000
TR2 married -> divorced	0.002	-	0.009	0.590	-	0.001
TR3 married -> widowed	0.000	-	-	0.124	-	-
TR4 divorced -> married	0.193	-	0.005	0.072	-	0.193
TR5 widowed -> married	0.084	-	0.003	0.051	-	0.428
TR6 parental home -> with a partner	0.000	-	-	0.000	-	-
TR7 parental home -> alone or with other persons	0.006	-	-	0.001	-	-
TR12 childless -> 1 child	0.000	0.000	-	0.000	0.000	-
TR13 1 -> 2 children	0.001	0.000	-	0.002	0.000	-
TR14 2->3 children	0.006	0.390	-	0.000	0.238	-
TR15 3->4 children	0.439	0.709	-	0.234	0.076	-
TR16 without children -> with children	0.000	-	-	0.000	-	-
TR17 with children -> without children	0.000	-	-	0.001	-	-

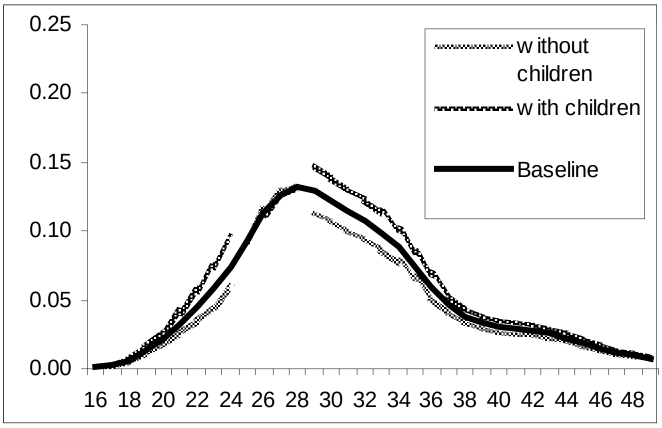
As an example of the output given by the method presented in this paper, we consider the transition TR1 (never married-married). First of all, we specify the needed information for transition rate calculations adapted to the ISTAT 2003 data. Similar indications could be defined for the other transitions. Fig. 5 shows the baseline risk and the relative risks by sex, level of education and the presence of children in the household.

Transition	Event	Events that implies the exit from observation	Cases in the analysis	Starting age of exposure	Final age of exposure
TR1 (never-married → married)	First marriage	None	Who did not experience marriage before *agesw*	*agesw* (age at the beginning of the window of observation)	Age at marriage or Age at the interview

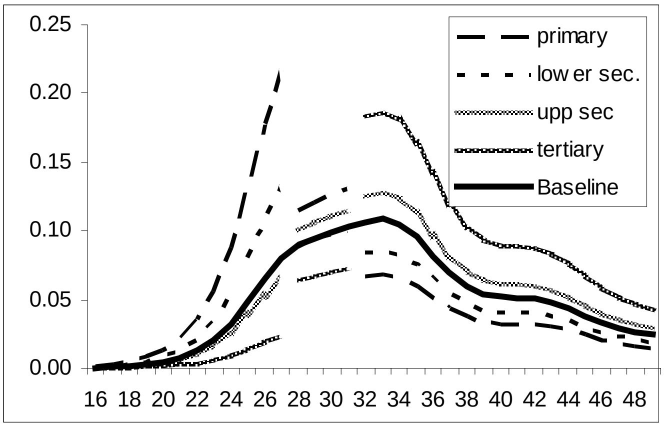
Figure 5 Transition TR1 (never married->married). Baseline and relative risks according to sex, level of education and presence of children in the household. Italy. ISTAT 2003. (Events observed in the window of observation: 1306 women and 1279 men)



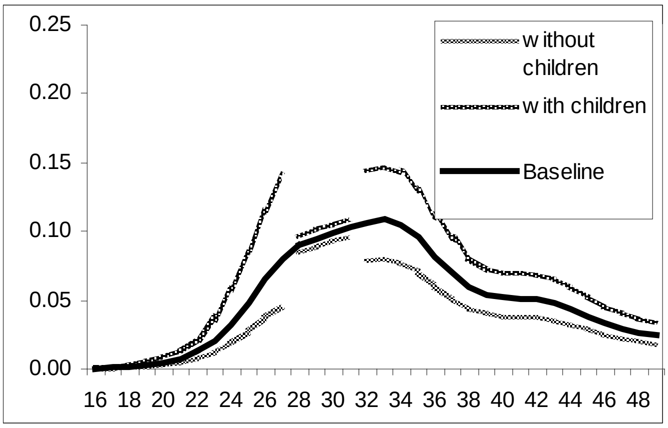
Women. Italy. Level of education. Relative risks			
	15-24	25-28	29-50
primary	2.237	1.657	1.046
lower sec.	2.108	1.273	0.779
upp sec	0.712	0.782	1.163
tertiary	0.298	0.607	1.055



Women. Italy. Children in the household. Relative risks			
	15-24	25-28	29-50
without children	0.785	1.012	0.876
with children	1.273	0.989	1.142



Men. Italy. Children in the household. Relative risks			
	15-27	28-31	32-50
primary	2.694	1.277	0.624
lower sec.	1.648	0.983	0.791
upp sec	0.799	1.122	1.177
tertiary	0.282	0.71	1.721



Men. Italy. Children in the household. Relative risks			
	15-27	28-31	32-50
without children	0.573	0.939	0.739
with children	1.744	1.065	1.353

5.2 Age profiles of family and fertility events in the Netherlands

We used data from Netherlands "Fertility and Family Survey" (FFS-NL). The 8145 interviews were carried out between February and June 2003. Since 1974, Statistics Netherlands organizes the Netherlands Fertility and Family Survey (FFS-NL) collecting longitudinal information on leaving home, cohabitation, marriage, and childbearing. Differently from the Italian data, in this case the exact date of birth is available, as well as the information on the date of each event, in the format month-year. Nonetheless, for some individuals it could happen that the information on the month is missing, since sometimes only a specification for the year of occurrence is provided. In this case, we input a random month extracted from a discrete uniform distribution $U(1,12)$.

For the Netherlands, data limitations and adaptation are similar to the Italian case. For example, we do not have all the required information for the analysis of all the transition proposed in Section 2. For the Dutch case we do not show detailed schemes for every transition since we still refer to the schemes proposed in Section 4 except for the following:

- children's dates of births are available only for women

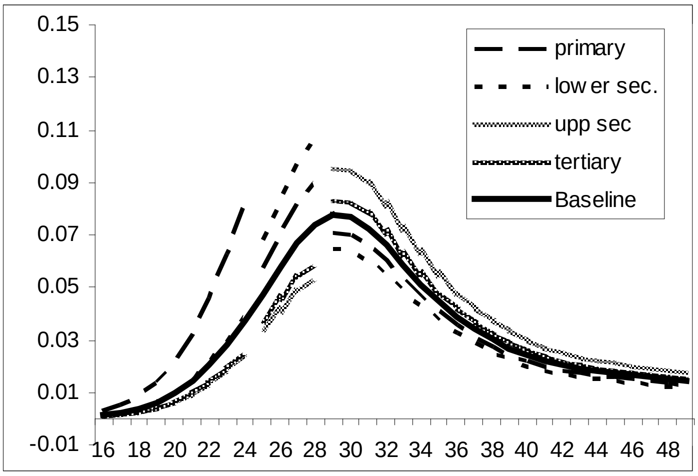
- in the FSS-NL children's dates of exit (or death) are not available. Thus, we cannot study transitions related to the presence of children in the household. Besides, we have no information for evaluating the time-varying variable "with children in the household (Yes/No)".

Briefly, for the Netherlands we can analyze the following transitions: TR1 (never married → married); TR2 (married → divorced); TR3 (married → widowed); TR4 (divorced → married); TR5 (widowed → married); TR6 (parental home → with a partner); TR7 (parental home → alone or with other persons); TR12 (childless → 1 child) (only women); TR13 (2nd birth) (only women); R14 (3rd birth) (only women); TR15 (4th birth) (only women).

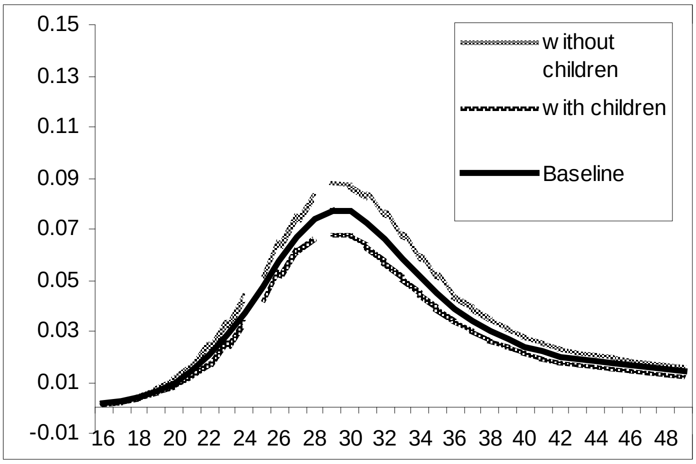
Table 5 Analysis of Deviance. Significance Chi-square test (p-value) comparing model without covariate and model with education (Column 1); model with education and model with "married or not" (column 2); model with education and model with "with/without children in the household" (column 3) by sex and transition

	Women			Men		
	education	**married**	**with children**	**education**	**married**	**with children**
	(1)	**(2)**	**(3)**	**(1)**	**(2)**	**(3)**
TR1 never married -> married	0.006	-	0.687	0.213	-	-
TR2 married -> divorced	0.67	-	0.36	0.046	-	-
TR3 married -> widowed	0.822	-	-	-	-	-
TR4 divorced -> married	0.269	-	0.413	0.241	-	-
TR5 widowed -> married	-	-	-	-	-	-
TR6 parental home -> with a partner	0.024	-	-	0.843	-	-
TR7 parental home -> alone or with other persons	0.000	-	-	0.000	-	-
TR12 childless -> 1 child	0.000	0.000	-	-	-	-
TR13 1 -> 2 children	0.000	0.000	-	-	-	-
TR14 2->3 children	0.012	0.124	-	-	-	-
TR15 3->4 children	0.011	0.446	-	-	-	-

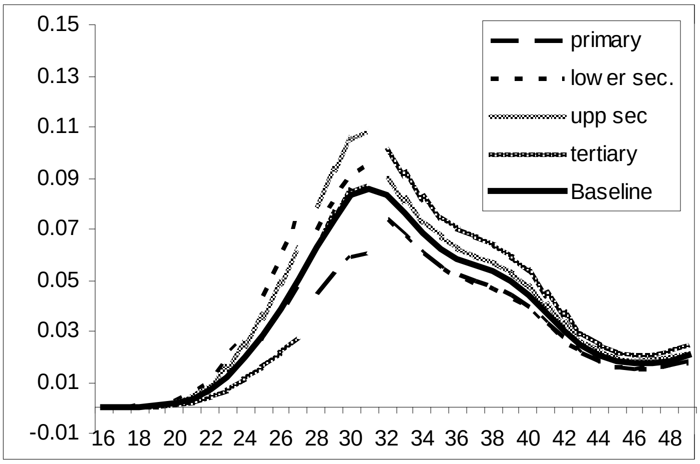
Figure 6 Transition TR1 (never married->married). Baseline and relative risks according to sex, level of education and presence of children in the household. Netherlands. FFS 2003. (Events observed in the window of observation: 295 women and 312 men)



Women. Netherlands. Level of education. Relative risks			
	15-24	25-28	29-50
primary	2.223	1.212	0.913
lower sec.	1.036	1.44	0.833
upp sec	0.647	0.719	1.23
tertiary	0.671	0.797	1.069



Women. Netherlands. Children in the household. Relative risks			
	15-24	25-28	29-50
without children	1.153	1.109	1.141
with children	0.868	0.901	0.876



Men. Netherlands. Level of education. Relative risks			
	15-27	28-31	32-50
primary	0.958	0.707	0.889
lower sec.	1.529	1.103	0.879
upp sec	1.237	1.26	1.067
tertiary	0.552	1.017	1.198

Transition-specific age profiles are estimated separately for men and women and the effect of educational level is considered. As time-varying covariates, we can consider "married (Yes/No)" and "having had first birth (Yes/No)" as an approximation of the missing covariate "with children in the household (Yes/No)".

p-values associated with the Null Hypothesis that a specific covariate does not increase the fit of the model, are shown in table 5. The meaning of this table is the same as table 4. Results suggest that, among women, education could be left out of the models in TR2,TR3, TR4; "with children" could be left out in all the three transitions that included this covariate whereas "married" could be excluded in TR14 and TR15. Among men, the introduction of education has a significant effect only for TR2 and TR7. We cannot consider time varying covariates for men since information on their child births is not available in the initial dataset.

6. Discussion

In this paper we proposed a general method that estimates age profiles for the main transitions experienced by individuals in the field of living arrangement and fertility. The method, called MAPLE (Method for Age Profile Longitudinal Estimation), starts with the specification of the required micro-level data and includes a data processing routine that leads to the estimation of regression models. These models allows, for each transition, to evaluate the smoothed age profile and the relative risks given by time-fixed and time-varying covariates.

The analytical strategy can be segmented in four steps. The first step consists in the specification of the required micro-data for the evaluation of the age profiles. For this purpose, we indicated all the information that the starting data files should contain in order to apply MAPLE. The second step relates to data preparation and includes the computation of ages at various events, the specification of the *window of observation* and of the *episode* (period at risk) for each individual. In the third step a *transition-specific data matrix* is computed. Within the window of ages, the events experienced and the time spent in a specific state contribute to the calculation of events and time of exposure for all the individuals in the sample. Starting from a data file where each record is a set of dates and status variables relating to the i-th individual, we obtain a matrix for each transition where the single row refers to a specific age x and containing the number of events and time of exposure. In the last step, we analyzed how an observed set of events and time of exposure can be modelled by a smoother function, obtaining age profiles. In particular, we developed GAM (Generalized Additive Models) that permit to evaluate the smoothed age profile (as transition rate baseline) and, at the same time, to estimate the effects of a vector of covariates as multiplicative changes from the baseline. Finally, we presented an application of MAPLE to Italy and the Netherlands.

In conclusion, we would like to underline the flexibility of this method. Firstly, it can be applied to every setting where micro-level data on transitions are available from a large-scale representative survey. Secondly it could be used for different kind of transitions, even for those not strictly related to fertility and living arrangements. Thirdly, we can specify different covariates both time-constant and time-varying over the life course. Moreover, based on regression model the method allows to evaluate confidence intervals and to test hypotheses. The whole method is written in R software and it could be easily recalled as an R function in order to be applied to real data.

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Session 2: Mortality

Chair: Nico Keilman

AN APPROACH TO IMPROVE THE CONSISTENCY OF MORTALITY PROJECTIONS OBTAINED BY THE LEE-CARTER METHOD

Dalkhat Ediev[1]

It is argued that the LC method for projecting mortality may result in distrotions of age profile of the mortality and, in general, will tend to underestimate life expectancy. Adjustments to the method are proposed, which improve consistency of projections. Also, in view of inability of the LC model to cope with empirical data, an alternative extrapolation method is proposed, which is based on separate extrapolation of age-specific mortality with consistency adjustments imposed in order to avoid inconcistent divergence of death rates.

1. Adjustmments to the LC-model

The Lee-Carter method (Lee and Carter 1992) for projecting mortality is based on reducing the variety of age-specific profiles of mortality dynamics to a single time-dependent function $k(t)$. The method may be summarized by the following relation:

$$m(x,t) = e^{a(x)+b(x)k(t)+\varepsilon(x,t)} \tag{1}$$

where $m(x,t)$ is the death rate at age x at time t, $a(x)$, $b(x)$, $k(t)$ are the age and time-dependent model parameters to be estimated from the data, and $\varepsilon(x,t)$ is the error term. Function $a(x)$ reflects main age-specific regularities in mortality, $k(t)$ is a parameter reflecting the level of mortality, and $b(x)$ reflects how mortality at different ages responds to changes in the overall mortality. The mortality index $k(t)$ is usually projected to the future using extrapolative techniques or random-walk models.

Despite relying on a single time-dependent function $k(t)$, the model is flexible enough to reflect differential mortality dynamics at different ages. In fact, it is very close to independent extrapolation of age-specific log-mortality rates. Commonly, the $k(t)$ function is projected linearly (i.e., $k(t) = \alpha + \beta \cdot t$ In this case the LC-method is merely the same as separately linearly projecting the logs of age-specific rates:

$$\lambda(x,t) = \ln m(x,t) = a(x) + b(x)(\alpha + \beta \cdot t) = \big(a(x) + b(x)\alpha\big) + b(x)\beta \cdot t. \tag{2}$$

Here β and α are the slope and intercept for the $k(t)$ function; $b(x)\beta$ and $a(x)+b(x)\alpha$ give slopes and intercepts for trends of age-specific log-mortality rates. This property of the method was pointed to already at time of the original publication by Lee and Carter (McNown 1992).

Due to the aforementioned property, the method tends to amplify irregularities in the age profile of mortality and, eventually, may lead to inconsistent projection results. Apart from structural biases of the LC model, which are discussed later on, the amplification of irregularities by itself may result in downward biases in projected life expectancies (in fact, this applies to any extrapolative method). Therefore, it must be suggested, at least, to use some smoothing techniques in order to minimize random irregularities in projected mortality age profiles.

Apart from the problems related to randomness amplification in LC-projections, there is another potential structural problem of the model, which may result in systematic distortions of the mortality structure and level. The problem is also related to the fact that model (1) is close to independently projecting age-specific log-mortality rates at different rates derived from historical observations. In particular, in past decades infant mortality declined at a considerably higher rate compared to mortality at oldest ages[2]. Yet, it might be reasonable to expect that future mortality improvements for children

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2 This applies also to a much longer period in past: usually, mortality at younger ages declined faster.

will be less intensive compared to the rate of mortality improvements at old ages. Meanwhile, the LC method based on historical trends will still project faster mortality improvements at youngest ages and slower than average reductions in mortality at old ages. Apart from structural biases this may lead to significant underestimation of life expectancy for most of the ages, as the role of infant and young-age mortality in current levels of life expectancy is negligible, while mortality decline at old ages is crucial for further gains in life expectancy. This feature of the method is general: it will tend to underestimate future improvements in life expectancy. That is because it is based on past experience and projects better improvements for those ages, which showed better mortality reductions in the past. These ages, however, will have less impact on life expectancy in future, as life expectancy will be more sensitive to mortality improvements at ages, which have not yet benefited from mortality improvements.

To make the point clear, one may look into how mortality improvements at different ages are projected under (1), which (omitting the error term) is equivalent to the following relation:

$$\hat{\lambda}(x,t) = \ln \hat{m}(x,t) = a(x) + b(x)k(t). \tag{3}$$

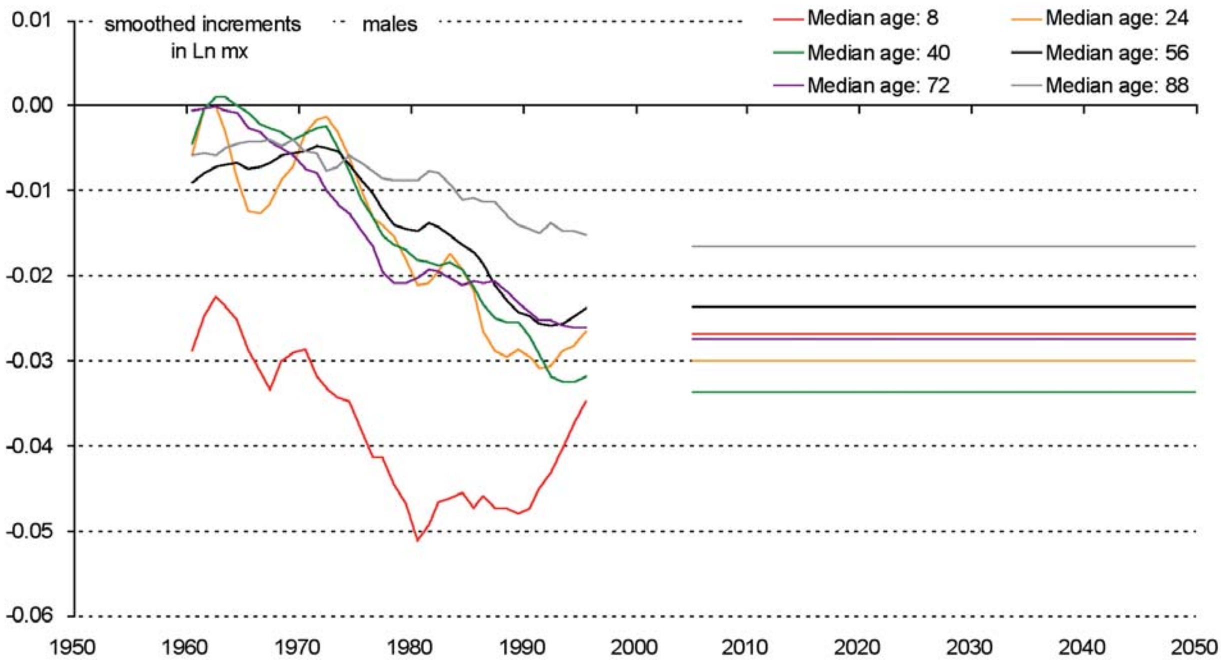
First order increments of (3) give rates of mortality improvements:

$$\Delta(x,t) = \hat{\lambda}(x,t+1) - \hat{\lambda}(x,t) = b(x)(k(t+1) - k(t)). \tag{4}$$

This expression indicates that the LC model assumes proportionality among age-specific rates of mortality reductions. In fact, the $b(x)$ function is close to a weighted average of increments in log-mortality rates per increment in the $k(t)$ function, with weights proportional to the square of the latter increments (see the appendix for explanation). Although the assumption about proportionality of log-mortality reductions might be relevant for the past, it may not be granted forever. In projections, when $k(t)$ is extrapolated at some constant rate of change derived as an average from the past, age-specific mortality reductions (4) are also constant and equal to the weighted average age-specific mortality reductions from the past experience.

Figure 1 depicts age-specific improvements of logarithms of death rates for Austrian males in 1951-2005, with age groups aggregated into six large groups (of 16 years length each, i.e., 0-15 full years, 16-31 years, etc.) and also averaged across time with 20-year moving average. One may observe that mortality reductions at young ages were in general faster if compared to reductions at other ages. This is also reflected in the $b(x)$ function of the LC model, which reflects faster mortality improvements at ages below 15, as well as slower mortality decline at medium ages (45-55) and oldest ages (80+) imputed into the projection structure. From the time series we notice that the age profile of mortality reductions was not stable (unsmoothed data are more explicit about that), and it does not seem to persist in the long run in the same frozen shape as it is implied by the LC model. Thus, the assumptions of the LC-model imply only remarkably pessimistic trends in mortality improvements for most of ages above 15. A more optimistic assumption for the future could be setting eventually similar relative improvements for all age groups.

Figure 1 Smoothed first order increments of logs of age-specific death rates averaged across wide age groups (of 16 years length each, ages shown on the graph correspond to medians of the age groups). For smoothing the time series 20-year long moving averaging is used. Austria, males, 1951-2005 data and projections by the LC method



In order to address problems caused by the structure of the LC model, the following model is suggested, of which the LC model is a special case[3]:

$$\eta(x,t) = a(x) + k(t) + v(t)\beta(x). \tag{5}$$

In addition to the $k(t)$ function responsible for changes in the *level* of mortality, model (5) includes the $v(t)$ function responsible for changes in the *age structure* of mortality improvements.

Model (5) turns into the Lee-Carter model if $v(t)$ and $k(t)$ functions are linearly related to each other. Similarly, the model turns into the generalized Gompertz model if functions $a(x)$ and $\beta(x)$ are linearly related to each other. If neither of these assumptions is true, however, both the Lee-Carter and Gompertz model tend to systematically bias the death rates.

Parameters in (5) may be estimated both in a way similar to the LC model (using the SVD) and, more conveniently, they may also be estimated on the basis of some scaling agreements and simplifying assumptions. We develop these agreements in such a way, that parameters of model (5) and of the LC model are explicitly related: the models share the same functions $a(x)$ and $k(t)$, and $\beta(x)$ equals to the deviation of $b(x)$ from its average. Under these agreements, the LC model is equivalent to model (5) if $v(t)$ is *equal* to $k(t)$[4].

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[3] The general model proposed embraces, apart from the Lee-Carter model, other mortality models too, e.g., the generalized Gompertz model.

[4] Details of estimation procedures are given in the Appendix A. Our estimation procedure is simpler compared to the SVD procedure. In principle, this might result in somewhat less effective estimates of the model functions, as SVD is optimal in terms of minimizing sum of squares of residuals. However, one should note that in practical calculations SVD is not used straightforwardly. Rather, it is supplemented by different adjustments and corrections (e.g., smoothing $a(x)$ and $b(x)$ functions using splines or otherwise, post-estimation fitting of $k(t)$ to observed deaths or life expectancies, etc.) Therefore, the actually used SVD is also not optimal in terms of fit to initial log-mortality rates and its usage might become pointless if there are other, simpler procedures with the same or better performance.

After parameters in model (5) are estimated, they may be projected under different assumptions depending on the particular situation encountered by the statistician. One option is to apply extrapolative procedures, as it is done in the LC model. While it seems that this approach can project satisfactorily the mortality level $k(t)$, it might not be the best choice for projecting the structural indicator $v(t)$. In particular, while the level of mortality may theoretically be improved indefinitely, developments in the age structure of mortality seem to be less variable and should meet some consistency requirements. For example, mortality at ages above 40 is indeed an increasing function of age, which imposes restrictions on the age structure of mortality. Setting $v(t)$ equal to $k(t)$, as it is implied by the LC model, will eventually result in inconsistent age profiles of projected mortality. E.g., in case of Austria the model results in unrealistic distortions of mortality at adult ages: it suppresses mortality reductions at ages 45-55 and above 80, with resulting over-Gompertzian mortality growth at ages above 60. Eventually, the method produces a profile with mortality at ages 45-55 being higher than at ages 60-70.

One should note that model (5) is rather flexible and may fit very well to *historical* data as well as the LC model do. Furthermore, the structural function $v(t)$ estimated from historical data will always be close to the function $k(t)$ of main changes in mortality (see Appendix B to the Chapter for explanations). However, this does not imply that the same should be projected to the future. Both functions are close to each other *within* the interval of data fitting, while the structural parameter $v(t)$ behaves more stably beyond the fitting interval. This reflects a good performance of extrapolative models when *data fitting* is concerned, while they tend to exaggerate the changes in mortality structure in *projections*. Indeed, for mortality fitting in a time period of T years the LC uses a standard mortality age profile (the $a(x)$ function) and additionally about T+100 parameters. Such a number of parameters is already too high, as, given we have the standard mortality profile, the mortality profile for each year might be fitted more or less well using two parameters (as it is suggested, e.g., by the Brass model). Furthermore, these approximately 2T mortality parameters are also inter-correlated if there is a trend in mortality dynamics (e.g., assuming linear or other two-parametric trend, one may think of about only four independent parameters capable of fitting the data). Hence, it is easy for the aforementioned models to fit the data, which, however, does not imply robustness of projections derived from these models. To make projections more robust, one should somehow limit the model's flexibility in projections in order to prevent model peculiarities derived from fitting needs and irrelevant to the future from affecting the projection.

Restrictions on changes in the $v(t)$ in the future may be imposed using model (5) in order to avoid inconsistencies in the projected structure of mortality. Such restrictions may be developed from consistency conditions. Another option might be to imply some smooth trajectory of $v(t)$ changing from the initial trend to some eventual constant level. In fact, this eventual level v^* of $v(t)$ determines eventual age structure of mortality. Hence, having some assumptions $\hat{m}^*(x)$ about eventual age structure of mortality, one may explicitly derive v^* using, e.g., OLS procedures, and apply some smooth trends of $v(t)$ converging to v^*[5].

The approach described above might be prominent in a situation, when ultimate mortality structure may be approximated by a mortality profile observed from another population, with more advanced longevity. If this is not the case, however, it might not be that easy to set reasonable assumptions for $m^*(x)$ and v^*. In such a case one may use the following procedure, which may be considered as an adjustment to the LC projection. Let all the LC functions and the projection for the $k(t)$ be somehow obtained. Taking into account the aforementioned flexibility of the models addressed here and also the closeness of $v(t)$ and $k(t)$ functions within the data fitting period, we may set these two functions identical within the data fitting period and assume that projected changes in $v(t)$ – unlike $k(t)$ – will gradually slow down. In particular, the following scenario might be used for $v(t)$:

$$v(t_0) = k(t_0), \qquad \Delta v(t) = \Delta k(t) e^{-(t-t_0)/\tau}, \; t \geq t_0, \tag{5}$$

where $\Delta v(t) = v(t+1) - v(t)$ and $\Delta k(t) = k(t+1) - k(t)$ are changes of the two functions over time and τ is a convergence parameter.

Scenario (5) will result in a smooth convergence of age-specific improvements to some average level, which could be a better assumption about future developments of mortality, if the age differentials in mortality improvements in the past may not be indefinitely prolonged to the future.

[5] Considerations about convergence to the eventual mortality structure may, alternatively, be used to derive $\beta(x)$ function. In that case dynamics of $v(t)$ will be obtained as a consequence of scaling agreements. E.g., setting $v = 0$ for mortality pattern determined by the age effect function $a(x)$ and $v^* = 1$ for converged structures, one may set $\beta(x) = \ln(m^*(x)) - \overline{\ln(m^*(x))} - (a(x) - \overline{a(x)})$.

The adjusting procedure makes the projection non-diversive in terms of improvements of age-specific death rates and is prone to inconsistencies. Yet, *imputing fully convergent trajectories of age-specific mortality improvements might also result in significant systematic biases in projections due to ignoring the long-persisted differences observed in the past.* In part, this problem may be solved using the appropriate level for the convergence parameter τ, which corresponds to the duration of time, when age-specific differentials in mortality improvements are assumed to persist. Alternatively, one may distinguish two kinds of age-specific differentials in mortality improvements: those expected to persist forever and those to be eliminated in future. This may be done using the following model, which has nearly the same structure as model (5):

$$\eta(x,t) = a(x) + k(t)(1+\beta_0(x)) + v(t)\beta_1(x). \tag{6}$$

$\beta_0(x)$ corresponds to age-specific differences in mortality dynamics, which continue during the projection; $\beta_1(x)$ corresponds to differences which, under the same adjustment procedure as suggested above, disappear in time.

$\beta_0(x)$ must be constructed in such a way that it does not result in inconsistencies. For example, it might be estimated from separate LC-type model fitted to as long as possible period of time. Another way would be to derive $\beta_0(x)$ from $\beta(x)$ function of the main LC-type model, e.g., smoothing it using monotonic splines or otherwise making it monotonic in order to avoid crossovers of age-specific death rates in projection. E.g., in some cases it may be obtained as a smoothed average of the followinf two monotonic estimates[6]:

$$\beta_0(x) = \max_{y \ge x}\{\beta(y)\} \text{ or } \beta_0(x) = \min_{y \le x}\{\beta(y)\}. \tag{7}$$

Note, that these two estimates may be used as upper and lower-bound estimates for the intervals of projected death rates. The average of both estimates in (7) may be used as a central estimate[7].

Having estimated $\beta_0(x)$, the $\beta_1(x)$ function may be obtained by subtracting $\beta_0(x)$ from $\beta(x)$ estimated from the data for the period relevant to the projection:

$$\beta_1(x) = \beta(x) - \beta_0(x). \tag{8}$$

Similar ideas may be used to obtain coherent mortality projections for two sexes or several regional populations. For this purpose a multi-regional generalization of the LC method has been proposed in the literature (Li and Lee 2005), which is based on introducing similar functions in model (1) for all populations concerned except for the standard mortality profile $a(x)$, which is population-specific; also, in case of regional projections, the model introduces region-specific time-age interaction additives. Such an approach to projecting the mortality of both sexes using the same $k(t)$ function implies *instant* convergence of mortality improvements for both sexes, which may neglect recent trends. One may also use, however, an approach similar to (5), which allows for temporary continuation of observed differences between sexes with respect to mortality improvements, with exponential convergence in future:

$$k_i^*(t_0) = k_i(t_0), \qquad \Delta k_i^*(t) = \Delta k^*(t) e^{-(t-t_0)/\tau}, \; t \ge t_0, \tag{9}$$

here index 'i' refers to a subpopulation of interest (i.e., to sex or to region), $k_i(t)$ marked with asterisk are those obtained after applying the convergence adjustment, and $k^*(t)$ is a common function characterizing overall mortality dynamics, e.g., the average of functions $k_i(t)$ obtained for each subpopulation separately.

Another improvement to the model may also be proposed, which better addresses mortality dynamics in the past in case of *accelerated* or *decelerated* mortality improvements. The LC model projects to the future weighted averages of the past age-specific mortality reductions. The method is not able to reflect mortality dynamics with different accelerations of mortality reduction at different ages. In particular, although infant mortality in Austria was decreasing faster compared to the overall level of mortality (as it may be seen from $\beta(0) > 0$ or $b(0) > 1$), its advantage compared to the overall

[6] This method seems to work more or less effectively when the original $\beta(x)$ function is already nearly monotonic. This might not be the case, however, when a very short data period or a period with no sustaining trends of age-specific mortality dynamics are concerned. In any case, it seems that expert judgment may not be fully avoided.

[7] In any case, a provision should be made for guaranteeing that $\overline{\beta_0(x)} = 0$, as this follows from scaling agreements we use for estimation purposes. Otherwise, the convergence procedure, in addition to structural adjustments, will also result in altering the pace of mortality reductions.

decrease in mortality was gradually leveling off[8]. The LC model is not able to fit such trends, as it assumes linear relation between the overall mortality reflected in $k(t)$ and all the age-specific log-rates, while in some cases this linearity is not the case. In order to address different acceleration/decelerations of log-mortality at different ages, one may use non-linear models in terms of the parameter k. In particular, we propose the following *quadratic* model:

$$\eta(x,t) = a(x) + k(t)(1+\beta(x)) + k^2(t)\gamma(x) + \varepsilon(x,t). \tag{10}$$

As model (10) is even more flexible than the LC model, it may also result in divergent and eventually inconsistent trends in age-specific mortality. Hence, it is better to apply to projections obtained by (10) the same adjustment procedure proposed above, e.g.:

$$\hat{\eta}(x,t) = a(x) + k(t) + v(t)\beta(x) + v^2(t)\gamma(x), \tag{11}$$

where $v(t)$ is given by (5).

2. Alternative to the LC method: Separate extrapolation of age-specific mortality

In view of structural problems of the LC model and also in view of its similarity to separate linearly extrapolation of age-sex-specific log-mortality rates, as well as of its non-transparency, we address here an alternative approach, when age-sex-specific rates are explicitly and separately extrapolated into the future. First, we start with examining the relevance of the very structure of the LC model to the data.

As it was mentioned in another part of the work, the LC model is not able to fit to mortality dynamics with different age-specific *accelerations* in mortality improvements. In that case some non-linear models might be more adequate. Besides, if mortality dynamics at different ages is significantly different, approach based on using the average log-mortality rate as an indicator of the overall mortality level might be inappropriate (and this is essentially the case of the LC model). In case of Austrian male population, e.g., one may note significantly different timing in mortality reductions at different ages. In particular, mortality decline at ages around 55 was relatively stable after the war, while it has recently been accelerated both at younger and older ages. It is impossible to fit to such dynamics within the LC framework as average mortality level reflected by the $k(t)$ function becomes not equally relevant to mortality developments at all ages. The case of Russian mortality is even more striking: in past decades, overall mortality was increasing, while mortality at young ages was decreasing. Projecting the Russian mortality based on the LC methodology would result in ever-increasing mortality at middle ages with ever-decreasing mortality at young ages. Even more, would we manually set the overall mortality to decrease in projection (by choosing a proper scenario for the $k(t)$ function), that would automatically result in increasing projected child mortality. In such situations, when different age groups have exhibited significantly different trends, regressors other than the average log-mortality rate might be more useful.

In addition to these concerns about the structure of the model, study of correlations between variations of age-specific log-mortality rates also suggests that, perhaps, the framework of the LC model might be not the best one to address actual mortality dynamics.

To test more formally, whether underlying assumptions of the LC model are valid, one may study dynamics of age-specific log-mortality improvements, which may be presented under the LC model as follows:

$$\Delta(x,t) = \ln\left(\frac{m(x,t+1)}{m(x,t)}\right) = b(x)\cdot\Delta(k(t)) + \varepsilon(x,t+1) - \varepsilon(x,t), \tag{12}$$

where $\varepsilon(x,t)$ are independent error terms. Since the model suggests random walk-type dynamics for the $k(t)$ function, variations of age-specific mortality improvements should be significantly and evenly correlated for model to be valid.

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[8] Actually, nonlinear relation of log-mortality rate at young ages to $k(t)$ is a reflection of accelerated mortality decline at old ages, while mortality decline at young ages might be stable. Such a pattern of mortality dynamics may better be addressed when regressors other than $k(t)$ are used to model age-specific mortality, e.g. time index, standardized mortality rates or life expectancies. We discuss these possibilities separately, in the corresponding section of the work.

Empirical study, however, contradicts to this assumption. Fig. 2 depicts correlation coefficients between age-specific increments in log-mortality rates estimated from data on mortality of Austrian males in 1951-2005. When the long period, with pronounced overall changes in mortality is concerned, there is considerable correlation between mortality improvements at ages around 60 and above. However, even this kind of correlation does not support the LC model, which, at best, may be applied to ages beyond 60 only. When more recent period with more uniform mortality dynamics is addressed (from 1970 on), correlation is not significant enough even at higher ages.

This observation implies that the LC model is not relevant when, at least, short-term variations in mortality are concerned. It might be better to model directly age-specific mortality developments with taking explicitly the correlation between ages into account instead of using the method with random walk model for the mortality index $k(t)$. When trend extrapolation models are concerned, one may also look into weather assumptions of the LC model are consistent with medium-term developments of age-specific mortality trends. To do so, we present correlation coefficients between age-specific deviations of log-mortality rates from their linear trends in 1951-2005 for Austrian males (fig. 3). Correlations between residuals are more pronounced. Yet, correlations do not support the LC model assumptions even in this case. The model may reflect properly the overall mortality developments at ages around 30 and above (however, with a lack of fit to dynamics at ages around 50-55) in the period 1951-2005. When more recent time period (from 1970 on) is concerned, however, the model assumptions may be supported for ages around 75-90 only.

Figure 2 Correlation matrix for age-specific mortality reduction rates. Austria, males, unsmoothed data, 1951 to 2005

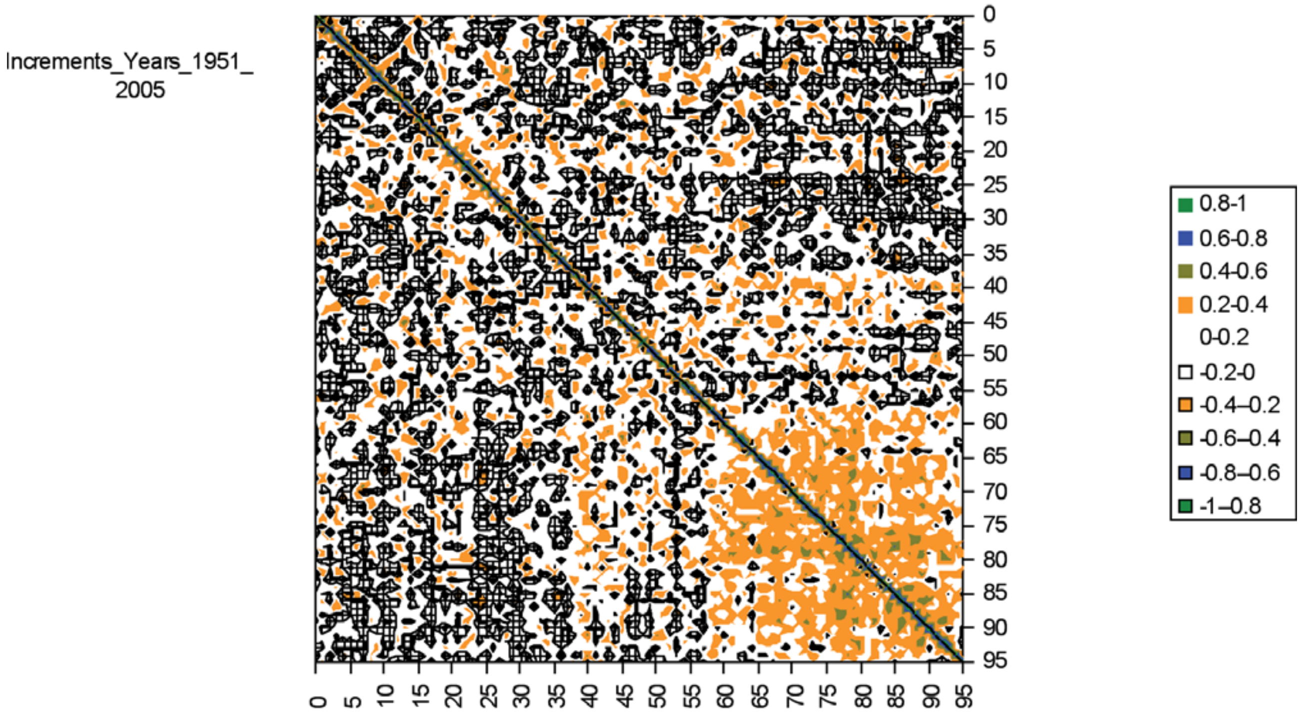
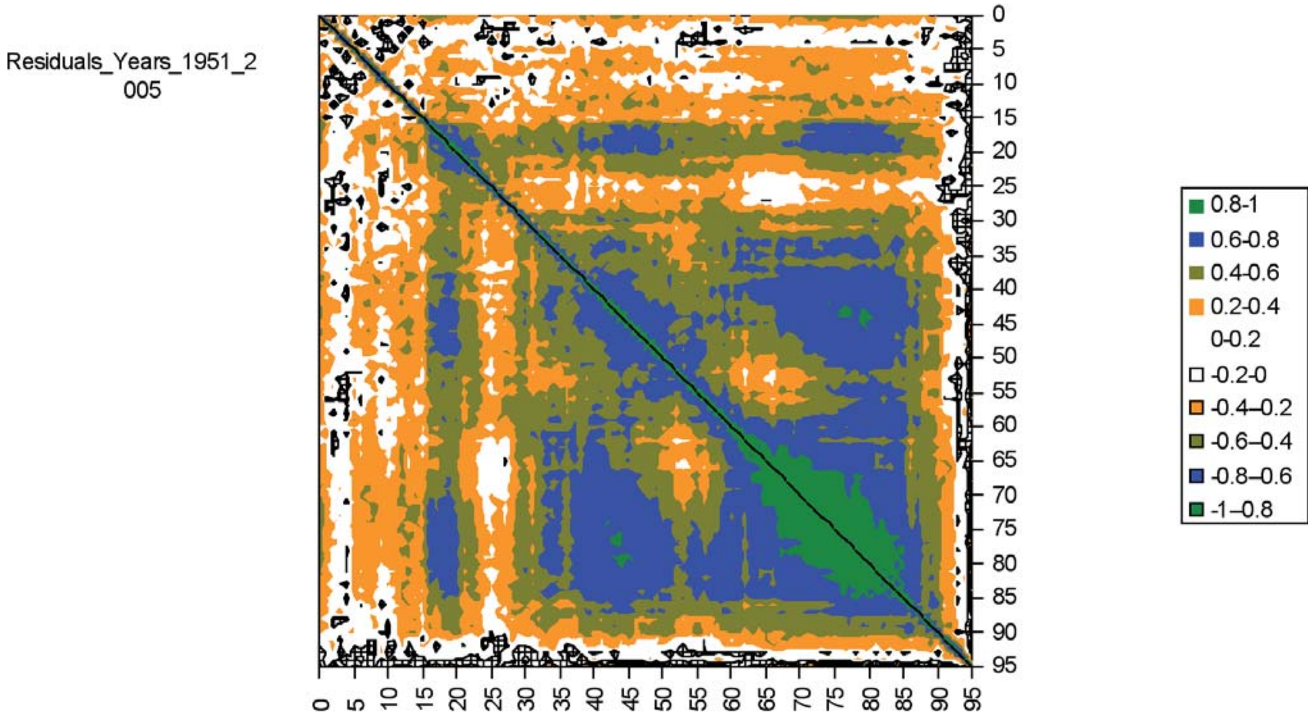


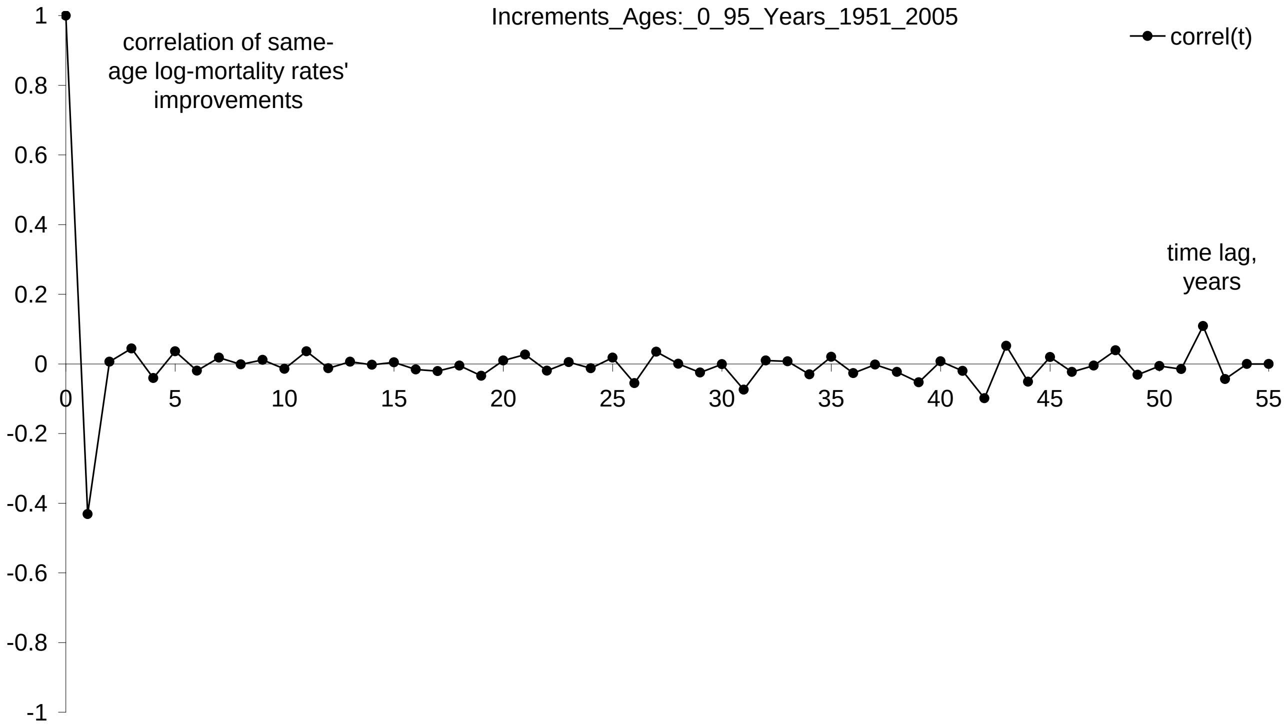
Figure 3 Correlation matrix for age-specific deviations of log-mortality rates from linear trend. Austria, males, unsmoothed data, 1951 to 2005



Based on the observations and concerns about the LC model presented, it might be a better option to look for other extrapolation techniques. Correlations between mortality improvements across time suggest that trend models, which assume existence of some steady trend of log-mortality rates with random and uncorrelated (at different points of time) deviations from it are perhaps more relevant to mortality dynamics than assumption about all the log-mortality rates being driven by a single mortality factor, which, in turn, follows random walk dynamics. Correlation coefficients between improvements in same-age log-mortality rates as the function of time elapsed between observations are presented on fig. 4. Results presented are obtained from all age-specific mortality improvements pooled into one sample; same patterns of correlations characterize dynamics of log-mortality rates for individual ages too. The only notable non-trivial correlation between changes in log-mortality rates is observed for adjacent years (time lag of one year), which is natural, as log-mortality rate at some year enters both in the mortality improvement of the same year and (with opposite sign) in the mortality improvement in the previous year. Hence, negative correlation between mortality reductions at adjacent years. Would there be no correlation at all between random terms in log-mortality rates at adjacent years, correlation between changes in log-mortality rates in adjacent years would be -0.5. In fact, it is somewhat higher due to slight positive correlation between mortality at adjacent years. We do not investigate further into possible causes of these correlations. These might be, e.g., correlation between deaths in one year with health in the previous one or assumptions on population at exposure used in estimating mortality coefficients, etc. In any case, correlations between mortality variations at adjacent years seem to be minor, while overall dynamics of log-mortality rates show existence of regular trends. Hence, simple regression models rather than random walk might be a better reflection of observed mortality dynamics. In these models different indicators might be used as independent variables or regressors. We discuss further down several such options. Simplest one might be to resume to separate (yet, not independent) linear extrapolation of age-specific log-mortality rates with some consistency adjustments. One may derive projections from independently estimated linear trends for age-specific log-mortality rates[9]:

[9] In some occasions *non-linear trends*, e.g., parabolic ones, might be more relevant. We do not consider them here, however, both due to more complicated consistency adjustments to be developed and also because linear trend model seem to be sufficient when the model is fit to relatively short recent period only. In particular, the linear model shows a good performance when $a(x)$ is fit to data for the latest available year.

Figure 4 Correlation coefficients between changes in same-age log-mortality rates as function of time lag between observations, with all the age-specific rates pooled together. Austria, males, unsmoothed data, 1951 to 2005



$$\hat{\eta}(x,t) = a(x) + b(x)t\,. \tag{13}$$

Here we denote by cups the trend values.

Usually, due to fluctuations and temporary variations in trends of log-mortality rates, age-specific intercepts and slopes $\hat{a}(x)$ and $\hat{b}(x)$ will show some random variations across age. As fluctuations of these parameters may be interpreted as not relevant to mortality trends, one may smooth them using either splines or moving averaging procedures. We use moving averages of five observations with no smoothing for starting and final points.

Examples of estimated (by OLS) and smoothed parameter functions for males are presented on figs. 5 and 6. Intercepts $a(x)$ simply present some basic age pattern of mortality. Slopes $b(x)$ reflect different paces of mortality improvement in past. Short-term random variations, seemingly caused by mere chance, are eliminated by smoothing. Despite this smoothing, however, model parameters may result in inconsistencies being used in projections without adjustments. To understand that, one may compare mortality reduction rates at, say, ages of 52 and 65. As it is seen from fig. 6, mortality was recently declining faster at age 65. Therefore, if this pattern is prolonged to remote future, it will result in lower mortality at age 65 than at age 52. Similarly, projected mortality for males might become lower than for females at some ages. Such inconsistencies arise if projection incorporates transient temporary developments in mortality dynamics which, in fact, are to disappear in future.

Figure 5 Age-specific intercepts a(x) of models (13): estimated and smoothed. Austria, males, unsmoothed data, 1970 to 2005

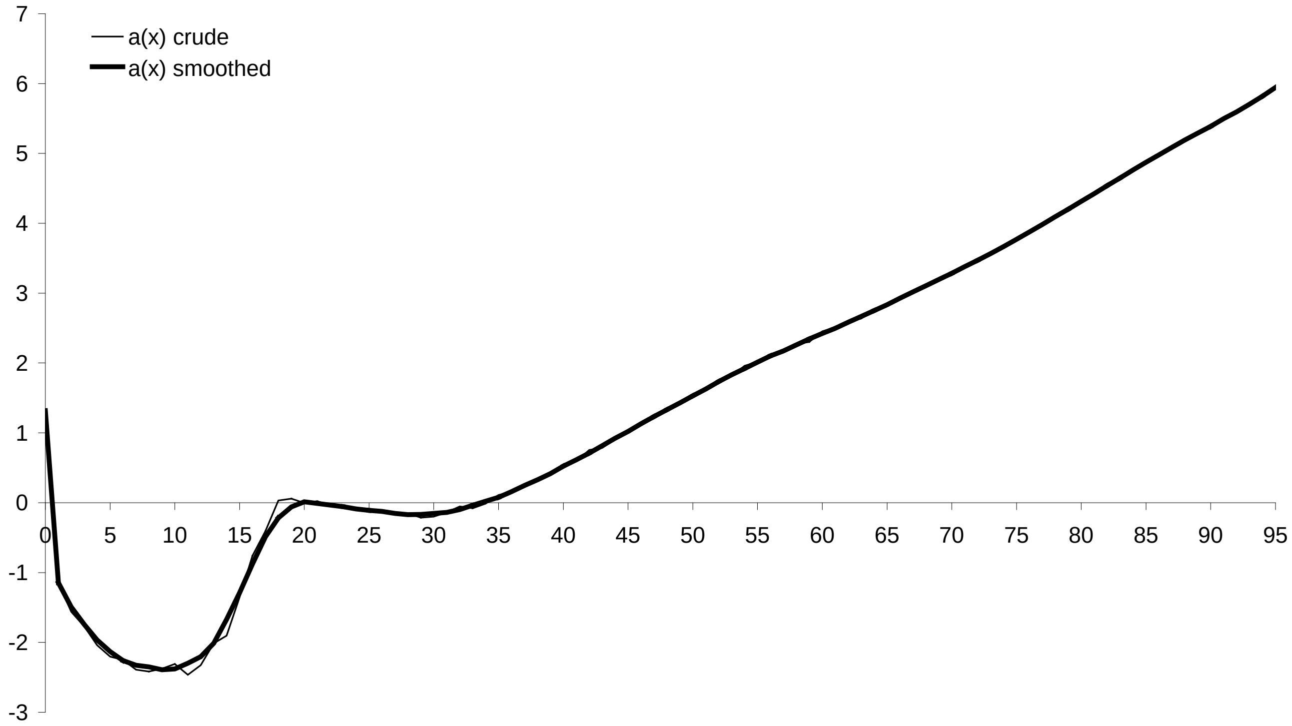
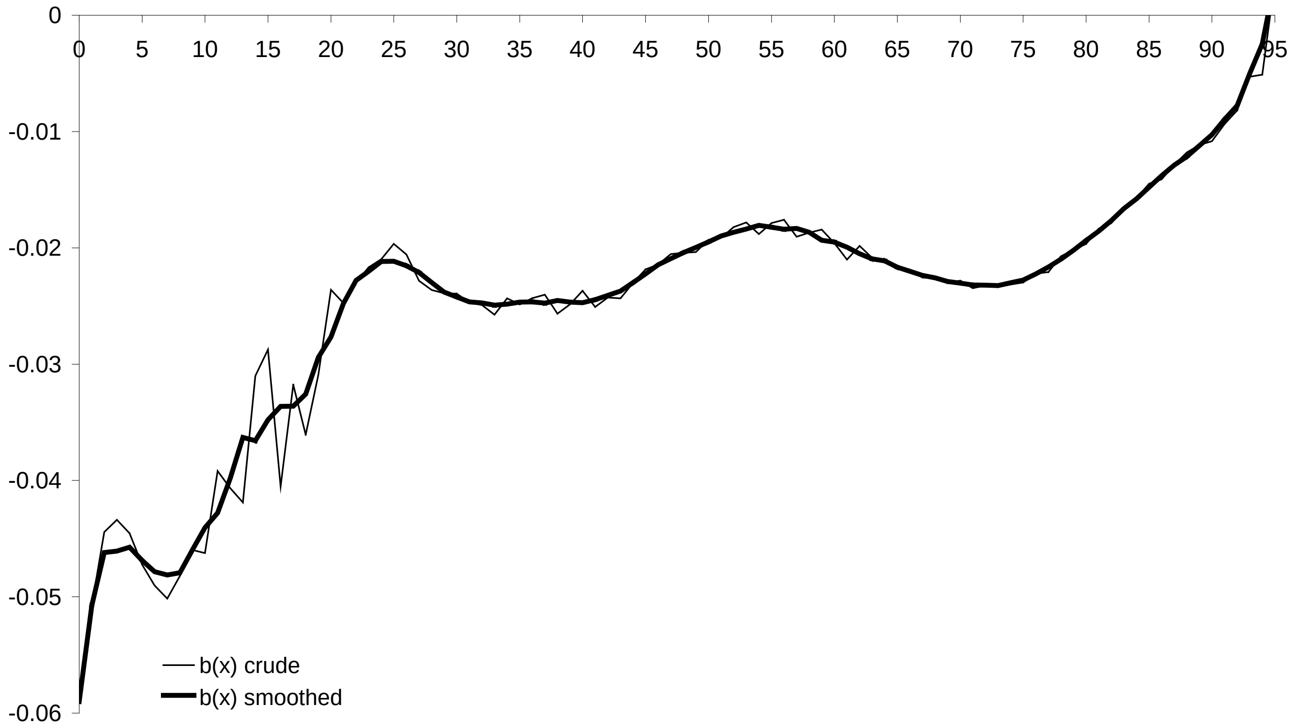


Figure 6 Age-specific slopes b(x) of models (13): estimated and smoothed. Austria, males, unsmoothed data, 1970 to 2005



In order to make the projection more consistent and relevant to long-term developments, we propose the same type of adjustments as were proposed in case of the LC model. In particular, we propose to differentiate between two parts of mortality improvements – one that persists in long run and another, which is temporary:

$$\hat{\eta}(x,t) = a(x) + b_0(x)t + b_1(x)v(t). \tag{14}$$

$b_0(x)$ are long-term slopes, which are to be used in the projection without any further adjustments and, hence, should not result in inconsistencies. $b_1(x) = b(x) - b_0(x)$ are 'inconsistent' parts of estimated slopes (i.e., observed in recent past), which may not be used in long-run and are of transitory nature. $v(t)$ is a function used to model transition from recently observed mortality trends to eventual (consistent) trends. At the starting point of projection, as well as in the data fitting period, this function equals to the main regressor (time index t in the case addressed here) but later on, its change rate decreases and, eventually, $v(t)$ converges to some asymptotic value. As in the case of adjusting the LC projections, we propose the following model for the transition function:

$$v(t_0) = t_0,$$
$$\Delta v(t) = \Delta t \cdot e^{-(t-t_0)/\tau} = e^{-(t-t_0)/\tau},\ t \geq t_0, \tag{15}$$

where $\Delta v(t) = v(t+1) - v(t)$, t_0 is the beginning of the projection period, and τ is a convergence parameter.

As for separating consistent and inconsistent parts of the slopes $b(x)$, it might be done in several ways. Based on observed faster mortality improvements at younger ages, one may assume, for example, the consistent long-term slopes $b_0(x)$ to monotonically increase with age (being negative, i.e., to monotonically approach closer to zero at higher ages)[10]. Such a profile for long-term mortality dynamics will usually be consistent, as it will not result in mortality at old ages being lower than that at younger ages, if that was not the case in recent observations. This assumption is supported in the aforementioned case of Austria by the overall tendency of estimated slopes $b(x)$ to increase with age (see fig. 9). Similarly, one may assume that long-term mortality improvements for male population are to be lower in magnitude compared to those for female population.

Based on monotonic assumption of $b_0(x)$, this function may be derived, e.g., by applying some monotonic splines to $b(x)$. We use here another approach, which is based on deriving upper and lower bounds for $b_0(x)$ and deriving the function as smoothed average of both bound estimates[11]:

$$b_0^{MAX,males}(x) = \max_{y \leq x}\left(b^{males}(y), b^{females}(y)\right),\quad b_0^{MAX,females}(x) = \max_{y \leq x}\left(b^{females}(y)\right),$$
$$b_0^{MIN,males}(x) = \min_{y \geq x}\left(b^{males}(y)\right),\quad b_0^{MIN,females}(x) = \min_{y \geq x}\left(b^{males}(y), b^{females}(y)\right). \tag{16}$$

Then, for each sex, the central estimate for $b_0(x)$ is a smoothed average of both upper and lower bound estimates:

$$b_0(x) = SmoothedAverage\left(b_0^{MAX}(x), b_0^{MIN}(x)\right). \tag{17}$$

Apart from the time variable, other indexes may also be used as independent variables in trend models for the dynamics of log-mortality rates. In particular, life expectancy at birth might be an important option, both due to its relevance to the mortality level and also to facilitate non-divergent multiregional projections, when life expectancies achieved by populations more advanced in mortality reductions are used as targets for mortality projections at other populations.

When life expectancy rather then the time index is used as a predictor, the gap between upper and lower estimates for $b_0(x)$ becomes narrower and projection becomes more robust. This is natural, as life expectancy at birth already bears some important information about mortality level. However, two shortcomings are to be taken into account. Firstly, life

10 Indeed, this must apply, at least, to medium and old ages. In the simplified method proposed here we assume monotonicity for all the age scale, although this may not be the case for all populations.

11 Similar to the case of the LC model, the method proposed here seems to work more or less effectively when the $b(x)$ function is already nearly monotonic. This might not be the case, when a very short data period or a period with no sustaining trends of age-specific mortality dynamics are concerned. Also, some expert judgment about $b_0(x)$ is unlikely to be avoided.

expectancy at birth should be projected first to be used as a predictor of age-specific death rates. Secondly, dynamics of the life expectancy at birth might be relevant to mortality dynamics at ages with high and changing mortality only, while it might be less sensitive to changes in mortality at ages with very low mortality or when these changes are small compared to changes at other ages. In a cense, usage of life expectancy at birth is opposite to using the average log-mortality rate (which is the case for the LC model) as a mortality predictor, as in the first case more stress is put on ages with high and rapidly decreasing mortality, while changes in average log-mortality are weighting all the ages evenly and, hence are biased towards middle and young ages with low mortality.

Another problem with using the life expectancy at birth as predictor for mortality in (18) is that the life expectancy used as an input variable of the model and the other one, which is computed from projected age specific rates may differ substantially[12]. Perhaps, this problem is not of major importance, as input estimates for life expectancy may be neglected. Also, they may be iterated until the projected life expectancy converges to the necessary level. Alternatively, regressions (18) may be developed in terms of age-specific life expectancies.

Another regressors, which might be used as mortality predictors are: standardized mortality rates, weighted averages of age-specific mortality or log-mortality rates, etc. Choice of proper regressor depends on its availability, on ability to reliably project it, and on relevance of its dynamics to mortality developments at ages of interest. Despite variety of choices, our study shows that different extrapolative models – being estimated on the same data – usually result in nearly the same projections.

3. References

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[12] In case of Austrian males, e.g., difference between linearly extrapolated life expectancy at birth and one, which is obtained from using this value as inpu in (7) is about 2.2 years, i.e., about 2.7%.

APPENDIX. ESTIMATION PROCEDURES.

The LC model as well as alternatives proposed above allows a certain degree of freedom in choosing the parameters, as these models are over-parameterized. We propose scaling assumptions the models, which are consistent with those proposed by Lee and Carter (1992):

$$\overline{k(t)} = \overline{v(t)} = \overline{\beta(x)} = 0, \tag{B1}$$

hereinafter upper lines denote averages across the data used to estimate the model parameters. Given assumptions (B1), it is easy to derive the following estimates for general model (16) as well as for the LC model:

$$a(x) = \overline{\eta(x,t)}^{(t)}, \tag{B2}$$

$$k(t) = \overline{\eta(x,t) - a(x)}^{(x)} = \overline{\eta(x,t) - \overline{\eta(x,t)}^{(t)}}^{(x)}, \tag{B3}$$

where we indicate the variable across which the average is taken next to the averaging line. Another option, well discussed in the literature, might be to set parameters in such a way, that the model fits exactly to the last observed year:

$$a(x) = \eta(x, t_0), \tag{B4}$$

$$k(t) = \overline{\eta(x,t) - a(x)}^{(x)} = \overline{\eta(x,t)}^{(x)} - \overline{a(x)}. \tag{B5}$$

Having obtained estimates (B2)-(B5), we may write the following expression:

$$\delta(x,t) \stackrel{def}{=} \eta(x,t) - (a(x) + k(t)) = v(t)\beta(x) + \varepsilon(x,t), \tag{B6}$$

where $\varepsilon(x,t)$ are final residuals. Note that the same may be applied to the LC model after noting that $b(x) = 1 + \beta(x)$. In principle, the rest of parameters may be derived using the SVD procedure. However, one may use more simple procedures described further down. We suppose that improvements, if any, which might be obtained by replacing the simplified procedure by the SVD, are minor compared to the structural biases of the LC model itself.

In the simplified procedure we assume that residuals $\varepsilon(x,t)$ do not correlate to $k(t)$ function and propose the following estimates for the $\beta(x)$:

$$\beta(x) = \frac{\overline{\delta(x,t)k(t)}^{(t)}}{\overline{k^2(t)}^{(t)}}. \tag{B7}$$

Finally, we estimate the $v(t)$ in order to better fit the data in terms of minimal sum of residuals' squares (i.e., applying the OLS procedure):

$$v(t) = \frac{\overline{\delta(x,t)\beta(x)}^{(x)}}{\overline{\beta^2(x)}^{(x)}}. \tag{B8}$$

The procedure described–apart from its simplicity–has an advantage that it is fully consistent with the simplified alternative to SVD method proposed by Lee and Carter for estimating the parameters of their model. In particular, estimates (B2) and (B3) will be identical to those obtained for the Lee-Carter model. The $\beta(x)$ function (B7) will be identical to the $b(x)$ in the Lee-Carter model after subtracting the average value of the last one:

$$\beta(x) = b(x) - \overline{b(x)} = b(x) - 1. \tag{B9}$$

The only difference is in $v(t)$ function: the Lee-Carter approach proposes setting it identical to the $k(t)$ function, while in general this function–estimated from (B8)–may deviate from $k(t)$, which might be crucial for projections as it is discussed in the main part of the work.

In fact, $v(t)$ and $k(t)$ are close to each other if residuals (B6) at different years are not correlated:

$$v(t)=\frac{\overline{\delta(x,t)\dfrac{\overline{\delta(x,\tau)k(\tau)}^{(\tau)}}{\overline{k^2(\tau)}^{(\tau)}}}^{(x)}}{\overline{\left(\dfrac{\overline{\delta(x,\tau)k(\tau)}^{(\tau)}}{\overline{k^2(\tau)}^{(\tau)}}\right)^2}^{(x)}}=\overline{k^2(\tau)}^{(\tau)}\frac{\overline{\delta(x,t)\overline{\delta(x,\tau)k(\tau)}^{(\tau)}}^{(x)}}{\overline{\left(\overline{\delta(x,\tau)k(\tau)}^{(\tau)}\right)^2}^{(x)}}=$$

$$\overline{k^2(\tau)}^{(\tau)}\frac{\sum_{\tau}k(\tau)\sum_{x}\delta(x,t)\delta(x,\tau)}{\overline{k(\tau)\sum_{t1}k(t_1)\sum_{x}\delta(x,t_1)\delta(x,\tau)}^{(\tau)}}\approx\overline{k^2(\tau)}^{(\tau)}\frac{k(t)}{\overline{k^2(\tau)}^{(\tau)}}=k(t). \qquad \text{(B11)}$$

An alternative procedures for $b(x)$ and $\beta(x)$ functions may be derived from observation that it equals to some weighted average of changes in log-mortality rates per change in $k(t)$ function:

$$b(x)=\frac{\overline{\delta(x,t)k(t)}^{(t)}}{\overline{k^2(t)}^{(t)}}+1=\frac{\overline{\delta(x,t)k(t)+k^2(t)}^{(t)}}{\overline{k^2(t)}^{(t)}}=$$

$$\frac{\overline{(\eta(x,t)-a(x))k(t)}^{(t)}}{\overline{k^2(t)}^{(t)}}=\frac{\overline{\dfrac{\eta(x,t)-\eta[k=0]}{k(t)-0}k^2(t)}^{(t)}}{\overline{k^2(t)}^{(t)}}. \qquad \text{(B12)}$$

This relation may be put into more explicit form if one considers increments of log-mortality rates:

$$\Delta\eta(x,t)=\eta(x,t+1)-\eta(x,t)=\Delta k(t)+\Delta v(t)\beta(x)+\Delta\varepsilon(x,t)\approx\Delta k(t)b(x)+\Delta\varepsilon(x,t). \qquad \text{(B13)}$$

Based on this relation, one may propose the following alternative procedures for estimating the $b(x)$ (and, hence, the $\beta(x)=b(x)-1$) function:

$$b(x)=\frac{\overline{\Delta\delta(x,t)\Delta k(t)}^{(t)}}{\overline{(\Delta k(t))^2}^{(t)}}=\frac{\overline{\dfrac{\Delta\delta(x,t)}{\Delta k(t)}(\Delta k(t))^2}^{(t)}}{\overline{(\Delta k(t))^2}^{(t)}}. \qquad \text{(B14)}$$

This method might be statistically more sound as it weights higher those periods with more significant changes in the overall level of mortality, while (B7) and (B12) weight observations according to the cumulated change of mortality from some reference level, the latter corresponding to $k(t)=0$.

From the aforementioned link of $b(x)$ to changes in age-specific log-mortality rates as compared to the changes in the overall level of log-mortality, one may also develop the following simple procedures:

$$b(x) = \frac{\delta(x,t_2) - \delta(x,t_1)}{k(t_2) - k(t_1)}, \qquad \text{(B15)}$$

here t_1 and t_2 - some time points from within the period covered by data. In case of using (B15), the LC model will exactly match mortality profiles both at t_1 and t_2.

MORTALITY AND LONGEVITY PROJECTIONS FOR THE OLDEST-OLD IN PORTUGAL

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* *The analyses, opinions and findings contained in this paper represent the views of the authors and are not necessarily those of Statistics Portugal.*

ABSTRACT

The mortality decline observed in developed countries over the last decades significantly increased the number of those surviving up to older ages. Mortality improvements are naturally viewed as a positive change for individuals and as a substantial social achievement for societies, but create new challenges in a number of different areas, ranging from the planning of all components of social security systems to labour markets. Understanding mortality and survival patterns at older ages is crucial. In this paper, we compare the results provided by a number of different methods designed to project mortality for the oldest-old in the Portuguese population. We identify the merits and limitations of each method and the consequences of their use in constructing complete life tables.

Keywords

Mortality, life tables, projection models, life expectancy

1. Introduction

Life expectancy at birth more than doubled in Portugal during the XX century. Based on all available demographic databases, historical trends show that both average and the maximum lifetime have increased gradually during the 20th century, with human life span showing no signs of approaching a fixed limit imposed by biology.

As in other developed countries, the mortality decline has been dominated by two major trends: a huge reduction in mortality due to infectious diseases affecting mainly young ages, more evident during the first half of the century, and a decrease in mortality at older ages, more pronounced during the second half. As a consequence, the number of those surviving up to older ages (e.g., 80 years and above) has increased significantly representing, in 2006, 4.9% (2.9%) of the Portuguese female (male) population.[1] Additionally, the number of deaths of the oldest-old accounts for an increasing proportion of all deaths, with reductions of mortality beyond these ages having a growing contribution to future gains in life expectancy.

Mortality improvements are naturally viewed as a positive change for individuals and as a substantial social achievement for developed countries. Nonetheless, this change poses a serious challenge in a number of different areas, ranging from the planning of all components of social security systems (e.g., public and private retirement systems, health care systems) to labour markets and economic models. In the insurance market, mortality improvements have an obvious impact on the pricing and reserving for any kind of long-term living benefits, particularly on annuities.

In view of these trends, it is important not only to have a clear understanding of mortality and survival patterns at older ages, namely about their age structure, but also about the population dynamics to which they are subject. In Portugal, as in most countries, population estimates and projections produced by the Statistics Portugal (INE – Instituto Nacional de Estatística) do not provide an age breakdown for the group aged 85 years and older. Although rough data on population estimates at these ages is available, except for censuses years they are considered unreliable, being biased by poor age reporting regarding both those alive and who die. Because of this, crude estimates of mortality rates by single year for people aged 85 and over may lack the required quality demanded for the construction of complete life span lifetables. To solve this problem we have to resort to projection models that describe appropriately the main mortality trends observed at these ages.

In recent years, the improvement of statistical data shed light on an apparently unexpected behaviour of mortality at advanced ages. Effectively, empirical evidence (e.g., Horiuchi and Wilmoth, 1998; Olshansky and Carnes, 1997; Wilmoth, 1995; Bongaarts, 2004; Gallop and Macdonald, 2005) shows that the rate of mortality increase at very old ages is neither increasing nor constant, but rather tends to decelerate from a certain age. In particular, the curve of mortality rates, in logarithmic scale, presents a concave shape for high ages, leading to a sort of “plateau”. This mortality pattern diverges from that stipulated, e.g., by the classical Gompertz law, for which mortality increases exponentially with age. The simple Gompertz law has proven to be a remarkably good model in different populations and in different epochs, and many subsequent mortality laws are indeed modifications of it, made to account for known deviations, for example, at very old ages. However, these latest developments reveal that the model can no longer provide a good description of data.

Various methodologies have been proposed for estimating mortality rates at oldest ages. Some generate population numbers from death registrations, which for the purpose of estimating the number of very old people are considered to be more reliable than population estimates derived from censuses. The most popular methods included in this category are the method of extinct generations and the survivor ratio method.

Other methodologies include fitting mortality curves over a certain age range, for which crude mortality rates may be calculated directly from data, followed by extrapolation. The incapacity of classic mortality laws to represent the modern behaviour of mortality at advanced ages, with a clear departure from trajectories generated by the classical Gompertz model has, in recent years, attracted the attention of researchers in both the demographic and actuarial area towards the development of alternative formulations that better capture this phenomenon. The number of alternative models proposed in the literature is vast and growing (see, e.g., Boleslawski and Tabeau (2001), Buettner (2002) and Pitacco (2004)). Among these, we investigate in this paper two versions of the logistic model, namely those suggested by Perks (1932) and Kannistö (1992), the method of Coale and Kisker (1990) and the recently proposed method of Denuit and Goderniaux (2005).[2]

The objective of this study is two-fold: (i) to compare the merits and disadvantages of different methods used to extrapolate mortality rates at older ages (ii) to measure the impact of these models in terms life expectancy calculations, briefly discussing the consequences of their use in mortality and longevity projections. The database used in this study was provided by Statistics Portugal and comprises the observed number of deaths given by age and year of birth, and the observed population size at December 31 of each year.

[1] According to the United Nations (2001), it is estimated that in 2001 the population of the oldest-old (i.e., those 80 years and older) represents 1.2% of the 6.1 billion inhabitants of the world, being the fastest growing segment of the population.

[2] Other formulations such as the Heligman-Pollard (1980) model, the logit model (Brass, 1971) and the Lindbergson (2001) method were reviewed but finally not tested.

The paper is organized as follows. Section 2 presents a brief characterization of the evolution of mortality patterns in Portugal. Section 3 gives an overview of the available population and death data and discusses some of the problems encountered in estimating mortality rates at old ages. Section 4 describes a number of approaches in estimating mortality rates at advanced ages. Section 5 presents the results and discusses the ability of each model in describing the mortality at these ages. Finally, Section 6 provides a short conclusion.

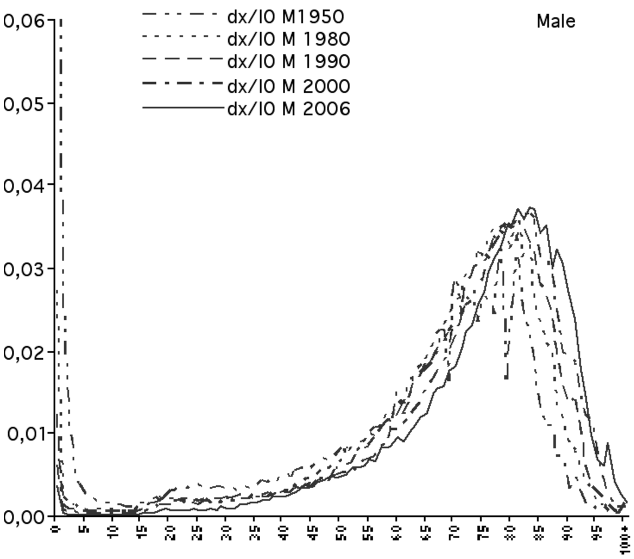
2. Mortality trends in Portugal

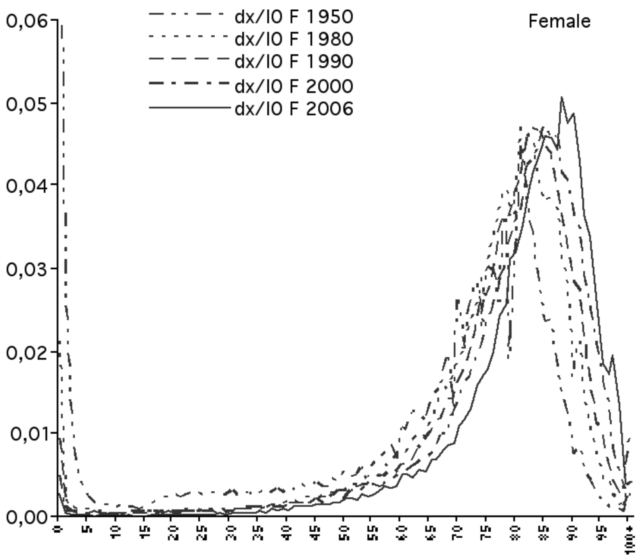
The XX century saw dramatic reductions in mortality rates at all ages in Portugal. Two major trends dominated the mortality decline during the last century: a huge decrease in infant mortality, more evident during the first half of the century, and a decrease in mortality at older ages, more pronounced during the second half. This clear change in mortality patterns led to an increase in the number of those surviving up to ages 80-85 years, with reductions of mortality beyond these ages having a growing contribution to future gains in life expectancy.

In 1950, around half of all deaths occurred under age 50. In the same year, infant mortality accounted for 19% of all deaths, whereas deaths at age 80 and over comprised only 13.8%. In 2006, there were 101 990 deaths of resident individuals in Portugal, the majority of which (81%) occurred at age 65 and over, whereas 47% of all deaths were registered at ages 80 and above. Deaths at ages under one year (infant mortality) represented only 0.3%.

In Figure 1 we represent the distribution of the number of deaths at different ages (i.e., of the graph of the function $x \mapsto d_x / l_0$) for a number of selected moments from 1950 to 2006. In both sexes, we can observe an increasing concentration of deaths around the mode (age of maximum mortality) of the distribution, as well as a shift in the mode towards older ages. As a result, Figure 2 shows a progressive rectangularization and expansion of the survival curve, most noticeably for the female population.

Figure 1 Curves of deaths ($x \mapsto d_x / l_0$), Portugal

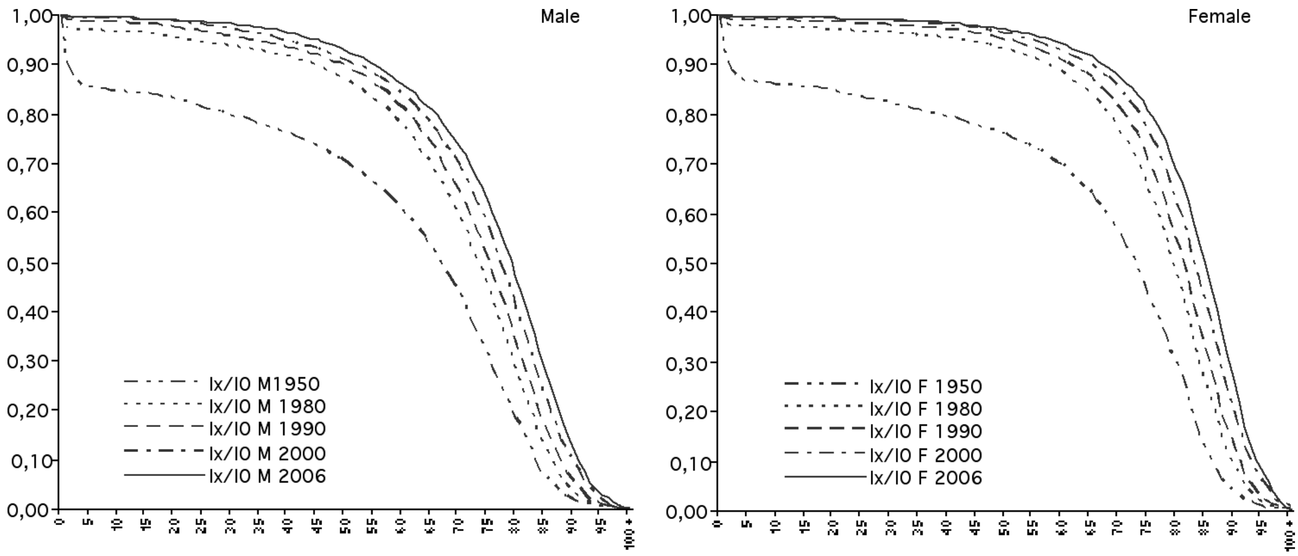




Source: Author's calculation based on data from Statistics Portugal, 1950-2006

The general downward trend in mortality rates at almost all ages means that an increasing proportion of the members of a given generation lives up to very old ages (around 70-75 years), shifting the survival function upwards and to the right to a more rectangular shape. At the same time, we can observe that the age of maximum mortality gradually shifted towards older ages (around 83 years for men and 88 years for women), in what is sometimes called the expansion phenomenon of the survival curve.

Figure 2 Survival function ($x \mapsto d_x / l_0$), Portugal



Source: Author's calculation based on data from Statistics Portugal, 1950-2006

To measure the importance of the rectangularization and expansion phenomena in the Portuguese population, we have calculated a number of indicators, namely the variance of residual lifetime, $Var[T]$, the corresponding coefficient of variation, $\sqrt{Var[T]} / E[T]$, the «entropy» of the survival curve (as defined by Keifitz (1985)), the median future lifetime and the inter-quartile range. Our results show, for both genders, a significant decrease in both the variance and the coefficient of variation of future lifetime T, a major decline in the entropy of the survival curve, a huge increase in the median future lifetime and a significant tightening of the inter-quartile range. All of this confirms the importance of the rectangularization and expansion phenomena in the Portuguese population.[3]

Table 1 shows the evolution of life expectancy at birth, $e_0(t)$, and at age 65, $e_{65}(t)$, over the period 1950-2006. The huge gains in life expectancy are evident for both men and women. Life expectancy at birth increased from 56.2 years for men and 61.2 years for women to 75.2 and 81.8 years, respectively. Men can now expect to live a further 19.0 years and women a further 20.5 years if mortality rates remained as estimated by 2006 life tables. It should be noticed, however, that gains in life expectancy were not uniform over this period, with the most expressive improvements being registered in the first 30 years of this sample. The rate of increase in life expectancy tends to slow down, mainly because future progresses will be achieved primarily by declines in mortality among older segments of the population.

Table 1 Life expectancy at birth and at 65 years old, Portugal

Year	$e_0(t)$		$e_{65}(t)$	
	Male	Female	Male	Female
1950	50,16	61,23	11,95	13,84
1960	60,85	66,38	12,54	14,71
1970	63,99	70,25	12,15	14,56
1980	67,81	74,81	13,10	16,06
1990	70,62	77,52	14,03	17,11
2000	72,89	79,90	15,18	18,64
2005	74,90	81,39	16,16	19,55
2006	75,15	81,75	16,31	19,78

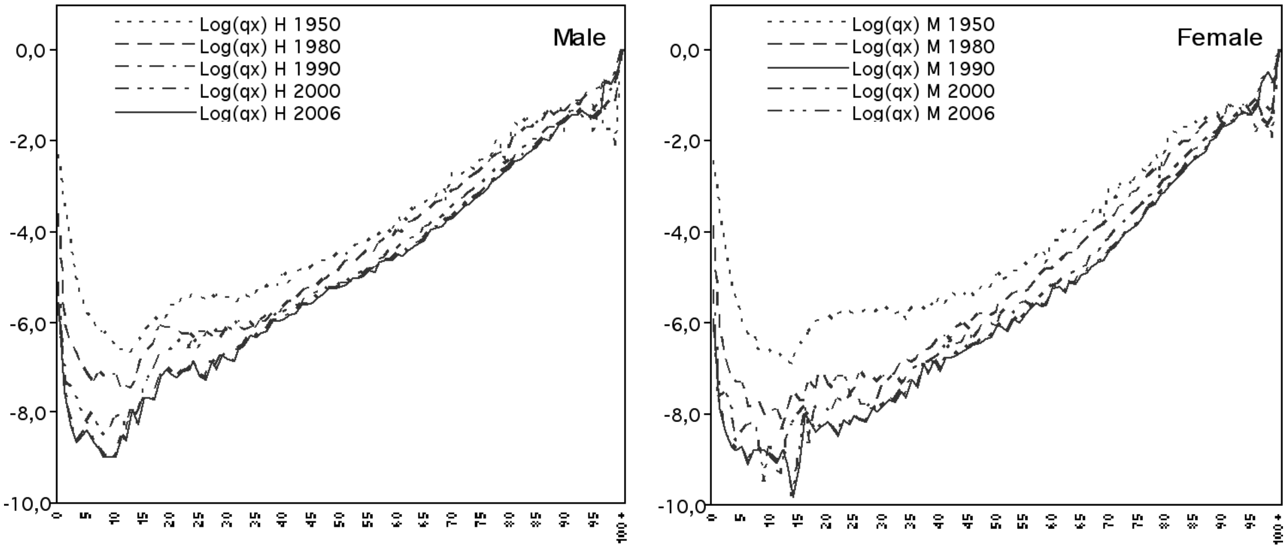
Source: Statistics Portugal, 1950-2006

[3] The complete set of results is not reported here due to space constraints but can be obtained from the authors upon request.

In Figure 3 we represent the mortality rates for a number of selected periods. The mortality hump at young ages, which represents the mortality due to accidents and violent causes of death, tends to spread over and to lose some importance both for men and women. It is represented by probabilities q_x particularly significant at ages between 15 and 30 years old, in consequence of the increased risk of violent deaths, most noticeably among the male population. Note also that by age 25-30, mortality follows its inevitable increasing trajectory with age at a more regular rhythm.

Finally, we note that the aged population differs significantly from the general population, others things being equal because of the proportion of women that comprises it. Effectively, even when there is a balanced distribution among male and female new births, gender differences in mortality translate into a preponderance of women at older ages. This proportion increases with age. In Portugal, as in most developed countries, the average gap in life expectancy between the sexes is roughly 6.6 years at birth and 3.5 years at age 65.

Figure 3 Mortality rates ($x \mapsto \ln(q_x)$), Portugal



Source: Author's calculation based on data from Statistics Portugal, 1950-2006

3. Data sources

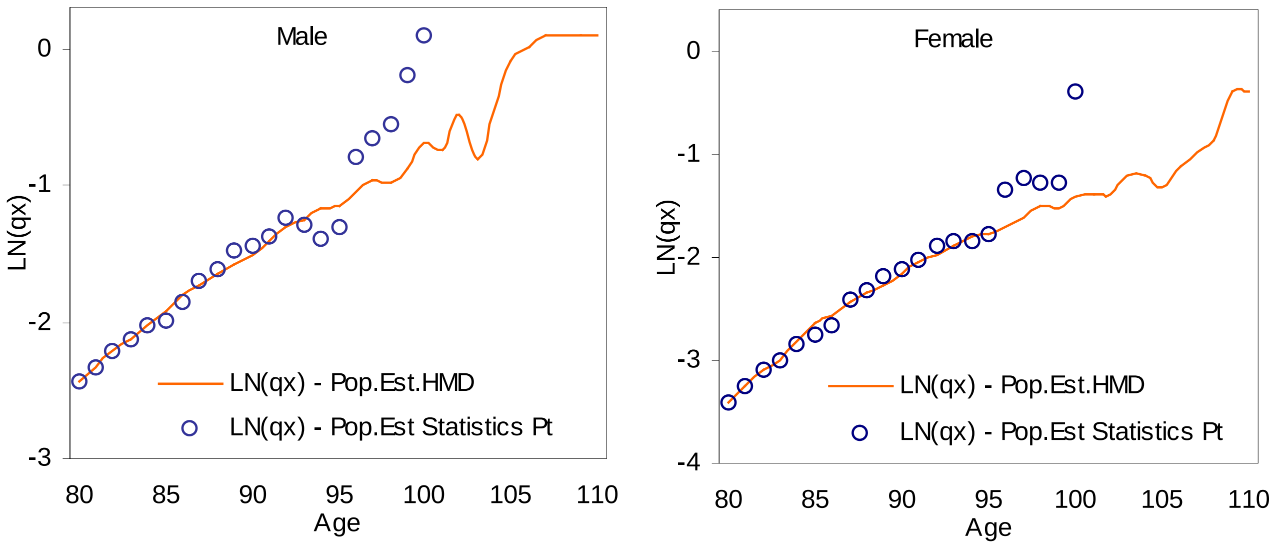
The database used in this study comprises two elements: the observed number of death $d_{x,t}$ given by age x, year of death t and, from 1980 onwards, also by year of birth, and population estimates $P_{x,t}$ at December 31 of each year. The data, discriminated by age ($x \in [0,99]$) and sex, refers to the entire Portuguese population and has been supplied by Statistics Portugal.

Deaths statistics are based on information collected at death registration by Civil Registration Offices. The declaration of death is a legal requirement, compulsory in Portugal since 1911 and based on official documents. The registry is exhaustive and the data are thought to be reliable, even for old people. However, for the very old some inconsistencies may persist in reporting age due to birth register problems.

The decennial population censuses provide the base figures from which official national resident population estimates for Portugal are derived. The latest census was carried out in 2001. Annual population estimates at 31st December, by age and sex, are obtained by rolling forward the estimates produced after a census using data on subsequent births, deaths and net migration. These rolled-forward estimates are generally subject to increasing error as they move further away from the last census. Population estimates are produced by sex and single year of age up to 99. However, they are officially disclosed only up to age 84, together with total figures for the group aged 85 years and older. This is justified by the fact that these figures may be heavily contaminated by random fluctuations, due to the small number of those surviving up to very old ages as well as to the probable misreporting of ages occurred at the censuses.

In Portugal, the calculation of crude age specific mortality rates at advanced ages suffers from several problems. The main issue concerns the quality and the availability of data on population estimates for ages 85 and above. Effectively, although data on deaths are in general of good quality, mortality rates may suffer from some inconsistencies at these ages, given that coherence between deaths and the number of those exposed at risk may not exist. Another concern refers to the degree of volatility observed in age-specific mortality rates at very old ages, a feature which affects the statistical significance of the results. To illustrate these problems, Figure 4 represents the mortality rates at age 80 and over for the period 2004-2006 based on official estimates and based on population estimates provided by the Human Mortality Database (HMD). Recall that the later are computed using the methods of extinct generations and almost extinct generations.[4]

Figure 4 Mortality rates for the oldest-old



Source: Author's calculation based on data from Statistics Portugal and Human Mortality Database (www.mortality.org)

Age specific probabilities of death q_x are calculated using three-year periods, by pooling deaths and exposures first and then dividing the former by the latter. Consider a two-year birth cohort in the age interval from x to $x+1$. Let $\dot{P}$ denote the sum of the January 1st population estimates for the two individual birth cohorts when they are aged x. Likewise, let $\dot{D}_L$ and $\dot{D}_U$ denote the sums of lower and upper triangle deaths within the same age interval for the same group of cohorts. Therefore, the probability of death for this two-year cohort is:

$$q_x = \frac{\dot{D}_L + \dot{D}_U}{\dot{P} + \dot{D}_L} \qquad (1)$$

The death rates m_x are calculated by first computing the corresponding exposure-to-risk under the assumption that deaths are distributed uniformly within Lexis triangles.[5]

[4] Recall that the method of extinct generations uses the available information on the number of deaths, by age and year of birth, from the death registrations, to reconstruct the surviving population, without using the census population at all. It is based on the assumption that when all the members of a given generation (people born in a given calendar year) have died, it is possible to reconstruct the numbers who were alive earlier, if the dates of death of everyone in that generation are known. This assumes that international migration can be ignored, which is the case if we confine to ages high enough so the migration flows are negligible. In practice, it is not necessary to wait until all the members of the generations concerned have died. By the time that the members of a generation have reached a given age (e.g. 100 years), only a small proportion of its original members will still be surviving and this proportion, that is, the ratio of the number of survivors who are still living to the members in the generation who died during the last years, can be estimated from the experience of previous generations. Multiplying this "survivor ratio" by the number of deaths that have occurred in a given generation during the last years, it is possible to obtain an estimate of the corresponding number of those who are still alive (survivors). Then, it is possible to reconstruct the past population by adding the estimated number of survivors to the number of registered deaths, generation by generation. This iterative procedure of reconstruction should be calibrated so that the population estimates obtained applying this method coincides with the values of the official organisms for the established maximum age (e.g., 85 years in Portugal).

[5] For more details see Wilmoth et al. (2005).

As we can observe, Figure 4 exhibits visible random fluctuations in mortality rates above age 92 for males and above 95 for females. Moreover, mortality rates based on population estimates calculated according to the method of extinct generations tend to diverge significantly at advanced ages, signalling data problems on official population estimates at these ages. In order to construct complete life tables, it was decided to remove fluctuations by smoothing crude estimates via a projection method.

4. Approaches in estimating mortality rates at advanced ages

4.1 The Coale-Kisker Method

The Coale-Kisker method, named after Coale and Guo (1989) and Coale and Kisker (1990), assumes that the exponential rate of mortality increase at very old ages is not constant, as stipulated by the classical Gompertz model, but declines linearly, a pattern empirically confirmed by the authors of this paper in Portugal and by a number of studies (e.g., Horiuchi and Wilmoth, 1998).[6] The Coale-Kisker method establishes that:

$$m_x = m_{79} \exp\left(\sum_{i=80}^{x_{\max}} k_i\right), \quad 80 \le x \le x_{\max} \tag{2}$$

where k_i denotes the rate of mortality increase (defined by $k_x = \ln(m_x / m_{x-1})$) at age x and $x_{\max}$ is the highest attainable age considered (110 years in their case). Coale and Kisker (1990) assume that k_x is linear above a certain age, 80 years in this case, that is:

$$k_x = k_{80} + (x-80)\cdot s, \quad x \ge 80 \tag{3}$$

In order to determine the slope coefficient s, the authors set an arbitrary value for m_{110}, namely $m_{110} = 1.0$ for males and $m_{110} = 0.8$ for females. The mortality differential by sex at age 110 was intentionally chosen to avoid a crossover between male and female mortality at advanced ages.

By inserting (3) into (2) and solving for s, we obtain

$$s = -\frac{\ln(m_{79}/m_{110}) + 31k_{80}}{465} \tag{4}$$

Mortality rates at age 80 and above are finally estimated by:

$$m_x = m_{79} \exp\left[\sum_{i=80}^{x} \left(k_{80} + (i-80)\cdot s\right)\right], \quad x \in \{80, 81, \ldots, 109\} \tag{5}$$

or simply by:

$$m_x = m_{x-1} \exp\left[k_{80} + (x-80)\cdot s\right], \quad x \in \{80, 81, \ldots, 109\} \tag{6}$$

The method assumes that the observed death rates around $x = 80$ are reliable and that k_{80} can be calculated from empirical data. In practise, we may need to smooth k_x around age 80 to eliminate irregularities. To perceive the influence of the boundary constraint set for the mortality rate at the highest attainable age, we have investigated whether changing the value of $m_{x_{\max}}$ modifies our results. Specifically, a number of different versions of the model has been tested considering $m_{x_{\max}} \in [0.8;\ 0.9;\ 1.0;\ 1.1;\ 1.2;\ 2.0]$ with limit age $x_{\max} \in (110, 120)$.

[6] Coale and Guo (1989) used this approach to close the extended version of the Coale-Demeny abridged (five-year age groups) model life tables. More specifically, the authors replace observed age specific death rates at old ages (85 years and over) by a sequence of death rates extrapolated for the age groups 85-89, 90–94, ..., 105-109, by assuming that the exponential rate of mortality increase at very old ages is not constant, but declines linearly.

4.2 The method of Denuit and Goderniaux

Denuit and Goderniaux (2005) recently developed a new method based on mortality rates q_x that imposes a closure constraint on life tables. Specifically, the method involves fitting the following log-quadratic regression model:

$$\ln \hat{q}_x = a + bx + cx^2 + \varepsilon_x \quad \text{with } \varepsilon_x \sim Nor\left(0,\sigma^2\right) \tag{7}$$

to data observed at advanced ages ($x \geq 75$ in our case), with the following two constraints:

$$q_{x_{\max}} = 1 \tag{8}$$

$$q'_{x_{\max}} = 0 \tag{9}$$

where $q'_{x_{\max}}$ denotes the first derivative of q_x with respect to age x, a, b and c are parameters to be estimated by OLS and $x_{\max}$ is a predefined highest attainable age.

By inserting (8) and (9) into (7), it can be shown that the model can be written as a function of a single parameter, i.e.,

$$\ln \hat{q}_x = \left(x_{\max}^2 - 2x(x_{\max}) + x^2\right)c + \varepsilon_x \quad \text{with } \varepsilon_x \sim Nor\left(0,\sigma^2\right) \tag{10}$$

Constraints (8) and (9) impose a concave shape to mortality rates at advanced ages and the existence of a horizontal tangent at $x = x_{\max}$. Constraint (9) aims to prevent an eventual decrease of the mortality rates at very old ages.

To understand the influence of the limit age on the performance of the model, we tested three different versions of (7) considering $x_{\max} \in \{110,115,120\}$. The final value for $x_{\max}$ is chosen to be the one that better describes the data.

To determine the age above which the original estimates $\hat{q}_x$ should be replaced by the fitted values generated by regression (7), we used an iterative procedure that runs model (7)-(8)-(9) over the age interval $x \in [x_0, 95]$, considering different values for x_0 (ranging from 70 to 90 years). To determine the "cutting-age x_0^*", we used the maximization of the determination coefficient R^2 as an optimum criterion. The tests carried out allowed us to identify ages 85 and 89 as generating similar values for R^2, so both have been considered as candidates in the choice process. To prevent the existence of discontinuities in the pattern of mortality rates in the neighbourhood of x_0^*, and to ensure a smooth transition between the original estimates and the adjusted values, some graduation is normally needed. In our case, we simply replaced initial estimates $\hat{q}_x$ within the age-interval $x = x_0^* - 5, \ldots, x_0^* + 5$ by their five-year geometric average.

4.3 The Logistic Model

The logistic function exhibits an "S" shape, that is, it grows quickly at first and then decelerates its progression, presenting a convenient asymptotic behaviour when it comes to model mortality rates at advanced ages. Formally, the logistic model for the force of mortality can be defined in general terms as (e.g., Thatcher et al., 1998):

$$\mu_x = \theta_1 + \frac{\theta_2 e^{\theta_3 x}}{1 + \sigma^2 \dfrac{\theta_2}{\theta_3}\left(e^{\theta_3 x} - 1\right)} \tag{11}$$

where $\theta = (\theta_1, \theta_2, \theta_2, \sigma^2)$ are parameters to be estimated.

The logistic model has been presented under a number of different versions of (11). In this paper, we adopt first the specification proposed by Perks (1932), defined as:

$$\mu_x = \frac{\theta_3 + e^{[\theta_0+\theta_1(x-80)]}}{1+e^{[\theta_2+\theta_1(x-80)]}}, \quad x \geq 80 \tag{12}$$

where $\theta_i \geq 0 \quad (i = 0,\ldots,3)$.

Note that the effect of the denominator in (12) is to flatten out the exponential increase of the Gompertz term in the numerator, noticeable at ages above about 80, which is now a well-established feature of mortality. Assuming that the number of observed deaths follows a Poisson distribution with parameter $\mu_x E_x$, i.e., $d_x \sim Poisson(\mu_x E_x)$, where E_x denotes the population exposed at risk, it can be shown that parameters $\theta = (\theta_0,\theta_1,\theta_2,\theta_3)$ are estimated by maximizing the following log-likelihood function:

$$\ln L(\theta) = \sum_{x=80}^{100} \left[-E_x\mu_x + d_x \ln\left(E_x\mu_x\right) - \ln\left(d_x!\right)\right], \quad x \geq 80 \tag{13}$$

4.4 The Kannistö Model

The model suggested by Kannistö (1992) is another special case of the logistic function, for which the logit transformation of the mortality rate can be express as a linear function of age. In this paper, we test a two-parameter version of the model defined by:

$$\mu_x = \frac{e^{[\theta_0+\theta_1(x-80)]}}{1+e^{[\theta_0+\theta_1(x-80)]}}, \quad x \geq 80 \tag{14}$$

where $\theta_i \geq 0 \quad (i = 0,1)$. Note that the model has an asymptote equal to one.

Assuming that the number of observed deaths follows a Poisson distribution with parameter $\mu_x E_x$, i.e., $d_x \sim Poisson(\mu_x E_x)$, where E_x denotes the population (centrally) exposed at risk, parameters $\theta = (\theta_0,\theta_1)$ are estimated by maximizing log-likelihood function (13).

5. Results

In this section we describe the results from implementing the approaches in estimating mortality rates at advanced ages described in Section 4. All methods were applied to crude death rates and probabilities q_x calculated according to definition (1), for two different periods (2003-05 and 2004-06) and for the three populations under study (men, women, both sexes).[7] The results obtained by the different approaches are presented in terms of projected value for q_x and in terms of computed estimates for the life expectancy at ages 0, 65, 80, 90 and 100. Finally, the projected values for q_x at ages 100, 115 and 120 years old produced by the different methods are reported, in order to perceive the ability of each model to suggest a highest attainable age to be considered when closing the life table. Because of space constraints, we report the results only for the complete population (both sexes).

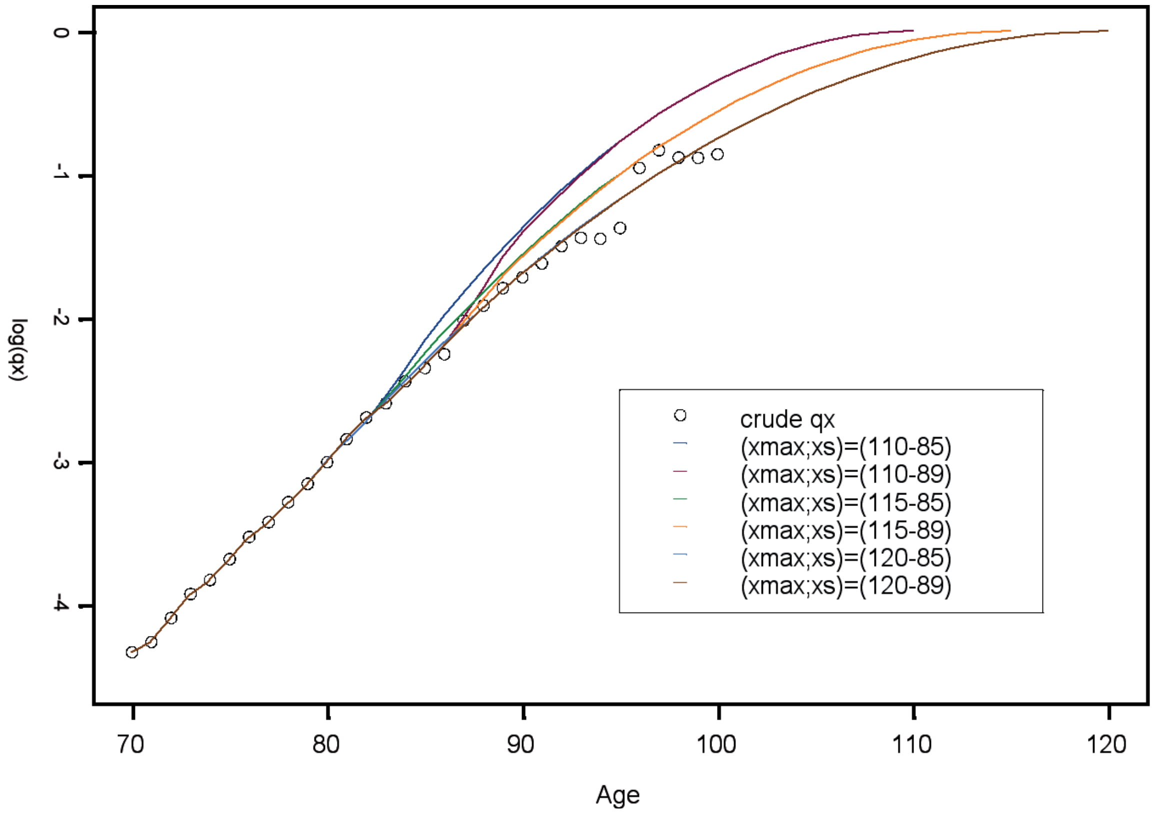
In Figures 5, 6, 7 and 8 we compare crude estimates of q_x with those generated by the different approaches. We can observe that crude estimates of q_x at advanced ages (above age 92 for males and above 95 for females) present a rather irregular behaviour, assuming, in some cases, a decreasing trend with age, a profile inconsistent with the expected trajectory for the evolution of the human mortality. In general terms, we note that mortality roughly increases with age, but the rate of mortality increase tends to decelerate from a certain age (85 to 90 years). This pattern translates into a

[7] All models were estimated with the help of S-PLUS 2000 and SAS packages.

graphical configuration characterized by a clear concave shape for the function $x \mapsto \ln(q_x)$, a result that is consistent with the findings of prior empirical studies on the behaviour of mortality at advanced ages.

The analysis of Figures 5, 6, 7 and 8 highlights that the goodness-of-fit of the alternative methods tested varies significantly. The method proposed by Denuit-Goderniaux (DG) seems to naturally extend the mortality rates observed at old ages. However, it is clear that the performance of the model is sensitive to both changes in the value of the limit age and of the age above which the original estimates are replaced by fitted values. For this particular population, the closest fit is attained by assuming that $x_{\max} = 115$ and that crude estimates are replaced by fitted values above age 85. The maximum attainable age considered should not ignore the maximum age for which deaths are registered at that moment in time.

Figure 5 Crude q_x and quotients extrapolated by Denuit-Goderniaux method



Notes: xmax = highest attainable age; xs = age above which the original estimates are replaced by the fitted values.

Figure 6 Comparison between crude q_x and quotients extrapolated by Coale-Kisker method

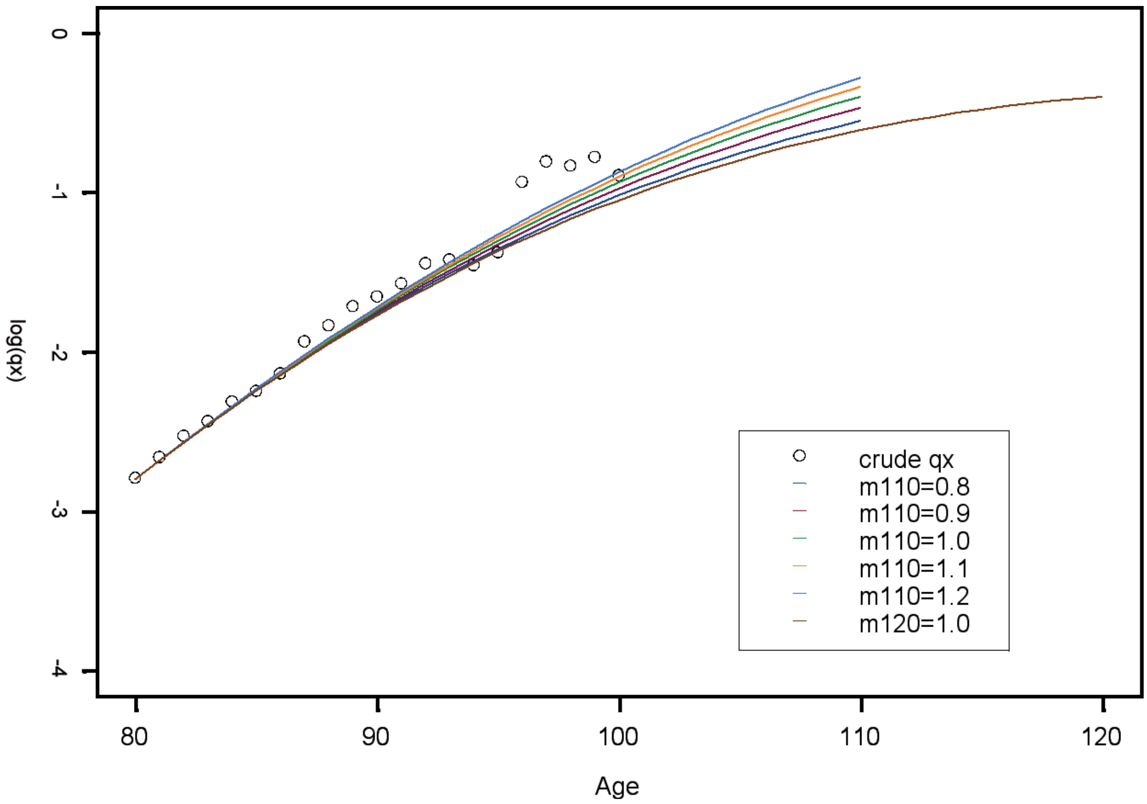


Figure 7 Comparison between crude q_x and quotients extrapolated by Kannistö model

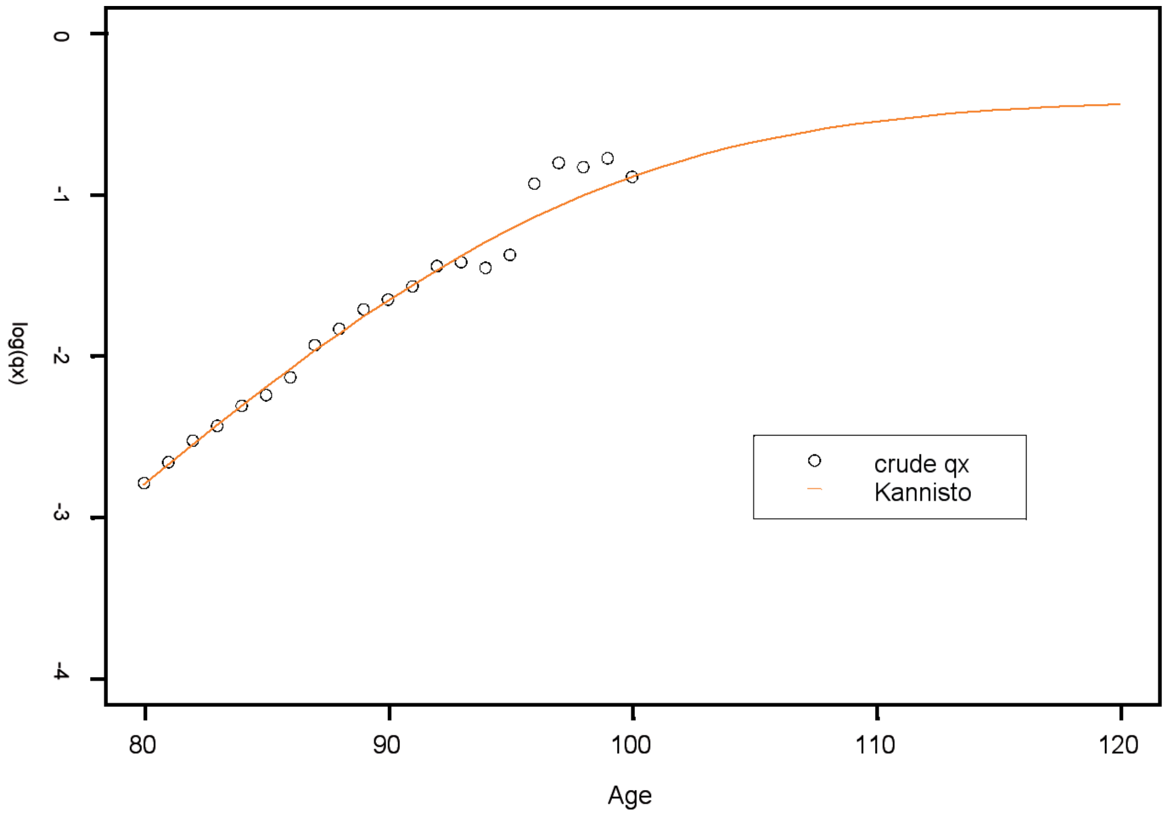
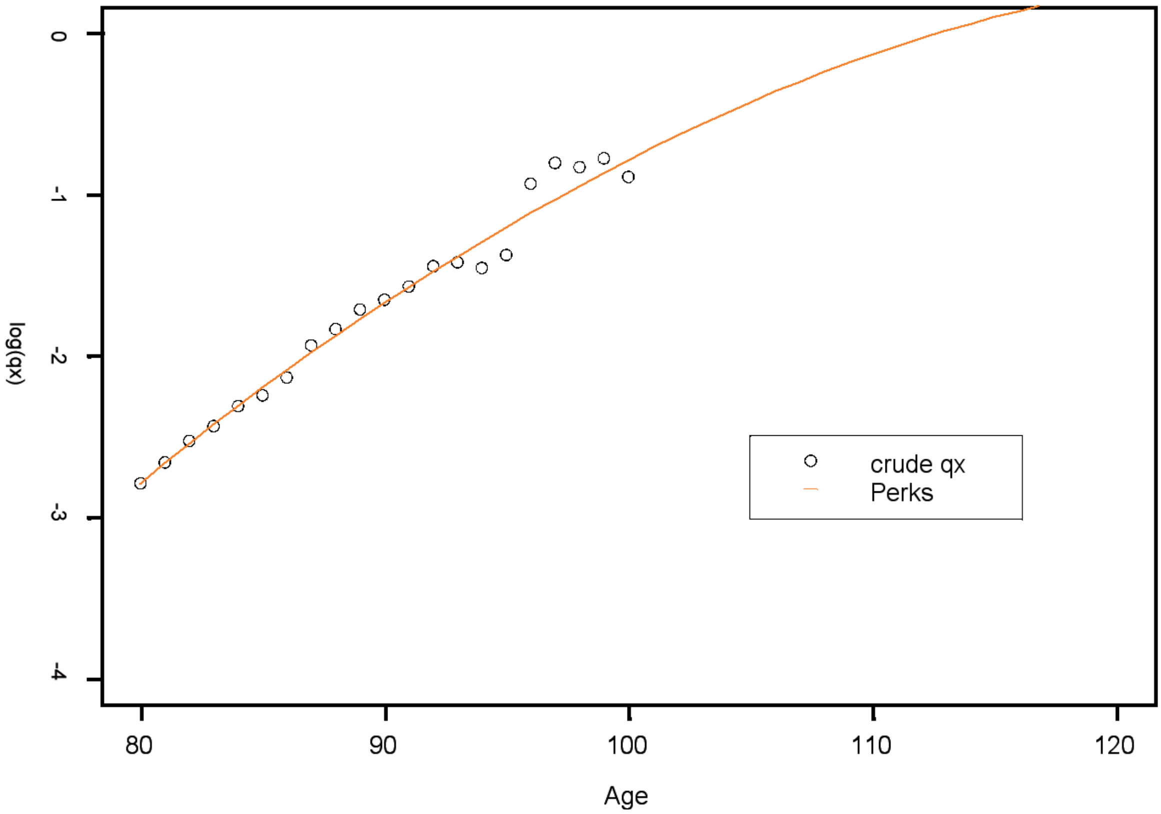


Figure 8 Comparison between crude q_x and quotients extrapolated by Perks model



The performance of Coale-Kisker (CK) method is quite reasonable but strongly depends on the pre-defined values for the mortality rate corresponding to the limit age $m_{x_{\max}}$ and on the limit age itself. Effectively, we note that the arbitrary values for m_{110} originally set by CK are not adequate to describe the mortality pattern observed in the Portuguese population. However, it should be stressed that the model is flexible enough to accommodate different mortality behaviours, and that by appropriately setting $m_{x_{\max}}$ we can replicate the empirical evolution of mortality across age.

The good performance of both DG and CK methods cannot be distanced from the fact that both methods impose constraints to the extrapolation process, which allow us to calibrate the models to the conditions observed at each moment in time, taking into consideration the maximum age for which deaths are registered in the population. The method suggested by Denuit-Goderniaux presents, however, an advantage over Coale-Kisker, since it allows us to set the limit age in advance, that is, gives us the chance to establish beforehand the age at which life tables should be closed. This means that we no longer need to close life tables by setting a probability of death for an open interval at and above $x_{\max}$.

To understand if the closing constraint $q_{x_{\max}} = 1$ imposed by Denuit and Goderniaux (2005) is reasonable from the point of view of the other models analysed, Table 2 reports the projected values for q_x at advanced ages generated by all models. As mentioned before, this is an important issue when it comes to produce complete life tables, and can be seen as an attempt to ascertain the differences between models that impose a closing constraint and models that merely extrapolate mortality rates observed in a given age interval. We note that models which are based on a simple extrapolation procedure convey values for q_x that resemble the behaviour of crude estimates up to a certain age, but in contrast produce unreasonable estimates at extreme ages. For example, we observe that the Coale-Kisker and the Kannistö models admit, with a quite high probability, that some members of the population will live beyond age 120.

Table 2 Projected values for q_x at advanced ages

HM	METHOD								
Age	Denuit-Goderniaux						Coale-Kisker	Kannisto	Perks
	xmax=110		xmax=115		xmax=120		m120	xlim=120	xlim=120
	xs=85	xs=89	xs=85	xs=89	xs=85	xs=89	1.0	xsub=80	xsub=80
110	1	1	0.943	0.943	0.839	0.839	0.545	0.575	0.870
115	1	1	1	1	0.957	0.957	0.616	0.617	1.095
120	1	1	1	1	1	1	0.667	0.641	1.298

On the contrary, in Figures 7 and 8 we observe that the logistic models of Kannistö and Perks basically extrapolate the mortality patterns conveyed by crude estimates, without any control over "natural" and observed mortality trends at old ages. Consequently, models tend to generate unexpected patterns at very old ages, either producing almost constant mortality rates (as it seems to be the case for the Kannistö model), either generating explosive trajectories (as it is the case for the Perks model) that are not confirmed by empirical studies, which show a deceleration in the rate of mortality increase at old ages. The instability of projected values means that the use of logistic models to extrapolate mortality patterns at old ages should be made with caution.

In Table 3 we can analyze the influence of the choice of the method of estimating mortality rates at advanced ages on the values of the complete life expectancy for a selected set of ages, namely $x \in \{0, 65, 80, 90, 100\}$. The methods proposed by Coale-Kisker, Kannistö and Perks tend to generate higher estimates for the life expectancy at all ages, since they seem to underestimate the evolution of mortality at older ages. The method of Denuit-Goderniaux produces more reliable estimates, and presents a more suitable trajectory for mortality, particularly in the case where the original estimates are substituted by the values adjusted from the 85 years of age. We note also, without surprise, that for the Denuit-Goderniaux method an increase in the highest attainable age translates into higher estimated values for e_x at all ages.

Table 3 Estimated life expectancy e_x for selected ages (in years)

HM	METHOD													
Age	Denuit-Goderniaux						Coale-Kisker						Kannisto	Perks
	xmax=110		xmax=115		xmax=120		m110					m120	xlim=120	xlim=120
	xs=85	xs=89	xs=85	xs=89	xs=85	xs=89	0.8	0.9	1.0	1.1	1.2	1.0	xsub=80	xsub=80
0	77.87	78.05	78.06	78.16	78.20	78.25	78.50	78.48	78.46	78.45	78.43	78.52	78.32	78.32
65	17.54	17.75	17.76	17.88	17.93	17.98	18.28	18.25	18.23	18.21	18.20	18.30	18.06	18.06
80	6.95	7.27	7.27	7.46	7.53	7.62	8.04	8.00	7.97	7.94	7.91	8.06	7.74	7.73
90	2.44	2.51	2.91	2.95	3.31	3.33	4.04	3.96	3.90	3.85	3.80	4.09	3.63	3.62
100	0.85	0.85	1.12	1.12	1.41	1.41	2.07	1.96	1.87	1.79	1.72	2.15	1.81	1.55

For the Coale-Kisker method, we can observe an inverse relation between $m_{x_{\max}}$ and the estimated values for e_x. Increasing the age for which $m_{x_{\max}}$ is set obviously increases the computed life expectancy.

6. Conclusions

In this paper, we compared the ability of a number of different methods to project mortality for the oldest-old in the Portuguese population in order to establish a sound methodology for the construction of complete life tables. Our results show that models which include a closing constraint seem to be perform better than models that merely extrapolate the mortality patterns observed in a given age interval. The significance of this is that by including closing constraints we no longer need to close life tables by setting a probability of death for an open age interval.

The method suggested by Denuit and Goderniaux presents the best results overall. The method is compatible with recent empirical studies showing that the rate of mortality increase tends to decelerate from a certain age, presenting a concave shape in the ($x, \ln(q_x)$) space, eliminates the possibility of decreasing mortality rates at advanced ages and is flexible enough to accommodate to mortality conditions observed at each moment in time. Future investigations should be able to confirm these conclusions on a broader basis, namely in other countries and for different moments in time.

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MORTALITY RATES IN POPULATION PROJECTIONS: A STOCHASTIC APPROACH TO INFERENCE

September 2007

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Abstract

At Statistics Sweden population projections are made for regions such as counties and municipalities. The size of the future population in one region depends on variables such as fertility, mortality and migration. Before making a population projection it is essential to determine whether the mortality in the region is the same or significantly different from the mortality in the whole country. This is done by comparing the observed number of deaths in the region (D) to the number of deaths that would be expected if the region would have the same mortality as the whole country (E). An often-used test statistic, referred to as the Standard Mortality Rate (SMR), is given by the ratio D/E. For regions with the same mortality as the whole country the expected value of SMR should be close to 1.

It is obvious that the values of both D and E are stochastic, but the method in use at Statistics Sweden today assumes that E is deterministic. In this paper two different approaches that does not restrict E to be deterministic are suggested: The first, and perhaps most obvious approach, is to determine the distribution of D/E under parametric assumptions. Using illustrating examples it will be shown that this approach provides shorter confidence intervals than the method in use today. The second approach suggested is to use the empirical distribution of the data. The value of the SMR as it is used in population projections today is based on deaths over a period of time, often several years. However, by constructing SMR for *each year* we get a number of observations that inference can be applied to. As can be seen in the illustrating examples this approach results in shorter confidence intervals in some cases but not over all.

1. Introduction

Statistics Sweden provides the public with a national population projection updated every year. However the interest of population projections is maybe even greater from smaller regions in Sweden that use population projections for economical decisions. Sweden has a total number of 21 counties. Most of them do some kind of population projection using the statistics provided by Statistics Sweden.

When making a population projection on a regional level the mortality is assumed to follow the same *pattern* as the projected national mortality. However, the *level* of mortality in a region is adjusted based on a comparison between the number of deaths observed (D) and the number of deaths expected (E). The expected number of deaths is simply what one would expect if the region would have the same mortality as the whole country given the population structure of the region. The comparison between D and E is given by the ratio D/E which is referred to as the Standard Mortality Rate (SMR). If a region has the same mortality as the whole country, the SMR should be close to 1. The ratio is used in population projections when adjusting the level of mortality in the region but can also be used for comparing mortality between regions.

The need of inference applied to the SMR is obvious because the numbers of deaths is stochastic. The inference done today at Statistics Sweden is based on the assumption that D is stochastic while E is deterministic. The aim of the authors of this paper is to find other approaches that do not restrict E to be deterministic. The results of the approaches suggested in this paper will be compared to the results of the method in use today using population statistics from 1969 to 2006.

The first approach considered is to find the distribution of D/E under parametric assumptions. The second approach is to use the empirical distribution of the data. A brief summary of how the SMR is used in the regional population projections at Statistics Sweden is presented in Section 2. The method in use today will be described in Section 3. In Section 4 the two approaches suggested are presented and in Section 5 illustrated using population statistics from 1969 to 2006. In Section 6 the results of the inference is presented in tables.

2. Introduction to the SMR

The SMR is given by D/E where D is the observed number of deaths in the region over a specific period of time and E is the expected number during the same period. D is easily observed while E is calculated using the probability of death in the nation and the mean population in the region. Hence, E is the expected number of deaths in the region if the region would have the same mortality as the whole country.

The SMR can be calculated for any period of time. In the regional population projections done today at Statistics Sweden the time period 1997-2006 is used. The SMR can also be used in any age constellation. At Statistics Sweden the SMR for males and females in the ages 20-64 and 65-90 years are calculated for each county. For the population younger than 20 or older than 90 years the SMR is not of interest when making a regional population projection because the number of deaths in these age groups is often low for small regions. However, due to the ageing population, Statistics Sweden is considering applying the SMR to the population older than 90 years.

When applying inference to the SMR a 95 percent confidence level is used when calculating confidence intervals and a one percent significance level is used in the significance tests.

We need to fix some notation and make some assumptions:

Let T_{gh} denote the mean population in the region g , $(g = 1,2,\ldots,G)$ that belongs to the age group h , $(h = 1,2,\ldots,H)$ during the time period considered. Further, let X_{gh} denote the number of persons in the region g belonging to age group h who died during the time period considered.

Our basic assumption is that $X_{gh} \sim Po(\lambda_{gh})$ for each g and h and that X_{gh} and $X_{g'h'}$ are independent if $g \neq g'$ or $h \neq h'$. The total number of persons who died in the region g during the time period is denoted by X_g, that is, $X_g = \sum_{h=1}^{H} X_{gh}$ and $X_g \sim Po(\lambda_g)$ where $\lambda_g = \sum_{h=1}^{H} \lambda_{gh}$.

The expected number of deaths in the region *g* during the time period is denoted by Y_g and is calculated as

$$Y_g = \sum_{h=1}^{H} \left(\frac{T_{gh}}{\sum_{g'=1}^{G} T_{g'h}} \sum_{g'=1}^{G} X_{g'h} \right).$$

If we let $w_{gh} = \frac{T_{gh}}{\sum_{g'=1}^{G} T_{g'h}}$ then an alternative expression of Y_g is given by

$$Y_g = \sum_{h=1}^{H} w_{gh} \sum_{g'=1}^{G} X_{g'h} .$$

The SMR for the region *g* is now defined as $SMR = \frac{X_g}{Y_g}$.

3. Method in use today

The perhaps easiest way to apply inference to the SMR for a region *g* is to assume that D_g is the outcome of a random experiment described by the stochastic variable X_g and that the stochastic variable Y_g is identically equal to its observation E_g (which we calculate in the same way as we calculate the variable Y_g but with observed mean populations and observed numbers of dead people). Hereby the problem of applying inference to the variable SMR_g reduces to the easier problem of applying inference to the variable X_g which we have assumed has the Poisson distribution with parameter λ_g .

A significance test for the expected value of SMR_g can be put up as follows. With the hypothesis

$$H_0 : \quad E(SMR_g) = 1 \quad \Leftrightarrow \quad \frac{E(X_g)}{E_g} = 1 \quad \Leftrightarrow \quad \lambda_g = E_g$$

$$H_a : \quad E(SMR_g) \neq 1 \quad \Leftrightarrow \quad \frac{E(X_g)}{E_g} \neq 1 \quad \Leftrightarrow \quad \lambda_g \neq E_g .$$

Since X_g is assumed to be Poisson distributed with mean λ_g the test statistic for this test becomes

$$z_g = \frac{\hat{\lambda}_g - E_g}{\sqrt{\sigma^2_{\hat{\lambda}_g}}} = \frac{D_g - E_g}{\sqrt{D_g}} .$$

If λ_g is large enough z_g can be regarded as an observation of a stochastic variable which is approximately standard normal distributed. The critical values when applying a one percent significance level is therefore -2,58 and 2,58. The result of this test applied to data for all counties in Sweden for the time period 1969-2006 can be found in the appendix.

If $\lambda_g \geq 10$, X_g can be approximately seen as having a normal distribution. A 95 % confidence interval for the expected value of X_g, i.e. for λ_g is therefore estimated by

$$\hat{\lambda}_g \pm 1{,}96\sqrt{\sigma^2_{\hat{\lambda}_g}} = D_g \pm 1{,}96\sqrt{D_g} .$$

To receive the confidence interval for SMR_g both the upper and lower limit of the interval above needs to be divided by E_g. Therefore the interval is given by

$$\frac{\hat{\lambda}_g \pm 1{,}96\sqrt{\sigma^2_{\hat{\lambda}_g}}}{E_g} = \frac{D_g \pm 1{,}96\sqrt{D_g}}{E_g}.$$

Confidence intervals for the SMR using this method can be found in Section 5 and in the appendix.

4. Suggested approaches

In this section we present two approaches that do not restrict E to be deterministic.

4.1 Inference under stochastic assumptions

As before we want to apply inference to $SMR_g = \frac{X_g}{Y_g}$. This time however we will not assume that $Y_g = \sum_{h=1}^{H} w_{gh} \sum_{g'=1}^{G} X_{g'h}$ is a constant variable. Instead we assume that all components in the SMR which describes number of people dying, i.e. the *X*'s, are stochastic variables and that all components which describe mean populations, i.e. the *w*'s, are just numbers. Besides this we make the same assumptions as in Section 2.

Taylor series expansion of SMR_g about $E(X_g)$, $E(Y_g)$ gives us the following expression:

$$\begin{aligned} SMR_g &\approx \frac{E(X_g)}{E(Y_g)} + \frac{1}{E(Y_g)}\left[X_g - E(X_g)\right] - \frac{E(X_g)}{E(Y_g)^2}\left[Y_g - E(Y_g)\right] \\ &= \frac{E(X_g)}{E(Y_g)} + \frac{1}{E(Y_g)}\left(X_g - \frac{E(X_g)}{E(Y_g)}Y_g\right). \end{aligned}$$

Thus

$$\begin{aligned} V(SMR_g) &\approx V\left\{\frac{1}{E(Y_g)}\left[X_g - \frac{E(X_g)}{E(Y_g)}Y_g\right]\right\} \\ &= \frac{1}{E(Y_g)^2}\left[V(X_g) + \frac{E(X_g)^2}{E(Y_g)^2}V(Y_g) - 2\frac{E(X_g)}{E(Y_g)}C(X_g, Y_g)\right]. \end{aligned}$$

We now want to construct an unbiased estimator for $V(SMR_g)$. For Y_g the following holds

$$E(Y_g) = E\left(\sum_{h=1}^{H} w_{gh} \sum_{g'=1}^{G} X_{g'h}\right) = \sum_{h=1}^{H} w_{gh} \sum_{g'=1}^{G} E(X_{g'h}) = \sum_{h=1}^{H} w_{gh} \sum_{g'=1}^{G} \lambda_{g'h}$$

and

$$V(Y_g) = V\left(\sum_{h=1}^{H} w_{gh} \sum_{g'=1}^{G} X_{g'h}\right) = \sum_{h=1}^{H} w_{gh}^2 \sum_{g'=1}^{G} V(X_{g'h}) = \sum_{h=1}^{H} w_{gh}^2 \sum_{g'=1}^{G} \lambda_{g'h}.$$

Unbiased estimators for $E(Y_g)$ and $V(Y_g)$ is therefore given by $\sum_{h=1}^{H} w_{gh} \sum_{g'=1}^{G} D_{g'h} = E_g$ and $\sum_{h=1}^{H} w_{gh}^2 \sum_{g'=1}^{G} D_{g'h}$ respectively.

Further it holds that

$$E(X_g Y_g) = E\left[\left(\sum_{h=1}^{H} X_{gh}\right)\left(\sum_{h'=1}^{H} w_{gh'} \sum_{g'=1}^{G} X_{g'h}\right)\right]$$

$$= E\left(\sum_{h=1}^{H} w_{gh} X_{gh}^2 + \sum_{h=1}^{H} w_{gh} X_{gh} \sum_{\substack{g'=1 \\ g'\neq g}}^{G} X_{g'h} + \sum_{\substack{h=1 \\ h'\neq h}}^{H} \sum_{h'=1}^{H} w_{gh'} X_{gh} \sum_{g'=1}^{G} X_{g'h'}\right)$$

$$= \sum_{h=1}^{H} w_{gh} \left(\lambda_{gh} + \lambda_{gh}^2\right) + \sum_{h=1}^{H} w_{gh} \lambda_{gh} \sum_{\substack{g'=1 \\ g'\neq g}}^{G} \lambda_{g'h} + \sum_{\substack{h=1 \\ h'\neq h}}^{H} \sum_{h'=1}^{H} w_{gh'} \lambda_{gh} \sum_{g'=1}^{G} \lambda_{g'h'}$$

and

$$E(X_g)E(Y_g) = \left(\sum_{h=1}^{H} \lambda_{gh}\right)\left(\sum_{h'=1}^{H} w_{gh'} \sum_{g'=1}^{G} \lambda_{g'h}\right)$$

$$= \sum_{h=1}^{H} w_{gh} \lambda_{gh}^2 + \sum_{h=1}^{H} w_{gh} \lambda_{gh} \sum_{\substack{g'=1 \\ g'\neq g}}^{G} \lambda_{g'h} + \sum_{\substack{h=1 \\ h'\neq h}}^{H} \sum_{h'=1}^{H} w_{gh'} \lambda_{gh} \sum_{g'=1}^{G} \lambda_{g'h'}$$

why

$$C(X_g, Y_g) = E(X_g Y_g) - E(X_g)E(Y_g) = \sum_{h=1}^{H} w_{gh} \lambda_{gh} .$$

An unbiased estimator for $C(X_g, Y_g)$ is therefore given by $\sum_{h=1}^{H} w_{gh} D_{gh}$.

If the Taylor approximation above is good, an unbiased estimator of it can be used as estimator for $V(SMR_g)$. We suggest the estimator

$$\hat{V}(SMR_g) = \frac{1}{E_g^{\ 2}}\left(D_g + \frac{D_g^{\ 2}}{E_g^{\ 2}} \sum_{h=1}^{H} w_{gh}^2 \sum_{g'=1}^{G} D_{g'h} - 2\frac{D_g}{E_g} \sum_{h=1}^{H} w_{gh} D_{gh}\right).$$

Using $\frac{D_g}{E_g}$ as an estimator for $E(SMR_g)$ an approximate 95 percent confidence interval can be constructed as

$$\frac{D_g}{E_g} \pm 1{,}96\sqrt{\hat{V}(SMR_g)}.$$

The hypothesis in the significance test are

$$H_0 : E(SMR_g) = 1$$
$$H_a : E(SMR_g) \neq 1.$$

As test statistic the following expression is used

$$z = \frac{\frac{D_g}{E_g} - 1}{\sqrt{\hat{V}(SMR_g)}}$$

where we assume that the test statistic z is approximately standard normal distributed. Confidence intervals and results of the significance test for the SMR using this method can be found in Section 5 and in the appendix.

4.2 Distribution of the data

In this approach the SMR is calculated for each year, age group and sex. If we assume that the SMR for each year are independent we have, for each age group and sex, the independent and random sample: $smr_1, smr_2, \ldots, smr_n$.

As an estimator of the SMR we use the mean of the observations

$$\overline{smr} = \frac{\sum_{i=1}^{n} smr_i}{n}$$

where $n = 1,2,\ldots$ are the years in the considered time period.

The estimator is approximately normal distributed according to the Central Limit Theorem if the observations are random independently and no fewer than 30.

The confidence interval for the SMR is given by

$$\overline{smr} \pm 1{,}96\sqrt{\frac{\hat{V}(SMR)}{n}}.$$

In the significance test we have the hypothesis

$$H_0 : E(SMR) = 1$$
$$H_a : E(SMR) \neq 1$$

where

$$z = \frac{\overline{smr} - 1}{\sqrt{\frac{\hat{V}(SMR)}{n}}}$$

is used as test statistic.

The critical values when applying a one percent significance level is -2,58 and 2,58. See Section 5 and the appendix for results.

5. Illustrating examples

As an illustrating example of the suggested approaches we calculate the SMR for men in the age groups 20-64 and 65-90 years. The difference in the confidence intervals can be seen in the graphs. The point estimates, lower and upper level of the confidence intervals can also be seen in the tables of the Appendix in section six. Point estimates marked with (*) indicates a significant difference from 1.

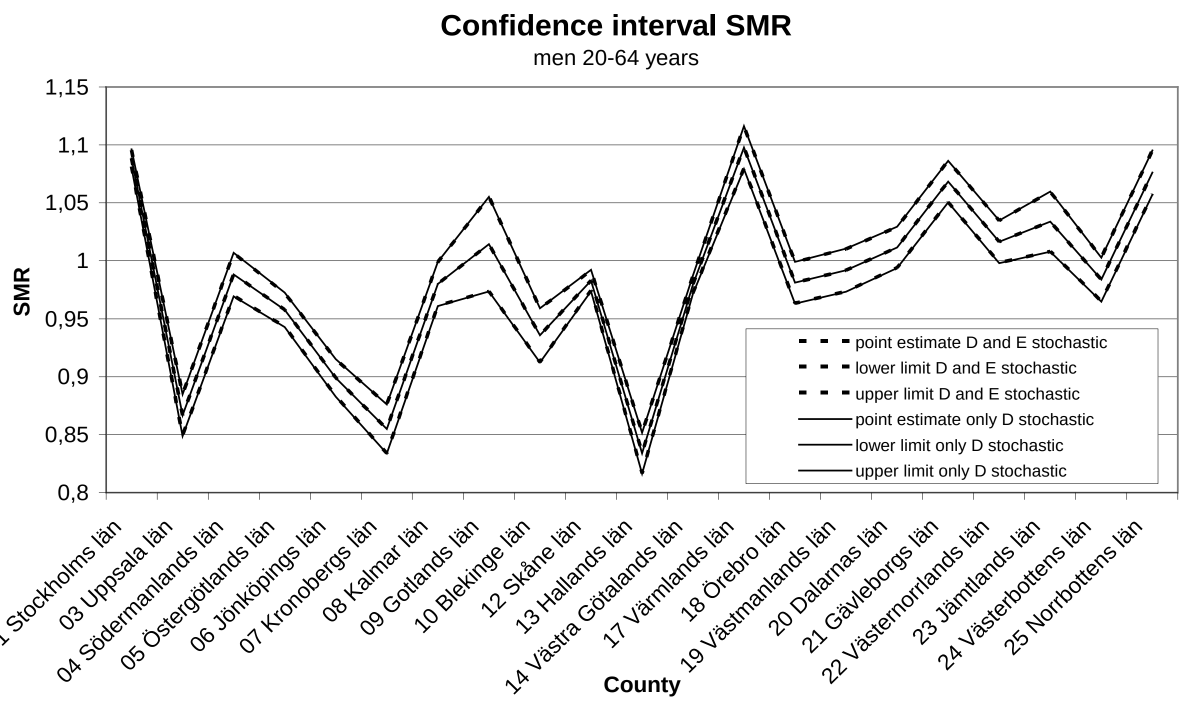
The illustrating examples show that the first approach suggested provides shorter confidence intervals than the old method. This is an argument why the confidence interval calculations should be performed using the distribution of D/E instead of, as in the old method, only using the distribution of D.

The second approach provides shorter confidence intervals in some cases but cannot be seen as an over all better approach.

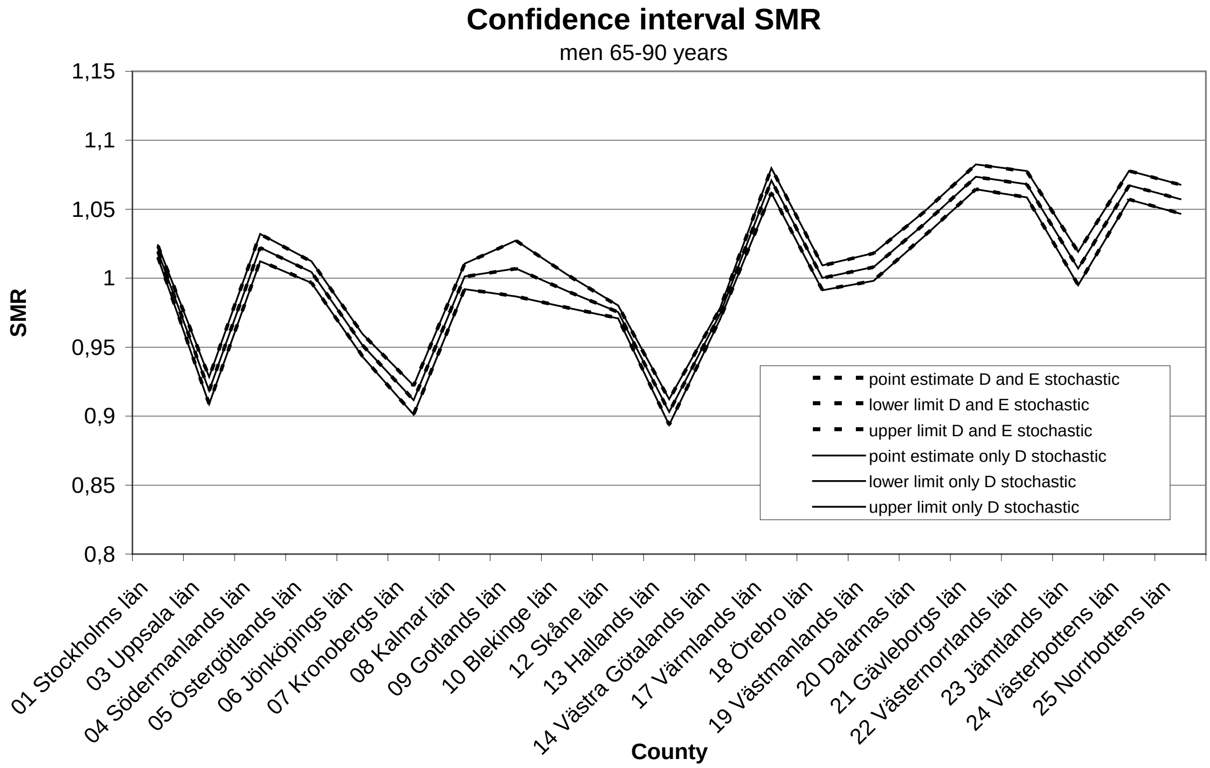
5.1 Using the inference under stochastic assumptions

The confidence intervals are calculated using the methods presented in section three and four. Over all we see shorter confidence intervals when assuming both D and E to be stochastic compared to the method when only D is assumed to be stochastic.

Graph 1

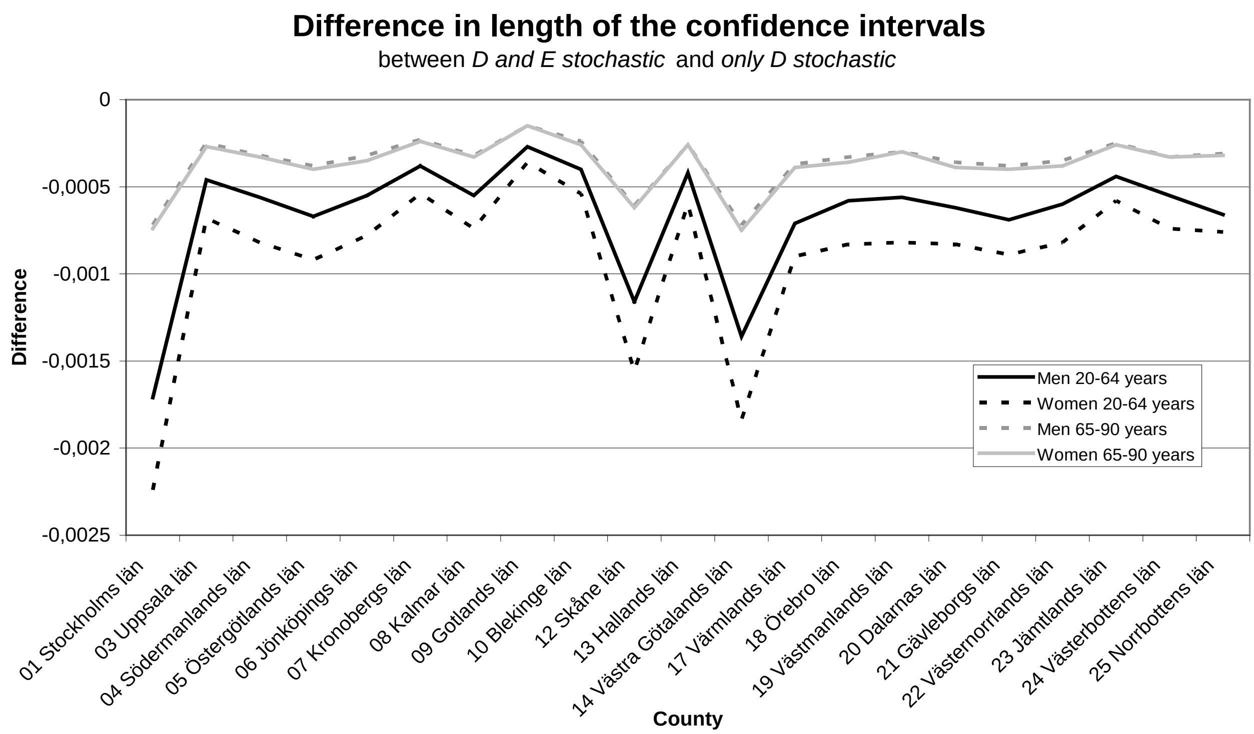


Graph 2



The difference between the confidence intervals is more obvious when comparing the difference in length.

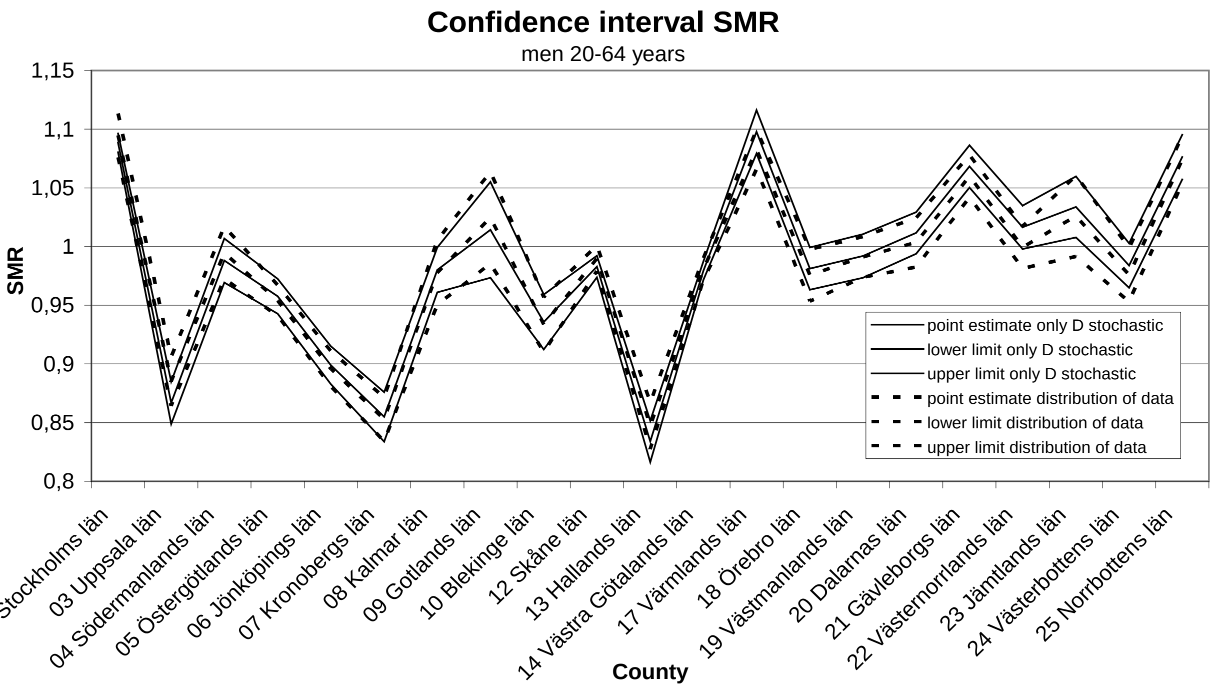
Graph 3



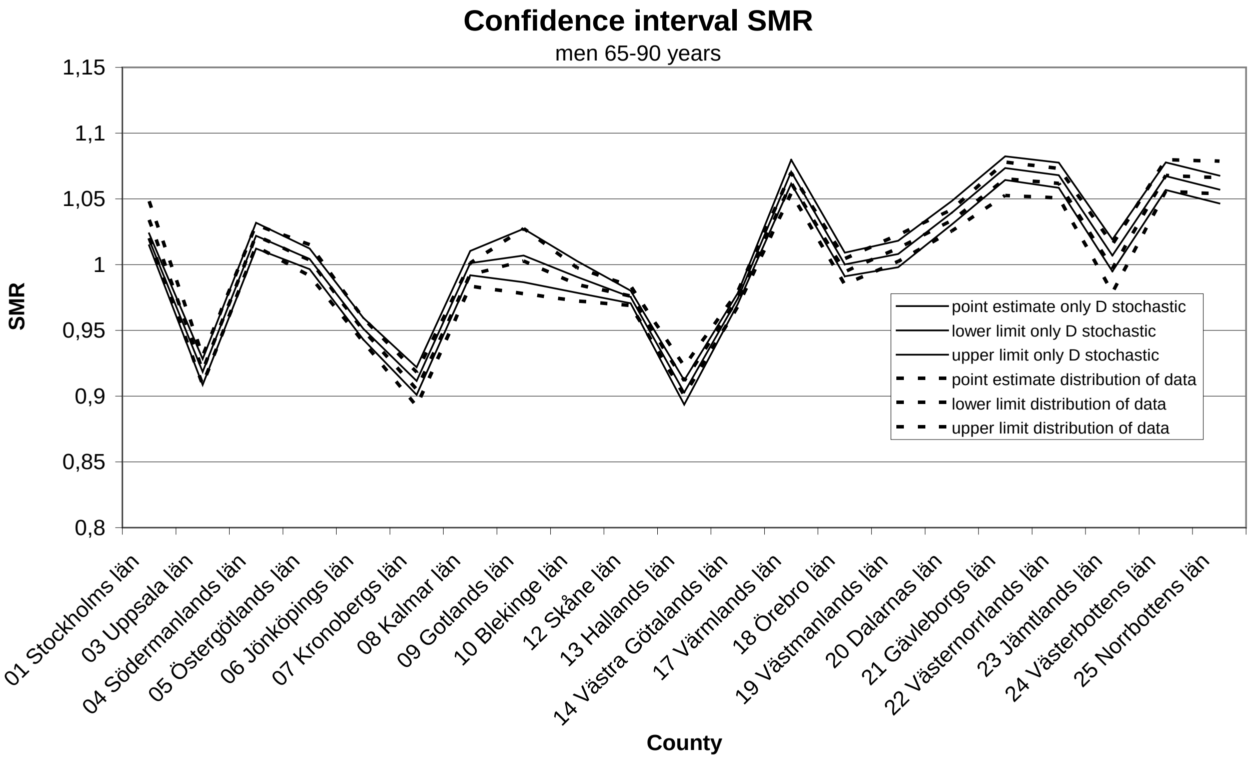
5.2 Using distribution of data

The confidence intervals are calculated using the methods presented in section three and four. The lengths of the confidence intervals when using the distribution of the data are not always smaller than the length of the confidence intervals calculated under the assumption that only D is stochastic. 22 out of 84 confidence intervals are shorter when using the distribution of the data.

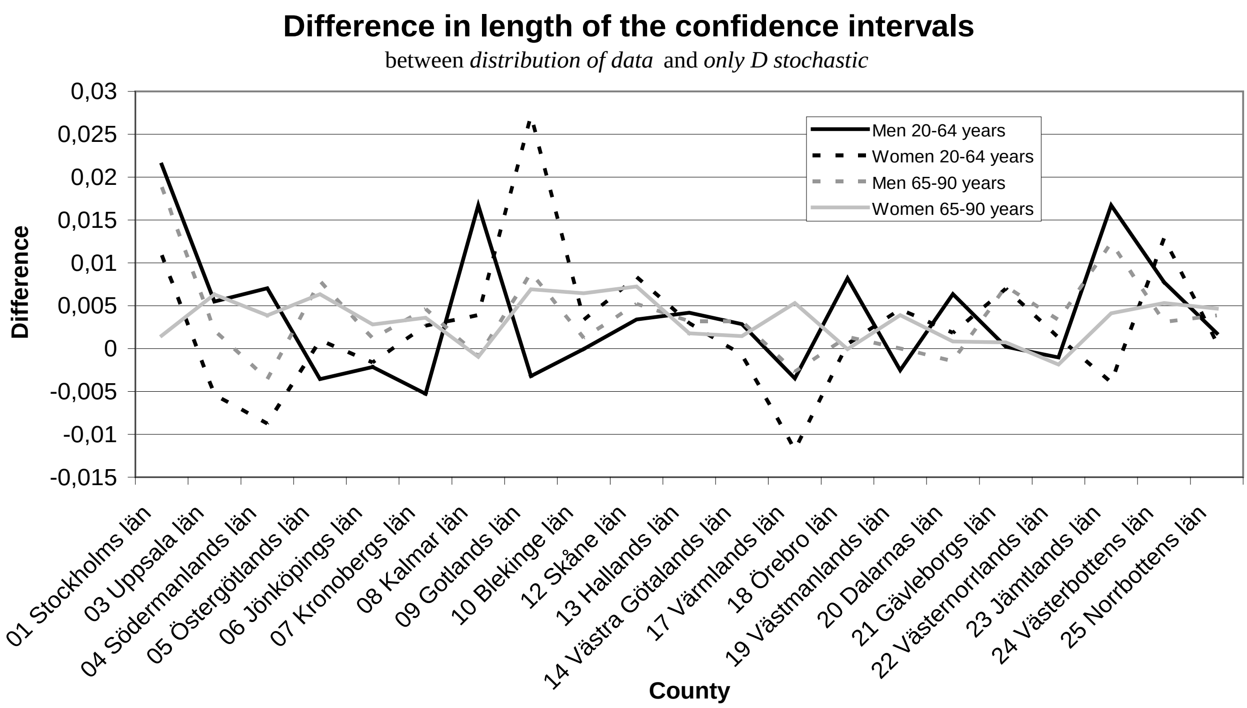
Graph 4



Graph 5



Graph 6



6. Appendix

SMR for men							
1969-2006	Method:	D and E - stochastic	confidence-interval	only D-stochastic	confidence-interval	distribution of data	confidence interval
region (conty)	age group	SMR	(SMR +/-)	SMR	(SMR +/-)	SMR	(SMR +/-)
Stockholm	20-64	1,08834*	0,00706	1,08834*	0,007915	1,0948*	0,018655
Uppsala	20-64	0,86685*	0,01781	0,86685*	0,01804	0,88507*	0,020785
Södermanland	20-64	0,98817	0,018465	0,98817	0,018745	0,99466	0,02227
Östergötland	20-64	0,95769*	0,01451	0,95769*	0,014845	0,95401*	0,01307
Jönköping	20-64	0,899*	0,01586	0,899*	0,016135	0,89583*	0,01506
Kronoberg	20-64	0,85488*	0,020945	0,85488*	0,021135	0,85273*	0,018505
Kalmar	20-64	0,98008*	0,018805	0,98008*	0,01908	0,97777	0,02744
Gotland	20-64	1,01432	0,04071	1,01432	0,040845	1,02463	0,039235
Blekinge	20-64	0,93577*	0,02316	0,93577*	0,02336	0,93344*	0,02333
Skåne	20-64	0,98297*	0,008635	0,98297*	0,009215	0,98999	0,01092
Halland	20-64	0,83401*	0,017455	0,83401*	0,017665	0,84701*	0,01976
V. Götaland	20-64	0,97945*	0,00726	0,97945*	0,00794	0,98055*	0,00936
Värmland	20-64	1,09778*	0,01811	1,09778*	0,018465	1,08234*	0,01672
Örebro	20-64	0,98115*	0,017715	0,98115*	0,018005	0,97557*	0,02211
Västmanland	20-64	0,99193	0,018295	0,99193	0,018575	0,99104	0,01731
Dalarna	20-64	1,01161	0,01746	1,01161	0,01777	1,00347	0,020945
Gävleborg	20-64	1,06836*	0,01766	1,06836*	0,018005	1,05985*	0,01811
Västernorrland	20-64	1,01644	0,018125	1,01644	0,018425	0,99935	0,01791
Jämtland	20-64	1,03382*	0,02579	1,03382*	0,02601	1,02598	0,034365
Västerbotten	20-64	0,98382	0,01871	0,98382	0,018985	0,97587*	0,022855
Norrbotten	20-64	1,0762*	0,018735	1,0762*	0,019065	1,07388*	0,019965
Stockholm	65-90	1,01922*	0,00416	1,01922*	0,00452	1,03401*	0,01393
Uppsala	65-90	0,91844*	0,00973	0,91844*	0,009855	0,91929*	0,010905
Södermanland	65-90	1,02202*	0,00972	1,02202*	0,00988	1,022*	0,008085
Östergötland	65-90	1,00436	0,007615	1,00436	0,007805	1,00322	0,011735
Jönköping	65-90	0,95133*	0,008095	0,95133*	0,008255	0,94999*	0,00884
Kronoberg	65-90	0,91147*	0,010425	0,91147*	0,01054	0,90513*	0,01285
Kalmar	65-90	1,00114	0,00909	1,00114	0,00925	0,99228	0,00879
Gotland	65-90	1,00695	0,020205	1,00695	0,02028	1,00261	0,024695
Blekinge	65-90	0,99056	0,011875	0,99056	0,011995	0,98483*	0,01261
Skåne	65-90	0,97538*	0,004435	0,97538*	0,00474	0,97609*	0,00733
Halland	65-90	0,90284*	0,009155	0,90284*	0,009285	0,91159*	0,01087
V. Götaland	65-90	0,97426*	0,003725	0,97426*	0,004085	0,97374*	0,005665
Värmland	65-90	1,07065*	0,00886	1,07065*	0,009045	1,06188*	0,00769
Örebro	65-90	1,0001	0,00887	1,0001	0,009035	0,99455*	0,00971
Västmanland	65-90	1,00805	0,00987	1,00805	0,01002	1,01241	0,010015
Dalarna	65-90	1,03947*	0,008705	1,03947*	0,008885	1,03351*	0,008155
Gävleborg	65-90	1,07337*	0,00886	1,07337*	0,00905	1,06516*	0,01266
Västernorrland	65-90	1,068*	0,009335	1,068*	0,00951	1,06175*	0,01117
Jämtland	65-90	1,00695	0,011985	1,00695	0,01211	0,99693	0,018265
Västerbotten	65-90	1,06727*	0,010275	1,06727*	0,01044	1,06776*	0,011985
Norrbotten	65-90	1,05715*	0,010385	1,05715*	0,01054	1,06603*	0,012465

SMR for men							
1969-2006	Method:	D and E - stochastic	confidence-interval	only D-stochastic	confidence-interval	distribution of data	confidence interval
region (conty)	age group	SMR	(SMR +/-)	SMR	(SMR +/-)	SMR	(SMR +/-)
Stockholm	20-64	1,05831*	0,009205	1,05831*	0,010325	1,06312*	0,015775
Uppsala	20-64	0,93558*	0,02488	0,93558*	0,02522	0,95117*	0,022525
Södermanland	20-64	1,04067*	0,025585	1,04067*	0,025995	1,03974*	0,021615
Östergötland	20-64	0,97409*	0,019715	0,97409*	0,020175	0,97207*	0,020655
Jönköping	20-64	0,93061*	0,02175	0,93061*	0,02214	0,92621*	0,021325
Kronoberg	20-64	0,91186*	0,02972	0,91186*	0,02999	0,91392*	0,031315
Kalmar	20-64	0,99639	0,02581	0,99639	0,02618	0,99202	0,028135
Gotland	20-64	0,99736	0,05474	0,99736	0,05492	1,00201	0,06849
Blekinge	20-64	0,93257*	0,031505	0,93257*	0,031775	0,93311*	0,03345
Skåne	20-64	0,97338*	0,0115	0,97338*	0,01228	0,97925	0,016455
Halland	20-64	0,86578*	0,02418	0,86578*	0,02448	0,8778*	0,025955
V. Götaland	20-64	0,98504*	0,009845	0,98504*	0,010765	0,9843*	0,0104
Värmland	20-64	1,05694*	0,02412	1,05694*	0,02457	1,05043*	0,01866
Örebro	20-64	1,02256	0,024325	1,02256	0,02474	1,018	0,025
Västmanland	20-64	1,03356*	0,025345	1,03356*	0,025755	1,03126*	0,02804
Dalarna	20-64	1,00576	0,023675	1,00576	0,02409	0,99651	0,025005
Gävleborg	20-64	1,04645*	0,023775	1,04645*	0,02422	1,0413*	0,027715
Västernorrland	20-64	1,02313	0,02468	1,02313	0,02509	1,00922	0,025725
Jämtland	20-64	1,02944	0,03551	1,02944	0,0358	1,02004	0,03381
Västerbotten	20-64	0,98007	0,025455	0,98007	0,025825	0,97342	0,032245
Norrbotten	20-64	0,98971	0,0249	0,98971	0,02528	0,9888	0,02559
Stockholm	65-90	0,96303*	0,003805	0,96303*	0,004175	0,96443*	0,00492
Uppsala	65-90	0,9433*	0,0104	0,9433*	0,010535	0,94339*	0,01369
Södermanland	65-90	1,0368*	0,010405	1,0368*	0,01057	1,04168*	0,0125
Östergötland	65-90	1,02487*	0,00809	1,02487*	0,00829	1,02666*	0,01145
Jönköping	65-90	0,97685*	0,008795	0,97685*	0,00897	0,97918*	0,01037
Kronoberg	65-90	0,93657*	0,011695	0,93657*	0,011815	0,93178*	0,013605
Kalmar	65-90	1,02363*	0,009995	1,02363*	0,01016	1,01938*	0,00967
Gotland	65-90	1,02175	0,021955	1,02175	0,02203	1,01924	0,02548
Blekinge	65-90	1,00521	0,012745	1,00521	0,012875	0,9992	0,016095
Skåne	65-90	0,93752*	0,004425	0,93752*	0,004735	0,93575*	0,008355
Halland	65-90	0,90037*	0,00994	0,90037*	0,01007	0,90892*	0,010955
V. Götaland	65-90	0,98167*	0,003925	0,98167*	0,0043	0,98051*	0,005015
Värmland	65-90	1,09488*	0,009625	1,09488*	0,00982	1,08933*	0,01247
Örebro	65-90	1,03581*	0,009585	1,03581*	0,009765	1,03141*	0,00973
Västmanland	65-90	1,02085*	0,010655	1,02085*	0,010805	1,03191*	0,01275
Dalarna	65-90	1,09009*	0,009615	1,09009*	0,00981	1,08845*	0,01022
Gävleborg	65-90	1,10843*	0,0096	1,10843*	0,0098	1,10288*	0,01016
Västernorrland	65-90	1,11186*	0,01022	1,11186*	0,01041	1,10786*	0,009475
Jämtland	65-90	1,05604*	0,013845	1,05604*	0,013975	1,04766*	0,01603
Västerbotten	65-90	1,10048*	0,011555	1,10048*	0,01172	1,1126*	0,014375
Norrbotten	65-90	1,06444*	0,01144	1,06444*	0,0116	1,07601*	0,01392

* indicates significant difference from 1 on a 1 percent significance level.

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Statistisk sentralbyrå
Statistics Norway

6 November 2007

Revised version

LIFE EXPECTANCY ADJUSTMENTS IN THE NORWEGIAN PENSION REFORM

Presented at the Joint Eurostat/ECE Work Session on Demographic Projections,

Bucharest, 10-12 October 2007

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Abstract

Many countries are reforming their pension systems because of an increasing proportion of old people. In some countries, including Norway, the monthly pension will depend on a flexible age at retirement and the remaining life expectancy when people retire, besides earned pension rights.

An important issue in this connection is whether the pension should depend on the projected cohort life expectancy for the retiring cohorts, or whether it should only depend on observed period values. In some countries the thinking is that use of projections would introduce a subjective element and also politicize the production of population projections. However, even if only period measures are used to compute the pensions, there is a fiscal need for projecting the future total annual expenditures for old-age pensions. Moreover, individuals approaching the retirement age would like to know how much pension they can expect to get, when they are considering when to retire. Thus, a pension system cannot entirely escape the need for projecting the life expectancy.

If *projected* life expectancies are used to compute the pensions, the methodology for making the mortality projections matters. Should simple and transparent methods be used or more sophisticated approaches like the Lee-Carter method? How often should the forecasts be made? Who should make theme and who should approve them? What happens if the real life expectancies improve faster than projected?

If *period* life expectancies are used to compute the pensions, some cohorts may claim that they will receive lower pensions than previous cohorts because of random changes in the life expectancy and that this is unfair. Thus, there is a need to develop robust and transparent statistical methods also when only period measures are used to avoid political and legal problems.

The paper will present the discussion and recommendations regarding the use of period versus projected life expectancy in the new pension system for Norway, which will be introduced in 2010. A brief description of how the life expectancy enters the computation of the pension will also be given.

1. Introduction

The increasing proportion of elderly has led many countries to reconsider their pension systems. The changing dependency burden will make it difficult or impossible in the future to provide pensions for the elderly of the population with regard to pensions, in addition to health care. An important element of many recent pension reforms has been to stimulate people to work longer and retire later. This can be achieved by introducing a flexible retirement age that rewards those who postpone their retirement. This has been done in countries like Sweden and Latvia.

If people can choose when they retire, their annual (or monthly) pension should depend on their remaining life expectancy, otherwise the pension system expenditures may grow beyond the sustainable. Basically, the pension rights or pension wealth for each individual who retires should be divided by the number of additional years they may expect to live.

The life expectancy is not constant, however. It has been increasing rapidly in many countries for many years, also for the elderly. Thus, the estimation of the annual pensions needs to take the increasing life expectancy into consideration. There are a number of concerns in this regard, both of a general and a practical nature:

- How should the remaining life expectancy be translated into an annual pension?
- What kind of estimate for the remaining life expectancy should the used, projected or observed values?
- Should estimates of the life expectancies used in the calculation of pensions be based on period or cohort mortality measures?
- There are significant random variations from year to year in life expectancy and survival probabilities for most populations, in particular for a relatively small population such as the Norwegian. Thus, there has to be some smoothing of the numbers. What method should be used for this? And on how many years should the smoothing be based on?
- How can the increasing life expectancy be built into the pension system?
- How should the difference in life expectancy between men and women be treated in a pension system? Moreover, should the pension system accommodate differences in life expectancy between other groups?

The present paper will look at these issues and explain how we are proposing to solve them in Norway.

2. The new pension system in Norway the divisor[1]

The pension wealth for a person at the age of retirement is the sum of annual entitlements over the life course in a defined contribution system. The expected present value of future pension benefits shall be distributed over the expected number of years lived after the person retires, discounting for income growth, inflation, interest rate and the probabilities of surviving from the age of retirement until each year of the pension period. Assuming that the nominal rate of interest is equal to the wage growth, the yearly pension may be calculated by dividing the earned entitlements by the life expectancy at the age of retirement.

The oldest cohort that can retire according to the new Norwegian pension system is the 1943 cohort, which may retire at age 67 in 2010, when the new system is planned to be implemented. The aim is that these persons should receive the same annual pension whether they retire according to the old or new system. For this reason it has been proposed that the pension benefits are standardized to the annual pension for persons born in 1943 who retire at age 67 in 2010. To achieve this, a divisor has been introduced.

If we, for simplification and for focusing on the demographic aspects of the new pension system, assume that the pension benefits are indexed at the same rate as the real wage rate, it can be shown that the divisor δ at pension age A for a person from cohort K can be estimated by:

$$(1) \qquad \delta_{K,A} = \frac{l_{62,A,K,K+60} \cdot e_{A,K,K+60}}{l_{62,67,1943,2003} \cdot e_{67,1943,2003}} ,$$

[1] This section is based on Brunborg et al. (2007) and Stølen (2007).

where

$e_{A,K,t}$ is the expected remaining life expectancy at age A for cohort K based on observations for year t, and

$l_{62,A,K,t}$ is the probability that a person from cohort K survives from the lower pension age 62 until pension age A based on the mortality observed in year t.

This standardization implies that the divisor for persons from the 1943 cohort who retire at age 67 in 2010 is identically equal to 1.

Thus, with some simplifying assumptions the divisor can be estimated as the quotient between life expectancies for the actual cohort and the 1967 cohort, weighted by survival probabilities, i.e. this form of the divisor is a function of purely demographic and no economic parameters.

Figure 1 shows the estimates of the divisor by age for three cohorts, 1943, 1963 and 1983. We notice the strong effect on the divisor from postponing the retirement. For the 1963 cohort it is reduced from about 1.42 to 0.95, by retiring at age 70 in stead of at age 62. This amounts to an increase in the annual pension at about 50 per cent. We also see that the divisor is higher for later cohorts, which is due to the assumed continued increase in the life expectancy.

Figure 1 Divisor by pension age for selected cohorts. Preliminary estimates, no smoothing, no indexation

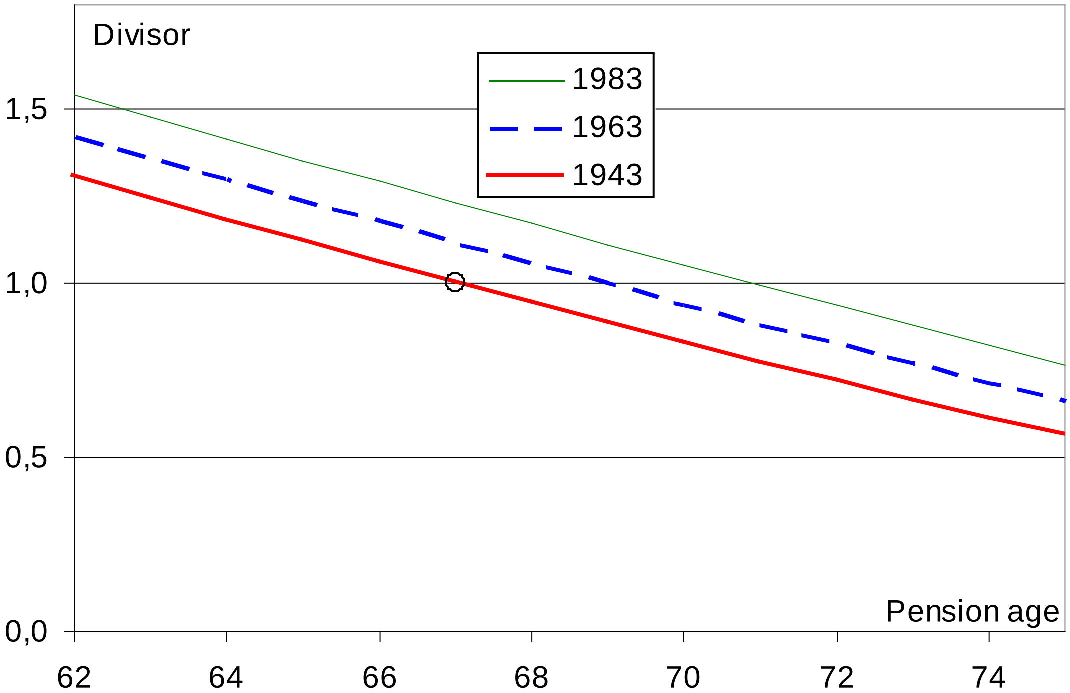
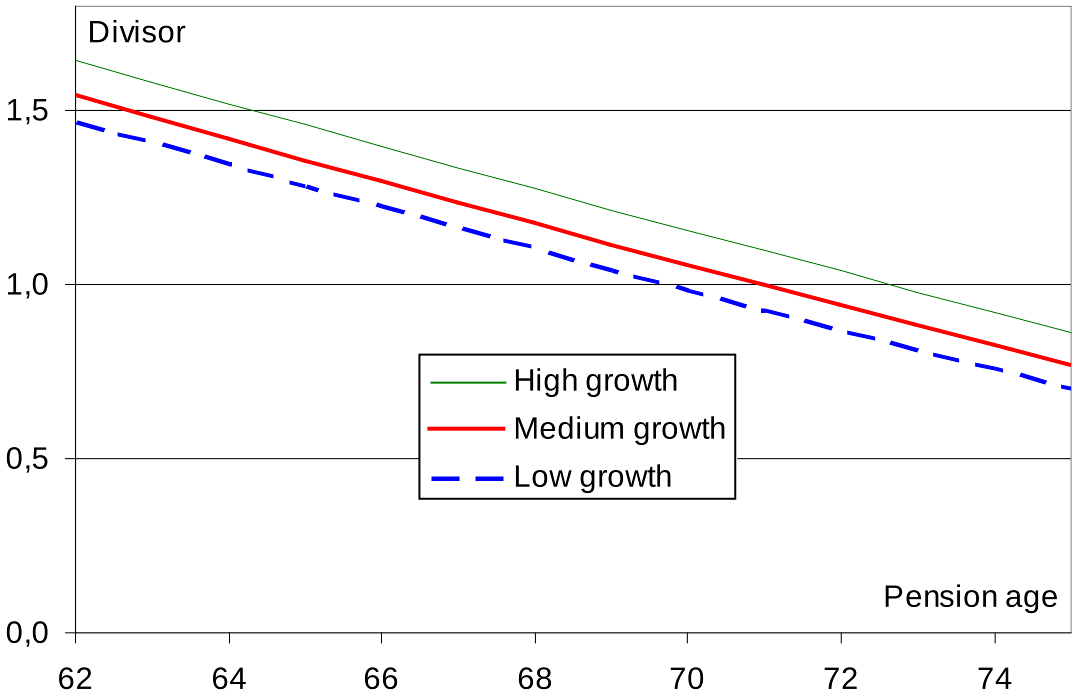


Figure 2 shows the divisor according to the assumptions in the most recent mortality forecast in the population projections for Norway (for 2005-2060). The divisor is larger, implying a smaller annual pension, the stronger the growth in future life expectancy.

Figure 2 Divisor for the 1983 cohort by mortality assumptions in the 2005-2060 population projections: Low, medium and high growth of the life expectancy. Preliminary estimates, no smoothing, no indexation



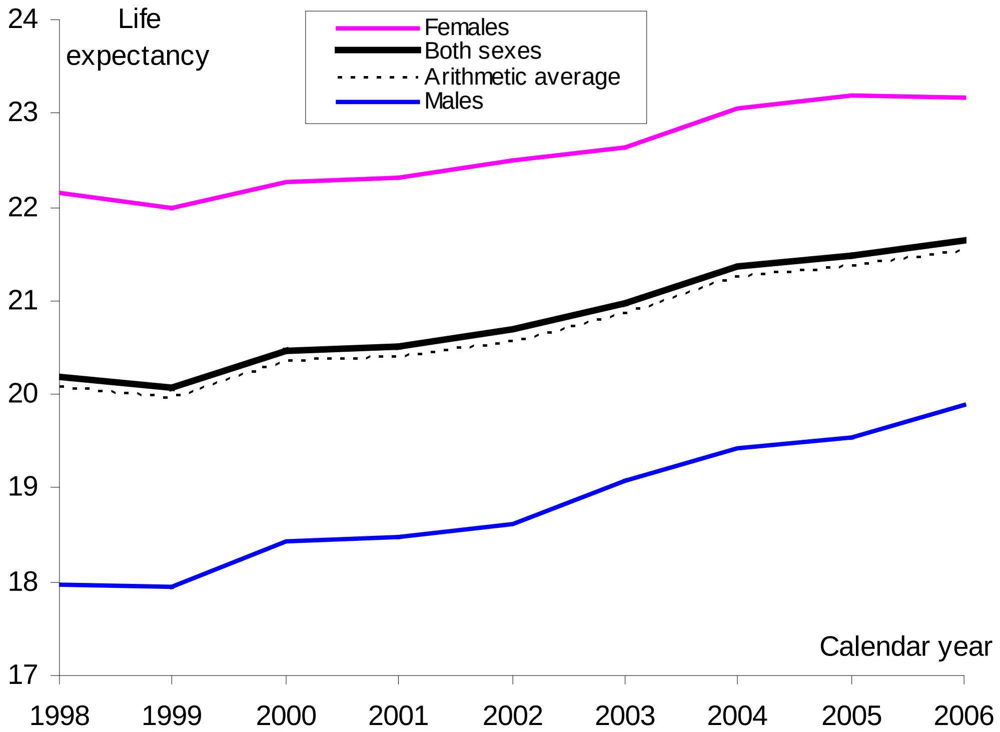
3. Gender neutrality

The Norwegian Parliament has decided that the divisor should be gender neutral. Thus, the large difference between male and female mortality should not affect the pension rules. It is easy to imagine the strong reactions if women would get a lower annual pension than men who retire at the same age because women can expect to live longer than men at all ages, given the same earned pensions rights. Adding to the likely uproar because of such a rule, would be the fact that women have on average much lower earnings over the life cycle than men, which would result in significantly lower pensions.

The male-female mortality differential is, however, one of the basic facts of life and demography (with a few exceptions historically and globally). Thus, mortality measures are almost always estimated for each sex separately. This has also been the case for Norway, until recently.

The simplest way of estimating the life expectancy for both sexes combined is to calculate the arithmetic average. This will usually yield acceptable results for the life expectancy at birth, but not for old ages where there are many more women than men. (In Norway today women constitute 59 per cent of the population over 67, the current pension age.) Thus, the best method would be to weight the age-specific death rates by the sex ratio at each age, or even better, to combine all data for men and women and treat them as one sex only. We have done this for Norway, see figure 3.

Figure 3 Life expectancy at age 62 for men, women and both sexes



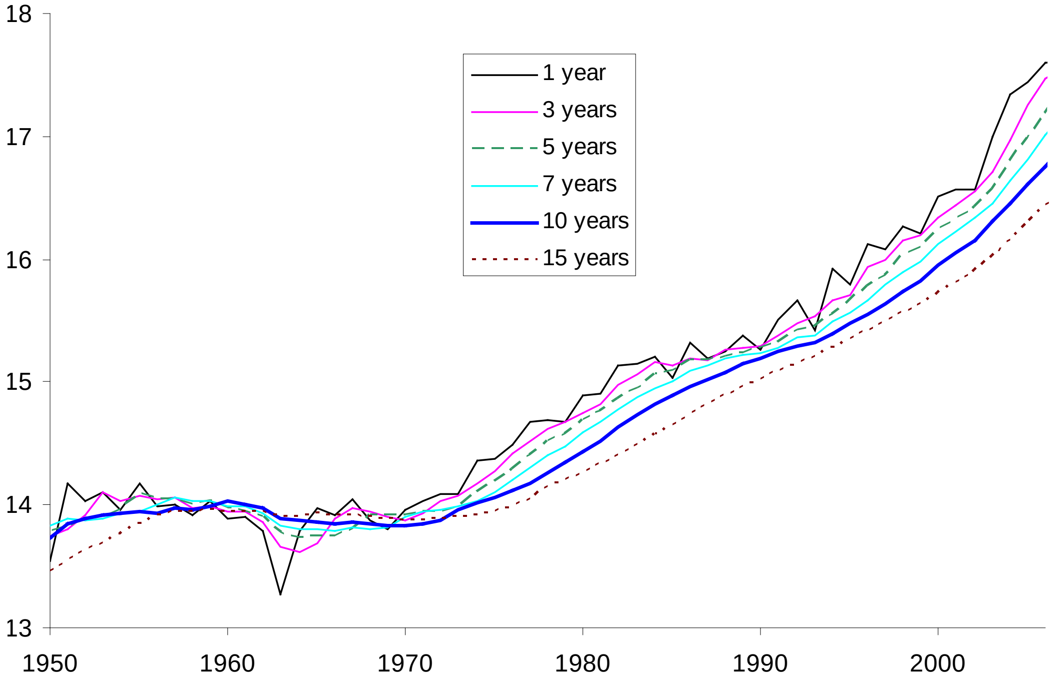
We notice that the arithmetic average for men and women is slightly lower than the life expectancy at age 62 for both sexes combined. Although the difference is small, only about 0,11 years, it could have a significant effect on annual pensions. (The figure also shows that from 2005 to 2006 e_{62} increased strongly for men (+0.36 years) whereas it declined slightly for women (-0.02 years). Using data for both sexes combined removes such anomalies.)

It is not impossible that the government or perhaps Statistics Norway could be taken to court by a group claiming unfairly treatment because poor or biased methodology was used to estimate the pensions, such as the arithmetic average. Because of the importance of this, Statistics Norway has started to estimate and publish life expectancy as well as other mortality measures for both sexes combined, in addition to men and women separately.

4. Smoothing issues

For a relatively small population like the Norwegian (4.7 mill as per 1. January 2007) there is considerable random variation in the mortality rates from year to year and from age to age, see figure 2. It is, therefore, necessary to smooth the life expectancies and survival probabilities that are used to estimate the pension divisor.

Figure 4 Mean remaining life expectancy for both sexes at age 67, 1950-2006, for different lengths of the observation period



In particular, we notice from figure 4 the declining trend in life expectancy from about 1950 to about 1970, which is due to the deteriorating mortality of men. Such trends should, of course, be reflected in the estimates of the divisor.

We have experimented with a number of smoothing methods, including 3, 5, 7, 10, 15 and 21-year moving averages, a 15-degree polynomial, and linear and quadratic regression analysis. We have looked at different time periods and done the smoothing for different mortality measures, including e_{62}, e_{67} and $l_{62,A,K}$ (the probability that a person from cohort K survives from the lower pension age 62 until pension age A). Figure 4 shows the results from smoothing the life expectancy over different time periods. The time trend is, naturally, smoother the longer the time period. The disadvantage of using long periods are, however, first, that estimates for recent years will be significantly lower than the most recent one-year estimates if the life expectancy is increasing more or less monotonically, as has been the case for almost two hundred years. Thus, we "lose" the observations for the last years, depending on the length of the smoothing period.[2] The use of consistently too low estimates does not matter, however, when we use the divisor to calculate the annual pensions. Secondly, with a long smoothing period it is difficult to detect new time trends, in particular a change to stagnating or declining life expectancy.

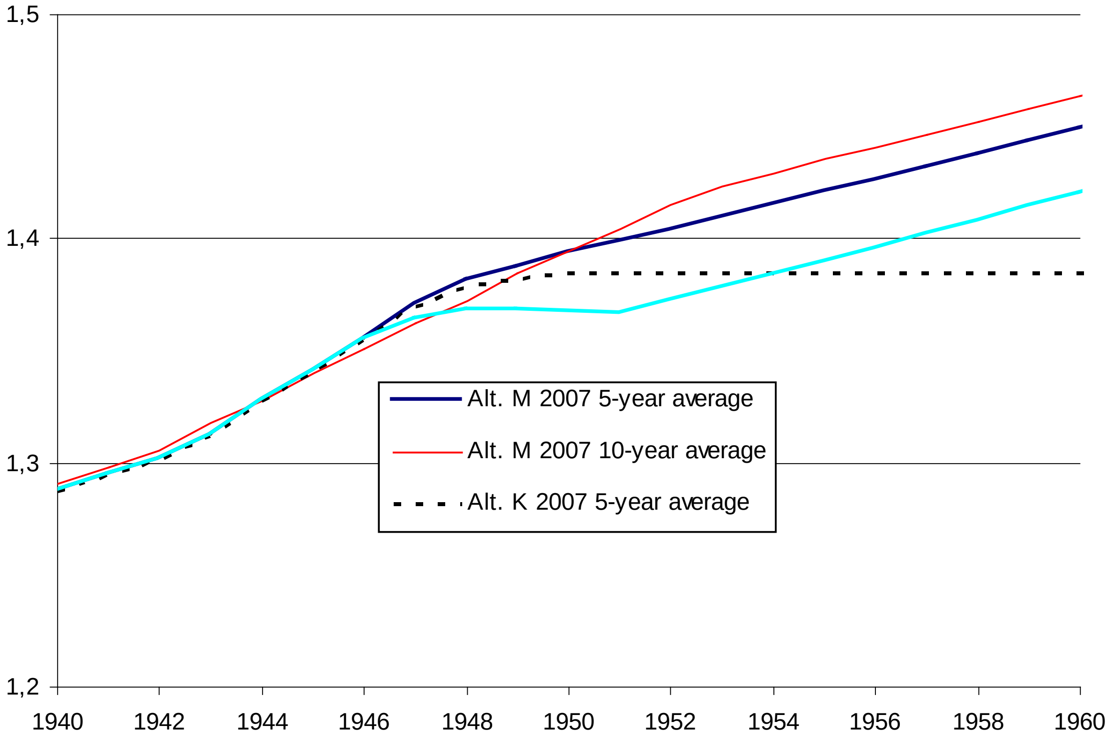
Sophisticated method may give smooth curves over time but have some drawbacks:

- Sophisticated methods are difficult to explain, both to politicians and to the public. It is hardly feasible to include a description of very complicated methods in a parliamentary report, for example.
- The estimates of the divisor should include the most recent available information. Thus, they should be reestimated every year. This might, however, change the estimates significantly.

[2] Note that the 5-year moving average yields exactly the same estimates as the average for the five last years, of course, and similarly for other smoothing period. The only difference between the 5-year moving average a_t and the average for the five last years for year t, is that the former cannot be estimated for years t and t-1, unless there is a special procedure for estimating the tails.

Figure 5 shows the divisor at age 62 for cohorts 1940-1960 by different assumptions about the development of life expectancy, and for 5- and 10-year averages. The use of 10-year instead of 5-year smoothing periods results in a smoother development of the divisor for cohorts born in the 1940s, the first to benefit from the new pension system, which is desirable. On the other hand, the use of a 10-year period leads to a higher divisor in the long run, which members of future younger cohorts will dislike. The life expectancy increased unusually fast in 2003 and 2004, which necessitated a long smoothing period. The 10-year average gives lower weight to these excetional years.

Figure 5 The divisor at age 62 for cohorts 1940-1960 by different assumptions about the development of life expectancy. 5- and 10-year averages. Preliminary estimates, no indexation



Our conclusion is that averaging over 5 to 10 years ensures a sufficiently smooth development, without having too serious disadvantages. In Sweden a 5-year average was chosen, but the population of Sweden is more than twice that of Norway, which reduces the stochastic variation.

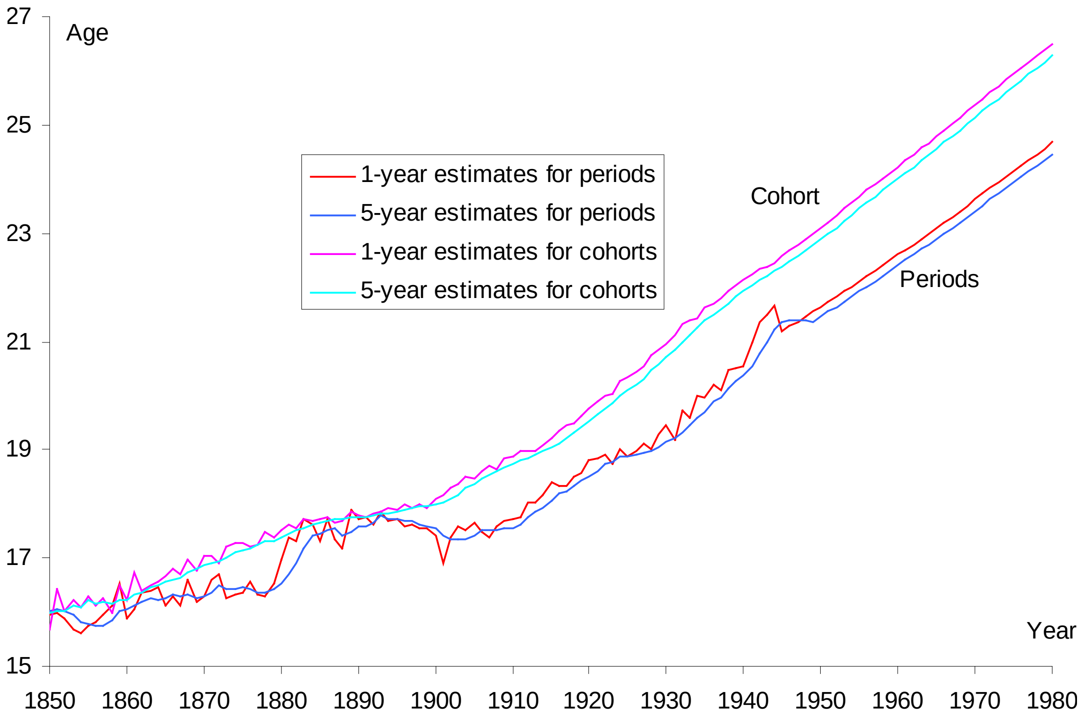
5. Cohort versus period mortality

The new pension system in Norway will be implemented in 2010. Thus, the first cohort that will be able to retire at the lowest new pension age 62 according to the new system is the 1948 cohort, which will turn 62 in 2010. In 2009, when the members of this cohort is considering whether to retire or not in the following year, the most recent available estimate of the life expectancy at age 62, e_{62}, will be the estimate based on observations for 2008, which will become available in April 2009. This estimate is made on the basis of data for persons 62 and older in 2008, i.e. those born in 1946 and before.

Figure 6 shows that the period e_{67} has increased rapidly for more than thirty years, after a decline from about 1950 to about 1970. (This decline was caused by an unfavourable mortality development for middle-aged and older men.) What about the *actual* number of years lived after age 62? It is likely that the declining death rates for older people in recent years will also affect the cohort life expectancy age ate 62. If a cohort lives much longer than the period e_{62} at the time of retirement, the pension fund needs to be distributed over more years, which may cause problems for the system.

The *cohort* life expectancy e_{62} for the cohorts that will soon retire, such as the 1948 cohort, will only be known after about 2050, when there is almost nobody left. Since this value is not available now it cannot, of course, be used to estimate pensions for the 1948 cohort. Instead we will have to rely on previous experience or projections of e_{62}, see figure 6. The upper graphs show that the cohort e_{62} has been growing steadily. The 1850 cohort lived on average 16 years after age 62 and the 1900 cohort about 18 years. We also see how the increasing mortality of men in the 1950s and 1960s led to a much slower growth of e_{62} for cohorts born between about 1880 and 1900. For more recent cohorts we do not know the final number yet since they have not completed their life cycle, but we have used the medium variant death rates from the most recent population projection for Norway, for 2005-2060 (Keilman & Pham (2005) and http://www.ssb.no/english/subjects/02/03/folkfram_en/.)

Figure 6 Remaining life expectancy at age 62 for periods and cohorts, 1- and 5-year estimates*



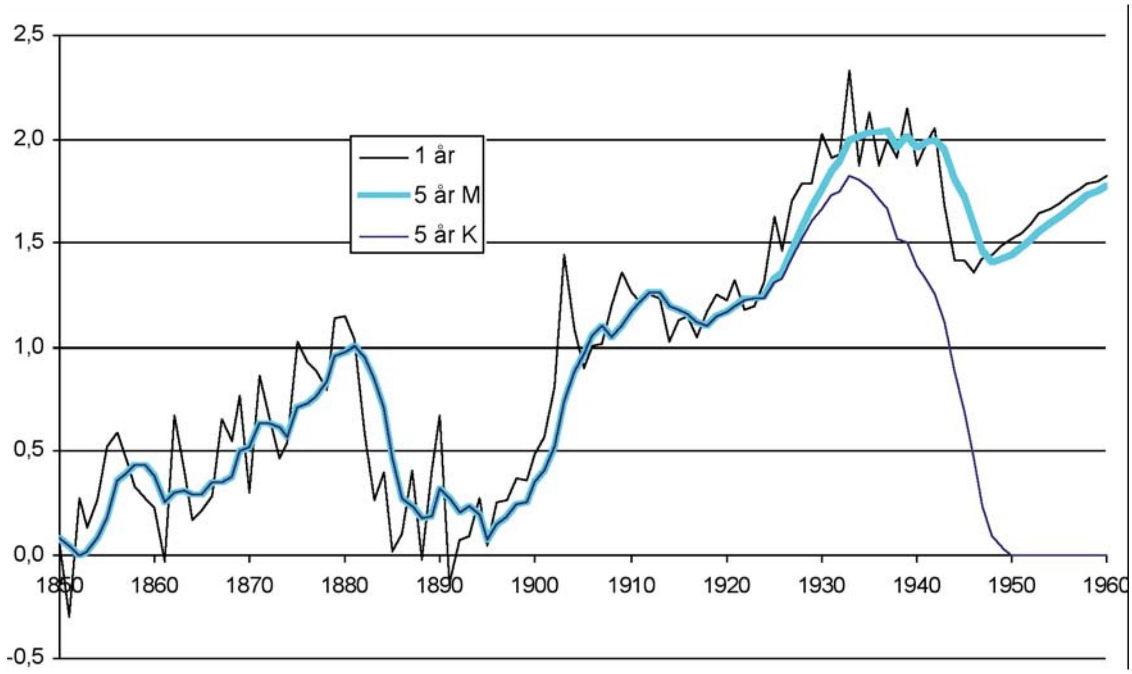
* The x-axis is the year of birth of the cohort. The period values are plotted for the year of observation minus 62.

The lower curves in figure 6 show the period e_{62} plotted against the year of observation minus 62, to enable comparison of the cohort and period life expectancy at age 62. The difference between the estimates of these is shown in figure 7. Until about 1915, when only observed values have been used, cohorts lived between 0 and 1 year longer than the period e_{62} at the time the cohort turned 62. After 1915 we have extrapolated the cohort e_{62} with death rates from three different projections: First, the medium variant from the 2005 projection, as in figure 6. According to this projection the cohorts will live between 1 and 2 years longer than the observed period value when they turned 62.[3] The upper curve in Figure 7 shows the difference between cohort and period life span according to the *highest* variant in the 2005 projections (H2005). The lowest curve shows the corresponding difference if the observed period death rates in 2006 remain *constant* throughout the projection period, i.e. until 2060 (K2006). By definition this difference will approach zero over time.

We conclude from this comparison that cohorts may live up to, and perhaps more than, two years longer than the observed life expectancies when the cohorts where able to retire. However, with the use of a divisor as in the proposed Norwegian pension system, it does not matter much if this difference is constant. But if it is changing over time, e.g. increasing as the tendency is, it would be more serious for the system.

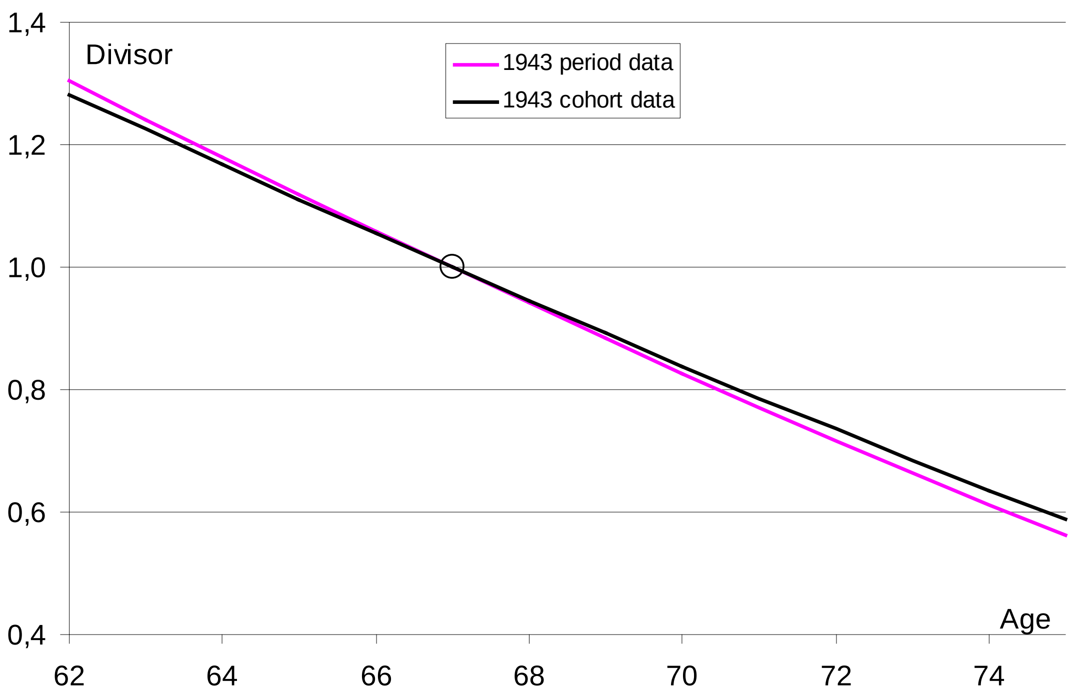
[3] The hump between the 1940 and 1950 cohorts is due to two factors: First, the unusually strong mortality decline since 2004, i.e. after the projections were made, especially for elderly persons. Second, the estimation of projected age-specific death rates from Lee-Carter parameters (Keilman and Pham 2005), introduced a discontinuity in the time series of the death rates between those based on past observations and those used in the projections.

Figure 7 Difference between cohort and period life expectancy at age 62, five-year estimates*



* The x-axis is the year of birth of the cohort. The period values are plotted for the year of observation minus 60. M is medium mortality forecast, K is constant mortality

Figure 8 Divisor based on period vs. cohort data for the 1943 cohort.
Preliminary estimates, no indexation



6. Projected or "observed" survival probabilities?

It seems natural to use projected values for the estimation of the divisor in the pension system. This would take care of some of the problems discussed above. There are, however, several problems using projected values:

1. Projections are uncertain. How can the uncertainty be taken care of in the pension system? The pension system administration needs to estimate the actual annual pension for each person who is retiring, depending on the divisor and the income history of each retiree. It cannot use confidence intervals for the pensions. Moreover, people who are considering retirement will most probably not be satisfied with confidence intervals for their annual pension.
2. Who should make the projection? In most countries the national statistical office would be the most qualified to do this. This would, however, introduce a strong political element into the projection activities. The statistical office could be accused of making life expectancy projections that have been designed to satisfy some certain interests, e.g. those of the government. Moreover, the statistical office could perhaps be used for making unrealistic projections.
3. There is no universally accepted method for making mortality projections. Some may argue that their method is better than the one being used. Moreover, the method should be relatively simple to explain and to use. Finally, reestimation of the projected values when new mortality observations become available might yield quite different estimates.
4. For several decades the life expectancy has been increasing in most countries. This will probably not continue to be the case for ever. It may, however, be difficult to detect in projections. If there is a period of declining life expectancy some cohorts might get a considerably higher pension than preceding cohorts, which they would probably consider unfair.

For these reasons most countries have chosen to base the life expectancy estimated in the pensions system on observations and not on projections. This has been done in Sweden, Poland, Lithuania, Italy and Germany (Palmer 2003; Lindell 2004). However, projected life expectancies are used in the USA and in Latvia, where the projections are made by a government statistical institution and reviewed by an independent committee of experts (Palmer 2003; Technical Panel on Assumptions and Methods 2003).

However, a pension system cannot entirely escape projections of the life expectancy. There are at least two reasons why population projections are required:

- First, to estimate the future fiscal implications of the pension system. The government needs to have estimates of the future pension revenues and expenditures and the expenditures will depend on how long the future retirees will live.
- Second, when people are approaching the minimum age of retirement they will consider whether they should retire or not and for this they will want to know what pension they can expect to get if they retire soon or if they postpone their retirement. For this it is necessary to have life expectancy projections to estimate the expected divisor and the expected pensions.

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Session 3: Population projections

Chair: Graziella Caselli

IV

A NEW TECHNIQUE FOR STOCHASTIC POPULATION PROJECTIONS

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Summary

The evolution of a population structure is determined by a sequence of birth, death, immigration and emigration phenomenon. Such a sequence of events can be seen as the realisation of a stochastic point process. It is difficult to study such a stochastic process analytically. On the contrary, it is possible to simulate the realisations of the process and hence the evolution of the studied population.

The basic hypothesis is that the sequence of events that determine the population's evolution is generated by a point process that is the combination of several independent Poisson processes, i.e. the birth, death, immigration and emigration processes.

Every Poisson process is characterised by its own instantaneous rate. It is known that by assembling more independent Poisson processes we obtain another Poisson process that has the sum of the rates of the components as a rate. On the other hand, in a Poisson process the waiting time for an event, beginning from an initial time or from the time when the latest event has occurred, is distributed following a negative exponential law.

These and other theoretical results allow us to simulate, for each year of study, the instant where the events occur and the type of event happening in each instant. Hence, year after year, the population's evolution.

The suggested simulation procedure gives us the estimates of the average values and of the standard deviation of all the parameters that characterise the population in each year of the studied period of time. In such a way we obtain important information on the accuracy of the projections and interval estimate techniques can be utilised.

Keywords

Poisson process, simulation, waiting time, combination of point processes, instantaneous rate of the realisation of the events, fertility rates, mortality rates, migratory rates.

1. Introduction and formulation of models

The methodological aspect of the proposal is based on the properties of the stochastic point processes of the Poisson type.

A point process is called Poisson if:

-The events occur singularly

-Prob {One event in [t, t+Δt)} = $v(t)\,\Delta t + o(\Delta t)$

-The events that occur in separate intervals are independent

The $v(t)$ function is called the instantaneous rate of the realisation of the events.

One population in evolution can be thought of as a point process built up from combinations of independent Poisson processes.

Every individual present within the population generates one or more independent processes such as: birth, death, migration. For example, a male of 35 years of age generates events such as death and emigration whereas a 27 year old woman generates events such as birth, death and emigration. We will suppose that each of such events derives from a particular Poisson process characterised by its own realisation rate that depends on age.

By assembling all the death processes related to single individuals we obtain the process of deaths. By assembling all the birth processes, we obtain the process of births. By assembling all the processes of emigration the process of emigrations. The immigration process, which does not depend on the population composition and whose rate constitutes an input for the entire procedure, is also considered.

The given model used to study the population's evolution is the one referred to the process of events obtained by assembling all the elementary processes occurring within the population.

It is difficult to study such a stochastic process analytically. On the contrary, it is possible to simulate some realisations of the process and, therefore, the evolution of the studied population.

2. The simulation procedure

We will study the population's evolution by simulating all the events that occur in each year of the studied period in the combined process. In order to do so we will remind you of some theorems concerning the Poisson processes.

Theorem 1: The process obtained through the combination of several independent Poisson processes is still a Poisson process.

Theorem 2: The realisation rate of a Poisson process made out of more independent Poisson processes equals the sum of the rates of the component processes.

The simulation procedure consists of the simulation of **when, which and how many** events arise within the combined process in each year.

When does an event arise?

Theorem 3: In a Poisson process the waiting time for an event, starting from an initial time or from the time when the latest event took place, is distributed according to a negative exponential law. That is, that it has a distribution function: $F(t) = 1 - e^{-\nu t}$ when ν is the realisation rate of the events.

It is known that if a number y arises from a uniform distribution (0,1), the value $t = F^{-1}(y)$ is distributed following a negative exponential law. Hence if y_i *(i=1,2,3,…)* is a sequence of values generated by a uniform distribution (0,1) $t_i = (-\log(1 - y_i))/\nu_i$ will constitute the sequence of the time that separates the events (ν_i is the sequence of the realisation rates that change due to the events that may have occurred before). Hence the events will occur at the times: $T_i = t_1 + t_2 + \ldots + t_i$

Which event will occur?

Theorem 4: If in an instant T_i an event in the combined process occurs the probability that such an event would derive from an assigned component process is given by the quotient between the realisation rate of the component process and the combined process realisation rate.

Hence, given the time T_i of an event in the combined process, we may simulate the type of event that occurred in T_i through a random trial on the distribution of the types of event which is defined by:

$$\Pr\{\text{One birth in } T_i\} = \lambda_i / \nu_i$$
$$\Pr\{\text{One death in } T_i\} = \mu_i / \nu_i$$
$$\Pr\{\text{One immigration in } T_i\} = \alpha_i / \nu_i$$
$$\Pr\{\text{One emigration in } T_i\} = \beta_i / \nu_i$$

where $\lambda_i, \mu_i, \alpha_i, \beta_i$ are respectively the rates of the processes of birth, death, immigration, emigration evaluated before the event in T_i and $\nu_i = \lambda_i + \mu_i + \alpha_i + \beta_i$ is the realisation rate of the combined process.

After having simulated the type of event using analogous procedures and using the same properties of the Poisson processes, we can establish:

- for a birth, the sex of the newborn and the age of the mother when delivering
- for a death, the age and the sex of the deceased
- for an immigration, the age and the sex of the immigrant
- for an emigration, the age and the sex of the emigrant

therefore obtaining a detailed description of the event.

How many events must we simulate?

The process of the generation of the events in one year ends when the time T_i in which the latest event should happen overtakes the value 1, that is the end of the year.

The update of the population structure at the end of the year

The recording of the obtained data allows for the updating of the population structure to be taken into consideration at the beginning of the following period.

Calculation of the parameters

The new population structure and the evolution that occurred during the year, may be described as a set of parameters that may be calculated on the basis of the events that happened during the same year. With the recorded data we may then determine for each year:

- the population distribution per five year age groups by sex
- the average population age by sex
- the number of births by sex
- the average age of the mothers at the delivering
- the number of deaths by sex
- the average age of the deceased by sex
- the number of immigrants by sex
- the average age of the immigrants by sex
- the number of emigrants by sex
- the average age of emigrants by sex
- ageing indexes
- economic dependence indexes
- replacement indexes
- other .. e. g. the maximum age of the living by sex

Dynamic of the parameters and calculation of the instantaneous realisation rates of the different types of events

The hypothesis that all the processes generating the events within the population are of the Poisson type allows us to characterise them through the instantaneous realisation rates of the events. The process is subsequently simplified by the ulterior hypothesis that the rates remain constant during the year.

Hence, for example, the instantaneous mortality rate for a man aged x, $\mu_m(x)$, supposed constant for the whole age x, will be defined by the relationship:

$$\mu_m(x)\Delta u = \Pr\{\text{A male aged } x \text{ dies within time interval } (x+u, x+u+\Delta u)\} \quad (0 \le u < 1)$$

The rate $\mu_m(x)$ may be calculated, as it is known, from a life table considering the relationship:

$q_m(x) = 1 - exp(-\mu_m(x))$, from which: $\mu_m(x) = -log(1 - q_m(x))$. Likewise $\mu_f(x) = -log(1 - q_f(x))$ will be the realisation rate of the event of death in a female aged x.

In the same way the following may be calculated:

- the realisation rates of birth events of a child of a mother aged x by fertility rates f(x): $\lambda(x) = -log(1 - f(x))$;
- the realisation rates of emigratory events of an individual aged x by migratory rates u(x): $\gamma_m(x) = -log(1 - u_m(x))$ and $\gamma_f(x) = -log(1 - u_f(x))$

In the case of immigration the realisation rate is supposed to be constant during each year and equals the average number of yearly immigrants by sex evaluated initially: say e_m and e_f.

In order to consider the mortality, fertility, and immigration evolution after the initial time, a further scenario is taken into consideration to represent the situation at the end of the period. The values of the parameters during the period are calculated through linear interpolation.

The realisation rates of the events referred to a population are easily obtained through the hypothesis of independence of the Poisson processes that generate events, summing up the realisation rates of all the individuals present in the population within a fixed time (Th. 2), hence:

- $\mu(t) = \sum_{x} [p_m(x,t) \cdot \mu_m(x) + p_f(x,t) \cdot \mu_f(x)]$

- $\lambda(t) = \sum_{x=15}^{49} p_f(x,t) \cdot \lambda(x)$

- $\gamma(t) = \sum_{x} [p_m(x,t) \cdot \gamma_m(x) + p_f(x,t) \cdot \gamma_f(x)]$

- $\beta(t) = e_m + e_f$

3. A simulation

The procedure of simulation foresees the generation of events related to a given number of years and therefore the estimate of the population structures year by year for a given number of years.

4. More simulations

The whole procedure is repeated for an assigned period of time generating, for each year of study, many estimates of the population structure. In such a way for every year of study and for each population characteristic we may calculate the respective average values and the standard deviations that give a precise estimate of the variability of the characteristic measures.

5. The data

The necessary data for the application of the procedure consists of:

A. mortality rates by sex

B. fertility rates

C. emigratory rates by sex

D. average number of immigrants by sex

E. immigrants age distribution by sex

With the aim of being able to catch the dynamic of such data and to explore the whole spread of possible scenarios of the data, two functions may be given for mortality rates and for fertility rates. The first refers to the beginning of the studied period of time and the second refers to the end of the same period.

Please note that the entire simulation procedure, being forced to simulate all the events occurring in each year of the studied period and for several times, requires a relevant quantity of processing that may be undertaken only through the utilisation of the modern means of calculating. For this purpose a processing program called PRODEST has been developed. Such a program allows us to obtain the desired results.

6. Simultaneous study of more populations

The procedure we just illustrated can be generally applied to the simultaneous study of several populations belonging to the same geographical area. As an example we may want to study the populations of Rome, distinguishing the district's populations that constitute Rome. Or we may want to study the population of Italy as a combination of the Italian regions populations.

Also in such cases the simulation procedure foresees the generation of all the events that occur in the whole geographical area studied which are characterised as described in the case of just one population. The only difference concerns the events related to the migratory movements that may occur both within the geographical area and in respect to the outside of the geographical area. Migratory movements towards populations within the same geographical area will correspond, as it is evident, to immigration movements in other units of the area. Whereas the migratory movements towards and away from the geographical area must be considered separately.

A migratory probability matrix, as an input data, is introduced to deal with such possibility. The rows of such a matrix are constituted by the migratory probability distribution from one unit of the area to another unit or away from the area.

In the simultaneous study of more populations the problem of the amount of calculations to be carried out becomes relevant. The MULTIPRODEST program helps in solving such a problem but as for populations of great dimension the processing time may be rather long.

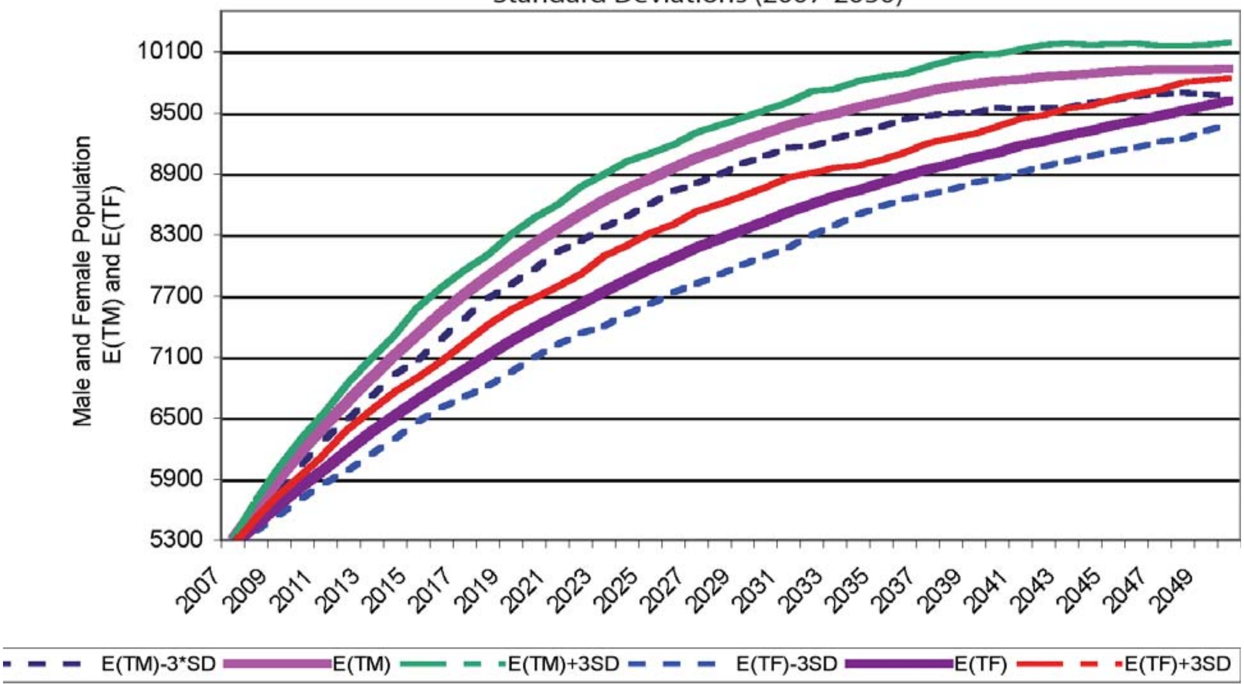
7. An example of demographic estimates with PRODEST

In the following pages we will show the results of demographic estimates regarding a small town close to Rome: San Cesareo. The period studied is from 2007 to 2050.

The average values of the estimate data are based on 21 simulations.

Figure 1 shows the average values for the total population of males (E(TM)) and of females (E(TF)) and ± 3 standard deviation intervals.

Figure 1 - Population Projections and Standard Deviations (2007-2050)



Tables 1 and 2 show the average values for both sexes by five year age groups.

Table 1 Average values for both sexes by five years age groups

YEAR	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54
2007	517,0	589,0	600,0	606,0	672,0	738,0	878,0	949,0	901,0	712,0	661,0
2008	552,2	579,8	615,3	635,7	696,7	774,6	926,9	993,0	975,9	753,4	682,6
2009	565,9	589,1	631,6	656,8	699,8	826,3	956,2	1035,6	1030,4	812,8	719,1
2010	597,0	597,6	630,3	656,0	746,4	842,1	1014,5	1040,7	1106,3	859,2	757,1
2011	613,6	600,2	652,8	668,8	751,1	870,9	1058,8	1095,0	1128,2	930,8	784,9
2012	605,8	627,5	681,0	680,9	735,2	919,8	1067,5	1165,4	1137,5	1001,5	806,7
2013	615,4	648,1	673,8	701,6	758,1	932,4	1095,5	1199,2	1165,9	1069,0	838,3
2014	626,7	655,1	676,9	719,5	776,9	935,2	1129,8	1220,5	1195,2	1117,6	884,8
2015	632,4	681,0	686,0	718,2	786,5	965,8	1140,2	1255,1	1203,3	1175,0	928,6
2016	637,5	696,7	686,2	734,1	798,8	963,9	1158,4	1289,5	1244,4	1195,1	989,0
2017	651,1	687,5	706,0	755,5	807,5	957,0	1187,0	1292,0	1295,5	1198,9	1052,8
2018	658,9	695,3	726,8	748,3	815,2	977,1	1189,9	1313,0	1322,7	1214,3	1111,3
2019	662,6	702,1	735,8	753,3	831,6	985,0	1203,1	1338,3	1339,9	1236,3	1151,3
2020	668,8	705,7	756,8	759,3	827,9	992,0	1218,7	1346,5	1366,2	1245,6	1195,9
2021	675,6	709,4	766,5	762,1	842,6	1001,3	1221,8	1356,3	1392,2	1277,4	1211,3
2022	680,1	722,0	758,4	778,9	856,3	1003,7	1221,1	1378,8	1392,0	1319,1	1210,5
2023	689,6	729,9	761,9	795,0	848,9	1016,2	1222,6	1378,0	1407,5	1343,5	1225,1
2024	698,5	729,7	761,6	807,2	852,3	1020,8	1222,2	1380,8	1422,5	1356,7	1245,6
2025	698,3	731,3	767,6	821,2	859,5	1012,0	1232,9	1387,0	1428,2	1377,1	1248,1
2026	700,0	734,4	772,0	824,9	858,1	1017,7	1234,7	1388,5	1428,0	1406,3	1269,2
2027	702,7	731,3	779,5	814,4	868,6	1033,2	1232,2	1389,1	1443,7	1403,3	1306,6
2028	700,0	739,4	780,6	810,6	883,4	1030,9	1224,6	1385,2	1445,7	1408,0	1328,7
2029	701,6	750,1	779,2	812,0	894,0	1026,4	1232,2	1385,9	1441,0	1424,3	1344,0
2030	697,1	750,7	779,7	821,7	905,4	1027,7	1223,1	1382,8	1444,8	1431,0	1361,9
2031	701,5	748,4	782,8	823,1	904,2	1026,0	1237,9	1382,3	1444,1	1429,2	1383,1
2032	699,3	748,1	783,0	828,4	898,8	1030,1	1243,2	1379,4	1448,2	1431,8	1380,4
2033	703,5	743,7	787,9	832,3	892,8	1037,2	1234,4	1372,3	1445,2	1435,9	1385,8
2034	708,9	743,3	798,0	827,7	892,2	1045,5	1224,2	1365,2	1441,5	1434,3	1396,7
2035	708,0	744,1	798,0	827,1	891,8	1052,4	1220,3	1359,2	1435,7	1433,9	1403,1
2036	713,4	741,4	799,2	833,8	895,4	1048,2	1215,9	1364,8	1432,3	1432,1	1400,0
2037	723,9	740,5	795,4	828,5	901,9	1043,4	1213,0	1368,6	1426,0	1436,0	1405,3
2038	718,8	744,5	785,5	834,1	901,5	1034,8	1219,1	1358,3	1414,8	1438,0	1405,6
2039	719,0	744,0	786,6	841,0	896,0	1028,0	1222,3	1348,2	1413,0	1429,5	1406,7
2040	715,3	747,6	779,5	842,1	897,0	1019,1	1220,9	1340,0	1412,9	1425,5	1405,0
2041	716,7	746,1	781,0	837,4	898,8	1028,7	1210,1	1331,8	1412,8	1420,1	1404,0
2042	712,5	751,1	778,9	830,9	895,7	1026,4	1201,2	1335,8	1409,1	1409,3	1409,9
2043	714,0	745,3	782,9	823,2	899,0	1024,2	1197,2	1335,9	1399,1	1404,9	1406,1
2044	708,6	751,9	779,2	826,1	898,9	1015,8	1188,3	1336,3	1391,0	1399,0	1401,8
2045	708,1	751,0	781,6	823,0	895,1	1021,1	1182,0	1334,7	1385,0	1394,9	1395,0
2046	707,0	750,3	784,8	822,8	894,2	1021,9	1177,7	1323,8	1374,6	1391,8	1391,6
2047	706,1	740,1	791,9	822,1	889,8	1019,8	1173,8	1313,3	1379,2	1381,7	1376,4
2048	704,4	741,6	788,6	823,0	884,9	1018,9	1173,9	1314,6	1376,8	1371,9	1369,0
2049	707,0	741,3	786,3	819,4	887,5	1022,5	1168,4	1300,0	1369,4	1371,7	1358,7
2050	711,6	737,9	782,0	821,0	881,4	1013,9	1177,0	1290,2	1357,8	1372,1	1355,6

Table 2 Average values for both sexes by five years age groups

YEAR	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99	>99	TOTAL
2007	630,0	588,0	529,0	426,0	283,0	180,0	61,0	20,0	5,0	0,0	10545,0
2008	682,6	579,9	556,0	439,5	316,4	207,3	60,0	27,9	5,4	0,0	11061,0
2009	709,0	575,8	600,8	436,6	363,1	211,1	79,7	37,4	6,6	0,0	11543,8
2010	720,8	583,8	633,1	477,2	366,0	225,3	100,8	43,0	5,7	0,4	12003,3
2011	713,3	632,4	620,0	499,5	386,2	254,8	125,5	39,7	7,5	0,2	12434,2
2012	731,3	679,8	622,5	532,8	409,9	263,0	143,0	40,3	9,3	0,3	12861,2
2013	747,0	727,8	616,3	557,5	415,5	290,2	159,1	41,2	12,0	0,7	13264,5
2014	780,1	752,3	607,0	595,1	415,6	326,8	163,3	52,8	13,8	0,9	13645,8
2015	807,9	762,2	615,7	622,4	451,9	326,0	176,2	62,4	15,0	0,9	14012,8
2016	835,2	753,0	659,6	612,7	472,9	343,4	200,1	74,0	13,1	1,5	14359,1
2017	851,1	768,2	701,7	611,8	497,1	363,5	205,0	83,9	14,8	1,6	14689,6
2018	881,2	780,3	742,0	606,0	511,8	373,0	221,9	94,7	16,9	1,8	15002,3
2019	920,6	805,2	763,9	601,0	541,5	373,9	244,9	96,7	23,5	2,3	15312,7
2020	956,0	831,4	772,2	609,5	564,0	399,0	243,9	105,9	25,7	2,6	15593,4
2021	1005,0	854,4	759,2	647,9	559,8	415,8	259,1	116,8	29,0	2,1	15865,9
2022	1058,4	869,5	771,9	682,5	565,1	429,7	274,2	119,7	31,8	2,6	16126,2
2023	1110,3	892,4	782,5	717,4	562,5	444,1	280,7	129,4	35,7	3,0	16376,2
2024	1143,3	926,5	804,1	737,9	555,5	471,9	283,2	143,3	37,2	4,3	16605,1
2025	1185,9	954,3	827,3	739,5	565,0	490,6	301,2	143,4	42,1	5,1	16818,0
2026	1204,3	997,8	849,3	730,0	597,7	487,7	315,3	153,8	48,3	5,3	17023,2
2027	1204,3	1046,0	862,3	741,4	629,1	495,8	327,9	162,1	48,6	5,9	17228,0
2028	1209,7	1094,0	877,7	753,8	656,0	494,8	338,9	167,9	52,0	6,3	17388,2
2029	1223,3	1116,9	906,8	772,1	668,5	489,8	360,2	173,1	56,4	8,3	17566,4
2030	1227,9	1160,5	928,5	794,3	667,8	497,7	376,8	183,8	56,0	9,6	17728,7
2031	1248,8	1175,1	967,4	812,6	661,2	524,2	378,4	193,6	63,8	10,4	17898,2
2032	1283,9	1174,9	1008,9	824,1	667,1	551,4	383,9	201,1	66,0	10,6	18042,8
2033	1299,3	1177,6	1053,5	831,1	681,0	576,6	382,6	209,7	71,2	10,9	18164,5
2034	1311,2	1187,5	1074,6	857,9	702,2	585,0	382,5	224,3	75,3	12,8	18290,8
2035	1323,9	1191,8	1117,5	872,5	724,0	585,6	390,9	238,6	78,7	13,9	18410,9
2036	1342,4	1205,2	1129,2	907,5	737,6	578,5	415,4	238,7	86,0	16,2	18533,2
2037	1341,6	1231,6	1127,5	945,9	748,9	587,9	442,0	242,3	90,0	17,0	18657,0
2038	1341,9	1249,8	1129,6	991,7	756,4	601,3	459,0	246,2	95,4	19,1	18745,4
2039	1347,7	1259,1	1134,4	1012,5	784,0	619,5	469,9	249,2	103,8	21,3	18835,5
2040	1352,7	1270,5	1132,9	1051,8	791,0	645,0	471,8	261,4	110,0	24,5	18916,5
2041	1350,5	1287,3	1147,2	1057,6	824,0	658,6	470,4	281,0	112,6	25,7	19002,4
2042	1354,1	1283,5	1171,6	1057,2	855,0	667,6	482,7	297,5	116,8	28,0	19074,8
2043	1358,5	1282,4	1190,1	1052,9	894,4	674,5	497,9	312,6	117,6	29,5	19142,4
2044	1357,5	1287,5	1195,0	1055,8	914,3	696,7	513,7	326,5	122,4	33,3	19199,4
2045	1359,1	1296,5	1206,2	1054,1	949,0	701,7	535,4	331,7	132,5	36,9	19274,6
2046	1359,8	1294,2	1226,0	1068,9	952,8	732,7	547,9	332,3	145,6	38,7	19339,1
2047	1364,7	1299,7	1218,8	1094,9	952,6	762,0	556,7	346,4	157,1	42,0	19389,1
2048	1362,9	1303,7	1220,0	1110,0	953,6	796,5	562,9	357,9	166,9	43,0	19444,7
2049	1357,6	1297,7	1226,8	1119,7	963,4	816,0	584,9	373,9	173,0	46,7	19491,8
2050	1350,6	1300,2	1233,8	1127,0	964,1	846,9	594,0	396,3	180,0	52,3	19545,6

Tables 3 and 4 show the standard deviations of the estimate values for the joint population by five year age groups.

Table 3 Standard deviations for both sexes by five years age groups

YEAR	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54
2007	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
2008	11,4	8,2	6,9	5,2	6,9	11,3	11,1	12,6	9,3	8,4	9,0
2009	17,9	13,4	8,3	10,5	11,3	19,3	20,0	18,2	12,0	11,7	13,5
2010	17,6	16,6	9,8	12,6	15,3	17,5	24,2	21,3	20,2	12,5	15,6
2011	23,5	15,4	12,9	12,6	16,7	24,8	31,9	23,1	22,6	16,9	13,7
2012	27,1	15,8	17,4	12,2	18,6	25,8	28,7	25,5	26,7	18,2	11,8
2013	26,4	21,0	20,0	11,0	17,8	28,3	21,9	25,5	21,1	16,2	17,3
2014	26,5	20,9	21,1	15,2	15,8	28,8	29,5	23,5	25,2	16,2	20,4
2015	29,7	21,4	23,6	17,6	14,0	30,3	33,1	27,3	22,9	23,9	20,0
2016	29,8	23,8	17,6	21,2	17,6	28,0	38,4	27,2	25,2	28,1	24,4
2017	26,8	30,9	15,1	25,7	17,8	25,4	36,9	31,4	25,4	27,5	26,4
2018	26,2	33,0	15,8	25,2	20,8	24,9	45,1	33,8	27,6	26,1	21,3
2019	25,4	33,9	18,0	31,7	16,4	21,5	40,3	33,1	26,5	31,8	19,0
2020	29,7	33,1	25,3	32,0	14,6	26,2	39,0	27,1	30,6	31,8	24,7
2021	26,1	31,4	30,1	29,4	22,8	34,3	31,2	25,9	30,9	32,5	25,8
2022	23,1	27,9	36,3	22,3	26,2	37,7	29,6	35,3	32,1	30,2	31,2
2023	28,1	25,4	37,2	16,6	29,0	33,3	33,3	49,5	32,1	29,9	29,5
2024	26,4	24,9	36,3	22,0	29,0	28,8	34,6	38,5	30,3	24,9	26,9
2025	22,5	24,1	32,8	29,2	25,9	25,3	31,7	46,8	23,3	31,1	22,4
2026	25,5	24,0	29,4	31,1	28,9	30,9	37,6	39,7	25,9	32,2	27,8
2027	27,1	24,3	26,3	35,3	25,0	30,0	40,2	34,5	36,4	29,4	29,4
2028	21,1	29,3	22,6	39,7	23,0	26,6	36,6	37,1	43,8	29,6	34,7
2029	20,9	24,6	28,3	35,5	26,1	32,9	33,4	31,4	41,3	29,9	35,0
2030	21,4	23,6	27,0	33,2	31,9	28,3	28,7	32,1	37,3	32,1	38,8
2031	24,0	23,3	24,0	29,6	32,9	26,3	33,7	38,4	30,9	22,1	35,8
2032	24,1	24,0	20,3	27,9	37,6	27,9	37,5	36,4	27,2	24,8	35,5
2033	20,0	23,4	29,2	25,1	41,5	25,7	31,4	32,4	27,9	37,3	35,3
2034	22,5	19,1	27,9	23,1	33,1	31,1	37,7	36,9	32,6	36,8	26,9
2035	15,7	21,7	22,2	30,7	26,5	35,1	35,8	33,9	30,7	33,8	30,1
2036	24,1	26,3	22,2	24,5	25,4	37,2	33,0	41,2	28,8	31,7	21,8
2037	25,3	26,3	19,9	24,3	26,2	38,5	35,5	45,2	30,8	31,1	27,4
2038	31,9	20,1	25,0	30,8	26,0	44,0	37,7	46,3	28,7	32,0	37,3
2039	27,4	23,1	24,5	31,5	24,7	39,7	37,1	40,9	33,6	31,7	37,5
2040	27,6	21,0	19,9	27,2	36,2	34,8	31,4	36,1	36,5	27,9	44,0
2041	27,5	23,4	29,2	23,4	28,3	29,4	39,4	35,3	43,1	25,3	40,1
2042	23,1	23,6	31,1	24,8	27,5	28,1	36,5	35,3	44,3	34,1	29,4
2043	29,4	29,5	28,3	23,7	28,8	34,2	43,5	40,0	40,8	31,7	34,0
2044	26,2	25,4	25,6	25,8	29,4	32,8	38,9	38,7	26,7	34,8	34,6
2045	27,5	22,9	23,0	27,4	23,0	30,3	35,5	31,5	29,1	32,3	27,4
2046	22,9	22,0	26,4	34,2	26,8	30,7	27,9	37,0	29,9	36,5	26,3
2047	17,4	23,0	27,9	30,7	23,6	29,4	34,0	30,3	34,6	37,7	27,4
2048	19,0	27,6	24,1	27,9	25,2	24,3	42,7	36,9	42,8	36,1	27,9
2049	21,6	22,1	23,0	26,4	28,0	26,2	36,3	30,4	35,2	26,5	30,3
2050	25,8	28,2	24,2	26,6	27,3	27,0	39,9	33,5	31,5	32,2	34,7

Table 4 Standard deviations for both sexes by five years age groups

YEAR	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95-99	>=100	TOTAL
2007	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
2008	7,0	7,0	6,0	5,7	4,4	4,8	2,8	2,1	1,6	0,0	32,2
2009	7,2	10,0	7,3	8,1	6,8	7,3	4,1	3,7	2,0	0,0	52,3
2010	10,5	11,3	13,0	10,7	9,5	9,4	4,9	4,3	2,7	0,7	64,6
2011	16,7	12,1	10,6	13,3	11,4	9,6	7,1	3,9	2,3	0,5	76,0
2012	22,5	12,2	14,5	14,4	12,2	9,9	7,6	3,9	2,2	0,5	106,3
2013	19,2	16,1	17,2	14,4	14,3	12,1	7,6	4,9	2,7	0,8	110,7
2014	22,4	18,0	14,3	12,0	13,6	10,3	8,2	6,2	3,2	0,8	114,7
2015	21,6	18,3	17,1	15,6	15,0	13,0	6,4	6,5	3,7	1,0	122,9
2016	23,2	19,8	14,8	18,8	14,8	16,9	10,6	7,5	3,7	1,4	121,6
2017	18,3	21,3	16,6	22,4	14,8	16,1	12,7	7,0	3,2	1,2	134,4
2018	19,1	18,6	18,2	22,9	15,5	16,4	12,1	5,5	4,5	1,5	137,4
2019	21,9	21,8	18,7	18,2	16,8	13,9	12,7	6,8	4,3	1,5	155,0
2020	23,3	23,1	22,3	15,9	22,6	14,3	11,9	8,5	4,9	1,0	151,8
2021	27,4	23,6	22,3	17,7	23,4	13,8	11,9	10,0	4,5	1,2	154,3
2022	24,5	19,7	22,2	19,4	21,3	15,2	12,2	11,9	4,4	1,4	165,2
2023	22,7	21,6	24,0	21,7	18,9	15,3	14,7	9,3	4,8	1,8	181,7
2024	21,1	29,7	27,2	21,1	21,6	15,0	12,7	9,4	4,9	2,0	177,0
2025	29,2	24,2	26,1	20,2	20,5	17,1	17,6	8,7	5,2	2,5	172,8
2026	29,7	27,6	28,2	17,8	19,9	16,4	14,0	9,9	6,8	2,5	165,8
2027	28,7	26,9	27,6	18,8	20,5	16,8	14,3	12,3	7,4	2,8	172,3
2028	26,7	25,1	28,7	24,2	25,8	18,3	11,5	10,6	6,4	2,4	175,2
2029	27,5	22,4	29,9	26,5	19,3	17,5	12,3	7,9	5,4	3,8	161,9
2030	26,5	25,2	27,6	33,3	15,9	22,2	17,5	10,7	6,9	3,6	164,8
2031	30,3	25,1	28,3	32,7	22,5	27,4	17,3	8,9	7,8	2,8	158,8
2032	31,6	27,5	24,0	28,8	24,4	21,6	16,4	8,2	6,5	3,3	161,8
2033	36,9	22,3	21,9	27,3	27,6	19,9	20,1	11,8	6,6	3,5	142,2
2034	38,0	24,0	22,3	23,2	29,1	14,0	19,1	13,9	6,1	3,8	117,8
2035	40,3	25,3	22,1	23,1	34,6	15,2	22,5	16,4	9,7	5,1	112,0
2036	36,8	29,1	27,2	22,3	28,8	21,5	23,5	16,0	8,1	3,9	107,2
2037	37,8	34,0	23,7	22,9	24,8	22,4	17,9	15,2	9,3	3,4	119,1
2038	26,8	37,5	23,8	25,1	22,5	27,1	15,1	14,5	10,2	4,3	118,8
2039	22,4	34,0	29,6	24,1	21,1	31,5	13,2	15,0	13,4	4,9	129,0
2040	29,7	30,8	22,3	24,3	24,5	36,8	16,0	14,0	14,0	5,5	131,4
2041	22,4	33,7	31,4	27,8	28,4	28,4	20,4	16,0	11,8	5,5	136,4
2042	33,8	33,7	37,3	22,3	24,8	22,8	20,7	11,4	12,0	5,7	134,3
2043	42,6	29,2	37,5	27,1	28,1	15,9	25,9	11,9	11,6	6,7	143,1
2044	37,6	21,9	32,5	29,9	30,0	18,0	27,7	14,2	12,6	5,8	133,7
2045	42,7	27,5	29,6	25,5	25,4	22,6	30,5	15,1	10,8	6,0	128,9
2046	41,2	24,0	33,1	31,6	31,0	23,0	26,2	18,4	9,6	6,5	126,2
2047	34,4	39,3	37,0	39,7	23,6	25,3	19,2	16,6	8,1	6,7	118,4
2048	33,2	47,2	28,6	37,2	30,5	28,6	16,1	22,9	9,9	6,1	121,3
2049	35,1	43,5	19,8	31,1	29,0	30,2	19,0	23,9	10,1	6,1	109,1
2050	31,1	42,7	30,2	31,1	26,6	29,2	22,1	24,7	11,0	6,0	105,7

Tables 5 and 6 show the average values for the joint population.

Table 5 Average values for the joint population

YEAR	0-4	5-14	15-24	25-49	50-64	65-79	80-99	>99	TOTAL	E(P)	I.V.
2007	517,0	1189,0	1278,0	4178,0	1879,0	1238,0	266,0	0,0	10545,0	39,3	88,2
2008	552,2	1195,1	1332,4	4423,7	1945,1	1311,9	300,6	0,0	11061,0	39,5	92,3
2009	565,9	1220,7	1356,6	4661,3	2004,0	1400,6	334,9	0,0	11543,8	39,8	97,2
2010	597,0	1227,9	1402,3	4862,9	2061,7	1476,3	374,7	0,4	12003,3	40,0	101,5
2011	613,6	1253,0	1419,9	5083,7	2130,6	1505,7	427,5	0,2	12434,2	40,2	103,6
2012	605,8	1308,5	1416,1	5291,8	2217,8	1565,2	455,8	0,3	12861,2	40,5	105,6
2013	615,4	1321,9	1459,7	5462,0	2313,0	1589,2	502,5	0,7	13264,5	40,7	108,1
2014	626,7	1332,0	1496,4	5598,3	2417,2	1617,6	556,7	0,9	13645,8	41,0	111,1
2015	632,4	1367,0	1504,7	5739,4	2498,7	1690,0	579,7	0,9	14012,8	41,2	113,6
2016	637,5	1382,9	1532,9	5851,4	2577,2	1745,2	630,6	1,5	14359,1	41,4	117,7
2017	651,1	1393,6	1563,0	5930,3	2672,1	1810,6	667,2	1,6	14689,6	41,6	121,3
2018	658,9	1422,0	1563,6	6017,0	2772,8	1859,8	706,5	1,8	15002,3	41,9	123,5
2019	662,6	1437,9	1584,9	6102,7	2877,1	1906,3	739,0	2,3	15312,7	42,1	126,1
2020	668,8	1462,5	1587,2	6169,0	2983,2	1945,7	774,4	2,6	15593,4	42,3	127,8
2021	675,6	1475,9	1604,8	6249,0	3070,7	1967,0	820,8	2,1	15865,9	42,5	129,7
2022	680,1	1480,3	1635,1	6314,8	3138,4	2019,5	855,4	2,6	16126,2	42,7	133,3
2023	689,6	1491,8	1643,8	6367,8	3227,9	2062,4	889,9	3,0	16376,2	42,9	135,6
2024	698,5	1491,2	1659,5	6403,0	3315,5	2097,4	935,6	4,3	16605,1	43,1	138,8
2025	698,3	1498,9	1680,8	6437,3	3388,3	2131,8	977,4	5,1	16818,0	43,3	141,8
2026	700,0	1506,3	1683,0	6475,2	3471,3	2177,0	1005,1	5,3	17023,2	43,5	144,5
2027	702,7	1510,8	1683,0	6501,6	3556,9	2232,8	1034,4	5,9	17228,0	43,7	147,9
2028	700,0	1520,0	1694,0	6494,4	3632,3	2287,5	1053,6	6,3	17388,2	43,9	150,9
2029	701,6	1529,4	1706,0	6509,9	3684,2	2347,5	1079,6	8,3	17566,4	44,1	154,1
2030	697,1	1530,4	1727,1	6509,2	3750,3	2390,6	1114,3	9,6	17728,7	44,3	157,9
2031	701,5	1531,2	1727,3	6519,6	3807,0	2441,2	1160,0	10,4	17898,2	44,5	161,9
2032	699,3	1531,1	1727,2	6532,8	3839,2	2500,1	1202,5	10,6	18042,8	44,7	166,6
2033	703,5	1531,6	1725,1	6525,0	3862,8	2565,6	1240,0	10,9	18164,5	44,9	170,8
2034	708,9	1541,3	1719,9	6510,7	3895,4	2634,6	1267,1	12,8	18290,8	45,0	174,0
2035	708,0	1542,2	1718,9	6501,5	3918,8	2714,0	1293,7	13,9	18410,9	45,2	178,8
2036	713,4	1540,6	1729,2	6493,3	3947,6	2774,2	1318,6	16,2	18533,2	45,4	182,3
2037	723,9	1535,9	1730,4	6487,0	3978,5	2822,2	1362,2	17,0	18657,0	45,6	186,0
2038	718,8	1530,0	1735,6	6465,1	3997,2	2877,7	1401,9	19,1	18745,4	45,8	191,2
2039	719,0	1530,5	1736,9	6441,0	4013,5	2930,9	1442,4	21,3	18835,5	46,0	195,4
2040	715,3	1527,1	1739,0	6418,3	4028,2	2975,6	1488,2	24,5	18916,5	46,2	200,2
2041	716,7	1527,2	1736,2	6403,5	4041,8	3028,8	1522,5	25,7	19002,4	46,4	204,1
2042	712,5	1530,0	1726,6	6381,8	4047,5	3083,8	1564,6	28,0	19074,8	46,6	208,6
2043	714,0	1528,1	1722,3	6361,4	4047,0	3137,4	1602,6	29,5	19142,4	46,8	212,8
2044	708,6	1531,1	1725,0	6330,3	4046,8	3165,1	1659,3	33,3	19199,4	47,0	216,9
2045	708,1	1532,6	1718,1	6317,7	4050,6	3209,3	1701,3	36,9	19274,6	47,2	220,9
2046	707,0	1535,1	1717,0	6289,7	4045,6	3247,7	1758,4	38,7	19339,1	47,3	225,1
2047	706,1	1532,0	1712,0	6267,8	4040,8	3266,2	1822,2	42,0	19389,1	47,5	229,3
2048	704,4	1530,1	1707,9	6256,0	4035,6	3283,5	1884,2	43,0	19444,7	47,7	233,2
2049	707,0	1527,7	1706,9	6232,0	4014,0	3309,9	1947,7	46,7	19491,8	47,9	237,4
2050	711,6	1519,9	1702,3	6211,0	4006,4	3324,9	2017,2	52,3	19545,6	48,1	241,8

Table 6 Average values for the joint population

YEAR	I.D.G.	I.D.A.	I.D.P.	I.R.	NATI	MORTI	E(Q)	IMMIG	E(IMM)	EMIGR
2007	23,3	20,5	43,8	97,0	110,3	73,5	72,5	850,3	34,2	371,1
2008	22,7	20,9	43,6	91,2	109,3	73,2	72,5	836,9	34,1	390,1
2009	22,3	21,6	43,9	87,7	121,0	82,2	73,2	828,9	34,3	408,1
2010	21,9	22,2	44,2	89,0	118,4	91,5	73,5	826,7	34,2	422,6
2011	21,6	22,4	44,0	94,6	121,4	92,4	74,5	837,1	34,3	439,1
2012	21,4	22,6	44,1	99,9	122,2	100,1	74,5	827,2	34,1	446,0
2013	21,0	22,7	43,6	103,8	124,0	104,4	75,2	825,0	34,3	463,3
2014	20,6	22,9	43,5	104,6	128,7	110,0	75,2	822,7	34,2	474,3
2015	20,5	23,3	43,8	106,2	126,7	113,0	75,3	811,0	34,4	478,4
2016	20,3	23,9	44,1	102,7	134,3	115,5	75,6	803,2	34,1	491,6
2017	20,1	24,4	44,5	101,8	132,1	121,7	76,1	800,5	34,3	498,1
2018	20,1	24,8	44,9	104,4	129,8	130,1	76,6	811,0	34,2	500,3
2019	19,9	25,1	44,9	107,0	135,7	132,7	77,2	792,7	34,1	515,0
2020	19,8	25,4	45,2	109,6	135,0	133,6	77,7	789,7	33,9	518,6
2021	19,7	25,5	45,2	112,2	141,2	140,0	77,5	785,3	34,3	526,1
2022	19,5	26,0	45,4	111,7	142,5	141,3	78,0	795,6	34,2	546,8
2023	19,4	26,3	45,7	112,3	139,2	147,3	78,1	780,3	34,3	543,4
2024	19,2	26,7	45,9	114,8	143,3	143,0	78,4	763,2	34,1	550,7
2025	19,1	27,1	46,2	116,3	140,0	151,3	78,6	765,4	34,5	548,8
2026	19,0	27,4	46,4	121,1	143,1	152,3	78,8	773,5	34,7	559,5
2027	18,9	27,9	46,7	128,7	137,3	160,8	78,6	743,0	34,3	559,5
2028	18,8	28,3	47,1	135,3	145,0	162,8	78,8	756,5	34,2	560,6
2029	18,7	28,9	47,6	137,7	140,4	164,2	79,3	750,4	34,4	564,2
2030	18,6	29,3	47,9	141,4	145,6	160,3	79,5	738,2	34,5	554,0
2031	18,5	30,0	48,5	142,9	142,7	170,0	79,7	732,5	34,4	560,7
2032	18,4	30,7	49,1	142,0	142,7	164,9	79,7	721,3	34,0	577,4
2033	18,5	31,5	50,0	141,6	146,3	171,3	79,8	723,0	33,9	571,8
2034	18,6	32,3	50,8	143,5	143,3	167,9	80,5	718,1	34,0	573,4
2035	18,5	33,1	51,7	144,3	147,0	172,2	80,2	717,4	34,2	569,8
2036	18,5	33,8	52,3	144,7	150,4	168,3	80,6	711,8	34,2	570,0
2037	18,5	34,5	53,0	148,9	143,0	170,7	80,7	704,4	34,4	588,3
2038	18,4	35,2	53,7	150,1	145,2	175,4	80,9	699,9	34,2	579,6
2039	18,5	36,0	54,5	149,9	141,6	170,4	81,0	690,5	34,2	580,8
2040	18,4	36,8	55,2	151,0	148,4	179,5	81,3	692,4	34,3	575,4
2041	18,4	37,6	56,0	153,8	145,9	176,0	81,4	681,5	34,3	579,0
2042	18,4	38,5	56,9	154,6	147,9	181,1	81,5	680,7	34,4	579,8
2043	18,5	39,3	57,8	155,9	142,9	179,2	81,7	677,7	34,2	584,3
2044	18,5	40,1	58,6	156,0	145,2	177,0	82,3	676,6	34,3	569,6
2045	18,5	40,9	59,5	157,7	145,6	180,1	82,7	666,3	34,3	567,2
2046	18,6	41,9	60,5	157,6	142,1	178,7	82,6	667,5	34,3	581,0
2047	18,6	42,7	61,3	158,3	146,0	182,1	83,6	669,7	34,3	578,0
2048	18,6	43,4	62,0	158,6	150,3	180,1	83,3	655,0	34,2	578,2
2049	18,7	44,4	63,1	158,6	150,0	179,0	83,0	649,2	34,6	566,5
2050	18,7	45,3	64,0	158,5						

Tables 7 and 8 show the standard deviations of the estimated values for the joint population.

Table 7 Standard deviation of the estimated values for the joint population per age groups

YEAR	0-4	5-14	15-24	25-49	50-64	65-79	80-99	100^+	TOTAL	E(P)	I.V.
2007	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
2008	11,4	9,0	8,5	20,3	13,1	8,0	6,1	0,0	32,2	0,1	0,8
2009	17,9	15,3	13,9	36,7	16,8	12,4	11,1	0,0	52,3	0,2	1,6
2010	17,6	16,0	18,3	43,3	20,1	17,9	13,4	0,7	64,6	0,2	1,8
2011	23,5	19,9	21,3	51,7	27,4	20,6	14,7	0,5	76,0	0,3	2,3
2012	27,1	23,0	21,5	60,1	31,0	24,5	15,1	0,5	106,3	0,3	2,5
2013	26,4	26,8	20,6	61,9	29,1	26,0	15,5	0,8	110,7	0,3	2,8
2014	26,5	32,6	27,0	57,8	31,7	22,2	14,0	0,8	114,7	0,2	2,3
2015	29,7	32,7	25,1	65,6	33,1	23,4	19,2	1,0	122,9	0,2	2,3
2016	29,8	28,4	26,2	75,9	37,7	26,7	23,6	1,4	121,6	0,2	2,2
2017	26,8	29,4	30,3	85,4	43,1	31,9	19,2	1,2	134,4	0,2	2,3
2018	26,2	35,4	32,2	87,2	35,8	32,9	19,6	1,5	137,4	0,3	2,8
2019	25,4	38,9	37,0	93,0	27,1	34,6	20,5	1,5	155,0	0,3	3,2
2020	29,7	46,3	38,5	90,7	31,2	43,4	17,2	1,0	151,8	0,3	3,4
2021	26,1	50,1	38,3	90,0	26,2	45,6	14,2	1,2	154,3	0,3	3,6
2022	23,1	45,3	33,5	86,1	32,7	38,4	18,1	1,4	165,2	0,3	3,6
2023	28,1	42,4	35,0	92,8	42,2	36,9	21,1	1,8	181,7	0,4	4,0
2024	26,4	43,7	40,6	83,2	48,4	35,4	22,3	2,0	177,0	0,3	3,4
2025	22,5	42,4	40,2	88,5	45,2	34,4	23,0	2,5	172,8	0,4	3,8
2026	25,5	39,7	46,9	75,1	49,1	38,8	20,7	2,5	165,8	0,3	4,0
2027	27,1	38,2	47,3	86,3	48,5	42,5	24,9	2,8	172,3	0,4	4,2
2028	21,1	39,3	51,8	86,5	61,1	44,8	23,3	2,4	175,2	0,4	4,3
2029	20,9	32,4	51,9	86,4	62,3	42,2	24,7	3,8	161,9	0,4	4,3
2030	21,4	38,8	48,4	82,0	57,9	44,5	29,6	3,6	164,8	0,4	4,7
2031	24,0	38,6	48,4	76,8	53,6	47,3	34,4	2,8	158,8	0,4	5,2
2032	24,1	36,4	43,2	71,4	58,7	39,5	33,0	3,3	161,8	0,4	5,0
2033	20,0	41,7	46,8	67,4	57,4	36,4	32,6	3,5	142,2	0,4	4,2
2034	22,5	32,3	41,2	62,1	55,6	36,7	34,5	3,8	117,8	0,3	3,4
2035	15,7	31,6	41,8	64,8	57,7	34,0	33,3	5,1	112,0	0,3	3,5
2036	24,1	37,5	35,8	63,1	51,5	34,0	33,5	3,9	107,2	0,4	3,9
2037	25,3	36,1	33,3	64,2	56,3	24,7	32,3	3,4	119,1	0,3	3,0
2038	31,9	35,8	41,3	67,4	58,1	32,7	36,0	4,3	118,8	0,3	3,7
2039	27,4	39,6	42,6	72,0	61,2	40,2	40,9	4,9	129,0	0,3	3,9
2040	27,6	33,4	48,6	65,9	63,9	40,2	49,1	5,5	131,4	0,3	4,2
2041	27,5	35,6	43,6	64,6	61,3	50,4	46,1	5,5	136,4	0,3	4,1
2042	23,1	37,3	43,0	67,4	66,6	51,9	38,6	5,7	134,3	0,3	3,8
2043	29,4	38,1	38,7	72,0	67,5	60,9	33,1	6,7	143,1	0,3	4,2
2044	26,2	34,4	28,1	85,0	65,1	56,0	29,3	5,8	133,7	0,3	4,3
2045	27,5	31,2	32,2	78,2	60,8	55,7	32,8	6,0	128,9	0,3	4,7
2046	22,9	34,3	39,3	80,1	54,8	52,2	27,0	6,5	126,2	0,3	4,5
2047	17,4	36,2	37,9	65,4	64,8	55,8	26,9	6,7	118,4	0,3	4,2
2048	19,0	33,1	37,5	76,0	65,5	50,9	28,8	6,1	121,3	0,3	4,7
2049	21,6	28,7	40,0	70,6	56,3	44,6	32,6	6,1	109,1	0,3	4,7
2050	25,8	35,6	35,4	71,4	58,3	54,5	34,5	6,0	105,7	0,3	4,9

Table 8 Standard deviation of the estimated values for the joint population per age groups

YEAR	I.D.G.	I.D.A.	I.D.P.	I.R.	NATI	MORTI	E(Q)	IMMIG	E(IMM)	EMIGR
2007	0,0	0,0	0,0	0,0	9,0	7,3	3,4	26,5	1,0	12,1
2008	0,2	0,1	0,3	1,5	11,3	7,7	2,9	25,9	1,3	20,2
2009	0,3	0,2	0,4	1,6	8,5	7,8	3,0	34,3	1,5	17,4
2010	0,3	0,3	0,3	2,5	14,1	10,1	3,2	26,7	1,6	17,5
2011	0,3	0,3	0,3	2,8	9,1	12,3	2,8	33,3	2,1	20,5
2012	0,3	0,4	0,5	2,4	11,5	9,6	4,0	34,7	1,4	18,5
2013	0,4	0,4	0,5	2,9	12,0	10,9	2,8	24,5	1,5	19,8
2014	0,4	0,3	0,6	3,4	10,0	11,4	2,3	21,6	1,5	17,4
2015	0,4	0,3	0,6	3,7	11,0	10,3	2,4	24,8	1,3	21,6
2016	0,4	0,3	0,6	4,4	12,2	12,5	3,0	34,5	1,5	21,0
2017	0,4	0,3	0,6	4,9	9,7	9,9	2,3	28,4	0,9	27,7
2018	0,4	0,3	0,6	4,4	10,6	13,2	2,5	17,7	1,4	22,2
2019	0,5	0,3	0,6	4,7	11,2	11,6	3,1	19,5	1,7	28,4
2020	0,5	0,4	0,7	4,6	11,7	14,9	3,0	23,6	1,2	19,6
2021	0,5	0,4	0,6	4,3	8,9	8,4	1,7	24,5	1,5	19,1
2022	0,4	0,3	0,5	3,8	11,7	11,6	2,1	33,9	1,2	26,4
2023	0,5	0,4	0,6	3,1	9,9	14,5	2,0	26,0	1,4	19,9
2024	0,4	0,4	0,6	4,3	11,4	10,2	2,4	22,5	1,4	23,1
2025	0,4	0,4	0,6	4,1	10,7	9,0	2,2	27,7	1,5	29,4
2026	0,5	0,4	0,7	5,4	11,3	11,8	3,1	24,5	1,5	19,1
2027	0,5	0,5	0,7	6,4	10,4	14,2	2,3	28,1	1,3	26,4
2028	0,4	0,4	0,6	7,2	9,7	15,1	2,9	36,9	1,4	24,0
2029	0,4	0,4	0,6	5,7	9,7	11,2	2,2	28,9	1,5	21,8
2030	0,5	0,4	0,6	4,9	10,5	10,8	2,1	28,7	0,9	19,8
2031	0,5	0,4	0,6	4,4	12,3	13,3	2,7	29,3	1,6	22,0
2032	0,4	0,4	0,5	5,0	11,5	12,6	1,5	31,3	1,9	28,1
2033	0,3	0,4	0,5	3,4	10,0	12,8	2,2	21,0	1,8	22,1
2034	0,3	0,4	0,5	3,3	9,4	11,0	2,6	32,6	1,1	26,4
2035	0,3	0,5	0,5	6,3	12,2	16,2	2,2	28,7	1,1	19,6
2036	0,3	0,5	0,6	6,2	10,9	13,9	2,6	21,8	1,5	23,6
2037	0,2	0,4	0,5	7,7	12,3	13,9	1,7	27,4	1,1	23,8
2038	0,3	0,4	0,5	7,7	8,8	16,4	2,4	23,3	1,4	26,5
2039	0,3	0,5	0,7	7,2	11,0	13,5	1,9	32,4	1,7	19,7
2040	0,4	0,5	0,7	6,6	13,0	13,3	2,6	26,6	1,6	20,2
2041	0,4	0,5	0,8	5,6	11,9	12,7	2,1	28,2	1,5	18,0
2042	0,4	0,5	0,8	5,4	14,1	9,3	1,9	23,7	1,4	19,6
2043	0,4	0,5	0,8	4,8	9,2	13,3	2,1	22,1	1,4	25,2
2044	0,4	0,5	0,7	5,3	10,5	10,4	2,5	25,6	1,7	21,5
2045	0,4	0,5	0,7	5,8	14,2	12,7	2,5	26,0	1,6	25,9
2046	0,3	0,5	0,7	7,7	10,5	14,6	2,0	25,8	1,4	24,6
2047	0,4	0,6	0,8	8,3	9,7	15,1	2,0	24,4	1,6	24,3
2048	0,3	0,5	0,7	9,3	12,7	16,2	2,2	24,3	1,5	22,8
2049	0,3	0,5	0,6	8,5	11,1	15,8	2,2	28,4	1,7	21,9

For lack of space separate results for male and female populations are not shown.

Legenda:

E(P): average age of population

ETAMAX: maximum age of the living population

I.V.: old age index

I.R: replacement index

NATI(E): number of male births (female) within the year

MORTI: number of male deaths (female) within the year

E(Q): average age of the deceased within the year

IMMIG: number of male immigrants (female) within the year

E(IM): average age of the male immigrants (female) within the year

EMIG: number of male migrants (female) within the year

E(EM): average age of the male migrants (female) within the year

For the women:

Figli: number of children born in the population within the year

EMP: average age of the delivering females who have had children within the year

For the joint population:

I.D.G.: index of dependence for the youth population

I.D.A.: index of dependence for the elderly population

I.D.P.: total index of dependence (youth and elderly)

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Organized in cooperation with the National Institute of Statistics of Romania (INSSE),
Bucharest, 10-12 October 2007

POPULATION PROSPECTS OF THE LOWEST FERTILITY WITH THE LONGEST LIFE: THE NEW OFFICIAL POPULATION PROJECTIONS FOR JAPAN AND THEIR LIFE COURSE APPROACHES[1]

Ryuichi Kaneko[2]

Abstract

In this paper, first I briefly discuss the results and methods of the new round of the official Population Projections for Japan released in December 2006. They provide a sketch of expected demographic changes over a 50 year period from 2006 to 2055. The eventual total fertility rate is 1.26 (ranging from 1.06 to 1.55), and the female life expectancy is 90.3 (ranging from 89.2 to 91.5). As a result, the population is expected to fall by 30% by 2055, and the proportion of the elderly is to rise up to as high as 40.5%, which is twice the 20.2% of 2005, already the world's highest. The projections are unique not only in showing the world's lowest fertility assumptions with the highest life expectancy, but also in their sophisticated life course approach in constructing assumptions on vital rates. Through this framework, they provide measures for the projected life of women via the multistate life table techniques applied to the projected population. For instance, life time probability of childlessness and having no grandchildren are estimated as 38.1% and 50.2%, respectively, in cohorts born in 1990. The average life time spent in never married status increases to 42.5 years (or 47% of the life expectancy) in cohorts born in 1990 from 25.3 years (31%) in those born in 1950. These measures indicate that long, but less-reproductive and non-familial lives prevail among new generations, resulting in a drastic increase in elderly who have no offspring or family in the current sense.

1. Introduction

In this paper, first I discuss the results and methods of the new round of the official Population Projections for Japan released in December 2006. They provide a sketch of expected demographic changes over a 50 year period from 2006 to 2055, indicating a persuasive view that a substantial population decline with unprecedented population aging is an unavoidable part of the future of the society. The projections are unique not only in showing the world's lowest fertility assumptions with the highest life expectancy, but also in their sophisticated life course approach in constructing assumptions on vital rates. In this connection, I attempted to construct the multistate life table for the projected life of Japanese women to obtain their life course measures mainly by ultimate family status. The measures include life time probability of never marrying, childless, having no grandchildren and so on. The probabilities incorporate incidences from premature death before the events. The average life time spent in each family status such as never married state, childless state, and only-child state, are also examined. For example, the life time probability of childlessness and having no grandchildren are 38.1% and 50.2% respectively in cohorts born in 1990. They were 18.4% and 22.2% in cohorts born in 1950. As well, the average life time spent in never married status increases to 42.5 years (or 47% of the life expectancy) in cohorts born in

[1] This is a paper presented to the Joint Eurostat-UNECE Work Session on Demographic Projections (Bucharest, 10-12 October 2007).

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1990 from 25.3 years (31%) in those born in 1950. These changes set off a drastic increase in the near future of elderly who do not have offspring to live with or rely on in this society. I also briefly discuss possibilities for the life course approach to be the basis of the next framework of population projection.

2. Population Prospects

Based on the results of the 2005 population census and the newly obtained vital statistics, the National Institute of Population and Social Security Research announced a new population projection for Japan in December 2006. In this section, the outlines of this projection are briefly explained.

The projection covers the total resident population of Japan which is also the target population of the Census. The projection starts from the population at the time of the 2005 Census, and covers the period up to 2055, enumerating the population as of October 1 each year. It also includes calculations of the population up to 2105 in order to examine the long term demographic development assuming constant vital rates at the level of 2055.

The population (classified by sex and year of age) is projected through the cohort component method with assumptions on vital events and international migrations based on the past statistical trends. Because of the uncertainty in future movements of birth and death, three assumptions are made for each factor to produce a range of forecasts for the future population by means of nine variants, i.e. 3 x 3.

The assumed total fertility rate in 2055 is 1.26 for the medium fertility variant, 1.55 for the high variant, and 1.06 for the low variant. These are a set of the world's lowest fertility assumptions for official population projections. The life expectancy at birth in 2055 is 90.34 years and 83.67 years respectively for females and males for the medium mortality variant, 89.17 years and 85.41 years for the high variant, and 91.51 years and 84.93 years for the low variant. These are the mortality assumptions of the world's highest life expectancy.

When the results of the medium fertility variant are combined with the medium mortality level, the total population is projected to fall from 127.8 million in 2005 to 89.9 million in 2055 (Table 1, Figure 1). This is a loss of 37.8 million or 30% of the initial population. Initially, the decline takes place slowly, but after 2039 it accelerates to a pace of more than one million every year. The population changes that occur over the 50 year period are significantly unevenly distributed across age groups. The age group under 15 reduces by 10.1 million, the working age group (age 15 to 64) by 38.5 million, while the group of the elderly, aged 65 and over, increases by 10.7 million. The uneven changes of the three age groups results in an age structure that is totally different from the starting population. In 2050, the proportion of children under 15 is down to 8.4 per cent from 13.8 per cent in 2005. The working age group 15 to 64 is reduced to 51.1 per cent from 66.1 per cent in 2005. And the proportion of the elderly doubles from 20.2 per cent in 2005 to 40.5 per cent during the next 50 years (Table 1, Figure 1, see also Figure 2).

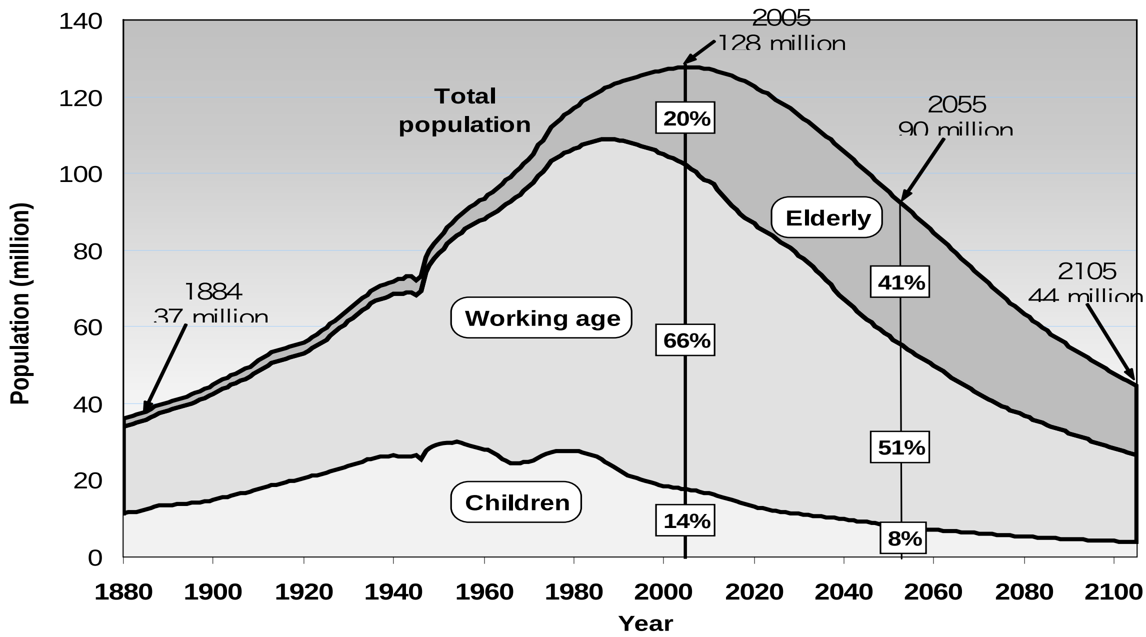
Table 1 Projected Future Population and Proportion by Age Group, 2005-2105: the Variant with Medium-Fertility, Medium-Mortality Assumptions

Year	Population in 1,000				Proportion (%)			Dependency ratio (%)		
	Total	0-14	15-64	65+	0-14	15-64	65+	Total	Child	Old-age
2005	127,768	17,585	84,422	25,761	13.8	66.1	20.2	51.3	20.8	30.5
2010	127,176	16,479	81,285	29,412	13.0	63.9	23.1	56.5	20.3	36.2
2015	125,430	14,841	76,807	33,781	11.8	61.2	26.9	63.3	19.3	44.0
2020	122,735	13,201	73,635	35,899	10.8	60.0	29.2	66.7	17.9	48.8
2025	119,270	11,956	70,960	36,354	10.0	59.5	30.5	68.1	16.8	51.2
2030	115,224	11,150	67,404	36,670	9.7	58.5	31.8	70.9	16.5	54.4
2035	110,679	10,512	62,919	37,249	9.5	56.8	33.7	75.9	16.7	59.2
2040	105,695	9,833	57,335	38,527	9.3	54.2	36.5	84.3	17.2	67.2
2045	100,443	9,036	53,000	38,407	9.0	52.8	38.2	89.5	17.0	72.5
2050	95,152	8,214	49,297	37,641	8.6	51.8	39.6	93.0	16.7	76.4
2055	89,930	7,516	45,951	36,463	8.4	51.1	40.5	95.7	16.4	79.4
2060	84,592	6,987	42,778	34,827	8.3	50.6	41.2	97.7	16.3	81.4
2070	73,488	6,158	36,325	31,005	8.4	49.4	42.2	102.3	17.0	85.4
2080	63,387	5,304	31,505	26,578	8.4	49.7	41.9	101.2	16.8	84.4
2090	54,925	4,600	27,674	22,651	8.4	50.4	41.2	98.5	16.6	81.8
2100	47,712	4,093	24,144	19,475	8.6	50.6	40.8	97.6	17.0	80.7
2105	44,592	3,856	22,631	18,105	8.6	50.8	40.6	97.0	17.0	80.0

Source: NIPSSR(2006), Population Projection for Japan: 2006-2055, the medium-fertility and medium-mortality variant. The results for the period 2056-2105 are ancillary calculation with constant vital rates as of 2055.

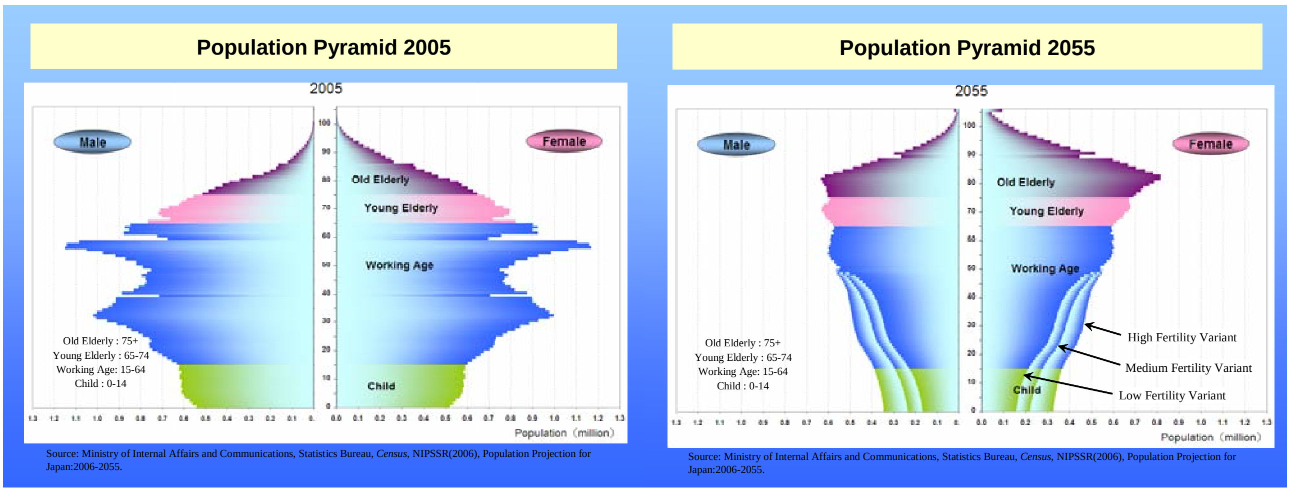
The amount of population loss over the 50 years ranges from 28.2 million in the high fertility with low mortality-variant to 45.4 million in the low fertility with high-mortality -variant. The proportion of elderly ranges from 36.3 per cent in the high fertility with high mortality-variant to 44.4 per cent in the low fertility with low mortality-variant. The ancillary calculation of the population in 2105 with vital rates assumed constant at the 2055 level is 44.6 million or 35 per cent of the initial population in the medium fertility with medium mortality-variant. The assumption of low fertility combined with high mortality results in the smallest total population of 33.6 million or 26 per cent of the starting population, while the result of the high fertility with low mortality-variant is the largest at 62.7 million or 49 per cent of the 2005 population size.

Figure 1 Growth and Reduction of Population of Japan: 1880-2105



Source: Ministry of Internal Affairs and Communications, Statistics Bureau, Census, NIPSSR(2006), Population Projection for Japan: 2006-2055, the medium-fertility and medium-mortality variant.

Figure 2 Population Pyramids of Japan: Present and Fifty Years Later



Source: Statistics Bureau, Census 2005, NIPSSR (2006), Population Projection for Japan: 2006-2055 (three fertility variants with medium-mortality).

3. Assumptions

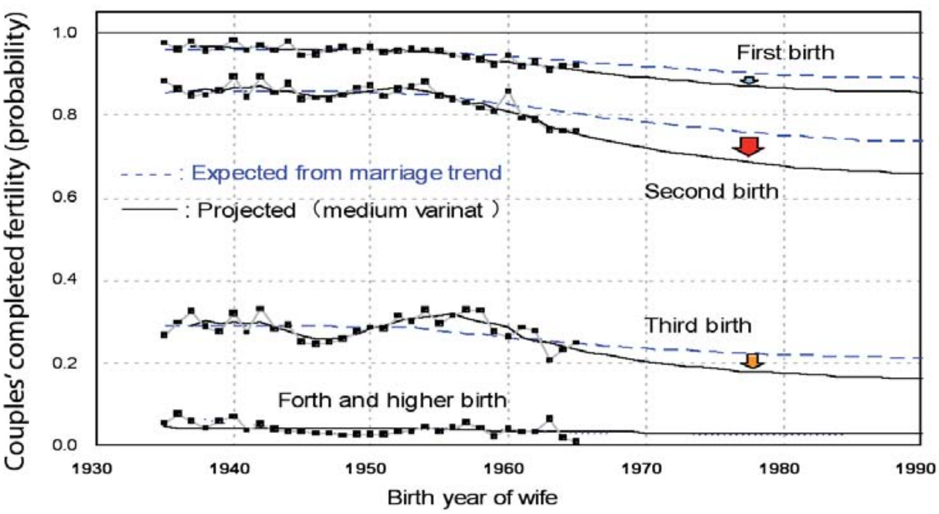
This unprecedented population comes out from assumptions of the world's lowest fertility prospects combined with the longest life expectancy. The eventual total fertility rate is 1.26 (ranging from 1.06 to 1.55), and the female life expectancy 90.3 (ranging from 89.2 to 91.5). How could it be possible for fertility to remain so low for a period of some 50 years? How could the life expectancy be highest?

4. Fertility Assumptions

Fertility assumptions underlying the projection were made on the basis of the cohort-fertility method, or the life course approach. That is a statistical projection of the level of completed fertility and the birth timing of each female birth cohort including those who have not yet completed their reproductive processes. Future annual age-specific and total fertility rates, which are required for the projection with the cohort component method, can be obtained by converting the cohort rates into the period rates. The age-specific fertility rates of cohorts were estimated or assumed separately by birth order using models with parameters for lifetime birth probability, birth timing and some other traits of the process.

For cohorts that had completed a substantial part of their reproductive processes, the entire processes were constructed by estimating the parameters of an empirically adjusted mathematical model through a statistical method (described later). For younger cohorts for which scant or no data were available, the fertility schedules are generated through reconstruction of reproductive life course formed by such behaviors as marriage and marital childbearing. Assumptions are set up with the following four parameters of reproductive behaviours; (1) the mean age at first marriage, (2) the proportion of never married, (3) the completed number of births from married women, and (4) the coefficient of divorce, bereavement and remarriage. Each of the parameters is projected according to trend derived from recorded data compiled for cohorts so that the completed life courses of future generations are assembled. For example, estimates and prospective trends of couple's birth probabilities by birth order, which sum up to (3) the average completed number of births from the couple, is illustrated in Figure 3. In the figure, the broken lines indicate trends of the expected life time probabilities of having a child of each birth order for first marriage couple resulted only from changes in marriage pattern. They show somewhat downward trend reflecting the trend of marriage delay. The solid lines indicate trends of the probabilities in medium fertility assumption in which the reductions from changes in couple's reproductive behaviors in addition to marriage delay are took into account according to the observed trends through the national fertility surveys. For more information on the construction of fertility assumptions, see elsewhere (Iwasawa and Kaneko 2007).

Figure 3 Expected and Prospective Trends of Couple's Probability of Having Birth of Each Order by Wife's Birth Year



Source: NIPSSR (2006), the Thirteenth National Fertility Survey, 2005.

The assumptions on those four parameters of the female cohorts born in 1990 are shown in Table 2 for three alternative projection variants, i.e. Medium, High and Low[3]. All of the assumed values of the components change to the same direction for fertility reduction even in the high variant, resulting in substantial decreases of the total fertility rate (TFR)[4].

Table 2 Assumptions for Measures of Fertility Components and Total Fertility Rates for the Reference Cohort (born in 1990)

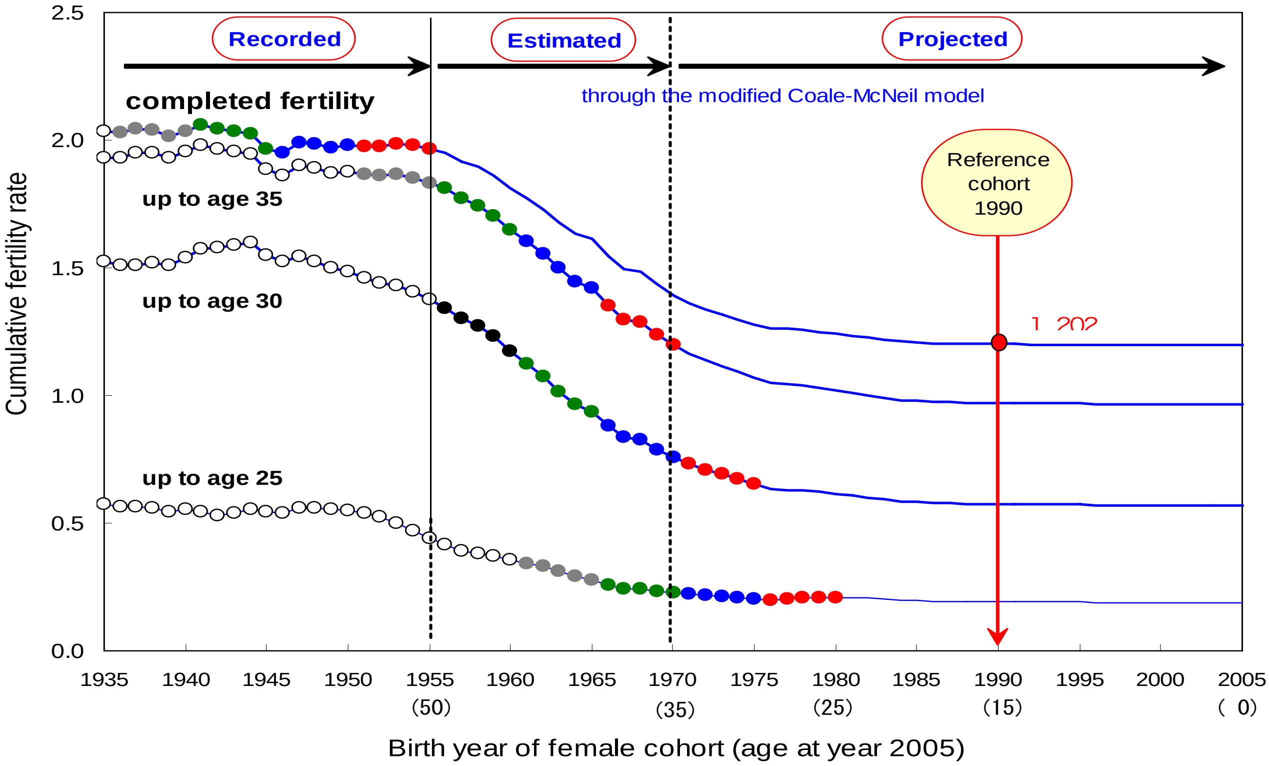
Measures of Fertility Components	Observed: cohort born in **1955**		Assumptions of Population Projection: cohort born in **1990**		
			Medium	**High**	**Low**
(1) Mean age at first marriage	24.9	⇨	**28.2**	27.8	28.7
(2) Proportion never married at age 50	5.8 %		**23.5 %**	17.9 %	27.0 %
(3) Couples' completed fertility	2.16		**1.70**	1.91	1.52
(4) Effect of divorce, widowhood and remarriage	0.952		**0.925**	0.938	0.918
Cohort Total Fertility Rate (Japanese women)	1.94	⇨	**1.26 (1.20)**	1.55 (1.47)	1.06 (1.02)

The parameters are translated into fertility schedules separately by birth order through a demographic model called the Generalized Log-gamma model (an extension of the Coale-McNeil model) with empirical adjustments specific to unique Japanese patterns (Kaneko, 2003). The resulting cohort trends of the cumulative fertility rate are shown in Figure 4 in the solid lines along with the recorded values represented by dots. As stated above, these cohort fertility rates were converted into the period fertility rates as assumptions of the cohort component procedure. The transitions of assumed TFR are shown in Figure 5 along with projected number of births which exhibit extensive reductions.

[3] The cohorts born in 1990 served as a reference whose values of parameters were most carefully examined.

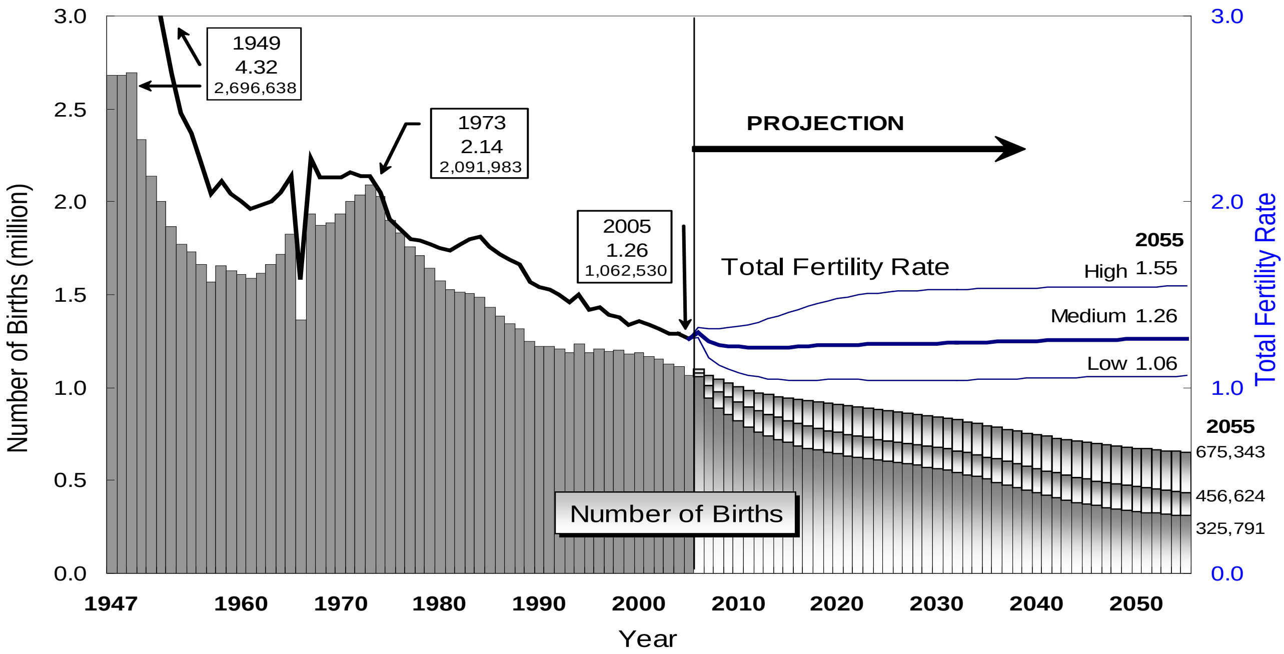
[4] This construction is applied only for Japanese women. The fertility rates of women with foreign nationalities are produced in relation to those of Japanese, using the observed relationships to be fixed for the future.

Figure 4 Recorded and Projected Cohort Trends of the Cumulative Fertility Rate at Selected Age: 25, 30, 35, and Completed (Age 50)



Source: NIPSSR (2006), Population Projection for Japan: 2006-2055 [the medium-fertility assumption].

Figure 5 Number of Births, and Total Fertility Rate in Japan
Trends and Prospects: 1947-2055



Source: Ministry of Health, Labor and Welfare, Vital Statistics. NIPSSR (2006), Population Projection for Japan: 2006-2055 (the three fertility variants with medium-mortality assumption).

The assumption building through estimation and projection of cohort measures of life course components of fertility in the projections enables us to construct the projected life course in relation to the relevant life events. I constructed the multistate life tables for the projected life by family status. Those are examined in the next section. The results indicate that less-reproductive and non-familial lives prevail among today's young and future generations, reflecting rapid transformation of partnerships and family formation patterns observed in the current cohorts.

5. Mortality Assumptions

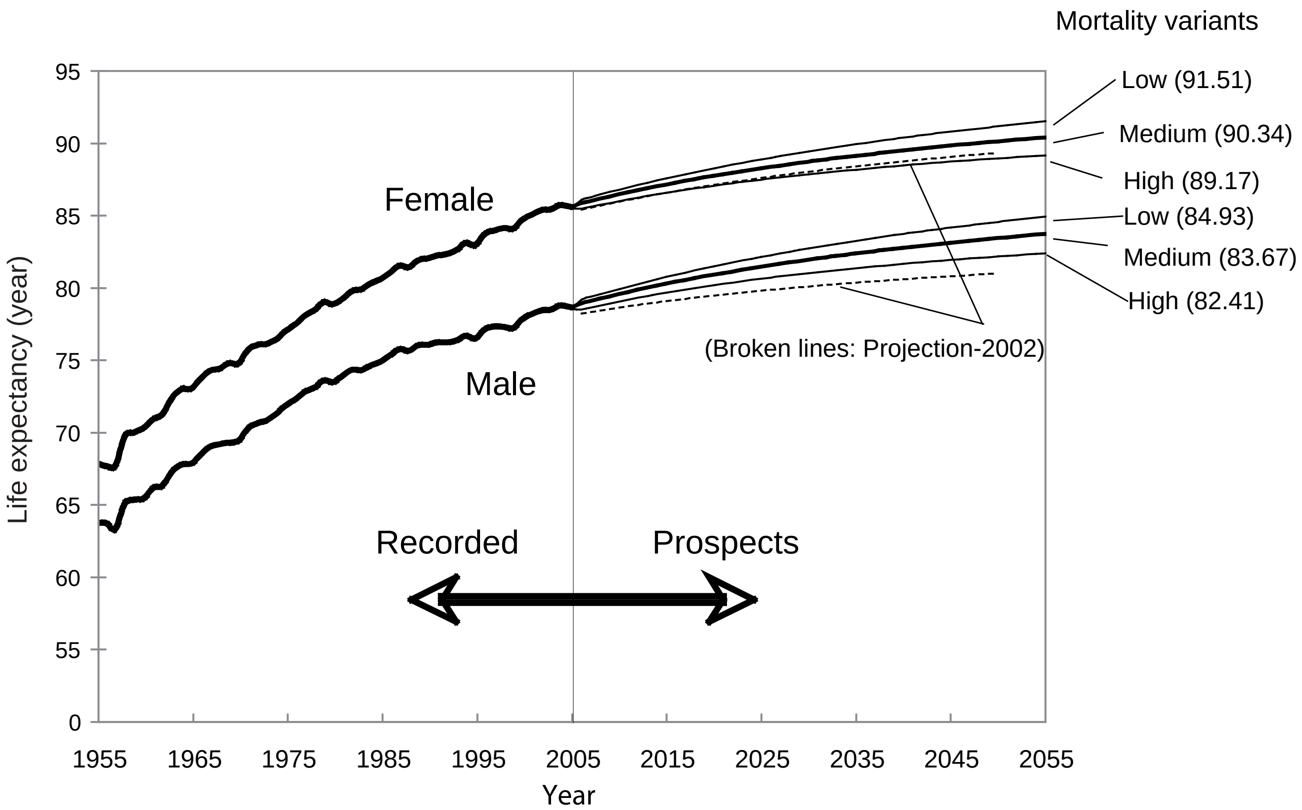
The Lee-Carter model was adopted as a basis to construct future annual life tables. The procedure is, however, modified by introducing a new technique called the shifting logistic model method (Bongaarts 2005), which identifies improvements in the mortality rates as shifts of the aging process toward older ages. This modification is a reflection of the actual mortality trend observed in Japan as a continuing gain of life expectancy especially in old ages. Combining the Lee-Carter model with the shifting logistic model is considered to be a better way of accounting for this trend, and in fact exhibits more sensible age patterns of future mortality than those from the Lee-Carter model alone.

Because of the unpredicted life expectancy gains in recent years of low mortality countries including Japan (Oeppen and Vaupel 2002, Tuljapurkar et al. 2000), a higher degree of uncertainty was anticipated for the future of mortality. This was coped with by making multiple assumptions just like those in fertility. The high and low variants of mortality, however, are derived from the boundaries of the 99 per cent confidence interval of the mortality level parameter of the Lee-Carter model (denoted by k_t in the original formulation).

According to the principal future life tables or the medium variant assumption of mortality, the life expectancy, which in 2005 was 78.53 years for males and 85.49 years for females, is expected to extend to 79.51 years for males and 86.41 years for females by 2010, 81.88 years for males and 88.66 years for females by 2030, and to 83.67 years for males and 90.34 years for females in 2055.

The assumed mortality rate for the high mortality variant will be higher, and consequently the life expectancy will be shorter than for the medium variant. According to this assumption, the life expectancy in 2055 will be 82.41 years for males and 89.17 years for females. Similarly, in the low mortality assumption, the mortality rate will be lower, and therefore the life expectancy will be longer than in the medium variant. The life expectancy by 2055 according to this assumption will be 84.93 years for males and 91.51 years for females. The assumed course of life expectancies is shown in Figure 6.

Figure 6 Trends and Prospects of the Life Expectancy: 1955-2055



Source: Ministry of Health, Labor and Welfare, *the Abridged Life Table*. NIPSSR (2006), *Population Projection for Japan: 2006-2055.*

6. Life Course Construction

The life-course construction is a characterized approach employed in the projection for Japan since the 1990's. It requires a good deal of quality data and a somewhat complicated model system. Series of the census, vital statistics, and micro data from national representative fertility surveys were brought together into play in the construction of fertility assumptions. It is often the case with population projection that excessive complications do not contribute to accuracy. However, our experience indicates that complexity to some extent would serve, since it provides detailed information on the way life of the future could be. It offers many distinct traces to improve the models through monitoring and contrasting the actual drift of the measures with those projected. In addition, it contributes to fulfilling accountability on preposition of the projections to the public. I briefly discuss the roles, uses, prospects and some limitations of the approach as well.

I attempted to construct the multistate life table for the projected life of Japanese women to obtain their life course measures mainly by ultimate family status. The measures include life time probability of never marrying, childless, having no grandchildren and so on. The probabilities incorporate incidences from women's immature deaths before the events. Average life time spent in each family status such as never married state, childless state, and only-child state, are also examined.

In Table 3, the woman's life time probabilities and distributions by family status are indicated for female birth cohorts born in every five years since 1950 through 1990. The cohorts born in 1950 and 1955 had completed their reproductive life processes by the time of projection, thus their figures are regarded as actually recorded. On the other hand, the cohorts born in 1960 and after have not yet completed the processes, and their figures are all for projected life by the assumption based on the trends of relevant parameters, though length of the projected period varies by cohort to cohort in relation to their age at projection.

Life time probability of a woman who is eventually marrying, assessed at her birth, is 86.4% for anyone among cohorts born in 1950. The figure gradually decreases from one cohort to the next until 75.7% for a woman born in 1990. These figures are somewhat lower than those calculated from the nuptiality rate among the fertility assumption, since the former includes effects from premature death before marriage.

The probability of never marrying, childlessness and having no grandchildren are 24.3%, 38.1% and 50.2% respectively in cohorts born in 1990 (see also Figure 7). If compared with those in preceding cohorts, these figures indicate rapid prevalence of less-reproductive and non-familial life styles toward an unprecedented level in this society.

Average life times spent in certain family status for female cohorts born in 1950-90 are presented in Table 4, with their proportion in the life expectancies. For instance, the average life time spent in never married status increases to 42.5 years (or 47% of the life expectancy) in cohorts born in 1990 from 25.3 years (31%) in those born in 1950. Figure 8 illustrates that the amount of life spent in never married state will drastically increase for Japanese women of young generations.

The changes in woman's life course cause tectonic movements in population composition as well. In Table 5 and Figure 9, I indicated the composition of female population by family status at three chronological times, year 2005, 2030, and 2055. They give us a manifest view that these life course changes observed above initiate the drastic increase of elderly who do not have any offspring in this society of the near future. 30.9% of women age 65 and higher do not have children in 2055, and 8.0% in 2005. Although only the situations for women are indicated here, the view should be expanded over the whole nation.

Table 3 Woman's Life Time Probabilities and Distributions by Family Status: Perspectives from the Medium Variant for Cohorts Born in 1950-90

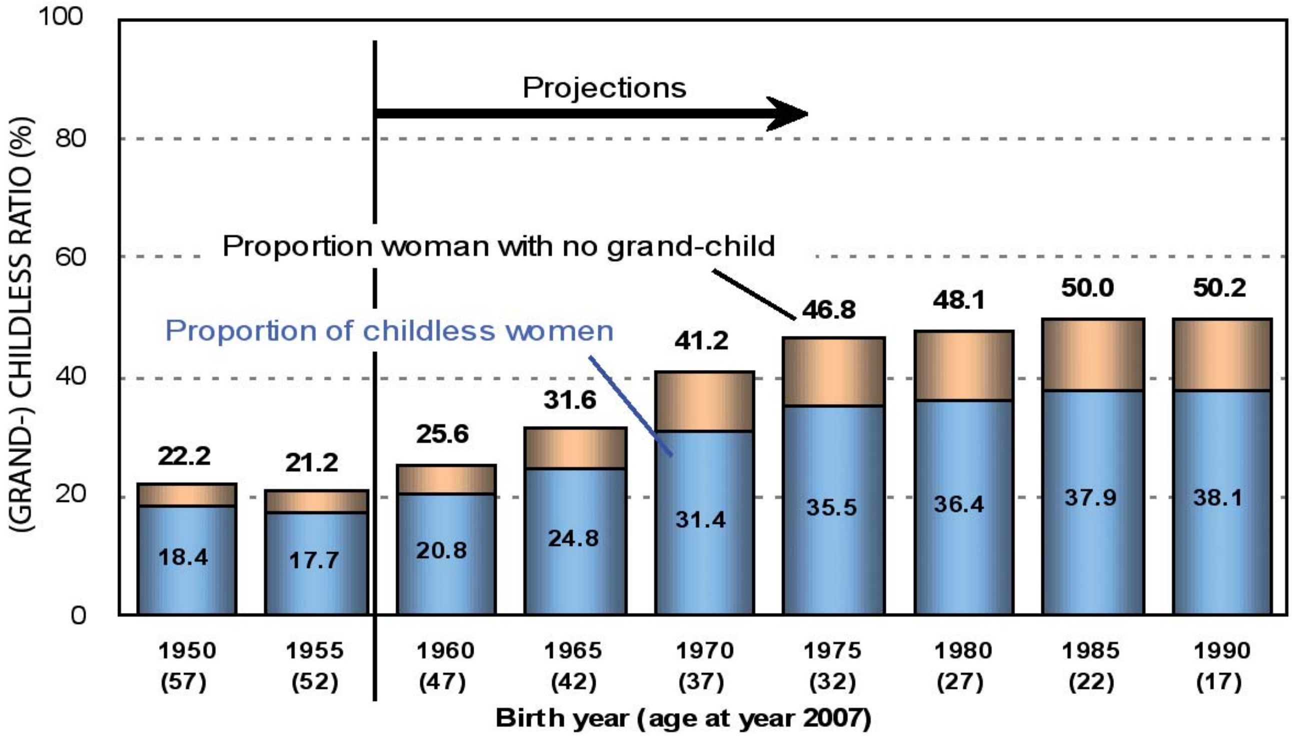
(%)

	Birth year of woman's cohort								
	Recorded		Projected						
	1950	1955	1960	1965	1970	1975	1980	1985	1990
Life time probability of woman … (at birth)									
Marrying	86.4	88.8	87.1	85.6	82.1	78.3	76.4	75.7	75.7
Having 1st birth	81.6	82.3	79.2	75.2	68.6	64.5	63.6	62.1	61.9
Having 2nd birth	70.4	71.1	65.8	58.6	49.8	45.7	44.8	43.9	43.9
Having 3rd birth	23.6	26.7	24.1	19.1	14.7	12.8	11.7	11.3	11.2
Having 4th and higher birth	4.2	4.7	4.5	3.8	3.2	2.7	2.3	2.0	1.9
Never marrying	13.6	11.2	12.9	14.4	17.9	21.7	23.6	24.3	24.3
Childless	18.4	17.7	20.8	24.8	31.4	35.5	36.4	37.9	38.1
Never having 2nd child	29.6	28.9	34.2	41.4	50.2	54.3	55.2	56.1	56.1
Never having 3rd child	76.4	73.3	75.9	80.9	85.3	87.2	88.3	88.7	88.8
Never having 4th child	95.8	95.3	95.5	96.2	96.8	97.3	97.7	98.0	98.1
Life time distribution of woman by number of child (at birth)									
Childless	18.4	17.7	20.8	24.8	31.4	35.5	36.4	37.9	38.1
Never married	13.6	11.2	12.9	14.4	17.9	21.7	23.6	24.3	24.3
Ever married	4.8	6.5	7.9	10.5	13.5	13.8	12.8	13.6	13.8
Only child	11.2	11.2	13.3	16.5	18.7	18.8	18.8	18.1	18.0
Two children	46.8	44.4	41.8	39.5	35.2	32.9	33.0	32.6	32.8
Three children	19.4	22.0	19.6	15.3	11.5	10.1	9.4	9.3	9.3
Four and more children	4.2	4.7	4.5	3.8	3.2	2.7	2.3	2.0	1.9
Net Reproduction Rate	87.5	90.0	84.5	76.3	66.3	61.2	59.6	58.1	57.9
No grandchild	22.2	21.2	25.6	31.6	41.2	46.8	48.1	50.0	50.2

Life time proportion of woman (without mortality effect = directly derived from fertility assumption)									
Never married	5.0	5.8	9.3	12.0	16.2	20.4	22.6	23.5	23.5
Childless	10.3	12.7	17.5	22.7	30.0	32.8	35.7	37.1	37.4
No grandchild	12.1	15.0	21.3	28.8	39.3	42.9	46.8	48.9	49.4

Source: From the projection 2006, medium-fertility and medium-mortality variant. The life time proportions of woman never married and childless (without mortality effect) are officially provided numbers. Other numbers are calculated by the author from the assumption. The sex ratio at birth for the net reproduction ratio is an officially provided assumption and is 105.4 (fixed value from average over year 2001-05).

Figure 7 Childless and Non-grandchild Ratio among Women by Cohort: The Medium Assumption for Female Cohort born in 1935-1990



Source: From the projection 2006, medium-fertility and medium-mortality variant. Proportions are calculated by the author from the assumption of the projection 2006, medium-fertility and medium-mortality variant.

Table 4 Woman's Average Life Time Length of Period Spent in Each Family Status: Perspectives from the Medium Variant for Cohorts Born in 1950-90

(year)

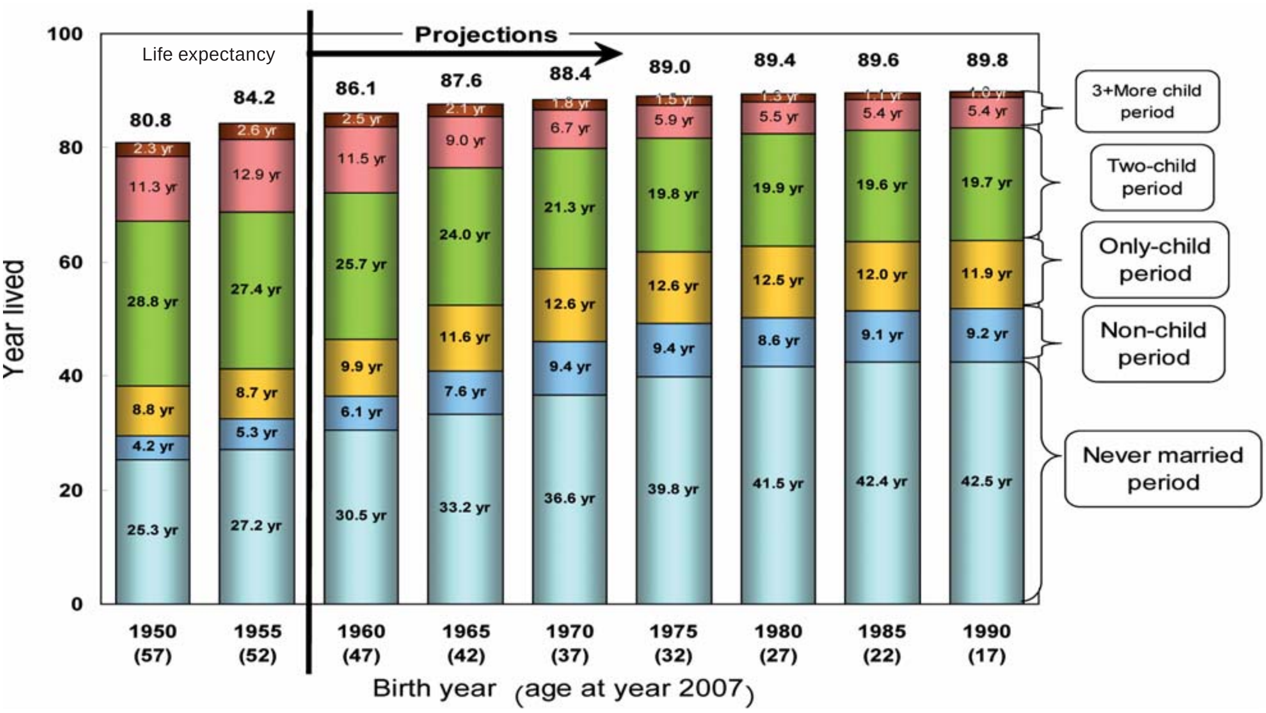
	Birth year of woman's cohort								
	Observed		Projected						
	1950	1955	1960	1965	1970	1975	1980	1985	1990
Average life time length of period spent in each family status									
Life expectancy	80.8	84.2	86.1	87.6	88.4	89.0	89.4	89.6	89.8
Never married	25.3	27.2	30.5	33.2	36.6	39.8	41.5	42.4	42.5
Childless	29.5	32.5	36.5	40.9	46.0	49.2	50.2	51.4	51.7
Never had 2nd child	38.3	41.2	46.4	52.5	58.7	61.8	62.7	63.5	63.7
Never had 3rd child	67.1	68.7	72.1	76.5	79.9	81.6	82.6	83.1	83.4
Never had 4th child	78.5	81.5	83.6	85.5	86.7	87.5	88.1	88.5	88.8
Ever married	55.4	57.0	55.6	54.3	51.8	49.2	47.8	47.3	47.3
Having Child(ren)	51.2	51.7	49.6	46.7	42.4	39.8	39.2	38.2	38.1

(%)

Proportion of life time spent in each family status									
Life expectancy	100	100	100	100	100	100	100	100	100
Never married	31	32	35	38	41	45	46	47	47
Childless	37	39	42	47	52	55	56	57	58
Never had 2nd child	47	49	54	60	66	69	70	71	71
Never had 3rd child	83	82	84	87	90	92	92	93	93
Never had 4th child	97	97	97	98	98	98	99	99	99
Ever married	69	68	65	62	59	55	54	53	53
Having Child(ren)	63	61	58	53	48	45	44	43	42

Source: From the assumption of the projection 2006, medium-fertility and medium-mortality variant. The life expectancies are officially provided numbers. Other numbers are calculated by the author.

Figure 8 Woman's Average Life Span and Its Composition by Family Status for Birth Cohorts born in 1950-1990



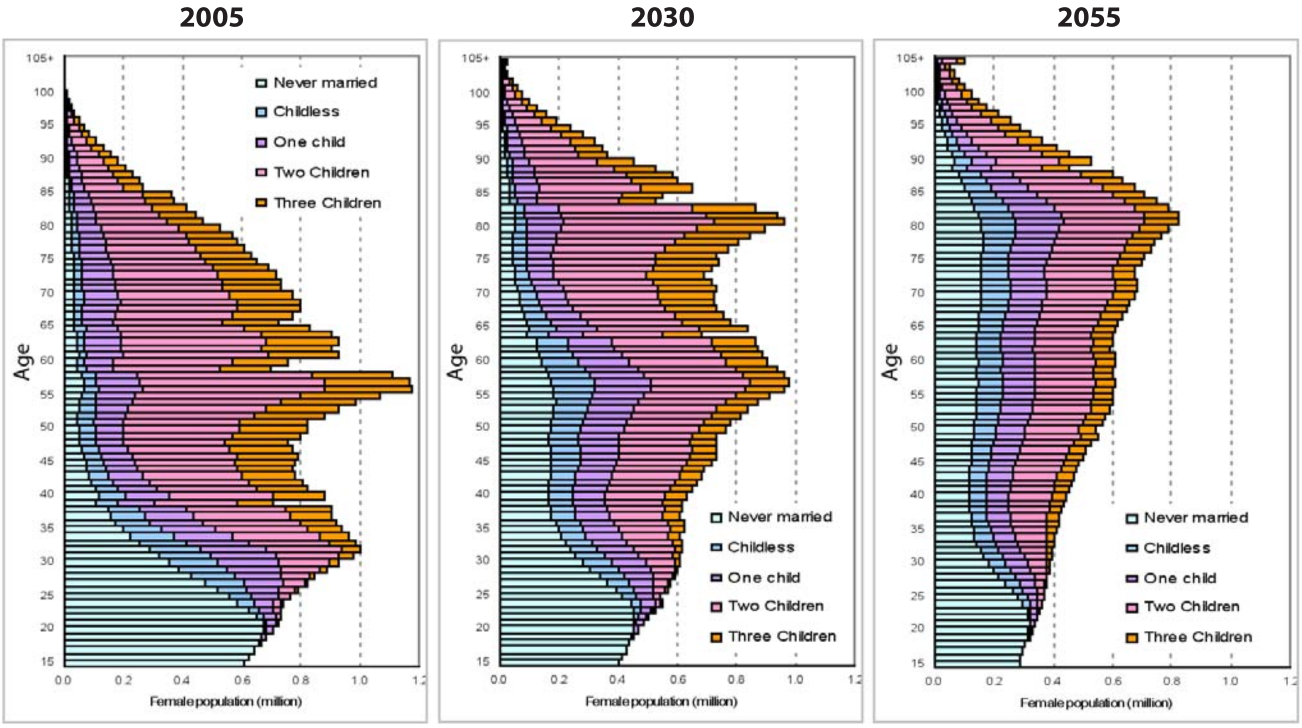
Source: From the projection 2006, medium-fertility and medium-mortality variant. The life expectancies are officially provided numbers. Other numbers are calculated by the author from the assumption.

Table 5 Composition of Woman in Each Age Group by Family Status: Perspectives from the Medium Variant in Year 2005, 2030, and 2055

Age Group	2005			2030			2055		
	Proportion of woman …								(%)
	Never married	Childless	Less than two children	Never married	Childless	Less than two children	Never married	Childless	Less than two children
15-19	99.2	99.3	99.9	99.2	99.4	99.9	99.2	99.4	99.9
20-24	88.4	91.9	97.9	89.2	92.6	98.1	89.3	92.6	98.1
25-29	57.5	72.4	88.6	62.6	75.2	89.3	62.8	75.3	89.4
30-34	29.3	46.1	68.1	38.5	54.0	73.4	38.5	53.9	73.4
35-39	17.6	30.0	48.5	28.6	41.9	60.3	28.7	41.9	60.3
40-44	11.4	20.7	36.1	25.1	37.9	55.9	25.1	37.9	56.1
45-49	7.3	14.6	27.4	23.3	36.5	55.1	23.9	37.5	55.7
50-54	5.0	11.5	23.3	21.9	35.0	54.1	23.6	37.5	55.7
55-59	5.2	9.6	22.2	17.9	32.2	51.7	23.6	37.5	55.7
60-64	4.6	7.5	20.7	14.3	26.8	44.8	23.5	37.4	55.6
65-69	4.1	7.7	22.4	10.7	20.4	35.8	23.5	37.4	55.6
70-74	4.4	8.1	23.1	7.2	14.6	27.4	22.9	36.5	55.1
75-79	4.4	8.1	23.1	5.0	11.5	23.3	21.9	35.0	54.1
80-84	4.4	8.1	23.1	5.2	9.6	22.2	17.9	32.2	51.7
85-89	4.4	8.1	23.1	4.6	7.5	20.7	14.3	26.9	45.0
90-94	4.4	8.1	23.1	4.1	7.7	22.3	10.8	20.6	36.1
95-99	4.4	8.1	23.1	4.4	8.1	23.1	7.4	14.9	27.9
100+	4.4	8.1	23.1	4.4	8.1	23.1	5.1	11.3	23.2
15+	22.8	29.8	43.6	24.8	34.3	49.4	28.9	41.2	58.1
15-49	41.6	51.4	65.3	48.0	58.9	73.4	48.2	59.0	73.4
65+	4.3	8.0	22.9	6.3	12.2	25.6	18.2	30.9	48.8

Source: From the projection 2006, medium-fertility and medium-mortality variant. Numbers are calculated by the author.

Figure 9 Composition of Female Population by Family Status: Perspectives from the Medium Variant in Year 2005, 2030, and 2055



Source: From the projection 2006, medium-fertility and medium-mortality variant. Numbers are calculated by the author.

7. Discussion

In this paper, first I described results and methods of the latest Japanese official Population Projections. Besides their unprecedented demographic perspectives on population decline and aging through the world's lowest fertility assumptions with the highest life expectancy, the projections provide some pictures of people's life course changes making use of their life course approaches employed mainly in constructing fertility assumptions. Hence, I attempted to build multistate life tables for the projected life of Japanese women in relation to family status. As a consequence, it is revealed that historically unparalleled increases both in proportion of never married and childless women and in average life time spent in those statuses would be witnessed within the scope of the next few decades. It also sets up an expansion of elderly people who have no offspring or family. These insights in individual aspects of the population projection should rouse public awareness as to the necessity for fundamental alteration in life course related institutions, in addition to reforms of macro socioeconomic organizations such as the labor market or social security system. For example, the society would not be able to rely any more on individual families caring for elderly in the manner used up to today.

The life course approach to making assumptions of vital rates requires significant amount of quality data and sophisticated models. There are arguments that the complexity of models does not necessarily contribute to precision in prediction, especially for systems consisting of many factors. Population projection, in particular, can be carried out with a jump-off population and three assumed vital rates (fertility, mortality, and migration). Incorporation of other components such as marriage makes the model complex, and requires additional assumptions on the future course of the component whose future is often more uncertain than those of the basic three factors. Seeking too much reality in projection models often leads to a morass of technical difficulties with little gain in accuracy. However, to the extent that each rate consists of several behavioral factors which change disjointedly, preparing different assumptions for their future courses is essential for sensible prediction[5]. Therefore, better policy about the degree of reality to seek in a population projection depends

[5] For example in fertility, coupling and having babies among couples are separate behaviors and their propensities have their own trends.

on what kinds of data are available and how confidently we see the future of each component involved, providing the present knowledge and technology. Then, what is the direction we should take to expand "the present knowledge and technology?"

We live in an era of difficulty in forecasting the demographic future of the society due to unpredictable developments in all vital rates. In many countries of the developed world, the traditional cohort component method with naive vital assumptions has continually shown its limitations along with the development of institutional changes called the Second Demographic Transition (Lesthaeghe 1994, Van de Kaa 1987) in the last quarter of the former century[6,7]. Demographers have increasingly become aware of projection's uncertainty, and some new "paradigms" have emerged as solutions. Among them, the probabilistic population projection is a most pronounced exemplar that enhances projection's practical applicability explicitly indicating its inherent uncertainty. It bestowed the scientific outline on population projection. Beside techniques that specify uncertainty, however, we should seek frameworks to reduce uncertainty on the other side. The life course approach or statistical life course construction of relevant cohorts should be the basis of the novel framework for that function, since it offers distinct traces by each lifetime behavior to improve the models through monitoring and contrasting the actual drift of the measures for them. The measures are free from annoying disturbances, so called tempo effects, and rarely violently fluctuate.

The present study combined with the above discussion suggests that the life course approach in population projection may deserve all efforts to overcome the difficulties that have been preventing it from working such as unavailability of the data it requires and the model complexities it induces. It provides essential information on people's life in the upcoming society, and is an effective basis for more reliable demographic prediction tools. The instruments are ample. The event history models, micro-simulation techniques possibly with agent-based design, and decision making theories, for instance, should play central roles in developing such a framework. However, what is most required for the approach to work is cohort data of many aspects. In most cases, an enhancement of the statistical systems and starting a new series of survey may be required. Public consent should be necessary. For that purpose, international cooperation is indispensable, and developing a communal outline for data collection may be effective.

8. Conclusion

According to the multistate life table constructed from the principal assumptions of the latest official population projections of Japan, the life time probability of childlessness and having no grandchildren among women born in 1990 are respectively 38.1% and 50.2%, while the corresponding proportions are 18.4% and 22.2% in those born in 1950. Similarly, the average life time spent in never married status increases to 42.5 years (or 47% of the life expectancy) in cohorts born in 1990 from 25.3 years (31%) in those born in 1950. These changes set off a drastic increase of elderly who do not have offspring to live with or rely on in this society of the near future.

The life course approach in population projection provides rich information on people's life in the upcoming society on the one hand, and on the other, it should be a promising basis of a new "paradigm" of the projection in the era of the Second Demographic Transition. The development of the new framework should be accompanied by enhancement of the data collection via upgrading national statistical systems toward demographic formulation of individual life courses.

[6] The precursors in demographic projection experienced similar difficulties during the first Demographic Transition accompanied with the post-war baby boom.

[7] The author thinks it reasonable that the shifts in mortality improvement recognized as the fourth stage of the Epidemiologic Transition (the age of delayed degenerative diseases, Olshansky and Ault 1986) being experienced by the most developed countries since the late 1960's should be included in the same stream of the Second Demographic Transition to the extent that they were concurrent phenomena unexpected in the context of the first Demographic Transition.

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REPLICATING THE OFFICIAL POPULATION PROJECTION FOR SWEDEN USING A TIME SERIES APPROACH

Gustaf Strandell,
Statistics Sweden, 2007

1. Introduction

The official population projection for Sweden is produced at Statistics Sweden every third year and the result is presented in a publication; see for example Statistics Sweden, 2006. The official population projection for Sweden follows the traditional cohort projection method and is thus an example of what we in this paper will call a deterministic population projection. It is however widely recognized that there is a need for statistically assessing the precision in a population projection and several approaches have been suggested for estimating the future error boundaries for a population projection. In the past, as well as in the present, in Sweden, as well as in many other counties, the method in use has been the scenario approach in which the demographic components are assumed to enfold according to high or low constant levels.

In response to current demographic discussions concerned with estimating the precision of population projections a Statistics Sweden Development Project was undertaken. The aim of the development project was to study the relatively new branch of this area which has brought theory and application of stochastic processes into main focus in demographic contexts, (see e.g. Alho, 1990, Goldstein, 2004 and Keilman, Pham and Hetland, 2002) and to apply these techniques to Swedish population data. The development project was documented in the Statistic Sweden publication *Stochastic Population Projections for Sweden,* Hartmann and Strandell, 2006.

During the course of the development project a number of stochastic population projections for Sweden were produced, based on different assumptions and techniques, albeit not in direct connection with the official population projection. It is the belief of the author of this paper that stochastic population projections are seldom produced in direct connection with official population projections produced around the world. The general understanding at Statistics Sweden has been that the deterministic and the stochastic techniques constitute altogether different approaches which can not be coordinated and which necessarily gives incoherent results.

The main conclusion of this paper is that the results of the Swedish official population projection from 2006 could have been attained using a stochastic time series approach. Specifically we illustrate how the 2006 Swedish official population projection emerges as a mean of a large number of extrapolations of past demographic time series behaviour. In the terminology of this paper we thus replicate the official population projection for Sweden with a stochastic population projection. In order to maintain simplicity and transparency the stochastic population projection has been produced using standard one-dimensional time series models which are easily explained and applied.

Our time series approach also leads to assessment of error boundaries for the official projection, error boundaries which we believe are more precise then those which are attained using the classical scenario approach. These error boundaries also demonstrate, for example, the fact that it is much more difficult to forecast the young population than the older population of Sweden.

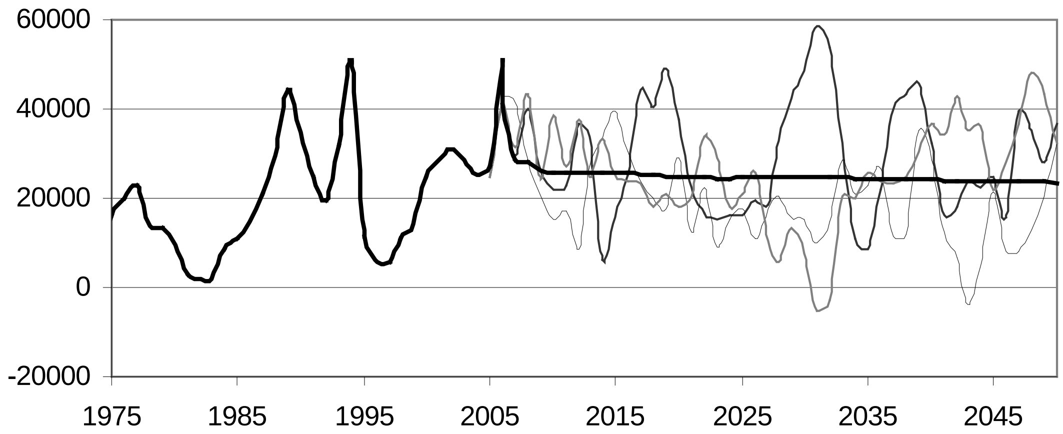
2. Stochastic and deterministic population projections at Statistics Sweden

The purpose of this section is to give a description of the difference between deterministic and stochastic population projections, as these concepts are understood at Statistics Sweden. The description given here is deliberately non technical and elementary.

The progress over time of a population obeys some rather strong deterministic rules. All individuals in the population will die at some point, and those who doesn't die gets one year older for each year, only women can get can give birth to new individuals and only at certain ages, etc. In all population projections that are made at Statistics Sweden these deterministic facts are incorporated at the simulation stage. The simulation process proceeds one year at the time and from the population in the previous step the current number of births, deaths and in- and out migrants are calculated and everyone who survives gets one year older. The simulated demographic components for the current year and the population from the previous year are then used to calculate the population for the current year.

For the simulation process to be feasible we need projections of the future mortality, the future fertility and the level of the future migration from which we can determine rates, risks and probabilities. In producing these projections the stochastic and the deterministic techniques follow different routes. Diagram 2.1 contains observed net migration in Sweden 1975-2005 and projections of the net migration as they can look in a deterministic and a stochastic population projection.

Diagram 2.1. Stochastic and deterministic projections of net migration in Sweden



The net migration in Sweden during 1975-2005 is characterized by heavy fluctuations with high tops and deep valleys, and it is not hard to believe that the future net migration will also fluctuate. However, we can not say with certainty exactly when the future tops and valleys will occur or how high or deep they will be. The probability of guessing the right future net migration also gets smaller as we try to look further into the future.

The thick black curve in the diagram stretching from 2006 to 2050 shows how the net migration has been projected in a deterministic projection at Statistics Sweden (in the official projection made at Statistics Sweden the in migration and the out migration are projected separately, the net migration shown here has been calculated from those projections). Since the producers of this projection do not know (at least with certainty) the placement or the shape of the future tops and valleys they have put the net migration at a reliable even mean level. In the official population projection for Sweden even mean levels are projected for the mortality and the fertility in a similar way.

The fluctuating thinner curves which stretches from 2006 to 2050 in Diagram 2.1 shows some projections of the net migration from a stochastic projection. In producing the stochastic projection we want the projected time series to have the same statistic characteristics as the time series we wish to extrapolate into the future. We want the stochastic projections to reflect both the horizontal trend and the fluctuations around the trend which are present in the observed time series in the diagram. However, again, we can not know neither the shape nor the placement of the future tops or valleys. Therefore, instead of projecting just one possible future for the net migration we project a big number of possible futures which all follow the same trend and contain the same amount of variation, but in which the width, the height and the placement of the future tops and valleys are determined at random.

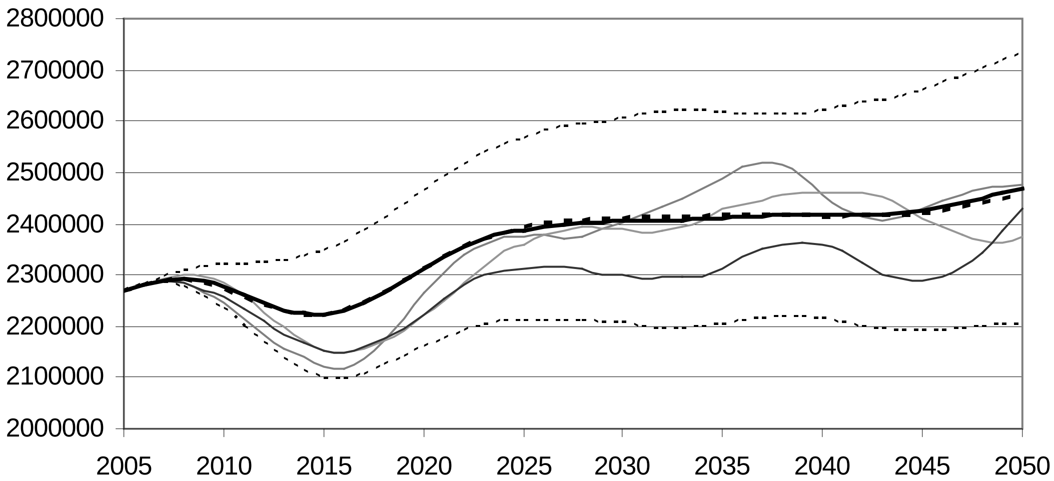
In a stochastic population projection we use tools from our time series toolbox to produce a big number of triples of time series according to the philosophy described above. Each triple consists of one projection of the future net migration, one projection of the future fertility and one projection of the future mortality. Each triple is then put into the population simulation machinery which manufactures a possible future development of the population. The raw product of a stochastic population projection consists therefore of an entire family of population projections.

To conclude, we note that the difference between the deterministic and the stochastic approach can be expressed in terms of when in the production process we turn to considering mean values. In a deterministic projection we start by projecting future mean values for the underlying demographic components. These mean values are then used in the simulation of the future development of the population. In a stochastic projection, in order to keep the simulation of the population's development as close to reality as possible, we want the projections of the future development of the demographic components to have the same characteristics as there observed counterparts. This results in a large number of population projections, from which we can calculate a mean projection at the end of the production process.

If the purpose of our projection activities only is to project a reliable future population progression in the mean, then this can be achieved just as well by any of the two methods described above. However, the stochastic approach results in a much larger set of data then the deterministic approach, and this extra data contains extra information which can be put to good use.

Diagram 2.2 shows the projected number of people aged 0-20 in Sweden for the years 2006-2050 and the corresponding observed number for 2005. The thick black line in the middle actually consists of two lines, one broken line which shows the result of the Swedish official population projection 2006 and one non broken which shows the mean result of the stochastic population projection to be presented in the next section. Needles to say, the two lines almost coincide. The area between the two broken lines constitutes 95 % prediction intervals for the stochastic projection. Since the results of the two projections coincides so closely we claim that the prediction intervals from the stochastic projection can be seen as 95 % prediction intervals for the official Swedish population projection.

Diagram 2.2. Stochastic and deterministic projections of population of Sweden age 0-20



The stochastic projection on display in Diagram 2.2 has been produced using time series models which has been fitted to observed data for the demographic components for the last 30 years, i.e. 1976-2005. The prediction intervals could hence be understood as follows: Suppose that the trends which has been observed in data for the demographic components for the last 30 years will not radically change during the projection period, and suppose that the future random deviations from the trends in form of variation will be of the same type and the same size as those which has been observed. For each year during the projection period the probability is then 95 % that the indicated prediction interval will contain the real observed population that year. The prediction intervals are thus reliable as long as the statistic properties of the demographic components will not change drastically during the prediction period.

3. Methods, assumptions and results

In this section we present the stochastic population projection which according to our claim replicates the Swedish 2006 official population projection. We first discuss the methods we have used to project the fertility, the net migration and the mortality and we then present the results of the projection and compare these to those in the official projection.

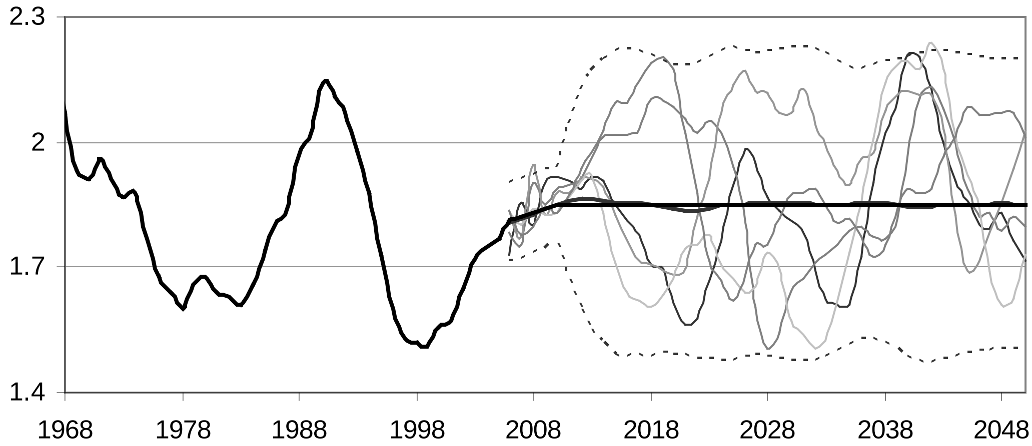
The overall level of the fertility rates in our stochastic projection is controlled by one single parameter, the total fertility rate (TFR). The fertility scheme which is used together with the TFR to calculate age specific fertility rates for each year during the projection period is kept constant during the projection period (we use the same constant fertility scheme as in the official projection).

To project the future TFR we use an AR(2) model:

$$TFR_t = a_1 TFR_{t-1} + a_2 TFR_{t-2} + \mu(1 - a_1 - a_2) + e_t .$$

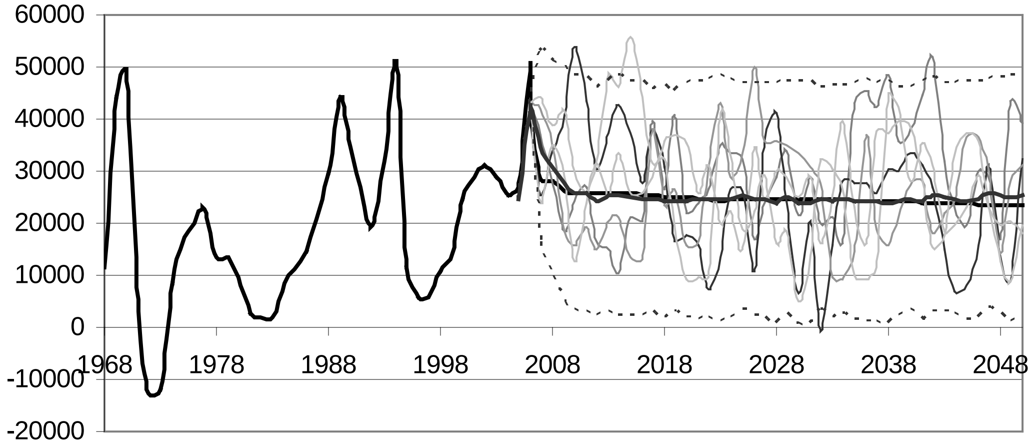
We first fit the model to the observed TFR for the years 1976-2005. We then make a small adjustment of the μ parameter in order for the projection of the TFR using the AR(2) model to give results consistent with those in the official population projection. The fitted and adjusted model is then used to produce 500 projections of the TFR, where we simulate the error term e_t with independent random (Gaussian) noise with the residual variance from the fit of the model. The result is shown in Diagram 3.1. The two almost coinciding lines in the middle of the diagram shows the projected TFR in the official projection and the mean of the 500 projections made in our stochastic projection. The area between the broken lines constitute 95 % projection interval for the TFR.

Diagram 3.1. Stochastic and deterministic projections of TFR in Sweden



In our stochastic projection we do not treat in migration and out migration separately (in the official projection they are however treated separately). Instead, we work directly with net migration. The net migration is treated as an exogenous variable in the sense that the projected size of the future population (or any other "outer" variable) does not influence the projection of the net migration. The future net migration in the stochastic projection is projected in almost complete analogy with how we project the TFR. To project the net migration we use an AR(2) model which we fit to observed data for the years 1976-2005. The μ parameter of the AR(2) model is then fine tuned in order for the model to produce projections which in the mean coincide with the corresponding net migration in the official projection. The result is shown in Diagram 3.2. The net migration is finally divided among ages and sexes according to a migration scheme which is kept constant during the projection period and which coincides with the corresponding migration scheme in the official projection.

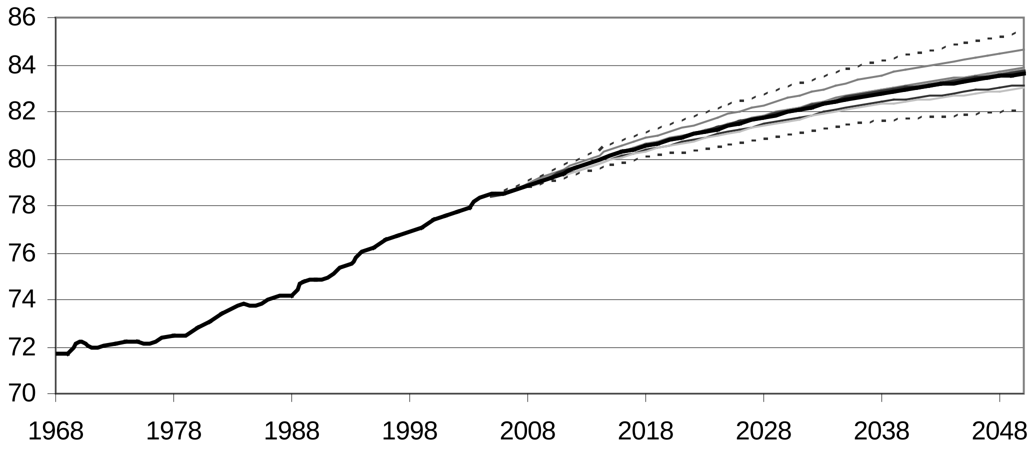
Diagram 3.2. Stochastic and deterministic projections of net migration in Sweden



In our stochastic projection we use a Brass model to model the mortality (see e.g. Brass, 1971 and Brass, 1974). The future age and sex specific death rates are controlled in the Brass model by two parameters, alpha and beta (actually one alpha and one beta per sex). When the Brass model is used in a population projection the usual course of action consists of several steps. First, values of alpha and beta for past years are calculated from observed mortality data. The resulting time series for alpha and beta are then projected into the future using some time series models which has been fitted to the calculated alphas and betas. Finally, the future values of alpha and beta are transformed back to future mortality data.

In our stochastic projection we have chosen a different path along which we project future mortality data. Although in technical terms we do use the Brass model, the main idea behind our approach is illustrated here by giving reference directly to the life expectancy at birth. Both the TFR and the net migration have been treated above as stochastic processes which follow a stationary horizontal trend where the difference between the years seems to be mostly due to random jumps up or downs. The life expectancy at birth is, as can be seen from Diagram 3.3, a process of a different breed. The difficulty in projecting future values of this series is not primarily about the placement and shape of the future tops and valleys. Instead, the problem seems to be to find the future direction of the curve describing the life expectancy. The life expectancy in Sweden will with high probability continue upwards but the question is at what rate.

Diagram 3.3. Stochastic and deterministic projections of life expectancy at birth for males in Sweden



In our stochastic projection we have based our projection of the life expectancy on the projection of the future life expectancy in the official projection. Let $Life(off, y)$ $y = 2006,\ldots,2050$ denote the projection of the life expectancy in the official population projection. Our stochastic projection of the life expectancy consists then of 500 projections of the form

$$Life(k, y) = Life(off, y) + d(k)(y - 2005) + e(k, y), \qquad k = 1,\ldots,500 \quad y = 2006,\ldots,2050,$$

where the 500 numbers *d(k)* and the 22500 numbers *e(k,y)* are chosen randomly and independently. In other words, we produce 500 new life expectancies by adding linear functions with random coefficients of direction, *d(k)*, and by adding a small noise term, *e(k,y)*, to the projected life expectancy in the official population projection. An evaluation study of deterministic predictions of the life expectancy at birth undertaken at Statistics Sweden has shown that a deterministic projection ten years into the future has a mean error of one year. The spread of our random coefficients of direction, *d(k),* has been decided according to this.

Using our 500 projections of the TFR, our 500 projections of the net migration and our 500 projections of the life expectancy at birth (actually of alpha and beta) and the population simulation machinery mentioned in Section 2 we now iterate 500 population projections, i.e. 500 projections of the number of children born, the number of people deceased and the population divided after age and sex. Diagrams 3.4-3.8 display the outcome. The thick lines in the middle of the diagrams always consist of two lines, one showing the mean value of the 500 iterations in our stochastic projection and one showing the corresponding prediction from the Swedish official 2006 population projection. We claim that these lines coincide closely enough for us to say that we have replicated the official projection with a stochastic projection. The thinner non broken lines in the diagrams shows some individual projections from our stochastic projection and the area between the broken lines constitute 95 % projection intervals.

Diagram 3.4. Stochastic and deterministic projections of the number of children born in Sweden

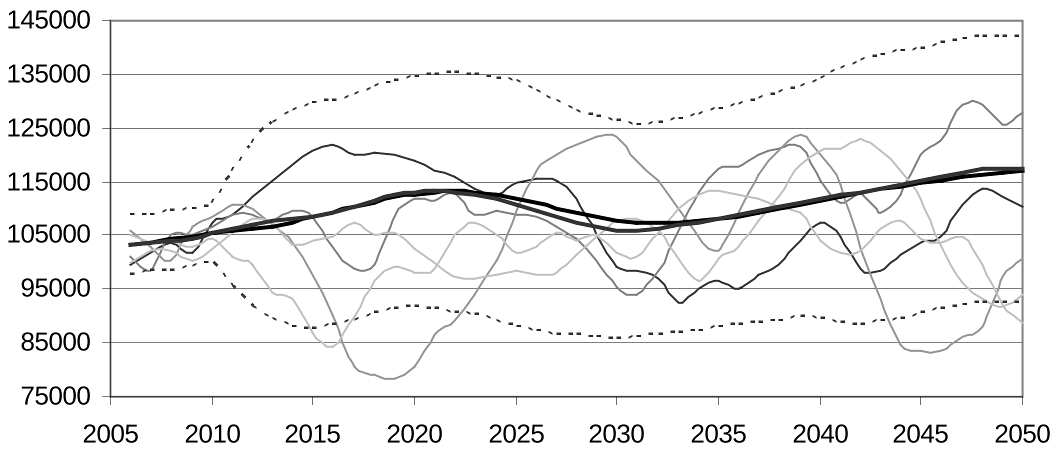


Diagram 3.5. Stochastic and deterministic projections of the number of people deceased in Sweden

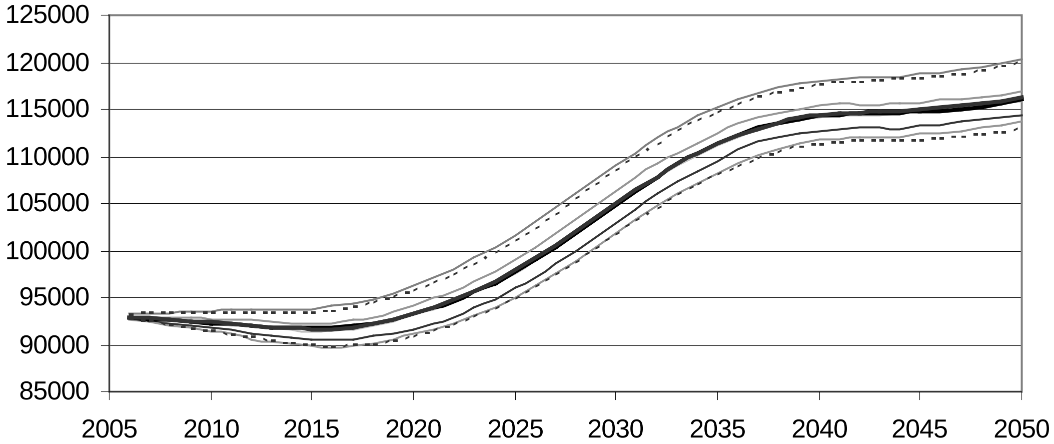


Diagram 3.6. Stochastic and deterministic projections of the total population in Sweden

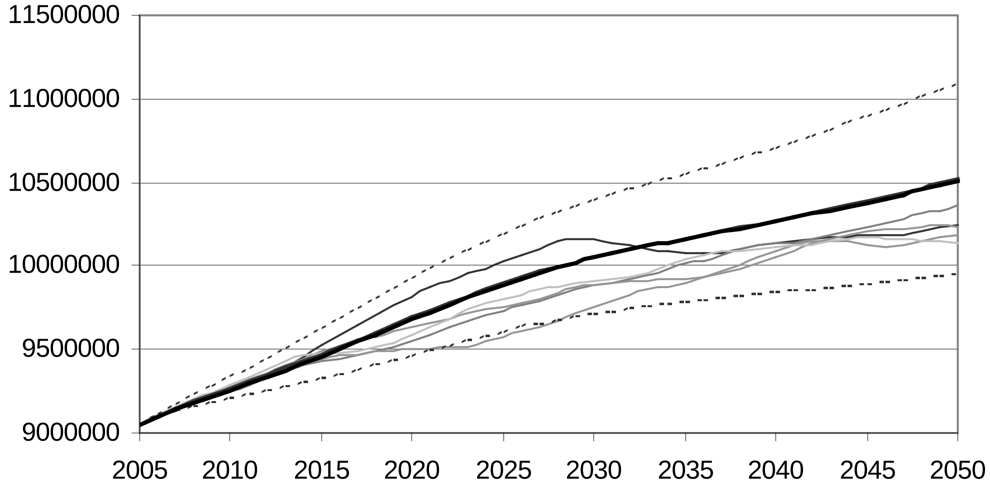


Diagram 3.7. Stochastic and deterministic projections of population males by age in 2050.

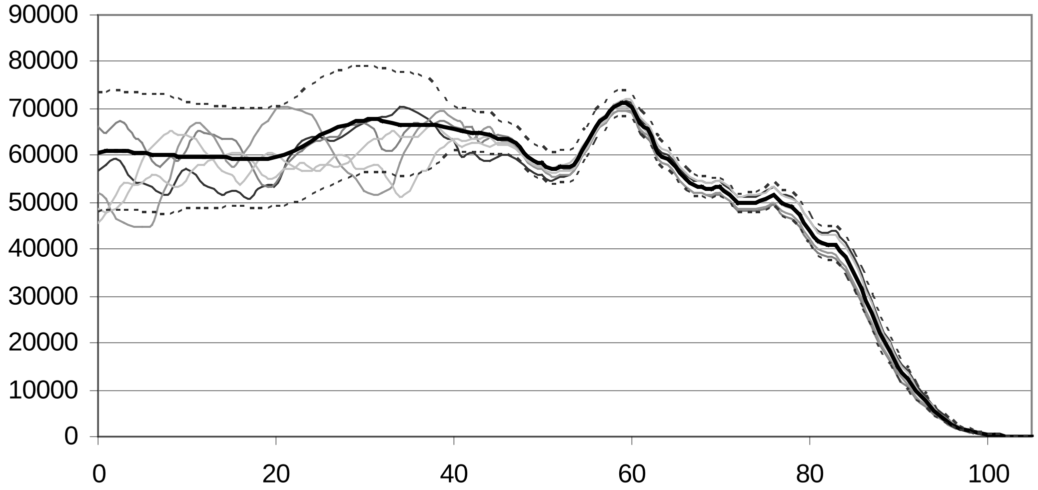
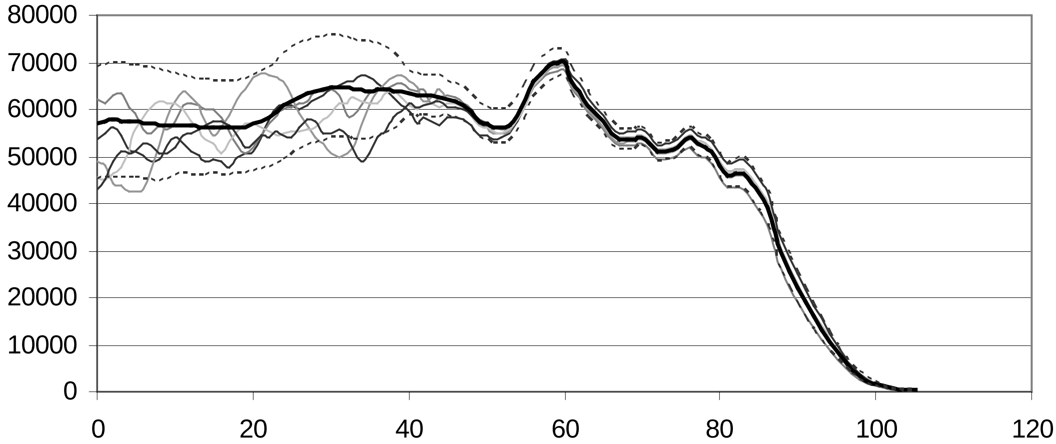


Diagram 3.8. Stochastic and deterministic projections of the population females by age in 2050



From Diagram 3.7 and Diagram 3.8 we conclude that it is much more difficult to say anything certain about the number of young people in Sweden in the year 2050 than it is to guess the number of old people that year. This means that care for older people during the next 45 years or more can be planned today (as long as the only variable under concern is the number of people) whereas care and education for younger people can only be planned a few years into the future.

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POPULATION FORECAST AND THE IMPACT ON THE LONG TERM GROWTH POTENTIAL

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1. General considerations

Romania – as the majority of the new EU Member States – has many socio – economic gaps as against the European average, gaps that represent the potential negative factors for the future development of the Romanian economy and also for the European economy as a whole. Fulfilling the objectives from the Lisbon Strategy, namely the European Union to become the economy with the highest competitiveness degree in the world, requires a higher contribution of each member state to the European economy development.

As a consequence, both for the less developed countries and also for the European Union, the national economies convergence and the gaps decrease represent the main objectives. Only with a higher contribution of the member states for the European economic growth and with an improvement of the competitiveness for each economy, the Lisbon Strategy objective can become a reality.

For Romania, an accelerated progress requires a permanent assessment of the production factors and based on this, promoting the economic policies in order to use in an efficient and balanced way the resources.

Starting from the truth that the labour force represents the most important production factor, Romania's progress and competitiveness targets can't be achieved if the labour force is not fully efficient, not only from the quantitative point of view, but especially from the qualitative one.

In order to use efficient the human potential, it is necessary to be taken into account the fact that the human resource in Romania has many specific characteristics, especially structural and qualitative.

The main particularities in Romania regarding the labour resource are: a relatively low participation on the labour market and an occupational structure below the potential.

The labour market in Romania is characterized by a relatively high inactivity rate, high unemployment rates especially for 15-19 and 20-24 age groups, a high percent of early retirement and a significant employment rate in the agricultural sector, which is characterised by employment relationships unregulated by tax provisions.

Taking into account also the signals regarding the ageing process, it results that in Romania, as compared to the other countries, the pressures from the human factor on the long term economic growth are not so high.

Consequently, the NCEF forecast is not pessimistic like forecasts exacerbating the demographic trends role.

2. Potential GDP – an answer for measuring the impact of the human resources on the economic growth

Ensuring a sustainable economic growth on the long term, in accordance with the existent potential and without affecting the main economic correlations represents one of the fundamental objectives of the European Commission. The Lisbon Strategy itself is an answer for this objective. From this point of view, the Stability and Convergence Programmes want to put in evidence those macroeconomic policies that ensure the medium and long term sustainability of the public finances, as a premise of a durable economic development.

The potential growth can be defined as the growth ratio corresponding to the maximum utilization of the production factors, supposing that only the labour factor is limited on the long term.

The structural method for the estimation of the production functions supposes, generally, that the production technology ca be written as a 1st degree homogenous function of an efficient employment, so including the technical progress. The technical progress is supposed to save both the labour and the natural resources. An efficient work corresponds to the multiplication between the labour productivity and the employment, and can be written taking into account the active population and the unemployment rate.

From this perspective the potential GDP and the output gap assessment became a demanded process in the methodology elaboration of the Convergence Programme. Furthermore, in the latest years the European Commission has been preoccupied with the establishment of a unitary computing method, in order to facilitate the comparison between countries. Based on the Ecofin Council conclusions from 12th of July 2002 and 11th of May 2004, the production function method has become the reference method for the output gap reckoning.

An analyse of the potential GDP evolution during 1997-2006 based on historical data regarding the production factors, namely the labour force and the capital stock, reveals the fact that in the labour force evolution, expressed through employment, there is a suddenly decrease in 2002, due to the discontinuity point in the data series, this as a consequence of the statistical estimation methodology change. In the same period the series regarding the physical capital had an increasing trend after 1999, materialised in positive increasing rhythms of the GDP starting 2000. In these conditions, we may appreciate that in the latest years the Romanian economy potential improved mainly due to the modernisation of the capital stocks.

Regarding this, there are relevant the following trends:

- The domestic investments and especially the private ones recorded high dynamics, with 2 digit values; for example in 2005 the investment from the private sector increased by around 25% and in 2006 by about 18%; the investment rate has started to reach the maximum value from the developed countries or from the countries that have succeed in reducing the economic gaps (countries from Asia, and also Spain, Portugal), namely almost 25% in 2006;
- Romania has started to be one of the most important destinations for the capital inflows; in the last 3 years the foreign direct investments surpassed EUR 20 billion, as against EUR 10 billion during 1990-2003;
- The important restructure of the industrial output towards the capital goods, which is already accepted as a second Romania's industrialisation; the cars building industry not only has increased its contribution to renew the capital stocks, but it has also become the main activity for the Romanian export.

On medium and long term the potential GDP depends more on the labour productivity and active population growth rhythms. The human factor intensity – including here the qualitative pluses brought by education and research – is as more important as the demographic constraints will be higher in Romania, too.

The model proposed in order to analyse on the long term the impact on the potential growth uses the methodology also used by the European Commission, which is based on the production function. So, we have considered that the Cobb - Douglas production function suits the best with the demand of this analyse.

As far as the labour force is involved, the potential employment has been computed based on NAIRU applied to active population (labour force). NAIRU results from the Hodrick – Prescott filter application to the unemployment rate series.

$\ln Y = a\ln N + (l\text{-}a)\ln K + \ln TFP$

or: $y = an + (l\text{-}a)k + tfp$, in a logarithmic form, where:

Y – potential GDP;

N – potential working age employment;

K – capital stock;

TFP – total factors productivity.

As an unobservable macroeconomic indicator, the potential GDP is used in the macroeconomic policies analyse and projection, not directly, but through the output gap. The output gap represents the difference between the actual GDP and the potential GDP, seen as a share in the potential GDP. In other words, the output gap represents the percent deviation of the actual GDP from its potential level and has the following formula:

Output gap = (Actual GDP – potential GDP)/potential GDP -100

There are two types of output gaps if we take into consideration the sign of the difference (actual GDP – potential GDP):

- Expansionist output gap (*actual GDP < potential GDP*);
- Recession output gap (*actual GDP > potential GDP*).

3. The utilisation of the labour resources and the long term economic growth

The macroeconomic forecasts for the period 2007-2020 take into account a positive vision regarding the domestic and international environment. The business environment keeps stable and the economic growth of Romania's main trade partners will not follow a descending trend. Romania's EU accession will accelerate the social and economic development. Both the domestic potential of the capital and of the labour force will favour a continuous and sustainable economic growth. The important investment process from the previous years and also the forecasted one has been reflected in the modernization and the increase of the production factors.

For the assessment of the correlation between the labour market potential and the durable economic growth, the following hypotheses have been considered (exogenous variables):

- Demographic perspective which aren't encouraging; even with the perspective of improving the reproduction health and infant health, Romania's total population will decrease by about 700 thousand persons in the forecasting period;
- The ageing population process will be a constant phenomenon, but not so relevant at the beginning of the period; however, it was taken into consideration that the population over 64 years old will be higher in 2020 by about 400 thousand persons, as against 2007;
- Maintaining the economic growth rhythm at a certain level in order to ensure at the end of the period a value close to EU average level; such an objective is possible if the economic growth will exceed 6% annually, in accordance with the potential GDP.

In Romania there are two phenomena, relatively divergent: a decrease of the labour resources and an increase of the employment necessary. The NCEF forecast - based on the potential GDP model – shows that during 2006-2020 the working age population will decrease by about 200 thousand persons and the labour force necessary will increase by 700 thousand persons.

Nevertheless, it is possible that the labour force deficit to maintain the 2006 value, 450 thousand persons respectively and even to have a light decrease (by about 10 thousand persons). The trend, representing an improvement of the activity rate, is possible because of the fact that nowadays in Romania there is an important number of the working age population that are not active. There are about 5.5 billion people.

Another positive element in order to improve the activity rate until 2020 from 63.7% in 2006 is represented by the labour resources flows stopping towards the more developed European countries. We foresee that in the second part of the period (probably after 2015) the labour force international movement sold will become positive, also because the main coverage resource of the deficit will become an external labour resource, as against the actual situation when the deficit is covered by the population over 64 years old.

In this context is worth to be mentioned that the global deficit is not a sum of the territorial deficits or the occupational deficits. From this perspective, if the progresses regarding the labour market mobility won't be significant it is possible that the labour force necessary and the deficit to be higher. The increased mobility (the occupational flexibility) – due to a more efficient implementation of the employment policies – will represent the major factor in order to ensure the structural improvement of the employment and based on this a higher qualitative contribution of the human factor to the economic growth.

For the next period it is foreseen an orientation of the rural area population towards the non agricultural activities. The process will be stronger due to the increase of the economic performances in the output and services sector but also due to the opportunities provided by the diversification of the intervention instruments in order to ensure the competitiveness in these areas. In this context, it is estimated that the balance point between the employment rate in the rural area and the one in the urban area will be reached in 2009.

Due to the structural changes, the share of the employment in agriculture has decreased and in the services and constructions sector it has increased. For the next period, these trends will continue. The employment in agriculture will have a 16% share in the total employment in 2020 as compared to 30.5% in 2006, meaning a more qualitative utilization of about 1.3 million persons.

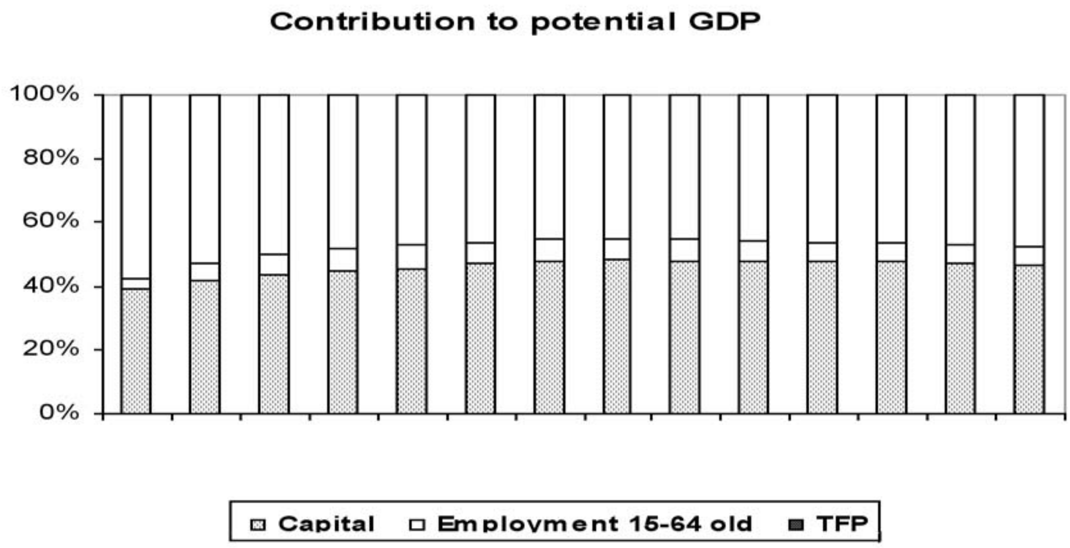
Table 1 The forecast of the labour resources utilisation on the long term is as follows:
thousand persons

	2006	2013	2020	Changes 2020/2006	
				persons	%
Working age population	15057	14975	14870	-187	-1.2
Available labour resources	9588	10110	10273	685	7.1
Working age active population	9588	10110	10273	685	7.1
Labour force deficit	-453	-415	-347	106	-23.4
Working age employment	8860	9470	9709	849	9.6
Working age population activity rate - %-	63.7	67.5	69.1	-	-
Working age population employment rate-%-	58.8	63.2	65.3	-	-

Both the activity rate and the employment rate will improve in the next period. If in 2006 the working age population activity rate was 63.7%, for the period 2007-2020 it is foreseen an increase of 5.4 percentage points as against 2006.

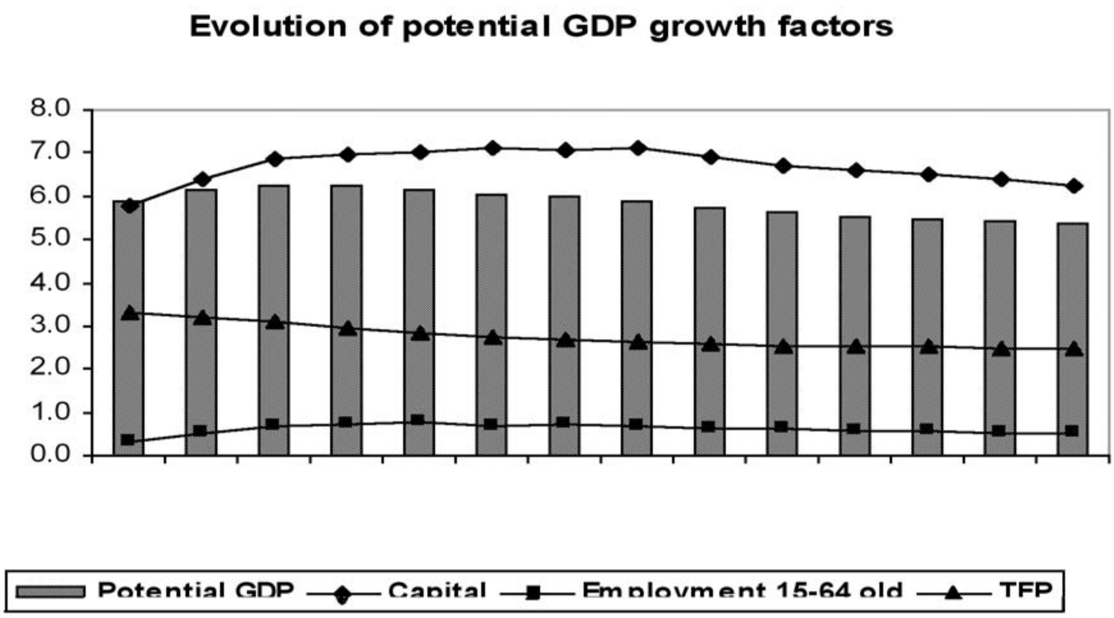
The working age population employment rate will increase from 58.8% in 2006 to 65.3% in 2020, due to the durable job creation process and due to equilibrium between the occupational flexibility and working place security.

The decrease of the unemployment rate was and will be, too one of the main preoccupations of the Romania's Government. In 2006, the unemployment rate was 7.3% and will follow a descending trend in the future until 4.8% in 2020. This decrease will be possible due to a more efficient implementation of the labour force employment policies.



Consequently, from the potential growth perspective, the long term evolution is characterised through:

- An increase of the employment rate of at least 8 percentage points during the considered period;
- A decrease of the unemployment rate to the levels closed to those considered to be the natural unemployment rate;
- A constant and sustained increase of the total factors productivity, as a result of the process of assimilating the performing technologies and amplifying the innovation capacity of the Romanian economy; Keeping the high level of redistribution of the national income in the population's advantage in order to achieve the income convergence and thus the standard of living from the European Union.



The impact of this evolution - measured through the direct contribution of the human factor to the growth potential – can reach 0.3 - 0.4 percentage points as compared to 0.1 percentage points in 2006 and also to the negative contribution until 2005. There can be also added the effect of the labour productivity and implicitly the total factors productivity, as an expression of a higher quality labour force.

A TOOL FOR PROJECTING AGE PATTERNS BASED ON A STANDARD AGE SCHEDULE AND ASSUMPTIONS ON RELATIVE RISKS USING LINEAR SPLINES: TOPALS

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1 November 2007

1. Introduction

Using a multistate model such as MicMac[1] for making scenarios requires that assumptions need to be specified on the future values of many parameters, *viz.* transition rates distinguished by age, sex, forecast year and covariates like level of educational attainment. Particularly the distinction by age implies that many parameter values need to be specified. One solution is to specify model curves describing the age patterns and specify assumptions on the future values of the parameters of the model curve instead of values of rates for all separate ages. This paper introduces TOPALS (Tool for projecting age patterns using linear splines), a method that is capable of describing all kinds of age curves. The parameters can be interpreted easily and the fit of the curve to the data is good. The basic idea is to choose an age schedule that captures the general pattern of the demographic process and to model deviations from this curve by age-specific relative risks[2] which are modeled by a linear spline function. The values of the spline function are estimated by dividing the average values of the transition rates for successive age groups by the average values of the standard age schedule of transition rates for the corresponding age groups. The standard age pattern can be calculated by averaging an age pattern over countries (e.g. the EU15 average) or by estimating some model age schedule (e.g. the Heligman-Pollard model for mortality).

The idea of modeling deviations from a standard age schedule was developed by Brass (1974). Brass assumes a linear relationship between a double logarithmic transformation of the age pattern to be fitted and a double logarithmic transformation of a standard age schedule. The two parameters can be estimated by linear regression. One problem of this approach is that the parameters lack a clear interpretation. Zeng Yi et al. (2000) propose an alternative method for estimating the two parameters of the Brass relational model, based on the median age and interquartile range. TOPALS is more flexible than the method proposed by Zeng Yi et al., produces a better fit and is at least as simple. By using a linear spline function TOPALS is flexible in two respects. First, it can describe all kinds of age curves. Secondly, the user can choose the desired level of goodness of fit and degree of smoothness. This paper illustrates TOPALS by fitting curves of age-specific mortality and fertility rates for Italy and the Netherlands using the EU15 average as standard age schedule.

TOPALS can be used for making projections of future changes in age-specific rates by specifying assumptions about changes in the average values of the relative risks for successive age groups. Three approaches can be followed. First, assumptions can be made on the future values of the relative risks related to the standard age schedule in the base year. Secondly, the standard age schedule can be projected into the future using a random walk model with drift and a linear spline function describing changes in the age pattern over time. Assumptions on the relative risks can be specified relative to the projected standard age schedule. Thirdly, the age pattern in the base year can be used as standard age schedule. The latter approach produces projections that are similar to the Lee-Carter method. The three variants of TOPALS will be illustrated by projecting age-specific mortality rates for Italy and the Netherlands for the year 2050.

[1] "TOPALS is developed within the project MicMac: Bridging the micro-macro gap in population forecasting; see for more information: www.micmac-projections.org."

[2] Throughout the paper we will use the term 'relative risk' as a rather general term to describe ratios of rates and risks.

TOPALS allows to take into account the effects of covariates by modeling different age patterns for different categories of a covariate. For example, one may assume different age patterns of the transition rates for different levels of educational attainment. The paper shows how the linear spline function of relative risks can be used for this purpose.

2. Using TOPALS to estimate age profiles

We assume that a standard age schedule of transition rates is given. The age profile of transition rates for a given country or population category can be estimated on the basis of relative risks, i.e. on the ratio between the transition rates of that country or population category and those according to the standard age schedule. The relative risk at age x for country i (or population category i) is equal to:

(1) $$r_{i,x} = \frac{q_{i,x}}{q_x^*}$$

where q_x^* is the transition rate at age x according to the standard age schedule. All rates discussed in this paper are distinguished by sex. However, for the sake of readability we omit subscripts indicating sex. The age pattern of the relative risks can be described by a linear spline function. This is a piecewise linear curve. The ages at which the successive linear segments are connected are called 'knots'. The relative risks at each age can be estimated by the linear spline function:

(2) $$\hat{r}_{i,x} = a_i + b_{i,0}(x-m) + \sum_{j=1}^{n} b_{i,j}(x-m-k_j)D_j$$

where $D_j = 0$ if $x - m \le k_j$ and $D_j = 1$ otherwise, m is the minimum age, $x \ge m$, k_j are the knots, n is the number of knots, a_i and $b_{i,j}$ are parameters to be estimated.

The knots can be fixed a priori or they can be chosen in such a way that the fit of the spline to the data is optimal. In the latter case a non-linear estimation method is required, *e.g.* a non-linear least squares method. If the location of the knots is fixed a priori, a_i and $b_{i,j}$ can be estimated by linear regression. However, these parameter values are difficult to interpret. Therefore we suggest to estimate the linear spline in a different way. The values of $\hat{r}_{i,k_j}$ at the knots k_j are set equal to the average values of $r_{i,x}$ for successive intervals. $\hat{r}_{i,k_1}$ is set equal to the average value of the $r_{i,x}$s in the interval $1 \ldots (k_1+k_2)/2$, $\hat{r}_{i,k_2}$ is set equal to the average value of the $r_{i,x}$s in the interval $1+(k_1+k_2)/2 \ldots (k_2+k_3)/2$, etc. The values of a_i and the $b_{i,j}$s can be estimated from the values of $\hat{r}_{i,k_1}$, $\hat{r}_{i,k_2}$, $\hat{r}_{i,k_3}$, etc. by solving the following equations:

(3) $$\begin{aligned} \hat{r}_{i,m} &= a_i \\ \hat{r}_{i,k_1} &= a_i + b_{i,0}k_1 \\ \hat{r}_{i,k_2} &= a_i + b_{i,0}k_2 + b_{i,1}(k_2-k_1) \\ \hat{r}_{i,k_3} &= a_i + b_{i,0}k_3 + b_{i,1}(k_3-k_1) + b_{i,2}(k_3-k_2) \\ &etc. \end{aligned}$$

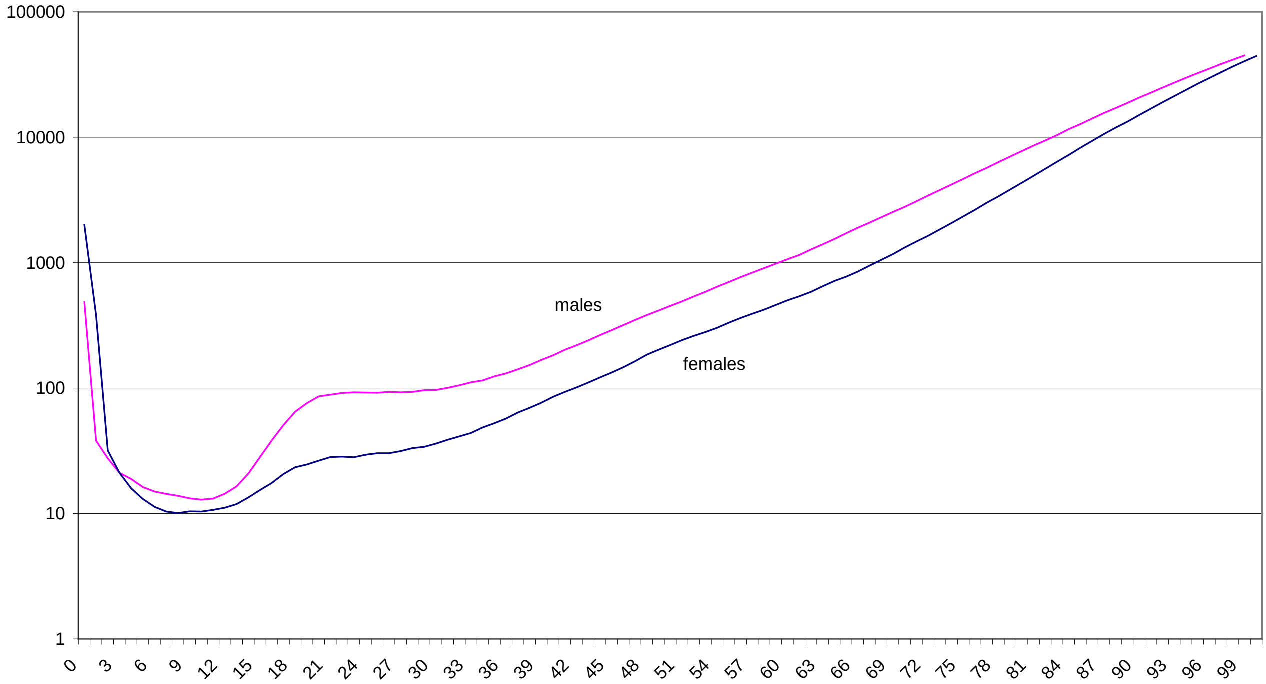
The transition rates for country i are estimated by multiplying the relative risks which are estimated by the linear spline function ($\hat{r}_{i,x}$) by the transition rates according to the model age schedule (q_x^*):

(4) $$\hat{q}_{i,x} = \hat{r}_{i,x} q_x^* .$$

3. Modeling age patterns of mortality rates

The standard age schedule that we use to model age patterns for mortality rates is the (unweighted[3]) average of the EU15 countries in 2003, for men and women separately. The age-specific mortality rates used in this paper are calculated from data on age-specific deaths and population numbers published by Eurostat. Figure 1 compares the age-specific mortality rates for men and women in 2003 (on a logarithmic scale).

Figure 1 Age-specific mortality rates, EU15 average, 2003



[3] The age-specific mortality rates based on the weighted EU15 average with population size of all countries as weights, hardly differ from the unweighted average. On average the weighted average age-specific mortality rates are 1 percent lower than the unweighted averages. The logarithms of the age-specific mortality rates differ even less: by only 0.2 percent.

Figure 2 compares the age-specific mortality rates for Italian women and men in 2000 (solid lines) with the EU15 average in 2003 (dotted lines). The overall age pattern for Italy is similar to the EU15 average, but in general the Italian mortality rates are lower than the EU15 average. Figure 3 compares the Dutch mortality rates to the EU15 average. Again, the age patterns seem rather similar, but mortality rates for Dutch men in their 20s and 30s are relatively low.

Figure 2 Age-specific mortality rates, Italy 2000 and EU15 average 2003

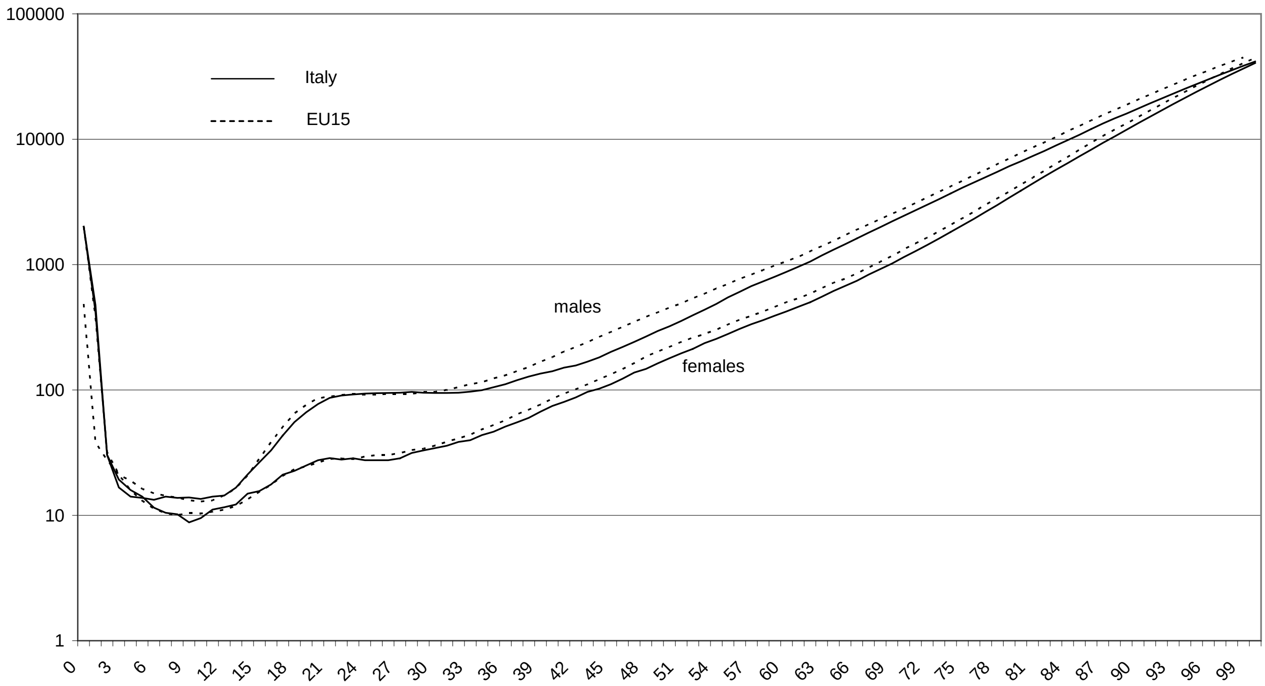
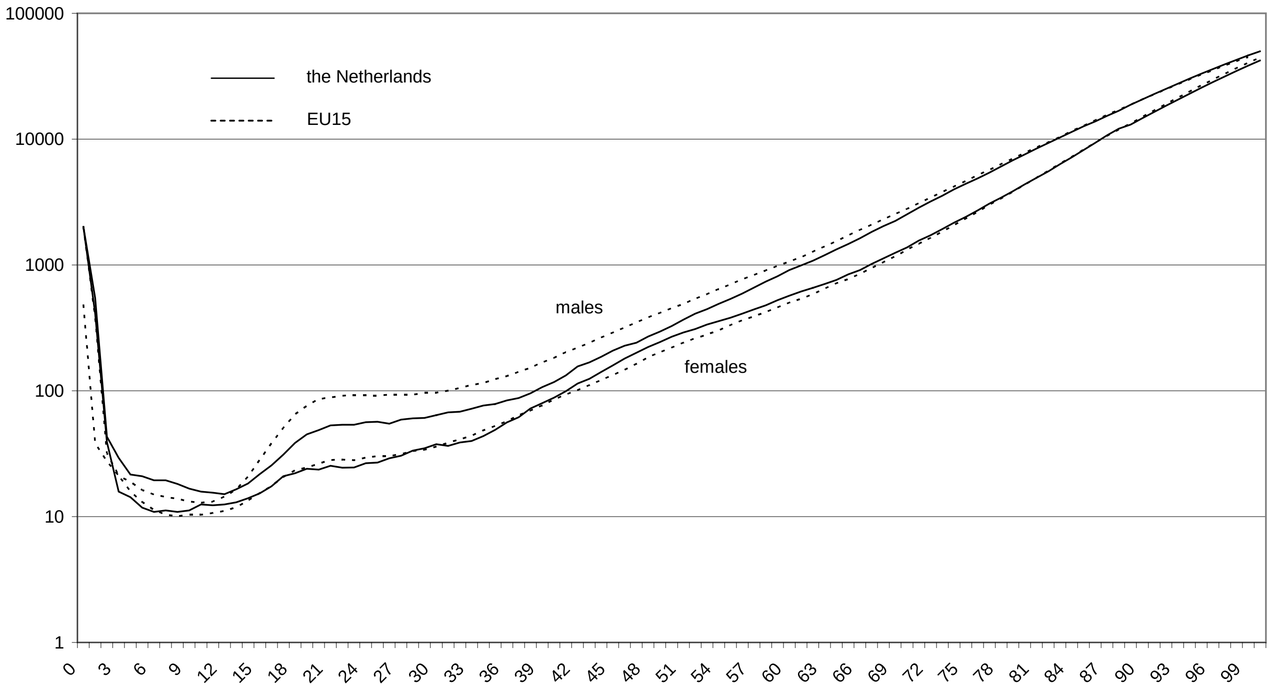


Figure 3 Age-specific mortality rates, the Netherlands and EU15 average, 2003



The solid lines in figures 4a and 4b show the ratios of the age-specific mortality rates of Italian men and women compared with the EU15 average. The figures show that particularly the mortality rates for both Italian men and women in their 40s are clearly lower than the EU15 average and the differences at young and old ages are smaller.

The dotted line is a linear spline function. As discussed in section 2 the linear spline function is estimated on the basis of average values for successive age intervals. For each 10-year age group the average mortality rates for Italian men and women and for the EU15 averages for men and women are calculated. As the mortality rates for age 0 are considerably higher than for other ages, this age group is taken as separate age category. See table 1. For each age group the ratio of the Italian average mortality rate and the EU15 average is calculated. See table 2. The linear spline is estimated by setting the value for age 5 equal to the average of the quotients for the age group 1-10, the value for age 15 to the average of the quotients for age group 11-20, etc. For age 90 the value is set equal to the average of the quotients for the highest age group. These are the knots of the linear spline function. For the other ages the values of the linear spline can be calculated on the basis of the formulas given in section 2. This corresponds with linear interpolation between ages 5 and 15, 15 and 25, etc. The figure shows that these linear splines describe the age pattern of the relative risks rather well. One benefit of this approach is that the parameters of the spline (i.e. the values in table 2) can be interpreted easily: they simply indicate to what extent age-specific rates in different age groups are lower or higher than the EU15 average.

Figure 4a Relative mortality rates, (EU15 2003 =1), Italy, men, 2000

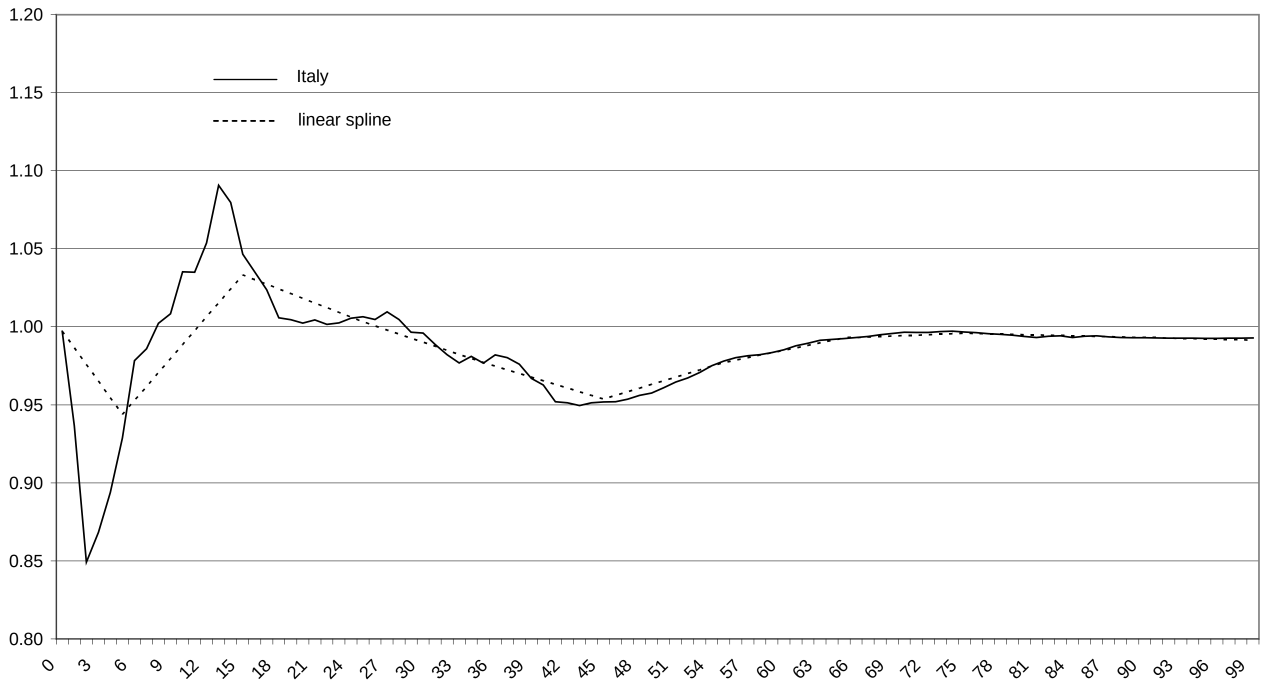


Figure 4b Relative mortality rates, (EU15 2003 =1), Italy, women, 2000

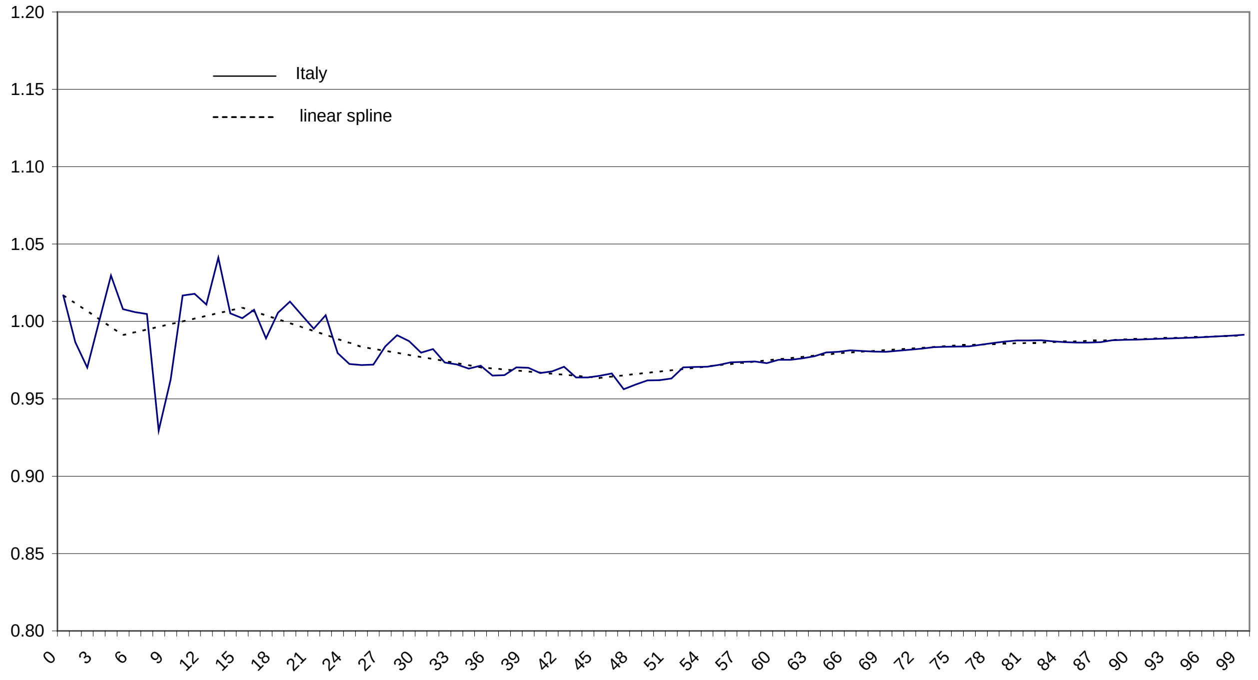


Table 1. Age-specific mortality rates (per 100,000), 2003						
	EU15		Italy*		Netherlands	
age	men	women	men	women	men	women
0	486	382	477	423	546	410
1-10	19	15	16	14	22	15
11-20	41	19	44	20	31	19
21-30	93	32	94	30	59	30
31-40	133	63	118	56	92	63
41-50	313	163	241	134	245	195
51-60	750	385	651	326	659	443
61-70	1869	953	1779	829	1819	1022
71-80	5111	2996	4922	2659	5424	3057
81+	22945	19968	21361	17936	25370	19268
* Figures for 2000						

Table 2. Relative mortality rates, 2003 (EU15 average = 1)					
	Italy*		Netherlands		
age	men	women	men	women	
0	0.98	1.11	1.12	1.07	
1-10	0.83	0.97	1.15	1.03	
11-20	1.08	1.02	0.77	0.98	
21-30	1.01	0.94	0.63	0.95	
31-40	0.89	0.88	0.69	1.00	
41-50	0.77	0.83	0.78	1.20	
51-60	0.87	0.85	0.88	1.15	
61-70	0.95	0.87	0.97	1.07	
71-80	0.96	0.89	1.06	1.02	
81+	0.93	0.90	1.11	0.96	
* Figures for 2000					

Figures 5a and 5b show the rate ratios and linear splines for Dutch men and women. The relative risks for Dutch men in their 20s and 30s are low, whereas mortality rates at older ages are higher than the EU15 average. For Dutch women the mortality rates for most ages are higher than the EU15 average.

Figure 5a Relative mortality rates (EU 15=1), the Netherlands, men, 2003

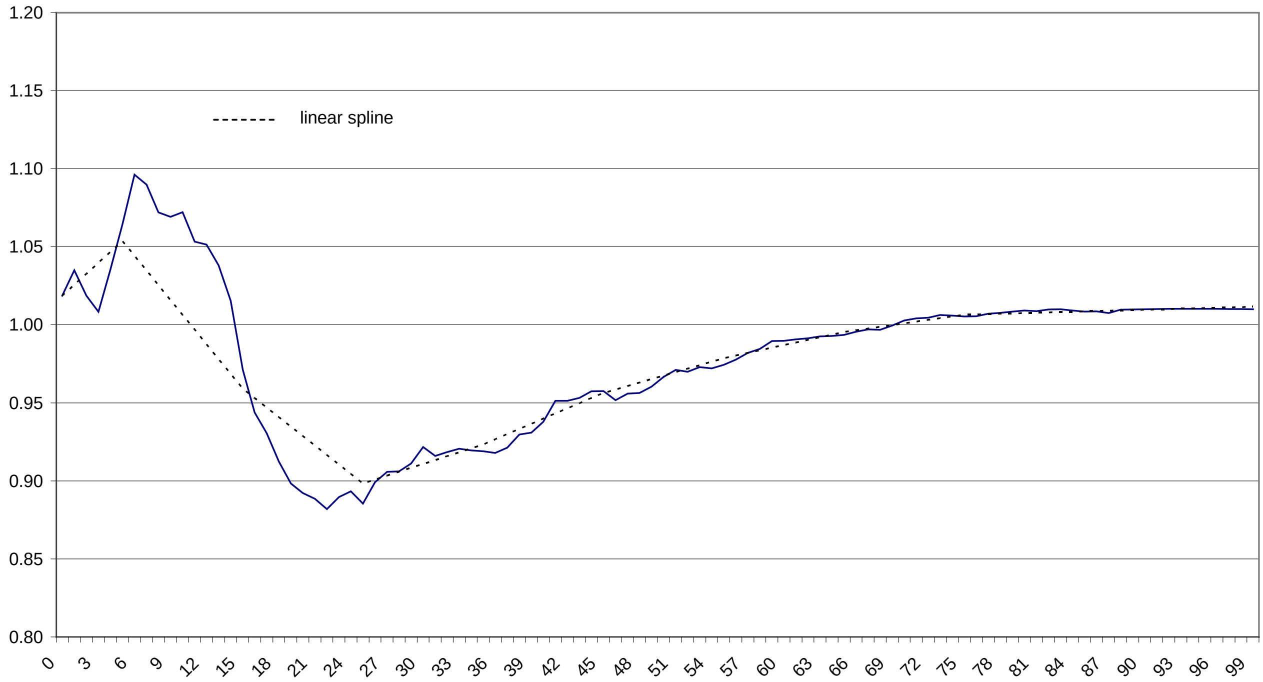
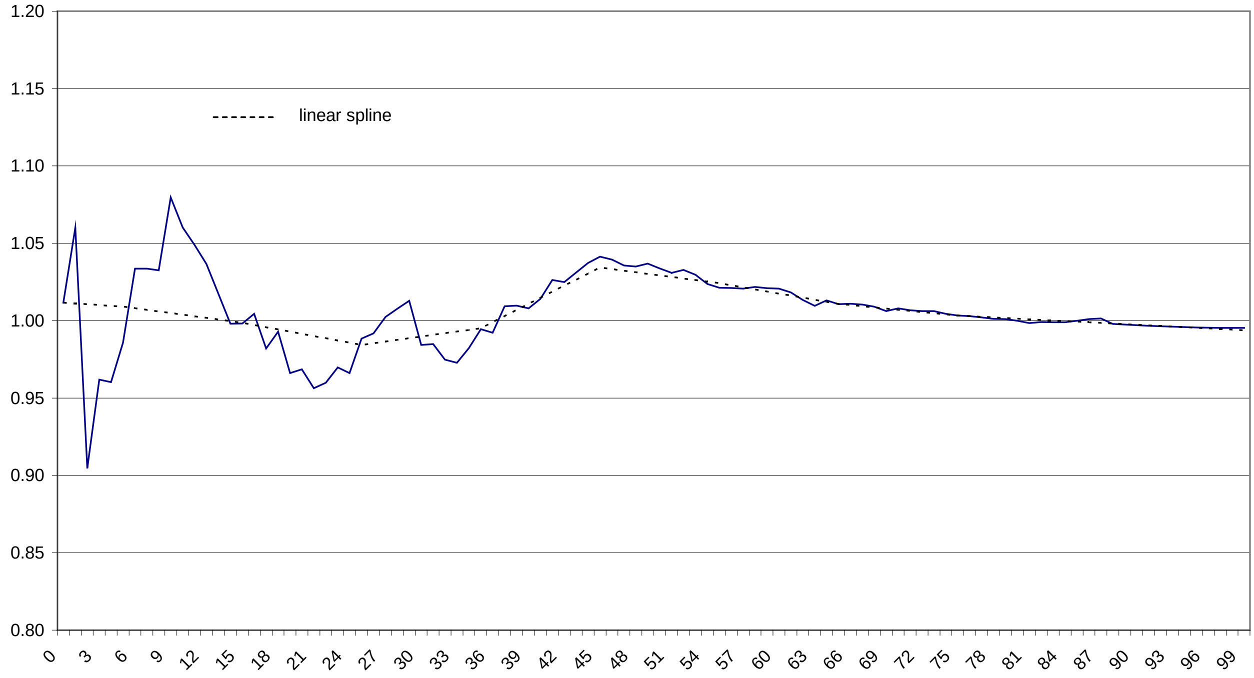


Figure 5b Relative mortality rates (EU 15=1), the Netherlands, women, 2003



Multiplying the values of the rate ratios estimated by the linear splines by the EU15 age-specific mortality rates provides the fitted curves of age-specific mortality rates. Figure 6 shows that the fit for both Italian men and women is good. Figure 7 shows the fit for Dutch men and women which is satisfactory as well. Thus based on the same standard age schedule, *viz.* the EU15 average, age-specific mortality rates for both Italy and the Netherlands can be modeled rather accurately.

Figure 6 Age-specific mortality rates, the Netherlands, 2003

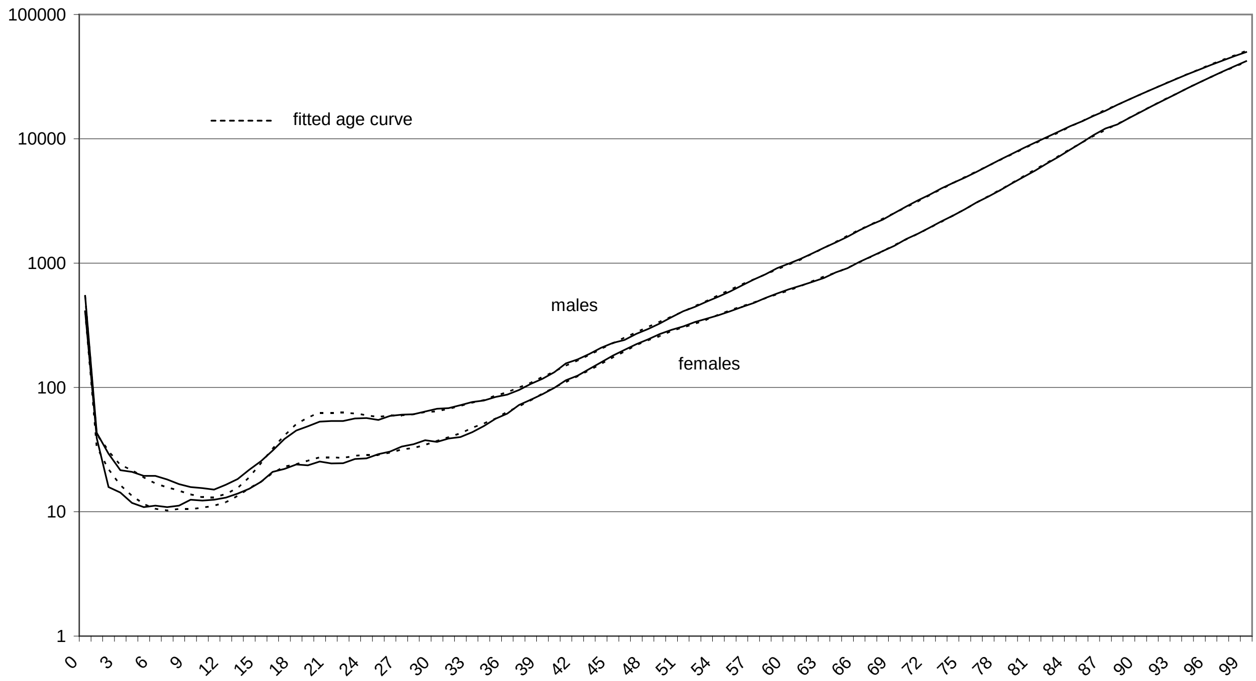
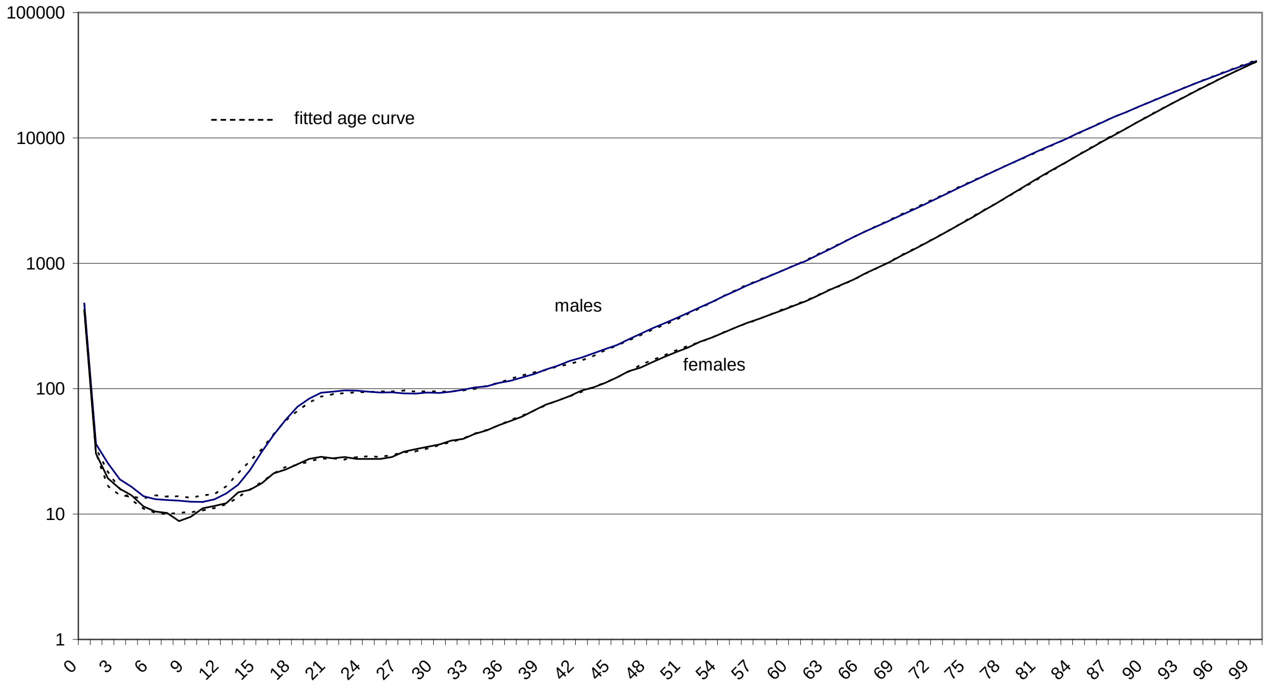


Figure 7 Age-specific mortality rates, Italy, 2000



4. Modeling age patterns of fertility rates

In order to illustrate the flexibility of the method we fit age patterns of fertility rates which have quite a different shape than mortality age curves. Figure 8 shows age-specific fertility rates for Italy and the Netherlands as well as the (unweighted) EU15 average. The overall pattern of the curves is similar, but there are clear differences as well. At most ages fertility rates in the Netherlands are higher than in Italy. Moreover, the age curve of the Netherlands is more peaked than the EU15 average. Furthermore the peak of the curve in both the Netherlands and Italy is at a slightly higher age than the EU15 average.

Figure 8 Age-specific fertility rates, Italy, Netherlands and EU15 averages, 2005

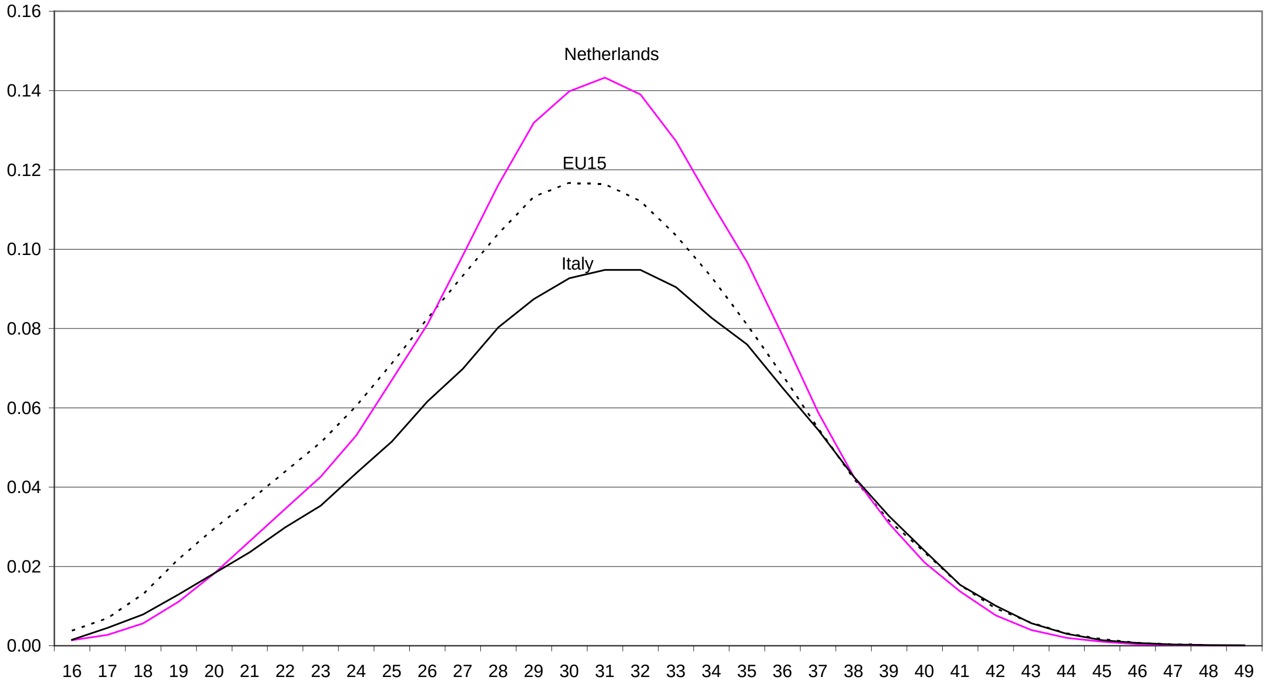
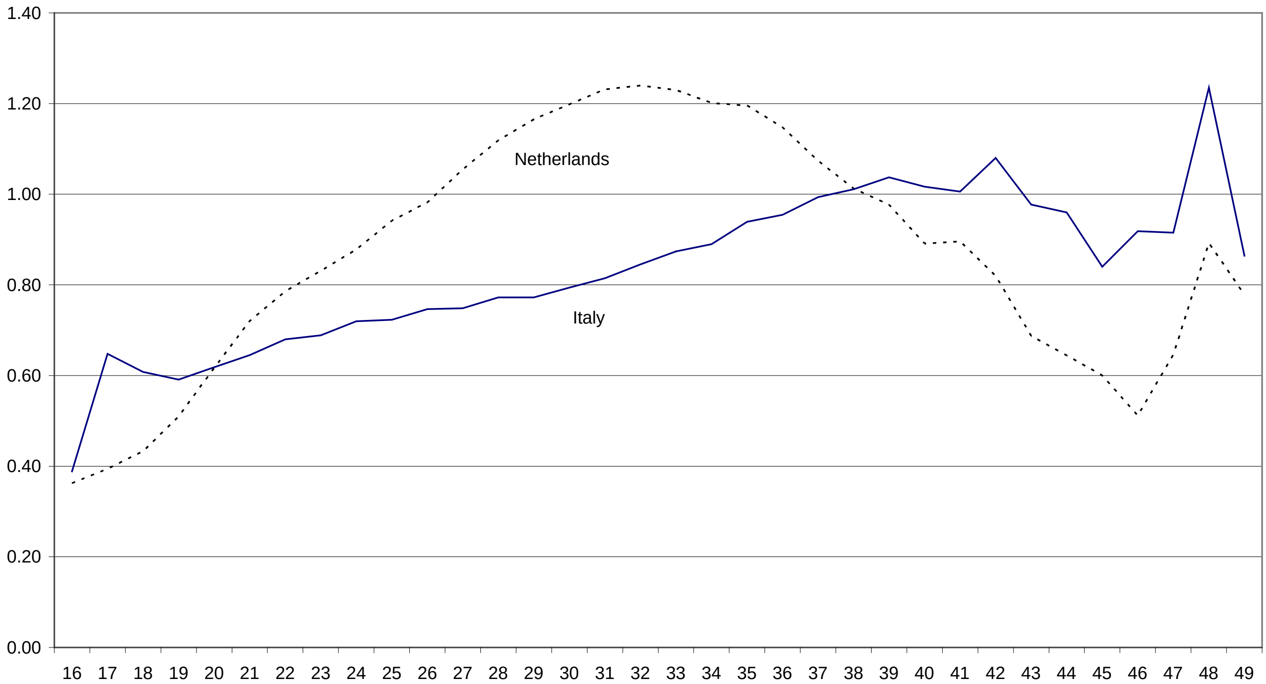


Figure 9 shows the ratios of the age-specific fertility rates of Italy and the Netherlands compared with the EU15 average. The figure shows that between ages 27 and 38 the fertility rates in the Netherlands are higher than the EU15 average, whereas for younger and older ages the opposite applies. For Italy for ages younger than 38 years fertility rates are lower than the EU15 average, whereas for the oldest ages fertility rates for Italy are on average equal to the EU15 average.

Figure 9 Relative fertility rates (EU15=1), Italy and Netherlands averages, 2005



For fitting a linear spline function to the relative risks shown in figure 9 relative fertility ratios for six age groups are calculated. See table 3. On the basis of these ratios fertility age curves for Italy and the Netherlands are fitted using the method described in section 2. Figures 10a and 10b show that the fitted curves describe the age-specific fertility rates in both the Netherlands and Italy rather accurately.

Table 3. Age-specific fertility rates, 2005

	per 1,000 women			Relative fertility rates (EU15 = 1)	
age of mother	EU15	Italy	Netherlands	Italy	Netherlands
16-20	15	9	8	0.60	0.52
21-25	53	37	45	0.70	0.85
26-30	102	78	113	0.77	1.11
31-35	101	88	124	0.87	1.22
36-40	44	44	46	0.99	1.05
41+	4	4	3	1.01	0.80

Figure 10a Age-specific fertility rates, the Netherlands, 2005

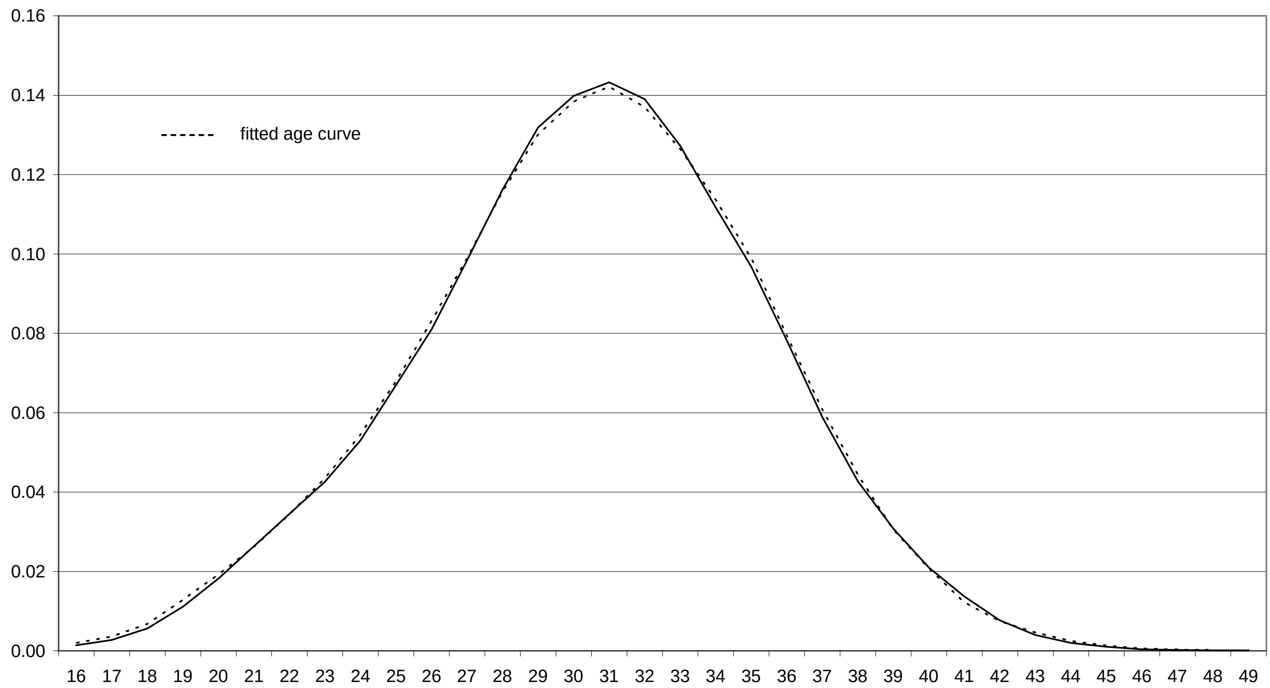
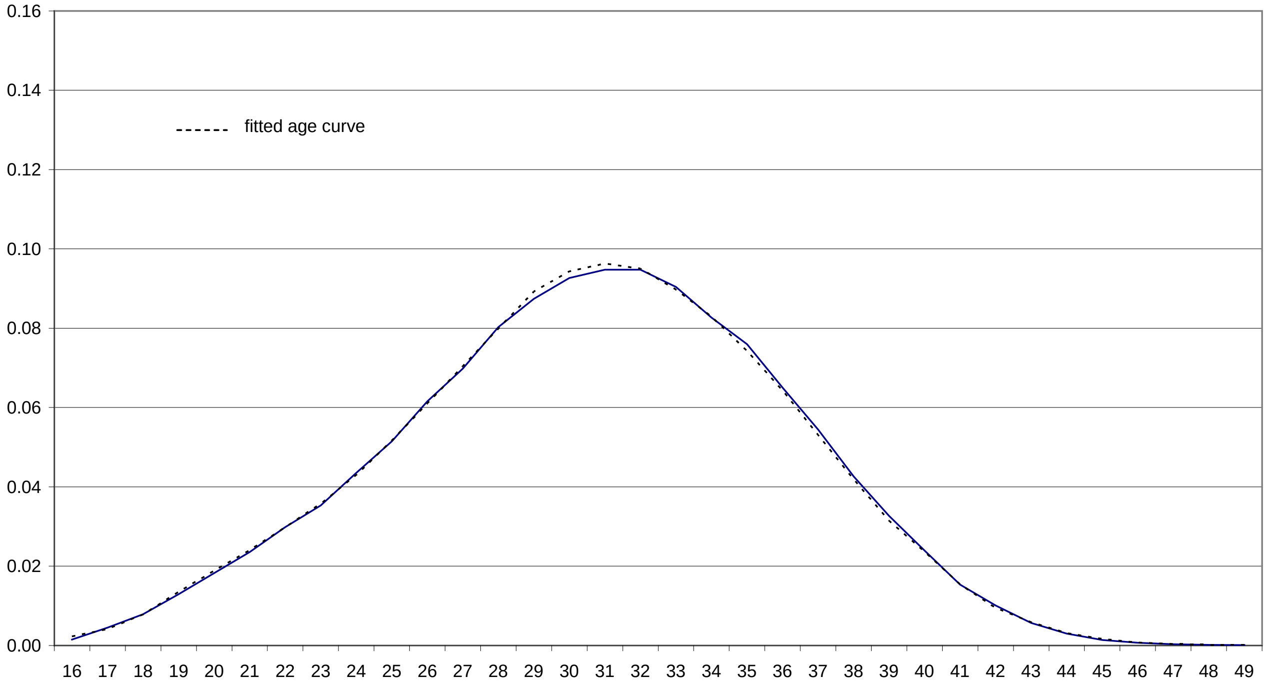


Figure 10b Age-specific fertility rates, Italy, 2005



5. Using TOPALS to project future age-specific mortality rates

Using the estimated relative risks for making projections on future values of age specific mortality rates three alternative procedures may be followed. First, one may make assumptions on future changes in the values of the relative risks for 10-year age groups and multiply these values by the age specific mortality rates according to the standard age schedule in the base year. Secondly, one may make assumptions about the future changes in the age-specific rates according to the standard age schedule and assumptions about the future values of the relative risks compared with the projected age schedule. In the second variant one may assume that the relative risks will remain constant. Alternatively one may assume that the relative risks will change. For example, one may assume that age-specific mortality rates of various countries will converge by assuming that the relative risks of each country will change in the direction of a value of 1. We will also discuss an alternative to the second variant in which we use the age profile of each individual country in the last observation year as standard age schedule rather than the EU15 average. In this variant the relative risks describe future changes in the age pattern of a country compared to the age pattern in the base year of that same country. The latter variant produces projections that are similar to the Lee-Carter method.

5.1 *Variant 1: changes in relative risks*

In our first variant of using TOPALS the projections of age-specific rates are based on the same standard age schedule that is used for fitting the age profile. Projections are based on assumptions on changes in the values of the relative risks. The values of the relative risks in year *t*+*T* can be projected by:

(5) $$\hat{r}_{i,x,t+T|t} = \hat{p}_{i,x,t+T|t}\hat{r}_{i,x,t}\ ,$$

where $\hat{r}_{i,x,t+T|t}$ is the projection of the value of $r_{i,x}$ in the year $t+T$ based on observations up to and including year t; $\hat{r}_{i,x,t}$ is the estimate of the relative risk at age x in year t based on the linear spline function (2); and $\hat{p}_{i,x,t+T|t}$ determines by how much $\hat{r}_{i,x}$ is assumed to change between t and $t+T$. The values of $\hat{p}_{i,x,t+T|t}$ are estimated by a linear spline function similar to (2). Thus assumptions on the values of $p_{i,x,t+T|t}$ need to be specified for the averages of age groups rather than for all separate ages and the linear spline is used to estimate the values for each separate age. The future values of the age-specific mortality rates can be projected by:

(6) $$\hat{q}_{i,x,t+T|t} = \hat{r}_{i,x,t+T|t}\, q^{*}_{x,t}\ .$$

By way of an example we formulate assumptions on changes in $r_{i,x}$ between 2003 and 2050 by specifying values of $p_{i,x}$ which correspond closely with changes in the age patterns of mortality rates assumed in the EUROPOP baseline scenario for 2050. For Italy EUROPOP assumes a stronger decline of mortality rates for women than for men, whereas for the Netherlands a smaller decline for women than for men is assumed. Particularly for elderly Dutch women only little reduction of mortality is assumed. EUROPOP assumes a strong decline of mortality rates for Dutch men in their 50s and 60s and a much smaller reduction for men aged 70 or over. For Italy a strong decline in mortality is projected for women between ages 40 and 80 and for men between 40 and 70. For Italian men in their 20s and 30s only a small reduction is assumed, whereas for both Italian men and women a relatively small reduction is assumed for the eldest ages. Overall, EUROPOP assumes that the relatively low mortality rates in Italy will become even lower, while the relatively high mortality rates for Dutch women are assumed to decline very little. In line with this scenario we specify assumptions on the values of $p_{i,x}$, i.e. the factor by which the relative risks in the base year are multiplied in order to project the relative risks for the year 2050 (see table 4). On the basis of these assumptions, values of the relative mortality rates (compared with the EU15 average for 2003) for 2050 are calculated (see table 4). These values are used to calculate linear splines which are multiplied by the age-specific EU15 mortality rates for 2003 in order to calculate age-specific mortality rates for 2050.

Table 4. Assumptions on changes in relative mortality rates between 2003 and 2050

	Italy*		Netherlands	
age	men	women	men	women
0	0.60	0.60	0.50	0.50
1-10	0.30	0.30	0.30	0.20
11-20	0.30	0.40	0.50	0.40
21-30	0.90	0.70	0.75	0.60
31-40	0.90	0.50	0.75	0.75
41-50	0.40	0.40	0.60	0.75
51-60	0.30	0.40	0.50	0.75
61-70	0.30	0.30	0.50	0.70
71-80	0.50	0.30	0.70	0.70
81+	0.60	0.60	0.75	0.80

* changes between 2000 and 2050

Figures 11 and 12 show the age-specific mortality rates for Italy and the Netherlands in 2050 which are projected on the basis of the assumptions specified in table 4 (solid line) as well as the age-specific mortality rates according to the EUROPOP baseline scenario (dotted line). The overall patterns are similar. Due to the fact that our approach is based on a smooth standard age schedule, the projected age patterns are smoother than those according to the EUROPOP scenario, which is based on separately projecting mortality rates for individual ages.

Figure 11 Age-specific mortality rates, Italy, 2050

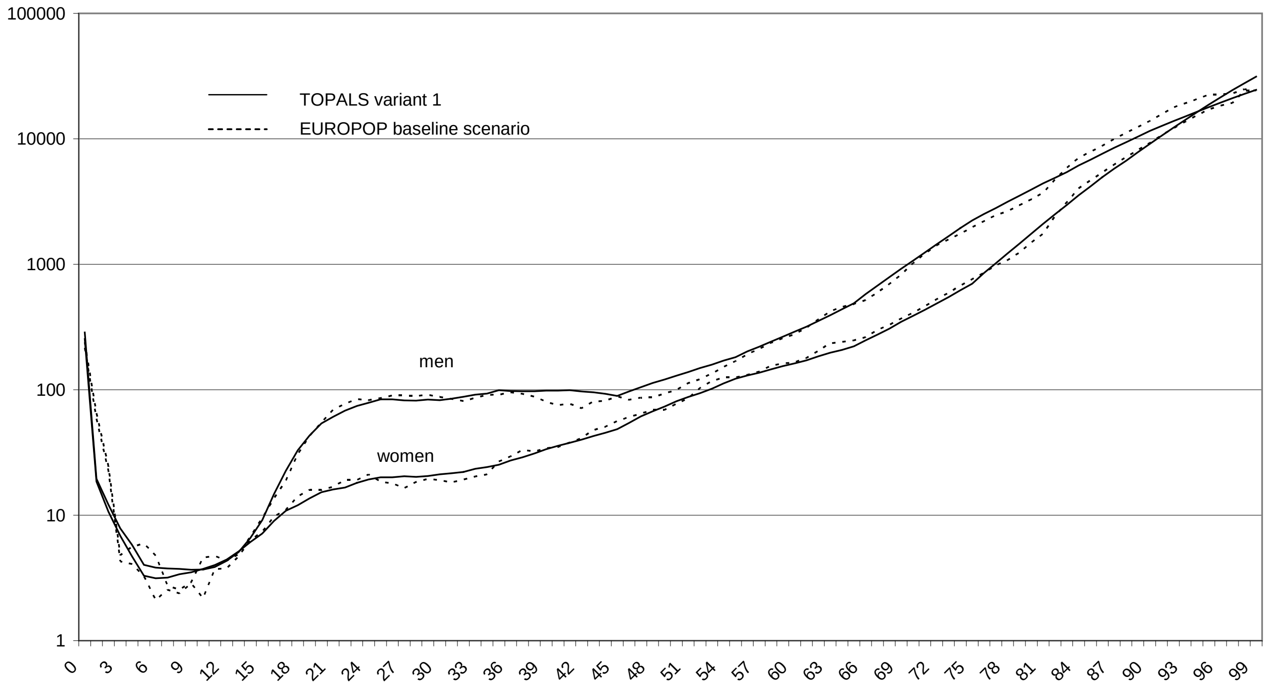


Figure 12 Age-specific mortality rates, the Netherlands, 2050

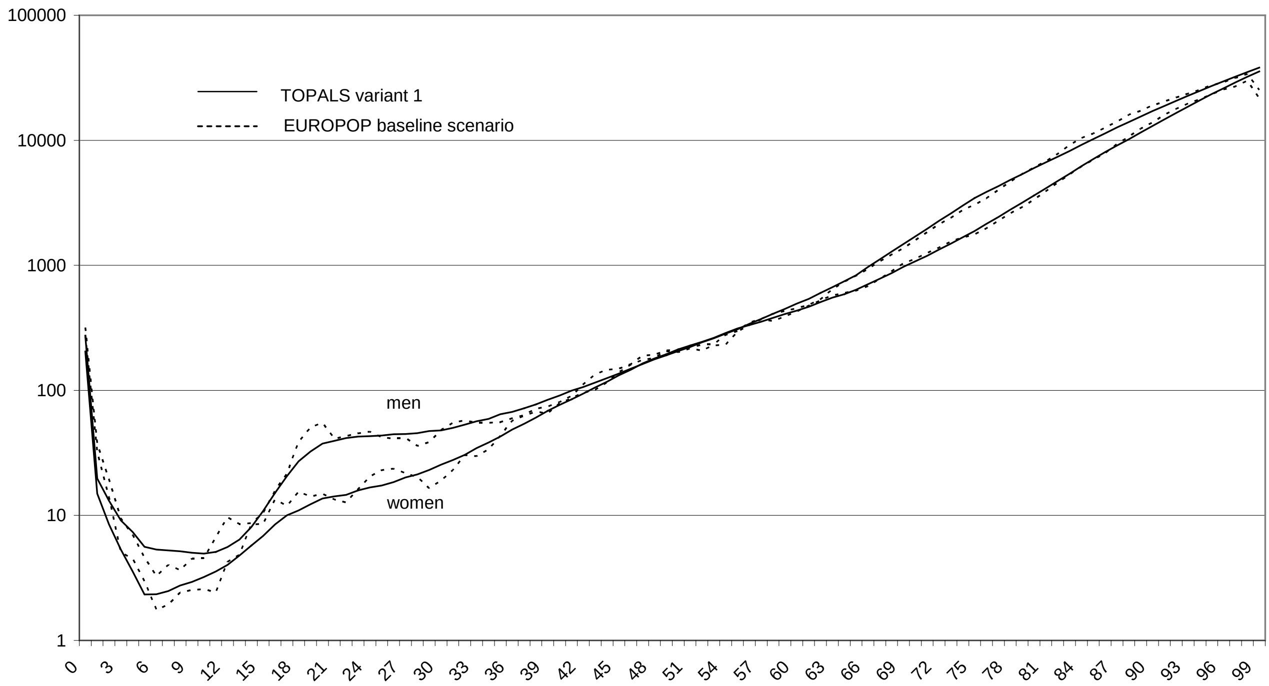


Table 5 shows the life expectancy at birth calculated from the projected age specific mortality rates. According to the EUROPOP baseline scenario life expectancy at birth in Italy will increase more strongly than in the Netherlands. As a consequence the differences between both countries will increase. Thus no convergence is assumed. This can be explained by the fact that past changes are projected into the future. During the last decades mortality rates have declined more strongly in Italy than in the Netherlands. As a consequence a projection of these trends into the future leads to a stronger projected increase in life expectancy in Italy than in the Netherlands.

Table 5. Life expectancy at birth						
	EU15		Italy		Netherlands	
	men	women	men	women	men	women
1985	71.1	77.7	71.8	78.2	72.6	79.2
2003*	75.2	80.9	76.1	82.0	75.7	80.4
2050						
EUROPOP			83.6	88.8	80.2	83.6
TOPALS variant 1			83.6	88.8	80.2	83.6
TOPALS variant 2	83.2	86.3	83.8	87.2	83.2	86.2
TOPALS variant 3			85.0	89.5	81.2	82.7
* Italy: figures for 2000						

5.2 Variant 2: changes in standard age schedule

Our second variant of using TOPALS for making scenarios consists of two steps. First we make assumptions on future changes in the standard age schedule. Secondly, we make assumptions about the future values of the relative risks compared with the projected standard age schedule. In line with the widely applied Lee-Carter method for projecting mortality (Lee and Carter, 1992) we may assume that the standard age schedule can be projected by using a random walk model with drift. This model projects a linear change. By applying the model to the logarithms of mortality rates the model projects a constant relative change.

The random walk model with drift is

(7) $$\ln q^*_{x,t} = \ln q^*_{x,t-1} + d_x + e_{x,t}\,.$$

where $q^*_{x,t}$ is the mortality rate according to the model age schedule at age x in year t, d_x is the 'drift' and $e_{x,t}$ is the error term with $E(e_{x,t}) = 0$ and $E(e_{x,t}\, e_{x,t+i}) = 0$ for $i \neq 0$. The value of d_x can be estimated by the average change in $\ln q^*_{x,t}$ over the last n years.

According to this model the standard age schedule for year $t+T$ can be projected by

(8) $$\ln q^*_{x,t+T} = \ln q^*_{x,t} + Td_x\,.$$

since $E(e_{x,t}) = 0$.

Separately projecting $q^*_{x,t+T}$ for each x will produce a rather irregular age pattern. Therefore linear splines of the age pattern of changes d_x are estimated, similarly to estimating the splines of the relative risks according to eq. (2):

(9) $$\hat{d}_x = a + b_0(x-m) + \sum_{i=1}^{n} b_i(x-m-k_i)D_i\,,$$

where a and b_i can be estimated similarly as in eq. (3) replacing r by d.

The standard age schedule for year $t+T$ can be projected by:

(10) $$\hat{q}^*_{x,t+T|t} = \exp(\ln q^*_{x,t} + T\hat{d}_x)\,,$$

where $\hat{q}^*_{x,t+T|t}$ is the projection of q^*_x for the year $t+T$ based on observations up to and including year t.

For country i age-specific mortality rates for the year $t+T$ can be projected on the basis of assumptions on the values of the relative risks $r_{i,x,t+T}$ compared with the projected standard age schedule:

(11) $$\hat{q}_{i,x,t+T|t} = \hat{r}_{i,x,t+T|t}\hat{q}^*_{x,t+T|t}\,,$$

where $\hat{r}_{i,x,t+T|t}$ is the estimate of the value of $r_{i,x,t+T}$ based on a linear spline function similar to (2). Thus assumptions on the values of $r_{i,x,t+T}$ need to be specified for age groups rather than for all separate ages (similarly to the assumptions specified in table 4). The linear spline is used to estimate the values for each separate age.

The relative risks in the year *t+T* may be assumed to be equal to those in the year *t*:

$$(12) \qquad \hat{r}_{i,x,t+T|t} = \hat{r}_{i,x,t} \, .$$

Alternatively one may assume that mortality rates of separate countries are converging (Li and Lee, 2005). This implies that the values of $r_{i,x,t+T}$ are closer to 1 than the values of $r_{i,x,t}$. This can be modeled by a partial adjustment model:

$$(13) \qquad \hat{r}_{i,x,t+j|t} = \hat{r}_{i,x,t+j-1|t} + \alpha_i (1 - \hat{r}_{i,x,t+j-1|t})$$

where $0 \leq \alpha_i \leq 1$ and $j = 1, 2, \ldots T$. Note that eq. (12) is a special case of eq. (13) assuming $\alpha_i = 0$.

This method is illustrated by projecting the average age-specific mortality rates for the EU15 countries. Eq. (10) is used for projecting the age-specific mortality rates of the EU15-countries for the year 2050. The values of d_x are estimated on the basis of changes in the age-specific mortality rates between 1985 and 2003. Figures 13a and 13b show the projected age-specific mortality rates for the EU15 average for men and women respectively and compare them with the 1985 and 2003 age patterns. Table 5 shows that according to these projections life expectancy at birth for the EU15 average would increase to 83.2 years for men and 86.3 years for women.

Figure 13a Age-specific mortality rates, men, EU15 average, 1985, 2003 and 2050

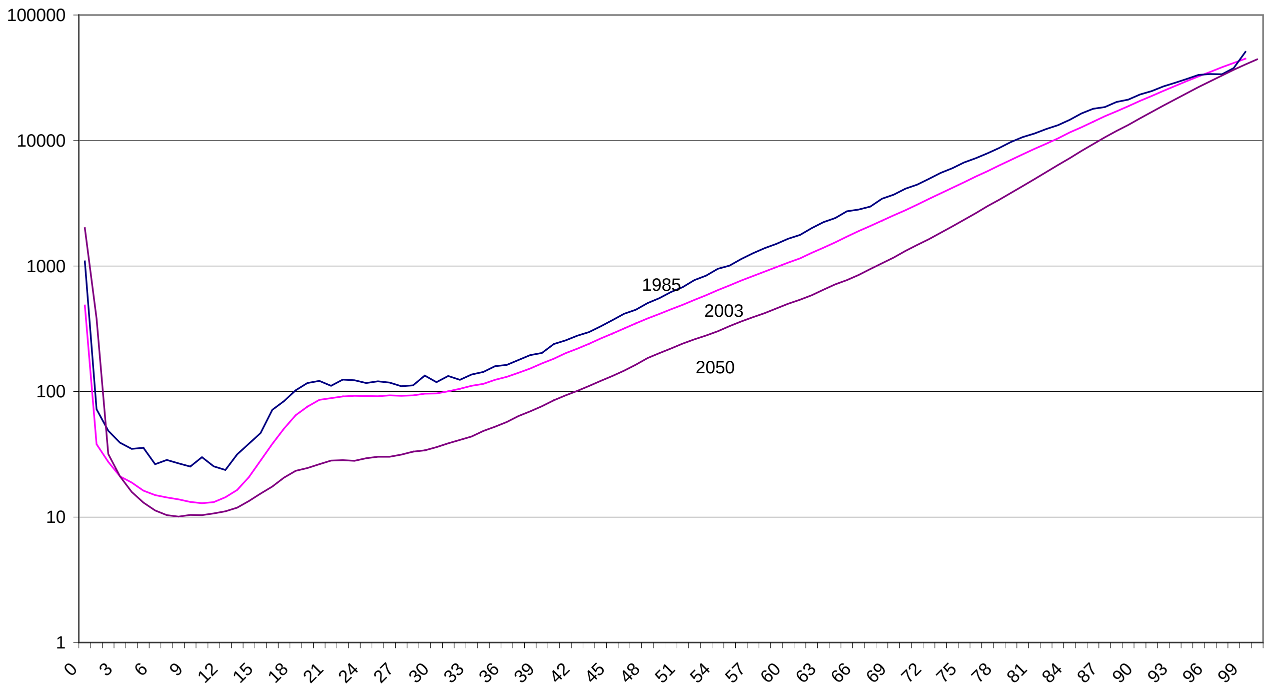
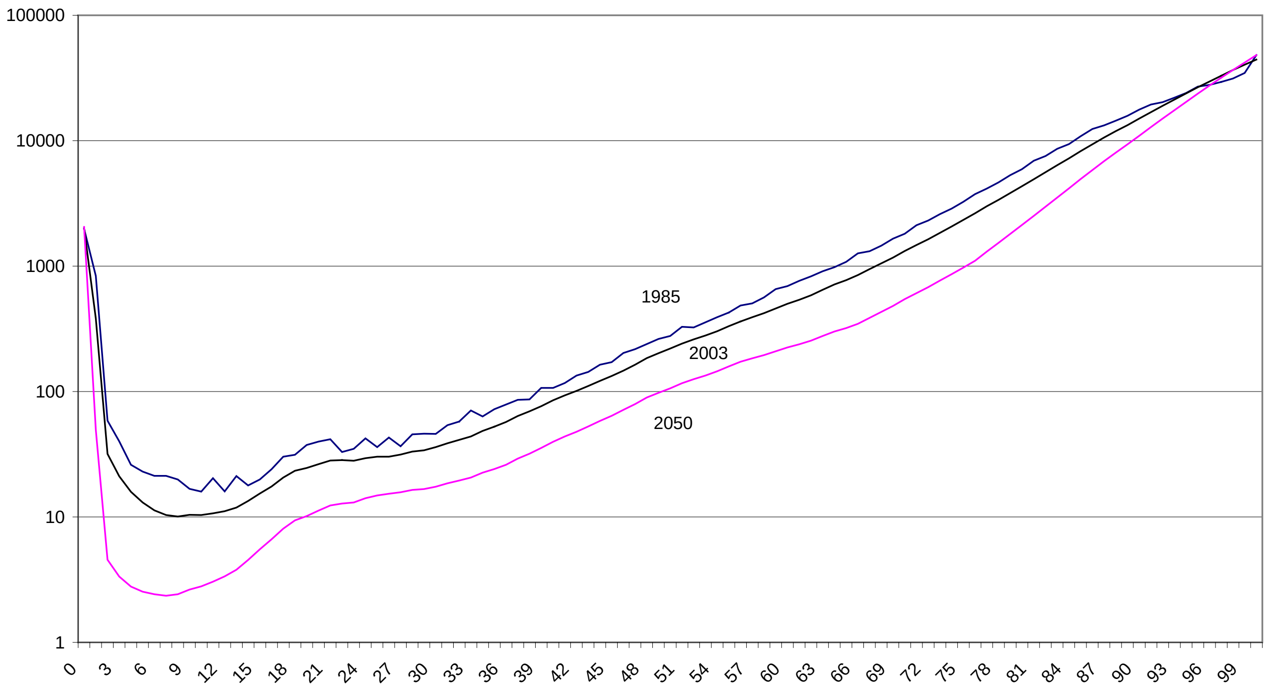


Figure 13b Age-specific mortality rates, women, EU15 average, 1985, 2003 and 2050



Based on these projections age-specific mortality rates for Italy and the Netherlands are projected assuming the relative risks in 2050 to be equal to those in 2003 (*i.e.* using eq. 12). Figures 14a and 14b show the projections for Dutch men and women respectively. The solid lines describe the projections calculated from eq. (11).

5.3 Variant 3: using age profile in base year as standard age schedule

We compare the projections of variant 2 with an alternative variant in which we do not use the EU15 average as standard age schedule but rather the age pattern of each individual country in the last observation year. We call this *Variant 3*. The projections of Variant 3 are based on equations (14) and (15) instead of equations (7) and (10):

(14) $$\ln q_{i,x,t} = \ln q_{i,x,t-1} + d_{i,x} + e_{i,x,t}$$

(15) $$\hat{q}_{i,x,t+T|t} = \exp(\ln q_{i,x,t} + T\hat{d}_{i,x}),$$

where $\hat{d}_{i,x}$ is the linear spline estimate of the drift for age x.

The projections according to Variant 3 are similar to the projections by the Lee-Carter method. The main difference is that Variant 3 projects a smooth age pattern, whereas the Lee-Carter method tends to produce an irregular age schedule.

The Lee-Carter model is

(16) $$\ln q_{i,x,t} = \alpha_{i,x} + \beta_{i,x}\mu_{i,t} + \varepsilon_{i,x,t},$$

where $\varepsilon_{i,x,t}$ is an error term with E($\varepsilon_{i,x,t}$) = 0 (Lee and Carter, 1992). This method describes age-specific death rates as the sum of an age-specific component that is independent of time and another component that is the product of a time-varying parameter reflecting the general level of mortality, and an age-specific component that represents how rapidly or slowly mortality at each age varies when the general level of mortality changes (Lee, 2000). Future values of $q_{i,x,t}$ can be projected by making projections of future values of $\mu_{i,t}$, as α and β do not vary with time and the expected value of ε equals zero. Projections of $\mu_{i,t}$ can be made by a random walk model with drift:

(17) $$\mu_{i,x,t} = \mu_{i,x,t-1} + c + \eta_{i,x,t},$$

where $\eta_{i,x,t}$ is an error term with E($\eta_{i,x,t}$) = 0 (Lee and Carter, 1992).

Thus the projection of μ for the year t+T can be calculated by:

(18) $$\hat{\mu}_{i,x,t+T|t} = \hat{\mu}_{i,x,t+T-1|t} + c\,.$$

Projections of $q_{i,x}$ for the year t+T can be calculated from (16):

(19) $$\ln \hat{q}_{i,x,t+T|t} = \alpha_{i,x} + \beta_{i,x}\hat{\mu}_{i,t+T|t}\,.$$

Subtracting (16) from (19) gives

(20) $$\ln \hat{q}_{i,x,t+T|t} - \ln q_{i,x,t} = \beta_{i,x}(\hat{\mu}_{i,t+T|t} - \mu_{i,t})\,.$$

From (18) it can be derived that

(21) $$\hat{\mu}_{i,x,t+T|t} = \mu_{i,x,t} + cT\,.$$

Substituting (21) in (20) yields

(22) $$\ln \hat{q}_{i,x,t+T|t} = \ln q_{i,x,t} + \beta_{i,x}cT\,.$$

Thus $q_{i,x,t+T}$ can be projected by:

(23) $$\hat{q}_{i,x,t+T|t} = \exp(\ln q_{i,x,t} + \beta_{i,x}cT)\,.$$

If it is assumed that $\beta_{i,x}c = \hat{d}_{i,x}$, the Lee-Carter projections are exactly equal to the projections by Variant 3 according to eq. (15). Usually the values of $\beta_{i,x}$ are estimated by singular value decomposition, assuming $\Sigma_x \beta_{i,x} = 1$ and $\Sigma_t \mu_{i,t} = 0$. As for each age x a separate value of β is estimated, the Lee-Carter method tends to produce a rather irregular age pattern of death rates. By using a linear spline, Variant 3 produces a smooth age curve.

Figure 14b shows that the mortality rates of Dutch women projected by Variant 2 are lower than the rates projected by Variant 3. The explanation is that during the last decades mortality rates for Dutch women have declined more slowly than the EU15 average, thus the projection of changes observed in the Netherlands (Variant 3) results in a smaller decline than the projection of changes in the EU15 average (Variant 2). Figure 14a shows that for Dutch men the projections at old ages differ between both variants since mortality rates of elderly men in the Netherlands have declined more slowly than the EU15 average.

Figure 14b Age-specific mortality rates, the Netherlands, men, 2050

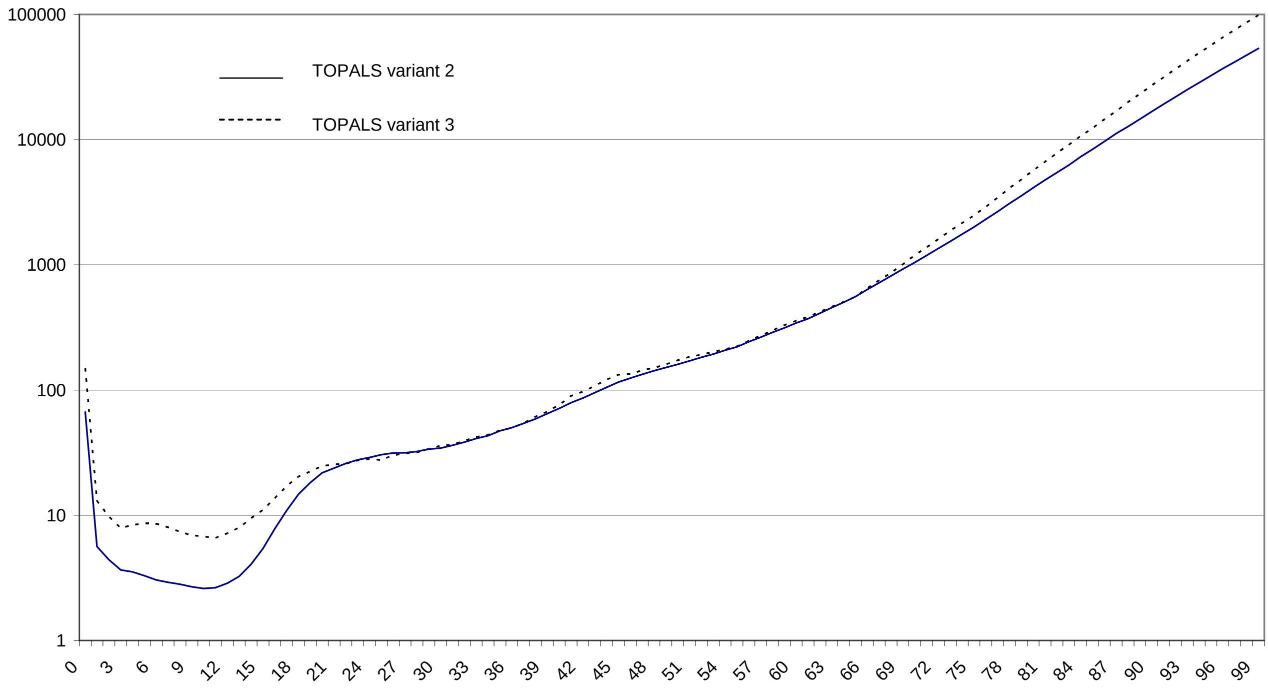


Figure 14b Age-specific mortality rates, the Netherlands, women, 2050

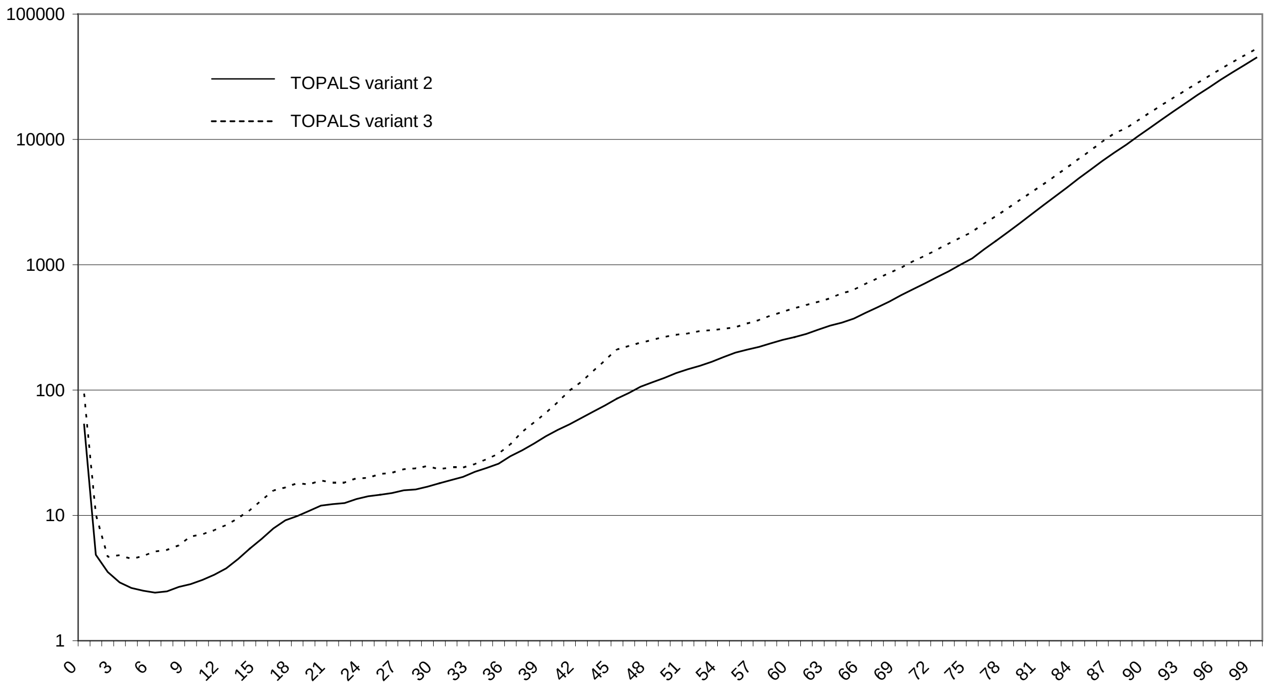


Table 5 shows that using Variant 2 life expectancy at birth for Dutch men and women in 2050 will be almost equal to the EU15 average. According to Variant 3 life expectancy would be considerably lower. Note that in using Variant 2 the relative mortality risks of the Netherlands compared to the EU15 average in 2050 are assumed to be equal to those in 2003. Thus even though no convergence of the age-specific mortality rates is assumed, there is a converging tendency of life expectancy at birth of Dutch men and women to the EU15 average. The convergence of life expectancy can be explained by the fact that if mortality rates are lower and relative risks stay equal, the relative risks will have a smaller effect on the level of life expectancy at birth. This can be illustrated as follows. The EU15 average life expectancy of men in 2003 equals 75.2 years (see table 5). If in country *A* all age-specific mortality rates would be 20 percent lower than the EU15 average, life expectancy at birth would be 77.7 years, thus 2.5 years higher than the EU15 average. The projected life expectancy of the EU15 average in 2050 equals 83.2 years (table 5). If in country *A* all age-specific mortality rates would still be 20 percent lower, life expectancy in country *A* would equal 85.1 years, 1.9 years lower than the EU15 average. Thus the difference in life expectancy between country *A* and the EU15 average in 2050 would be smaller than in 2003, even though the relative differences of the age-specific mortality rates between country *A* and the EU15 average in 2003 and 2050 would the same. This mechanism is comparable to the fact that if age-specific mortality rates decline by a constant rate, the increase in life expectancy declines.

Figures 15a and 15b show the projections of mortality rates for Italian men and women according to Variant 2. For young ages the projections of Variant 2 are lower than if Italian mortality rates are extrapolated (i.e. Variant 3) whereas the opposite applied to older ages. Table 5 shows that following Variant 2 life expectancy at birth in Italy in 2050 will differ less from that in the Netherlands than according to the EUROPOP baseline scenario. Note that the converging tendency of life expectancy also applies to the difference between men and women. The main cause of the projected narrowing gap between life expectancy of men and women is that during the last decades mortality rates of men have declined more strongly than those of women. This trend is projected into the future. Moreover, as mortality rates will continue to decline, differences in life expectancy at birth will decline because of the same mechanism that causes convergence between countries as described above.

Figure 15a Age-specific mortality rates, Italy, men, 2050

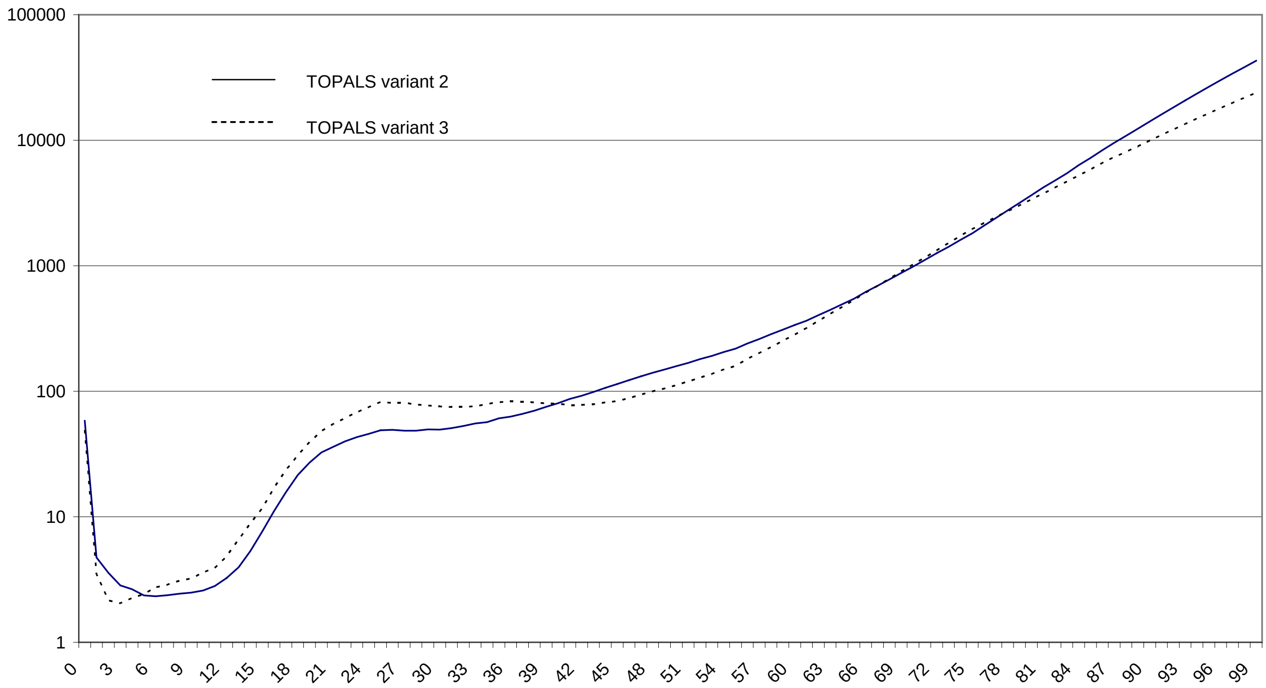


Figure 15b Age-specific mortality rates, Italy, women, 2050

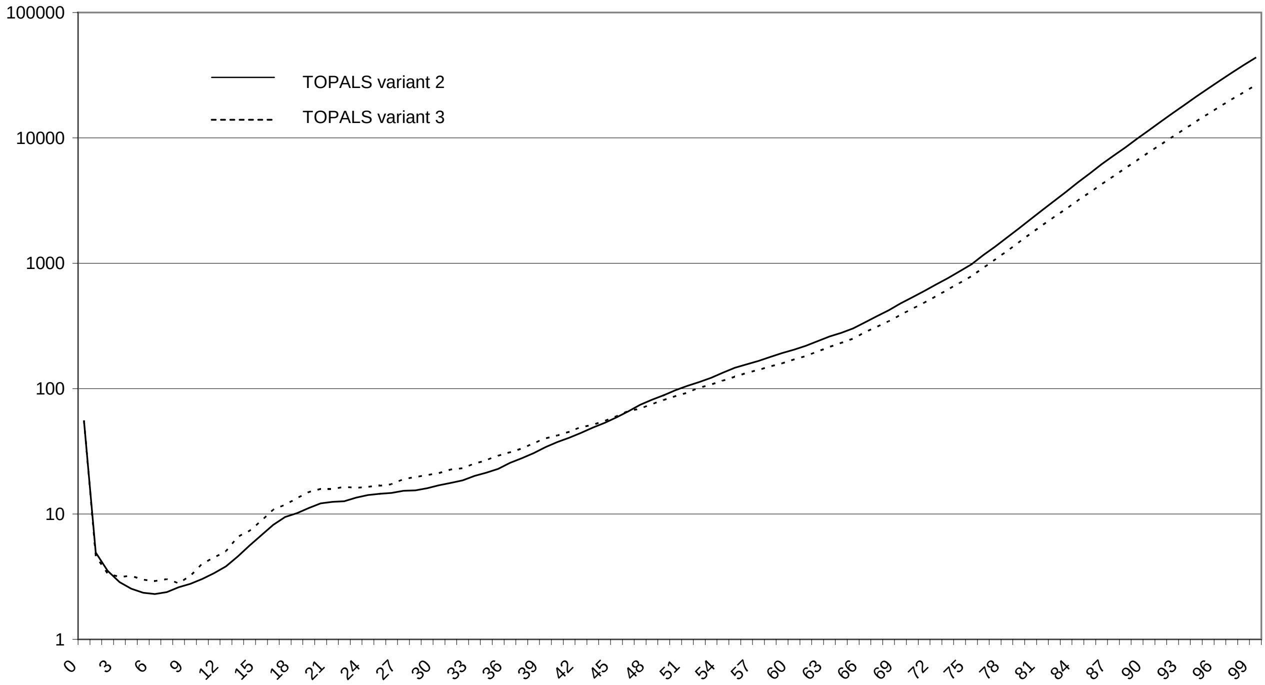


Table 6 shows the life expectancies at birth for all EU15 countries in 2050 projected by Variants 2 and 3 and compares these with the EUROPOP baseline scenario. The table shows clearly that the projections produced by Variant 2 for the separate countries are more closer to each other than those produced by Variant 3 and than the EUROPOP scenarios. The standard deviation of the life expectancies at birth in 2050 according to Variant 2 equals 0.6 for men and 0.7 for women. In 2003 the standard deviation was 1.0 for men and 1.1 for women. Whereas the projections by Variant 2 for 2050 have a smaller standard deviation than that observed in 2003, the opposite is true for Variant 3. According to Variant 2 there is a difference of 2.3 years between the countries with the lowest and highest life expectancy at birth for men in 2050. In 2003 this was 3.9 years. For women Variant 2 projects that the difference between the minimum and maximum life expectancies decreases from 3.8 years to 2.5 years. In contrast Variant 3 projects that the difference between the minimum and maximum values will increase to 6.3 years for men and 8.0 years for women.

Table 6. Projections of life expectancy at birth in 2050

	men				women		
	TOPALS		EUROPOP		TOPALS		EUROPOP
	Variant 2	Variant 3			Variant 2	Variant 3	
Austria	83.3	87.2	83.6		86.6	88.4	87.7
Belgium	82.7	82.0	82.3		86.3	85.9	88.3
Denmark	82.7	80.9	80.9		86.3	84.3	83.7
Finland	82.6	84.4	81.9		86.2	86.5	86.5
France	83.7	85.9	82.7		87.8	90.7	89.1
Germany	83.3	83.3	82.0		86.5	87.5	86.9
Greece	83.7	81.5	80.3		85.3	83.8	85.1
Ireland	83.0	84.6	82.4		86.2	86.2	87.0
Italy	83.8	85.0	83.6		87.2	89.5	88.8
Luxembourg	82.5	82.0	81.6		86.3	84.4	86.6
Netherlands	83.2	81.2	80.2		86.2	82.7	83.6
Portugal	82.0	81.1	80.4		85.6	86.0	86.6
Spain	83.6	82.0	81.4		87.2	88.2	87.9
Sweden	84.3	84.5	83.3		87.3	87.1	86.5
United Kingdom	83.3	83.8	82.9		86.4	86.2	86.6
Average	83.2	83.3	82.0		86.5	86.5	86.7
Minimum	82.0	80.9	80.2		85.3	82.7	83.6
Maximum	84.3	87.2	83.6		87.8	90.7	89.1
Standard deviation	0.6	1.9	1.2		0.7	2.2	1.6

6. The effects of covariates

The level of mortality rates may vary between population categories. For example, people with a high level of educational attainment tend to have considerably lower mortality rates than people with low educational levels. These differences by level of educational attainment may differ by age. For example, at middle ages these differences may be smaller than at older ages. This may be explained by a selection effect. If low-educated people with bad health have relatively high mortality risks at middle ages, relatively few low-educated people with bad health survive to older ages and consequently relatively many low-educated people at older ages have a good health.

TOPALS can simply be extended to take into account differences between population categories. The mortality rate for category y of variable Y is modeled by

(24) $$\hat{q}_{i,x,y} = \hat{c}_{i,x,y}\hat{r}_{i,x}q^{*}_{x},$$

where $\hat{c}_{i,x,y}$ is an estimate of $c_{i,x,y}$ based on a linear spline function similar to (2) and $c_{i,x,y}$ indicates to what extent the mortality rate for category *y* differs from the average level. If two covariates are included, the mortality rate for category *y* of variable *Y* and category *z* of variable *Z* is estimated by:

(25) $$\hat{q}_{i,x,y,z} = \hat{c}_{i,x,y,z}\hat{r}_{i,x}q^*_x.$$

If the effects of the two covariates on mortality are assumed to be independent, the mortality rate can be estimated by

(26) $$\hat{q}_{i,x,y,z} = \hat{c}_{i,x,y}\hat{c}_{i,x,z}\hat{r}_{i,x}q^*_x.$$

Assuming independence of the effects of variables *Y* and *Z* on mortality, assumptions on the future level of mortality of persons in category *y* of variable *Y* and category *z* of variable *Z* can be made by

(27) $$\hat{q}_{i,x,y,z,t+T|t} = \hat{c}_{i,x,y,t+T|t}\hat{c}_{i,x,z,t+T|t}\hat{q}_{i,x,t+T|t},$$

where $\hat{q}_{i,x,t+T|t}$ is projected by eq. (6) or eq. (11), and $\hat{c}_{i,x,y,t+T|t}$ and $\hat{c}_{i,x,z,t+T|t}$ indicate to what extent future mortality rates for categories *y* and *z* are assumed to differ from the future average age pattern. If the effects of both covariates are assumed to be interdependent an interaction term can be added to eq. (27). If it is assumed that the differences in the level of mortality between population categories will diminish, the future values of *c* are expected to move to 1. This can be modelled similarly to eq. (13). A constraint in specifying the values of $\hat{c}_{i,x,y,t+T|t}$ and $\hat{c}_{i,x,z,t+T|t}$ is that the average of $\hat{q}_{i,x,y,z,t+T|t}$ over the categories of *y* for each *i* and *x* as well as over the categories of *z* should equal $\hat{q}_{i,x,t+T|t}$. Thus:

(28) $$\sum_y p_{i,x,y,t+T|t}\hat{c}_{i,x,y,t+T|t} = 1 \quad \text{and} \quad \sum_z p_{i,x,z,t+T|t}\hat{c}_{i,x,z,t+T|t} = 1$$

where $p_{i,x,y,t+T|t}$ is the projected fraction of persons in category *y* of the variable *Y* in year *t+T* for country *i* and age *x* and $p_{i,x,z,t+T|t}$ is the fraction of persons in category *z* of the variable *Z*.

The method is illustrated by modeling the effect of level of educational attainment on mortality rates for the Netherlands. Empirical analyses suggest that mortality differences between persons with high and low levels of educational attainment are particularly high at middle ages and decline for the oldest age groups. We assume that mortality rates of people with a low level of educational attainment at middle ages will be 50 percent higher than the average and that mortality rates of people with a medium level of educational attainment will equal the average level. See table 7. In addition we assume that 20 percent of the population aged 20 years or over has a low level of educational attainment and 50 percent has a medium level. Than eq (28) implies that the relative mortality rates of people with a high level of educational attainment at the middle ages will be 30 percent lower than the average. This can be calculated easily as follows:

(29) $$p_L c_L + p_M c_M + p_H c_H = 1,$$

where p_L is the proportion of the population with a low level of educational attainment and p_M and p_H are the proportions with medium and high levels respectively. Then if assumptions are specified on c_L and c_M, c_H can be calculated from (29) as follows:

(30) $$c_H = \frac{1 - p_L c_L - p_M c_M}{p_H},$$

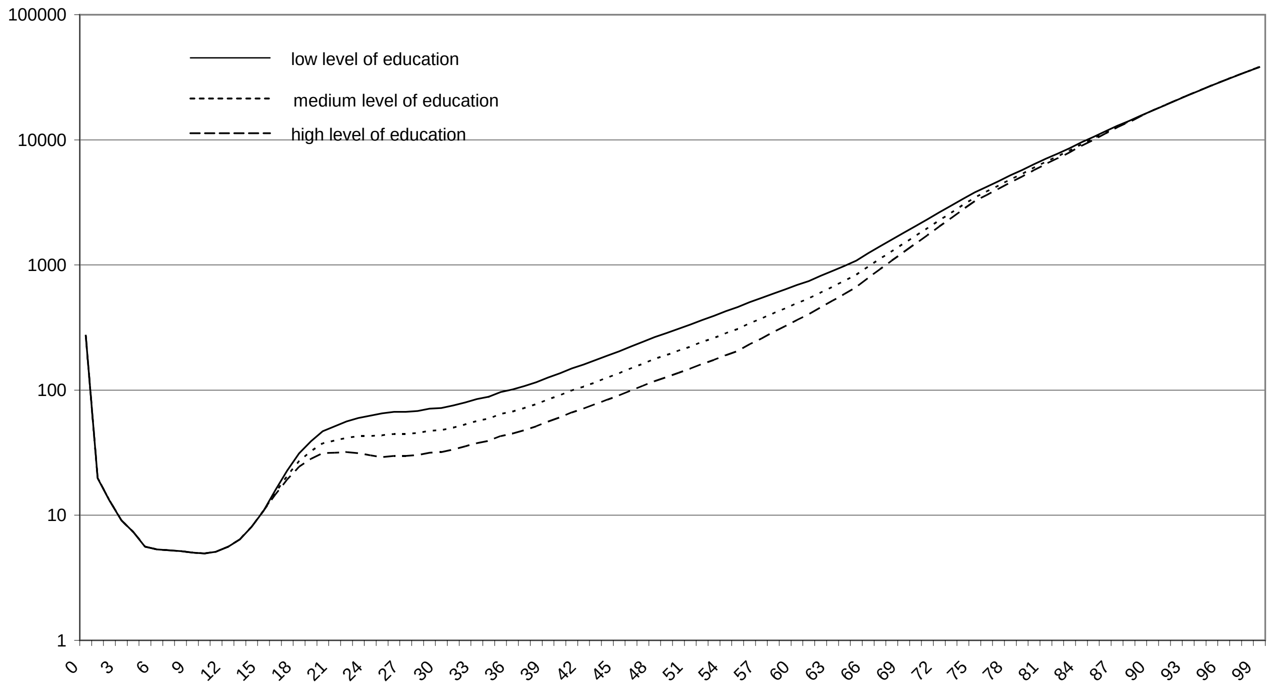
Instead of making assumptions on the values of c_y relative to the average level, one may make assumptions of c_y relative to one reference category. For example, if a medium level of educational attainment is used as reference category, c_M in eq. (30) equals one[4]. On the basis of these assumptions age-specified mortality rates for people with low, medium and high levels of educational attainment for Dutch men in 2050 are projected using eqs. (27) and (30). See figure 16. According to these age-specific mortality rates life expectancy at birth of a man with a low level of educational attainment would equal 78.6 years and that of a highly educated man would equal 81.4 years.

[4] In this example the value of the reference category equals the average value. This is not necessary. If the values c_y are regarded as relative risks compared with a reference category, the constraint (28) can be omitted. In that case the standard age schedule describes the age-specific rates for the reference category rather than for the average population.

Table 7. Assumptions about relative mortality rates by level of educational attainment

	distribution of population by educational level			relative mortality rates		
age	low	medium	high	low	medium	high
0	100%	0%	0%	1.0	1.0	1.0
1-10	100%	0%	0%	1.0	1.0	1.0
11-20	50%	50%	0%	1.0	1.0	1.0
21-30	20%	50%	30%	1.5	1.0	0.7
31-40	20%	50%	30%	1.5	1.0	0.7
41-50	20%	50%	30%	1.5	1.0	0.7
51-60	20%	50%	30%	1.5	1.0	0.7
61-70	20%	50%	30%	1.3	1.0	0.8
71-80	20%	50%	30%	1.1	1.0	0.9
81+	20%	50%	30%	1.0	1.0	1.0

Figure 16 Age-specific mortality rates, by level of educational attainment, men, Netherlands, 2050



7. Conclusions

This paper introduces a simple, but flexible method for making projections of age-specific rates: TOPALS (Tool for projecting age patterns using linear splines). The method consist of two steps. First, the age-specific rates are fitted to a standard age schedule using relative risks which are modeled by a linear spline function. Second, assumptions are made on the future values of the relative risks. Using a standard age schedule has the benefit that assumptions about a limited number of parameters need to be specified rather than about all individual age-specific rates. Using a linear spline function to model relative risks has the benefit that it is flexible, as it allows to describe all kinds of variations compared to the standard age schedule, while still producing a smooth age curve.

This paper illustrates TOPALS by fitting age-specific mortality and fertility rates for Italy and the Netherlands. The (unweighted) EU15 average is used as standard age schedule. Even though the age patterns of mortality and fertility are quite different, TOPALS is capable of fitting both age curves accurately.

In using TOPALS for making projections three approaches can be followed. These are illustrated by projecting age-specific mortality rates for the year 2050 for Italy and the Netherlands. In Variant 1 assumptions about changes in the relative risks are specified. By using linear splines it is sufficient to make assumptions about average changes for age groups rather than assumptions about changes in relative risks for each age separately. This variant is illustrated by making assumptions which correspond to the EUROPOP baseline scenario. In Variant 2 the standard age schedule is projected into the future. For this purpose a random walk model with drift is used. By using linear splines projections for age groups rather than for each individual age have to be made. One benefit is that this tends to produce a smoother age pattern. Subsequently assumptions need to be made about the future values of the relative risks. They may be assumed to remain constant, but one may also assume that they will change, for example that the differences between countries will decline. It should be noted that even if the relative risks are not assumed to change and thus the (relative) differences in age-specific mortality rates between countries will not change, this method will project declining differences in life expectancy. This is caused by the non-linear relationship between life expectancy and age-specific mortality rates. If the level of mortality rates is lower, the same relative difference in mortality rates will produce a smaller difference in life expectancy. In Variant 3 the age-specific rates of each country in the last observation year are used as standard age schedule. Assumptions on future values of relative risks indicate how the age pattern will change. This variant tends to produce more differences in projections of life expectancy between countries than Variant 2. The projections by Variant 3 resemble the projections by the well-known Lee-Carter method, the main difference being that Variant 3 tends to produce a smoother age pattern than the Lee-Carter method.

TOPALS can be extended by taking into account effects of covariates. On the basis of assumptions about differences in age-specific rates by categories of a covariate, different age patterns can be modeled. One benefit of using linear splines is that this allows to take into account that the effects of covariate may vary between age categories. Nevertheless the method produces a smooth age pattern.

8. References

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Joint EUROSTAT/UNECE Work Session on Demographic Projections
Bucarest, 10-12 October 2007

POPULATION FORECASTING VIA MICROSIMULATION: THE SOFTWARE DESIGN OF THE MICMAC-PROJECT[1]

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1. Introduction

This paper describes design considerations and the general layout of the microsimulation software of the project 'MicMac - Bridging the micro-macro gap in population forecasting', which is funded by the European Commission under the 6th Framework Programme.

In microsimulation life-courses of individuals are projected by randomly drawing their trajectories from a stochastic model, which portrays the propensity for individual transitions between relevant demographic states during life (Willekens, 2005). These simulated life-courses are collected in a virtual population and inference on future population development can be made by analysing this virtual population. If the underlying model realistically describes individual behaviour, then rich and detailed future population characteristics can be derived from the analysis of the aggregated simulated life-courses.

This procedure has several key ingredients: A stochastic model that is able to characterize individual behaviour over the life-course in settings that can be rather complex. Data sources, statistical models, and corresponding estimating procedures that allow to derive the empirical input for the microsimulation, that is, the estimated transition rates. And software that combines the input, allows to incorporate assumptions about future behavioural and institutional changes easily, performs the actual life-course simulations, and provides the simulation results in a format that will allow detailed further analysis.

The microsimulation software that is developed as part of the MicMac-project shall serve all these purposes. It will contain a pre-processor to facilitate the estimation of relevant transition rates from data. Then the so called Mic-core will perform the simulation according to the underlying multistate model. Finally, a so called postprocessor will provide tools for presentation of results.

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The following section will first summarize some general considerations for the software design. Then we will briefly describe the underlying multistate model, followed by a description of the Mic-core. A summary of the current state of development, which is still in progress, and an outlook on features still to be implemented will conclude the paper.

2. General considerations

When designing software, several decisions have to be made that will have far-reaching impact on its potential usefulness, on how easily it will be accessible, and consequently on how widely, or not, it can serve the purposes of its users. As the MicMac-project is to provide methodology that is not linked to a specific problem, but that can be used for applications in a very general multistate framework, this flexibility needs to be anticipated in the software design. The development of the MicMac-software is guided by the following principles:

- No additional expenses should be linked to the use of the Mic-software. That is, we will use free software with no license restrictions. Furthermore, the software should be available for as many current operating systems as possible. No particular commercial product will be needed to run the Mic-software.
- Transparency of the product is mandatory. Therefore we will use open-source software so that the users can follow our steps and are put in the position of easily making adequate changes according to their needs. Additionally, open-source software will allow to extend the current features of Mic to settings we have not (yet) anticipated.
- The software shall be computationally efficient to handle the potentially large number of individual life-courses with state-spaces of considerable size.
- The MicMac-project intends to provide tools for the users, without forcing them to the solutions we suggest. This implies that we will use well-defined interfaces between the Mic-core, which provides the actual simulation results, and the software that creates input rates as well as the software employed to analyse the created output.

As a consequence of these considerations we decided to use R as software tool for the pre-processor as well as the post-processor, which is available as Free Software under the terms of the Free Software Foundation's GNU General Public License (R Development Core Team, 2007). R provides an environment for statistical analyses and graphics, and it runs on UNIX platforms, Windows and MacOS. The combination of up-to-date statistical methodology and well-designed publication-quality plots make it an ideal candidate for the pre- and post- processor. Nevertheless, we are aware that R may not be familiar to many prospective users. Therefore, even though we encourage its use, we design our software so that users can continue with their preferred tools and will only have to provide the input rates in a specified format. The same applies to the simulated life-courses, which are simply stored in a database for further evaluation.

The Mic-core, which will be described in section 4, is the only part that will be closed to the user and it will be programmed in JAVA. The Mic-core will request no other interaction by the user than the provision of the transition rates in a well-defined format and a simulation frame, basically providing the starting population and the time horizon for the microsimulation. This is done so that the simulation is performed in a computationally efficient way with a little overhead as possible. If necessary, JAMES also allows distributed computation.

3. Model of the individual life-course

The MicMac microsimulation is based on a continuous-time multistate model (Willekens, 2006). Each individual occupies one of I potential states, which are collected in the state-space $S = \{1,\ldots,I\}$. This state-space is determined by the problem to be studied and will contain the relevant demographic states that need to be considered. In Mic we usually include death as one of the potential transitions, hence include 'dead' as one (absorbing) state in the model. Furthermore, there are two key time scales, which both are considered as continuous, namely the age x of an individual and calendar time t. (A third time-scale, the time since entry into the current state, is not yet considered in MicMac's multistate-model. An extension to this setting poses no particular problem to the simulation approach though.)

An individual enters the population in state $i_0 \in S$ at age x_0 and calendar time t_0. We allow individuals to enter either by birth, in which case $x_0 = 0$, or by immigration at age $x_0 \geq 0$. When and to which state the next transition will happen is determined by the age- and time-dependent transition rates $\lambda_{ij}(x,t)$. These rates define the instantaneous risks of an individual, who is currently in state *i*, aged *x*, at calendar time *t* to make a transition to state *j*. All states $j \neq i$ for which $\lambda_{ij}(x,t) > 0$ compete to be the state visited next. For the actual performance of the simulation these transition rates $\lambda_{ij}(x,t)$ will have to be provided by the user.

While the age-dependence will usually be estimated from appropriate surveys or population data, the dependence on calendar time will be determined by assumptions on how future development in these age-specific rates is expected to be. These can be derived from formal forecasts or expert judgement, and the Mic-core will not be restricted to any specific kind of projection model. In the simplest case current rates will be assumed to prevail in the future, hence removing the dependence on t, i.e. assuming that $\lambda_{ij}(x,t) = \lambda_{ij}(x)$.

To each rate $\lambda_{ij}(x,t)$ corresponds a random sojourn time S_{ij}, whose distribution is uniquely defined by the transition rate. Therefore random numbers from these sojourn-time distributions can be simulated. Which of the (usually several) competing states will be visited next is determined in the following way: Random sojourn times are simulated for all competing states, for which $\lambda_{ij}(x,t) > 0$. The 'winning' state is the state j^*, for which the actual random time $s_{ij^*} < s_{ij}$, for $j \neq j^*$. Once the next state is determined, we know that the individual entered this state j^* at age $x_0 + s_{ij^*}$ at calendar time $t_0 + s_{ij^*}$. The corresponding rates for a transition out of j^* can again be determined from the provided $\lambda_{ij}(x,t)$. The procedure is repeated until either the individual enters the absorbing state 'dead' or until the pre-defined time-frame of the simulation is reached.

The simulation of the random sojourn times is achieved by the relation that links the distribution function of a positive random variable to its hazard rate. This relation is

$$F(s) = P(S \leq s) = 1 - \exp\{-\Lambda(s)\},$$

where $\Lambda(s) = \int_0^s \lambda(u)\,du$ is the so called integrated or cumulative hazard (Kalbfleisch & Prentice, 2002). In this way we can simulate random numbers from the distribution of *S* by drawing random numbers from a standard uniform distribution $U[0,1]$ and inverting the distribution function $F(s)$, see Robert & Casella (1999). In short this leads to

$$s = \Lambda^{-1}(-\ln u),$$

where *u* is a random number from the standard uniform distribution. The inversion of the integrated hazard Λ^{-1} is possible if the corresponding rate λ is positive, i.e. the respective transition is possible. The actual inversion need not have a closed-form solution, but rather may have to be obtained numerically. It is, however, particularly simple if the integrated hazard is piecewise linear. In the Mic-software we use piecewise linear approximations to integrated hazard rates.

4. Simulating the virtual population: the Mic-core

The actual microsimulation of the MicMac-project is implemented in JAVA, based on *JAMES, a Java-based Agent Modelling Environment for Simulation*, developed by the Research Group of Modelling and Simulation at the Computer Science Department, University of Rostock (Himmelspach & Uhrmacher, 2007; Himmelspach et al., 2007). This environment allows flexible definitions of simulation environments, and the multistate model described in the previous section has been incorporated in the framework of *JAMES*.

The virtual population is combined from the individual life-courses that are generated during the run of the microsimulation. To perform such a run the following parameters have to be set:

- A starting time t_s , an ending time t_e ,[3] and
- a starting population of size N_s , where for each individual the age and the occupied state at time t_s must be given.

After the frame for the simulation has been defined, the simulation is started. This is done by first determining the next transition times and the next states visited for all individuals in the initial population. All individuals, whose next state is not 'dead', are then lined up in the so called event-queue according to their times for next transition. Figure 1 shows an example of such an event-queue.

The actual run of the experiment is performed by repeating the following steps, which are solely depending on the event queue:

- The first element of the event-queue, the so called 'head', is dequeued.
- The next event-time and the next state is determined for the head of the queue.
- If the new state for the head is 'dead', then the individual is not enqueued again.
- *Otherwise the individual is enqueued at the correct position in the event-queue.*
- *The previous new state becomes the current state, and the newly determined next state is stored.*
- The new head of the event-queue is picked and the procedure is continued until either the end-time t_e is reached or the event-queue is empty.

Figure 1 Example of an event-queue

Individual: 56	Individual: 257	Individual: 669	….
s.current: 5	s.current: 1	s.current: 1	….
trans.time: 12.6.2007	trans.time: 1.9.2007	trans.time: 4.12.2007	….
s.next: 6	s.next: 8	s.next: 2	….

To give an example, consider the situation depicted in Figure 1. In the first step, individual No. 56 would be picked and its next transition time would be determined. If the next transition would be assigned for, let's say, October 1, 2007, and if the scheduled transition would not be death, then individual No. 56 would be enqueued between individual 257 and 669. Individual No. 257 would be the new head of the event-queue and would be dealt with next. If, however, individual No. 56 would be schedule to have the next transition on, e.g., August 25, 2007 (and again the target state would not be absorbing), then it would be enqueued at position number one again, and would be dealt with in the next step. If the transition would have been death of the individual, it would disappear from the event-queue. All transitions of all individuals are stored in a database for further analysis.

5. Conclusion and Outlook

At present the Mic-core allows the simulation of life-courses of independent individuals according to rates that are age-dependent, but not time-dependent yet. The inclusion of transition rates that vary over calendar time is a simple extension of the current design and poses no technical challenge whatsoever. New individuals are added to the population when childbirth occurs to female individuals in the virtual population.

[3] The ending time of a simulation run alternatively can be defined implicitly by the last individual of the starting population entering the absorbing state 'dead'.

Another feature that is not yet implemented is the creation of 'linked lives', that is the formation of partnerships where the attributes, i.e. the state, of the partner changes the transition rates of an individual. Currently individuals are considered independently, and no marriage market is implemented. However, the design of a mate-matching algorithm, which assorts partners according to some key characteristics is already in progress.

In this context one must bear in mind though, that many of these extensions will also ask for the corresponding data so that these diverse rates and attributes can be estimated. Besides the technical aspects of the software implementation these data requirements, which can be quite extensive, pose one of the major challenges to realistic microsimulation approaches.

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Session 4:
Household projections

Chair: Vasile Ghetau

V

eurostat

Preliminary version

ON FUTURE HOUSEHOLD STRUCTURE[1]

15 August 2007

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Summary

We have computed a probabilistic household forecast for Norway to 2030. We have combined a probabilistic population forecast, which predicts numbers of men and women by age, with a probabilistic forecast for the shares of persons who have a particular household position, given age and sex. Point predictions for the shares were computed by means of a deterministic multistate household forecast model. Variances and covariances were estimated from observed forecast errors in an old household forecast. We have restricted ourselves to private households.

We find that prediction uncertainty for future numbers of married couples, cohabiting couples, and one-person households, as indicated by the coefficient of variation, is rather low. Lone parents and other private households show much larger prediction uncertainty.

Key words

Household forecast, probabilistic forecast, population forecast, random shares, Norway

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[1] This paper is based on the joint work of the international research group "Changing family patterns in Norway and other industrialized countries" at the Centre for Advanced study at the Norwegian Academy of Science and Letters in Oslo during the academic year 2006/2007.

1. Introduction

Knowledge about the future number of households, their composition, and changes therein is important for many purposes. Support policies for the elderly depend on numbers of elderly persons who live alone or with others (Grundy 2001; Glaser et al. 2003). Housing planners use estimates of household size and household growth in the future (Holmberg 1987, King 1999, Muller et al. 1999). A recent concern from environmental studies relates to the strong growth in the number of households: Even when the size of a population remains constant, more households imply a larger demand for resources. Household members share space, home furnishings, transportation and energy, leading to significant economies of scale. For instance, members of two-person households in the United States in 1993–94 used 17% less energy per person than one-person households did (O'Neill and Chen 2002).

The number and composition of households in the future is uncertain, but some developments are more probable than others. The standard way to deal with uncertainty is to formulate two or more alternative scenarios for household dynamics or other demographic developments in a household forecast, and trace their consequences for household size and structure (e.g. Jiang and O'Neill 2006). The drawback is that uncertainty is not quantified, and that the results are inconsistent from a statistical point of view (Lee 1999). Therefore, there is a need for probabilistic household forecasts, which give us prediction intervals for future numbers of households and for the household statuses of individual persons.

The purpose of this paper is to show how existing statistical methods can be combined for computing probabilistic household forecasts. To the best of our knowledge, there is only one earlier published example of such a forecast. Alders (1999, 2001) computed a probabilistic household forecast for the Netherlands until the year 2050. His deterministic multistate projection combines fertility, mortality, migration, marriage, and marriage dissolution with shares that distribute the population by age, sex, and marital status over six household positions. Parameters for expected values were obtained from observed time series. Next, uncertainty distributions and uncertainty parameters were assumed on intuitive grounds. Some of the parameters were disregarded, for instance correlations between the sexes, across ages, and across time. As opposed to Alders' approach, we estimate the uncertainty parameters from data on observed forecast errors for an old household forecast.

First, we discuss conceptual issues (Section 2). We argue that there are no generally accepted definitions for the concepts of household and family. We give our working definitions and discuss measurement problems. In Section 3, we briefly describe deterministic and probabilistic forecasts of households and population, and sketch our approach based on random shares. Sections 4 and 5 give two empirical applications of these ideas for the case of Norway. In the concluding section we argue that the method of random shares also can be applied to probabilistic forecasts in other fields, for instance regional forecasts and health forecasts.

2. Conceptual issues

2.1 Definitions

Studies that analyse numbers of households and families are confronted with problems of definition and of measurement. We start with the notion of "family". Different disciplines approach the definition problem differently. Economists usually ignore it (e.g. Rosenzweig and Stark 1997; Ermisch 2003), while demographers take a pragmatic approach based on available data and the definitions therein (e.g. Keilman 2005). Among sociologists, there is considerable controversy over just what constitutes a family. In earlier times, there was consensus on depicting families: sociologists used the notion of "nuclear family", i.e. a married couple and their minor children, all living apart from other kin (Moen and Forest 1999). This went back to Durkheim's definition of 1921 (Hoffmann-Nowotny 1987; see also International Encyclopaedia of the Social Sciences Vol. 5 of 1968, page 303). However, concomitant with life-course changes, definitions of the family have been broadened to recognize a range of forms. However, there is considerable debate about a number of elements in the classical definition. For instance, does a cohabiting couple qualify for being considered as a family? How about same-sex couples? Is the presence of children a necessity? Does the family definition also cover lone parents? See Settles (1999) for a discussion.

Following Moen and Forrest (1999), this paper takes an inclusive view of the diversity of families and households. Our definition allows considering variations in household and kinship arrangements, as men and women move in and out of various living arrangements. We define a family as a group of two or more persons who live in the same dwelling, and who are related by marriage, cohabitation, blood, or adoption. Hence, we include same-sex couples, childless couples, lone parents, three generation families, and reconstituted families in our definition. A household consists of all persons who live in the same dwelling. Thus, a household may consist of just one person, or it may be a multi-person household. In many cases, the multi-person household consists of a family (possibly with other household members). However, we also allow multi-person households that are not families; for instance two unrelated students who share the same dwelling.[2]

The family and household definitions agree with international statistical practice, see the recommendations formulated by the Conference of European Statisticians (2006). We also follow these recommendations when we define the household positions that individuals may occupy at any given point in time: child, cohabiting partner, married partner, lone parent, and other. Child refers to a blood, step- or adopted son or daughter younger than 25 years of age (but regardless of marital status) who lives in the household of at least one of the parents, and who has no partner or own child(ren) in the same household. Young adults aged 25 or over, as well as persons younger than 25 with a partner or an own child, who live with their parent(s) belong to the category other. The latter category also includes persons who live in a multi-person household but who have no relationship (parent-child, or partner in consensual or marital union) to the other household members. A cohabiting couple (i.e. partners living in a consensual union) is understood as a couple that has a marriage-like relationship while not being married to each other, irrespective of the partners' sexes. Cohabiting persons can have any marital status (including married; in that case they are married to different partners). The category of married couples consists of those who are currently married and live together with the spouse. A lone parent lives together with one or more children as defined above, but without a spouse or cohabiting partner.

The household positions defined here characterize individual persons. Knowing the household positions of the household members, the household type follows immediately: a household may be a married couple household, a cohabiting couple household, a lone parent household, a one-person household, or a household of type "other". Due to data limitations (see below), we will only include private households, and thus ignore institutional and collective households.

2.2 Measurement problems

Given a certain set of definitions for households and families of various kinds, it is not always clear how to operationalize these, and next how to measure their numbers. Together, we refer to these problems as "measurement problems". These measurement problems imply that we are often unable to assess real household or family trends, but that we obtain measurements that reflect those real trends only to a certain extent. Here are a few examples.

1. The notion of "living in the same dwelling" figures prominently in our definitions of household and family, but it is problematic. It is based on an individual's place of usual residence (PUR). In our empirical application, we will use Norwegian data from three sources: the Census of 2001, the Survey of Living Conditions, and a household forecast that was computed in the 1990s. The Census is based on the *de jure* definition of PUR, whereas the forecast attempted to reflect the actual (*de facto*) PUR. We expect differences in PUR for some categories of the population, in particular for young adults. Largely, the census followed the rules of the Norwegian Population Register. One of these rules implies that a person who leaves the parental household will not be registered at the new address in Norway, unless he or she marries, gets a child, or receives the major part of his or her income from labour. Moreover, some elderly persons regularly move from a private to an institution and back, for instance because of health problems. It is not always clear whether their PUR is the private or the institutional household.

2. Although most households can clearly be classified as either private or as institutional, the distinction between these two types is not always sharp, because of intermediate forms such as assisted living etc.
3. It is not always clear whether two persons are a cohabiting couple, in particular (but not only) when they are of the same sex. The notion of "marriage-like relationship" leaves room for many subjective interpretations, which may differ from one situation to another, and even between the partners.
4. A lone parent who starts cohabiting with a new partner may still be classified as a lone parent, for instance because the new partner does not take parental responsibility for the children.

[2] This is the so-called dwelling definition of a household; see Conference of European Statisticians (2006).

3. Forecasting tools

3.1 The method of random shares

We have computed an illustrative probabilistic household forecast for Norway. The forecast applies to the period 2004-2030. It gives predictive distributions for

- the population broken down by five-year age group (0-4, 5-9, …, 85-89, 90+), two sexes, and the six household positions for individuals in private households defined in Section 2.1: child, living in consensual union, living with spouse, living alone, living as lone parent, and other;
- numbers of private households of five types, defined in Section 2.1: cohabiting couple household, married couple household, one-person household, lone parent household, and other private household.

We have combined two statistical methods:

1. a probabilistic *population forecast*, which gives the predictive distribution of the population broken down by sex and age, irrespective of household status;
2. predicted random shares, which distribute future population numbers by age and sex randomly over *household positions*.

We have used an existing probabilistic population forecast for Norway. This forecast was obtained earlier by means of the scaled model of error (Alho and Spencer 2005), in the framework of the UPE-project (Alho et al. 2006). Methodological details of the UPE forecast can be found at the web site *http://www.stat.fi/tup/euupe/*, which also contains also forecast results for Norway and 17 other European countries, including details of age and sex for ten-year intervals to 2050. In the empirical part of this paper, we focus on the predictions of the random shares.

Outline of the method

Notation

x: age $(0,1,\ldots\omega)$

s: sex (1=men, 2=women)

t: time

j: household position of an individual person $(1,2,\ldots J)$.

$V(j,x,s,t)$: a random variable which gives the population in household position j, age x, sex s at time t.

$W(x,s,t)= \Sigma_j V(j,x,s,t)$: a random variable which gives the population age x, sex s, at time t, irrespective of household position.

$\rho(j,x,s,t) = V(j,x,s,t)/W(x,s,t)$: a random variable which reflects the share of persons age x sex s, who are in household position j at time t. $\rho(j,x,s,t) > 0$, $\Sigma_j \rho(j,x,s,t) = 1$.

Problem

We have a probabilistic forecast for the population $W(x,s,t)$ for future years t. We want to specify a probability distribution for the shares $\rho(j,x,s,t)$ for future years. Given a probability distribution for $\rho(j,x,s,t)$, we compute a probabilistic forecast for $V(j,x,s,t)$ as $V(j,x,s,t)= \rho(j,x,s,t).W(x,s,t)$

Approach

We suppress x and s for notational convenience. In Section 4 we present two empirical specifications for the stochastic processes that predict the shares $\rho(j,t)$: a multivariate random walk, and random walks based on continuing fractions. The main characteristics of the two approaches are as follows.

Multivariate random walk. By definition, the random shares $\rho(j,t)$ are restricted to the interval [0,1]. Their sum over j is 1. A multinomial logit model transforms the ρ-variables into ξ-variables, i.e. $\xi(j,t) = \ln\{\rho(j,t)/\ \rho(1,t)\}$, with $\xi(1,t) = 0$. Thus, household position j=1 is selected as the reference category. The back transformation is $\rho(j,t) = \exp(\xi(j,t))/[\ \Sigma_j \exp\{\xi(j,t)\}]$. We specify a multivariate random walk for the $\xi(j,t)$, with expectations (point predictions) $\hat{\xi}(j,t)$. The point predictions are computed by means of a deterministic multistate household forecast; see Section 3.2. The variances for the household positions and the covariances across household positions are estimated from observed errors for the shares $\rho(j,t)$ in an old household forecast (Section 4.2).

Random walks based on continuing fractions. The main features of this method are the following. We order the household positions j according to some criterion that reflects their importance in a way to be discussed later. For the most important household position (j=1) we define the simple logit transform of $\rho(1,t)$ as $\xi(1,t) = \ln\{\rho(1,t)/[1-\ \rho(1,t)]\}$ and construct a univariate random walk for $\xi(1,t)$. Next, we compute the fraction of the second share relative to all shares except for the first one, i.e. $\lambda(2,t)= \rho(2,t)/\{\ 1 - \rho(1,t)\}$, compute the logit transformed value of $\lambda(2,t)$, and construct a univariate random walk in the logit scale. We continue this way with the remaining shares. By construction, the fraction $\lambda(j,t)$ expresses the probability that a person's household position at time t is j, given that it is not 1,2, …, j-1. Thus the univariate random walks in the logit scale are independent across household positions.

For both specifications we also include estimates of correlations across ages and across the sexes. Given the specification of the stochastic process for the random shares, we use simulation to find the multivariate distribution of the shares $\rho(j,t)$ in the future (Section 5).

3.2 Household forecasts: point predictions for Norway

This section presents the point predictions $\hat{\rho}(j,t)$, obtained from a deterministic multistate household forecast. We have updated a household forecast for Norway, which was computed in the 1990s (Keilman and Brunborg 1995). The new forecast applies to the period 2002-2031; we used Van Imhoff's program LIPRO version 4.0 (*http://www.nidi.knaw.nl/en/projects/section2/270101/*) for multistate household models to compute it (Van Imhoff and Keilman 1991).

Household projection models developed in demography over the past few decades are primarily of the headship rate type, in which the dynamic processes of household formation and dissolution, which underlie changes in household structure essentially, are treated as a black box. LIPRO ("Lifestyle projections") is a dynamic household projection model, which explicitly focuses on the flows underlying household changes. It is based on the methodology of multistate demography, but includes several extensions to solve the particular problems of household modelling. The LIPRO model is of the recursive type $V_{t+1} = P_t.V_t + Q_t.I_t$. Here V_t is a vector of the population at time t, broken down by household position, age and sex, I_t is a vector of immigrants during the time period (t, t+1) with the same format as V_t, while P_t and Q_t are square matrices that contain time dependent transition probabilities for the population V_t and for immigrants I_t, respectively. These transition matrices are functions of the forecast parameters, i.e. birth rates, death rates, emigration rates, and rates for household events. All rates are specific for age, sex and household position (in addition to time). The jump-off population for the LIPRO forecast is V_0. Below we report briefly how we arrived at estimates for the jump-off population on 1 January 2002, and for the forecast parameters for the first forecast period, i.e. 2002-2006.

Jump-off population. The jump-off population of the new household forecast is based on observed data taken from the Population and Housing Census of Norway of 3 November 2001. The data consist of a three-way table of the population in private households broken down by sex, five-year age group, and the following six household positions: "child", "cohabiting", "married", "living alone", "single parent", and "other". The Census uses the household definition given in Section 2. Since childless couples (either married or cohabiting) have much higher risks of dissolving their relationship than couples with children, we have additionally distinguished between cohabiting persons with and without children, and likewise for married persons – this way we defined eight household positions. For each combination of sex and five-year age group, we used data from Statistics Norway's Survey of Living Conditions (panel waves of 2000, 2001, and 2002;

see below) to compute the shares necessary for splitting up cohabiting persons and married persons into the two groups. We have assumed that the numbers thus obtained apply to the household structure of the population of Norway as of 1 January 2002.

Household events. A household event is defined as an immediate change from one household position to another one. Given the eight household positions, there are 56 theoretically possible events. Many of these imply events that are impossible in practice. For instance, a person cannot change immediately from "married, no children" to "cohabiting, with children". One or more intermediate positions are necessary, for instance "married, with children" and "lone parent". Other events are so rare that we have omitted them for practical reasons.

We have estimated occurrence-exposure rates for household events from the Survey of Living Conditions (SLC). The SLC is a panel survey with annual waves in the spring of each year since 1997; see Normann (2004). We have used data from the six waves conducted between 1997 and 2002.[3] The target population each year was the population aged 16-79 living in private households. The sample size in 1997 was 5000. In later waves, additional persons were drawn from new members of the target population (either immigrants or new persons aged 16). Between 1997 and 2002, 5525 persons were drawn. Among these, 2562 persons took part in all six waves. Remaining persons either refused to respond in one or more rounds, or they left the target population (due to death or emigration; individuals aged 80 years and over were retained in later waves). Response rates were relatively low among the over 80. Otherwise, response was of good quality.

Because the SLC only covers private households, institutional households are not included in this paper.

Households in the SLC are defined slightly differently compared to the Census: a household consists of persons who live in the same dwelling and who have common housekeeping. We assume that the household dynamics in those households is the same as that among households defined according to the household dwelling definition that is used for the Census.

Each respondent reported about the relationship with the other members in his or her household at each interview. We coded the respondent's household position according to the eight categories mentioned above and identified household events by comparing the household positions at subsequent interviews.[4] Due to both left censoring and right censoring, a number of events imply a change from, or to, an unknown household position. Added over all five calendar years, we obtained 3645 events of 27 different types, and 22462 years of exposure. We computed occurrence exposure rates for each of the 27 events, specific for sex and five-year age group. For a number of household events, the age patterns looked irregular, or even unreasonable. In those cases we had to take executive decisions to obtain reasonable patterns. We used occurrence exposure rates from the previous household forecast (see below) in some of the adjustments.

External events. In addition to household events, we distinguish the following external events: birth, death, immigration, emigration, entrance into an institutional household, and exit from an institutional household.

Birth rates broken down by mother's age (five-year age groups) and household position were taken from the previous household forecast and adjusted proportionally in such a way that the predicted number of live born children during the years 2002-2006 would correspond to the number that Statistics Norway has registered for that period.

Death rates by five-year age group and household position were estimated based on data from the Norwegian population registers. Øystein Kravdal kindly supplied us with deaths and exposure times broken down by marital status, age and sex for the years 1995-99.[5] We have applied

- death rates for persons with marital status "currently married" to persons with household position "cohabiting" or "married", irrespective of the number of children in the household;
- death rates for never married to persons with household position "child", "living alone", or "other";
- death rates for the divorced to persons with household position "lone parent".

We call these death rates "initial death rates", to be used later.

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[3] Starting in 2003, the SLC has a very different format compared to earlier years, and data from before 2003 are not directly comparable to the later data.

[4] Individuals may, in principle, experience two or more household events during one calendar year. But this is rare, because a one-year time interval is rather short. We have disregarded these multiple events.

[5] Mortality data by marital status for Norway are not available for more recent years.

The target population for this household forecast is the population in private households. Thus, mortality in institutions is disregarded. There are reasons to believe individuals who live in an institution have higher mortality than those who live in private households. Therefore, the (initial) death rates for the whole population cannot be used for persons in private households. The previous household forecast applied to *both* private households and institutions. Its results for the period 1990-1994 indicated that 28.7 per cent of all deaths take place in institutions. We have assumed that this is also the case in 2002-2006. Next, we adjusted the initial death rates such that the number of deaths that the model predicted for the period 2002-2006 was equal to 71.3 per cent of all registered deaths for that period.

Entrance to and exit from institutions was combined with *international migration* into one forecast parameter (for each combination of age, sex, and household position) of *net entries*. The net entries were expressed as absolute numbers (contained in the vector I_t above). We obtained values for the net entries for the period 2002-2004 in several steps.

1. Numbers of net migrants for the period 1990-1994 broken down by age, sex, and household position taken from the previous forecast were proportionally adjusted such that they summed to 83,832, which is the total net immigration registered by Statistics Norway for the years 2002-2006.
2. Initial death rates applied to the jump-off population resulted in an initial number of deaths for the period 2002-2006, specific for age, sex, and household position.
3. The set of death rates obtained after reducing the number of deaths by 28.7 per cent (see above) resulted in a second set of deaths.
4. The difference between the initial number of deaths (point 2) and the reduced number of deaths (point 3) can be interpreted as the number of exits from the population in private households to the institutionalized population. The implicit assumption in this case is that a person who leaves a private household occupies a place in an institution immediately after the death of an institutionalized person. The total number of such exits from private households in the period 2002-2006 amounted to 60,425 persons, or 12,000 persons per year. The census indicated that 35,000 persons lived in an institution in 2001. Thus, the annual turnover is about one-third.
5. For each combination of age, sex, and household position we subtracted the number of exits from the population in private households from the net immigration numbers (point 1). The result was a set of net entries that showed positive numbers for the young and the middle-aged segments of the population, and negative numbers for the elderly.

Consistency. Parameters for household events and for external events, when applied to the jump-off population, result in certain numbers of events of each type. For some of these events one wants to require that their numbers are constrained to certain values. This is what is generally known as the consistency problem in multistate household forecasts; see Van Imhoff and Keilman (1991) and Van Imhoff (1992). We speak of internal consistency when the numbers of two or more household events are related. An example is that the numbers of new marriages of men and women during a given period should be equal[6]. External consistency applies to the case where the sum of a set of external events is constrained to be a fixed constant. For instance, the total number of births, irrespective of mother's age or household position, may be required to be a certain number.

LIPRO includes a feature that takes account of internal and external consistency relations. LIPRO's consistency algorithm assumes that the relations are linear. We defined 18 consistency relations: 15 for household events related to couple dissolution and to events experienced by children and by parents, and one each for the total number of births, deaths, and net entries. LIPRO adjusted the parameter values for household events and external events in such a way that all relationships were fulfilled.

Entry into a consensual union and marriage are <u>not</u> among the 15 events for which we formulated consistency requirements. There are two reasons.

1. Numbers in the jump-off population are not consistent. The census shows slightly more married men than women who live with their spouse: 834,583 men and 834,225 women. For cohabiting couples the numbers are 204,109 (men) and 204,155 (women).

[6] Assuming that the net effect of marriages across country boundaries is zero.

2. Observed data for new marriages are not consistent. Marriage data from Statistics Norway for the year 2005 show that among the registered population, there were 22,932 men who married, against 20,474 women. When we restrict ourselves to marriages where both partners are registered in the country, there were only 18,976 couples who married in 2005. The surplus of 1918 men implies 10.1 per cent of 18,976 couples. In 1990, the surplus was only 1.1 per cent – the share has increased regularly from that level to the current 10.1 per cent. The regular increase is largely due to increasing numbers of men of Norwegian nationality who marry a woman from Thailand, the Philippines or Russia (Daugstad 2006). At the same time, net immigration numbers are not consistent. During the years 1985-2006, the surplus of married men (compared to married women) fluctuated strongly between a minimum of -2,600 in 2002 and a maximum of 2,887 in 1987. Business cycles in the Norwegian economy and associated fluctuations in labour migration from EU-countries may explain the variations in the surplus of married men in net migration. Comparable data for cohabiting persons are lacking, but there is no reason to believe that the inconsistencies are less than those for new marriages and net migration for married persons.

Point predictions for household position shares. We have used the exponential version of LIPRO and harmonic mean consistency type to compute a household forecast for the years 2002-2032. We constrained total numbers of births, deaths (net of deaths in institutions) and net entries to corresponding numbers taken from the Medium Variant of Statistics Norway's 2005-based population forecast; see *http://www.ssb.no/english/subjects/02/03/folkfram_en/*. Parameters for household events were assumed constant over the forecast period (except for possible adjustments due to consistency). This is not to imply that we are convinced that there will be no change in rates for household events in the future. We simply do not have good enough data to extrapolate a possible trend in these events.

Figure 1 summarizes the main results. It gives point predictions of shares for eight household positions for the forecast period, irrespective of age and sex. The year 1990 (Census information) has been added for reference. The figure shows a continuation of changes in household and family structures that have gone on for several decades now: it becomes increasingly more likely for an individual to live alone or in consensual union, and much less likely to live with a spouse and children (Keilman 2005). A new trend is that elderly women are less likely to live alone in the future than they are nowadays, and more likely to live with their husband; compare Figures 2 and 3. The same trend applies to shares of elderly men who live with their spouse (but at a higher level) and elderly men living alone (at a lower level). This new trend is due to improved longevity of both sexes, but with steeper increases in life expectancy among men than among women (Meslé 2004).

3.3 Stochastic forecasts of population: selected UPE-results for Norway

Stochastic population forecast results for Norway were taken from the project "Uncertain Population of Europe" (UPE), which gives predictive distributions of the population by sex and age in 18 European countries (the 15 members of the European Union pre-2004, plus Norway, Iceland, and Switzerland) for the period 2004-2050; see Alho et al. (2006) and Alders et al. (2007). The UPE-project started from analyses of the historical developments in fertility, mortality, and migration in the 18 countries, in which the time trend in these variables were separated from random deviations from the trend. Estimates were given for both the time trends and the random deviations in the future, which made it possible to calculate a cohort component forecast in terms of probability distributions. Below we give a few illustrative results for Norway (see *http://www.stat.fi/tup/euupe/no11_results_nor.html*).

For Norway, a population of 5.26 million is expected for 2030, very close to Statistics Norway's figure in its most recent official population forecast made in 2005. The latter forecast amounts to 5.37 million (forecast variant with medium population growth; see *http://www.ssb.no/emner/02/03/folkfram/tab-2005-12-15-01.html*).The 80 per cent forecast interval in 2030 stretches from 4.90 to 5.65 million. There is an 80 per cent probability that the share of the population aged 65 years and over in 2030 will be between 19.5 and 23.3 per cent – at present it is 14.7 per cent.

4. Analysing observed errors in the shares

4.1 Data

We have computed empirical errors in the shares $\rho(j,t)$ emerging from the household forecast that was published in the early 1990s (Keilman and Brunborg 1995). It has jump-off year 1990 and results for 1995, 2000, 2005 and later. All dates are pr. 31 December. As before, we have restricted ourselves to private households. We have analysed predicted shares for six household positions: "child", "cohabiting", "married", "living alone", "single parent, one or more children", and "other".[7] Henceforth, we label these household positions as CHLD, COH, MAR, SIN0, SIN+, and OTHR, respectively. The predicted shares were evaluated against empirical shares observed for the years 1997, 2001, and 2002. This gave us empirical forecast errors, defined as observed shares relative to predicted shares. These forecast errors are approximate, because the empirical data contain sample errors and measurement errors, as discussed in Section 2. We have the following data:

- the 1997-wave and the 2002-wave of the Survey of Living Conditions. The data have sample errors. We interpolated forecasted shares between 1995 and 2000, and between 2000 and 2005.
- data from the Census taken on 3 November 2001 are compared with corresponding forecast results. There are two problems. First, household definitions differ between the forecast (the housekeeping definition of a household, similar to the SLC; see Section 3.2) and the census (dwelling definition). Second, the Census uses the *de jure* definition of household position, whereas the forecast attempted to reflect the *de facto* situation. This creates problems for some categories, in particular for young adults who *de facto* have left the parental household, but who are still registered at their parents' address (see Section 2.2). We interpolated forecasted shares between 2000 and 2005.

4.2 Empirical errors

The previous household forecast gives us empirical errors for lead times of 6¼ (Spring 1997), 10¾ (3 November 2001), and 11¼ (Spring 2002) years ahead. Figures 4-6 compare, for a selection of the six household positions, predicted shares from the household forecast with observed shares from the SLC (Spring 1997 and Spring 2002) and the Census (November 2001). Two general observations emerge, both of them in accordance with what one could expect. First, forecast errors increase over time. Second, the errors observed from the Census are much smaller than those from the SLC, since the latter include sample and other survey errors. The forecast predicted too few young cohabitants (Figure 4) and young adults living alone (Figure 6), as opposed to too many children living with their parents, and far too many young adults in household position "Other" (not shown).

To a large extent these errors can be explained by the favourable economic development around the turn of the century, which led to many young adults living independently. These errors are less visible in 2001 (Census data) than in 2002 (survey data), because the census is based on the *de jure* household position. Many young adults are registered as living in the parental household, while *de facto* they have left home. The forecast predicted too high mortality and thus too many widows and widowers. This explains the underprediction of the number of elderly married men and women (Figure 5), and the overprediction of the number of elderly men and women who live alone (Figure 6).

The group "Other" consists for a large part of adults aged between 25 and 30 who live with their parents (unrelated persons who share a dwelling belong to this group, too). Thus there is a large degree of complementarity between the groups "Child" and "Other". Indeed, when we combined children and persons with household position other into one category, the age patterns for this combined category became were more regular than those for the separate groups.

Observed shares of lone parents aged 60 and older are generally higher than predicted shares for this group (not shown). This is probably due to an artefact in the empirical data, in which elderly persons who live together with an adult child are still categorized as lone parents, even when their youngest child is over 25 years of age. For this reason, we have ignored forecast errors for single parents beyond age 60.

[7] As opposed to the deterministic household forecast in Section 3, we have not distinguished childless married or cohabiting couples from those who live with children, because this would create additional errors in the data for 2001.

In Section 3.1 we have described the two types of stochastic processes for the errors, namely a multivariate random walk (based on a multinomial logit transformation for the errors) and an approach using continued fractions (based on a simple logit transformation). These two specifications by no means exhaust all possible specifications, but they serve to illustrate that the probabilistic household forecast depends strongly on the stochastic model that one selects.

We have applied the multivariate random walk model to error data from the two SLC rounds and the Census. The continued fractions approach is limited to Census data. Hence, in the latter approach we avoid the idiosyncrasies of the error data from the sample surveys. In particular, the error patterns based on the SLC for the groups "Child" and "Other" are difficult to explain. For the remaining household positions, the errors are generally larger in the SLC than in the Census, as noted above. We believe that the continuing fractions approach gives the best possible representation of the underlying data of the two approaches, given the constraints with which we are working. The multivariate random walk application should be considered as a sensitivity analysis.

4.3 Multivariate random walk

Variances and covariances. At any point in time, we write the share $\rho(j,t)$ as the product of the predicted share and a multiplicative error term: $\rho(j,x,s,t) = \hat{\rho}(j,x,s,t).e_{\rho}(j,x,s,t)$, with $e_{\rho}(j,x,s,t) > 0$. The multiplicative error $e_{\rho}(j,x,s,t)$ in the share $\rho(j,x,s,t)$ translates into an additive error in the $\xi(j,x,s,t)$-variable, i.e. $\xi(j,x,s,t) = \hat{\xi}(j,x,s,t) + e_{\xi}(j,x,s,t)$, where $\hat{\xi}(j,x,s,t)$ is the expected value (point prediction) of $\xi(j,x,s,t)$, and $e_{\xi}(j,x,s,t)$ is the error in ξ . Since household position j=1 is the reference category, we see that $e_{\xi}(j,x,s,t) = \ln(e_{\rho}(j,x,s,t)/e_{\rho}(1,x,s,t))$: the error for share j in the ξ-scale equals the log of the error for that share in the original ρ-scale, relative to the ρ-error for share 1. Similarly, the point prediction $\hat{\xi}(j,x,s,t)$ equals $\ln(\hat{\rho}(j,x,s,t)/\hat{\rho}(1,x,s,t))$. We model the error variances and covariances as a function of household position j and time t, and predict them for lead times other than 6¼, 11¾, and 12¼ years ahead.

Household position "living alone" is the reference category. There are 297 observations: 12 for "child", 81 both for "cohabiting" and for "married", 49 for "lone parent", and 74 for "other". With only 297 observations we can not entertain a very rich model. We have assumed the following.

We assume that the errors are normally distributed with zero expectation and covariance matrix $\Sigma^2_{\xi}(t)$, the elements of which only depend of household position j and of time t, independent of sex and independent of age. For t = 6¼ and t = 12¼, each observed error can be written as the sum of a sample error and a prediction error. For t = 11¾, there is no sample error. We assume that the two types of error are independent. Prediction errors are assumed to behave like a random walk, the innovation variance of which is written as π_j^2. The corresponding covariance matrix for all five household positions together is written as Π^2. Thus, the prediction error covariance matrix at time t is $t.\Pi^2$. The sample error has a constant variance s_j^2, with covariance matrix S^2. Appendix 1 gives details of our estimations of the variances and covariances. We found covariance estimates that were small enough to assume independence in the random walks across household positions. Innovation variance estimates turned out to be equal to

CHLD	COH	MAR	OTHR	SIN+
0.1293	0.0404	0.0330	0.0916	0.1894

Correlations across ages and across the two sexes. For each household position, we estimated AR(1) the autocorrelation coefficient across ages (i.e. for neighbouring five-year age groups) and the correlation across sexes. Median values across household positions were 0.5224 and 0.5463, respectively.

4.4 Random walks based on continued fractions

For this particular application, we adopted the following sequence of positions:

(a) the share of MAR and SIN0 combined

(b) the share of MAR relative to those of MAR and SIN0 combined

(c) the share of COH relative to the sum of shares for COH, SIN+, CHLD, and OTHR

(d) the share of CHLD and OTHR combined relative to the sum of shares for CHLD, OTHR, and SIN+

(e) the share of SIN+.

MAR and SIN0 are the most important groups given their large shares (see Figures 5 and 6); by combining these two we stabilize the error patterns to a great extent. CHLD and OTHR are problematic, given the measurement problems for young adults noted earlier. We did not separate the groups CHLD and OTHR, but in our simulations, we have assumed that predicted shares for the combined group apply to the household position "Child" below age 25, and to the household position "Other" for ages 25 and over. SIN+ displays the most erratic patterns – it is left as a residual. Errors in both the youngest and the oldest age groups are difficult to explain, and hence we have limited the analysis to the age groups 25-59. We have assumed that simulated shares in the logit scale for the youngest and the oldest age groups have probability distributions that are the same as those for the age group 25-59 (except for age-specific locations that are determined by the point predictions).

Based on the sequence of positions listed under (a) – (e) above for the unconditional shares $\rho(j,t)$, we defined the conditional shares $\lambda(i,t)$ $i = 1,\ldots,4$ as follows (suppressing arguments for age x and sex s):

(a) $\lambda(1,t) = \rho(\mathrm{MAR},t) + \rho(\mathrm{SIN0},t)$

(b) $\lambda(2,t) = \rho(\mathrm{MAR},t)/\ \lambda(1,t)$

(c) $\lambda(3,t) = \rho(\mathrm{COH},t)/(1 - \lambda(1,t))$

(d) $\lambda(4,t) = (\rho(\mathrm{CHLD},t) + \rho(\mathrm{OTHR},t))/(1- \lambda(1,t) - \lambda(3,t))$

We used observed and predicted values of the unconditional shares ρ based on the Census to compute multiplicative errors in the conditional shares λ for $i = 1, \ldots, 4$, transformed these to the logit scale, and assumed a simple random walk for each of them. Estimates for the innovation variances computed by expression (A2) turned out to be 0.090, 0.098, 0.129, and 0.321. Moreover, we estimated the correlation between the sexes as 0.68, and that between neighbouring five-year age groups (assuming an AR(1) model) as 0.65.

5. Simulation strategy and illustrative results

For both approaches, we assumed normally distributed errors in the ξ-scale with zero mean and variances and covariances as estimated in Section 4. We drew 3000 error values specific for lead time (accounting for correlations across the sexes and across ages) from the assumed normal distributions, added point predictions, and transformed back to the ρ-scale. Since we applied the simulated shares to UPE results, lead time zero corresponds with 1 January 2003. This resulted in a set of 3000 random shares ρ, specific for sex, age, time, and household position. We applied the shares to corresponding UPE results, which we obtained from the UPE website.[8] We focus on the results from the method of continuing fractions, and we will compare a few of these with results from the multivariate random walk method. We inspected the following issues:

[8] The shares apply to private households only. In contrast, UPE results apply to all persons, both those in private and institutional households. The total number of persons in institutions in Norway has been fairly constant since the 1990s, about 35,000. The Census of 2001 shows that the majority (ca. 23,000) are aged 80 or over. We decided to ignore persons living in institutions, because we do not have good enough data to compute point predictions for their numbers. Thus, our forecast slightly overestimates the number of elderly in various households and the total number of households.

- Which household type k has largest prediction uncertainty? How fast does it increase with lead time?
- Which household position j has largest uncertainty? How does this depend of age, of sex, of time?
- How much does the prediction variance in population ($W(x,s,t)$) contribute to the variance in $V(j,x,s,t)$? How large part is due to the variance in the shares $\rho(j,x,s,t)$?
- How fast does the uncertainty in mean household size increase with lead time?

For the continuing fractions method, we computed empirical predictive distributions for three types of variables:

1. The population $V(j,x,s,t)$ in household position j, age x, sex s at time t. We selected men and women in three age groups (20-24, 50-54, and 80-84) and three future years (2010, 2020, and 2030), and computed the results for the following five household positions: "married", "living alone", "cohabiting", "child/other", and "lone parent".
2. The number of households $H(k,t)$ of type k at time t. These were computed for three future years (2010, 2020, and 2030) and five household types. The number of households consisting of a cohabiting couple equals half the total number of persons with that household position, and likewise for married couples. The number of one-person households equals the number of persons with that household position, and likewise for the number of lone-parent households. The number of other households was estimated as the number of persons with that position[9] divided by 2.5, which is the mean size of households of type other derived from the 2002 wave of the SLC.
3. Mean household size computed as $\sum_{j,x,s} V(j,x,s,t) / \sum_{k} H(k,t)$ for the years 2010, 2020, and 2030.

Households by type. Table 2 summarizes our results for households of various types. The forecast predicts a 28 per cent growth in the number of private households between 2002 and 2030, from 2.026 million to an expected 2.587 million. The UPE forecast expects the population to increase by only 16 per cent, and thus mean household size drops from 2.21 in 2002 to 2.04 in 2030. Forty per cent of the households in 2030 can be expected to consist of only one person, up from 36 per cent in 2002.

Household types that have the smallest innovation variances (i.e. married couples and one-person households; see Section 4.4) have the lowest relative forecast uncertainty as measured by the CV. At the same type, these household types are the most numerous. Relative uncertainty increases regularly between 2010 and 2030.

Table 2 Average value, coefficient of variation (CV), and lower and upper bounds of 80% prediction interval of numbers of households and mean household size in 3000 simulations

	Married couple	One-person household	Cohabiting couple	Lone-parent household	Other private household	All private households	Mean household size
2010							
Average	876,790	828,010	267,348	183,530	42,142	2,197,820	2.15
CV	0.042	0.078	0.108	0.172	0.217	0.019	0.018
80% low	830,542	744,952	231,010	143,924	30,972	2,145,600	2.10
80% high	923,086	911,772	304,906	225,313	54,406	2,251,148	2.20
2020							
Average	918,529	926,450	317,853	199,615	37,138	2,399,585	2.07
CV	0.068	0.117	0.150	0.246	0.348	0.035	0.030
80% low	839,540	791,250	257,524	141,334	21,874	2,292,957	1.99
80% high	999,560	1,068,251	379,653	264,160	54,068	2,509,124	2.15
2030							
Average	955,644	1,043,608	344,575	207,263	35,892	2,586,981	2.04
CV	0.093	0.142	0.181	0.297	0.451	0.052	0.046
80% low	841,856	863,320	266,457	134,089	17,619	2,418,441	1.92
80% high	1,069,151	1,242,075	427,838	288,355	58,376	2,758,973	2.16

[9] That is, the number of persons with household position child or other who are 25 years or older.

Household positions of individuals. We have constructed Table 3 to inspect prediction uncertainty for individuals in different household positions. The table gives the coefficient of variation (CV), which reflects relative uncertainty. Three factors have an impact on the CV. First, there is a strong effect of lead time, as CV's almost without exception increase regularly between 2010 and 2030. Second, the effect of innovation variances of the various household positions is rather strong. However, and this is the third factor, the latter effect is sometimes mediated by the effect of the level (point prediction) of the particular share. For instance, married men and women were given a low innovation variance, but at ages 20-24 where their shares are low, the CV for married men and women is rather high. In this age group, household position CHLD/OTHR (in practice "child") has the lowest uncertainty among the five household positions. Note also age 80-84. Men are likely to live with their spouse (household position MAR), which gives a low CV, while women frequently live alone (SIN0), which results in a low value for their CV.

Table 3 - Coefficient of variation for predictive distributions, men and women in three age groups and five household positions

	20-24	50-54	80-84	20-24	50-54	80-84
		Men			Women	
2010						
MAR	0.312	0.072	0.069	0.300	0.071	0.166
SIN0	0.167	0.217	0.223	0.178	0.230	0.110
COH	0.321	0.288	0.292	0.256	0.280	0.341
CHLD/OTHR	0.106	0.711	0.323	0.150	1.051	0.283
SIN+	1.139	0.331	1.252	0.969	0.258	1.213
2020						
MAR	0.489	0.113	0.128	0.466	0.113	0.224
SIN0	0.246	0.312	0.337	0.261	0.347	0.194
COH	0.453	0.415	0.389	0.358	0.409	0.440
CHLD/OTHR	0.163	1.585	0.626	0.223	1.886	0.565
SIN+	2.093	0.429	2.176	2.000	0.374	1.857
2030						
MAR	0.611	0.160	0.205	0.602	0.152	0.280
SIN0	0.311	0.380	0.395	0.329	0.417	0.245
COH	0.559	0.494	0.489	0.444	0.480	0.507
CHLD/OTHR	0.218	1.969	0.878	0.290	2.465	0.962
SIN+	2.822	0.540	2.368	2.252	0.479	1.748

The results in Table 3 are based on the predictive distributions for *numbers of persons* in various household positions. A similar table can be computed based on *shares* for these household positions. The striking result is that the coefficient of variation for a certain share, almost without exception, is only slightly lower than that for the corresponding number of persons. For instance, Table 3 gives a CV equal to 0.152 for the predictive distribution of married women aged 50-54 in 2030. The corresponding CV based on shares equals 0.146. Hence only 0.006 points of the CV of 0.152 are due to uncertainty in the number of women aged 50-54 in 2030, and the remaining 0.146 are due to uncertainty in the share for household position MAR. How can this be explained?

The number of married women aged 50-54 in 2030 is found as the product of the corresponding share and the total number of women of that age. Write $V = \rho.W$ for short. Since the share ρ is independent of the population W, an approximate value of the variance in V (by the Delta theorem) is $\mathrm{Var}(V) \approx \mathrm{E}^2(W)\mathrm{Var}(\rho) + \mathrm{E}^2(\rho)\mathrm{Var}(W)$, which leads to $\mathrm{CV}^2(V) \approx \mathrm{CV}^2(\rho) + \mathrm{CV}^2(W)$. The table below gives estimated values of the terms in these expressions, based on 3,000 simulations.

E(W)	Var(W)	CV²(W)	E(ρ)	Var(ρ)	CV²(ρ)	CV²(V)
151,213	40,741,661	0.001782	0.610966	0.007958	0.021319	0.023112

Of the two CV's, that of ρ determines the CV of V almost entirely in this case. The sum of the two squared CV's amounts to 0.023101, close to the exact value in the last column. Thus, we conclude in general that when forecast uncertainty is expressed in terms of the CV's of the predictive distributions, the factor with the largest CV (share ρ or population W) contributes most to the uncertainty in the population for a specific household position. In many cases, the share ρ has the largest CV, but there are exceptions, in particular for the young and the old. For instance, men aged 80-84 in 2030 have a CV equal to 0.1456, while the CV for the married share of these men is 0.1424. (The square root of their sum equals 0.2037, which is close to the value in Table 3.)

Consistency. When we draw random shares for married or cohabiting men and women from their predictive distributions, there is no guarantee for consistency between resulting absolute numbers for the two sexes. The discussion in Section 3.2 shows that such consistency is not necessary, because in practice there are small differences: the Census of 2001 shows 358 more married men than married women who live with their spouse, and 46 fewer cohabiting men than cohabiting women. We have used a correlation equal to 0.68 across the sexes. What is the impact of this correlation on the differences between future numbers of married or cohabiting men and married or cohabiting women? Table 4 shows simulation results for correlations equal to 0.68 (the reference value), 0.75, and 0.95.

Table 4 - Difference between number of men and women by household status, averages and standard errors across 3000 simulations

	Correlation 0.68		Correlation 0.75		Correlation 0.95	
	MAR	COH	MAR	COH	MAR	COH
2010						
Average	-11,023	1,443	-10,992	1,439	-10,883	1,433
Standard error	641	490	581	439	357	235
2020						
Average	-19,000	-3,140	-18,933	-3,241	-18,672	-3,623
Standard error	1,010	778	915	696	564	371
2030						
Average	-31,902	3,834	-31,775	3,654	-31,302	2,967
Standard error	1,388	1,017	1,262	910	802	484

On the whole, an increase in the correlation between the sexes decreases the average difference only marginally, and the standard error of the mean substantially. Note also that differences between men and women are much larger in the forecast than in the base data. They increase with forecast lead time, and the standard errors indicate that the expected differences are significantly different from zero. Yet, compared to the numbers of married couples (about 900,000; see Table 3) and cohabiting couples (about 300,000) the differences are small enough to be ignored.

Comparison with multivariate random walk. The empirical specification of the multivariate random walk is based on observed forecast errors from both the SLC and the Census. Because of sample and other survey errors in the SLC it is reasonable to expect larger uncertainty in random share forecasts that are driven by the multivariate random walk, then those driven by the continued fraction method. The estimates for the latter are solely based on forecast errors derived from the Census. A comparison of predictive distributions for shares derived from the two methods confirms this. As an example we present Box-and-whisker plots for the distributions of shares for men and women in three age groups and two household positions in 2030; see Figure 7.

Central tendencies in the share distributions are the same (as they should be), but the dispersions in the shares based on the multivariate random walk are clearly much larger than those based on continued fractions. The boxes, the edges of which reflect the first and the third quartile are longer, and the whiskers, the endpoints of which indicate minima and maxima of the distributions, stretch further towards zero and one. In other words, the continued fractions method results in less uncertain forecasts, since the predictive distributions are relatively narrow.

6. Conclusions

We have computed a probabilistic household forecast for Norway. We have combined a probabilistic population forecast, which predicts numbers of men and women by age, with a probabilistic forecast for the shares of persons who have a particular household position, given age and sex. Point predictions for the shares were computed by means of a deterministic multistate household forecast model. Variances and covariances were estimated from observed forecast errors in an old household forecast. We have restricted ourselves to private households.

We find that prediction uncertainty for future numbers of married couples, cohabiting couples, and one-person households, as indicated by the coefficient of variation, is rather low. Lone parents and other private households show much larger prediction uncertainty.

The method that we have used can also be applied to other forecasts in which the population is distinguished by a certain characteristic, in addition to age and sex. We have used household position, but other applications could include health or disability status, region of residence (cf. Wilson and Bell 2007), labour market status, etc. Such applications require that one specify a stochastic process for the random shares, which distribute the population over the various states, given age and sex. A historical deterministic forecast can be used to estimate the parameters of the stochastic process. The random shares are to be combined with the results of a probabilistic population forecast. The latter type of forecast exists for many industrialized countries.

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APPENDIX 1

The purpose of this appendix is to outline our estimation strategy for the variances and covariances of the multivariate random walk.

Variances

Observations for 1997 and 2002 consist of sampling errors and prediction errors. The observations for 2001 consist of prediction errors only. Hence we write

$$\text{(A1)}\quad t.\pi_j^2 = \frac{\sum_{x,s}\left(e_{\xi}(j,x,s,t)\right)^2}{n_{j,t}-1} - \hat{s}_j^2 \quad \text{for } t = 6¼ \text{ and } t = 12¼,$$

$$\text{(A2)}\quad t.\pi_j^2 = \frac{\sum_{x,s}\left(e_{\xi}(j,x,s,t)\right)^2}{n_{j,t}-1} \quad \text{for } t = 10¾.$$

Here $n_{j,t}$ is the number of observed errors for household position j at time t. Note that we use the Mean Squared Error (MSE) to estimate the prediction variances. In order words we do not subtract the means and thus use conservative estimates of the variances.

Using the Delta theorem, and assuming a multinomial distribution for empirical shares, we compute an approximate value for the estimated sample variance $\hat{s}_j^2$ for household position j as $(\tilde{\rho}_j + \tilde{\rho}_1)/(\tilde{\rho}_j\tilde{\rho}_1 n)$, $j = 2,3,\ldots,6$.[10] Here $\tilde{\rho}_j$ is the empirical share for household position j, $\tilde{\rho}_1$ is the empirical share for the reference group "living alone", and n is the size of the population group to which the shares apply. These sample variances vary strongly across ages, because the empirical shares have very distinct age patterns; cf. Figures 4-6. Population sizes were approximately 200 given age and sex. The median values across age groups and the two sexes turned out to be as follows.

estimated sample variance, median values					
	CHLD	COH	MAR	OTHR	SIN+
1997	0.0942	0.1370	0.0646	0.4551	0.2520
2002	0.1115	0.1804	0.0744	0.4542	0.2540

The sample variances are approximately the same in the two years. We subtracted the median values above from the MSE's for 1997 and 2002. Together with the MSE's for 2001 this resulted in the following values

	CHLD	COH	MAR	OTHR	SIN+
1997	0.7786	0.2313	0.1993	0.6009	2.4217
2001	1.3240	0.3790	0.2782	0.7246	1.6197
2002	1.8680	0.6299	0.5371	1.4879	1.7757
column sum	3.9706	1.2402	1.0147	2.8134	5.8171

The MSE's for children are relatively large, because we have only four data points for each year. The MSE's for lone parents are high because this household position is difficult to predict. The values vary a great deal across household positions.

[10] See Appendix 2 for a proof.

We divided each number in the table above by its column sum, and ran an OLS-regression without constant with these 15 values as dependent variables, and lead times as independent variables. The resulting OLS-estimate of the coefficient was 0.0326, with a standard error of 0.0026. Residual sums of squares across three observations were lowest for "child" (0.0076), "cohabiting" (0.0181), and "married" (0.0288), and largest for "other" (0.0326), and "lone parent" (0.0649). When multiplied again with the column sums above, this gives us the following estimates of innovation variances:

CHLD	COH	MAR	OTHR	SIN+
0.1293	0.0404	0.0330	0.0916	0.1894

The random walks for the errors in the shares of household positions "married" and "cohabiting" have small innovation variances, compared to those for the other household positions.

Covariances

We proceeded in a similar manner as for the variances above. There are ten covariances to be analysed, one for each pair (i,j), $i, j = 2,3,...6; i < j$. First we estimated sample covariances $\hat{s}^2_{i,j}$. A first-order approximation for the latter covariances is $1/\tilde{\rho}_1 n$ for every i and j; see Appendix 2. There was very little variation across ages, sexes, and time (years 1997 and 2002 only) in this approximate covariance value, except for extreme ages, which had high values. The median value was 0.0465.

Next we computed mean cross-products of errors similar to the mean squared errors in expressions (A1) and (A2). The numerator is the sum of cross-products of errors. The denominator is $\{\min(n_{i,t}, n_{j,t})\text{-}1\}$. In order to obtain an acceptable number of observations, we pooled the data across the three calendar years. Even then, some mean values are based on unacceptably few data points. Mean cross products for the three years combined are given in Table 1. We used these values as an estimate of the corresponding error covariances. The table gives also estimates of the correlations, and the number of data points involved. Correlations are essentially zero, except for covariances (CHLD,MAR) and (CHLD,SIN+). But the latter two estimates are based on too few data points to be trusted. Thus the data do not suggest any particular dependence of the errors across household positions, and we shall assume that they are independent.

When the median sample covariance of 0.0465 is subtracted from the estimated covariances in Table 1, the resulting values are still small enough to assume independence across household positions. Thus the covariance matrix $\Sigma^2_\xi(t)$ is estimated as $\hat{\Sigma}^2_\xi(t) = t\hat{\Pi}^2$, with innovation variances equal to

$\hat{\Pi}^2 =$

CHLD	COH	MAR	OTHR	SIN+
0.1293	0	0	0	0
0	0.0404	0	0	0
0	0	0.0330	0	0
0	0	0	0.0916	0
0	0	0	0	0.1894

APPENDIX 2

The purpose of this appendix is to derive expressions for sample variances and covariances of the ξ_j-variables, when values for the shares ρ_j are estimated from a sample survey.

The share ρ_j for household position j is estimated as $\tilde{\rho}_j = n_j / n$, where n_j represents the number of persons in household position j, and n the total number of persons irrespective of household position. There are J household positions. We assume a multinomial distribution for the counts $n_{j.}$ Hence the variance of $\tilde{\rho}_j$ is estimated as $\tilde{\rho}_j(1-\tilde{\rho}_j)/n$, and the covariance between any two shares $\tilde{\rho}_i$ and $\tilde{\rho}_j$ as $(-\tilde{\rho}_i\tilde{\rho}_j/n)$, $i \neq j$.

The multinomial logit transformation implies that $\xi_j = \ln(\rho_j / \rho_1)$, $j = 2,3,\ldots,J$. Household position $j = 1$ is taken as the reference. Hence $\dfrac{\partial \xi_j}{\partial \rho_j} = \dfrac{1}{\rho_j}$, $j \neq 1$, whereas $\dfrac{\partial \xi_j}{\partial \rho_1} = -\dfrac{1}{\rho_1}$. Then the Delta theorem gives the following approximate estimated value for the estimator of the variance of ξ_j:

$$\operatorname{Var}(\tilde{\xi}_j) \approx \left(\left(\frac{1}{\rho_j}\right)^2 \operatorname{Var}(\rho_j) + \left(\frac{-1}{\rho_1}\right)^2 \operatorname{Var}(\rho_1) - 2\frac{\operatorname{Cov}(\rho_1,\rho_j)}{\rho_1\rho_j} \right)_{\rho_i=\tilde{\rho}_i} .$$

Inserting expressions for variances and covariance, we find

$$\operatorname{Var}(\tilde{\xi}_j) \approx \frac{\tilde{\rho}_1 + \tilde{\rho}_j}{\tilde{\rho}_1\tilde{\rho}_j n} .$$

To find the covariance between $\tilde{\xi}_2$ and $\tilde{\xi}_3$, we write first $\xi_2 = \ln(\rho_2 / \rho_1) = \xi_2(\rho_1, \rho_2, \rho_3)$, and

$\xi_3 = \ln(\rho_3 / \rho_1) = \xi_3(\rho_1, \rho_2, \rho_3)$.

Derivatives are $\dfrac{\partial \xi_2}{\partial \rho_1} = \dfrac{-1}{\rho_1}, \dfrac{\partial \xi_2}{\partial \rho_2} = \dfrac{1}{\rho_2}, \dfrac{\partial \xi_2}{\partial \rho_3} = 0$, and similarly $\dfrac{\partial \xi_3}{\partial \rho_1} = \dfrac{-1}{\rho_1}, \dfrac{\partial \xi_3}{\partial \rho_2} = 0, \dfrac{\partial \xi_3}{\partial \rho_3} = \dfrac{1}{\rho_3}$.

The Delta theorem gives

$$\operatorname{Cov}(\tilde{\xi}_2,\tilde{\xi}_3) \approx \left(\sum_i \sum_j \frac{\partial \xi_2}{\partial \rho_i} \frac{\partial \xi_3}{\partial \rho_j} \operatorname{Cov}(\rho_i,\rho_j) \right)_{\rho_i=\tilde{\rho}_i}$$

, with $\operatorname{Cov}(\rho_i,\rho_i) = \operatorname{Var}(\rho_i)$.

Inserting expressions for derivatives and covariances, we find $\operatorname{Cov}(\tilde{\xi}_2,\tilde{\xi}_3) \approx \dfrac{1}{\tilde{\rho}_1 n}$. The multinomial logit transformation implies that any pair $(\tilde{\xi}_i,\tilde{\xi}_j)$, $i \neq j$; $i, j \neq 1$ has positive covariance, which is independent of the shares $\tilde{\rho}_2$ and $\tilde{\rho}_3$, at least as a first order approximation.

Figure 1

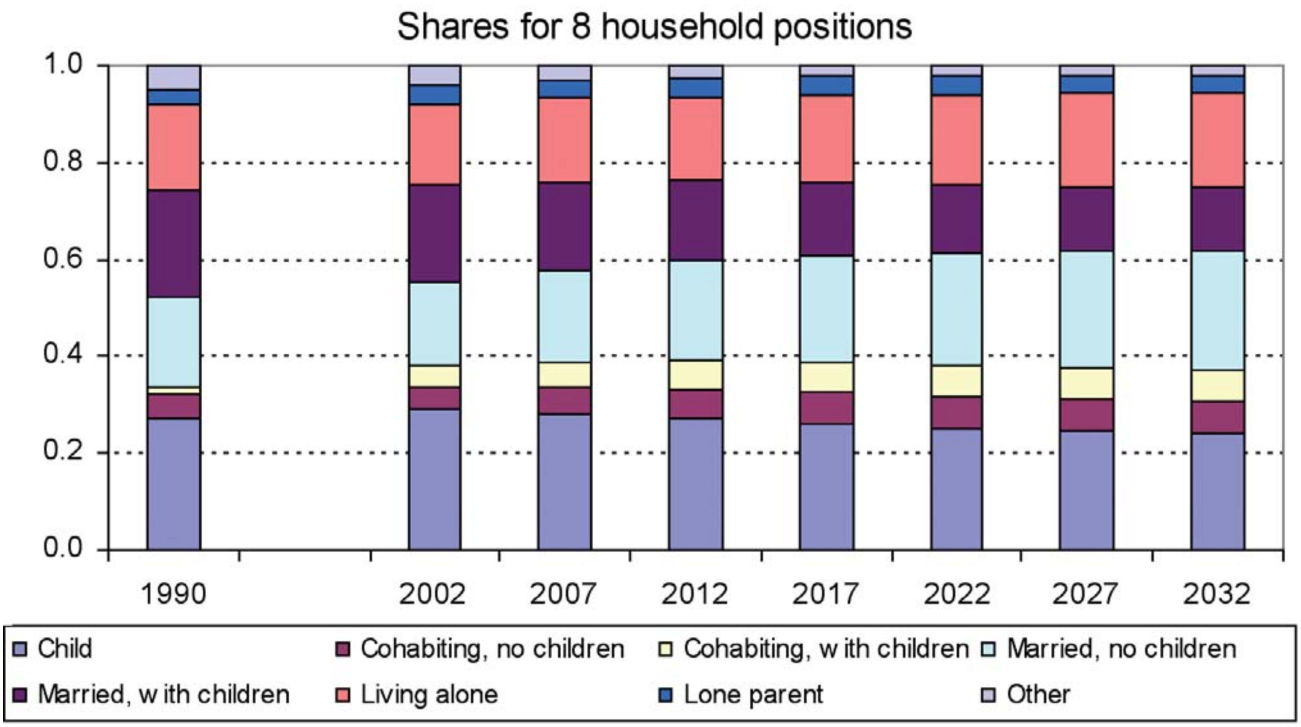

Figure 2

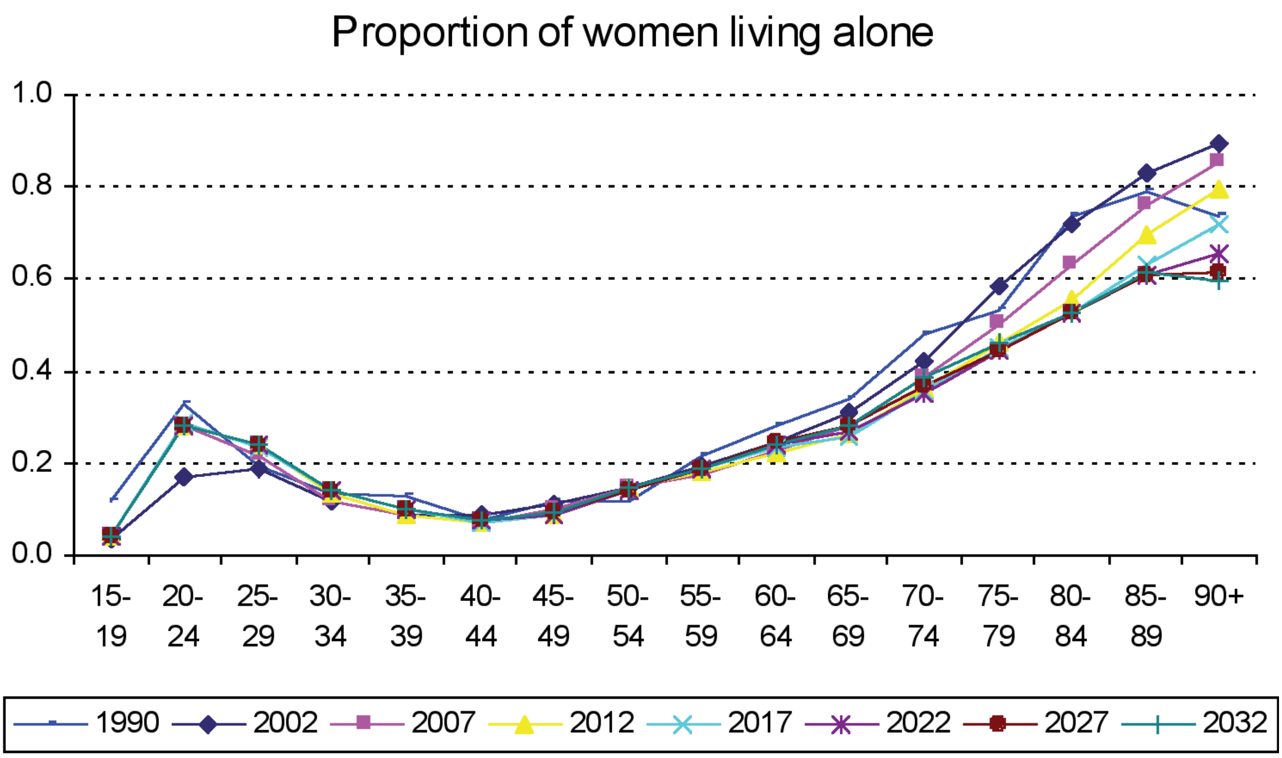

Figure 3

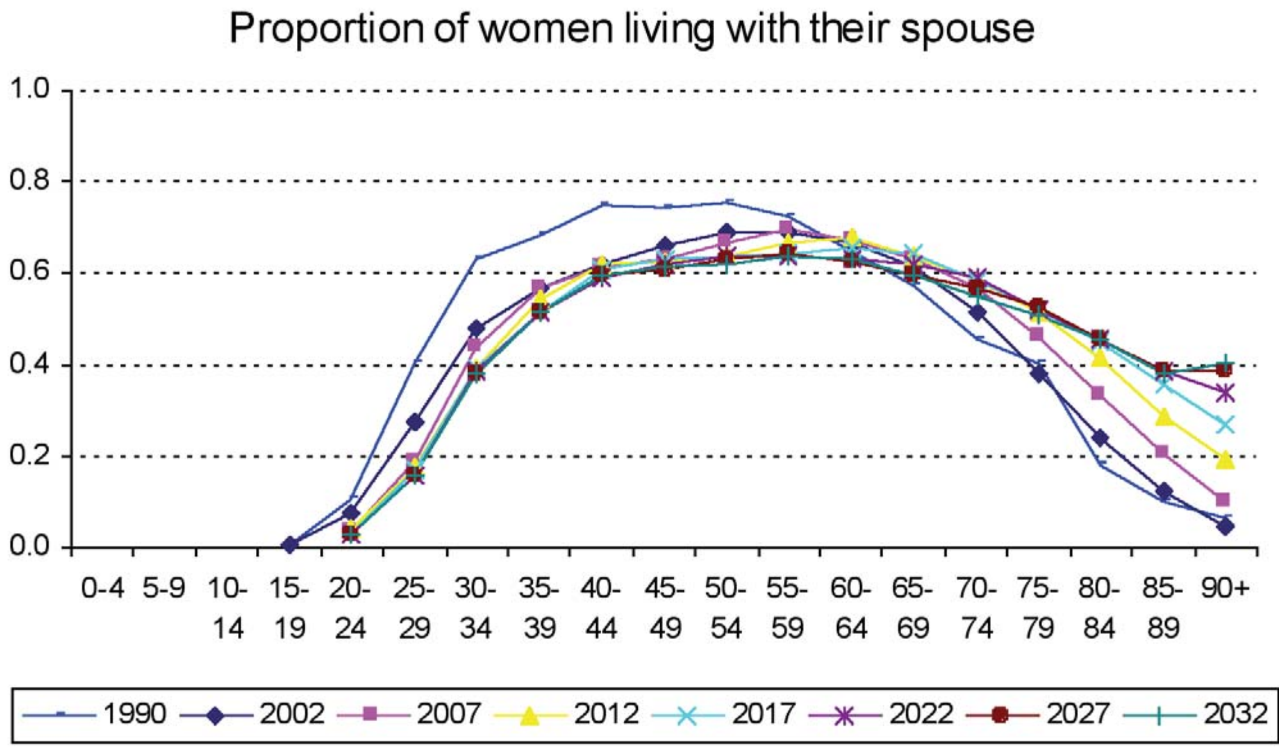
Proportion of women living with their spouse
1.0
0.8
0.6
0.4
0.2
0.0
0-4
5-9
10-14
15-19
20-24
25-29
30-34
35-39
40-44
45-49
50-54
55-59
60-64
65-69
70-74
75-79
80-84
85-89
90+
1990
2002
2007
2012
2017
2022
2027
2032

Figure 4

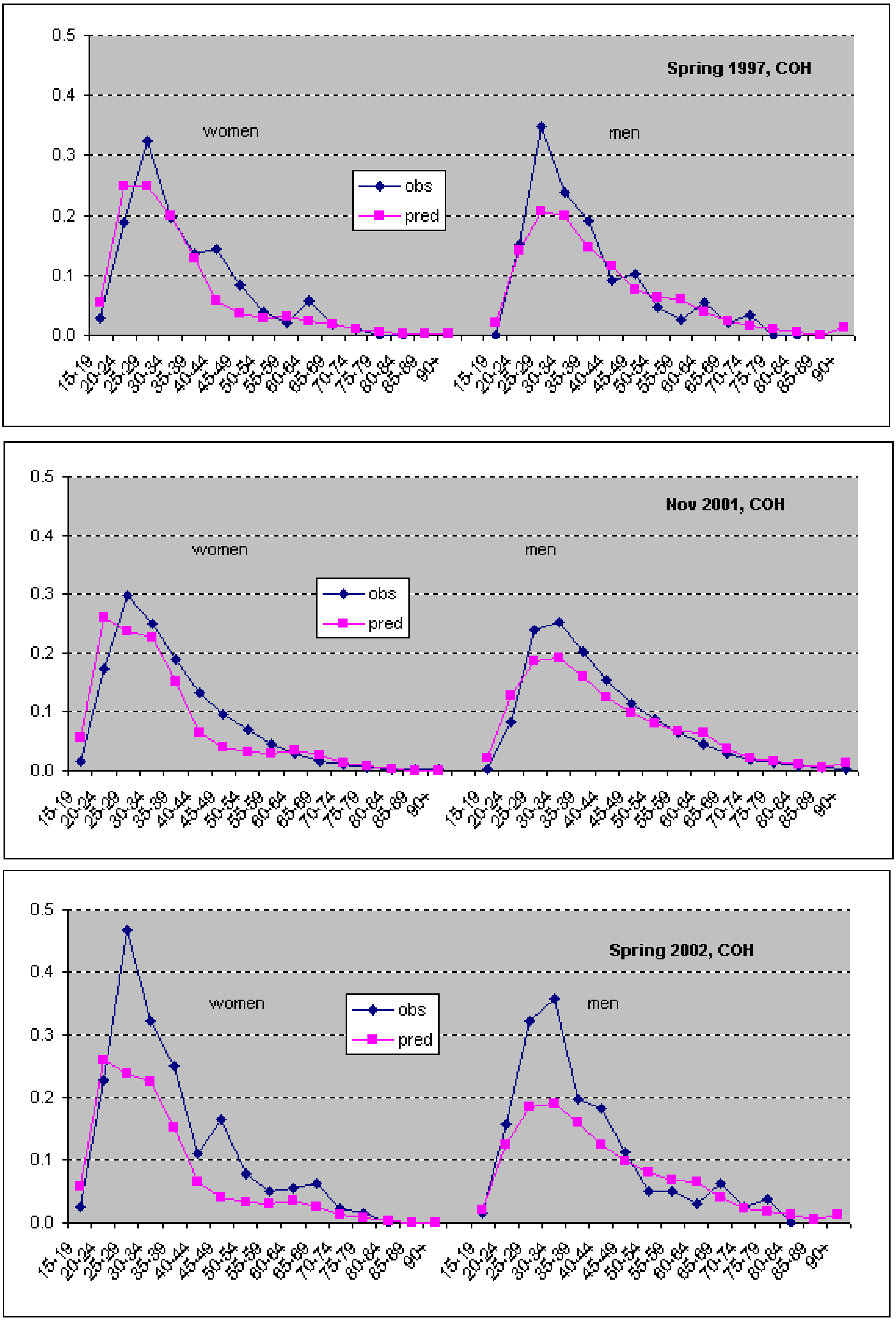
Spring 1997, COH
women
men
obs
pred
0.5
0.4
0.3
0.2
0.1
0.0
15-19 20-24 25-29 30-34 35-39 40-44 45-49 50-54 55-59 60-64 65-69 70-74 75-79 80-84 85-89 90+
Nov 2001, COH
women
men
obs
pred
Spring 2002, COH
women
men
obs
pred

Figure 5

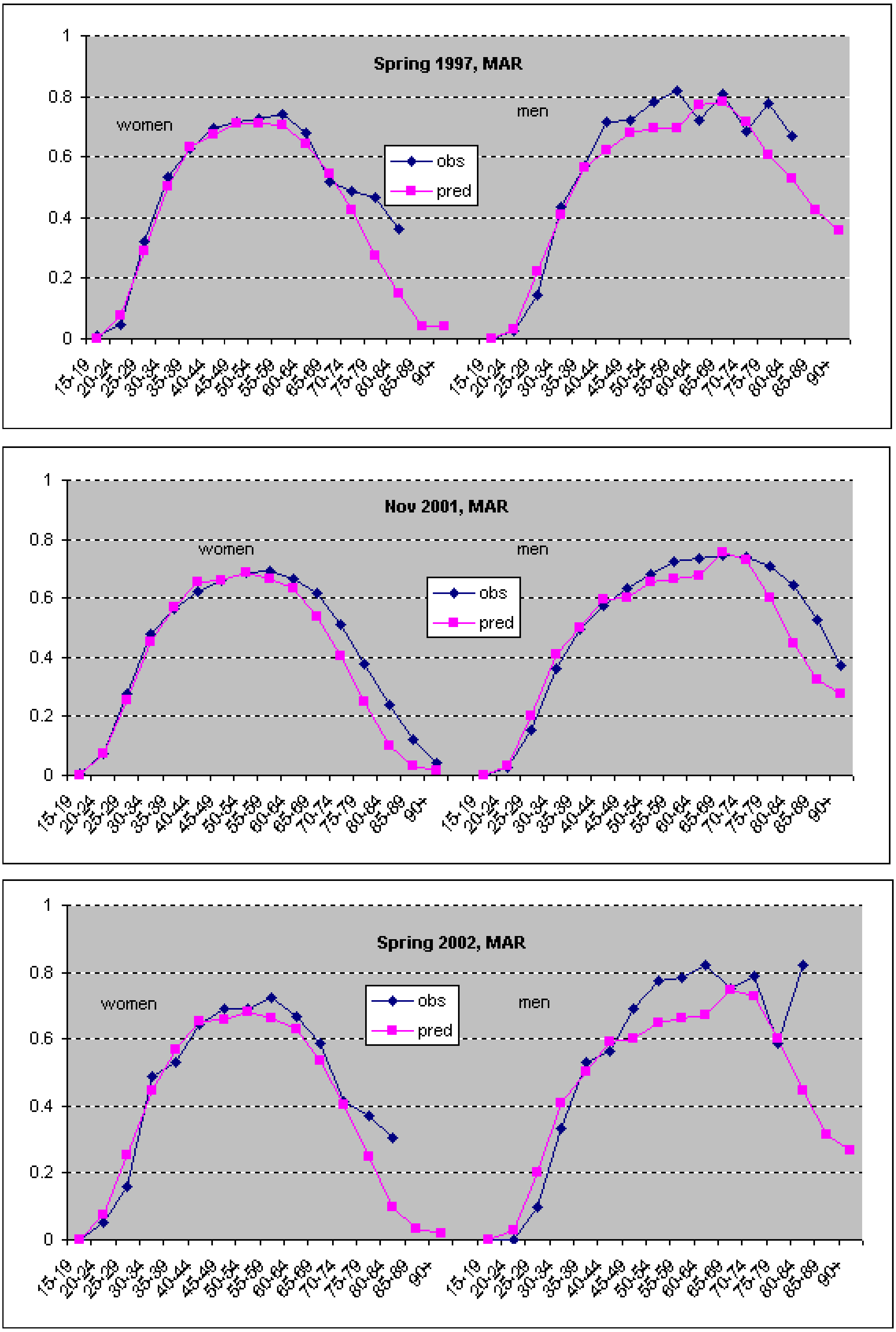
Spring 1997, MAR
women
men
obs
pred
1
0.8
0.6
0.4
0.2
0
15-19
20-24
25-29
30-34
35-39
40-44
45-49
50-54
55-59
60-64
65-69
70-74
75-79
80-84
85-89
90+
Nov 2001, MAR
Spring 2002, MAR

Figure 6

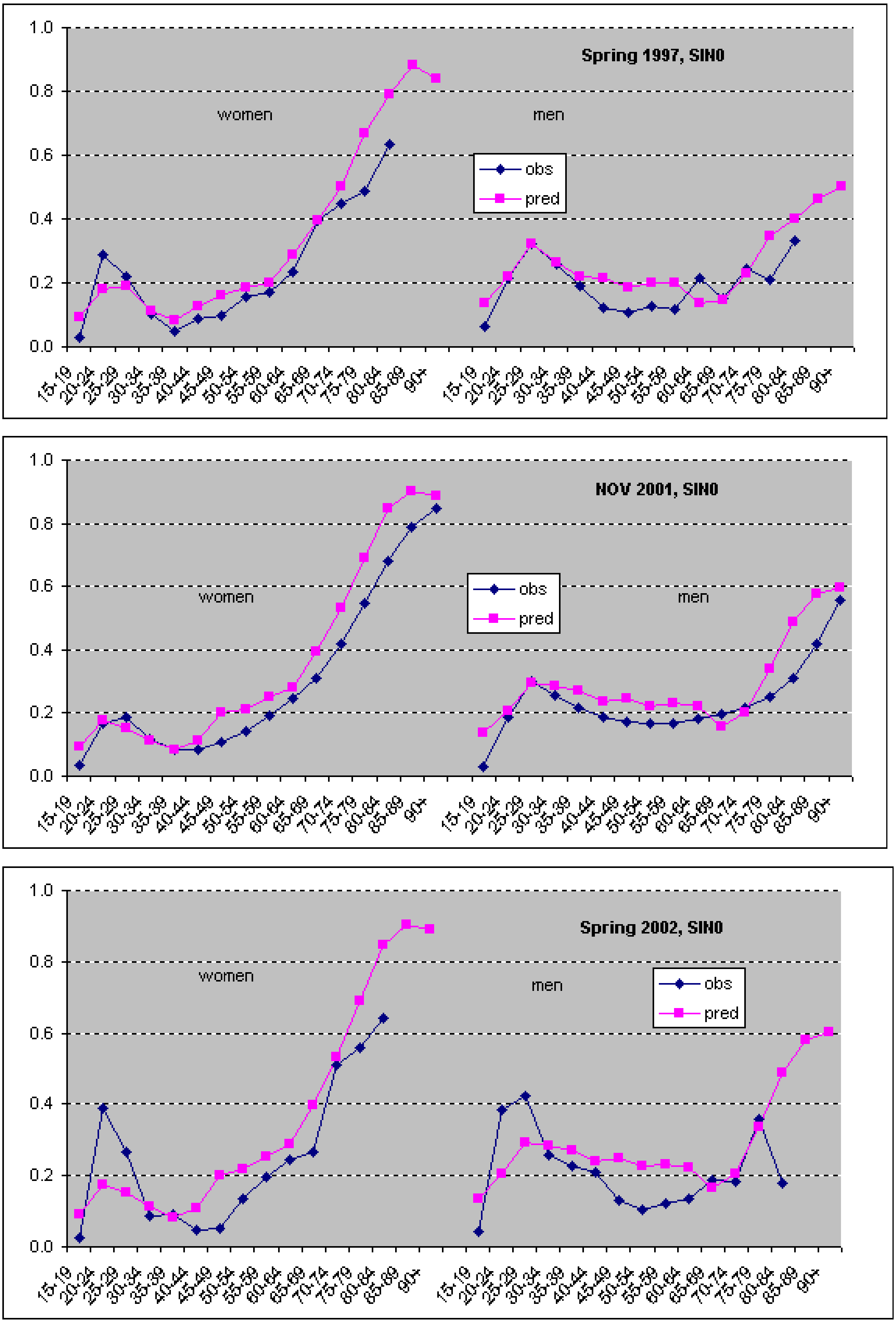
Spring 1997, SINO
women
men
obs
pred
NOV 2001, SINO
women
men
obs
pred
Spring 2002, SINO
women
men
obs
pred
1.0
0.8
0.6
0.4
0.2
0.0
15-19
20-24
25-29
30-34
35-39
40-44
45-49
50-54
55-59
60-64
65-69
70-74
75-79
80-84
85-89
90+

Figure 7a Box-and-whisker plots for shares in household position "married" in 2030, men and women, three age groups

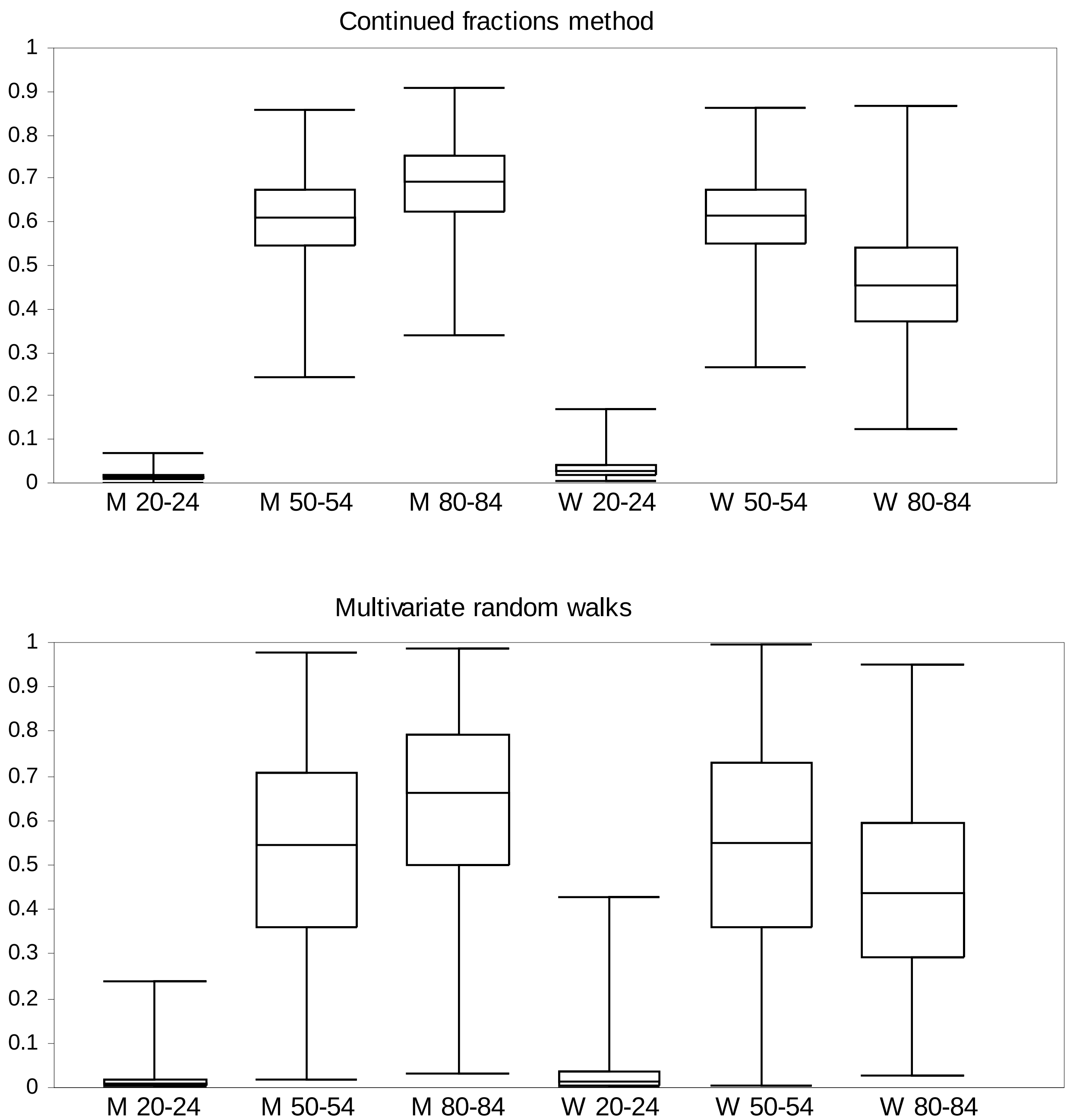

Figure 7b Box-and-whisker plots for shares in household position "living alone" in 2030, men and women, three age groups

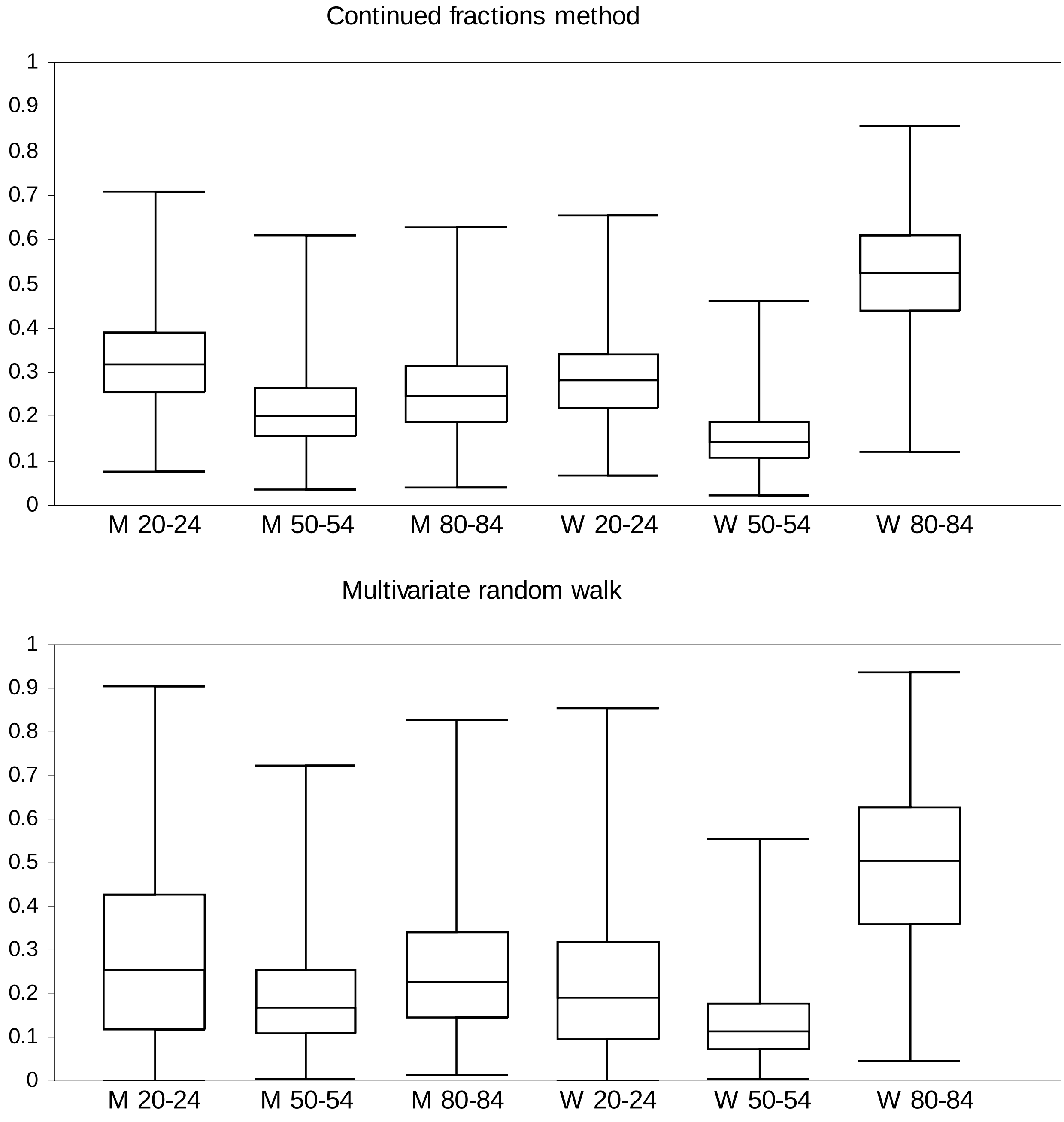

Table 1 Number of observations (n), mean cross products (mcp) and correlations (corr) for errors $e_{\xi}(j,x,s,t)$

	CHLD,COH	CHLD,MAR	CHLD,OTHR	CHLD,SIN+	COH,MAR	COH,OTHR	COH,SIN+	MAR,OTHR	MAR,SIN+	OTHR,SIN+
n	11	5	11	3	75	71	49	68	49	44
mcp	0.0300	0.1286	0.1768	0.6954	0.0030	0.0008	0.0112	0.0002	0.0091	0.0009
corr	0.0484	0.2344	0.0827	0.4729	0.0077	0.0013	0.0101	0.0005	0.0122	0.0008

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TOWARDS A DYNAMIC HOUSEHOLD PROJECTION MODEL FOR THE NETHERLANDS

Coen van Duin

Paper for Joint Eurostat/UNECE Work Session on Demographic Projections: 10 – 12 October 2007, Bucharest, Romania

1. Introduction

Statistics Netherlands biennially produces a set of long-term demographic projections. These projections are made in a three stage process. First, the national population forecasts are made, which project the population by age, gender and country of origin. Next, the household projections are computed. These project the population by household position and marital status, as well as the number of households by type and composition. Finally, the regional population and household projections are made. The regional projections are produced jointly by Statistics Netherlands and the Netherlands Institute for Spatial Research (RPB) and have a forecast horizon of twenty years. The national projections currently extent until 2050.

Compared to the population forecasts, household projections at Statistics Netherlands have a relatively short history. The first ones were published in 1992 (De Beer, 1992). The projection model was, in the terminology of Kuijsten and Vossen (1988) a mixture of a static and a dynamic macrodemographic model. Transitions between marital statuses were modelled and in that sense the model was dynamic. Transitions between household positions were not modelled. In this sense, the model was static. Since the unit of analysis was subpopulations rather than individual persons, it was a macro model. The projection method was purely demographic. No socio-economic or other non-demographic determinants were taken into account.

Over the years, a number of adjustments have been made to the original model. In 1993, the model was extended to include projections of the composition of households by size and by number of dependent children in the household (De Jong, 1994). In 2001, a cohort-perspective was introduced into the static part of the model (De Jong, 2001). In the latest round of household projections, an additional (static) sub model has been added which projects the household positions for first and second generation immigrants by country of origin (Van Duin, 2007).

Despite these adjustments, the present household model remains a hybrid, with static and dynamic elements. For the new round of projections early 2009, Statistics Netherlands aims to develop a projection method where transitions between household positions are also modelled. This article discusses some issues in the development of the new model. In section 2, the current model is described in some more detail. Section 3 describes the register-based household data on which the new projections will be based. Section 4 outlines the preliminary choices for the new projection method. Section 5 focuses on a particular problem in the formulation of assumptions for the transition probabilities: the effect of increasing (healthy) life expectancy on pair dissolution among the elderly.

2. The current household projection method

Statistics Netherlands' current household projection model consists of four sub models, which are run consecutively.

In the first stage, the population by marital status is projected using a macro simulation model. The input for this model are marriage probabilities, divorce risks, birth- and death rates and net migration numbers by marital status. The population by marital status is computed by generating marital status transitions, deaths and births in the start-of-year population, and by applying net migration numbers, in order to obtain the end-of-year population Small corrections are then made to impose consistency with the total population by age and gender from the national population projection. Also, consistency relations between marital status transitions for men and women are enforced.

Next, the population by household position and marital status is projected using a static macro model. This static model in essence distributes the projected population by age, gender and marital status over the household positions. Persons with the household positions "single", "single parent" or "head of household of type Other" are reference persons in their household. Women with the household position "Living with a partner" are also counted as reference persons. The number of households by type is calculated from the number of reference persons.

In the third stage the household composition by size and number of dependent children are projected using a dynamic macro model (De Jong, 1994). The transitions included in this model are birth of children and children leaving the parental household.

The fourth sub model is again static. It projects the number of first and second generation immigrants by household position. The projection is based on assumptions about how the age and gender-distributions of the household positions for the immigrant groups will change compared to those for the total population (convergence or divergence). Consistency with the results and assumptions of the household projection for the total population is imposed.

The multi-stage structure of the projections may seem overly complicated. Partly, it is a result of the fact that different parts of the model were developed at different times. It is less time consuming and saver to add a new sub model at the end of the projection-chain than to change a well-functioning existing model. A disadvantage of this kind of staged process is that information generated at the later stages is not used at the earlier stages. For instance: the influence of a changing household structure on the number of births is not explicitly modelled in the current method, since population projections are made first and household projections second.

An advantage of the multi-stage structure is that the most reliable projections are made first and provide a reference frame for the projection of more uncertain quantities. Even though household-developments have an influence on the number of births, fertility behaviour is usually more predictable than household formation. This makes it unlikely that explicitly modelling the influence of expected trends in household composition on the number of births will improve the fertility forecast.

Another advantage of the multi-stage approach is that optimal use can be made of the available data. The marital status model uses the long time series that are available on marital status transitions. Similarly long time series exist on births by parity and age of the mother. These data are exploited in the macro simulation model which projects households by size and number of children. Reliable data on transitions between household positions was, until recently, scarce. This is the main reason why a static projection model for household positions continues to be used.

3. Register-based household data

In the period that the first household forecasts at Statistics Netherlands were made, data concerning household positions was only available from surveys. The projection of 1992 used data from the Household Demand Survey (WBO). This survey, held every four years, collected information from some 50 thousand households. Among other variables, it contained information on household positions at the time of the survey and on the household position a year earlier. Later projections used data from the Labour Force Survey, for which data was collected every year from 100 thousand persons.

In the early 1990s, data from the Household Demand Survey was used for dynamic household projections in the Netherlands. These were the first projection with the micro simulation model PRIMOS (Heida, 1991) and a number of demographic scenario's (one of which, termed "the realistic scenario", could be considered a projection) with the macro model LIPRO (Imhof and Keilman, 1991). Both of these dynamic projections used the retrospective data from the survey to estimate transition probabilities. Not each transition probability in the model could be estimated directly from the data in the survey, which made it necessary to use a number of simplifying assumptions. Sampling errors in the data were another problem for these early dynamic projections.

The introduction of the electronic municipal population register at the end of 1994 meant that a near-integral source of information became available. The data in the electronic register go back to the year 1995, which means that, at the moment, time series over a period of more than a decade are available. Since 2001, Statistics Netherlands publishes household statistics based on the information in this register.

Unlike the housing demand surveys, the population register does not contain the variable "household position". It does contain data on address, age, gender, marital status and family relations (spouse, parents, children) of registered persons. This data is used by Statistics Netherlands to construct a households register (Harmsen, 2003). First, persons are grouped together by address. Next, a derivation scheme is used to determine which groups of persons at the address constitute a household, and what the household positions of the persons in these groups are.

The derivation scheme is partly deterministic. If only one person is registered as living at an address, the household position is determined to be "single". If a married couple is living at the same address, their household positions are determined to be "partner in a married couple". If two unrelated, unmarried persons move to a new address at the same date, they are classified as cohabiting. Using these types of decision rules a household position can be determined for about 93% of the cases.

In about 7% of the cases, various household positions are possible given the data in the register. In these cases, a probabilistic imputation scheme is used. This scheme takes into account the age difference of the people living at the address, their gender, their marital status, possible family relations between the people at the address and whether the address is in a rural or urban area.. The parameters of the imputation model are estimated from survey data.

For each new year, the probabilistic imputations are performed independently of those for the previous year. As a result, artificial transitions between household positions would be found if data from consecutive years were compared at the level of individual persons. In one year, two persons living at the same address might have been classified as cohabiting, while in the next year, the positions for the same two people at the same address would be imputed as single. The probabilistic fluctuations caused by the derivation scheme only lightly affect transversal quantities such as total number of persons per household position per year, because the fluctuations average out. They do however limit the usefulness of the data for longitudinal analyses.

To overcome this problem, a scheme is used where the imputed household positions in previous years are made consistent with those from the most recent year (Witvliet, 2002). The philosophy behind this procedure is that the data from the most recent year will be the most accurate, since it is based on the largest amount of information. For instance, if two persons that are living at the same address get married, it is very likely that they were cohabiting at this address before they got married. If their household positions were imputed as "single" in the previous year, this should be corrected retrospectively.

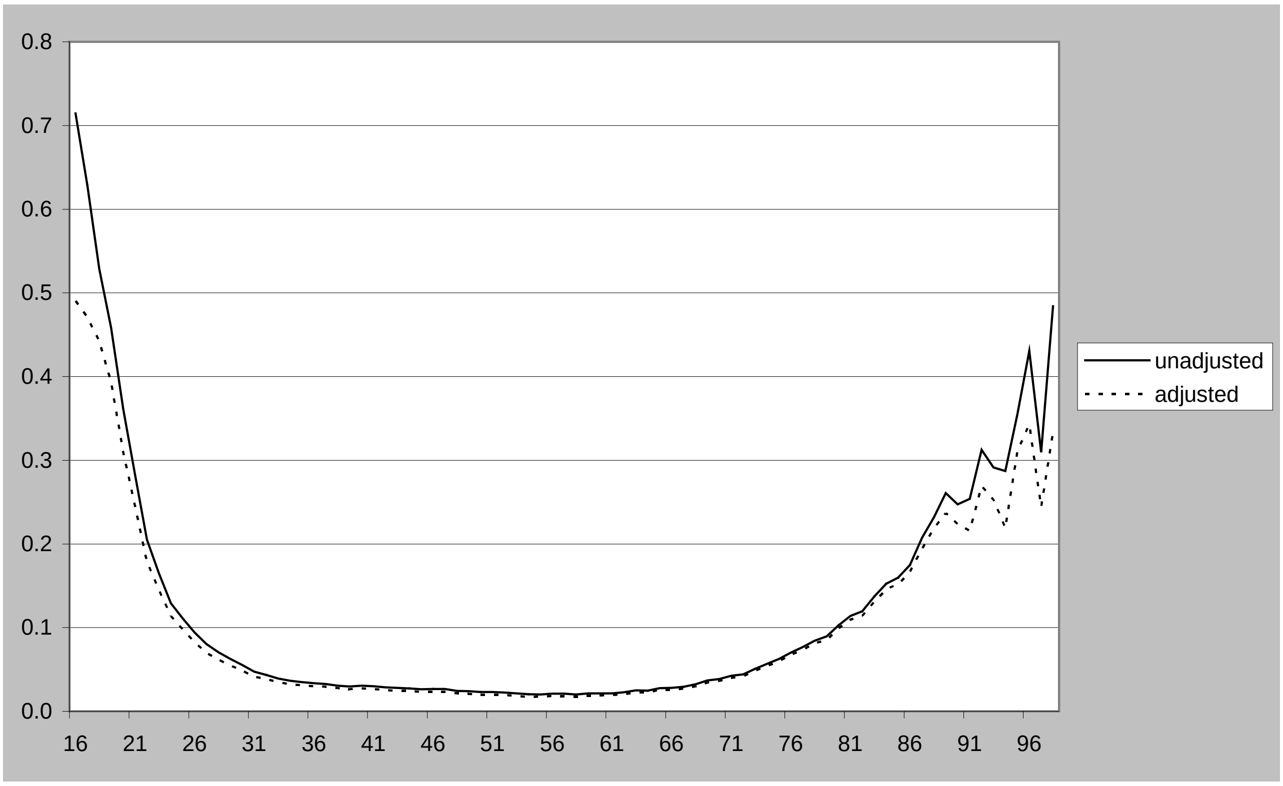
After the household positions for a new year have been derived, the imputed positions for the previous year are adjusted to conform with those for the new year. The adjustments introduce a small bias, since imputation errors where cohabiting people are imputed as singles are more likely to be discovered (for instance if they marry, or if they move together to a new address) than errors where two single people are imputed as cohabiting. Therefore, a second round of adjustments is applied which restores the original shares of the various household positions in the population. This second round of adjustments also assures that the adjusted numbers of persons per household position for the previous year differ little from the numbers that were derived, and published, a year earlier.

After the imputations in the previous year have been adjusted, those in the year before that are adjusted in the same way. This procedure is repeated until the first year in the register is reached. It has been shown that the method is stable and that the number of adjustments does not grow as one adjusts data from years that are further in the past.

Figure 1 shows the yearly exit probability from the state Living with partner for women in 2005, calculated from unadjusted and adjusted household register data. The adjustments reduce the estimated number of transitions at the young and old ages. At intermediate ages, the adjusted and unadjusted transition probabilities differ very little.

The register-based household data provide a rich source of information on household transitions. A large number of background variables is available, such as age, gender, country of birth (of registered persons and their parents), date of immigration (for immigrants), number and age of children, etc.. Because of the large mass of data, analyses at quite detailed levels are feasible. Also, the coupling between the household- and the population register allows us to investigate the relation between household transitions and demographic events, such as migration, births and deaths. Since part of the data is based on imputation rather than direct observation, the imputation procedure that was used should always be taken into account when interpreting the results of analyses on this data set.

Figure 1 Yearly exit probabilities from the state Living with partner (women, 2005)



The register-based household data have been used by Statistics Netherlands and external users for a wide variety of demographic analyses. The regional projection models PRIMOS (ABF research) and PEARL (Statistics Netherlands/ Netherlands Institute for Spatial Research) also use it. The data is also employed in a micro simulation model that is being developed as part of the MicMac project (De Beer, 2006).

4. Outlines of the new projection method

Statistics Netherlands aims to develop a fully dynamic household projection method which exploits the longitudinal information in the household register. One advantage of such a method is that it yields projections for flows between household positions. With the development of the regional projection model PEARL, producing national projections on such flows has become urgent. Since PEARL models household transitions, it needs national projections for these flows on which to base the time-development of the household transitions at the regional level.

The new projection method will be based on a dynamic macro model. It will therefore model transitions between demographic states on the basis of transition probabilities. These transition probabilities can depend on a number of background variables. Certainly, they should depend on age and gender, possibly on other variables such as ethnicity or number of children. There are no plans to include non-demographic variables, such as for instance level of education, into the projection model.

Since we aim to produce the same output as the current projection model, the states that are distinguished in the model should preferably correspond at least to those for which projections are now made. That means that we require a distinction by seven household positions, crossed with four marital states. The household positions distinguished in the current projection model are

1. Single
2. Living with partner
3. Living in parental home (dependent child)
4. Single parent
5. Head of private household of type “other” (meaning: not a single person household, a household headed by two partners or a single parent household)
6. Other member of private household
7. Living in institution

The four marital states are Never married, Married, Widowed and Divorced. As some of the 28 combinations of household position and marital status are very rare, there will be room to reduce the number of states somewhat. On the other hand, it may be necessary to include additional states in order to improve the performance of the model. The expected developments in the household structure for various subgroups in the population should be taken into account when defining the states in the projection model (Prinz, 1994).

Apart from a projection of the number persons by household position and marital status, Statistics Netherlands also publishes projections about the number of households by size, by number of children and by age of the youngest child living in the household. These projections are made using the third component of the current projection model: the dynamical model for household composition. This model distinguishes females by age, parity (0,1,2,3,4+ children) and by the ages of the eldest and youngest child still living in the household. Introducing all this additional background information into the dynamic projection model for household positions would very much enlarge the state space. At present, we therefore aim for the new model to replace only sub models 1 and 2: the dynamical model for marital status projections and the static model for the projection of household position by marital state. In time, it should be able to replace sub model 4 as well (household positions of first- and second generation immigrants). Since children are attributed to households in a separate procedure, it is not necessary to split the position Dependent child into sub states for different household types, as was done in the LIPRO household projection for the Netherlands of 1991 (Imhoff, 1991).

For the technical implementation of the model, there are several options. In the research stage, we will probably be using simple prototypes programmed in excel. For the longer term, it seems more attractive to use an established model such as the LIPRO model of the Netherlands Interdisciplinary Demographic Institute, or an adjusted version of PEARL, rather than reinventing the wheel. Because of its flexibility, the LIPRO model will probably also be used in parts of the development process.

5. Time dependence of pair dissolution among the elderly

Once a model has been specified and implemented, a next challenge is to specify the set of "most probable" transition probabilities on which to base the projection. Typically, assumptions about the time development of the transition probabilities used in a projection will be quite conservative. One will assume that the probabilities will remain as they are, that a decreasing or increasing trend in (a summary measure of) the transition probabilities will continue for some more years and than stabilize, or that the transition probabilities will return from the current value to an average level over the past (say) ten years.

Death rates are an exception to this rule. Most population projections assume a reduction of the death rates right up to the projection horizon. If the deceased person is living with a partner, his or her death will have generated a household transition of the partner. This means that, especially at the higher ages, it may be insufficient to use a no-change assumption for the exit probabilities from the state Living with a partner.

Another case where a no-change assumption is likely to be insufficient is the transition from living in a private household to living in an institution. In the Netherlands, there is a strong trend for people to remain living independently rather than moving to a retirement home up to increasingly higher ages. There are good reasons to assume that this trend will continue. An increase in healthy life expectancy will mean that people will become dependent on institutional care only at higher ages. Also, with an increase in the population of the elderly, there will be financial pressures to enable this group to continue living outside an institution for as long as possible.

As a first analysis into the time-dependence of transition probabilities, we look at the effect of expected changes in death rates for men and in the risks for men to move to an institution on the separation risks for women.

It could be argued that we do not need to introduce time-dependence into the a priori exit probabilities as a result of the partner dying or moving to an institution. After all, the outlined problem is one of consistency between transitions for men and women. If the projection model imposes consistency between the number of partnership dissolutions between men and women (as LIPRO does), then a decrease in the death/institutionalization probability for men will result in a decrease in the posteriori pair dissolution probabilities for women, and vice versa. A problem with this approach is that the changes in death and institutionalization rates are very age-specific, while most consistency algorithms, also those implemented in LIPRO, only impose consistency between numbers of transitions at an aggregated level. In other words: a reduction of the probability for men to die at age 80 would lead to an adjustment in the probability for women to lose their partner at age 20.

Figure 2 shows the yearly exit probabilities for women from the state Living with a partner for the year 2005, distinguished by the household position at the end of the year (institution or private household). Between the ages of 40 and 55, the exit probabilities for women from this state decrease with age, as a result of a decreasing risk of separation. Above the age of 55, the exit risk starts to rise. For ages of 80 years and older, a large part of this increase is due to women moving into an institution.

Figure 2 Yearly exit probabilities for women from the state Living with a partner by target state (2005)

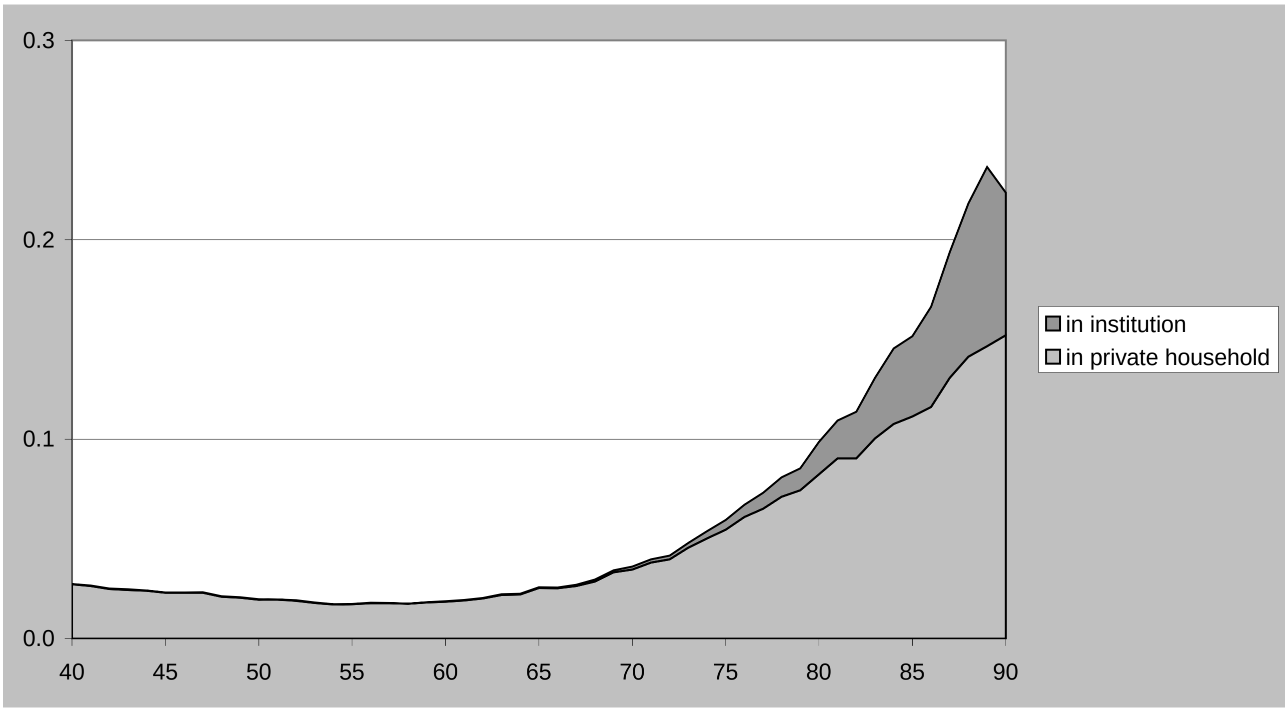
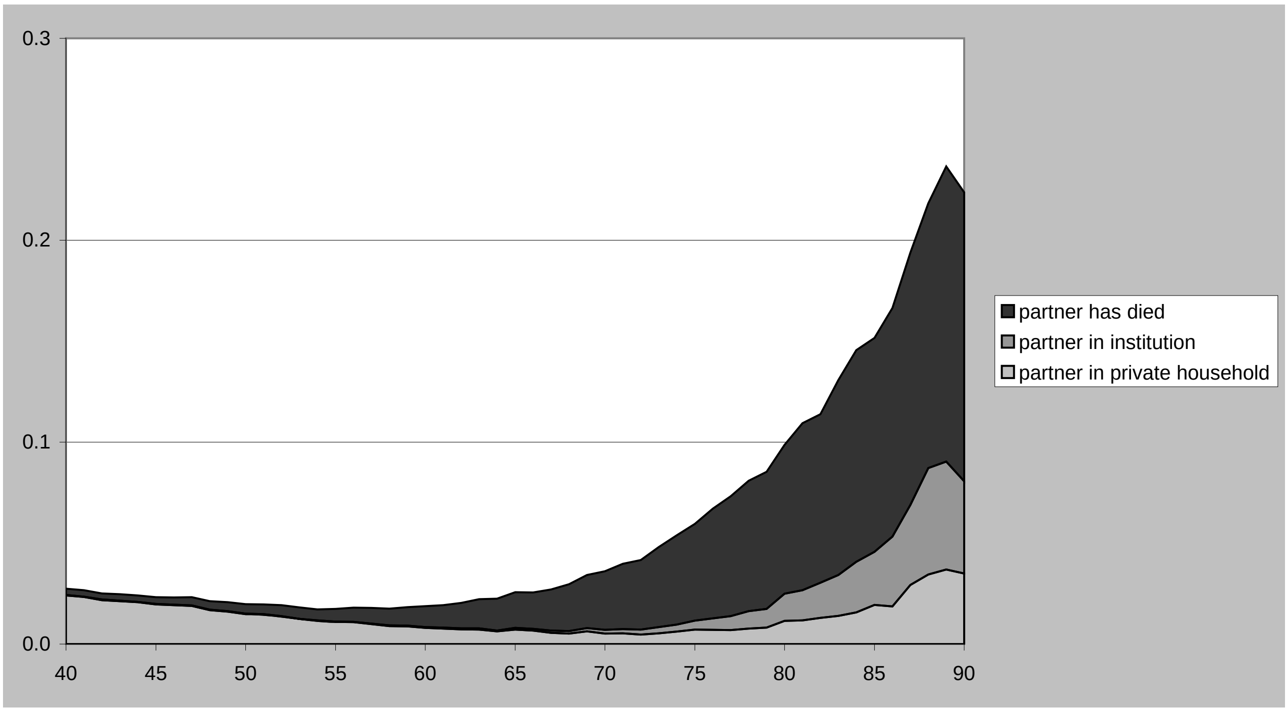


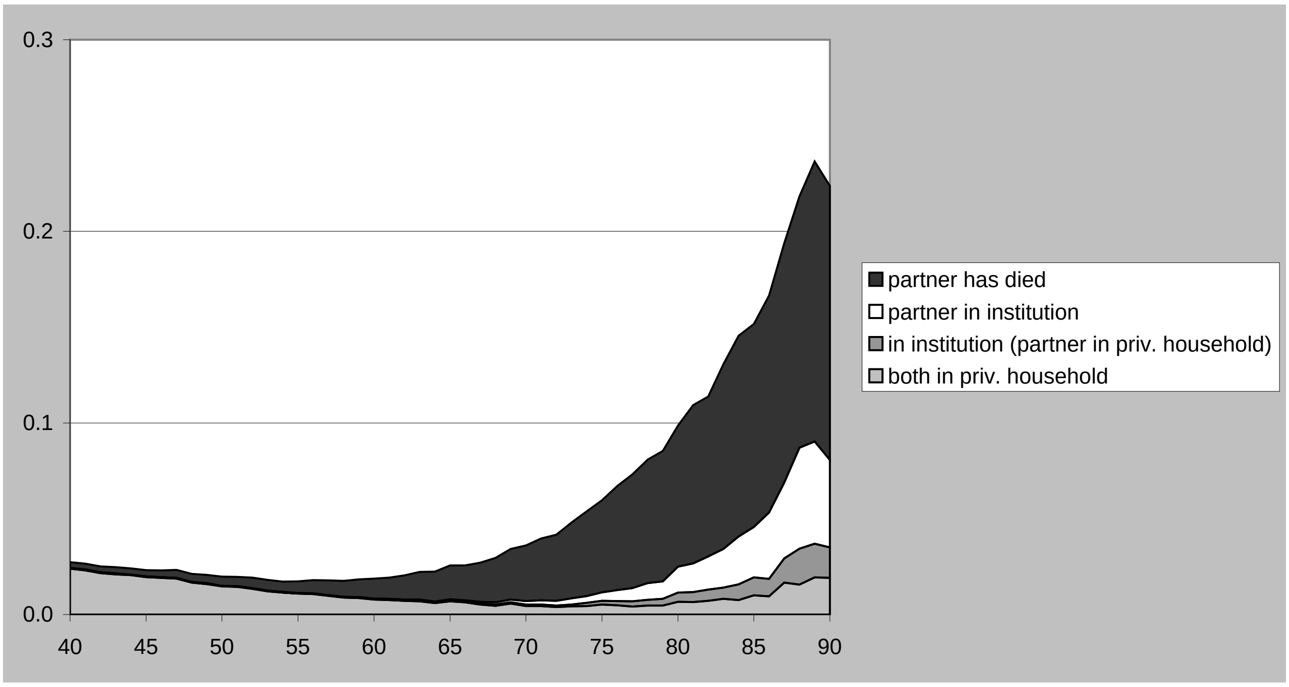
Figure 3 again shows the yearly exit probabilities for women from the state Living with a partner, but now distinguished by the state of the partner at the end of the year (dead, in an institution or in a private household). It can be seen that the rise of the exit probabilities between the ages of 55 and 75 can be almost completely attributed to an increase in the risk of the partner dying. At ages of 75 and over, an increase in the institutionalization risk of the partner also begins to play a role.

Figure 3 Yearly exit probabilities for women from the state Living with a partner by target state of the partner (2005)



In figure 4, the exit probabilities are distinguished simultaneously by the end-of-year state of the women and of their partners. It can be seen that nearly the entire increase in the exit probabilities can be explained in terms of the partner dying, the partner entering an institution, or the woman herself entering one. Clearly, the expected changes in these processes could have a large effect on the household positions of the elderly.

Figure 4 Yearly exit probabilities for women from the state Living with a partner by target state of the woman and her partner (2005)



In order to quantify this effect, we can adapt a procedure that is being used to estimate the widowing risk in the dynamic marital status projection model. The following estimate is used

$$p_{wid.}(x, female, married) \approx p_{death}(x + \Delta_x, male, married),$$

where $p_{wid.}$ is the age specific yearly risk to be widowed and p_{death} the age specific yearly risk to die, x is the age of the women and Δ_x is an age-dependent shift which transforms the death-risks curve for married men into a widowing-risks curve for married women.

Figure 5 shows the age-specific widowing risk for women, and the shifted and unshifted mortality risk for married men. The risks are calculated for the period 1995-1999. The shifted mortality curve for men provides quite a good estimate for the widowing curve for women.

Figure 5 Female widowing risk and mortality risk of married males, 1995-2000

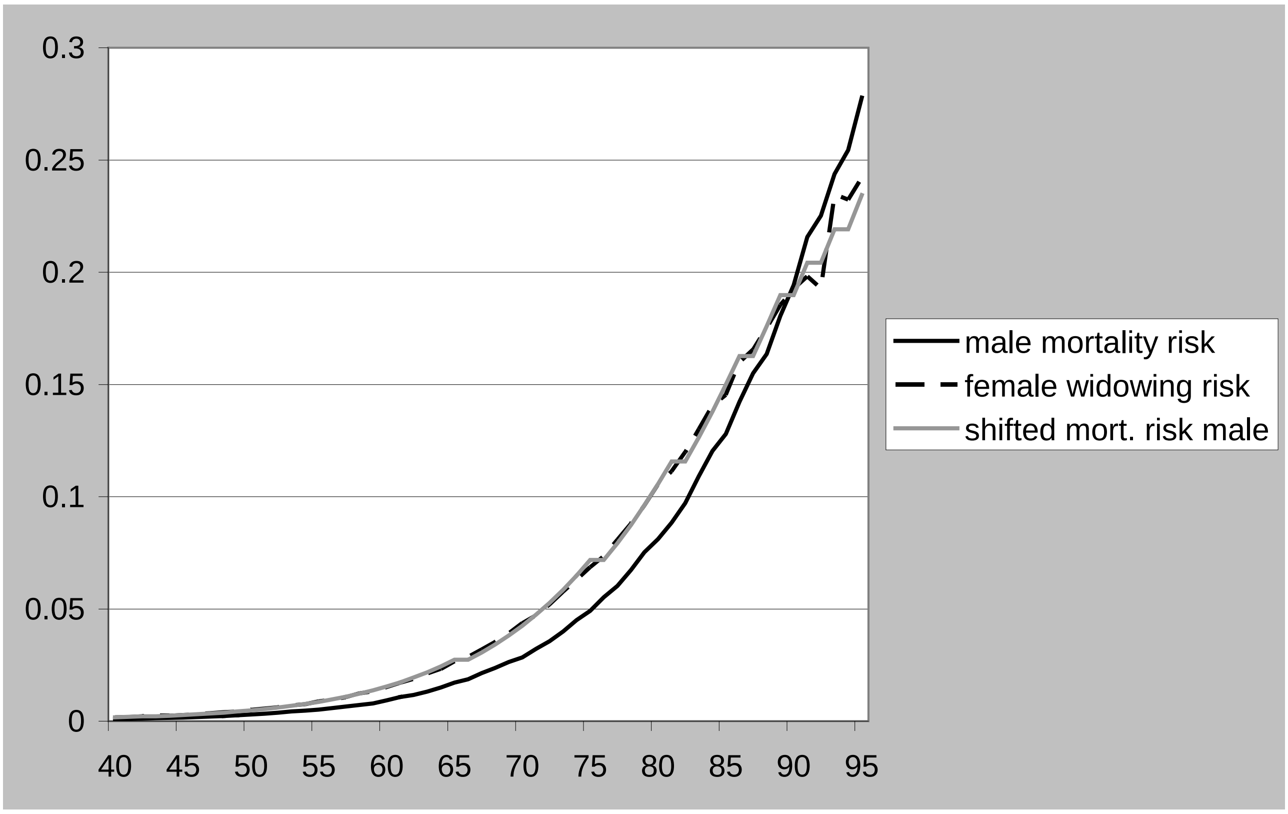
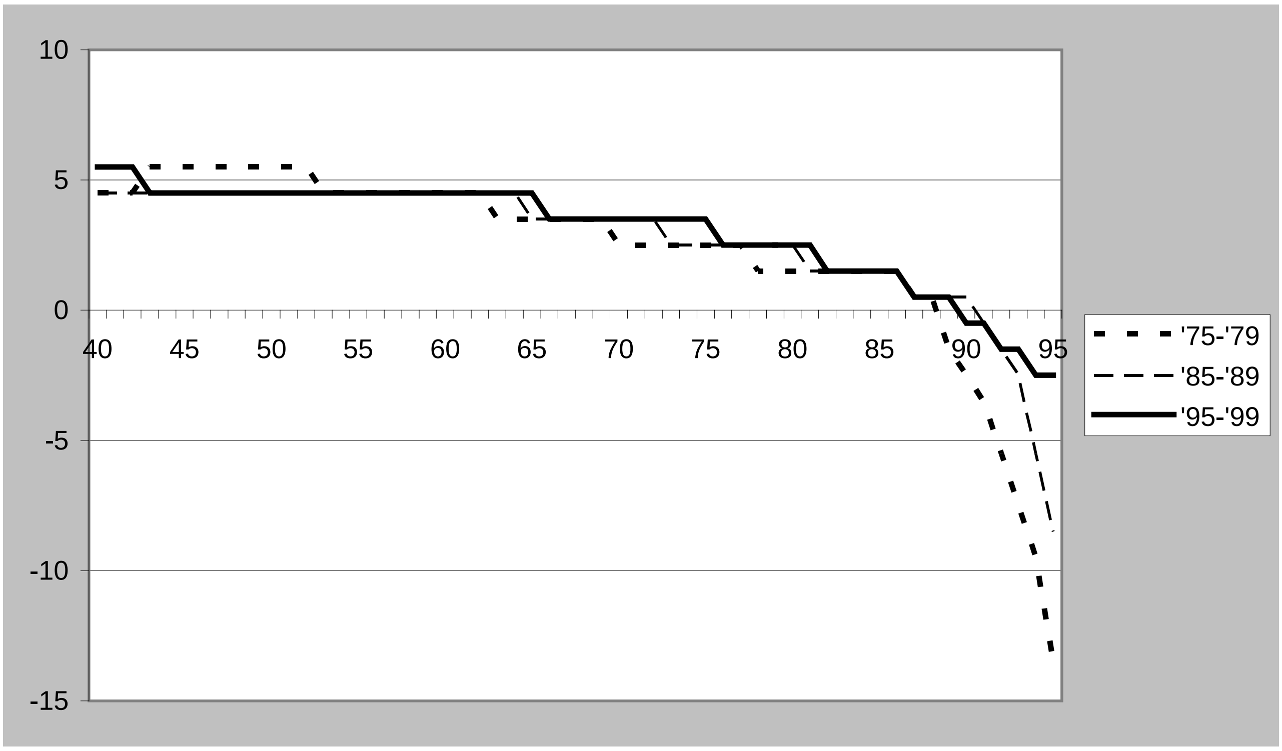


Figure 6 shows the shift parameter Δ_x calculated on data from the periods 1975-1979, 1985-1989 and 1995-1999. For middle-aged women, the age shift is between 4 and 5 years. In the Netherlands, the average age difference between the male and female partner in a marriage is 2-3 years. However, middle aged women with relatively older partners are more likely to become widowed, which is why the age shift is larger than the age difference for marrying couples. At the oldest ages, the shift parameter becomes negative. This is due to selection: a woman who is still not widowed in her 90s is more likely to have a younger husband than a married woman of 40.

There is some time dependence in the shift parameters which can be explained by the selection effect becoming weaker as the life expectancy of (married) men increases. Upton the age of 85, the time dependence of the shift parameter is weak. Much of the effect of reduced mortality for men on the widowing risks for women can therefore already be captured in a model that uses time independent shift parameters.

Figure 6 Age-shift parameters calculated from data from three periods



Using age-shift parameters, we can relate, in a similar way, the mortality risks of men that are living with a partner (married or unmarried) to the exit probability for women from this state because of the death of their partner. We can do the same for the risk of men entering an institution. We label the event that a partner dies "event 1" and the event that the partner moves to an institution "event 2". The probabilities of these events for women can be estimated as

$$p_1(x,\ female, living\ with\ partner) \approx p_{death}(x + \Delta^{(1)}{}_x, male, living\ with\ partner)$$

$$p_2(x,\ female, living\ with\ partner) \approx p_{institution}(x + \Delta^{(2)}{}_x, male, living\ with\ partner)$$

and we can estimate the time development for the female separation risks as

$$p_1(x,\ female, living\ with\ partner,\ j_1) \approx$$

$$p_1(x,\ female, living\ with\ partner,\ j_0)\,\frac{p_{death}(x+\Delta^{(1)}{}_x, male, j_1)}{p_{death}(x+\Delta^{(1)}{}_x, male, j_0)},$$

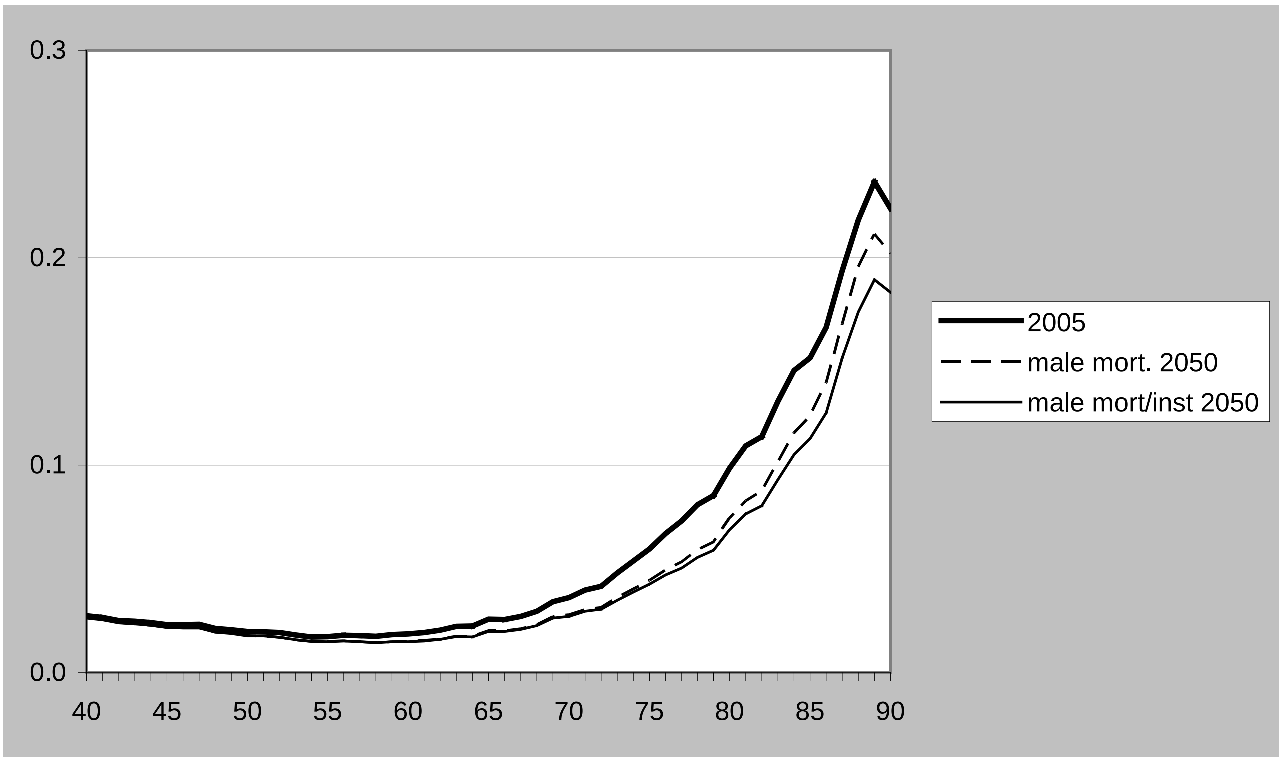
$$p_2(x,\ female, living\ with\ partner,\ j_1) \approx$$

$$p_2(x,\ female, living\ with\ partner,\ j_0)\,\frac{p_{institution}(x+\Delta^{(2)}{}_x, male, j_1)}{p_{institution}(x+\Delta^{(2)}{}_x, male, j_0)}.$$

Here, we have made an additional assumption that the relative change in p_{death} and $p_{institution}$ for men living with a partner is the same as for men not distinguished by household position.

To get a feeling for the size of these effects, we calculate the exit probability from the state Living with partner for women based on adjusted risks for these two events. We adjust the risks for event 1 by using the mortality risks for men in 2005 and the projected risks in 2050. As an approximation, we use the age-shift parameters for widowing. The risks for event 2 are adjusted by assuming that the risk for men to move to an institution is reduced by 40%. This is roughly in line with the assumptions for the proportion of persons living in an institution in 2050 in the latest household projections. Since the same reduction factor is applied to all ages, the age-shift parameters for event 2 have no influence on the results. Figure 7 shows the adjusted exit probabilities.

Figure 7 Exit probability from state Living with partner for women, 2005, with adjusted risks for event 1 (dashed line) and for events 1 and 2 (thin solid line)



The assumed trends in mortality for men and in the age at which men move to an institution would result in a shift, to higher ages, of the separation risk-curve for women with some 3-4 years. This is not a small effect. Therefore, it seems necessary to take into account the age specific relation between these transitions for men and the separation transitions for women.

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PROBABILISTIC HOUSEHOLD PROJECTIONS BASED ON AN EXTENSION OF HEADSHIP RATES METHOD WITH APPLICATION TO THE CASE OF RUSSIA

Sergei Scherbov and Dalkhat Ediev[1]

The paper presents a probabilistic method for projecting the number of households and their distribution by size. The method combines probabilistic population projection with probabilistic headship rates model. For distributing the households by size we use recently developed models for conditional proportions of households of different sizes among households of the same or bigger size. Models are approbated on the case of Russia with fertility scenario assuming considerable success of demographic policies recently introduced in the country. Parameters for household models are estimated from the 1994 microcensus sample using bootstrap procedures. Our results show significant changes in future distribution of private households in Russia. Also, despite overall decline in the number of households, they imply persisting shortage of housing infrastructure for four-person households. Typically these would be households of two parents with two kids, i.e., families put into the focus of recently introduced demographic policies.

1. Introduction

Household projections are important for planning purposes and also for analyzing the implications of population dynamics for consumption, labor, ecology, etc. (MacKellar, et al. 1995, O'Neill and Chen 2001, Perz 2001, Prskawetz, et.al. 2004). In some areas, like housing and urban planning, projections of distribution of households by size are of key importance, as they are relevant to decisions involving substantial long-term public and private expenditures. It is important for such applications to have better knowledge about trends expected in the future and also about *uncertainty* accompanying such trends. Understanding the uncertainty of households' prospects is important, as it is not always possible to easily and quickly adjust investment decisions to deviations from the projected trend. This issue is also important for developing demographic and taxation policies oriented on family and households' composition.

This work utilizes recent advances in modeling households' distribution by size and also in probabilistic projections to develop probabilistic household projections for Russia in 2005-2050. The case of Russia deserves special attention for several reasons. During the last century the country has passed through many dramatic social and economic disturbances, which imprinted deeply into the population age structure and have serious implications for demographic prospects (Ediev 2001). Almost inevitable depopulation of the country and changes in age structure may have significant and, some times, contradictive effects on prospects of households of different sizes. At the same time, the country is facing an urgent need for better planning and improving the infrastructure and living arrangements in particular. Hence the importance of understanding the prospects and uncertainty of household dynamics in future.

2. Methodology and data

This work is based on an extension of the conventional headship rates method (United States National Resources Panning Committee 1938, United Nations 1973). Several rationales support such choice. Firstly, these are the headship rates method and its extensions, which are widely used by governmental agencies, despite progress in more sophisticated modeling of households. Age-specific headship rates happen to be a remarkably stable indices, which vary only moderately despite

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significant demographic developments observed in many populations. Changes in fertility and mortality have only a limited effect on headship rates, and it is population age structure, which is a primary source of variations of households number and distribution (see the appendix for some analytical results in support for this view and implications for stable population). At older ages mortality and morbidity may play more significant role as a factor of headship rates' dynamics. Yet, this effect may be neglected in a study of the overall number of households, which this work is devoted to.

The need for probabilistic household forecasting has been acknowledged elsewhere (J.de Beer and M.Alders 1999[2], Leiwen and O'Neill 2004). De Beer and M.Alders forecast uncertainty in future number of households introducing a number of assumptions regarding institutional population, probability of changes in the age at leaving the parental home, assumption about the conditional probability of changes in the percentage of people living alone etc. In order to derive these assumptions a very good information base should be available. From the data available for Russia deriving such distribution would require too much of subjective judgment. Leiwen and O'Neill proposed an extension of the headship rates method introducing age- and household size-specific headship rates. The latter rates were proposed to be derived as functions of demographic indicators, such as propensity of leaving home, marriage, divorce, fertility rates, and mortality. Such an approach seems to be promising, as it is much less demanding data and model assumptions compared to micro-simulation approach and also allows addressing the role of demographic events in households' formation. In some applications, however, there might be not enough data for the model. Also, the extension to the method may require special reconciliation procedures in order to guarantee internal consistency of the projection, which may limit its application especially in probabilistic projections. E.g., total population in private households obtained from their model distribution by size might be inconsistent with the size of actual population in private households. Another potential drawback is usage of parameters, which are quite volatile and involve non-trivial correlations between them. E.g., size/age-specific headship rates may vary considerably across time and regions, depending on prevailing fertility levels, while age-specific headship rates derived regardless household characteristics are usually much more stable, i.e., the former rates are negatively correlated. Usage of such model parameters may worsen performance and robustness of the probabilistic model and increase demand for data availability and quality.

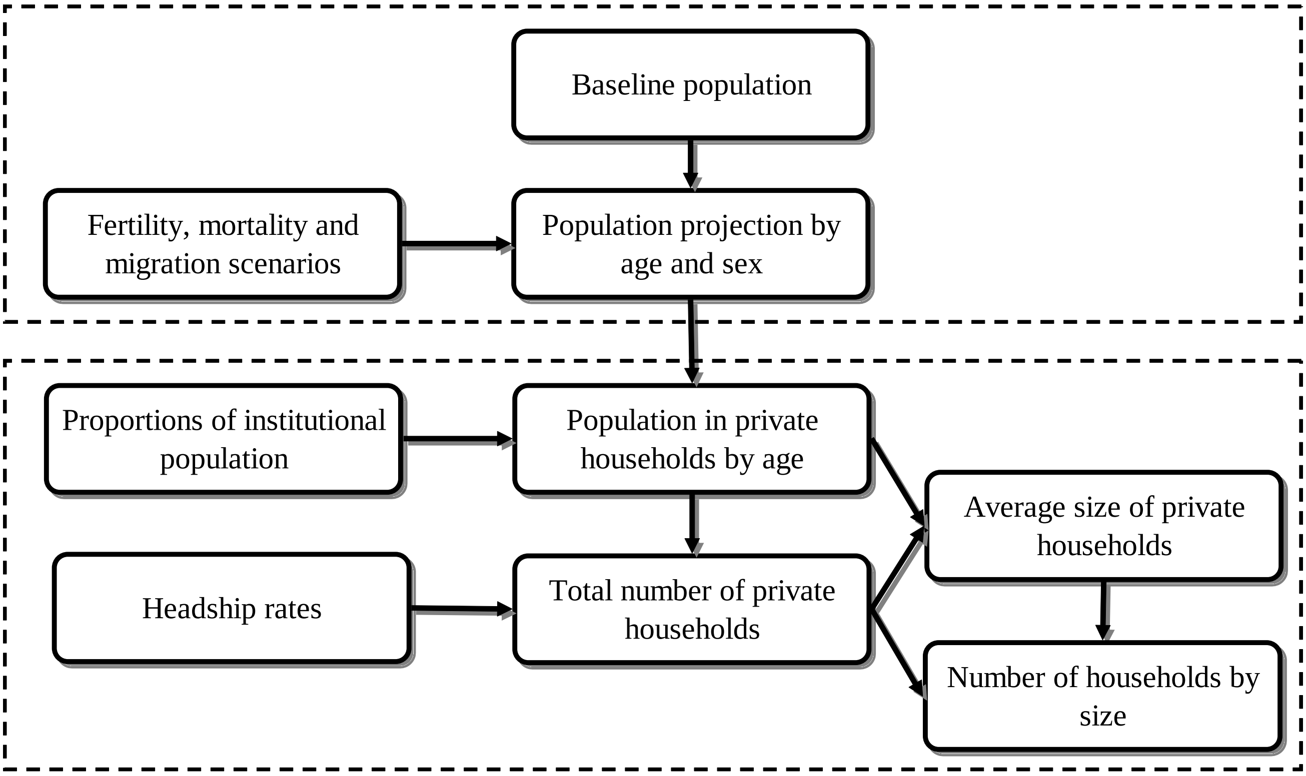
Here we present an approach, which is also based on the extension of the headship rates method. The extension we use is based on deriving distribution of households by size from the overall average size of households, which, in turn, is derived from the conventional age-specific headship rates. The approach was proposed by Gisser (1986a, 1986b) and used in Austrian household projections since then. Advantageous feature of the approach is that the average size of households indirectly reflects demographic developments, even though headship rates might be insensitive to those developments. In particular, changes in fertility assumed in population projection will, in fact, affect population size and, thereby, the average size of households. Unfortunately, like many other extensions of the headship rates method, the approach may eventually result in inconsistent projections, and special reconciliation procedures are to be used, which somewhat limits its usage in probabilistic projections. E.g., sum of proportions of households of different sizes may deviate from one, or population totals obtained directly from age structure or from the distribution of households by size may differ considerably. Merits of the headship rates approach may be used to a wider extent, however, based on recent developments of models for conditional shares of households among households of the same or larger size and for average sizes of such households (Ediev 2007), see details further down in the text.

We use conventional age-specific headship rates (eventually, generated at random, however, as it is described below) to derive the number of households from the projected population by age and the average household size. Then we apply conditional shares approach to derive the number of households by size.

General scheme of household projections adopted in this paper is presented at Diagram 1. Basically, two tasks are identified: making population projections and projecting number of households by size, based on population projections. In case of probabilistic projections this sequence is repeated a given number of times.

[2] Probabilistic population and household forecasts for the Netherlands Joop de Beer and Maarten Alders

Diagram 1 Projection of the number of households



2.1 Population projections

Population projections were prepared using a probabilistic approach. This approach has been already applied successfully in many instances to project population at national, macro-region and global levels (Lutz et al. 1997; Lutz and Scherbov 1998; Keilman et al. 2002; Lutz et al. 2003; Lutz and Scherbov 2004, etc.). There are mainly three approaches to probabilistic projections that are proposed in the scientific literature. The first approach is based on the time-series analysis of past vital rates, the second approach is based on the analysis of past projection errors and the third one is based on expert opinion. A good overview of these approaches is given by Bongaarts and Bulatao (2000) and H. Booth (2006). Those three approaches are not mutually exclusive but often complementary. In particular, the expert judgement is implicitly or explicitly considered in all of them. The third approach, the one actually adopted here, uses expert's opinion explicitly. The expert-based population projections were first proposed in the scientific literature by Lutz et al. (1996). Further use and development of the method can be found in Lutz et al. (1997), Lutz and Scherbov (1998), Lutz et al. (1999, 2001, 2004).

There are many sources of uncertainty in the future developments of fertility, mortality and migration. Recent introduction of a new demographic policy in Russia makes situation even more uncertain. The main aim of the policy was to increase the number of second births. It is not clear what would be the reaction of population to the new measures aimed at fertility stimulation. In is not clear whether the number of second births will increase in cohorts of women or simply a shift in the birth calendar will occur without essential change in the completed fertility of cohorts.

In our projections we assumed that population policy will bring certain positive results. We assumed that this will lead, first, to shortening of interval between 1st and 2nd birth and, second, to increase of the number of second births by 50 percent. Those assumptions result in the increase of projected mean value of period TFR to 1.5 in 2008, peak at the value of 1.76 in 2014 and declines afterwards remaining constant at the level of 1.7 starting from 2027. The range of uncertainty in 2050 covers the interval of TFR from 1.25 to 2.15 children per woman.

In case of life expectancy we assumed that lower end of 90 percent range corresponds to no future increase of life expectancy both in case of males and females. The upper end corresponds to growth in life expectancy of about 2 years per decade for females and 2.8 years per decade for males thus decreasing the gap that exists between life expectancy of males and females. This result in mean predicted values of life expectancy in 2050 equal to 71.3 and 81.7 years of life for males and females correspondently.

Mean predicted value of the number of net migrants was considered constant and equal to 126 thousands people coming annually to Russia. The range between 0 and 256 thousands people assumes to cover 90 percent of all the future outcomes of net migration

In order to generate the required distributions of the future path of fertility, mortality and migration, we adopt the method used by Lutz et al. (2001).

The starting year of projections was 2005. The data on population, fertility, mortality and migration for this year were utilized Age specific fertility rates were preliminary smoothed using mixed Gamma distribution function. Age-specific mortality rates are smoothed using Heligman-Pollard mortality schedule. Projections were made for single year age groups and are thus carried out on a yearly basis

2.2 Deriving the number of households by size

Next step after making population projections is to obtain the total number of private households. To do that we apply age-specific institutional population proportions to the projected population in order to obtain population living in private households. And next, we apply age specific headship rates to population living in private households in order to get the total number of private households (Diagram 1). Proportions of the institutional population were fixed at the level observed in census 2002[3]. Headship rates are derived from 1994 micro-census data using the bootstrap method. In order to avoid biases caused by artificial geographic compositions generated in bootstrap, we pull stratified samples, pooling together regions with similar average sizes of households. Two groups of regions were defined: those containing regions with average household size below 3 and above 3 members.

Based on the generated number of households, the average size is calculated as the ratio of the number of households to the population in private households, and the α -method from (Ediev 2007) is applied size after size:

$$\nu_{k/k+} = e^{-\alpha_k \eta_k}, \tag{1}$$

where $\nu_{k/k+}$ is the conditional share of households with k members among households of the same or larger size, η_k is average size of such households minus k, and α_k are model parameters. The parameters α_k are obtained from regressions against average size of the households, which are also derived from the bootstrap procedure based on stratified data of 1994 micro-census.

Procedure starts with smallest households, i.e., one-person households, and the average size for households of the next size is obtained recurrently by subtracting the number and the population residing in the households of the preceding size.

$$\eta_{k+1} = \frac{N_{k+} - k \cdot H_k}{H_{k+} - H_k} - (k+1) = \frac{\eta_k}{1 - \nu_{k/k+}} - 1, \tag{2}$$

here H_k and H_{k+} are the numbers of households of size k and of the same or larger size; N_{k+} is population residing in households with k or more members.

3 Population in institutional households comprise 1.6 percent of all the population. This percentage varies across age and sexes.

3. Results

We used two different approaches in developing our projections. First approach was based on directly applying headship rates obtained from Russian microcensus. Our probabilistic projection set contained 1000 simulations. For each simulation we stored age-specific population distributions for every projected year. Then, we applied fixed age specific headship rates obtained for Russia as a whole to each population composition, thus deriving the total number of households[4]. In the next step we calculated average size of the household for each simulation and distributed the total number of households by the number of households of each size. Since we used probabilistic age-specific population distributions, we also obtained probabilistic distribution of the number of households of different size.

In a second approach, the algorithm of the distribution of the number of households by size was similar, except we used random headship rates. They were obtained using the bootstrap procedure described above.

Resultant distributions of households by size were close in these two approaches with random headship rates approach having slightly higher variance. Thus we will present results only for a case when we applied random headship rates.

3.1 Population and Households: general overview of prospects

First of all let us look at the probabilistic population projection for Russia. At Figure 1 we present the fractals of this distribution. As we observe from this figure, there is virtually no chance for population growth in the future. This is predefined by a very low fertility and high mortality levels. Low fertility will have also implications for the total number of households. The total number of households is also projected to decline in a long term (Figure 2).. In a short term, next 10-15 years, the median number of households is going to slightly increase even though fertility level is low. After that period a steady decline is expected and by 2050 the number of households may fall to 47 million from 52.5 million in 2005. The 95% uncertainty range will spread from 41.5 to 52.4 million of households in 2050.

[4] Since we are interested in population living in a private households, we adjusted projected population with proportion of people living in private households obtained from 2002 census.

Figure 1 Probabilistic population projection

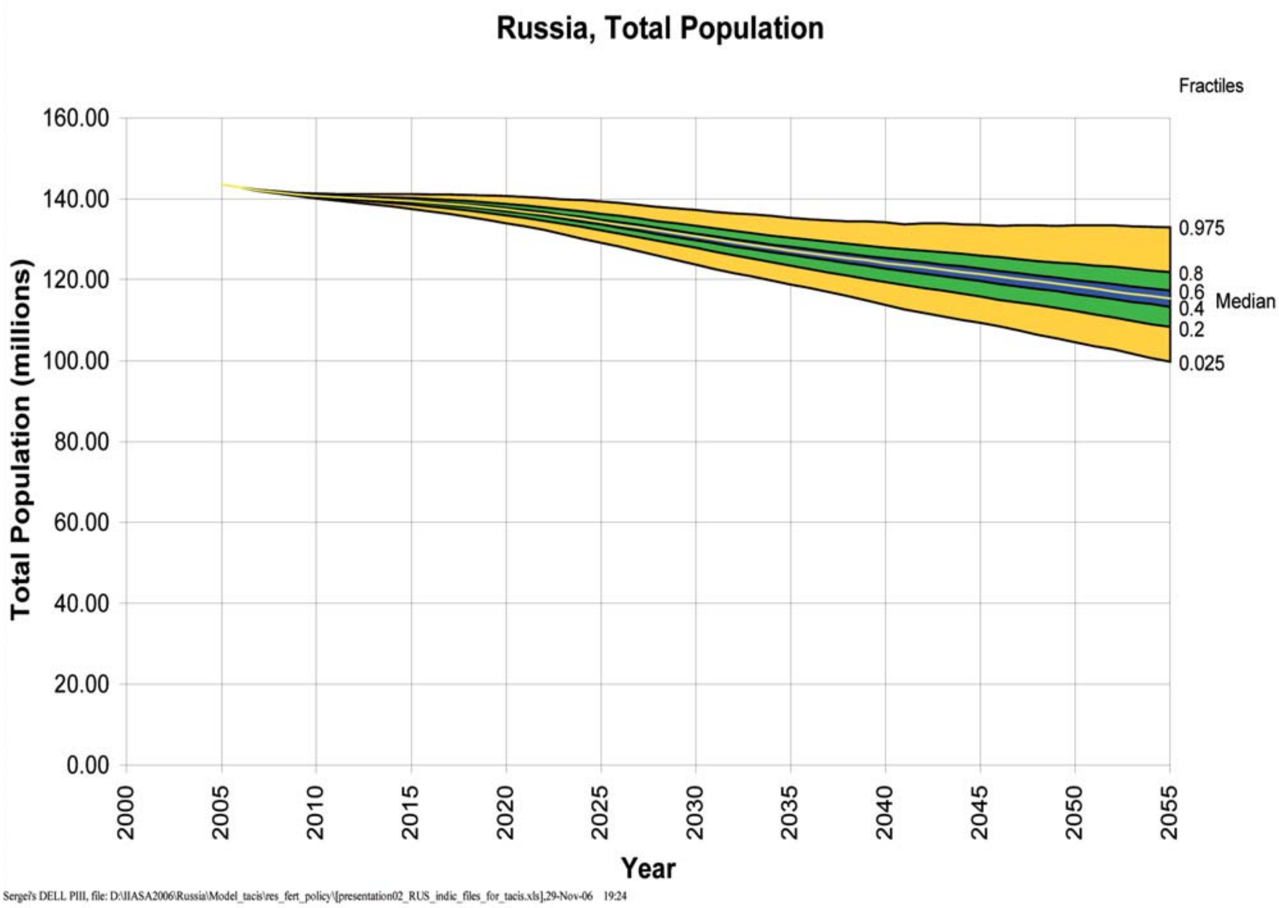
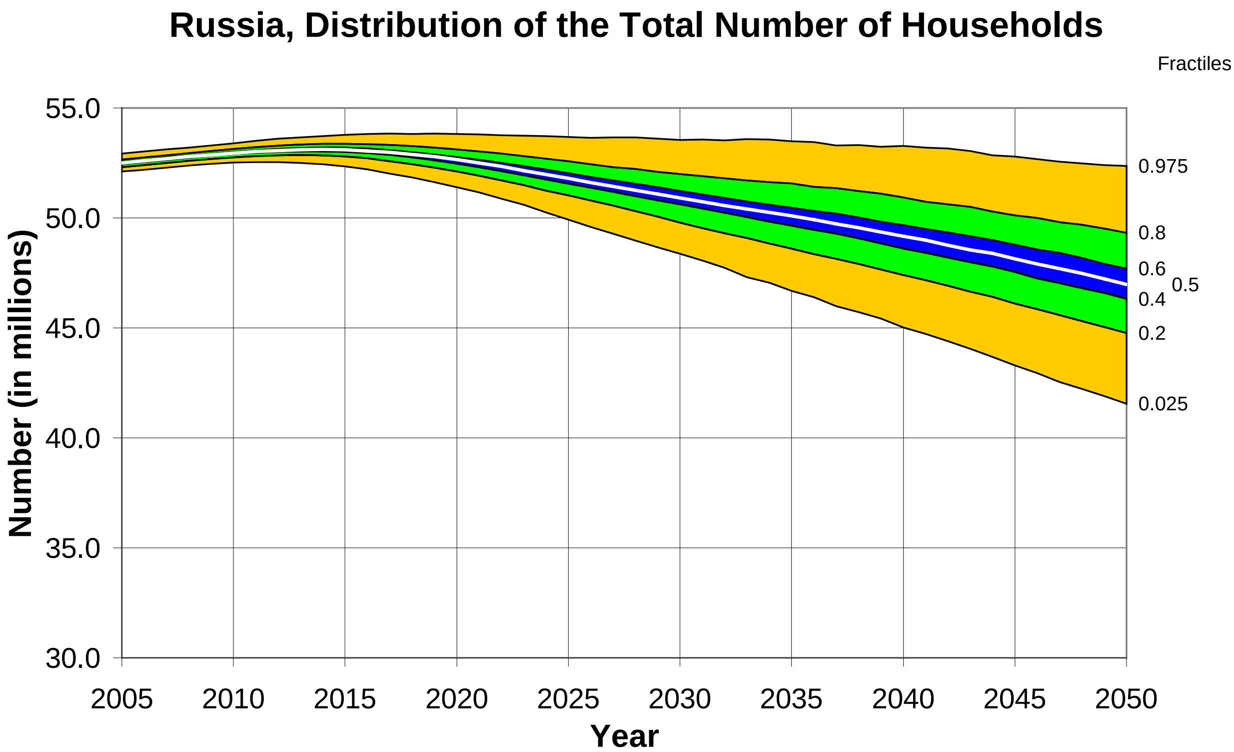
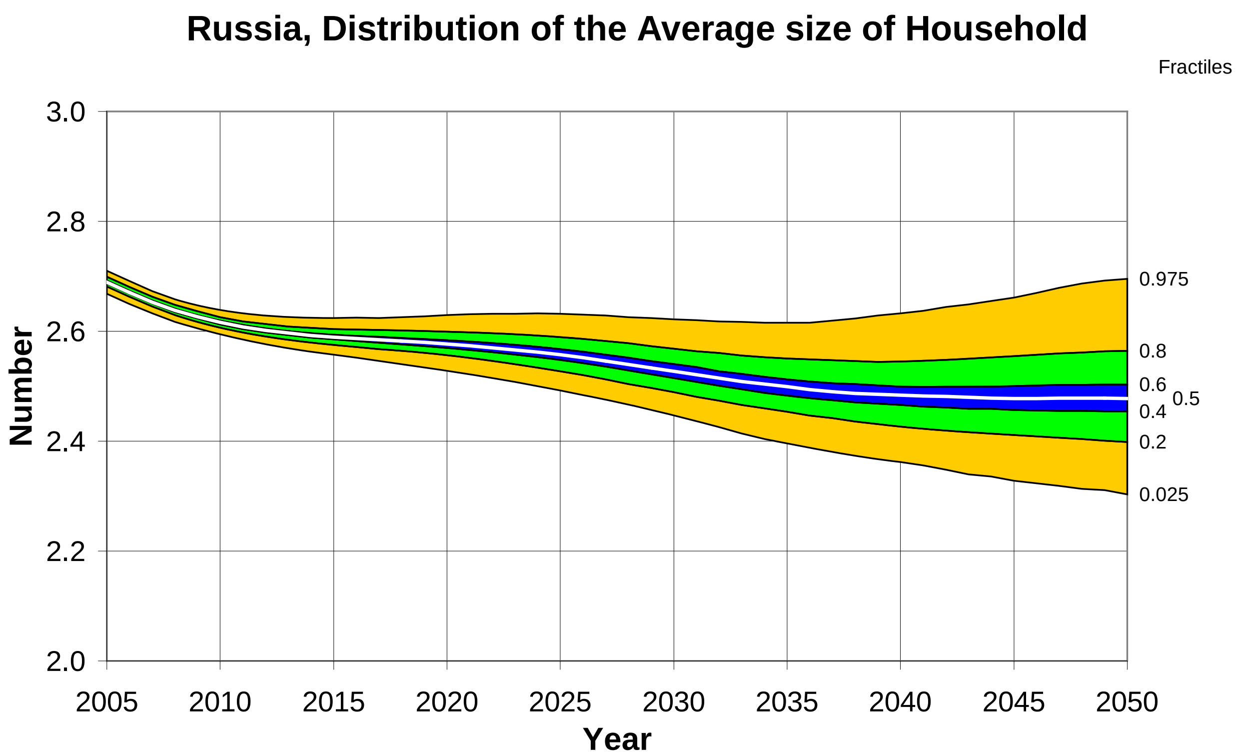


Figure 2 Distribution of the total number of households, projection 2005-2050, Russia



Not only the total number of households is projected to decline, but households will become smaller. (Figure 3). Median household size falls from 2.69 in 2005 to 2.5 in 2035 and after that stays almost constant. In 2035, 95% prediction interval includes households with sizes between with 2.4 and 2.6 members. In general the decline in fertility level leads to the decline of an average household size.

Figure 3 Distribution of the average household size, Russia, 2005-2050



Even though the total number of households is expected to decline, we may expect diverse trends if we study the dynamics of households of different sizes (figures 4-8). In the near future we may observe the rise in the number of households of size one from 11.5 million in 2005 to almost 13 million in 2035 (Figure 4). Households of size 2 and 3 show either no change or a very slight decrease in the near future with a moderate decrease by 2050 (figures 5 and 6). The strongest decline will be observed in households with four and more members (figures 7 and 8). We may expect that households of size four will decline by 20% and of size 5 by 60% by 2050. Typically that would be households consisting of two parents and two or three children.

Figure 4 Distribution of households of size one, Russia, 2005-2050

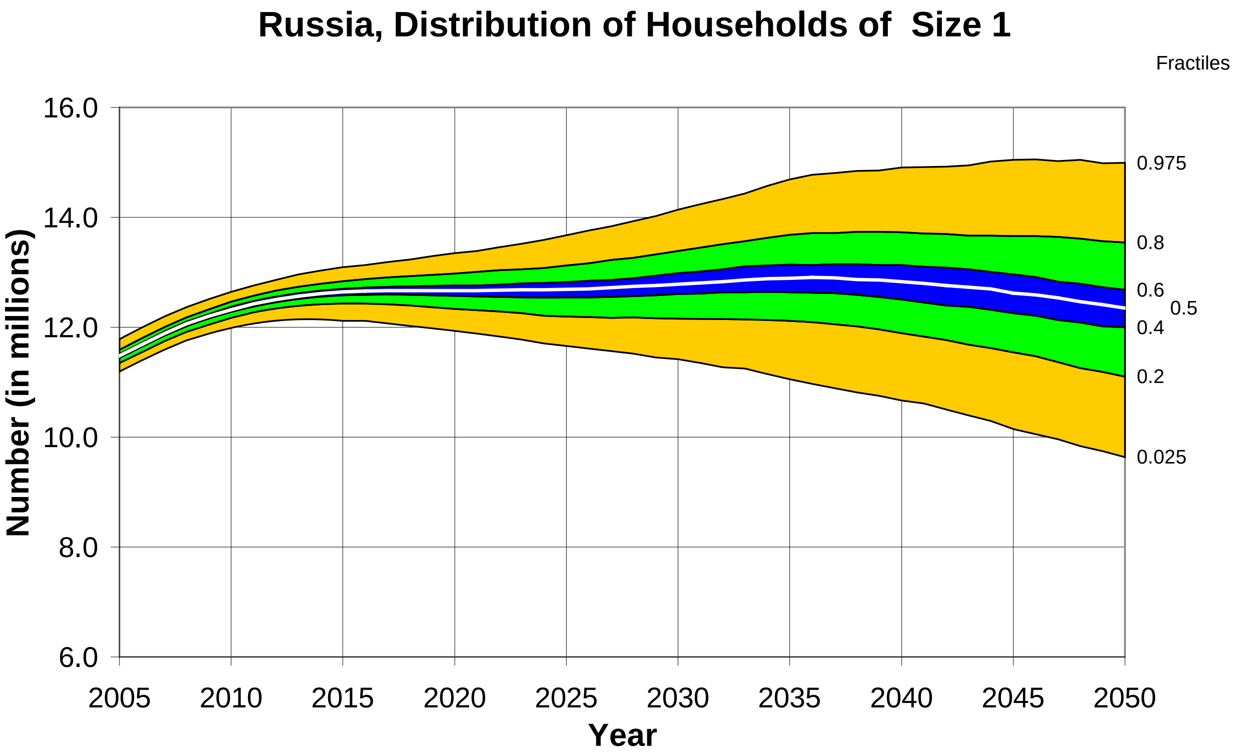


Figure 5 Distribution of households of size two, Russia, 2005-2050

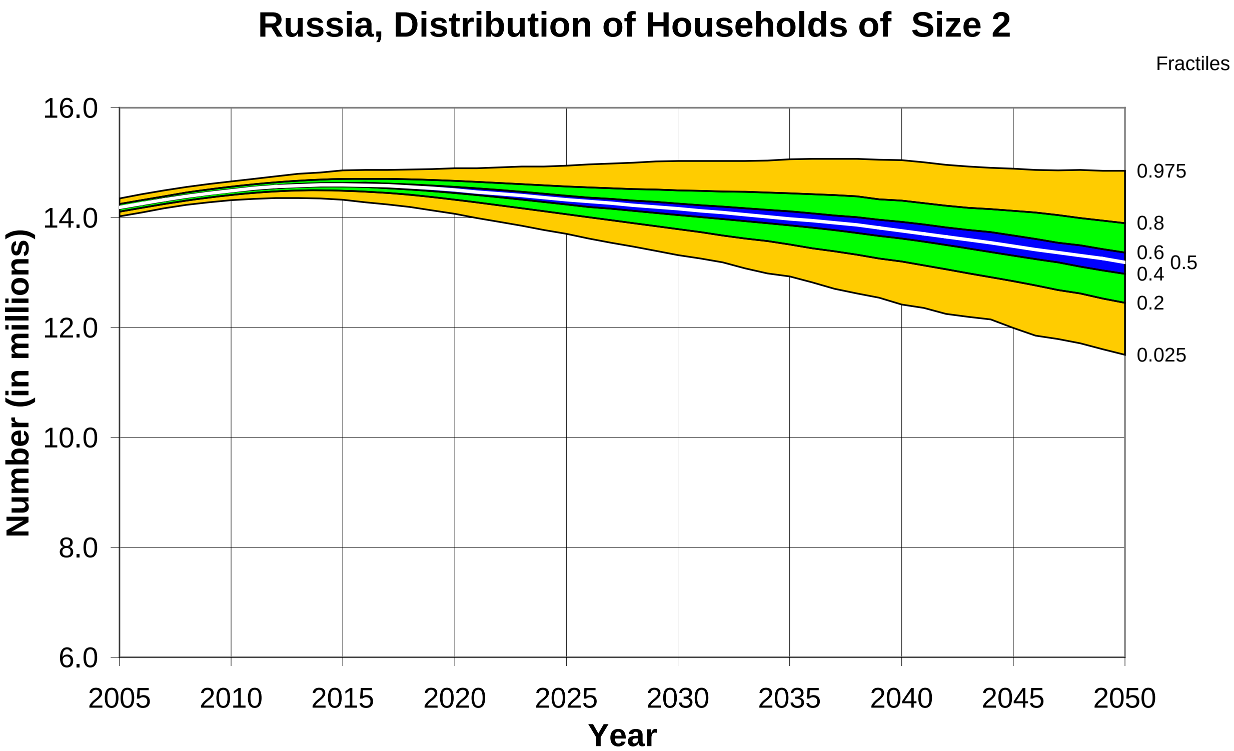


Figure 6 Distribution of households of size three, Russia, 2005-2050

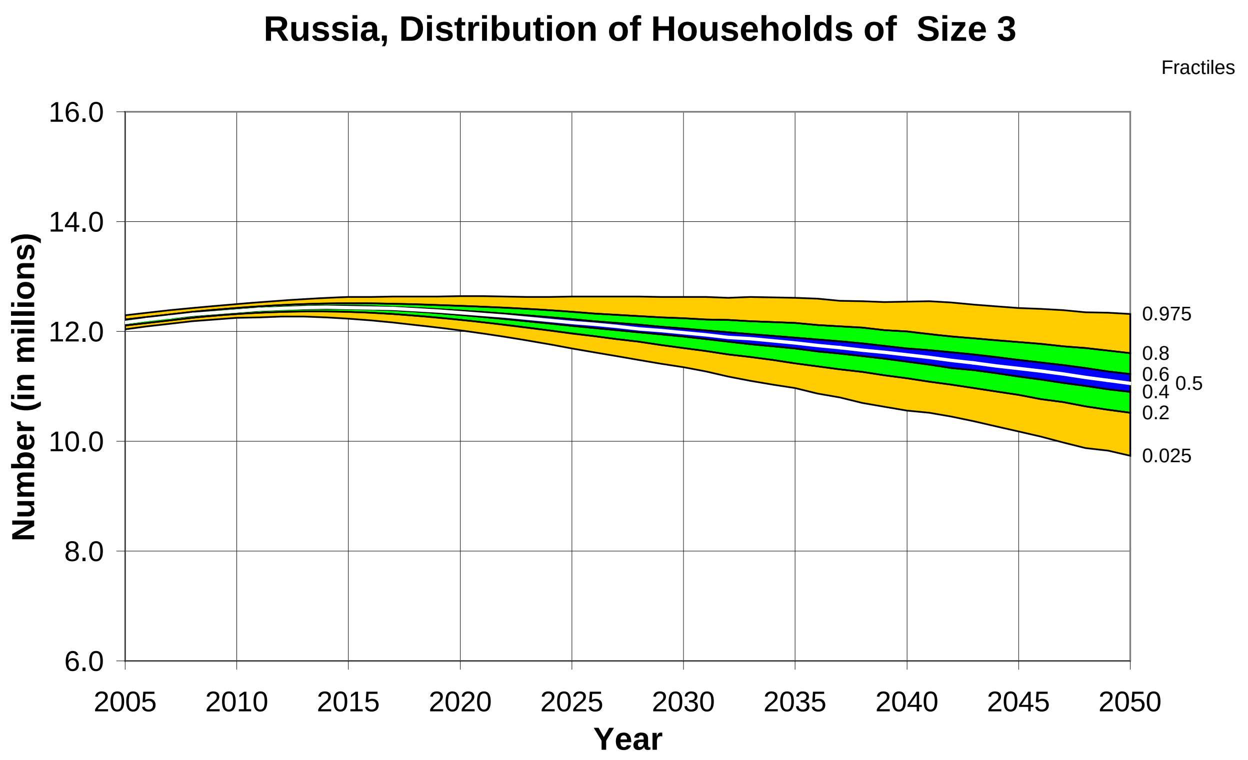


Figure 7 Distribution of households of size four, Russia, 2005-2050

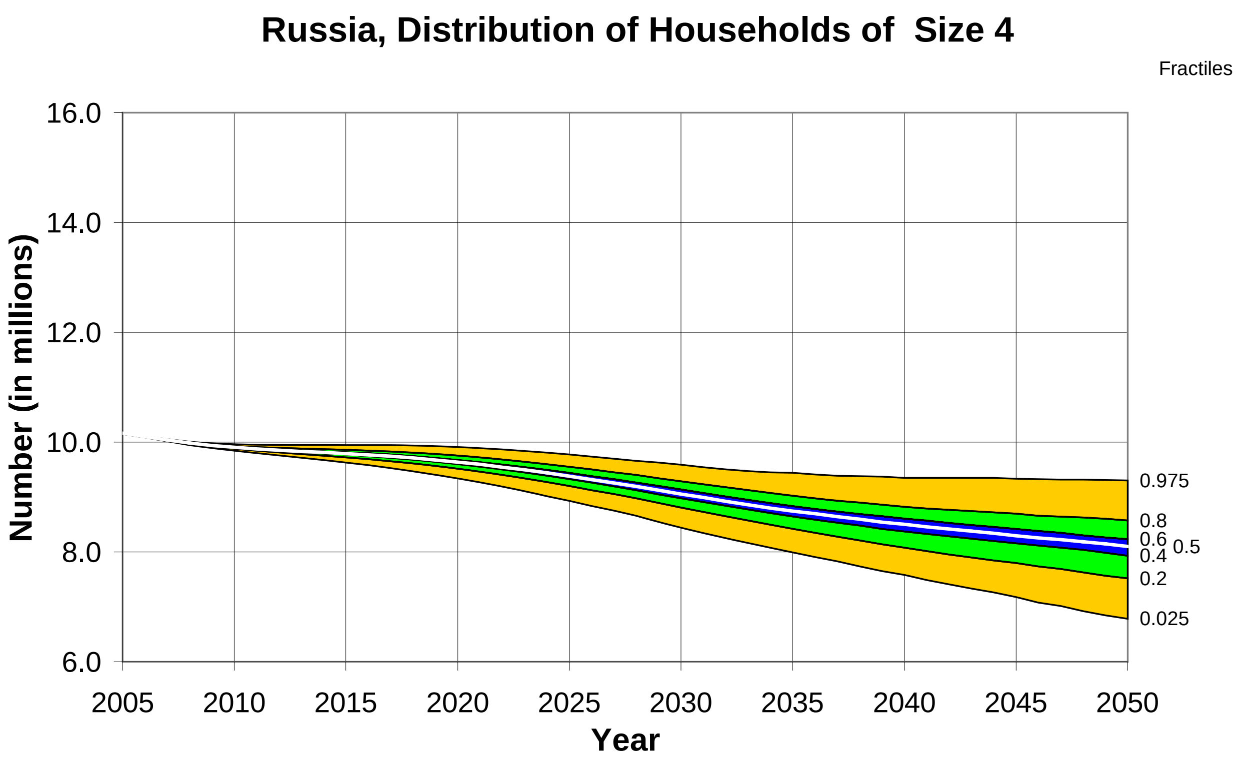
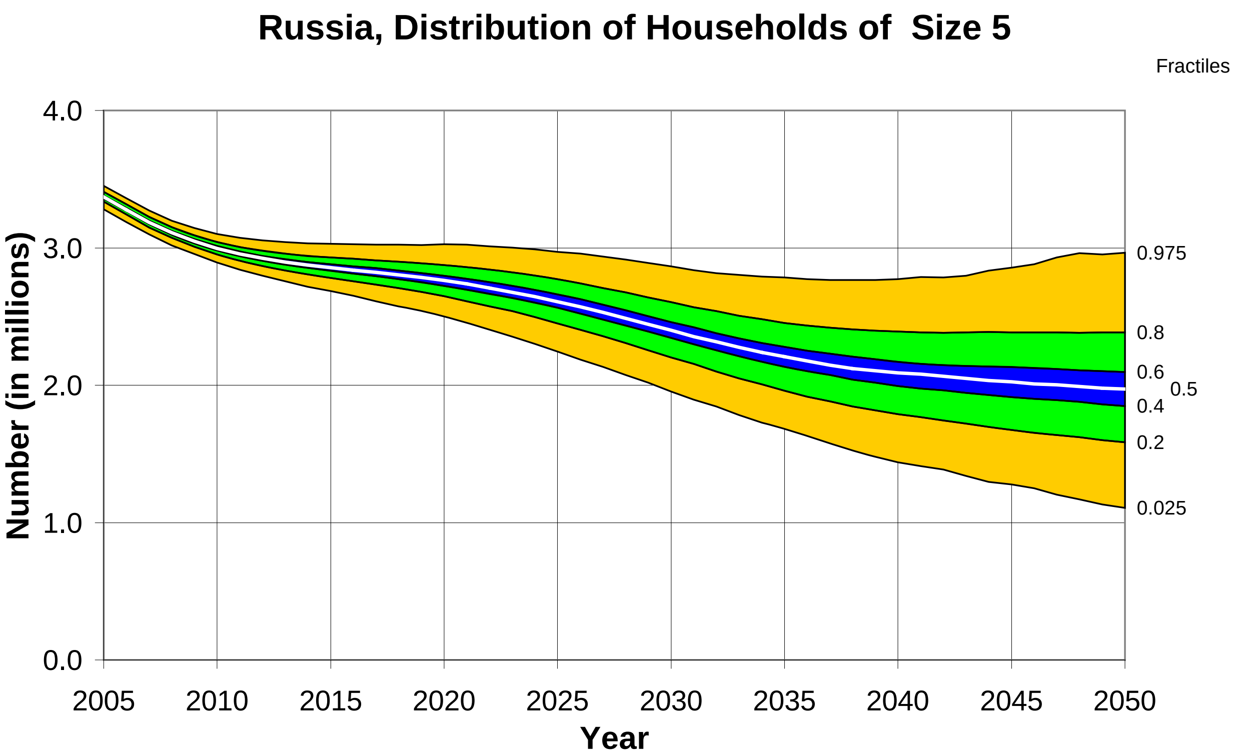


Figure 8 Distribution of households of size five, Russia, 2005-2050



Another way to look at the future distribution of households by size and to track the uncertainty associated with those distributions, is to present distribution of households by size for a particular time point (figures 9-10). From Figure 9 we may observe that there is virtually no chance that the number of households with four and more members will higher in 2025 than it was in 2005. In 2050 similar statement could be made regarding households with 3 and more members (Figure 10).

Figure 9 Distribution of households by size five, Russia, 2025

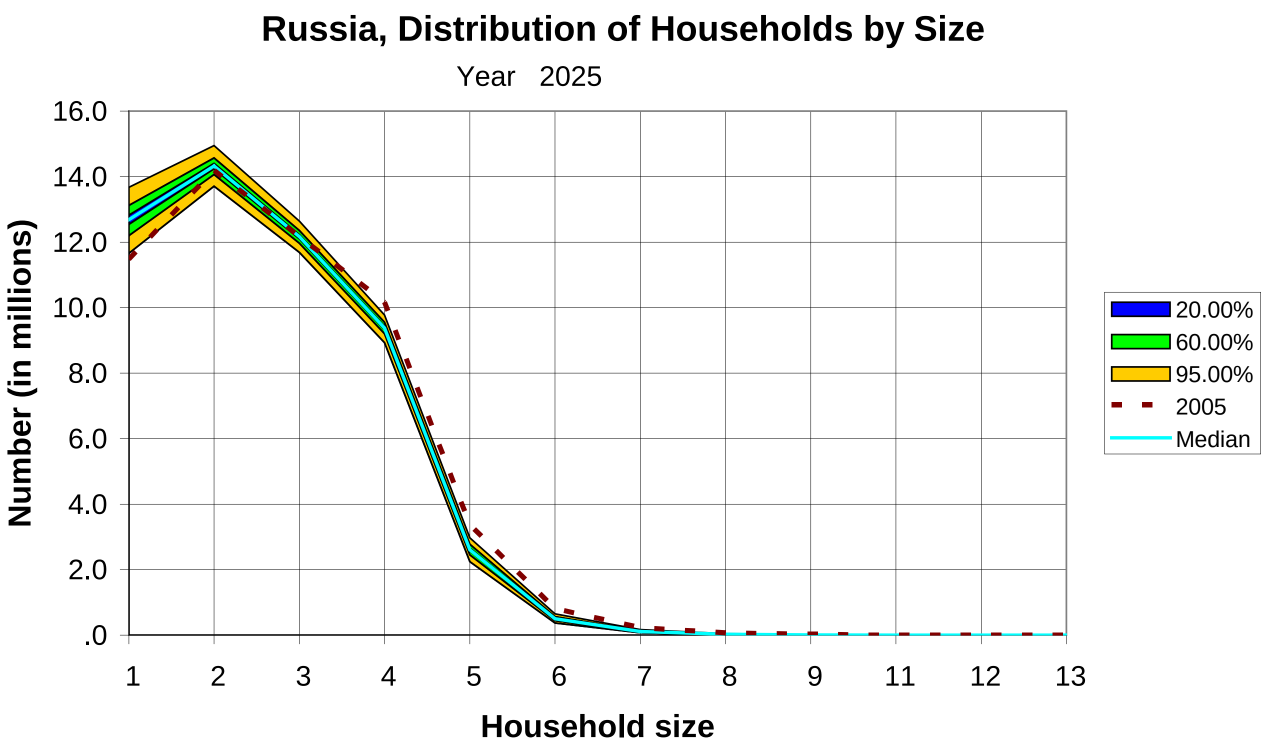
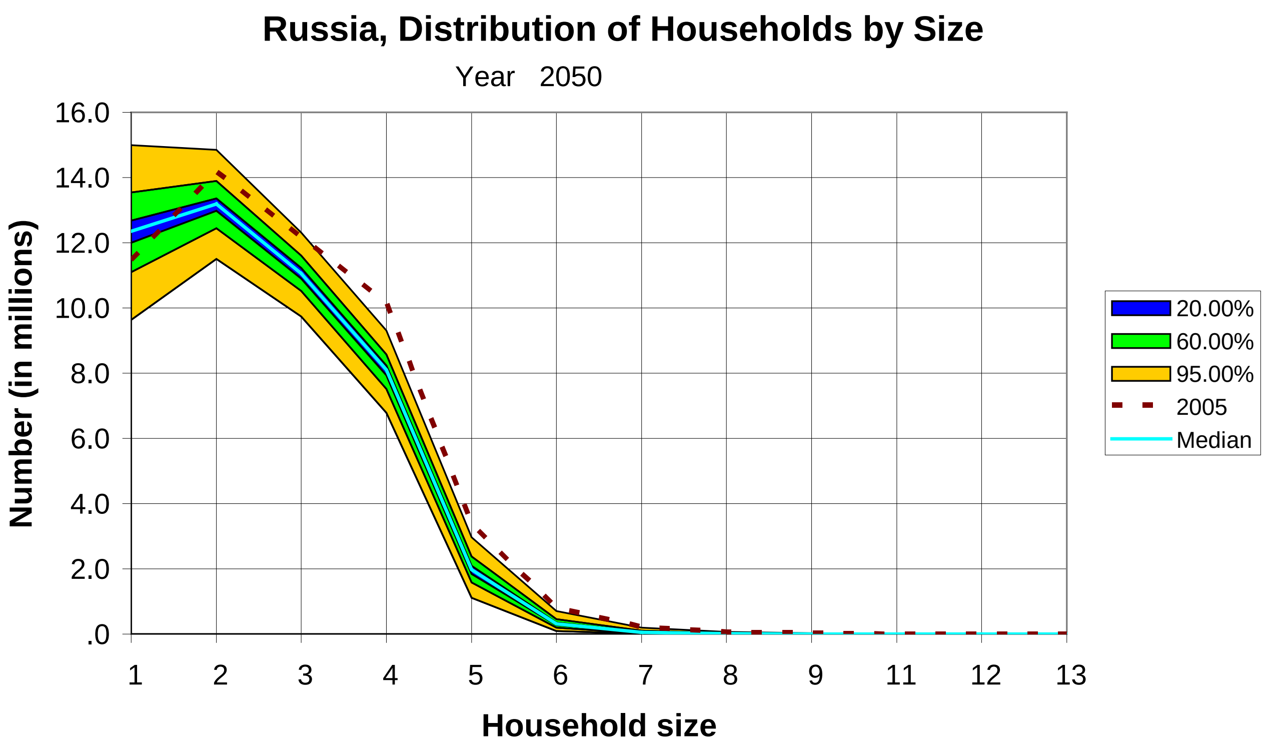


Figure 10 Distribution of households by size five, Russia, 2050



4. Conclusions

In this paper we presented the first probabilistic projections of the number of households in Russia. How this projections could be used? What type of questions are we able to answer having these results?

One of an extremely important issues in Russia today is availability of housing. Many families and households live in apartments where several people share one room. However, there exist social norms of housing per person, depending on the size of household[5]. Using those norms and assuming that they stay constant in the future, it is relatively easy to calculate the probabilistic demand for housing in Russia. At Figure 11 we present results of these calculation. With dotted line markers we designated existing availability (in 2002) of housing in millions of square meters that is occupied by households of different size. With vertical bars we present the demand for housing by household size calculated using social norms standards.

As we see from the graph, households with one or two members occupy even more housing space, then would correspond to social norms. There might be several reasons for that. First of all the distribution of housing is extremely uneven. Two households of the same size may live in a very different housing conditions. However one of the explanations of excessive available housing is that many of households of this type consist of elderly person living alone. Usually this person will have a bigger apartment, since at certain time he was living together with a spose and probably children. Children left home, spouse died and apartment or house (usually in rural area) is occupied by one person.

The most alarming situation is with availability of housing for households with four members. Typically that would be two patents living with two children. Since demographic policy adopted in Russia today is aimed at a second child, housing facilities for households consisting of four members, should be available. If the situation with housing availability does not improve, even though the number of households of size four is expected to decline, the shortage of housing will still be present, unless the policy will be developed to construct houses for households with four and more members (Figure 12). And even if in 2025 there might be enough of existing housing for households with 5 and more members, due to a very strong decline in the number of such households, the lack of housing for households with four members almost certain will be there if there is no considerable increase of housing stock for households with four member.

[5] Decree of the President of the Russian Federation Nr. 425 of April 28, 1997 "On Housing and Utility Sector Reform in the Russian Federation"

Figure 11 Demand and supply for housing, Russia, 2005

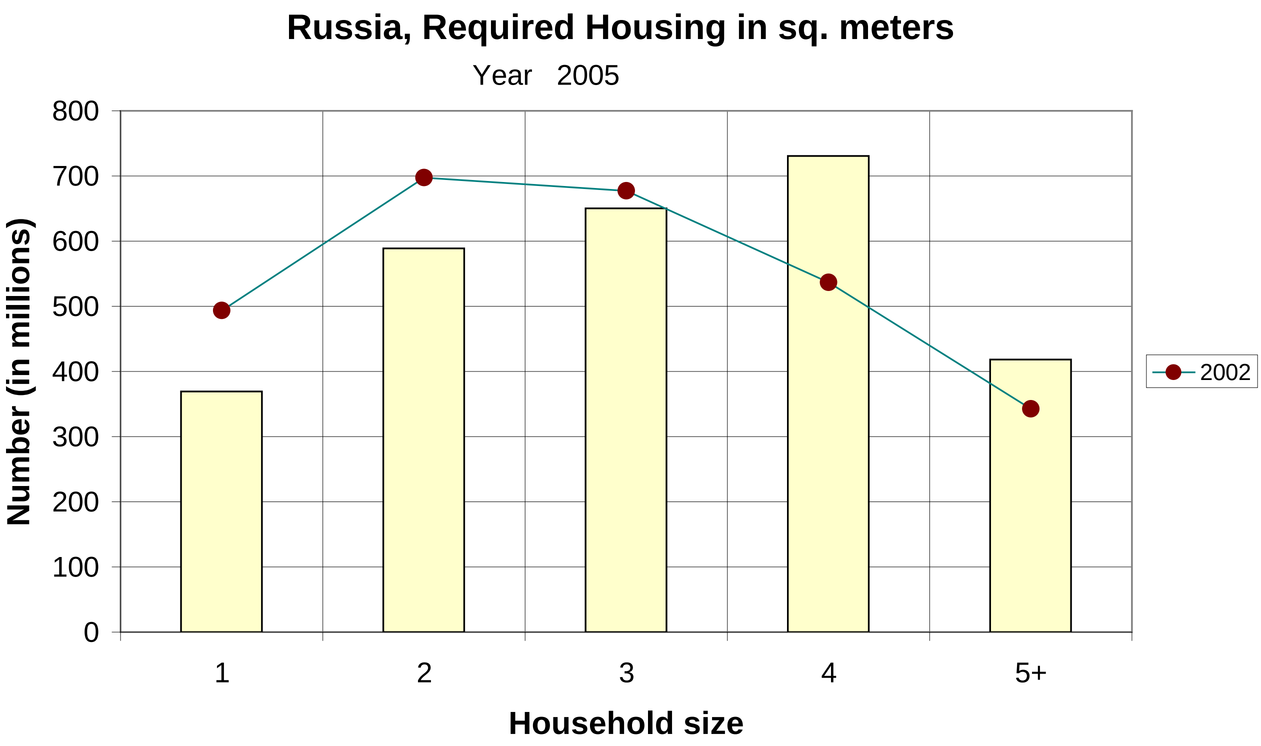
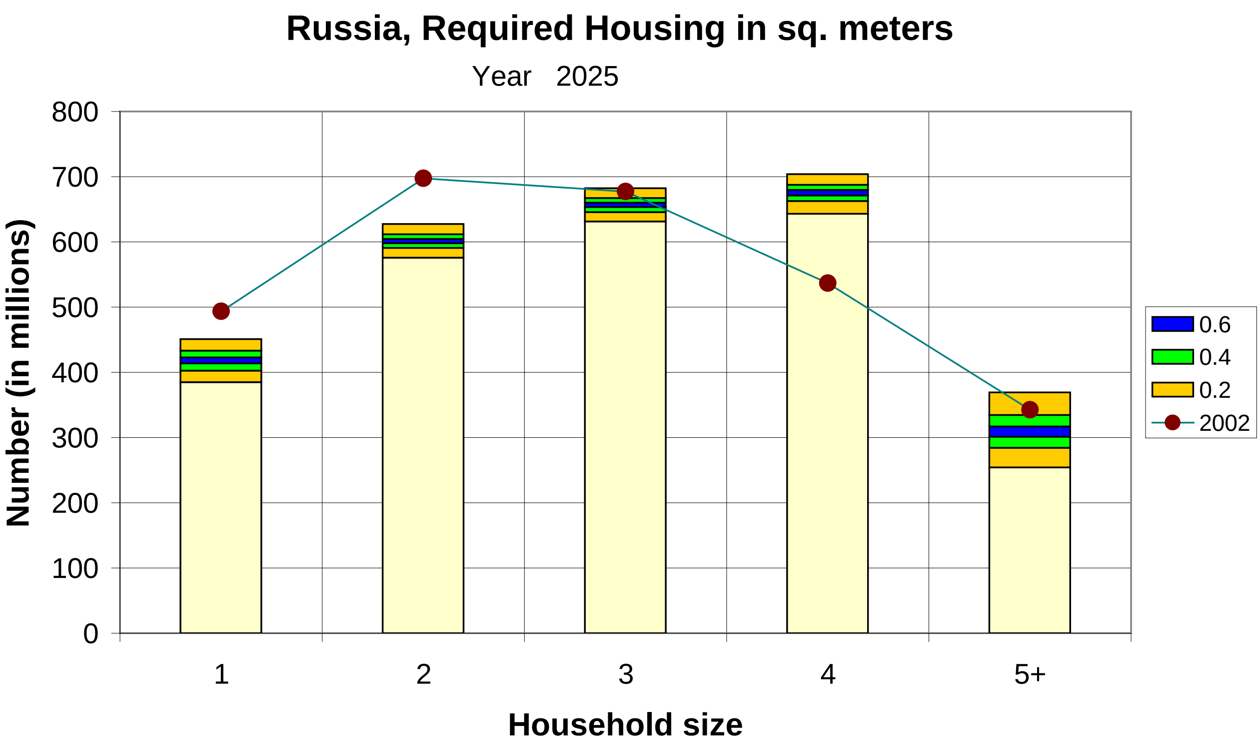


Figure 12 Demand and supply for housing, Russia, 2025



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APPENDIX. PRIVATE HOUSEHOLDS BY SIZE IN STABLE POPULATION

1. Headship rates

Let $P(x;t)$, $H(x;t)$, $M(x;t)=P(x;t)-H(x;t)$, and $h(x;t)=\dfrac{H(x;t)}{P(x;t)}$ be the population in private households, household heads, non-head members of households, and headship rates at age x[6]. We suppose the following simplified model determining evolvement of these functions. Dynamics of the number of heads is determined by mortality and also by formation of new households. Death of the head implies that all other household members move to other existing households, rather than forming a new household[7]. Secondly, formation of new households is through separation from existing households and happens at some fixed age-specific rates $g(x)$. Thirdly, we apply same age-specific death rates to both heads and non-head members. These simplifying assumptions allow separating the two processes and lead to the following differential equation for the population of non-head members of households:

$$\frac{\partial}{\partial t}M(x;t)+\frac{\partial}{\partial x}M(x;t)=-\mu(x;t)M(x;t)-g(x)M(x;t). \tag{A1}$$

From Eq. (A1), which is written in terms of the non-head population only, it is possible to derive the following relation for that population:

$$M(x;t)=M(0;t-x)e^{-\int_0^x(\mu(y;t-x+y)+g(y))dy}=P(x;t)e^{-\int_0^x g(y)dy}, \tag{A2}$$

where we suppose that there are no heads of age zero (i.e., $P(0;t)=M(0;t)$) and use the following traditional relation for the dynamics of the size of birth cohort:

$$P(x;t)=P(0;t-x)e^{-\int_0^x \mu(y;t-x+y)dy}. \tag{A2}$$

The population of heads may be obtained as the difference between the population total and the non-head population:

$$H(x;t)=P(x;t)-M(x;t)=P(x;t)\left(1-e^{-\int_0^x g(y)dy}\right). \tag{A3}$$

[6] For the sake of simplicity and also to avoid uncertainty related to the sex of the household head, we do not address sex, although it may be added to the study.

[7] In fact, emergence of new households due to the death of the head of existing household may indirectly be reflected in the model proposed through the age-specific rates of changing status from “non-head” to “head”.

This allows obtaining the headship rates:

$$h(x;t) = \frac{H(x;t)}{P(x;t)} = 1 - e^{-\int_0^x g(y)dy} . \tag{A4}$$

Hence, headship rates are constant and do not directly depend on the reproduction regimen of the population as long as the age-specific rates of separating to new households are fixed. This result may be extended to the case of varying rate of new households' formation $g(x;t)$:

$$h(x;t) = 1 - e^{-\int_0^x g(y;t-x+y)dy} . \tag{A5}$$

In this more general case, again, mortality and fertility are not directly involved in headship rates. In the model proposed the reverse transitions from 'head' status to 'non-head' status were neglected. Hence, solutions (A4) and (A5)–under non-negative rates of transition from non-head status to the head status–are ever increasing by age. In real populations there is slight decrease in headship rates for oldest-old ages, as elderly may join households of their kin instead of continuing keeping their own household. However, this decline in headship rates may also reflect more options for stating the 'household head' in census in households with several generations cohabiting together and also reflect cohort effects on headship rates.

In any case, headship rates seem to be much less sensitive to variations in reproduction regimes compared to, say, population size and age structure. This explains the remarkable stability of headship rates in human populations and also provides a rational in support of the headship rates method. This point is also supported by empirical data (e.g., Leiwen and O'Neill 2004, Ediev 2007): age-specific headship rates are remarkably stable, when no details concerning the household size or type are concerned.

2. Average size of households

Due to relatively less sensitiveness of age-specific headship rates to changes in reproduction regimen, one may study consequences of stable populations' age structures for number, average size, and distribution of households by size assuming some fixed age profile of the headship rates.

Let $h(x)$ be the headship rate at age *x*, which we assume to be fixed for all populations to be considered. Average size of households, which–under the model proposed–determines their distribution by size, may be written as follows for the stable population:

$$n^s = \frac{\int_0^\omega Bl(x)e^{-\rho x}dx}{\int_0^\omega Bl(x)e^{-\rho x}h(x)dx} = \frac{\int_0^\omega l(x)e^{-\rho x}dx}{\int_0^\omega l(x)e^{-\rho x}h(x)dx} , \tag{A6}$$

here B are births in the stable population, $l(x)$ is the survivorship function, ρ is the Malthusian parameter (or Lotka's coefficient), and ω is the maximum lifespan.

Headship rates are nil for children and grow rapidly to the level about 0.6 by age of 25-30. Therefore, one may use the following approximate for headship rates in (A6) in order to simplify the relation:

$$h(x) \approx \begin{cases} 0, & x \le x_{\min} , \\ h^*, & x > x_{\min} . \end{cases} \tag{A7}$$

Substituting this into (A6), we have:

$$n^s \approx \frac{\int_0^{\omega} l(x) e^{-\rho x} dx}{h^* \int_{x\min}^{\omega} l(x) e^{-\rho x} dx} = \frac{\int_0^{x\min} l(x) e^{-\rho x} dx + \int_{x\min}^{\omega} l(x) e^{-\rho x} dx}{h^* \int_{x\min}^{\omega} l(x) e^{-\rho x} dx} =$$

$$= \frac{1}{h^*} \left(1 + \frac{\int_0^{x\min} l(x) e^{-\rho x} dx}{\int_{x\min}^{\omega} l(x) e^{-\rho x} dx} \right). \tag{A8}$$

This expression indicates that there is a lower limit for average size of households of stable populations:

$$n^s \geq \frac{1}{h^*}. \tag{A9}$$

For usual case of headship rates of about 0.6 at most of adult ages, this implies, that *average size of households in stable population may not be lower than about* 1.67, which–given the models proposed for households distribution by size–has apparent implications for limiting the proportions of households of different sizes.

Expression in the right-hand side in (A8) depends on mortality and on reproduction regimen of the population. To make these relations more explicit, let us use the following simplifying approximation. Let us consider, that survivorship function may be approximated by a piece-wise constant function:

$$l(x) \approx \begin{cases} 1, & x \leq e_0, \\ 0, & x > e_0, \end{cases} \tag{A10}$$

here e_0 is the life expectancy at birth. Using this approximation, one may get from (A8):

$$n^s \approx \frac{1}{h^*} \left(1 + \frac{\int_0^{x\min} e^{-\rho x} dx}{\int_{x\min}^{e0} e^{-\rho x} dx} \right) = \frac{1}{h^*} \left(1 + \frac{1 - e^{-\rho x_{\min}}}{e^{-\rho x_{\min}} - e^{-\rho e_0}} \right) = \frac{1}{h^*} \frac{e^{\rho e_0} - 1}{e^{\rho (e_0 - x_{\min})} - 1}. \tag{A11}$$

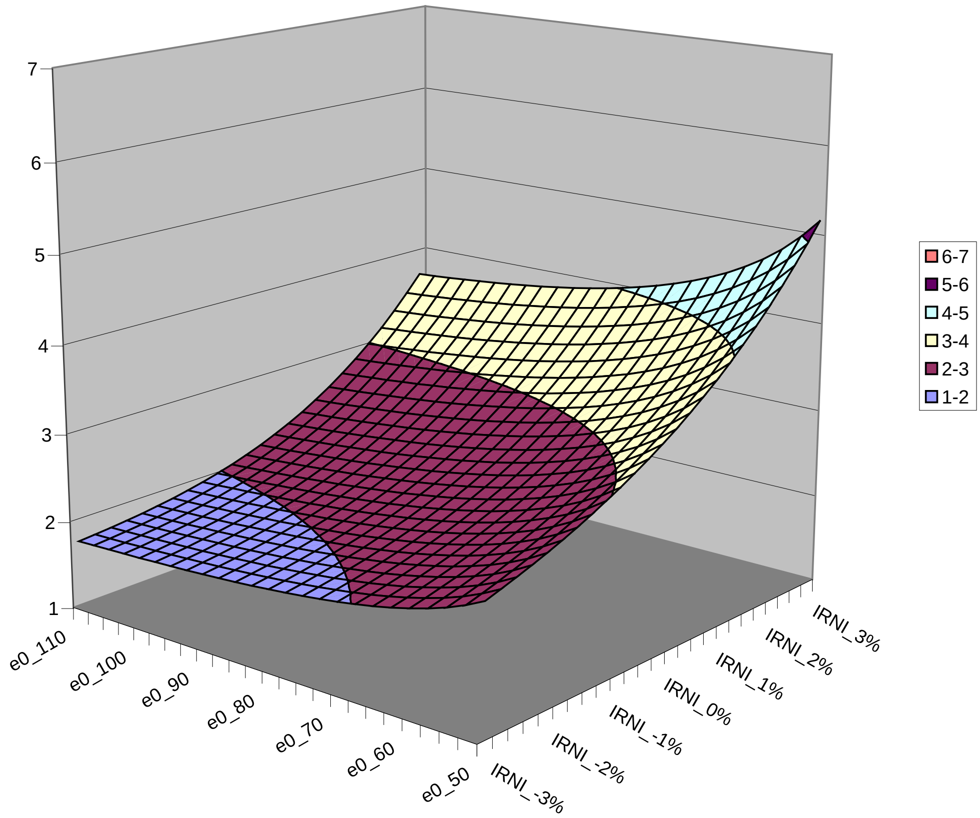
For stable populations with reproduction close to simple replacement, i.e., with Lotka's coefficient close to zero, it is life expectancy at birth, which is the main factor of variations in average size of households:

$$n^s \approx \frac{1}{h^*} \frac{e_0}{e_0 - x_{\min}}, \tag{A12}$$

when $\rho e_0 << 1$.

For wider range of stable populations, one may use (A11) to study the variations of the average size of households[8]. Figure A1 presents results of calculations using $x_{\min} = 25$, $h^* = 0.6$. The figure shows explicitly that the main factors of declining average sizes of households were improvements in life expectancy and fertility decline – both processes tightly linked in the process of demographic transition. Hence, demographic transition itself–apart from cultural changes and reassessments of family values–has caused decline in average size of households. Note, however, that at the first stages of the transition, when mortality decline results in improvements of Lotka's r, average size of households might be relatively stable or even growing. Later on, however, decline in households' average size must follow.

Figure A1 Approximates of average size households in stable population as a function of life expectancy at birth and of Lotka's coefficient (IRNI)



3. Distributions by size

Distribution of households by size may be derived from their average size as it was proposed elsewhere (Ediev 2007) and is described in the paper.

Figure A2 presents results of estimating the proportions of households of different sizes in stable populations with varying fertility and with life expectancy fixed at the level of 80 years. Figure A3, on contrary, presents results for stable populations with varying mortality and with replacement fertility, i.e., in fact, for stationary populations. Changes in population age structure associated with fertility decline have negative effect on proportions of households with four and more members, and positive effect for proportions of one- and two-person households. Proportion of households with three persons, however, varies only moderately even within the remarkably wide range of fertility levels analyzed. Rise in

[8] Numerical simulations show that the approximation (A11) works pretty well and provides results very close to those obtained directly from (A6).

life expectancy has nearly the same effect on households' distribution by size. Hence, simultaneous fall in fertility and rise in life expectancy, which was observed for many populations enhances effects of both processes on households' dynamics. In particular, it is the mere change in population age structure during the demographic transition processes, which seems to be the main factor of emergence of the modern distribution of households with declined share of large households and dramatic growth of the share of one-person households. This is illustrated on figure A4, which presents proportions of one-person households for stable populations with different combinations of fertility and life expectancy.

Figure A2 Proportions of private households of sizes 1 to 5 in stable population as a function of Lotka's coefficient (IRNI) under life expectancy at birth fixed at 80 years

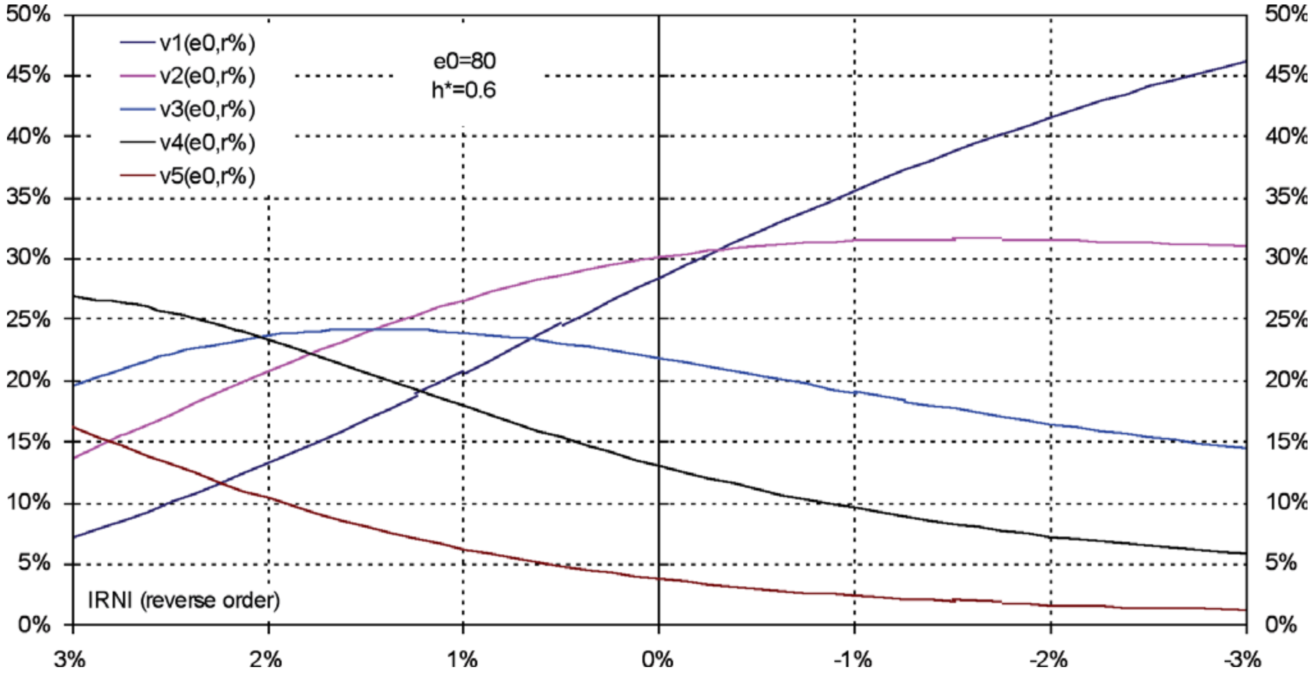


Figure A3 Proportions of private households of sizes 1 to 5 in stable population as a function of life expectancy at birth under the replacement fertility

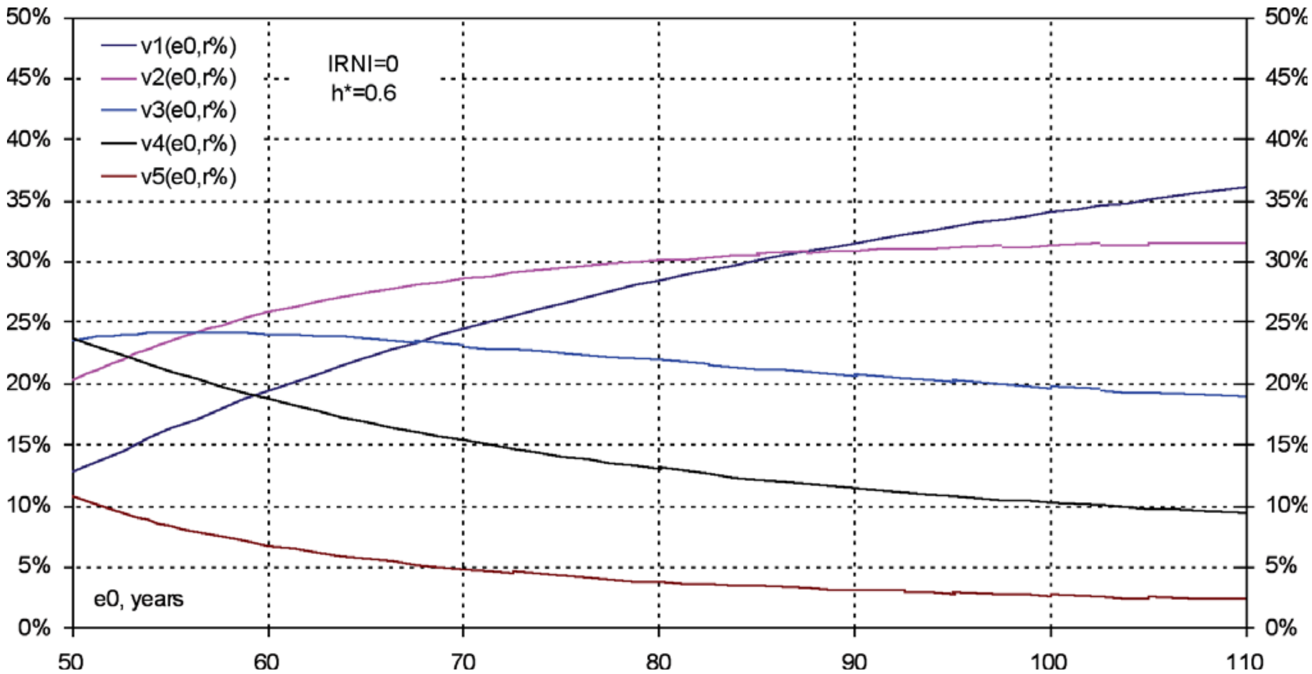
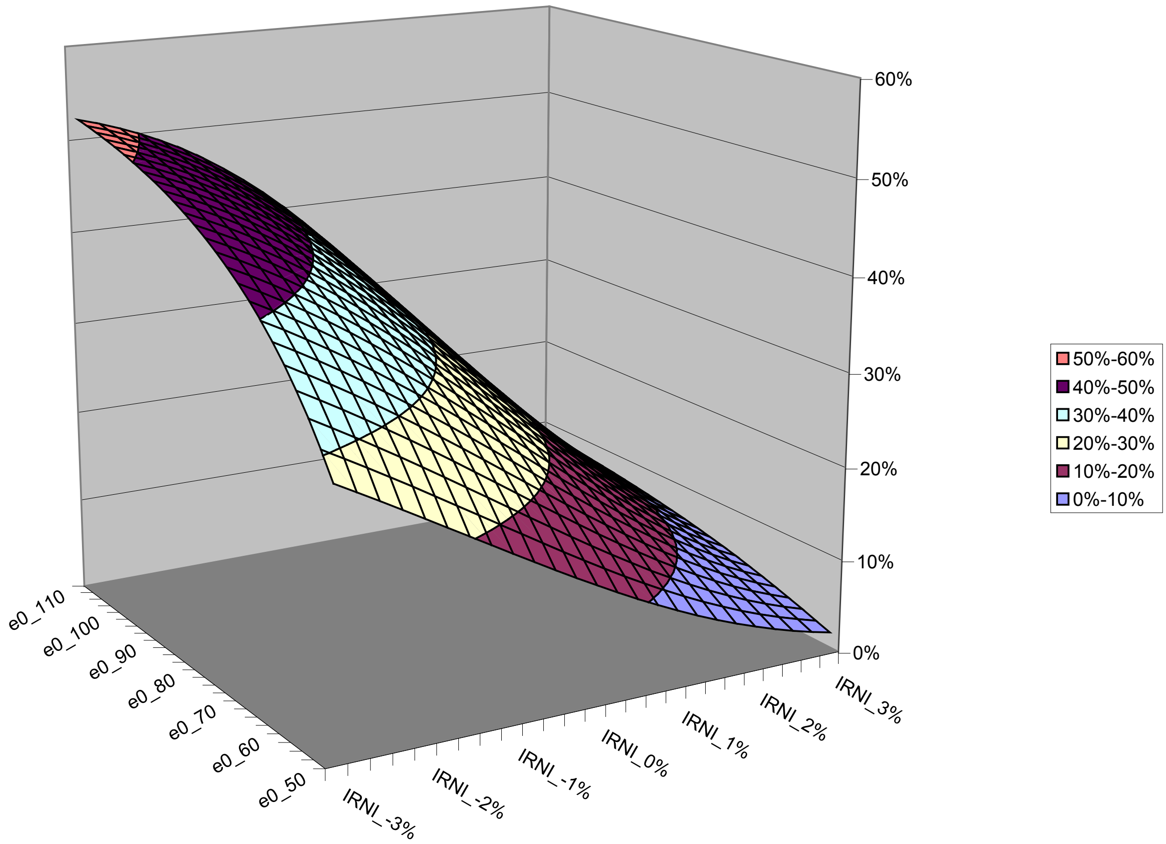


Figure A4 Proportion of single-person households in stable population as a function of life expectancy at birth and of Lotka's coefficient (IRNI)



Session 5: Specific Projection Issues

Chair: Frans Willekens

VI

UNITED NATIONS STATISTICAL COMMISSION and ECONOMIC COMMISSION FOR EUROPE

STATISTICAL OFFICE OF THE EUROPEAN COMMUNITIES (EUROSTAT)

CONFERENCE OF EUROPEAN STATISTICIANS

Joint Eurostat/UNECE Work Session on Demographic Projections

(Bucharest, 10-12 October 2007)
Agenda item 8: Specific projection issues

BAYESIAN MODEL SELECTION IN FORECASTING INTERNATIONAL MIGRATION: SIMPLE TIME SERIES MODELS AND THEIR EXTENSIONS

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Central European Forum for Migration and Population Research

Abstract

The paper aims to present selected ideas concerning the application of Bayesian time series analysis to international migration forecasting. In the methodological part, the discussion focuses on the Bayesian framework for formal selection of forecasting models, which is one way of accounting for the uncertainty of model specification. In this approach, the subjective *a priori* knowledge of the researcher concerns not only the parameters, but also particular models as such. Both types of prior beliefs are subsequently transformed into knowledge *a posteriori* on the basis of information obtained from the sample of observations.

The theoretical discussion is illustrated in the paper by an empirical example of forecasts of both-way migration flows between Poland and Germany for 2005–2015, based on the aggregate data series from German population registers. The analysis covers three sets of forecasting models: simple stochastic processes – sub-models of ARMA(1,1), extensions of an AR(1) model to simplest cases with non-constant conditional variance, as well as propositions assuming a linear analogy to post-accession migratory developments in countries that joined the European Community earlier (Portugal and Spain). In each case, the outcome of the formal model selection in the Bayesian framework allows for the identification of models supported by the data at hand. The Bayesian framework also enables to interpret the results with respect to uncertainty of the forecasted phenomena in coherent, probabilistic terms.

1. Introduction

The paper aims to present selected ideas concerning possible applications of Bayesian time series analysis in forecasting international migration among selected European countries. The Bayesian paradigm ensures the formality of inference, while allowing to include the *a priori* expert judgement in the analysis, alongside with the observations. Hence, the

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former can supplement the data-based information for small samples characterising many time series of within-European migration. Such a research problem fits into the methodological framework of stochastic population forecasts, where uncertainty assessment forms a key issue (Keilman, 1990; Lutz, Sanderson and Scherbov, 2004), only that within the Bayesian approach probability is defined in subjective terms, as a measure of belief.

Additionally, the paper aims to contribute to the debate on simplicity and complexity of models used in population forecasting (e.g., Smith, 1997) through applying formal model selection techniques. Such methods constitute one possible way of assessing and controlling the uncertainty of model specification.

In the first, methodological part (Section 2), the discussion focuses on the Bayesian framework for a formal selection of forecasting models. The theoretical background is illustrated in Section 3 by an empirical example of forecasts of both-way migration flows between Poland and Germany for 2005–2015, for three alternative classes of models. Section 4 briefly touches upon the issue of robustness of forecasts against selected changes in the assumed prior distributions. Finally, in Section 5, the main conclusions from the analysis are offered.

2. Bayesian methodology of formal model selection

In the Bayesian approach to formal model selection, the subjective *a priori* knowledge of the researcher concerns not only the parameters, but also particular models as such. Both types of prior beliefs are subsequently transformed into knowledge *a posteriori* on the basis of information obtained from the sample of observations. The result of the selection procedure consists thus of posterior probabilities of the choice of particular models, given the data.

In forecasting, the Bayesian approach consists in calculating a predictive probability distribution of the vector of future values of the variable of interest, $\mathbf{x}^F$, yielded by a particular forecasting model M on the basis of an observations vector $\mathbf{x}$. The estimation involves the *a posteriori* information about the vector θ of the parameters of M given data $\mathbf{x}$, $p(\theta \mid \mathbf{x})$, which is obtained from the Bayes' theorem as:

$$p(\theta \mid \mathbf{x}) = p(\theta) \cdot p(\mathbf{x} \mid \theta) \,/\, p(\mathbf{x}). \tag{1}$$

In (1), $p(\theta)$ denotes subjective knowledge *a priori* about the parameters θ, embodied in a form of a probability distribution; $p(\mathbf{x} \mid \theta)$ is the likelihood of data given θ; and the normalising constant $p(\mathbf{x})$ is found by integrating $p(\mathbf{x} \mid \theta) \cdot p(\theta)$ over the whole parameter space Θ. Given the above, the predictive probability distribution of $\mathbf{x}^F$ is calculated as:

$$p(\mathbf{xF} \mid \mathbf{x}) = \int p(\mathbf{xF}, \theta \mid \mathbf{x})\, d\,\theta = \int p(\mathbf{xF} \mid \theta, \mathbf{x}) \cdot p(\theta \mid \mathbf{x})\, d\,\theta, \tag{2}$$

integration being carried out over the whole Θ (Zellner, 1971: 29).

Following Osiewalski and Steel (1993), let $M_1, \ldots, M_m$ denote m non-nested, mutually exclusive models, adding up to the whole model space $\mathbf{M}$. Assuming their *a priori* probabilities $p(M_i)$, which reflect the researcher's subjective intuition about plausibility of particular models for a specific forecasting task, the respective *a posteriori* probabilities, given the data vector $\mathbf{x}$, are obtained from the Bayes' Theorem as:

$$p(M_i \mid \mathbf{x}) = \frac{p(M_i) \cdot p(\mathbf{x} \mid M_i)}{\sum_{k \in \mathbf{M}} p(M_k) \cdot p(\mathbf{x} \mid M_k)}\,. \tag{3}$$

The models can be assumed *a priori* as equiprobable, with $p(M_i) = 1/m$ for all i, or follow the Occam's razor principle (meaning that "entities should not be multiplied unnecessarily")[2]. In the latter case, simpler models are preferred over more complex ones, which can formally be reflected for example in prior probabilities such that $p(M_i) \propto 2^{-l_i}$, where the symbol '∝' denotes proportionality and l_i is the number of parameters in the i-th model (*idem*).

[2] Merriam-Webster Online Dictionary: www.m-w.com/dictionary/occam's razor, as of 23.07.2007.

In the model selection problems, the Bayes' Theorem is applied twice: to update the prior distributions of parameters θ_i for all models M_i given the data **x**, following (1), and at the same time to obtain the posterior probabilities of particular models using (3). In order to accommodate the problem within the framework of the Gibbs sampling procedure, which is often used for numerical computations in Bayesian analyses, the *Model Choice via Markov Chain Monte Carlo* (MC^3) algorithm of Carlin and Chib (1995) can be applied. In the current study, the method has been implemented within the WinBUGS 1.4 software environment (Spiegelhalter *et al.*, 2003).

The proposed procedure consists in an iterative sampling from the full conditional distributions for model-specific parameters θ_i and the model index μ, repeated sequentially until convergence to an ultimate solution is reached. The full conditional distributions are given by the following equations (Carlin and Chib, 1995: 475–477):

$$\begin{cases} p(\theta_i \mid \theta_{j\neq i}, \mu, \mathbf{x}) \propto \begin{cases} p(\mathbf{x} \mid \theta_i, \mu = i) \cdot p(\theta_i \mid \mu = i) \text{ for } \mu = i \\ p(\theta_i \mid \mu \neq i) \qquad \text{for } \mu \neq i \end{cases} \\ p(\mu = i \mid \theta, \mathbf{x}) = \dfrac{p(\mathbf{x} \mid \theta_i, \mu = i) \cdot p(M_i) \cdot \prod_{j\in\mathbf{M}} p(\theta_j \mid \mu = i)}{\sum_{k\in\mathbf{M}} [p(\mathbf{x} \mid \theta_k, \mu = k) \cdot p(M_k) \cdot \prod_{j\in\mathbf{M}} p(\theta_j \mid \mu = k)]}. \end{cases} \tag{4}$$

The model parameters θ_i are thus either sampled using a standard Gibbs procedure if $\mu = i$ or drawn from pre-defined linking densities ("pseudo-priors") $p(\theta_i \mid \mu \neq i)$ otherwise. The latter can be for example the preliminary estimates of model-specific posteriors $p(\theta_i \mid \mu = i, \mathbf{x})$ (*idem*). As in all Markov Chain Monte Carlo methods, the first S iterations of (4), until convergence, are discarded (the "burn-in" phase), while further N are used to estimate the posterior distributions of the parameters and the model index (Casella and George, 1992: 168).

3. Empirical application to Polish–German migration forecasts

The empirical example offered in the current section concerns forecasts of both-way migration flows between Poland and Germany for 2005–2015, based on the aggregate data series from the German population registers and from the Eurostat. The variable under study is a log-transformed annual migration rate, that is, the number of migrants per 1,000 inhabitants of the sending country, denoted as $m_t = \ln(Mig_t / Pop_t \cdot 1{,}000)$. For all presented model selection procedures, the Occam's razor priors $p(M_i) \propto 2^{-l_i}$ have been assumed.

The presented analysis covers three distinct sets **M** of forecasting models, here treated separately due to different sample vectors **x**. Firstly, simple stochastic processes of different complexity and features – subclasses of ARMA(1,1), are studied. The sample is composed of 14 observations from the period 1991–2004, after socio-economic transformations in Poland and East Germany, to ensure an unchanging institutional setting. Five models are considered:

$$\begin{aligned} &M_1: m_t = c + \varepsilon_t \text{ [oscillations around a constant];} \\ &M_2: m_t = c + m_{t-1} + \varepsilon_t \text{ [random walk with drift];} \\ &M_3: m_t = c + \phi\, m_{t-1} + \varepsilon_t \text{ [AR(1) process];} \\ &M_4: m_t = c - \theta\, \varepsilon_{t-1} + \varepsilon_t \text{ [MA(1) process];} \\ &M_5: m_t = c + \phi\, m_{t-1} - \theta\, \varepsilon_{t-1} + \varepsilon_t \text{ [ARMA(1,1)].} \end{aligned} \tag{5}$$

The random term ε_t follows a Normal distribution $N(0, \sigma^2)$. Priors for the constants are diffuse, $c \sim N(0, 100^2)$, while the ones for the AR and MA components are more informative, with $\phi, \theta \sim N(0.5, 1^2)$, reflecting prior beliefs in a likely stationarity or time-reversibility of the relevant processes. Precision $\tau = \sigma^{-2}$ of the random term was assumed to follow a Gamma distribution $\Gamma(0.25, 0.25)$ for migration from Poland to Germany ($E[\tau] = 1$, $Var[\tau] = 4$, very low precision), and $\Gamma(4,0.4)$ for the opposite-direction flows ($E[\tau] = 10$, $Var[\tau] = 25$, higher precision). These assumptions reflect prior beliefs in an uncertain character of migratory processes.

Secondly, extensions of an AR(1) model to cases with non-constant conditional variance are analysed, including ARCH(1), GARCH(1,1) and the simplest stochastic volatility (SV) models. In this case, the sample ranges over 20 years from 1985 to 2004, in an attempt to capture three periods of different institutional background and potentially also migration volatility: before, during and after the system transformation. Four models based on an autoregressive process AR(1), $m_t = c + \phi\, m_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim \mathrm{N}(0, \sigma^2_t)$ are analysed, where:

$$
\begin{aligned}
&M_6: \sigma^2_t = \sigma^2 \text{ [reference model with constant variance];}\\
&M_7: \sigma^2_t = k + \alpha\, \varepsilon^2_{t-1} \text{ [AR(1)-ARCH(1)];}\\
&M_8: \sigma^2_t = k + \alpha\, \varepsilon^2_{t-1} + \beta\, \sigma^2_{t-1} \text{ [AR(1)-GARCH(1,1)];}\\
&M_9: \ln(\sigma^2_t) = k + \gamma \ln(\sigma^2_{t-1}) + \zeta_t;\ \zeta_t \sim \mathrm{iiN}(0, \rho^2) \text{ [simplest AR(1)-SV].}
\end{aligned}
\tag{6}
$$

The priors for c and ϕ are the same as before, while for computational reasons for the remaining parameters it was assumed that: $\alpha, \beta, \gamma \sim \Gamma(10, 20)$, $1/\rho^2 \sim \Gamma(10, 1)$, as well as $k \sim \Gamma(1, 0.1)$. Thus, the proposed class encompasses the simplest processes either with deterministic (ARCH / GARCH) or stochastic (SV) change in conditional variance, compared with the constant-volatility reference model.

Finally, a proposition is examined (Kupiszewski, 1998) that there may exist an analogy between the Polish–German migration following the Poland's 2004 accession to the European Union, and migratory developments in countries that joined the EEC before, in particular, Portugal and Spain. The sample covers 13 years (1992–2004), as such data on respective Iberian migration rates observed 18 years earlier were available. This analogy preserves the timing between the EEC/EU accession and the opening of German labour market for respective foreigners, in the case of Poles envisaged for 2011. Consider four models:

$$
\begin{aligned}
&M_{10}: m_t = c + \varepsilon_t \text{ [reference model with no analogy];}\\
&M_{11}: m_t = c + a\, m^{PT}_{t-18} + b\, z_t + \varepsilon_t \text{ [analogy to Portugal, PT];}\\
&M_{12}: m_t = c + a\, m^{ES}_{t-18} + \varepsilon; \text{ [analogy to Spain, ES];}\\
&M_{13}: m_t = c + a\, m^{IB}_{t-18} + b\, z_t + \varepsilon_t \text{ [analogy to both Iberian countries, IB].}
\end{aligned}
\tag{7}
$$

In $M_{10} - M_{13}$, autoregressive random components are assumed, $\varepsilon_t \sim \mathrm{AR}(1)$. A binary variable z_t in M_{11} and M_{13} removes an 1984 outlier for the Portuguese migration. The prior distribution for $a \sim \mathrm{N}(0.5, 1^2)$ assumes a likely existence on linear analogies between the respective migration flows.

The results of Bayesian model selection procedures for the three classes of models are summarised in Table 1 using the posterior probabilities $p(M_i|\mathbf{x})$. In a vast majority of cases, the selected models are the only ones, for which the data enhanced the prior beliefs. One important addition is the AR(1)-SV process for German–Polish migration: its posterior probability is over two times higher than the respective $p(M_i)$, also indicating significant data support to this model.

Table 1 Bayesian model selection results: posterior probabilities of various models

Migration flow	Subclasses of ARMA(1,1)					Extensions of variance				Models with analogy			
	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}
Poland → Germany	**0.42**	0.29	0.14	0.11	0.03	0.12	0.10	0.05	**0.74**	**0.69**	0.07	0.14	0.10
Germany → Poland	0.23	**0.49**	0.16	0.08	0.03	**0.71**	0.02	0.00	0.27	**0.88**	0.01	0.09	0.02
Model priors $p(M_i)$	0.31	0.31	0.15	0.15	0.08	0.50	0.25	0.13	0.13	0.40	0.20	0.20	0.20

Boldface denotes highest $p(M_i\,|\,\mathbf{x})$ in a given model class. Probabilities may not add up to unity due to rounding.
Source: own elaboration in WinBUGS.

Among the ARMA(1,1) sub-models, high posterior probabilities of the random walk processes and oscillations, with those for AR(1) being just around the respective priors, confirm the research intuition about a hardly predictable character of migratory processes. With respect to similarity to Iberian migration, none of the presented analogies has found enough data support. As the outcome of an analysis for models with different conditional variance indicate high posterior probabilities of the stochastic volatility models, it seems that this is not only migration, but also its own variability that can be perceived as random and uncertain. The SV models indicate that although the Occam's razor priors favour simpler models, the procedure can also point out to more complex ones if they have enough data support. In such way, the obtained solution to the "simplicity *versus* complexity" dilemma offered by the proposed method is not arbitrary, but based on the information provided by the statistical data at hand.

Selected results of forecasts yielded by particular models are presented in Table 2, which shows the predicted migration rates, $\exp(m_t)$ for 2005, 2010 and 2015, together with the jump-off observations for 2004. Along with the forecasted central tendencies, depicted by medians from the respective predictive distributions (2), uncertainty assessments are indicated using the 80-percent predictive intervals, based on the 10-percent and 90-percent quantiles from these distributions. Such an approach is widely applied in stochastic demographic forecasting[3].

From a demographic viewpoint, the obtained median trajectories are plausible and indicate a stabilisation of trends rather than rapid changes. Limits of the 80-percent predictive intervals are also generally reasonable, except for the random walk and AR(1) processes, which indicates a likely non-stationary character of the latter.

[3] Justifications are given for example by Lutz, Sanderson and Scherbov (2004: 37), including that "forecast distributions are themselves uncertain at the extremities. The 80 percent intervals are far more robust to the technicalities in the forecasting methodology than the 95 percent intervals."

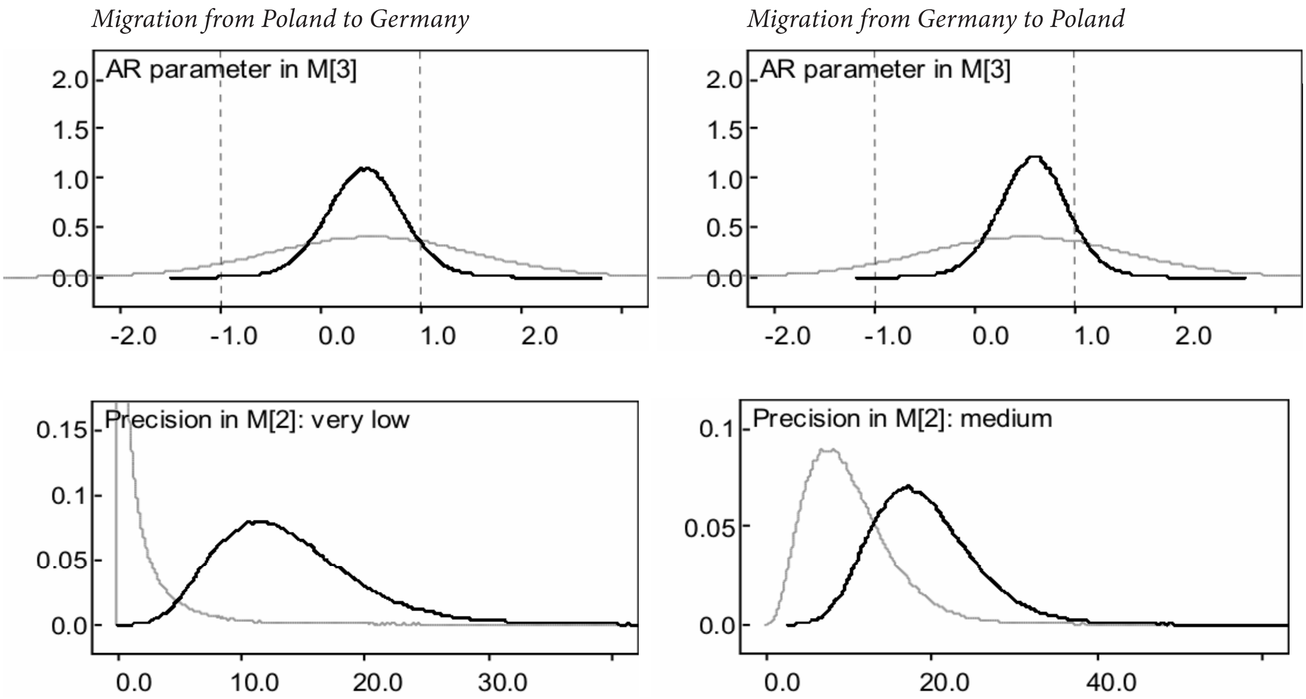
Table 2 Forecasts of exp(mt) for 2005, 2010 and 2015: various models

Model	**Forecasted exp(m_{2005})**			**Forecasted exp(m_{2010})**			**Forecasted exp(m_{2015})**		
	10%	50%	90%	10%	50%	90%	10%	50%	90%
Migration from Poland to Germany; exp(m_{2004}) = 3.65									
M_1: oscillation	**1.77**	**2.57**	**3.72**	**1.78**	**2.57**	**3.71**	**1.78**	**2.57**	**3.72**
M_2: RWD	2.49	3.64	5.33	1.23	3.62	10.71	0.69	3.61	18.80
M_3: AR(1)	1.99	3.00	4.53	1.64	2.67	5.02	1.56	2.63	5.17
M_4: MA(1)	1.80	2.80	4.21	1.72	2.60	3.94	1.72	2.60	3.96
M_5: ARMA(1,1)	1.97	3.09	4.81	1.60	2.68	5.29	1.53	2.65	5.46
M_6: AR(1)	2.33	3.63	5.66	1.53	3.60	9.01	1.37	3.60	10.73
M_7: - ARCH	2.03	3.12	4.73	1.63	2.55	4.13	1.60	2.52	4.08
M_8: - GARCH	2.43	3.45	4.73	1.78	2.99	5.58	1.66	2.89	6.15
M_9: - SV	**2.64**	**3.42**	**4.41**	**1.74**	**3.31**	**7.18**	**1.60**	**3.28**	**8.40**
M_{10}: no analogy	**1.96**	**2.99**	**4.41**	**1.71**	**2.62**	**4.04**	**1.71**	**2.63**	**4.05**
M_{11}: Portugal	1.99	2.99	4.42	1.79	2.93	4.88	1.77	3.24	6.22
M_{12}: Spain	1.97	2.94	4.31	1.77	2.76	4.34	1.85	2.96	4.80
M_{13}: both jointly	1.98	2.97	4.37	1.81	2.90	4.72	1.85	3.28	6.03
Migration from Germany to Poland; exp(m_{2004}) = 1.27									
M_1: oscillation	0.72	1.00	1.39	0.72	1.00	1.39	0.72	1.00	1.38
M_2: RWD	**0.91**	**1.25**	**1.71**	**0.48**	**1.18**	**2.90**	**0.28**	**1.12**	**4.36**
M_3: AR(1)	0.82	1.14	1.58	0.64	1.02	1.79	0.59	1.00	1.89
M_4: MA(1)	0.77	1.09	1.52	0.70	1.01	1.45	0.70	1.01	1.45
M_5: ARMA(1,1)	0.80	1.14	1.61	0.63	1.01	1.72	0.59	0.99	1.76
M_6: AR(1)	**0.90**	**1.26**	**1.76**	**0.69**	**1.24**	**2.36**	**0.63**	**1.24**	**2.71**
M_7: - ARCH	0.84	1.17	1.60	0.68	1.02	1.61	0.65	1.00	1.61
M_8: - GARCH	0.87	1.19	1.59	0.64	1.04	1.79	0.58	1.02	1.89
M_9: - SV	0.94	1.25	1.67	0.72	1.21	2.24	0.68	1.21	2.62
M_{10}: no analogy	**0.80**	**1.12**	**1.55**	**0.69**	**1.01**	**1.48**	**0.69**	**1.01**	**1.48**
M_{11}: Portugal	0.73	1.12	1.69	0.66	1.01	1.56	0.63	1.00	1.58
M_{12}: Spain	0.74	1.06	1.50	0.60	0.91	1.40	0.65	0.96	1.42
M_{13}: both jointly	0.71	1.05	1.54	0.61	0.94	1.45	0.69	1.03	1.53

Boldface denotes models with highest posterior probabilities under the Occam's razor priors.
Source: own elaboration; data for 2004: Eurostat / Destatis.

Figure 1 illustrates, how the sample data modified the prior distributions for two selected parameters: the autoregression coefficient ϕ in model M_3, and precision of the random term (τ) in the random-walk process M_2. Especially in the latter case, a shift towards higher precision (lower variance) of the statistical noise can be observed.

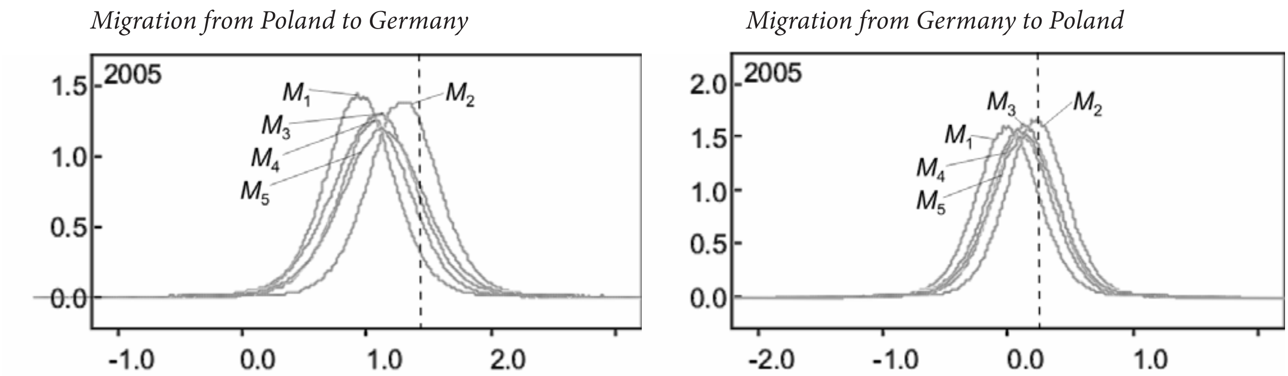
Figure 1 Comparison of prior and posterior distributions for selected parameters



Grey lines depict the distributions *a priori* and black ones – *a posteriori*. Dashed lines are the limits of unit circles.
Source: own elaboration in WinBUGS.

The predictive distributions of log-transformed migration rates yielded for 2005 by various models from the ARMA(1,1) class are illustrated in Figure 2, together with the actual observations for that year. In both cases the highest absolute *ex-post* errors have been obtained for the constant models (M_1), while the lowest ones – for the random walks (M_2). For migration from Germany to Poland a yet smaller error was yielded by the AR(1) model M_6.

Figure 2 Forecasts of "m_t" for 2005 yielded by various models from the ARMA(1,1) class, compared with the actual observations



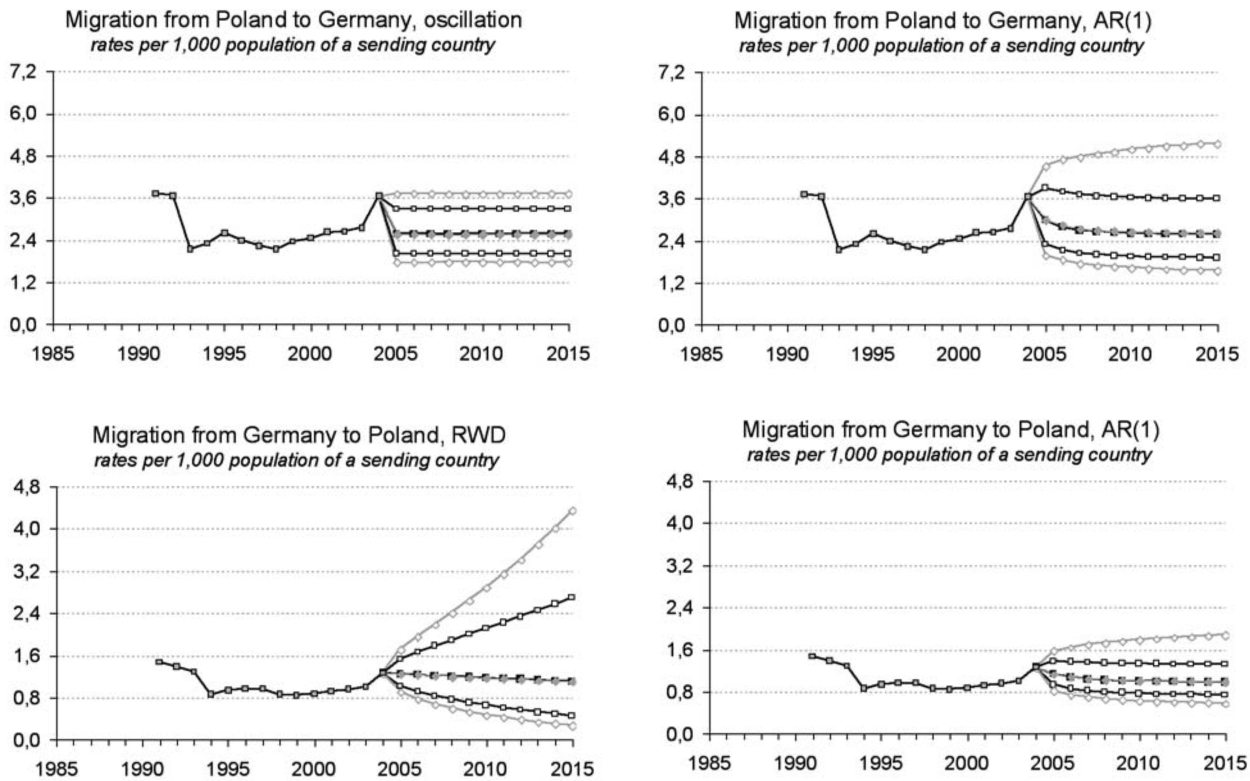
Dashed lines indicate the 2005 values of log-transformed migration rates, with $\exp(m_{PL\text{-}DE}) = 4.17$, $\exp(m_{DE\text{-}PL}) = 1.28$.
Source: own elaboration in WinBUGS, data for 2005: Eurostat/NewCronos.

4. Sensitivity of forecasts to selected changes in prior distributions

As Bayesian inference involves the use of judgemental prior distributions, it is important to assess the robustness of the outcome against changes in the latter. One possibility is to compare the impact of applying various competing priors on the obtained posterior or predictive distributions. In the Bayesian theory, attention is focused on "non-informative" priors, carrying as little statistical information as possible (Jeffreys, 1961). As an alternative, in practical applications "hardly informative" (vague) distributions can be used, as for example Normal N(0, D^2) for structural parameters, where D denotes an arbitrary large number, or Gamma $\Gamma(a, a)$ for precision parameters, with a carrying small values (Congdon, 2003: 2–3, 21).

In this paper, the analysis is limited to two aspects of sensitivity to changes in the prior distributions: for the parameters and models, both treated separately. Firstly, the robustness of forecasts yielded by a handful of models is assessed, that is, by a random walk with drift (RWD) for migration from Germany to Poland, oscillations around the constant for flows from Poland to Germany, and additionally for AR(1) in both cases. In order to obtain "hardly informative" priors for the parameters mentioned above, it has been assumed that $D = 100$ and $a = 0.001$. The results for the selected models are presented in Figure 3. Black lines depict trajectories for the medians and the 80-percent predictive intervals for the respective migration rates, exp(m_t), yielded for 2005–2015 under vague prior distributions, while grey lines – trajectories obtained under more informative priors, the same as defined in Section 3.

Figure 3 Forecasts of "exp(m_t)" for 2005–2015 under vague and informative priors



Grey lines denote medians, as well as 10-percent and 90-percent quantiles from the predictive distributions obtained under the informative priors for parameters, while black lines – under the vague ("hardly informative") ones.
Source: own elaboration; data until 2004: Eurostat / Destatis.

The forecasts yielded by particular models are visibly not robust against the suggested changes in priors, especially in the context of uncertainty assessments. Due to the shortness of time series, this outcome is consistent with the expectations. Nonetheless, the low-precision assumption is a natural premise in migration research, given the uncertain nature of the processes under study. In several instances the 80-percent predictive intervals obtained under vague priors are narrower than the intra-sample variability, and thus hardly realistic. In turn, in the case of the RWD model for migration from Germany to Poland, the intervals under informative priors are too wide, what can indicate that the prior beliefs in relatively low precision were too pessimistic. In all cases, under very modest sample-based information, the role of *a priori* judgement in migration forecasts seems crucial, especially for the *ex-ante* assessments of forecast errors.

Finally, robustness to model priors has been evaluated on the example of the ARMA(1,1) class, by assuming $p(M_i) = 0.2$ as an alternative to the Occam's razor distribution used before. For migration from Poland to Germany, the procedure yielded $p(M_1|\mathbf{x}) = \mathbf{0.32}$, $p(M_2|\mathbf{x}) = 0.22$, $p(M_3|\mathbf{x}) = 0.21$, $p(M_4|\mathbf{x}) = 0.16$ and $p(M_5|\mathbf{x}) = 0.10$, while for flows in the opposite direction, $p(M_1|\mathbf{x}) = 0.17$, $p(M_2|\mathbf{x}) = \mathbf{0.37}$, $p(M_3|\mathbf{x}) = 0.24$, $p(M_4|\mathbf{x}) = 0.12$ and $p(M_5|\mathbf{x}) = 0.10$. Although the selected models (respectively, oscillations and RWD) are the same, the posterior probabilities are slightly different than before. Such an outcome can be also attributed to small sample sizes.

Clearly, the presented basic sensitivity assessments do not exhaust all possible analytical options, which are worth addressing in a separate study focusing for example on the robustness of the outcomes of the model selection procedures against changes in the prior distributions for the parameters and models jointly.

5. Conclusion

The outcome of formal model selection in the Bayesian framework allows for identifying models with relatively highest data support. Results yielded by the selected models, in terms of predictive distributions and intervals, enable in turn to assess the uncertainty of forecasts of the variables under study. This is especially important in predicting international migration flows, given a hardly determinate nature of the processes in question.

Moreover, the Bayesian paradigm allows to incorporate prior expert knowledge into migration forecasts in a formal way, as suggested by Willekens (1994). This seems to be another advantage of the proposed approach in the context of predicting international migration within Europe, where in many cases only short series of data are available, carrying weak sample-based statistical information. As indicated by the analysis of the robustness of forecasts against selected changes in the priors, without judgemental assumptions about the low precision of the random terms, the *ex-ante* predictive intervals would be in many cases implausibly narrow.

The empirical examples also indicate that for Polish–German migration, among the three classes of models under study preference was given either to simple, unstructured processes like random walks or oscillations, or to models without any historical analogies, or finally to models with stochastic conditional variance (SV). These results support a research intuition about hardly predictable nature not only of international migration as such, but also of its uncertainty characteristics, related to heavy tails of the random term distributions. Notwithstanding, the key methodological conclusion from the presented analysis is a full support for including in migration forecasts formal assessments of uncertainty on various levels, including model specification, to which the Bayesian approach provides probabilistically-coherent tools.

Acknowledgements

Paper prepared within the research grant no. 03–34 of the Foundation for Population, Migration and Environment (BMU-PME) from Zurich, first presented during the conference "Dynamic Econometric Models" in Toruń, Poland, 4–6 September 2007 under the title *Bayesian Forecasting of International Migration in Europe: Selected Methodological Approaches*. The author is truly indebted to Prof. Jacek Osiewalski of the Cracow University of Economics for several ideas underlying the paper, many helpful remarks and discussions.

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CONCEPTION OF SPATIAL UNITS APPROPRIATE FOR REGIONAL POPULATION FORECASTS

Branislav Bleha

1. Introduction

There is no doubt that future population proceses and their reflection in number and, above all, structure of the population and consequently censal households, in number of pupils and students and in economic dependency ratio, is a relevant and essential scientific problem. We are convinced that geography (possessing all necessary prerequisites) is a science which should be - together with demography - one of the leaders in the field of creation of analyses and hypotheses of future population processes development in various territorial scale and it should also be able to deal with problems of territorial differences and differentiation or equalization of future processes and subsequently structures, as indicated by Vaňo (2004), Bleha (2005b). Our conviction can be supported by several arguments, presented in the paper. Except for other reasons, the regional forecasting is a component of scientific subject of demogeography (Bleha 2006).

In the paper we intend to pose a question of a set of territorial units appropriate for forecasting. There are two initial facts leading us to the problem stated above.

The first of them is an aspect of concordance of regional populations (in purely demographic interpretation) and really forecasted populations, which is rarely satisfied. The main reason is that forecasting within official administrative units is generally accepted. The second aspect is a consequent principal and practical question of decreasing rate of forecasting uncertainty, which could lead to higher accuracy and quality of forecasts. Definition of a projection of a regional population and its territorial delimitation is a highly demanding task, different in various geographical conditions. If we are not able to delimit spatially or use (up to a certain scale) demographically homogeneous populations, we can utilize other means, representing a basis for our hypothesis. A potentially high appropriateness of a certain type of functional regions for regional population forecasting represents our primary and key hypothesis that has been selected intuitively and which will be analysed and verified.

2. Practice in the Czech Republic and Slovakia

As noted by various forecasting (or projections[1]) experts, such as Kučera 1967, Kučera 1998 (in Bleha 2005), forecasts in Czechoslovakia and later in the successive separated republics were created for administrative regions and districts, respectively, effective at the time of the forecasting. The forecasts were subordinated to planning purposes of superior state administration authorities. Recently, several derived forecasts have been done (Vaňo, in Mládek et al., eds. 2006), such as labour force forecasts (Vaňo 2006). Based on a population forecast, they have been created for districts (corresponding with NUTS IV). Apart from the evident practical need and utilization of the forecasts in the respective administrative territorial units, their consistency from demographic point of view can be a matter of discussion. Nodality, functionality and aspects of proper territorial delimitation were analysed mainly by Bezák (1996a, b). This author observes several disruptions of basic principles (the ones of territorial effectiveness and territorial fairness) showing multiple districts of Slovakia as examples of these disruptions. Except for official forecasts we have noticed only one projection of future population development for a system of functional urban regions (FURs) by Bezák and Holická (1995).

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[1] For more details on projection – forecast relationship, see for example Pittenger (1976), Bleha (2006)

3. Overview of some territorial units potentially appropriate for regional forecasts

We will show three types of territorial forms which could theoretically be applied in regional analyses, using expamples of existing delimited regions in the Slovak Republic. The first framework is represented by regions of (local) migration defined by Bleha and Kurčík (in Mládek et al. 2006). In multiple cases the high rate of territorial concordance of boundaries of these regions with FURs has been confirmed, even if compared with some districts. Thus, the above mentioned facts confirm correctness of delimitation of daily commute regions and in some cases even confirm correctness of delimitation of the FURs and a strong correlation with migration processes. Moreover, this affirms the fact known from Bezák (2006), which could be classified as „high migration power" for short distances. However, we can find also territories were differences between these two types of regions have been identified. These differences may have different reasons, in any case they imply a need for their close research. We must emphasize that the only criterion for the regional delimitation was migration as a process and a pre-condition of territorial integrity, which could affect the final outcome.

Until now, the only framework of territorial forms relevant to the population issues has been presented by Mládek (Mládek, et al. 2006), introducing a demographic regionalization of Slovakia. This author´s approach was based on two partial demographic regionalization criteria of dynamics and age structure. Considering utilization of the cohort-component method, the age homogeneity of the delimited regions can be from the aspect of future hypotheses regarded as a certain „superstructure", being not necessary for the cohort-component method. The main attention will be paid to territorial distribution, size and nature (from the aspect of dynamics) of these regions. From demographic point of view, the above mentioned migration regions can be classified as semi-complex, while the demographic regions can be theoretically characterized by a higher level of complexity. Natural increase and age structure represent two main attributes that are combined to create types and subtypes of the regionalization described above. However, they lack the absolute complexity due to absence of migration aspect serving as a regionalization criterion. Absence of a detailed look at dynamic indicators seems to be a theoretical problem, too, although this probably was not the aim of the presented regionalization.

In the Demographic Research Centre, within the Infostat institute, a typology of Slovakia´s districts was published by Jurčová et al. (2006) utilizing the cluster analysis. Inasmuch it brings a typology of existing territorial units, this analysis is different from those mentioned above. In our opinion, the typology of (development of) demographic structures and processes represent the most appropriate way of preparation of future development hypotheses.

4. The FURs´ utilization in forecasting - foreign experience

Practice of creation of regional forecasts for official administrative units is typical also for most of west-European countries, but also in the U.S.A and Canada. However, we find differences in the rate of concordance (or identity) between unofficial units (considered as more geographical[2] ones) and official units. Moreover, we are interested if there exists a series (optimally continuous and an official one) of regional forecasts. Except for analysing literature and internet sources, a method of direct consultation with experts from relevant institutions (statistical offices, research institutes) was used to find out what is the utility degree and real ways of use of these forms. Apart from others, our analysis was based on a study by Bezák (2000) with a detailed description of formation and utilization of the FUR conceptions in multiple developed countries. However, the study does not show the way how these units are employed for regional forecasts.

Conception of daily commute regions first appeared in the United States of America. The idea was based on delimitation of functional regions with a defined relationship between the core and peripheries (although nodality is not the only aspect) bearing features of both metropolitan and non-metropolitan areas[3]. We must abstract from a more detailed description, because the emergence and diffusion of the conception in the Unided States was thoroughly descripted by the above mentioned author. In the U. S. A., these regions became officially employed statistical units, for which even population forecasts are created. In order to offer more details, we should mention that the *Metropolitan Statistical Areas (MSAa)* serve as officil units at several hierarchical levels. Moreover, the *Metropolitan Labour Areas (MLAa)* were delimited, too, showing a detailed picture of daily commute and being utilized officially. We did not succeed to gain any information

[2] I. e. units which more or less fulfil geographical comprehension of a region

[3] We must emphasize that it is impossible to describe all attributes of the FURs in this paper, including their complexity as well as their typological differentiation. For more details, see the references.

on existence of forecasts for these units. The reasons may be very prosaic. First, no data necessary for forecasting (such as tax rates, etc.) are available for these units, and second, borders of these units do not overlap with the statistical units, which makes it difficult to collect data. This is why the official forecasts are made for *municipalities and counties*. On the other hand, (some of) the MSAs can be composed by a mosaic of counties, which enables to create forecasts. In 2004, the U.S. Bureau of Economic Analysis (BEA) redefined so-called spatial economic units – BEA Economic Areas, reflecting shift in economic growth and population development of certain American regions and including recent conception of micropolitan areas (Johnson and Kort 2004). The redefined BEA economic areas (179 in the U. S. A.) are based on the micropolitan areas (defined first in 2004), CSAs (so-called combined statistical units having been created by clustering the CBSAs – the core units) and MSAs (the Metropolitan Areas). However, a certain instability is typical also for the statistical and scientific units, too, as a consequence of either statistical redefinitions or spontaneous shift of commute inflows and outflows. The territorial instability is not very welcome in terms of evaluation, comparison and practical use of series of regional forecasts.

In Japan, demographic databases are available for small statistical units (so-called *mesh*), although these do not represent natural spatial regions. Forecasts are made only for administrative units such as prefectures and municipalities, which cannot be considered as demographically homogeneous regions or functional regions.

In Australia, the Statistical Office (ABS) employs statistical *divisions* (or *districts* for cities without administrative function) as one of the statistical units. These are defined as areas being under influence of one or multiple cities and might meet our scheme of functional regions, on the other hand, they were not defined according to principles of the FUR. Besides, they do not cover the whole territory of Australia. This is fulfilled only in case of larger *statistical regions*. Thus we are not convinced about the correcntess of the FUR conception´s use for regional forecasting in this country.

Great Britain was the first country in European territory to receive and develop the American conception of the FURs. The recent elaboration of the FUR was realized by a scientific team of the Centre for Urban and Regional Development Studies (CURDS) in 2005, based on the 2001 census (Coombes et al. 2005). Several analyses using these scientific units are available (for example Rees et al. 1996), but they have never become official statistical or administratie units, which could possibly serve as a framework for continuous series of population forecasts.

According to Bezák (2000), the Netherland´s territorial units known as COROPs bearing features of functional regions respect borders of the provinces, or – more exactly – the COROPs represent clusters of municipalities within the provinces. In spite of this fact, no official forecasts for these units are available. Nevertheless, a prognosis for municipalities was published in 2006, which allows us to gain a certain picture of recent development in the COROPs.

Even if we make a detailed look at the neighbouring countries, no forecasts overlapping the FUR boundaries can be found. The statistical office of Austria employs these regions for data collection, but they have never been used for population forecasts. In Poland, the scheme of FUR was elaborated within a project of the IISA in the 1970s, but no official population forecasts have been presented for these units. In Slovakia, the model of FUR was first implemented by Bezák (1990) using the data from the 1980 census, later modified (e. g. Bezák 2001).

To summarize, we can state that the FUR conception has been introduced in multiple scientific fields as well as in practice (such as regional planning and policy, creation of administrative organization of a state´s territory), although an attempt to apply series of population forecasts for these regions has not been successful even in countries with highly developed statistics and demographic research.

European countries recently intend to work mainly with nomenclatural units of the NUTS system, which enables to make comparisons. A study presented by Kupiszewski and Kupiszewska (1999) is a positive example. These statistico-territorial units represent simultaneously administrative areas, on which they are based. Analysing how much the population character of these areas fits conceptions of population forecasting in individual countries is not a matter of this paper, as this seems to be a wider problem. However, we will try to make a theoretical analysis using the territory of Slovakia as an example, as shown below.

5. Regional populations and the Functional Urban Regions

Our intention is to solve the principle question of selection of territorial units that would be more appropriate for regional forecasting. What is the initial point of the problem? In our opinion, we should analyze the term „(regional) population" first. Regional population represents the best definition of a reproduction system in the process of forecasting.

Population´s inner homogeneity can be regarded as one of the principal attributes. A certain degree of homogeneity in terms of structures and processes should be the key feature of populations. Determination of an acceptable homogeneity degree and amount of demographical features (not necessarily corresponding within the population) can cause multiple problems in reality. One of the effects can be delimitation of units with „inconvenient" territorial extent as shown in the results of demographic regionalization.

In the Czech Republic, a socio-geographical and hierarchical regionalization using the 2001 census databases was done by Hampl (2005), based on several older regionalization stages coming from the 1991 census (Hampl 1996). Hampl (2005) respects commute to work and to school as the most frequent regional processes and utilizes them as the only processes in the complex socio-geographic regionalization. He indicates that especially commute to work has a significant integrating role, which confirms correctness of use of these attributes in application in the FUR model, although the Czech regional conception bears some specific features. If we take a look at the map of microregional (or, eventually, mesoregional) units of the Czech Republic according to the above cited author, from the demographic aspect, we can expect a relatively high degree of homogeneity at the lowest hierarchical level, abstracting from the very natural dichotomy centre (centres) – hinterland and city – countryside. In these forms we find strong centripetal relatioship materialized by the commute flows. Nevertheless, this is still not the evidence of the demographical homogeneity of the regions, although they are considered as socio-geographical regions or even (from socio-economic aspect) as complex regions.

Analyzing the heterogeneity and inconsistency of an administrative district from the aspect of fertility and mortality, with probability, that in the future a certain difficult-to-identify part of the district will start behaving in a non-standard way deviated from the empirical knowledge of the district as a whole and threaten the correctness of the prognosis through higher degree of uncertainty is approximately as difficult as in case of the FURs[4]. The situation is much different in case of migrations. The migrations, playing an important role at regional level (of forecasting) as identically stated by several authors (Bleha 2005b, Kupiszewski 1987, Champion et al. 1998, Both 2006) are more comfortably definable.

How can a commute inherently included in the process of delimitation of a region affect migration´s characteristics? Some authors distinguish between intra- and interregional flows, related to different reasons for migration. In the Slovak Republic, this fact is highlighted by Bezák (2006) in his analysis of migration flows. This fact comes from the substance of the FURs and the process of their regionalization, where the rate of uncertainty of a hypothesis about future migration is decreased by share of interregional flows expressed for example as a share of gross migration within the respective FUR. In each FUR, we can identifiy a certain share of intraregional flows which are predominantly not connected with move closer to the place of work, declared as the official reason for migration. This phenomena will be in the future considered as the principal factor in order to include migration and spatial redistribution of population at mesoregional level. Although in the future we will have to deal with the problem of decrease (increase) of individual FURs´ closeness, this reason – „the factor of including and disaggregation of migration" – is considered as an important theoretical argument for regional forecasting for this type of regions, as migration is exactly the factor that might affect accurateness and quality of the forecasts. In Slovakia, we can generally expect increase of labour force mobility. Identification of moves and expected higher redistribution of inhabitants is simpler within the conception of FURs due to the above stated reasons.

Regions defined in the system 91B or A, respectively, are quite often almost identical with the normative units, *de facto* no longer existing administrative districts for which so far only one more complex demographical analysis has appeared (Jurčová 2004). In comparison with the FUR 80 system, Bezák (2000) observes adaptation of the FURs´ borders to the administrative units and a significant geographical stability. The former tendency seems to be quite logical. Another matter is if this tendency was reflected in the forthcoming intercensal turbulent period with huge changes in commute to work. A satisfactory answer could be brought by a new system of the FURs elaborated according to the 2001 census. The facts mentioned above do not allow us to specify clearly which of the two systems (91A or 91B) is more appropriate for regional forecasting. Possibility to utilize bottom-up approach and thus possibility of multiple combinations are

[4] Although a mutual positive effect of daily commute and coherence on the one hand and a stronger isolation and homogeneity of the FURs on the other hand from aspect of natural reproduction (thus of its better comprehension in comparison with districts) cannot be excluded.

indisputable advantages. In specific parts of Slovakia, forecasting for large units would not be sufficient (for example the Bratislava FUR), so different approach will have to be used towards both, the core and its hinterland.

We have already mentioned the significant differences in population size. Considering the same hierarchical level, is future development in large territorial units easier to predict than the one in small ones? Do stochastic processes inherent in development of small units affect also the possibilities to create hypotheses about future development? Answers to these clear questions are not simple. Generally, larger population size of a territorial unit can balance out its heterogeneity and reversely, but only up to a certain degree. The factor of stochastic events and time fluctuations can be found often in case of local forecasts, made for cities, for example. Several FURs in both systems are smaller (in population size) than an average regional/district centre. This problem was quite frequently solved in forecasting prepared for former districts. In other countries, it is obvious to make forecasts for lower hierarchical units, being smaller in population size (counties in the U. S. A., municipalities in the Netherlands).

Table 1

Advantages and disadvantages of the FURs in regional forecasts in relationship with administrative units	advantage +, disadvantage -
cognition and research aspects	**++**
including migration	**++**
including fertility	**o**
including mortality	**o**
potential homogeneity	**+**
spatio-temporal stability	**+/o**
practical use in public administration and decision making institutions in Slovakia	**-**
practical use in relationship with the rest of the EU	**-**
access to statistical data for regional forecasts	**- -**

6. Conclusion

We have introduced several theoretical and methodological arguments for utilization of the FURs in regional forecasting. These regions are more closed/isolated and more geographical than administrative units. We also assume better opportunities to embrace migration and thus better quality of regional forecasts. Absence of a system based on the 2001 census databases seems to be a practical disadvantage, although a certain temporal stability of the regional structure identified in 1980 can be supposed. On the other hand, dynamics of the processes after 1989 call for delimitation of a new system using the fresh data and for verification of regional structure changes, if we intend to use them practically in regional forecasting. Existing dual (basically hierarchical) system offers the advantage of a possible variability in the process of regional forecasting. Consequently, it is possible to utilize regions from one or the other system on the principle of complementarity, if necessary.

Although the model of FURs seems to be a system highly stable in time, changes in intercensal periods stemming from shift in methodology or regional structure changes bring its potential drawbacks. However, in case that the changes of regional structure are only marginal and provided that the methods remain unchanged, the spatial scheme of the FURs can be stabilized. This creates good conditions for utilization of the FURs as a quality scientific basis for regional forecasts in the future. Absence of regular series of forecasts in other EU countries for territorial units delemited by the same methodology in their official national statistics appears as a strong disadvantage. This reduces their chance of being used for forecasting, likewise they have not been implemented into the spatio-administrative conception of the Slovak Republic.

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Joint Eurostat/UNECE Work Session on Demographic Projections
Organized in cooperation with the National Institute of Statistics of Romania (INSSE)
(Bucharest, 10-12 October 2007)
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LABOUR FORCE PROJECTION AT TERRITORIAL LEVEL IN ROMANIA

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Abstract

The transformations that have followed 1989 in the political system, economics and social life have produced changes in the demographic behaviour of the population.

Starting with 1990, the population of Romania decreased in number, from year to year, with an annual rhythm of 0.15%. Population's age structure in Romania shows a continuous, slow process of demographic ageing, principally determined by the decrease of the fertility which caused the reduction of the young population of 0-14 years and the growth of the old population of 65 years and over. The dynamics and structure of population have also contributed to mortality and extern migration. It seems differences in the age structure at a territorial level appear due to the different evolution of the demographic phenomenon's level and to the migration of the population (intern and extern), that occur in time. The negative effects of the ageing process upon the display of economic and social life, as well as on the future demographic evolution are visible and will accentuate in time, determining perturbations at the level of the school population, the economically active population and the old population.

At the same time, the number of employees in Romania has seriously decreased. Currently, less of half of the active population does not work or does, but illegally, in the country or abroad. By 2007, on the labour market there will enter the less numerous generations, and the number of employees will not grow semnificatively even in the case of an eventual constant economic growth.

This paper intends to emphasize the evolution of the economically active population and employment, on age groups, sex and regions. The volume of active population depends on the dimension of the age group 15-64 years and on the "intensity" which it participates with at the economic activity.

At the base of this paper there stands the evolution of population on age groups, sex and on regions, using the components method. Given the fact that the participation at the economic life is influenced by age, sex and regions, the activity and employment rates represent instruments of great detail in emphasizing the particularities and changes that have intervened in the population's participation at the economic activity. The analysis of activity and employment rates on age groups, sex and on regions, obtained from "Household Labour Force Survey (HLFS)", in the period 1996-2006, explained certain tendencies in their evolution.

The projections of the activity and employment rates, until 2050, imposes an analysis upon all elements, which, one way or the other can influence their evolution as well as the extrapolation of their tendencies.

The results of the projection of active population offer an image upon the future evolution of the number and structure of active population on age groups, sex and on regions.

1. Introduction

The labor force, which is represented by the economically active population of a country, is determined by the evolution of economical phenomenon and processes, but also by the complexity of demographic evolutions. By level and structure it cumulates the effects of technical and technological progress and it is determined by material, human and financial resources, available of potential, by prices, salaries, by the level of traditions and consume, and also by the economical politics. Of all these conditionings, we will rest upon the demographic one, which, although slightly visible, is extremely important through its implications.

The structural demographic mutations that have been registered at the level of total population are found in the offer as well as in the request of labor force. Thus, the structural and numerical modifications of the population and the demographic aging influence the level and structure of economically active population and its behaviors: employment and unemployment.

To continue, we are going to present data concerning the evolution's prognosis of economically active population and of employment for year 2025 and 2050. In the analysis of the main tendencies and hypothesis of the projected evolution there was insisted on emphasizing the modifications in the structure of population at working age (15-64), which represents the potentially economically active population.

The outcome of the projection of the economically active population creates an image of the future evolution of the umber and structure of these population segments, on age groups, gender and in a regional profile. The projecting of economically active population derives from the demographic projection of population, elaborated with the method of components.

The estimation of the economically active population, for the year 2050, was realized through two methods. For both methods we included the probable evolution of population at working age, divided on age groups, gender and regions. The projection has been made by applying the activity rates, and the employment rates on the population projected on age groups and genders, registered in each region.

2. Working age population

The population of Romania has faced, in the last few decades, some important modifications that are partly due to demographic transition and to the changes that took place in the political climate, changes that have greatly influenced the evolution of demographic phenomenon. Starting with 1990, the total population has been decreasing from year to year with an average annual rate of 0.2%. Throughout the period between 1992 and 2002, the population has been decreasing with 1.1 million inhabitants. The negative values of the natural birth rate, combined with the values of external migration sold have made the total population decrease with more than 268 thousand persons. The descendant evolution is not at all surprising, taking into consideration that all the information upon the natural move and the migration from the last 17 years show a well installed decrease.

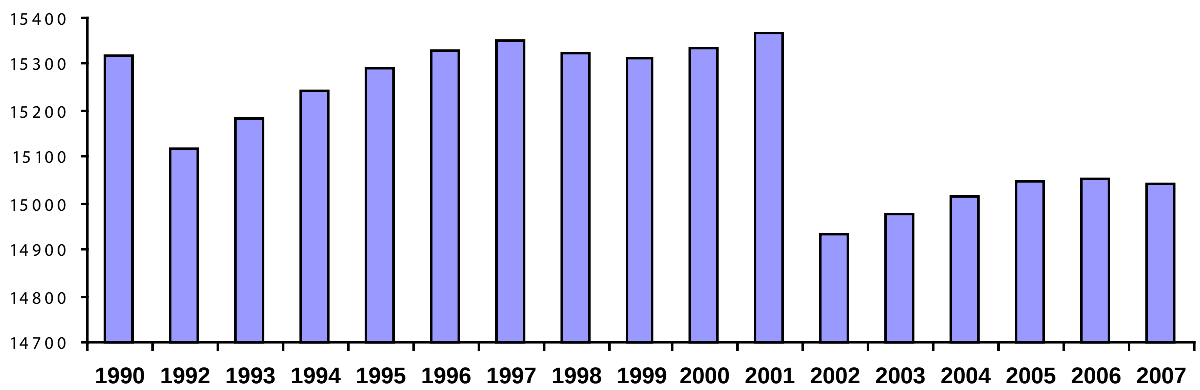
Population's age structure bears the characteristic print of an ageing population process, that is basically determined by the dropping number of births, which caused the decreasing of the young population of 0-14 years and the increase of population aged 65 and over. Therefore, this decrease, from a rate of 23.7% in 1990 to a rate of 15.4% in 2007, represented an important loss of 8.2 units.

The external migration and death rate have also contributed to the changes that took place in the population's dynamic and structure.

The old population number (aged 65 and over) has been increasing during this period, compensating for the "losses" suffered by the young population. The rate of the old population has increased from 10.3% (1990) to 14.9% (2007)

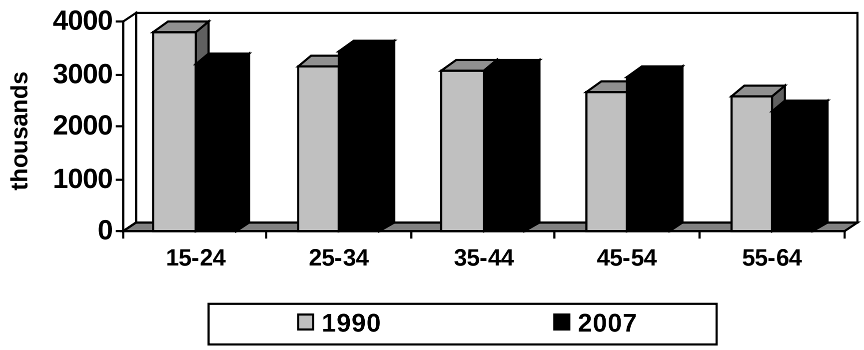
The working age population (15-64), the potentially active population, represents the main segment of the labor force offer. During this period, it registered less modification, oscillating between 66.0% (1990) and 69.8% (2007); even so, the evolution of the age structure at this subgroup has produced essential changes.

Figure 1 Working age population between 1990 and 2007



As the number of population aged 15-24 has continuously decreased from 16.4% (1990) to 14.9% (2007), the number of persons aged 35-44 has increased from 13,3% (1990) to 14.3% (2007).

Figure 2 Working age population by age group



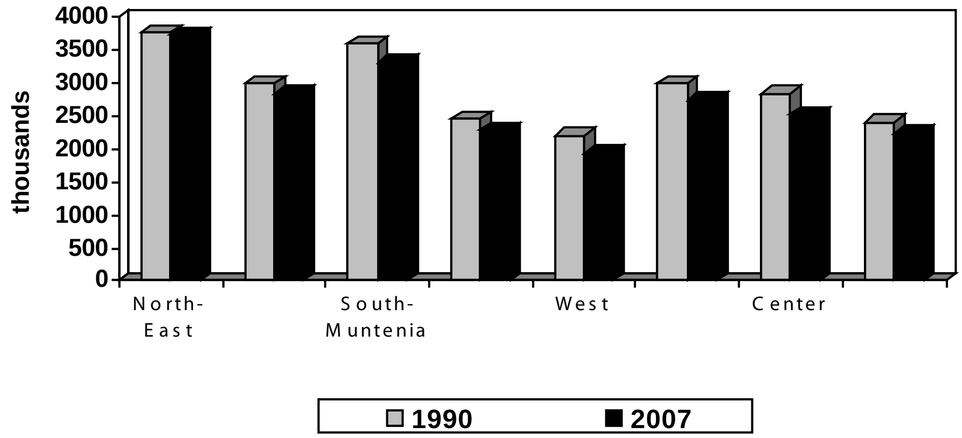
In January 2007 the working age population (15-64) was registering 15,043 thousand persons, representing 69.8% from the total population, 276 thousand persons less than in 1990 (15,319 thousand persons).

From a regional point of view, the decrease of the population has been realized differently, depending on the fertility, death and migration rates levels in every region.

The North-East region, registering 3,728 thousand persons (2007), is the biggest from the number of inhabitants point of view, representing 17.3% from the total population. Characterized by a high birth rate, the districts from this area (Suceava, Iasi, Botosani, Vaslui) have registered an increase in the period 1990-2001, followed by a decrease.

The region of South-Muntenia, with 3,305 thousand persons (2007), represents 15.3% of the country's population. In the period of 1990-2007 this region registered a fall of 7.5% in the population number.

Figure 3 Number of population by region

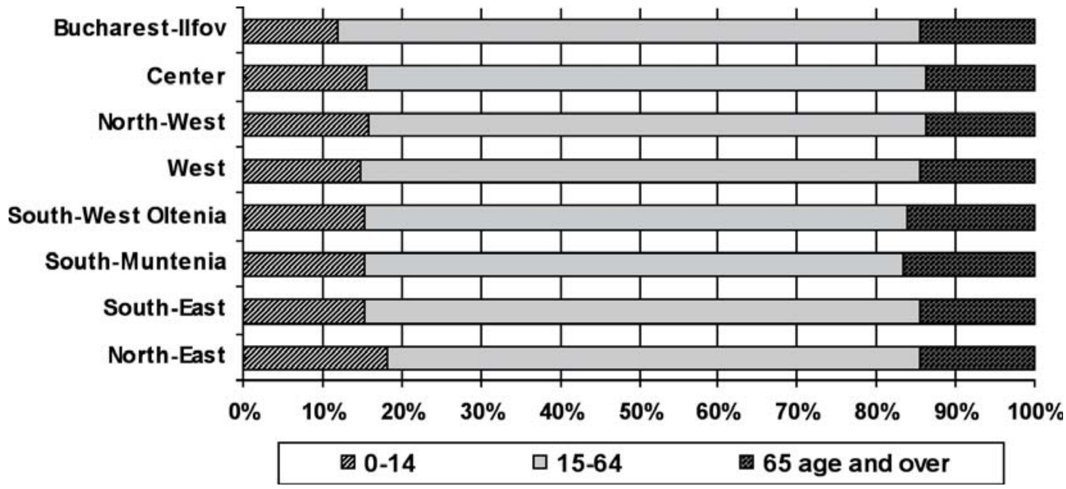


The South-East region, with 2,834 thousand persons (2007) and the North-West region, with 2,729 thousand persons (2007), have registered a decrease of 5.3% and 8.4% in the population number.

The West region, with 970 thousand persons in 2007, this figure representing 9% from the total population, is the smallest region but registering the highest decrease in the number of inhabitants 12.5%.

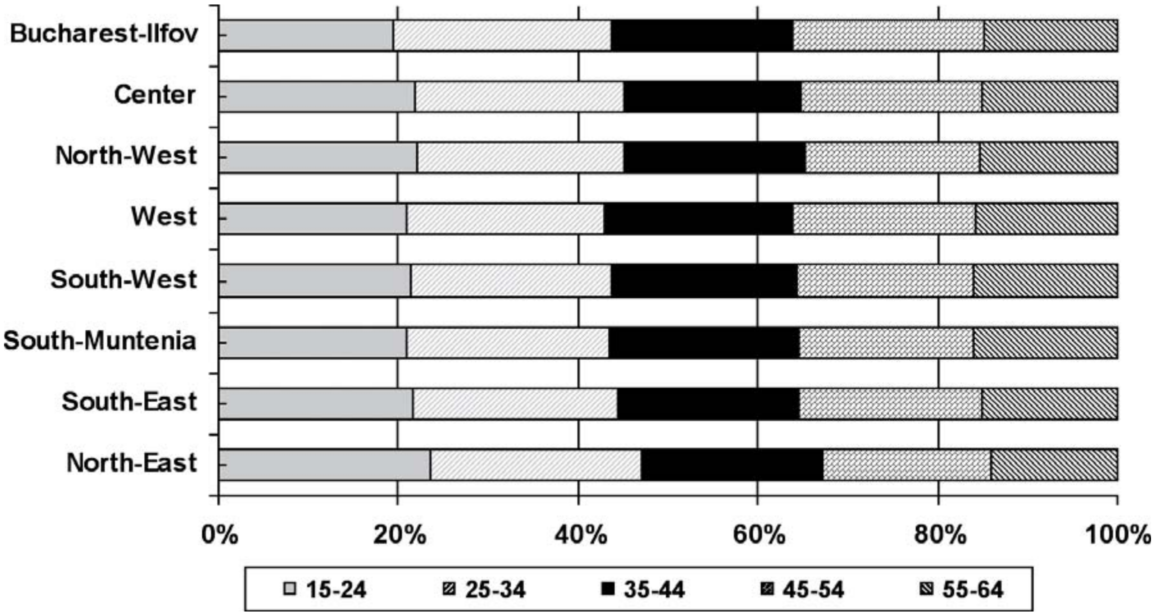
Bucharest-Ilfov is the highest urbanized region, which registered, in the last 17 years, a population decrease of 6.8%. Watching the structure of the population on regions and large age groups at January the 1st, 2007, we notice regions that are more "aged" (South-West Oltenia and South Muntenia) with old population of 16% and "younger" regions such as North-East, with the 0-14 years population with 18.1%, followed by North-West and Center, with 17%.

Figure 4 Population structure by region and large age group



The rate of the population aged 15-64 oscillates between 67.5% (North-East) and 73.7% (Bucharest-Ilfov). Also, the rate of the population aged 15-24 varies between 19.5% (Bucharest-Ilfov) and 23.6% in the North-East. The numerous generations born in the '70 and '80 have determined the growth of the age groups of 25-44, situated between 41% and 45%, in all regions.

Figure 5 Population structure by region and age group



The age group that registers the lowest rates, between 14% and 16%, is the population aged 55-64, which represents the less numerous generations, born after the World War II.

The projection of the economically active population is in fact derived from the demographic projection of the population. For this we use the "constant" variant of a demographic projection of the population on age, sex and regions of development, the base population being the one from 1st of July 2006; the values of the demographic phenomenon (birth rate, death rate, internal and external migration) registered in 2006 in each region.

The birth rate, the main component of the natural evolution of the population has registered, starting with 1995, a descendant evolution more obvious in the southern regions. The long term decrease of the birth rate led to the forming of a final decrease, in 2006, of approximately 1.3 children per woman in most of the regions. The total fertility rate in 2006 oscillated between 1.2 (in South-East and Bucharest-Ilfov) and 1.5 (North-East). One can remark the fact that the young generations post-pone not only building a family, but also giving birth to children. If the birth rate is a phenomenon extremely sensitive to changes of socio-economical and legislation factors, the population's death rate has more complex causal mechanisms, its evolution being characterized by a bigger inertia and a larger stability in time. The death rates, at older population, have decreased under the influence of mutations in the age structure and health state of the population. Although the number of deaths in the first year of life has had a tendency of decrease, the level registered in 2006 remains high in comparison with the one of developed countries (13.9 deaths under 1 year at 1000 live births).

At the moment, the level of life expectancy from a regional point of view, for the male population, varies between 68.39 years in the North-West and 70.73 years in Bucharest-Ilfov. For the female population, life expectancy oscillates between 75.33 years in the West and 77.15 years in Bucharest-Ilfov.

The internal migration fluctuations play an important part in the configuration of regional demographic typologies. Both the inner district and the outer district migration are influenced by the general level of regional economic development. Reducing the long distance migration in favor of the continuously growing short distances migration, leads to the apparition of areas with a larger capacity to maintain the inborn population. The North-West and Bucharest-Ilfov are the regions that attract population each year, especially young population situated in the 15-35 age group, therefore registering a positive rate. Being a main university center and due to the fact that the headquarters of the biggest companies are situated there, Bucharest-Ilfov is an area that acts like a magnet for labor force. Thus, in 2006, there have been registered the largest value of positive rates (+ 10,269 persons). The other regions present negative values of migrations, acting like 'donors' of both young and adult population.

From the total projection of population (2006-2025), there has been selected the population at working age (15-64 years), represented on age groups, sex and regions.

The fore coming evolution of the population at working age:

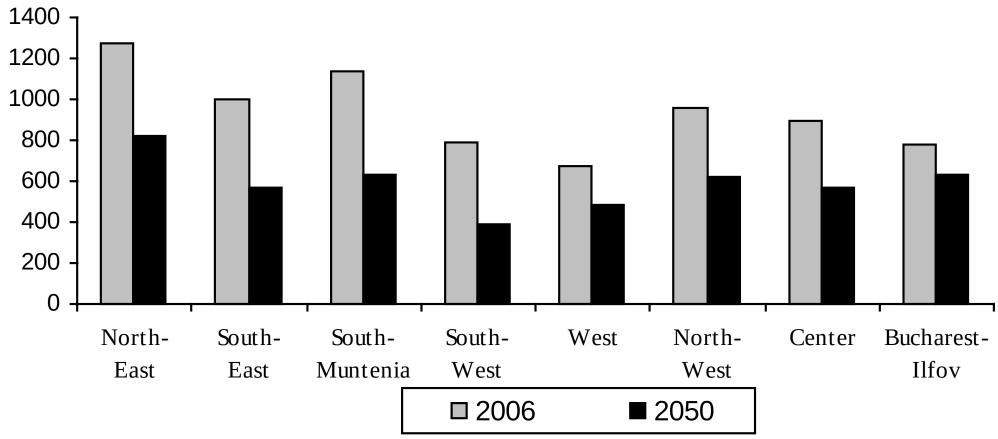
Table 1 Working age population by region between 2006 and 2050

-thousand persons-

Regions	2006	2025	2050	Modified 2050/2006	
				Absolute	Relative (%)
Age 15-64	**15062**	**13518**	**9479**	**-5583**	**-37,1**
North-East	2518	2326	1629	-889	-35,3
South-East	1994	1720	1123	-871	-43,7
South-Muntenia	2260	1959	1233	-1027	-45,4
South-West Oltenia	1573	1330	761	-812	-51,6
West	1367	1260	969	-398	-29,1
North-West	1924	1751	1264	-780	-34,3
Center	1789	1605	1144	-525	-36,1
Bucharest-Ilfov	1637	1567	1356	-281	-17,2

Population at working age (15-64 years) will reduce with 37.1%, down to 9,479 thousands until 2050. The drop will be moderate until 2015 and more accelerated towards the ending line of the projection horizon, the main factor of this evolution being the natural decrease.

Figure 6 Working age population by region in 2006 and 2050



The region of South-Muntenia registers the biggest drop of male, working age population. In 2050 it will have 505 thousand persons less, compared to 2006.

The female, working age population will register the biggest decrease in the South-Muntenia and South-East regions, with almost 522 thousand and 440 thousand persons less.

On the short term, until 2010, population aged 15-64 will maintain around 15 million. After 2010, the less numerous generations born after 1990 will add up to the population at working age. This is the reason why the 15-64 segment will gradually diminish, ending to represent between 15.4% and 15.6%, in 2025. "Old" groups from population at working age will become more and more numerous, creating, in time, a sort of misbalance on the work market. After 2015 this population segment will start reducing, as 'older groups', well represented in numbers, will integrate into the old population segment.

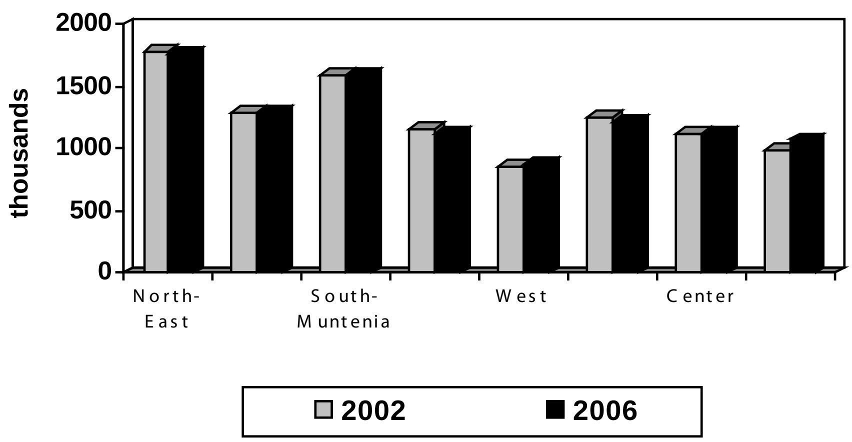
2. Economically active population

Economic activity includes all people, of both sexes, who supply, on a certain reference period, the necessary labor force for producing goods and services. Economic activity is composed of employment and unemployed population.

The number, structure, the level of education and professional forming as well as the modality of using the working force have a great influence upon the GDP, upon the economical, social and cultural development. At the same time, the proportions, the structure and the employment's evolution are determined by the cumulated actions of numerous factors. Some of these factors are the main rulers of the structural and qualitative dimension of the working force offer, as others determine the volume of the demand on the market. The transformations that the Romanian economy suffers impose the socio-economical changes, which generate mutations in the active population structure and quality.

Starting with 2002, the total of active population, 10,097 thousand persons, registered a decrease period, reaching 9,851 thousand persons in 2005, thus, the activity rate modifying from 63.6% to 62.4%. In 2006, there has been registered a slight growth in the numbers of active population (by 191 thousand persons), the level being 10,042 thousand active people. In comparison with 2002, in 2006 we can observe an increase of the active population by 7.8% in Bucharest-Ilfov region.

Figure 7 Economically active population by region in 2002 and 2006



At a regional level, the activity rate varies between 59% (in Center) and 66% (in North-East). Over 80% of the adults are active from an economical point of view and from the total population aged 50-64: 58%-60% are on the labor force market. In the care of the 65 year olds and over, the activity rate is situated between 33% and 35%. In the case of the young, male population, one of two is active from an economical point of view, as for the young women only one out of three is active.

The decrease of the working age population determines the reducing of active population. The general tendency of decrease of the economically active population reflects differently on the age groups level, from each region of development. For projecting the numbers of active population we utilized the activity rates, calculated on the dates took from 'AMIGO'.

The situation existent on the labour market in Romania, the evolution phenomena of employment, unemployment and non-economically activity represent the objectives of "The Household Labour Force Survey (AMIGO)". Starting with 1996, the household labour force survey is quarterly carried out, as a continuous research, thus allowing to get short-term date on the size and structure of labour force supply and to point out seasonal phenomena taking place on the labour market.

The last regulations of the European Commission have changed the definition of employment and unemployment coverage. For total harmonization with the European Union principles and methodologies in the field of employment and unemployment statistics, the questionnaires of the AMIGO survey have been redesigned in 2002.

We have worked on two hypothesis:

- "*constant scenario*" in which the activity rates from 2006 have maintained constant throughout the whole chosen period;
- "*medium scenario*" in which we have used the activity rates from 2002-2006 period.

Table 2 Economically active population by region in 2006, 2025 and 2050

-thousand persons-

Regions	2006	2025	2050	Modified 2050/2006	
				Absolute	Relative (%)
Constant scenario					
Total	**10042**	**9329**	**6560**	**3482**	**-34.7**
North-East	1757	1699	1241	-516	-29.4
South-East	1298	1154	756	-542	-41.8
South-Muntenia	1586	1392	886	-700	-44.1
South-West Oltenia	1119	992	602	-517	-46.1
West	871	831	622	-249	-28.6
North-West	1217	1162	835	-382	-31.4
Center	1120	1034	717	-403	-36.0
Bucharest-Ilfov	1074	1065	900	-174	-16.2
Medium scenario					
Total	**10042**	**9226**	**6488**	**-3554**	**-35.4**
North-East	1757	1721	1256	-501	-28.5
South-East	1298	1132	743	-555	-42.8
South-Muntenia	1586	1379	885	-701	-44.2
South-West Oltenia	1119	992	602	-517	-46.2
West	871	816	611	-260	-29.9
North-West	1217	1146	822	-395	-32.5
Center	1120	1013	703	-417	-37.5
Bucharest-Ilfov	1074	1027	865	-209	-19.5

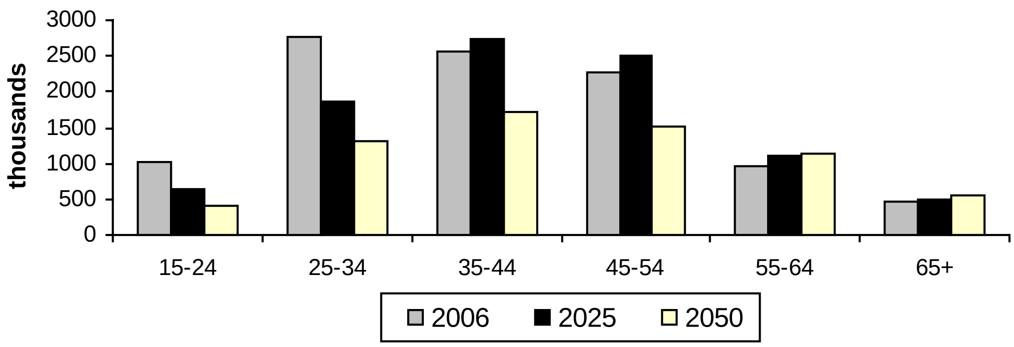
In both hypothesis the active population number will register a decrease, from 10 million today to 6,6 million in 2050 - in the 'constant scenario'- or down to 6,5 million - in the 'medium scenario'.

The decrease of the active population will be observed by all the regions, with smaller values in the western regions (North-West and West), the Center regions, North-East and Bucharest-Ilfov; large values will be registered in South-Muntenia region, an 'old' region from the point of view of population's age structure.

The slight increase form 2007-2010 period represents a certain conjuncture misbalance between enters and exits in and out of the active population: the less numerous generations born in the period World War II will exit and the numerous generations from the 80's will enter.

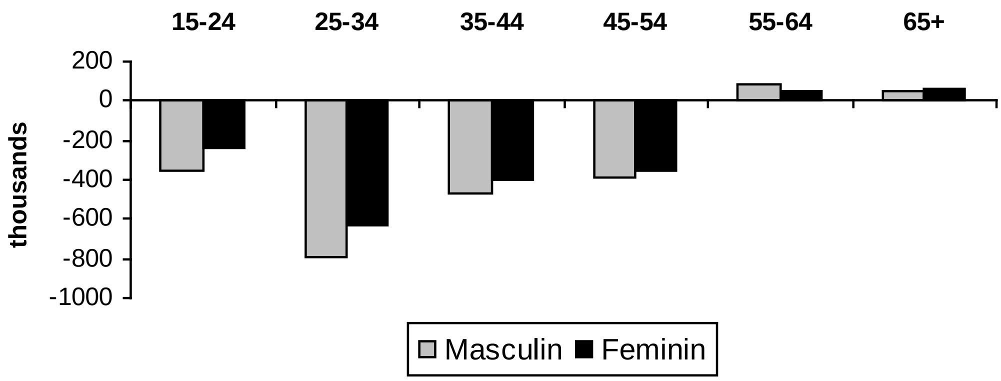
The dimension of the active population will be reduced due to several dynamics: active population increase for 40-64, 65 age group, but also over 65, and the decrease of the active population 15-24 and 25-34 age groups, made of the generations born after 1990. The main consequence of these structural changes is the growth of the medium age of active population, from 40 years in 2006 to 43 years in 2025.

Figure 8 Economically active population by age group
in 2006, 2025 and 2050



This misbalance will be felt also on the level of economical dependence report, therefore the 'economical weight' of the active population will grow, being determined by the continuous increase of the old population and to the decrease of the young population.

Figure 9 Increase/decrease of economically active population by sex, by age group
in 2006 and 2050



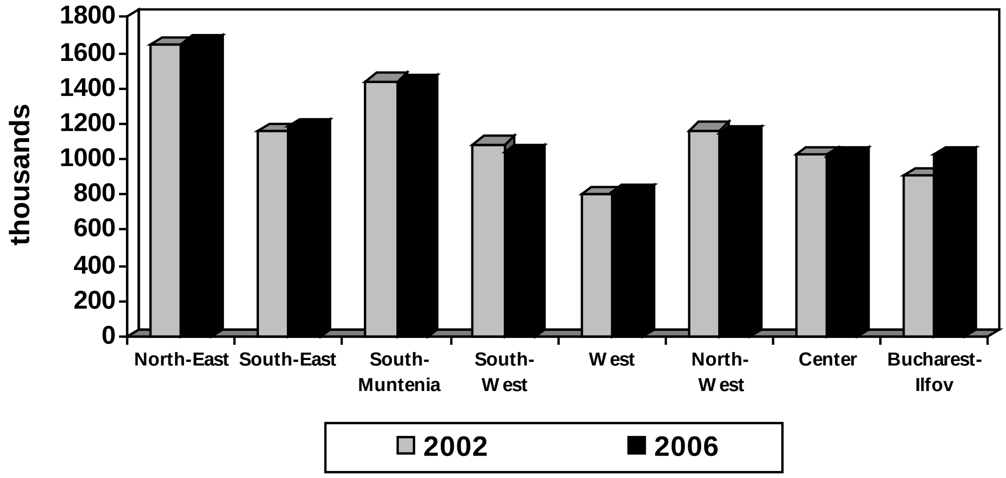
These differences will grow mainly on sex groups, but in the favor of male population, which number will constantly grow from the numbers of the total active population, from 55% to 55.2%.

3. Employment

The most important component of the active population is the employment population. This category is the effective producer of goods and services necessary to the whole society's existence. At the same time, the employment gives the tone for the labor force demand on the labor market. The labor force demand is determined by both the evolution for economical processes and phenomenon which have to do strictly with productive and nonproductive subsystems of society and also by the complexity of demographic evolution and the possible pressures of the offer. Population's consumption needs reflect in the quantity of work necessary to satisfy them, and this quantity finds its correspondent within the numbers of occupied population. Employment includes all the persons aged 15 and over, who have developed an economic activity producing goods or services, of at least an hour in the reference period (one week) in order to get income as salary, payment in goods or other benefices. It was adopted the standard criteria of "at least one hour" recommended by the International Labour Office (ILO) to define the employment in order to ensure the data comparability at international level.

In the 2002-2005 periods many modifications have been produced on the labor force market, a decrease of 87 thousand persons out of the occupied population being registered.

Figure 10 Employment by region in 2002 and 2006



The employment rate oscillated between 58% (2002) and 58.8% (2006), with differences on genders and lower averages, more in the case of women (between 52% and 53%) than in the case of men (63%-64%). From the age structure point of view, the biggest value is that of persons aged 25-34 followed by the age group 35-44.

Employment, part of the adult age group (24-54), is situated primarily in the urban environment, as for the 55 year old and over, they live mostly in the rural environment.

From the total value of employment almost a fifth is concentrated in the North-East region. At the opposite end, the West region is situated, with a value of almost 9.0%. In comparison with year 2002, in 2006 the employment increased with 114 thousand persons, in Bucharest-Ilfov, followed by North-East, South-East and West regions. The most accentuated decrease has been registered in South-West Oltenia.

The employment rate situated at working age (15-64) has reached the highest values in North-East regions and Bucharest-Ilfov, as the lowest values were in Center and South-East regions.

The modifications appeared at the values of the employment are determined by the changes registered at the level of the active population. These modifications reflect differently at the level of age groups from each region, due to the employment on age groups rates, corresponding to each development region.

For projecting the employment number, there have been used the employment rates calculated with data from AMIGO. We have worked with two hypotheses:

- "*constant scenario*", in which the registered occupation rates for 2006 have maintained constant on the whole chosen period;
- "*Medium scenario*" where there were used the occupation rates registered in the period 2002-2006.

Table 3 Employment by region in 2006, 2025 and 2050

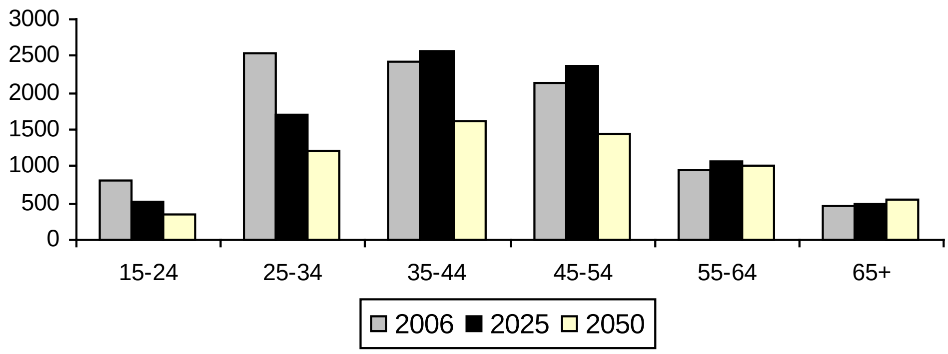
-thousand persons-

Regions	2006	2025	2050	Modified 2050/2006	
				Absolute	Relative (%)
Constant scenario					
Total	**9313**	**8724**	**6169**	**-3144**	**-33.8**
North-East	1653	1609	1183	-470	-28.4
South-East	1182	1059	698	-484	-40.9
South-Muntenia	1437	1276	819	-618	-43.0
South-West Oltenia	1039	931	571	-468	-45.0
West	815	783	587	-228	-28.0
North-West	1145	1100	793	-352	-30.7
Center	1019	947	659	-360	-35.3
Bucharest-Ilfov	1023	1019	861	-162	-15.8
Medium scenario					
Total	**9313**	**8625**	**6096**	**-3217**	**-34.5**
North-East	1653	1629	1197	-456	-27.6
South-East	1182	1042	688	-494	-41.8
South-Muntenia	1437	1271	821	-616	-42.9
South-West Oltenia	1039	935	573	-466	-44.9
West	815	766	574	-241	-29.6
North-West	1145	1084	781	-364	-31.8
Center	1019	934	651	-368	-36.1
Bucharest-Ilfov	1023	964	811	-212	-20.7

From the projection of employment we may observe its decrease from 9,3 million in 2006, to 6.1 million in 2050, in the 'constant scenario', and, to 6 million people in the 'medium scenario'.

Analyzing possible changes which can appear in the evolution of the employment on each region's level, we observe the much accentuated decrease between 40%-45% in the regions of South-East, South-Muntenia and South-West.

Figure 11 Employment by age group in 2006, 2025 and 2050



Bucharest-Ilfov will be the less affected region by this decrease.

As in the case of the active population, the decrease of the population aged 15-24 and 25-34 is also felt in the case of the occupied population. The increase from the 34-64 age groups will be notable.

Regardless of the region, the number of occupied population at working age will be larger for men than for women.

4. Conclusion

The activity rates for adult and more advanced ages might suffer changes that are hard to predict. There are too many unknown factors, especially those with origins in the present and future migration of labor force values, in the European space, as well as the ones determined by the economical evolution of the country in the new UE environment. A fact that seems to remain firm is the decrease of the activity rates of very young active ages, through the prolongation of the education period.

If the number and structure of economically active population will be influenced by the manner in which the modification of participating to economic activity on age groups, is still unknown; even so, we have sufficient elements to measure the impact which the evolution of birth rate will register in the last decades, and, especially its recoil after 1989. Apart from that, the potential changes that the birth rate might suffer in the next 20 years will not affect the dimension and structure on age groups and sex of the economically active population, but only its rate in the number of the total population and the report of economical dependence of youngsters.

PROJECTING ETHNO-CULTURAL DIVERSITY OF THE CANADIAN POPULATION USING A MICROSIMULATION APPROACH

Alain Bélanger[1], Éric Caron Malenfant[2], Laurent Martel[2] and René Gélinas[2]

Abstract:

Sustained immigration from non-European countries over the last two decades has contributed to a rapid increase in the ethnocultural composition of the Canadian population. From census to census, we observe an increase in the number and proportion of people belonging to ethnic, religious or linguistic minorities. These changes have many implications, for example, in terms of labour market integration, social services, racial and religious discrimination, social cohesion, or public institutions.

In this context, Statistics Canada developed a microsimulation model to project the ethnocultural composition of the Canadian population. The model allows the projection of several dimensions of the population such as its ethnic, religious, foreign-born and linguistic characteristics. The model takes as its starting point the 2001 Census complete microdata file (20 % sample) comprised of more than six million records. It uses a Monte Carlo process and the probabilities associated with each possible event to project the population according to a set of assumptions related to the components of population growth and the differential in demographic behaviours of subgroups. Development of the model required analysis of various data sources (census, surveys and vital statistics) to estimate its parameters, including differential demographic behaviours and intergenerational transfers of some characteristics.

Key results of the projections show that the ethno-cultural minorities would grow at a much faster rate than the rest of the population and would comprise most of the Canadian demographic growth until 2031. As a result, the share of the ethno-cultural minority groups in the population would continue to increase and more than one out of four Canadians would belong to a "visible minority group" by 2031. Ethnocultural diversity would continue to concentrate in the largest urban centers of the country, Toronto, Montreal and Vancouver. In Toronto for example, it is expected that near 60% of the population will belong to a visible minority group by 2031.

1. Introduction

Sustained immigration from non-European countries for the last two decades has rapidly increased Canada's cultural diversity, and is still doing so. The 2001 Census showed an increase in the numbers and proportions of immigrants, people belonging to visible minorities, allophones and people whose religion is non Christian (Statistics Canada, 2002, 2003a and 2003b). Canada's ethno-cultural makeup, especially in large urban areas, is changing rapidly, bringing political decision-makers to deal with a number of challenges, particularly in the areas of labour market integration, social services, racial and religious discrimination, social cohesion, or public institutions.

Differentials in demographic behaviors and settlement patterns (at time of arrival as well as after) of the newcomers tend to speed up the process of change. In particular, recent immigrants to Canada tend to show higher fertility, lower mortality because of the selection effect, and above all a much higher concentration in the largest metropolitan areas

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than the Canadian-born population. Immigrants' internal migration behaviors further intensify this concentration. This transformation may continue for the medium and long term if current trends are maintained in the forthcoming years. This paper is an attempt to chart some of these future transformations in terms of visible minority status[3], religious denomination, mother tongue and regional distribution under several scenarios of change through micro-simulation modeling. This paper should be viewed as a supplement to a report published by Statistics Canada in 2005 and titled *Population projections of visible minority groups, Canada, provinces and regions 2001-2017* (Bélanger and Caron Malenfant 2005). This report is available free on the Statistics Canada web site (www.statcan.ca).

The next section of the paper first explains why a microsimulation model was used and provides a general description of the model developed. It then describes the modules parameters, the assumptions and scenarios underlying PopSim, the microsimulation model specifically developed to realize these projections. It is followed by a brief analysis of the projections' key results for some of the projected characteristics. The paper concludes with a description of possible developments for PopSim.

2. Advantages of microsimulation and general description of the model developed

The increasing diversity in the place of birth of Canadian immigrants is expressed in various dimensions. Thus, to paint a portrait of the Canadian diversity in 2031 requires the simultaneous projection of a large number of individual characteristics: age, sex, marital status, place of residence, visible minority group, religious denomination, mother tongue, age at immigration, period of immigration. The traditional cohort component or the multistate projection models are not suited to the large number of characteristics needed to be projected.

Taking only one of these dimensions, let's say belonging to a visible minority group, using the cohort component method to project the future ethnic diversity of the population would necessitate projecting separately each of the 10 visible minority groups and adding the results of each of these sub-projections to another separate projection for the rest of the population to get the total. Each of these projections would have its own set of assumptions for fertility, mortality and migration to try to take into account some of the group's differentials in their demographic behaviors. From this base, prevalence rates or another distribution function could be applied to derive population numbers by religious denominations or mother tongue in a manner similar to the application of headship rates in household projections. Keeping track of age at immigration or period of immigration would generate even greater difficulties without providing satisfying results. In addition, results are likely to be inconsistent with each other as they will come from separate derived projections.

Using a more dynamic approach such as the multistate population projection model would rapidly turn out to be unmanageable given the size of the matrix needed to simultaneously project all possible transitions. In the case of the projections realized for this project, the transition matrix would have had to count billions of cells. Clearly, another projection model had to be developed.

Using a Monte Carlo process and the probabilities associated with each possible event, we can develop such a model through microsimulation. Microsimulation enables us to model complex demographic behaviors in a consistent and flexible way (Nelissen, 1991). The microsimulation model has several theoretical advantages over traditional projection models: 1) it can project a large number of characteristics simultaneously; 2) it allows for differentials in behaviors and thus can take into account findings in regards to fertility, mortality and migration differentials in the estimation of the projection parameters, and 3) it accounts for changes in population composition over the projected period.

This is why we developed PopSim, a dynamic, continuous time, longitudinal, event based, open, stochastic (Monte Carlo) spatial microsimulation projection model. Popsim, which used MODGEN[4], a microsimulation modeling language developed at Statistics Canada and based on C^{++}, is:

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3 Visible minority groups are defined according to the Canadian Employment Equity Act. Under this Act, members of visible minorities are "persons, other than Aboriginal persons, who are non-Caucasian in race and non-white in colour". The ten specific groups designated by the Act are: Chinese, South Asian, Black, Filipino, Latin-American, South East Asian, Arab, West Asian, Korean and Japanese, thus excluding all Aboriginal peoples and Whites. A question on visible minority groups (self-declared) has been asked in the Canadian censuses of 1996, 2001 and 2006.

4 Information on MODGEN is available on Statistics Canada's web site at www.statcan.ca.

- dynamic as opposed to static in the sense that population aging and growth are the result of a life-cycle behavioral model applied at the micro- level, while in a static model, population aging and growth are usually performed by re-weighting the initial database using exogenous information (usually, cell-based demographic projections);
- continuous time as opposed to discrete time;
- longitudinal as opposed to cross-sectional in the sense that the model simulates the entire life course of an individual until death, emigration or the projection horizon before starting a new case, while a cross-sectional model simulates the entire population over one time period (usually one year) and then moves to the next time period;
- event-based as opposed to time-based in the sense that individual characteristics as well as time are incremented conditionally to the outcome of other events while time-based models implement modules following a predetermined order (e.g., fertility before mortality or vice-versa);
- open as opposed to closed in the sense that the model allows for new individuals to enter the projected population through immigration, or to leave the population through emigration;
- spatial as opposed to a single region (national) model in the sense that the model simulates internal migrations between (currently) 29 Canadian metropolitan and non-metropolitan regions.

The advantages of such a design are that its dynamic and open features explicitly account for changes in population composition over the projected period. Its continuous time event based approach is the best way to deal with competing risks as it does not request a specific order in which the events have to occur, as is the case in discrete-time, time driven simulation models. Finally, its spatial feature allows for spatial analysis of the consequences of immigration, a key element of policy analysis in ethnic diversity and immigration studies.

Another advantage of PopSim is that it is designed so that it does not have to deal with the problem of the lack of robustness of the results encountered when the parameters are estimated from surveys with small samples and large variances (Caldwell, 1983) or when the limited sample size of the simulated population increases the Monte Carlo variance. PopSim doesn't face these limitations because it simulates the future life cycle of all the respondents to the 2001 Canadian census long form. The starting database contains therefore about 20% of the Canadian population or about 6 million records reducing the Monte Carlo variance to negligible proportions. In addition, to reduce the parameter's variance for most of the demographic modules, PopSim used vital statistics and census data to estimate the base risk of the occurrence of an event. In the case of fertility, for example, the base risk is computed from vital statistics and is entered as input into the microsimulation model in the form of a probability (of having a child) table by age and parity. Relative risks are then applied to the base risk to take into account the differential behaviours.

This approach also allows for scenarios to be built in a manner similar to the traditional cohort-component projection model. Thus, the base risk of a given component can be replaced by another base risk that corresponds to another assumption.

Figure 1 contains a diagram of the projection model used[5]. As in the traditional models, the population at time t+a results from the demographic changes that affected the population since time t. In the model used, an individual can change marital status, bear a child, move to another area of residence, die, emigrate to another country or age 1 year. A number of new individuals are also added over time by birth or by immigration.

For each individual, we calculate the probabilities that each of these possible events will occur on the basis of the individual's specific characteristics. For example, the risk of giving birth to a child will vary depending on whether the woman is a recent immigrant, belongs to a visible minority group or some other group, and so on.

Using a Monte Carlo process and the probabilities associated with each possible event, we can determine which event will occur first and compute the amount of time that will elapse before it occurs. When an event occurs (including an anniversary), the probabilities of all possible events are estimated again using the updated individual's characteristics and the potential events are reordered. Thus, each individual moves forward through time until he/she dies, emigrates or reaches the projection horizon.

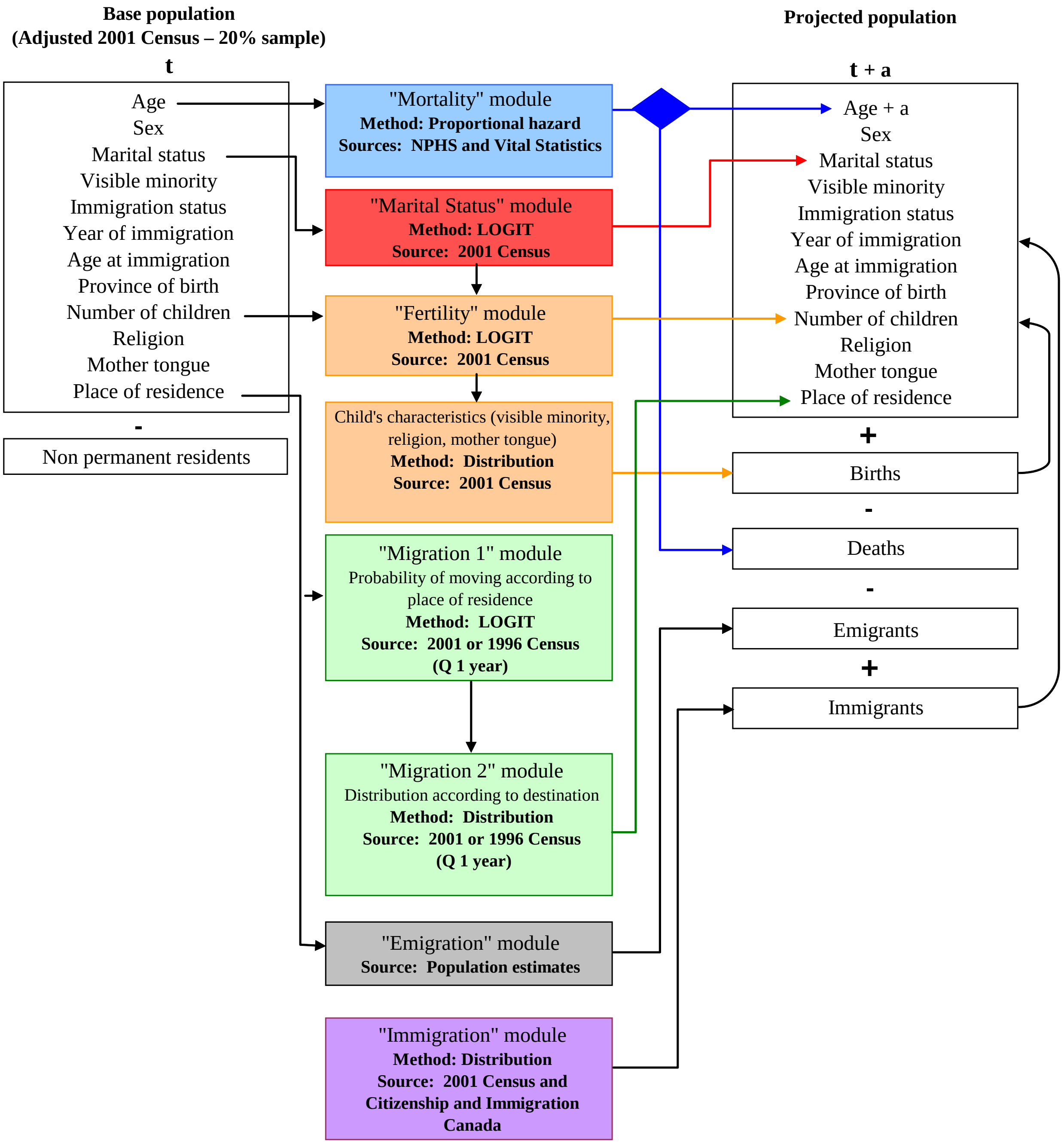
[5] This is a schematic representation since the model is actually continuous in time and event based. However, the figure captures in a single image all the methods and sources used to estimate the microsimulation model's parameters.

As shown in Figure 1, the model is composed of six modules, each one representing a demographic component. The parameters of the microsimulation model, particularly the equation for differentials in demographic behaviours, are usually obtain from logistic, proportion-hazard or multivariate regressions on either census or survey data, depending on which component is modelized. The next section describes how each module of Popsim was developed and how it takes into account differentials in behaviour.

3. Modules parameters

Previous analysis showed that fertility level will vary according to immigration status and duration of residence in Canada (Bélanger and Gilbert, 2003), visible minority groups (Caron-Malenfant E. and A. Bélanger, 2006) and religious groups (McQuillan, 2004). Analysis of 2001 Census data shows that, all other things being equal, women who immigrated in the previous 10 years were 19% more likely than other women to have borne a child during the year. This relative risk varies with birth order. When taking age, marital status, recent immigrant status, religion and place of residence into account, findings showed that, apart from Aboriginal women, who are a unique case, the most fertile women were those who reported belonging to the Black, Filipino or Arab visible minority groups (they were, respectively, 60%, 28% and 22% more likely than Whites to have a child under the age of 1 at home). Chinese, Korean, Japanese and West Asian women were the least fertile. Analysis of 1996 Census data produces similar results regarding the effect that ethnocultural characteristics have on fertility.

Figure 1 Schematic representation of the population microsimulation model



Moreover, given those differentials in fertility, and given that each group's share of the total population will vary, differential fertility is an important factor in the future population size, population age structure and population's composition. To fully take into account these differentials in fertility, the base risk of having a child (age-specific fertility rates by parity) will be increased or decreased based on modelled probabilities that vary according to the mother's age, parity, marital status, area of residence, immigrant status and length of residence in Canada, visible minority group and religious denomination.

The fertility module has a submodule that assigns characteristics to newborns and takes account of the intergenerational transfer of selected characteristics. For some characteristics, such as the region of residence, the mother's characteristic is assigned to the child, but because of interethnic marriages, for example, newborns do not necessarily have the same visible minority group, religious denomination or mother tongue as their mother. In this model, intergenerational transfers of mother tongue depend on the mother's mother tongue, immigrant status and area of residence; transfers of visible minority group depend on the mother's immigrant status and visible minority group; and religious denomination transfers are influenced solely by the mother's religious denomination. Those transfers of the ethnocultural characteristics are based on transition matrices computed with 2001 Census data. Finally, the child's sex is randomly imputed to ensure that, overall, the sex ratio at birth is maintained.

A number of studies have shown that immigrants have lower mortality than native-born Canadians (Ng. and all., 2005; Chen and all., 1996; Trovato, 1985). Their higher life expectancy is generally attributed to a selection effect, since landed immigrants in Canada are required to undergo a medical examination before they arrive in the country. Analysis of National Population Health Survey (NPHS, a longitudinal survey) data shows that immigrants who arrived within the last 10 years have a relative risk of dying of 0.35, which means that, all other things being equal, their death rate is a third that of the total population. For immigrants who arrived in Canada more than 10 years ago, the risk of dying is not significantly different of that of the rest of the population. Thus, in the projection model, the projected base risk of dying will be increased or decreased according to the immigrant status and length of residence in Canada of each individual in the microsimulation, the latter variable reflecting the fact that the selection effect fades over time. Survivorship proportions for the projected period were estimated by combining the relative risks based on National Population Health Survey data with projected age-, sex- and province-specific death rates projected using the Li and Lee (2005) mortality forecasting model.

The marital status transition module essentially creates an intermediate variable that provides a better estimate of fertility and, indirectly, internal migration, phenomena that are strongly affected by family composition. Three marital statuses were used: married, common-law and not in a union. Since transition probabilities for this component cannot be estimated from the census, multinomial logistic regressions were used to estimate parameters for redistributing the population according to the above statuses. In the model, the probability of being in one of the three statuses varies with age, the number of children at home, immigrant status and length of residence in Canada, visible minority group and area of residence. The marital situation of each individual is reassessed on the basis of how these characteristics have changed on each anniversary. This approach does not provide coherent marital histories for individuals, but it yields a plausible distribution of the population by marital status at least in the short term. Since marital status is merely an intermediate variable used to estimate fertility, this approach is considered satisfactory, but may necessitate improvements in future developments of the model.

Internal migration has little effect on the total size of the population at the national level, but it is one of the two components, international migration being the other, that have the greatest impact on the population's geographic distribution. A number of studies have shown that immigrants and non-immigrants exhibit differential behaviors with regard to the probability of internal out-migration and the choice of destination (Newbold, 1996).

Analyses of the probability of migrating and the destination of internal migration clearly indicate the importance of taking differential migration into account. For example, when the population is separated into groups defined by place of birth, we find that native-born Canadians who live outside their province of birth have the strongest propensity to migrate. Their probability of migrating is nearly three times that of Canadians who live in the province where they were born. They are also more mobile than very recent immigrants (those who arrived in Canada in the five years prior to the Census), who in turn are more mobile than other immigrants. Analysis of visible minority data also corroborates the assumption of differential migration for visible minority groups. Koreans are the most mobile, with a 40% greater chance of migrating than Whites, while Filipinos are the least mobile. With regard to choice of destination, it is worth noting that migrants who report belonging to a visible minority group are far more likely to move to Canada's largest urban centres (Toronto, Vancouver, Montreal, Ottawa, Hamilton, Calgary and Edmonton). About 65% of visible minority persons who change areas of residence move to one of these seven metropolitan areas, whereas 65% of Whites choose to settle elsewhere in Canada.

To take this differential migration into account, the migration module first estimates the specific probability of out-migrating for each individual by region of origin, age group, sex, presence of children in the family, visible minority status, mother tongue, place of birth and length of residence in Canada. Then a second module allocates a new region of residence (destination) to out-migrants on the basis of region of origin, their age group, visible minority status, mother tongue and place of birth.

The projection model sets the annual numbers of immigrants and assigns them individual characteristics (age, sex, area of residence, visible minority group, religious denomination, etc.) based on the characteristics of recent immigrants enumerated in the census. The immigration module's parameters can be defined so that the characteristics of future immigrants will be representative of the characteristics of immigrants who arrived during an earlier period, or so that the proportion of each visible minority group will be predetermined.

Lastly, the projections must consider not only the arrival of new immigrants, but also the departure of emigrants. The propensity to leave the country is related to the immigrant status and duration of residence in Canada (Michalowski, 1991). Studies show that new immigrants are more likely to emigrate again, either back to their country of origin or to another destination, especially the United States. Accordingly, the emigration module uses annual emigration rates by age and sex for each province and territory as the base risk to which a parameter that takes into account the higher propensity to leave the country of recent immigrants' is applied. More specifically, the base risks are multiplied by 2.4 for recent immigrants and by 0.67 for native-born Canadians, an estimate obtained from the Reverse Record Check study.

4. Assumptions and scenarios

As is generally done in a traditional population projection exercise, various scenarios were developed from a series of assumptions about the evolution of the components of population growth. The report from which this paper is a supplement (Bélanger and Caron-Malenfant, 2005) compares results from five different scenarios with varying fertility, immigration and internal migration assumptions. However, for the sake of brevity, results from only one scenario are presented in this paper in order to give an example of what PopSim can generate as outputs[6]. This scenario is similar to the reference scenario of the original publication. However, it differs from it in a few aspects.

First, the horizon is longer. The 2005 report projected the ethnocultural composition of the Canadian population from 2001 to the year 2017, in this paper the horizon is the year 2031.

Second, immigration levels are higher than in the previous reference scenario. Rather than a fixed number of immigrants, we assumed a fixed immigration rate of 7.0 per thousand, the average immigration rate observed during the 1990-2005 period. Although immigration rates were relatively stable in Canada during that 15 year period, past evolution of immigration levels shows that a reversal of trends can be both sudden and sizable. In addition, Canada could increasingly be looking at immigration to lessen some consequences of the inevitable aging of its population. Canada is one of the countries where the baby-boom was the most important. The first baby-boom cohorts are now reaching the age of retirement or, at least, ages where we observe reduced labour force participation. Immigration can be part of the solution to the projected increase in the labour force demand resulting from the increasing number of persons withdrawing from the labour market. As the Canadian population presents a positive growth rate during the projection period, we therefore assume that the number of immigrants increases across time from about 235,000 at the start of the projection to about 280,000 in 2031.

The model is built to allow the formulation of different assumptions concerning characteristics of future immigrants. In view of the specific objectives of these projections, immigration level has to be considered together with a set of sociocultural and geographic characteristics, including the immigrant population composition by visible minority group. In the scenario used in this paper, new immigrants are randomly chosen from the Census data base from among those who landed in Canada during the 1996-2001 period. As a group each new cohort of immigrants will show on average the characteristics (age at immigration, sex, place of residence, visible minority group and religious denomination) of immigrants who landed in Canada during that period.

Thirdly, life expectancy at birth would reach 81.9 years for Canadian males and 86.0 years for Canadian females in 2031. This assumption is slightly higher than the one used in the previous report, which assumed levels of 80.0 and 84.0 years in 2026 for males and females, respectively.

Other assumptions about fertility, internal migrations and marital status are the same as in the 2005 report (Bélanger and Caron-Malenfant, 2005). In Canada, the total fertility rate has been fairly stable, around 1.5 children per woman

[6] Readers interested in the other scenarios can contact the authors of this paper or demography@statcan.ca. The other scenarios show similar results although of a different magnitude: higher immigration and higher fertility assumptions translate into higher population growth and ethnocultural diversity while it is the opposite with lower fertility and immigration assumptions.

for a number of years (Bélanger, 2006) and the base risk of fertility estimated from the vital statistics reflects this level. While the total fertility rate is the same as the one used in the medium assumption of the most recent national, provincial and territorial projections (Bélanger, Martel and Caron-Malenfant, 2005), it should be noted that, unlike the latter, this projection model takes differential fertility into account.

Internal migration is a particularly important component for these projections since we are interested in the geographic distribution of visible minority persons. The assumption used is based on migration patterns observed between 2000 and 2001 from the Census.

Demographic statistics indicate that total emigration is fairly stable in Canada, at around 50,000 a year on average over the last 15 years. It also has a relatively minor impact on projected population size, though the higher propensity of young adults to leave the country may have a slightly larger effect on the population's age structure.

The projected emigration rates by sex, age and province of residence are the same as those used to produce the most recent projections for Canada, the provinces and territories (Bélanger, Martel and Caron-Malenfant, 2005).

5. Results

Table 1 contains population figures for visible minorities and the rest of the population on January 1, 2001, and in 2031 based on the projection scenario described above and the percentage change in those populations over the period considered.

Table 1 Population[7] (in thousands), percentage distribution and growth rate by visible minority status, Canada, 2001 and 2031

	2001		2031		
	Number	%	Number	%	Growth rate (per thousand)
Visible Minority	4,116	13,4	10,621	27,4	32.1
Rest of the population	26,616	86,6	28,165	72,6	1.9
Total	30,732	100,0	38,786	100,0	7.8

The data show that under the scenario considered, Canada's visible minority population could be 10,621,000 in 2031, or 2.6 times larger than in 2001, when it was estimated at about 4,000,000. This increase in the population extends the upward trend that saw the visible minority population go from 1.1 million in 1981 to 1.6 million in 1986, 2.5 million in 1991 and 3.2 million in 1996.[8]

The increase in the rest of the population would be much smaller. The population that does not report belonging to a visible minority group, estimated at 26,616,000, would rise to 28,165,000, an increase of about 6% in 30 years.

While the annual growth rate of the total population would be slightly lower than the current level, this would result from a dramatic differential in the growth rate of the two subpopulations. During the 30 years of the projected period, the visible minority population could grow at an average rate of 32 per thousand a year while the rest of the population would grow at a pace 17 times slower at about 2 per thousand a year.

By the end of this differential growth in 2031, more than one person in four (27.4%) would belong to a visible minority group in Canada (see Figure 1). In 2001, 13% of respondents reported belonging to a visible minority as defined in the Employment Equity Act; this was already an increase over the 11% who did so when the question was introduced in the 1996 Census. In 1981, the proportion of persons belonging to the visible minority groups within the Canadian population was 4.7%.

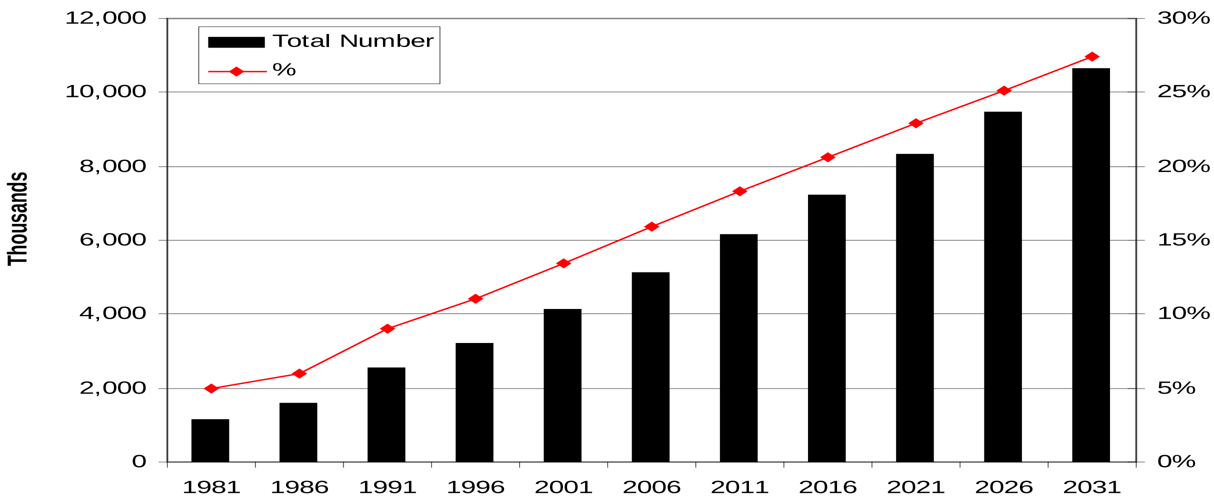
Among the factors that account for this more rapid growth in the visible minority population, the most important are unquestionably the sustained immigration and the high percentage of visible minority persons among the new arrivals in

[7] Excluding non permanent residents

[8] See the analysis that accompanied the release of 2001 Census data, at the following address: http://www.statcan.ca/english/census01/products/analytic/companion/etoimm/contents.cfm

the scenarios developed for these projections. Other factors include a higher fertility and a younger age structure, which result in fewer deaths, and a higher life expectancy for visible minorities than for the rest of the population.

Figure 1 Total number and proportion (%) of visible minorities observed (1981-2001) and projected (2001-2031) in Canada

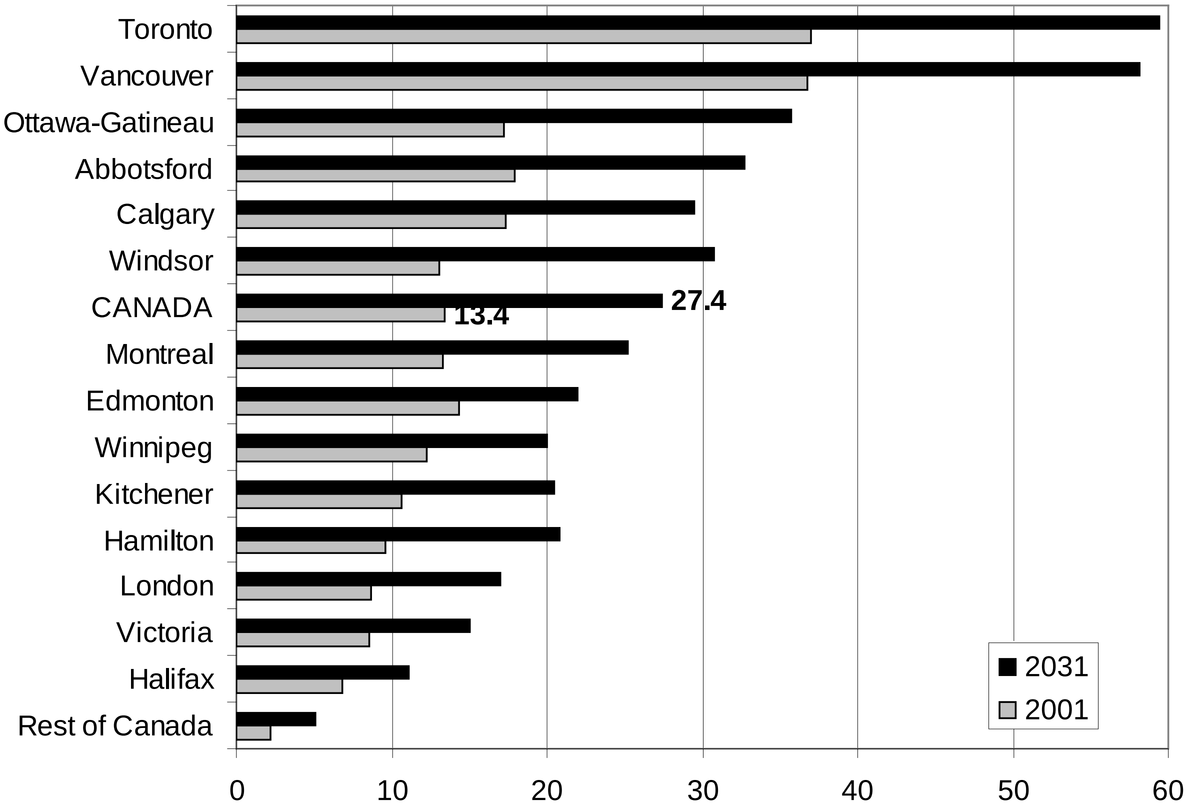


The data presented so far have shown that the evolution of Canada's ethnocultural diversity is particularly sensitive to immigration. Newly arrived immigrants are very selective in their destination choice when landing in the country, and Canadian immigration is increasingly an urban phenomenon. More than 70% of the immigrants who came to Canada in the five years preceding the 2001 Census chose to settle in one of the country's three largest census metropolitan areas: Montreal, Toronto and Vancouver.

Figure 2 shows that if the current trends continue, the evolution of Canada's ethnocultural portrait will not take place uniformly across the country. It is worth noting which of Canada's major urban areas have particularly high concentrations. The visible minority population is heavily concentrated in a small number of metropolitan areas. According to these projections the six largest metropolitan areas - Toronto, Montreal, Vancouver, Ottawa-Gatineau, Calgary and Edmonton - will present large proportions of visible minorities among their population in 2031. The cases of Toronto and Vancouver warrant special attention since, in 2031, almost 60% of their respective population would belong to a visible minority group. In this scenario, Toronto would have a population of 8,056,000 including 4,788,000 visible minority persons and Vancouver would count 3,239,000 people among which 1,883,000 would belong to a visible minority group.

It is also interesting to note that some metropolitan areas such as Abbotsford, Windsor, Kitchener, Hamilton and London show a rapid increase in their proportion of visible minorities between 2001 and 2031, some of them doubling their 2001 proportions. This results in part from a dissemination process of the visible minority population over time. These metropolitan areas are geographically close and economically linked to either Toronto or Vancouver, two of the largest points of entry for new immigrants.

Figure 2 Proportion of visible minority population by selected regions, Canada 2001 and 2031

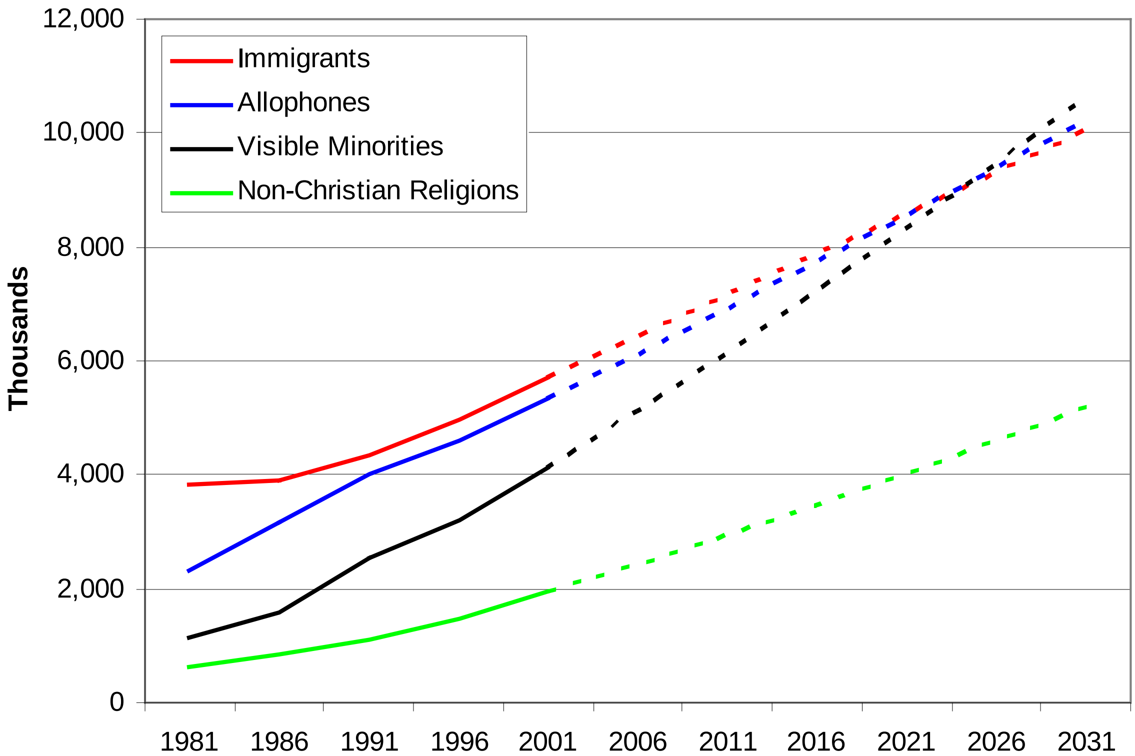


However, the dissemination process might be much slower in other areas of the country. Under the scenario developed for these projections, the rest of the country, which is composed mostly of non-metropolitan areas, but also counts smaller metropolitan areas such as St.John's, Saint John, Regina and Saskatoon, would remain fairly homogeneous in its ethnic composition. The region titled "Rest of Canada" on figure 2 represented 46% of the total population in 2001 and 38% in 2031, but it would count only about 5% of its population in a visible minority group. Its population would also be growing at a much smaller pace, 0.1 % per year compared to 1.7% and 1.5% for Toronto and Vancouver, respectively.

These findings are not without policy relevance. Clearly, if the current trends continue, Canada might well be facing increasing demographic challenges in areas that will be growing fast and will rapidly become more diverse while other regions will have to deal with a declining population, rapid aging, albeit with a more homogenous ethnic composition.

Figure 3 reveals other aspects of the transformation of Canada's ethnocultural landscape. The increasing importance of immigration as a component of population growth and the diversity in the countries of origin of new immigrants will also result in major changes in the number and proportion of foreign-born, of allophones (people whose mother tongue is neither English nor French) and of persons who are members of non Christian religions. According to these projections, the number of immigrants would increase from 5,700,000 in 2001 to 10,042,000 in 2031 when they would represent nearly 26% of the total Canadian population compared to 18.5% in 2001. The number of allophones would grow faster as their number is projected to increase from 5,328,000 in 2001 to 10,247,000 in 2031, rising from 17.3% to 26.4% of the total population. Finally, the number of persons who are members of non Christian denominations would increase from 1,959,000 to 5,200,000 during the projected period with their share of the total population doubling from 6.4% in 2001 to 13.4% in 2031.

Figure 3 Population of visible minorities, immigrants, allophones and members of non-Christian religious denominations, Canada, 1981 to 2031



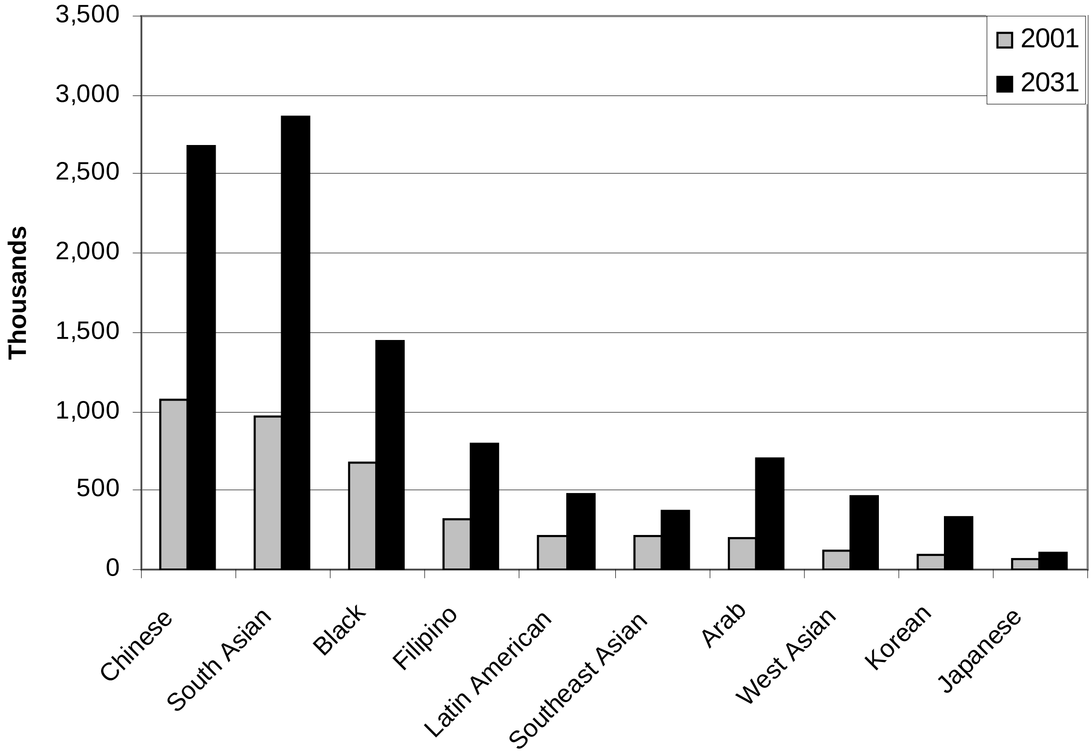
It is interesting to note that if in 2001 the number of immigrants was higher than the number of allophones or of persons belonging to a visible minority, in 2031 the number of immigrants would be lower than both, illustrating that projecting a single dimension of ethnocultural diversity is not sufficient to foresee the global changes to come. This is because past immigration was mostly from European countries with a large proportion coming from the United Kingdom. Immigrants who came prior to 1970 were therefore much more likely than newer immigrants to be non-visible (white), to have English as their mother tongue and to belong to a Christian religion. Increasingly, immigrants are coming from non-European countries and have ethnocultural characteristics that differ more from the characteristics of the majority. It is also due to the fact that in the model the visible minority group and the mother tongue can be transferred from the mother to their Canadian-born children, which, of course, is not the case for the immigrant status.

This will also have several policy implications as their integration into the Canadian labour market, as an example, might be more difficult than it was for former European immigrants, either because of discrimination based on their ethnocultural characteristics, or because of other factors such as reduced recognition of their academic credentials if they are from non-European or non-American countries, or because of lower proficiency in English or French.

It could also have consequences on urban planning. In places of rapid demographic and economic growth, land is scarce and expensive and it might become difficult to build new places of worship (Germain, 2004) or provide adapted sport and recreational facilities to a multiethnic population with different tastes and different needs (Poirier, Germain and Billette, 2006).

As noted earlier, the composition of cohorts of new immigrants as well as differentials in demographic behaviours, particularly fertility differentials between groups, could lead to differences in projected growth rates of subpopulations. PopSim also allows to analyse future changes in the ethnocultural composition of the population in more detail. Figure 4 shows the size of the various visible minority groups in 2001 and 2031.

Figure 4 Population by visible minority group, Canada, 2001 and 2031



As in 2001, the South Asian and Chinese groups would be the largest in 2031 with 2.86 and 2.67 million respectively, and together they would represent a little more than half (52%) of all visible minority persons.

They were already the largest groups in 2001, but their share of the total population was different then. While the Chinese group outnumbered the South Asian group in 2001, under this projection scenario, the latter may catch up to the former by 2031. As the South Asian group has higher fertility than the Chinese group and almost as big a share of immigration, the South Asian population would grow at a rate of 36.9 per thousand between 2001 and 2031 compared to a growth rate of 31.0 per thousand for the Chinese population.

After the South Asians and the Chinese, the two largest visible minority groups in 2031 would be two groups who, all other things being equal, were among the most fertile[9]: the Blacks and the Filipinos. Immigration would also play a role in the two groups' population growth, as they fall just below the Chinese and South Asian groups in their share of annual immigration, at 7% and 6%, respectively. The Black population could reach 1,445,000 in 2031. In 2001, it was estimated at 681,000. The Filipino population, estimated at about 321,000 in 2001, would grow to 798,000 by 2031.

However, the visible minority groups that would grow fastest between now and 2031 are the West Asian, Arab and Korean groups, with annual growth rates of 47.3, 42.6 and 41.9 per thousand, respectively. At the horizon of the projection, under the scenario considered, there would be 697,000 Arabs, 457,000 West Asians[10] and 326,000 Koreans in Canada. With populations of 200,000, 114,000 and 95,000 respectively, these visible minority groups together made up about 10% of the Canadian visible minority population in 2001, but their overrepresentation among the immigrants who arrived during the period used to develop the immigration assumptions, if maintained until 2031, could raise their share of the total visible minority population to 14%.

Immigrants from different origins tend to settle in different locations when they land in Canada. Vancouver receives a larger proportion of Chinese immigrants, Toronto is the destination of choice of South Asians while Montreal attracts proportionaly more Blacks and Arabs.

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[9] Excluding Aboriginals, who are not considered a visible minority group.

[10] Most West Asians born outside Canada are natives of Iran and Afghanistan.

Table 2 shows how the visible minority groups vary in their composition in the three largest metropolitan areas. In Toronto, South Asians would outnumber Chinese in 2031 by more than 50%, while in Vancouver, Chinese are by far the most populous visible minority group, more than doubling the size of the South Asian group, the second in importance. Montreal presents a more diversified portrait with both Chinese and South Asian groups smaller in population than the Black and Arab groups.

Table 2 Population[1] (in thousands) by region of residence and visible minority group, Canada, 2001 and 2031

	Montréal		Toronto		Vancouver		Rest of Canada	
	2001	2031	2001	2031	2001	2031	2001	2031
Total - visible minorities	461.3	1,084.4	1,788.7	4,787.7	754.6	1,883.2	1,111.3	2,865.8
Chinese	53.5	142.9	428.2	1,071.0	359.2	875.8	229.8	585.6
South Asian	57.1	133.5	501.8	1,634.5	174.3	393.6	230.8	700.2
Black	139.7	275.3	322.4	649.7	19.5	55.0	199.5	464.9
Filipino	18.2	45.7	140.1	344.1	60.0	176.0	102.8	231.8
Latin American	52.7	108.7	76.7	181.9	18.4	44.5	68.1	137.1
Southeast Asian	40.8	60.8	55.9	113.9	29.7	49.4	79.8	142.0
Arab	68.8	229.1	45.0	181.5	6.1	31.1	79.7	255.7
West Asian	11.7	37.7	56.0	232.9	22.6	88.4	24.1	98.5
Korean	3.5	10.2	42.0	128.9	26.1	84.9	23.4	101.8
Japanese	1.9	4.7	16.5	27.7	22.0	36.0	27.5	39.8
Others[2]	13.3	35.8	104.1	221.6	16.8	48.5	45.9	108.5
Rest of the population	3,018.6	3,213.1	3,045.1	3,268.0	1,295.6	1,356.3	19,256.8	20,327.3
Total	3,479.9	4,297.4	4,833.7	8,055.7	2,050.1	3,239.4	20,368.1	23,193.1

1. Excluding non-permanent residents.
2. Multiple visible minorities or not elsewhere identified.

This trend is likely to continue if new immigrants continue to establish themselves in the largest cities of the country. Results from the Longitudinal Survey of Immigrants to Canada (*The Daily*, September 4, 2003) are instructive concerning the possible reasons for new immigrants' choice of area of residence. According to the survey, 78% of newcomers "settled in areas where their network of friends and relatives lived." Among economic class immigrants who settled in Toronto, Montreal or Vancouver, the main reason for choosing their area of residence was that family members or friends were already living there. This reason ranks ahead of job prospects in all three areas.

6. Conclusion

These are only some of the results that can be generated from PopSim. As the unit of projection is the individual, the complete distribution of any of the projected population characteristics can be generated. Tables cross-tabulating any of the variables included in the model can therefore be easily created. Since the release of the Report in 2005 (Bélanger and Caron-Malenfant, 2005), Statistics Canada has received several demands for more detailed outputs from federal, provincial or municipal agencies and from academics. Results of these projections have proven to be useful for policy planning in several areas: city infrastructure, education and language formation needs, retirement and intergenerational challenges, labour market needs and changes, etc.

As useful as these projections can be, the model can be further developed to go beyond demographic modeling. This projection model is fairly detailed in its demographic components and takes into account ethnocultural differentials in demographic behaviours. However, the benefits of a microsimulation model are not fully encompassed. The policy relevance of the projection results can be much enhanced with the addition of some socioeconomic dimensions to the model.

Ongoing developments will simulate the educational achievements and the labour force participation of the Canadian population. These developments are expected to increase the policy relevance of the projection results and also improve their accuracy. Both education and labour force participation of new immigrants and visible minority groups are the subject of important research initiatives looking at social cohesion and immigrant integration in Canada. In addition, education and labour force are important explanatory variables for fertility (Becker, 1960; Macunovich, 1996) and internal migration decisions, while life expectancy varies by education level. Including these variables into the model should improve the accuracy of the demographic projections.

Accuracy of the projections can also be improved by adding modules dealing with interethnic marriages and intragenerational transfers of religion. The development of a full marriage market module would, for example, have advantages for the analysis of interethnic marriages as it would allow the tracking of second generation immigrants as well as the occurrence of multiple ethnic origins. Also, religious denomination in PopSim is not allowed to change over the life time of an individual. Using data from the Ethnic Diversity Survey which contains information on an individual's religious affiliation at different ages will permit the introduction of changes in religion over the life cycle and thus enhance the policy relevance of the projections.

7. Acknowledgements

The authors would like to thank Chantal Hicks, Geoff Rowe and Steve Gribble for their comments, suggestions and technical support related to this project"

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NEW TIMES, OLD BELIEFS: INVESTIGATING THE FUTURE OF RELIGIONS IN AUSTRIA AND SWITZERLAND[1]

Paper Presented at Joint Eurostat/UNECE Work Session on Demographic Projections, Bucharest 10-12 Oct 2007

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European demographic trends, including those of low childbearing are likely to change if there is a growth of religious groups with higher fertility. In order to investigate to which extent the religious composition matter for the future, we project the population by religion for Austria and Switzerland until 2050-2051. We take into account religion specific differences in fertility, religion-specific net migration, the rate of conversion between religions as well as transmission of religious beliefs from parents to children.

For Austria, we find that the proportion of Roman Catholics is likely to decrease from 75% in 2001 to less than 50% by the middle of the century, unless current trends in fertility, secularization or immigration are to change. The most uncertain projections are for those without religious affiliation: they could number as little as 10% and as many as 33%. The Muslim population—which grew from 1% in 1981 to 4% in 2001—will represent 14% to 26% by 2051. Religious change suggests that the size of the population as a whole could increase from 8 million up to 8.6 million by 2051.

In Switzerland, the Catholics and Protestants who represented together more than 95% of the population until 1970, and 75 % in 2000 are estimated to become less than 63% by 2050, and could even number as little as 42%. Those without religion are likely to be the fastest growing group, reaching between 13% and 33 % by 2050, from a level of 11% in 2000. There is also growth in the Muslim population, who are expected to reach a level of 8% to 11% by 2050, from an original level of 4% in 2000.

1. Motivation

The relative sizes of secular and religious populations belong to the most important social characteristics of a country. In the wake of religious change, family behaviour, including marriage and childbearing, is likely to be altered. The religious distribution of the population can also have political effects, including affecting the level of social cohesion and determining a country's foreign policy.

Most datasets on religion are based on surveys, and very few contain detailed information for the whole population over time, which make the Austrian and Swiss Census based datasets ideal for research. On the other hand, census data (which does not include degree of religiosity) may conceal differences in religious intensity between religious groups. Thus strong religious adherence and intensity could more frequently be a dominant identity trait in certain religious groups, while only a secondary identity characteristic in other religions.

For a brief review of national level religious projections, see Goujon et al. (2007). Fliegenschnee, Goujon and Lutz (2004) project the future size of the Protestant population in Austria. They foresee a substantial decrease, partly because of

[1] We would like to thank Statistics Austria and Swiss statistics, and particularly Stéphane Cotter, for providing the data, and Amela Ajonavic, Caroline Berghammer, Dalkhat Ediev, Marianne Fliegenschnee, Joshua Goldstein, Frank Landler and Wolfgang Lutz for making valuable comments and helping us with the data.

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secularisation and partly because of the conversion of children of mixed couples (where one partner is non-Protestant). The analysis reveals substantial differences especially between the capital Vienna and other regions in the rate of secularisation, where secularisation trends in Vienna are much stronger.

Barrett, Kurian and Johnson (2001) give extrapolations for the size of religions in most countries of the world. Their work for the Austrian projection suggests that the share of Christians would continue to decline, perhaps falling as low as 75% by 2050. This finding clearly contrasts with our results, which may be due to the fact that they are based on the 1991 census, and that they use simpler projection techniques. Our projections indicate that the share of Christians decreases below 75% for all scenarios.

Barrett, Kurian and Johnson (2001) also project the population of Switzerland by religion. They suggest that by 2050, the Christian population is likely to decline below 80%, while those without religion will grow beyond 15%. The projections we present here suggest a much more rapid fall in the proportion Christian due to a strong growth in the population share without religion and non-Christian religions.

Projections were also made in Switzerland in the framework of a larger study on religious groups' demographic characteristics in Switzerland (Bovay and Broquet 2004; Haug and Wanner 2000). However, these projections only go until 2020, and exclude the most rapid growing group – those without religion.

2. Religion, families, fertility and conversion

Regulating partnerships, sexuality and fertility is central in the teachings of most major religions, and religious beliefs can be powerful determinants of marriage, childbearing and divorce. For a discussion on the relation between religion and demographic outcomes, see Goujon et al. (2007). Religious texts and interpretations differ in their emphasis on marital obligations, divorce rights, fertility demands and individual self-determination. All religions encourage individuals to maintain their faith, and most support recruitment through conversion and discourage apostasy (though sanctions differ). In the European case, there is generally a high degree of freedom to convert without sanctions or to choose to live a life detached from religion (see e.g. Dubach and Campiche 1993).

Studies looking at the effects of religion are prone to be upward biased if variables that are associated with, although not caused by, religious beliefs affect demographic behaviour (Sander 1992). Religious influence is often mixed with politics and ideologies to such an extent that the effects of religion can rarely be estimated without considering the social, geographic and economic circumstances. Poverty, low education levels, resource availability and political stability could have strong effects on "demographic outcomes net of religion. This has lead some scholars to suggest that the role of religion may disappear (Cohen 1996). Nevertheless, religion has been found to have an independent effect, net of any economic and social influences, on various demographic and economic outcomes (Borooah 2004; Driver 1963; Lehrer 2005; Morgan et al. 2002). Furthermore, religious beliefs are likely to influence educational attainment, income levels and political stability. Therefore, assuming exogeneity of such variables could lead to a downward bias in the estimated influence of religion.

2.1 Fertility

Understanding differences in fertility patterns between religious groups is important as they have a reinforcing effect over time on the size of the different religions (lower/higher fertility leads to smaller/larger groups which in turn have fewer/more children).

The Bible encourages high fertility. It states (Bible, Gen 1:28), "And God blessed them, and God said unto them, Be fruitful, and multiply, and replenish the earth". The only permitted form of contraception in the Catholic church is the "calendar method" where one does not have sexual intercourse on the days of the month with the highest conception risk. However, although pronatalist measures may have had a historic impact among Austrian Christians, current church attendance is low and decreasing, and religious influences on fertility decisions has weakened over time (Berghammer 2003, also see Goujon et al. 2007).

Sander (1992) analyses American Roman Catholics born during the twentieth century and finds that for those born after 1920, fertility was no longer higher than for the Protestant majority. This may be caused by the fact that despite the clear opposition to contraception and abortion by the Roman Catholic church, as many as 87% of American Catholics believe that individual choice should prevail in family formation choices (Noonan 1986). Also, Roman Catholic European countries are currently well represented among those with "lowest-low fertility" (total fertility rate below 1.3), including Spain and Italy (United Nations 2004).

Islam is supportive of family formation, where marriage and childbearing represent central elements in the religion (Bakar 1995). Muslims' distinct childbearing patterns are evident in a number of different societies and many Muslims attribute their often relatively high fertility levels to their religion (Bovay and Broquet 2004; Borooah 2004; McQuillan 2004; Reynolds and Tanner 1995).

Although fertility levels of immigrants tend to be related to their country of origin (Abbasi-Shavazi 1998), the fertility rates of most immigrant groups tend to approach, if not converge to, the host country fertility patterns, which could decrease Muslim fertility in the coming decades (Nahmias 2004; Ram and George 1990).

2.2 Conversion to other religions and secularisation

Secularisation is the most important "conversion" flow. Most Catholics and Protestants who leave their respective churches become secular, while some convert to other religious groups. Crockett and Voas (2006) find that secularisation, at least in Britain, takes place across cohort lines, i.e., each generation being less religious than the previous one. In sociology the meaning of secularisation and how religion (belief, practice, membership, etc.) has evolved during the last decades is much discussed (e.g., Knoblauch 1999, Berger 1990, Luckmann 1991, Krüggeler 2001, 1989). Our study does not address the theme of religiosity, but understands secularisation and religious belonging only as the self-reported religious affiliation and not as a measurement of religious beliefs.

3. Religions in Austria and Switzerland: Past and present

A definition of the religious categories used in the projections is given in Table 1. We have grouped the smaller religions together in "other religions". Table 2 shows the religious composition for Austria 1900-2001 by age. Roman Catholicism is the largest religion for all age groups in all periods, while the bulk of those without religion are aged 30-60 years. During the last three decades, the fastest growth has been among those without religion, followed by the Muslims and the share with "other religions".

Table 1 Definition of the religious categories used in the projections

Categories	Census categories
Roman Catholic	Roman Catholic church
Protestant	Protestant church*
Muslim	Islamic religious community
"Other religions"	Other Catholic (united) churches Orthodox churches Other Christian communities Jewish religious community Hindus/Buddhists Other non-Christian communities Not indicated Others
Without religion	Individuals who declare no religious affiliation

Source: Statistics Austria ISIS, 2001 Census, and Swiss Statistics (2006a).
* The Swiss Protestant church includes only the census category " Eglise évangélique réformée". All other Protestant communities such as « Eglise évangélique méthodiste » and « Communautés néo-apostoliques » are included under the « other religion »category.

Table 2 Resident population in Austria by religion, 1900-2001

Year	Roman Catholic (%)	Protestant (%)	Muslim (%)	Other religions (%)	Without religion (%)	Unknown (%)	Total (absolute)
1900	91.6	2.7	0.0	5.4	0.2	0.2	6003780
1971	87.4	6.0	0.3	1.5	4.3	0.6	7491526
1981	84.3	5.6	1.0	2.0	6.0	1.0	7555338
1991	78.0	5.0	2.0	2.9	8.6	3.5	7795786
2001	73.6	4.7	4.2	3.5	12.0	2.0	8032926

Sources: Statistics Austria, Census 1900, and Statistics Austria ISIS

The religious composition of Switzerland 1900-2000 is shown in Table 3. In Switzerland, like in Austria, the same groups that were dominant in 1900 were also dominant in 1970 (for both years more than 95% of the population were either Catholic or Protestant). However, the Protestant share declined relative to the Catholic in this time period. Between 1970 and 2000, the share of the Protestants continued its decline, but also the Catholic church lost a substantial proportion of its adherents. By 2000, these two main religious groups only constituted 75 percent of the population. This was mainly due to secularisation causing a rapid growth in the proportion without religion, but also due to immigration of other religious categories, including Muslims, with higher fertility than the Christian and secularized population. The secularisation rates were high in both countries, but even higher in Switzerland, particularly among relatively young Protestants.

Table 3 Resident population in Switzerland by religion, 1900-2000

Year	Roman Catholic (%)	Protestant (%)	Muslim (%)	Other religions (%)	Without religion (%)	Total (absolute)
1900	41.0	57.8	0.0	0.6	0.0	3315400
1970	49.4	46.4	0.3	2.8	1.1	6269783
1980	47.6	43.9	0.9	3.8	3.8	6365960
1990	46.2	38.5	2.2	5.7	7.4	6873687
2000	41.8	33.1	4.3	9.8	11.1	7288010

Sources: Swiss Statistics (2006a).

3.1 Age structure

Austria and Switzerland share some developments with respect to age. The traditionally dominant religions, Catholicism and Protestantism, have, a relatively old age structure compared to other religious groups. On the other hand, the "without religion" group and also the Muslim population have relatively young age structures.

3.2 Marriage

Whether intra-religious marriages are common, and to what extent religion is transmitted from parents to children, is important in terms of determining the number of adherents in the longer term. For the Roman Catholic church in Austria, the impact of mixed marriages is not that important as 76% of Roman Catholic women who marry have a husband of the same faith. The picture is however very different for the Protestant population where 84% of Austrian Protestants marry non-Protestant partners, mostly Roman Catholics (56%) or persons without religion (23%). Lutz and Uljas-Lutz (1998) estimated that only half of the children of these couples become Protestant. Lutz (1985) shows that the religion of the mother is more important than the religion of the father for the transmission of religion from parents to children. For the smaller religious groups, there are generally fewer who marry within the same religion. Muslim women constitute an exception, where 85% of them have a Muslim husband, which could be because inter-religious marriages are strongly discouraged in Islam (Fitzgerald, Khoury and Wanzura 1976).

In Switzerland, around half of the Protestants marry a partner within their religious group, while for the Roman Catholics 60 % do so. In other religious groups it can be that it is very low like for the Buddhists or small Christian groups or very high like for the Muslims and some smaller Protestant groups who are mainly married to people with the same religious affiliation (Bovay and Broquet 2004). When parents share the same faith, their children have a relatively high probability of following their parents' religion. However, for children born to couples where one parent has a religious affiliation and the other one belongs to the group "without religion", only 72% are likely to be affiliated to the religion of the religious parent. The parent with a religious affiliation has more influence on the child's affiliation than the secular person, and more so if the woman is religious rather than the father (Bovay and Broquet 2004).

4. Projections

The projections of the population by religion status (from 2001 to 2051 for Austria and 2000 to 2050 for Switzerland) were created using the PDE Population Software[3], a simplified multi-state population projection program for states interacting with one another. States are defined by the user and can be regions, educational categories, ethnic or language groups, or other user-defined dimensions. The software requires data on fertility, mortality, migration and transition probabilities between states, by age and sex. This software has been utilised in recent years in many different settings (for a recent listing see Lutz et al. 2007*)*.

The inputs required for the projections are the following:

- Base year parameters: population by age, sex and religion status, age- and religion-specific fertility rates, age-, sex- and religion-specific mortality rates, age-, sex- and religion-specific net migration numbers, and transition rates between religion groups.
- Scenario assumptions as to the future of the parameters listed above.

The base-year populations for the two countries were taken from the 2000-2001 censuses. As mentioned above, we aggregated the population into five main religious categories: Roman Catholics, Protestants, Muslims, other religions and without religion.

4.1 Base-year fertility

Fertility differences by religion in Switzerland and Austria are given in Table 4. In both countries, Muslims have the highest fertility around 2.4 children per woman, while the seculars have the fewest with only 0.9 children in Austria and 1.1 in Switzerland. The TFR of Catholics and Protestants varies from 1.2 to 1.4. The "Other religions" group, with 1.4 to 1.7 children, conceals heterogeneity between different smaller groups, where the highest fertility of any group are the Hindus in Switzerland, which have 2.7 children. The Hindu population in Switzerland was, however, too small to be included as a separate entity.

Table 4 Total fertility rate by religion

	Austria 2001	**Switzerland 2000**
Roman Catholic	1.32	1.41
Protestant	1.21	1.35
Muslim	2.34	2.44
Other religions	1.44	1.74
Without religion	0.86	1.11
Total	1.33	1.50

Sources: Authors' calculations, Statistics Austria ISIS, and Bovay and Broquet (2004).

[3] The PDE Population Projection software has been developed by the World Population Program at IIASA and is available free of charge at http://www.iiasa.ac.at/Research/POP/pub/software.html

Lutz (1985) showed that in the early 1980s in Austria, mothers were predominantly responsible for passing on their religious beliefs to their children in mixed couples with Roman Catholic and Protestant members. A similar pattern was found in Switzerland (Bovay and Broquet 2004). Therefore, we assume that children have the same religion as their mothers, regardless of the type of union, mono-religious or mixed.

In the period 1981-2001, the TFR in Austria declined from 1.7 to 1.3. Fertility declined for all religious affiliations. For Switzerland between 1980 and 2000 the fertility rate dropped from 1.6 to 1.4 children per woman.

Despite the differences among religion groups the fertility patterns of all groups tend to follow similar trends, albeit at different periods of time and with different scales.

4.2 Base-year mortality

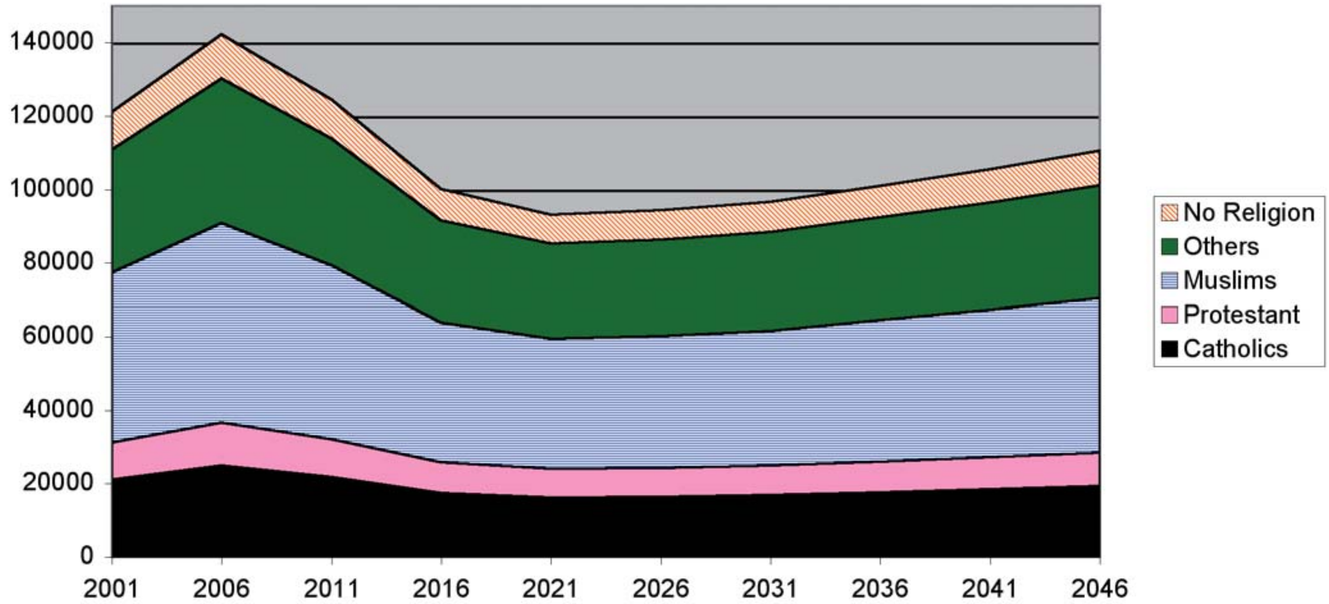
Mortality rates are extracted from life tables available at Statistics Austria and Swiss Statistics and are kept equal across all religious affiliations (Statistics Austria ISIS, Swiss statistics 2006a). Studies that control for confounding factors find that differences in mortality by religion are insignificant or small (Hummer et al. 2004; Strawbridge et al. 2000). It is also highly uncertain if any such longevity differences would remain over time. We therefore assume equal life expectancy between members of different religions.

4.3 Base-year migration

Migration is a key factor in the changing religious landscape for both Austria and Switzerland. The number of migrants has been high in recent years. In Austria as well as in Switzerland migrants came mostly from former Yugoslavia, Eastern Europe, Germany and Turkey. In Switzerland also a rather huge group from Sri Lanka migrated. Unfortunately we lack data on the faith of immigrants and/or emigrants.

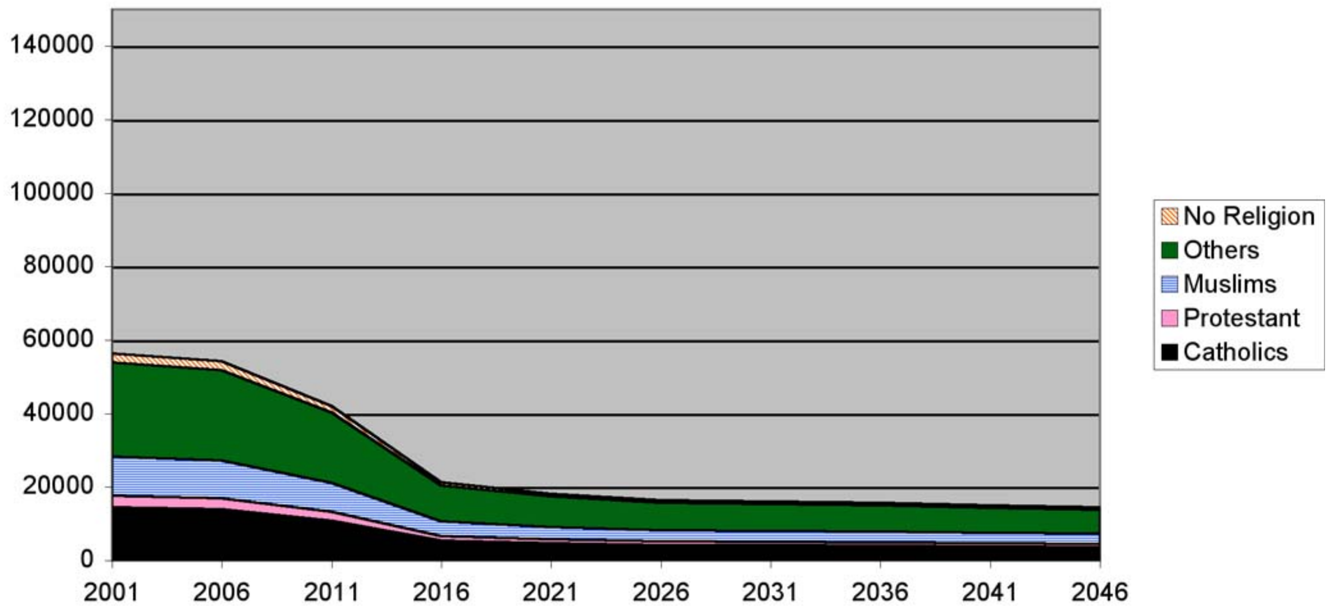
Religious affiliation was inferred from the country of origin. In a first step, we retrieved the number of in-migrants and out-migrants for the most important countries with the highest absolute net-migration for the period 1999-2004 for Austria and from 1998 to 2002 for Switzerland (different periods are due to data available. In a second step, we used the Central Intelligence Agency (CIA)-World Factbook (2005, 2007) that gives statistics on all the countries in the world to retrieve the shares of the population by religious affiliations. These shares were then applied to the flows for the periods to distribute the emigrants and immigrants according to the religious beliefs of their country of origin, see Figures 1a and 1b. The assumption that migrants have the same distribution as the rest of the population in their country of origin is, of course, quite daring. In certain cases, ethnic or religious conflicts could affect outbound migration of specifically persecuted groups. However, in the absence of better information, this was felt to be the best approximation method. The stocks of migrants obtained for the two flows were then disaggregated by age and sex according to the overall share by age and sex of inflows and outflows.

Figure 1a Religions affiliation of net migrants in Austria



Sources: Statistics Austria Yearbooks 2000 to 2006, CIA (2005) and authors' calculations.

Figure 1b Religions affiliation of net migrants in Switzerland



Sources: Swiss statistics (2006b), CIA (2007) and authors' calculations.

4.4 Base-year transitions

The transitions measure the flows occurring between religions, meaning moving from one religious affiliation to another one. Out of the twenty flows possible between the five religious categories, two seem predominant and will shape the future composition of the religious landscape of the countries.

Those flows reflect the progressing secularisation of the country, especially through the exodus of members of the Roman Catholic church and Protestant church. Abandonment of religion is much less common in Islam and to a lesser extent in "Other religions". Changes in the Muslim and 'other' religious categories will occur mainly through fertility and migration.

In Austria, the absolute number of members leaving the two main churches is accounted for by the Protestant and Catholic churches themselves. These were estimated to be around 34,000 on average per year for the period 2001-2006. This estimate may seem rather conservative in view of the higher number of Catholics leaving the Roman Catholic church in some particular years, i.e. 1995, 1999 and up to 52,000 in 2004. Those years were considered to be outliers caused by scandals in the Catholic church. The registration system does not allow for the same estimations in Switzerland, therefore we estimated the transitions based on the comparison of the 1990 and 2000 Catholic and Protestant populations, accounting for net-migration flows for the two religious categories. Results show slightly higher transition numbers for Switzerland compared to Austria with 31,000 persons leaving the Roman Catholic church and 24,000 leaving the Protestant church in the initial five year period.

4.5 Scenarios

The scenarios should help in answering the main questions we have about the future of religions in Austria in comparison to Switzerland. We developed a matrix of twelve scenarios emerging from the combination of several hypotheses on the different demographic determinants and the determinants of religious compositional change that is fertility, migration and transition/secularisation. The assumptions are developed below and summarized in Table 5.

Fertility[4]: "Stable fertility" scenario (Fs): Fertility by religion remains constant at the levels observed in 2001-2006 in Austria, and 2000-2005 in Switzerland.

"Converging fertility" scenario (Fc): Fertility by religion converges to a TFR of 1.4 children by 2026-2031, and remains constant afterwards. For Switzerland the TFR was assumed to convert to the level of 1.6 by 2025-2030. These TFRs are in accordance with the medium variants of both countries projections.

Mortality: One single trend, following the medium variant of population projections for mortality of the statistical offices.

Migration: "Medium migration" scenario (Mm): The net number of migrants to Austria and Switzerland follows the medium variant for population projections of the statistical offices. Under this scenario, in Austria net migration will increase strongly until 2011 and decline slowly thereafter. The net number of migrants is assumed to lay between 19,000 and 28,000. For Switzerland, net-migration will remain at 10-12,000 until 2010 and will go down to 4,000 in 2020, and slowly decline thereafter to reach 3,000 in 2050.

"High migration" scenario (Mh): Same as in Mm, except that the net number of migrants follows the high variant for migration stated in the population projections of the Austrian and Swiss statistical offices. The net number of migrants per year fluctuates between 27,000 and 38,000 during the 2001-2051 period for Austria. For Switzerland, after reaching 24,000 in 2011, the net number of migrants will rapid decline to 10,000 in 2020 and further to 2,000 in 2050.

Transition/Secularisation: "Constant secularisation" scenario (Tc): This scenario implies constant transition rates at the levels observed in 2001 for transitions of Catholics and Protestants to 'without religion' for Austria. In Switzerland, we chose to use the transition rates observed between the two censuses (1990 and 2000) as the high variant (see below) and to half those for the constant secularization scenario.

[4] Children are born in the same category as their mother. However, we do not consider what could happen within a more balanced religious composition of the country where the rate of mixed marriages may increase and affect the choice of one or no religion for the children.

"Low secularization" scenario (Tl): The transition rates converge to zero by 2026-2031 in Austria, and by 2025-2030 in Switzerland. After 2030-2031, everybody stays in the religious category they were born into.

"High secularisation" scenario (Th): The transition rates double the rates used in the "constant secularization scenario" between 2001-2006 and 2026-2031 for Austria, and between 2000-2005 and 2025-2030 for Switzerland, and remain constant afterwards.

Table 5 Summary of projection assumptions

<table>
<tr><th colspan="6">Fertility</th></tr>
<tr><td colspan="3">Stable
Fertility by religion remains constant at the levels observed in 2001-2006 (Austria) and 200-2005 (Switzerland).</td><td colspan="3">Converging
Austria: Fertility by religion converges to a TFR of 1.4 children by 2026-2031.
Switzerland: Fertility by religion converges to a TFR of 1.6 children by 2025-2030.</td></tr>
<tr><th colspan="6">Migration
(Based on official statistics)</th></tr>
<tr><td colspan="3">Medium
Austria: Annual net migration fluctuates between 19,000 and 28,000.
Switzerland: Annual net migration fluctuates between 3,000 and 13,000 per year.</td><td colspan="3">High
Austria: Annual net migration fluctuates between 27,000 and 38,000.
Switzerland: Annual net migration fluctuates between 2,000 and 24,000.</td></tr>
<tr><th colspan="6">Transition/Secularisation</th></tr>
<tr><td colspan="2">Constant
Austria: Constant transition rates at the levels observed in 2001.
Switzerland: Half transition rates observed between 1990 and 2000.</td><td colspan="2">Low
The transition rates converge to zero by 2026-2031 (Austria) and by 2025-2030 (Swizerland).</td><td colspan="2">High
The transition rates double by 2026-2031 (Austria) and by 2025-2030 (Swizerland).</td></tr>
</table>

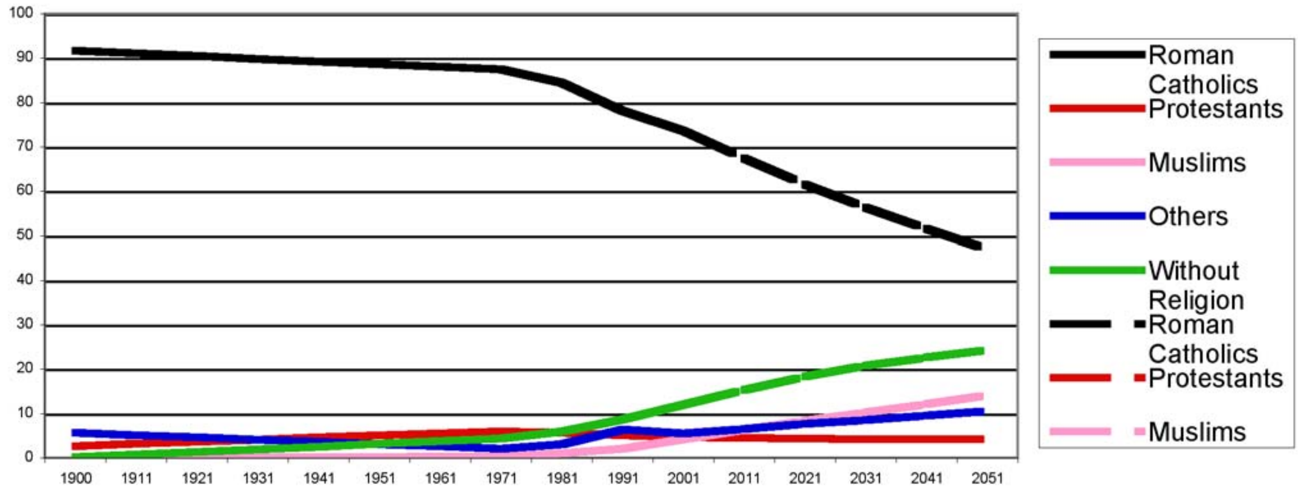
5. Projection Findings

The Austrian and Swiss populations will start shrinking in all twelve scenarios. The time at which this will happen varies within the period from 2015-2020 in Switzerland to 2031-2051 in Austria. The scenarios leading to higher population growth are those combining high migration with the different fertility and transition scenarios.

In terms of total fertility rate, the range would vary between 1.4 and 1.5 children in 2046-2051, in Austria, and between 1.5 and 1.6 in Switzerland in 2045-2050. In case of constant fertility differentials within religious categories, the total fertility rate would still increase because of the changing weights of the different religious categories with increasing weight of the more fertile groups (Muslims and other religions). The convergence of fertility scenario obviously leads to a smaller increase in fertility, to the target level of 1.4-1.6 children by 2030-2031. Secularization and the fertility behaviour of the secularized group will play an important role in determining future fertility levels.

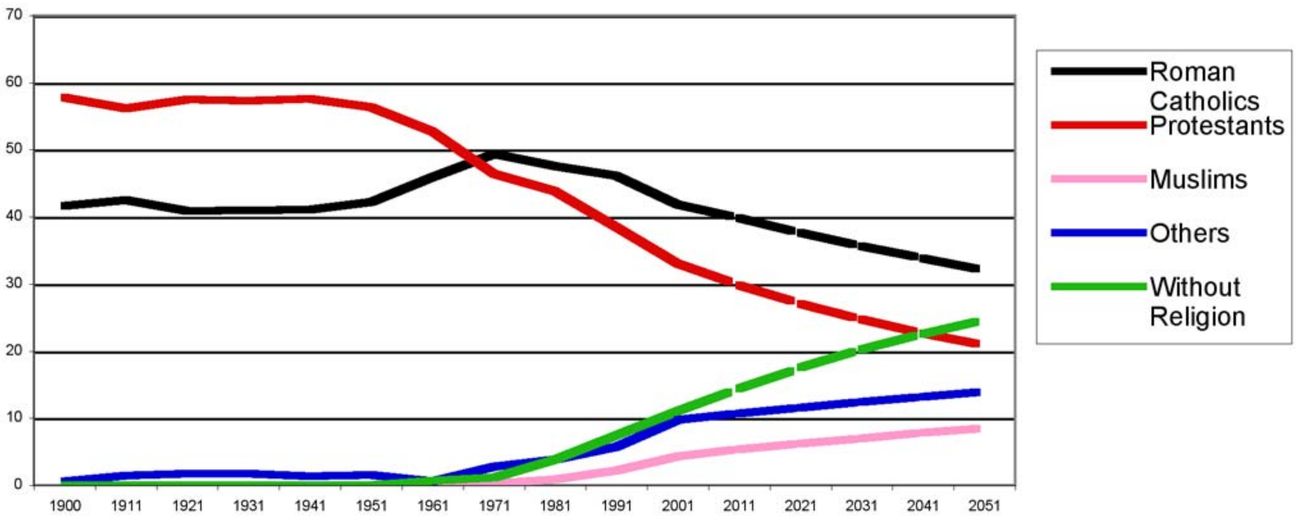
The different scenarios will affect the religious composition of the population. Figures 2a and 2b show the evolution of the share of all religion categories according to our "medium" variant (Converging fertility/Medium migration/ Constant secularization). Table 6 show more results for all scenarios. All scenarios show a severe decline of the until-now dominant religion groups, mainly due to secularization. As our migration scenarios entail a small proportion of Catholics and Protestants in the net number of migrants, migration is not influencing these shares very much. However, it is important to note that in both countries, the largest religious groups would still be the present dominant one: Roman Catholics in Austria, and Roman Catholics and Protestants in Switzerland. Most scenarios in Austria show a declining share of the Roman Catholics to levels below 50% by 2051. If the transition is kept constant at present levels, the proportion of Roman Catholics would drop to 45-47%, and it could fall below 40% if secularisation rates were to double between 2001 and 2051 (the lowest value is 37% according to scenario stable fertility/High migration/High secularisation). In Switzerland, the share of the Roman Catholics would decrease from 42% in 2000 to 26-38% in 2050, and that of the Protestants from 33% in 2000 to 16-25% in 2050.

Figure 2a Religions composition of the Austrian population in Percent 1900-2051, scenario FcMmTc



Sources: Statistics Austria ISIS, and authors' calculations.

Figure 2b Religions composition of the Swiss population in Percent 1900-2050, , scenario FcMmTc



Sources: Swiss statistics (2006a) and authors' calculations.

The projections show little change in the proportion of the population belonging to the Protestant church in Austria, which would fluctuate between 3 and 5% during the 2001-2051 period. However and contrary to what we have seen for the Protestant church in Switzerland, the Austrian Protestant church may witness a rise of its membership—from levels observed in 2001— if secularization rates were to decline to nil mostly due to the benefit of a higher share of Protestants in the immigrant population (above 8%) than in the emigrant one (7%) that can be attributed to immigration from Germany and Eastern European countries.

The future of the group 'without religion' will be logically highly dependent on secularisation rates among the Catholics and Protestants. The secularised population will be the second largest "religious group" by the middle of the century in most scenarios in both countries, and after the Roman Catholics. If the secularisation rate increases further (Th) the share of the population without any religion could be as high as 30-34% around 2050. The constant secularisation scenario gives an intermediate picture, in which the group 'without religion' will still grow considerably to 22-24% of the population by the middle of the century. Only if the rate of secularisation were to come to a complete stop by 2030-2031 (Tl) this category would stagnate around 11-15% during the 50-year projection period[5]. We should remind here that the trend observed in Switzerland between 1990-2000 was combed down for the secularization assumption.

Austria and Switzerland differ most in the increase in the Muslim population, which was our smallest religious category in both countries in 2000-2001, accounting only for 4% of the population. In Austria, the share could be as high as 18% in 2051—this, however, only in the case of stable fertility differentials and no convergence, as Muslim women have higher fertility compared to all other groups. The share of the Muslim population would increase to 14-15% in the case of a convergence of fertility rates to way-below replacement fertility as the rest of the Austrian. Although the share of the migrating population originating from Muslim countries in Switzerland is close to that of Muslims in Austria, approximately 20% in both cases, the main difference comes from the size of migration flows that is foreseen until the middle of the century. The Swiss projections assume an important decline in the net number of migrants and hence the proportion Muslims would only increase at most to 11% of the total population by 2050.

Migration will also increase the proportion following an "other religion" in both countries, but more intensively in Switzerland, where up to 17% of the population could belong to this group by 2050, compared to a share of 10% in 2000. This would be mostly due to the migration of Orthodox Christians from former Yugoslavia and Buddhists from Sri Lanka. In Austria, whereas in 2001, about 5% of the population were observing "other religions", up to 11% could do so in 2051.

Table 6 Share of the population by religious affiliation

		2000-2001					2050-2051				
Country	Scenario	Roman Catholics	Protestants	Muslims	Other religions	Without religion	Roman Catholics	Protestants	Muslims	Other religions	Without religion
Austria	FsMmTc	73.6	4.7	4.2	5.5	12.0	45.8	4.0	17.3	10.7	22.2
Austria	FsMmTh						37.4	3.4	17.4	10.7	31.1
Austria	FsMmTl						56.0	4.8	17.2	10.6	11.5
Austria	FsMhTc						45.0	4.2	17.9	11.2	21.8
Austria	FsMhTh						36.8	3.5	18.0	11.2	30.4
Austria	FsMhTl						54.9	5.0	17.8	11.1	11.3
Austria	FcMmTc						46.8	4.2	14.1	10.6	24.2
Austria	FcMmTh						38.1	3.6	14.1	10.6	33.6
Austria	FcMmTl						57.6	5.1	14.1	10.6	12.7
Austria	FcMhTc						46.1	4.4	14.7	11.1	23.8
Austria	FcMhTh						37.6	3.7	14.7	11.1	32.9
Austria	FcMhTl						56.5	5.2	14.6	11.1	12.5
Switzerland	FsMmTc	41.8	33.0	4.3	9.8	11.1	31.3	20.1	10.8	14.9	22.9
Switzerland	FsMmTh						26.6	16.9	10.8	15.0	30.8
Switzerland	FsMmTl						36.7	24.0	10.7	14.8	13.7
Switzerland	FsMhTc						30.7	19.3	11.4	16.6	22.1
Switzerland	FsMhTh						26.0	16.1	11.5	16.6	29.7
Switzerland	FsMhTl						36.0	23.0	11.4	16.5	13.2
Switzerland	FcMmTc						32.0	20.9	8.5	14.0	24.6
Switzerland	FcMmTh						27.1	17.4	8.5	14.0	32.9
Switzerland	FcMmTl						37.6	25.0	8.5	14.0	14.9
Switzerland	FcMhTc						31.5	20.0	9.1	15.6	23.8
Switzerland	FcMhTh						26.7	16.7	9.1	15.6	31.9
Switzerland	FcMhTl						37.0	24.0	9.1	15.6	14.4

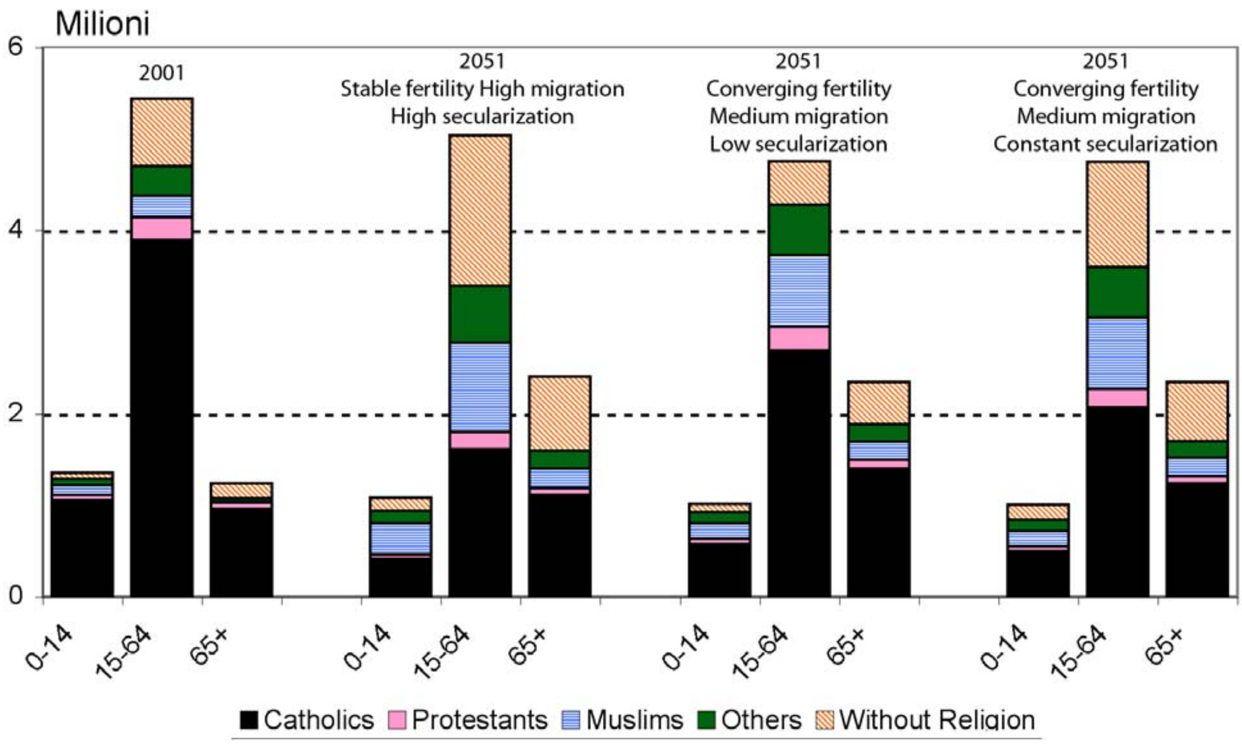
Source: Statistics Austria ISIS, 2001 Census, Swiss Statistics (2006a), and authors' calculations.

[5] We should remind here that the trend observed in Switzerland between 1990-2000 was combed down for the secularization assumption.

The religious landscape of the country will change greatly in the next 50 years and this will also have some repercussions at the age group level as can be seen from Figures 3a and 3b showing the division between different age groups representing the young, the working and the old generations. In Austria in 2001, the Catholics were dominant across all age groups, where the proportions of followers were 72% or higher. In 2051, if we consider the scenario with converging fertility/ medium migration/constant transition, whereas 53% of the 65+ age group will be Roman Catholics, only 43% of the working age population will have this belief. The 0-14 age group may consist of 17% Muslims whereas only 8% will have this affiliation among those aged 65+.

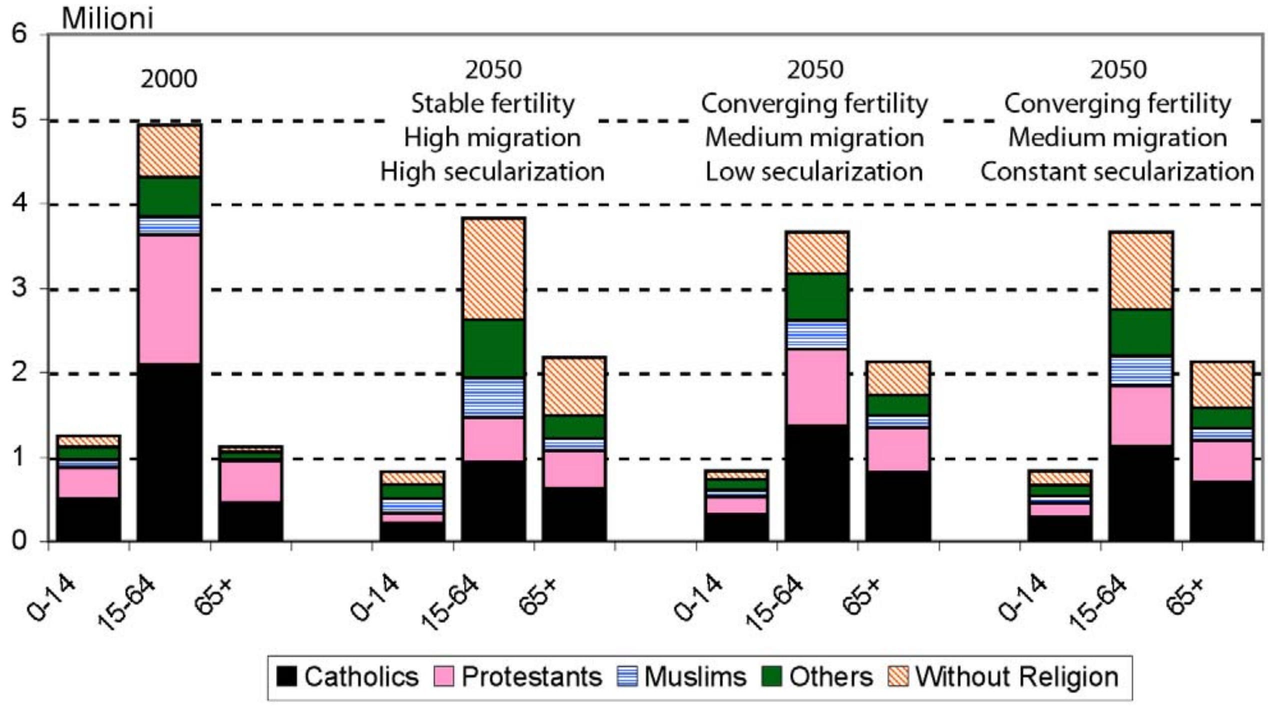
The differences could even be more extreme if migration and secularisation increase. Scenario stable fertility/high migration/high transition shows that in Switzerland only 38% of the working age population are Roman Catholics or Protestants compared to 50% of the old generation. In the same manner, the proportion of Muslims religion is more than three times higher in the young age group than in the old one (21% vs. 7%).

Figure 3a Religious affiliation in Austria by age groups for 2001 and three different scenarios for 2051



Sources: Statistics Austria ISIS, Census 2001, and authors' calculations.

Figure 3b Religious affiliation in Switzerland by age groups for 2000 and three different scenarios for 2050



Sources: Swiss statistics (2006a) and authors' calculations.

6. Conclusions

Our religious projections for Austria and Switzerland reveal both similarities and differences in terms of how the nations' religions change. The two countries are very similar in several respects. Both are central European, have relatively similar population sizes, age structure and overall fertility levels. It can be said that the two main Christian groups have a similar demographic structure in both countries. They are getting old because they have similar low fertility and people mainly leave the church when they are younger. In the small religious groups very often not only the members are increasing but also the fertility is higher.

Our projections show that neither Austria nor Switzerland will not have a secularised or a Muslim majority by 2050-2051. The secularisation assumptions that are constructed upon prevailing trends explain the massive increase in the proportion without religion in Austria, and in Switzerland. By 2050, we expect the secular population to represent between 11% and 34% of the Austrian population, and between 13% and 33% of the Swiss population.

By 2050/2051 our projections show that the Christian share will decrease to less than 63% both in Switzerland and Austria. In all scenarios they will still be the largest religious community in both Austria and Switzerland for the first half of the twenty-first century. However, in Austria we find that the share of Roman Catholics is likely to fall below 50% by 2051. Some scenarios even show a decline below the 40% line. The same is true in Switzerland where the share of the Roman Catholics could be as low as 26% in 2050, and that of the Protestants to 16%.

The Muslim population has already experienced a sharp increase; from 1% in 1980-1981 to 4% in 2000-2001, and by 2050-2051 will represent 14 to 18% of the population in Austria, and 8 to 11% in Switzerland. The lower share of Muslims in the religious landscape of Switzerland is mostly due to lower assumptions in the projected number of immigrants to Switzerland as compared to Austria. Other religious categories will increase their weight, and particularly in Switzerland where it could be as high as 17% in 2051 (11% in Austria in 2050) The rapid changes will also provoke imbalances with regard to the religious composition of the different age categories.

Fertility differences make a substantial difference for the size of the different religions, particularly in the longer term. However the fertility patterns are rather similar for both countries. Childbearing patterns also make a substantial difference with respect to the age structure. The secular population will grow considerably less due to its very low fertility levels, and their low fertility implies that their age structure will be considerably older in the future, and that their population share will shrink unless there is a continued inflow into this group due to continued secularisation. On the other end of the scale, the Muslim population will be the youngest, partly due to their relatively high fertility as well as smaller religious groups. The medium migration scenario from Swiss Statistics assumes that the immigration flow will be less than a fifth compared to the Austrian net immigration assumptions (from Statistics Austria). This has substantial effects on the composition of the population, as migration implies fast growth particularly among current minority religions in both countries. Migration is the most important reason for rapid growth among the Muslim community in both Austria and Switzerland, but also implies rapid growth among other groups, such as Orthodox Christians and Buddhists, particularly in Switzerland. In Austria, migration will mainly imply a rapid increase in the Muslim population if the religious composition remains the same. In Switzerland, migration will particularly influence the growth of the other religious categories.

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European Commission

Work session on demographic projections – Bucharest, 10-12 October 2007

Luxembourg: Office for Official Publications of the European Communities

2007 — 370 pp. — 21 x 29.7 cm

ISBN 978-92-79-04759-6