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A New Mixed Multiplicative-Additive Model for Seasonal Adjustment

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A New Mixed Multiplicative-Additive Model for Seasonal Adjustment

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Abstract:

Usually, seasonal adjustment is based on time series models which decompose an unadjusted series into the sum or the product of four unobservable components (trend-cycle, seasonal, working-day and irregular components). In the case of clearly weather-dependent output in the west German construction industry, traditional considerations lead to an additive model. However, this results in an over-adjustment of calendar effects. An alternative is a multiplicative-additive mixed model, the estimation of which is illustrated using X-12-ARIMA. Finally, the relevance of the new model is shown by analysing selected time series for different countries.

Keywords: Seasonal adjustment, calendar adjustment, over-adjustment, multiplicative-additive model, X-12-ARIMA

JEL-Classification: C 22

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1 Definition of the problem and the method of analysis

Seasonal adjustment is usually based on time series models which decompose an unadjusted series into the sum or the product of four unobservable components (trend-cycle, seasonal, calendar and irregular) – see section 2. This paper will demonstrate that these models are not suited to the seasonal and calendar adjustment of series with sharply pronounced seasonal fluctuations and trend-cycle movement, such as the production in the west German construction industry, which is extremely weather-dependent. After a brief presentation of the series, section 3.2 outlines the traditional criteria for determining the decomposition model, which result in an additive model. However, this over-adjusts for calendar effects (section 3.3). The following model offers an alternative

$$Y_t = D_t \cdot (C_t + S_t + I_t) \qquad (1.1)$$

where Y_t denotes the unadjusted data, D_t the calendar component, C_t the trend-cycle component, S_t the seasonal component and I_t the irregular component (section 3.5).

Further examples using domestic and international series confirm the usefulness of this model variant (section 4). In conclusion, it will be shown that it is possible to estimate the new time series model using X-12-ARIMA (section 5).

2 Traditional time series models

In a time series analysis, it is assumed that an unadjusted series (Y) may be decomposed into four unobservable components. The first of these is the trend-cycle component (C), which includes not just the long-term trend but also cyclical fluctuations. Then comes the calendar component (D), derived from the effects of working-day variations, for example. There is additionally the seasonal component (S),

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* The opinions expressed are those of the author and do not necessarily reflect the views of the Deutsche Bundesbank. I wish to thank Craig Humphreys, Robert Kirchner and David Findley for their valuable suggestions and help. Any remaining errors and shortcomings are, of course, my own.

which includes annual fluctuations that recur to almost the same degree in the same season. Finally, there is the irregular component (I), which includes all effects that cannot be explained using the trend-cycle, calendar or seasonal components. In theory, there is an infinite number of possible relationships between these components and the unadjusted data. In practice, however, a distinction is generally drawn between an additive and multiplicative approach.[1]

An additive model is based on the assumption that the sum of the components is equal to the unadjusted data. In particular, this means that the fluctuations overlapping the trend-cycle are not dependent on the series level

$$Y_t = C_t + D_t + S_t + I_t \, . \qquad (2.1)$$

However, a characteristic shared by the vast majority of time series seasonally adjusted by the Deutsche Bundesbank[2], Eurostat[3] and the US Census Bureau[4] is that there is a multiplicative relationship between the components. Hence, the absolute seasonal and calendar fluctuations depend on the series level

$$Y_t = C_t \cdot D_t \cdot S_t \cdot I_t \, . \qquad (2.2)$$

3 Seasonal and calendar adjustment of the production index for the west German construction sector

This section will develop calendar and seasonal adjustment of the production index for the west German construction sector for the period January 1980 to November 2003.[5] Weather-dependent construction output is an example of a time series with sharply irregular seasonal fluctuations. In addition, the series exhibits marked trend-cycle movements. The next section will include the results of transferring experiences gained here to pan-German construction output and other economic series, which are also characterised by sharp, irregular seasonal fluctuations.

1 For information on the time series decomposition of the various seasonal adjustment programmes, see (re X-11) Shiskin J., A.H. Young and J.C. Musgrave (1967), p 1, (re X-12-ARIMA) Census Bureau (2001), pp 153-155, (re TRAMO and SEATS) Gómez V. and A. Maravall (1996), p 56.

2 See Deutsche Bundesbank (1987), p 32.

3 See Fischer B. and C. Planas (2000), pp 177–178.

4 See Findley D.F., B.C. Monsell, W.R. Bell, M.C. Otto and B.-C. Chen (1998), p 129.

Figure 1 shows the production index for the west German construction sector between January 1980 and November 2003 (1995 = 100). It depicts a downward trend at the beginning of the 1980s, is then relatively flat, rises owing to German reunification in the early 1990s and has been falling since the mid-1990s. A characteristic feature is the repeatedly low output level in the coldest months of the year (December, January and February). The extent to which construction activity is hampered depends on the precise weather conditions – particularly the duration and intensity of frosty spells.[6] In the warmer months, the mid-year fluctuations can be accounted for by the timing of holidays, for example.

3.1 Fundamental considerations when determining a time series model

The next step is to determine the model on which to base the seasonal adjustment. A graphical representation is selected to highlight the degree of independence of the absolute calendar, seasonal and irregular components from the trend-cycle component. The AICC test available in the X-12-ARIMA programme is also applied in choosing the model.

Figure 2 is a scatter diagram between the trend-cycle level and the other components, with fluctuations between -50 and +30 index points. The diagram conveys the impression that the combined deviations of the absolute calendar, seasonal and irregular components are independent from the series level which, in line with the above criteria, suggests additive decomposition. Economically, the result can be interpreted as follows: that, on average, construction companies suffer equally high weather-related output losses in the colder months independent of the level of production and, vice versa, the warmer months contribute, on average, to an equally sharp improvement in production.

In addition to the graph, the X-12-ARIMA programme also includes the AICC test for selecting a model. The corresponding test variable is based on a comparison of

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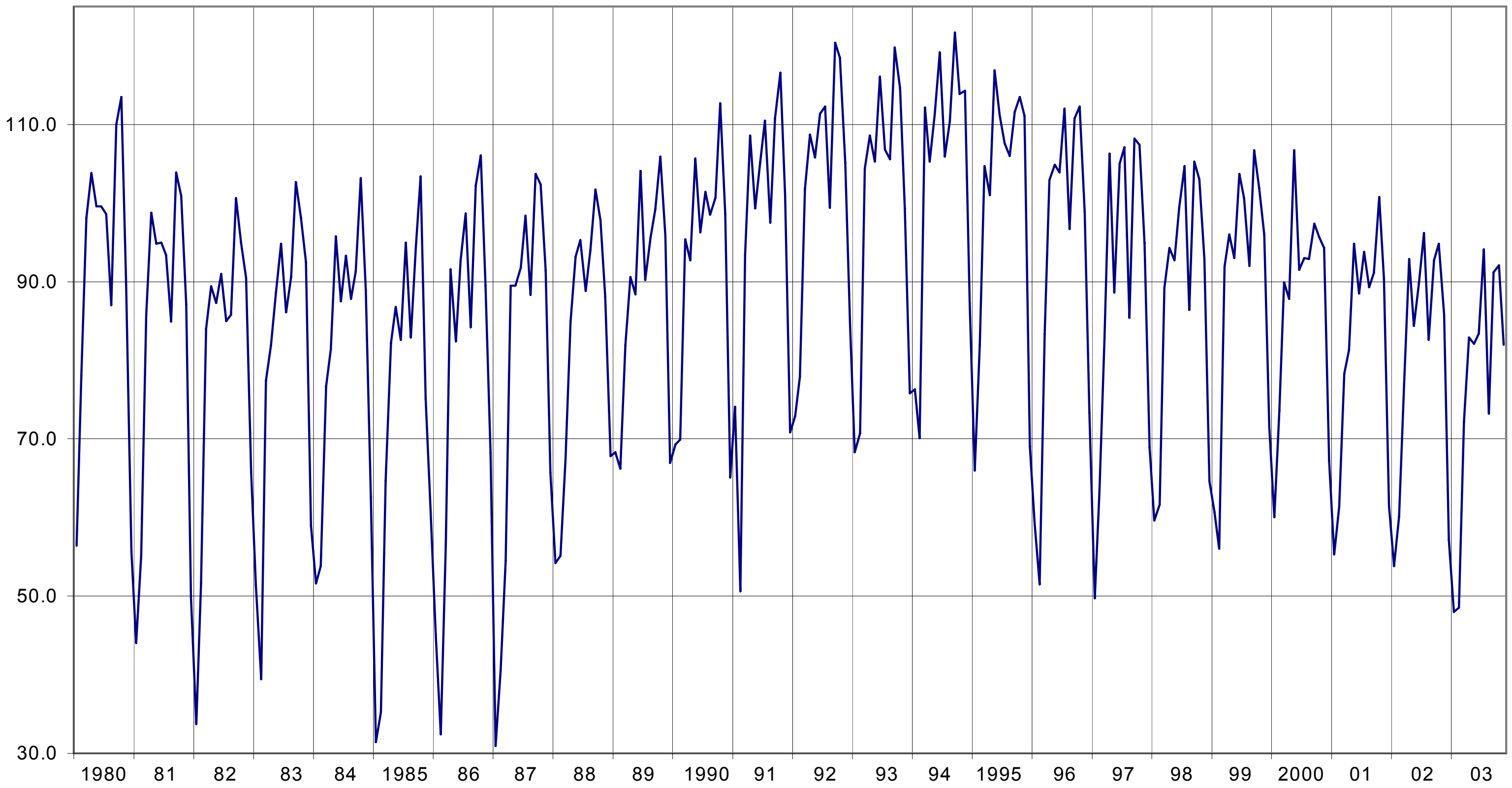
5 See Kirchner R. (1999), pp 48-58.

6 See Kirchner (1999), p 52.

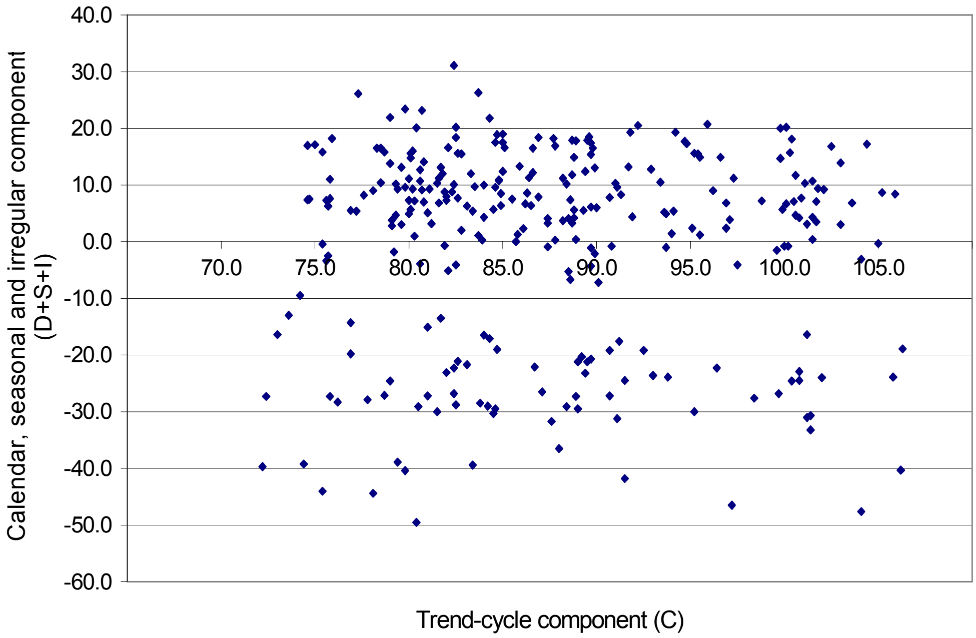
Figure 1: Production index for the west German construction sector
1995 = 100, unadjusted figures



the estimated values for the maximum likelihood function of the REGARIMA model described in section 3.2 for non-transformed unadjusted values and the model for logarithmically transformed unadjusted values. The transformation resulting in the highest possible maximum likelihood value is preferred. As these values are entered in the AICC test variable with a negative sign, the model specification with the smallest AICC value is considered the best.[7]

Figure 2: Scatter diagram mapping the trend-cycle component against the calendar, seasonal and irregular components in an additive decomposition of the production index for the west German construction sector

in index points



Selecting the transformation of the unadjusted values when defining the REGARIMA model is directly linked with a setting for the type of component decomposition. A REGARIMA model in which the unadjusted values are not transformed implies additive decomposition, whereas logarithmic transformation suggests a multiplicative approach. For west German construction output, the AICC test decides in favour of non-transformation (with a difference of 166 points between the test values) and thus points to an additive model for seasonal adjustment.

[7] One particular condition to ensure the tests are conducted correctly is that the differencing and the specification of the REGARIMA model outliers are identical (see Census Bureau (2001), pp 31-32).

This means that both the graphical analysis and the AICC test have favoured the additive approach over the multiplicative approach.

The next section will show, however, that additive decomposition can cause problems with calendar adjustment.

First, though, we need to analyse the approach to estimating the calendar effect in more detail.

3.2 Estimating calendar effects

Calendar adjustment is conducted in Census X-12-ARIMA using a REGARIMA model. Accordingly, a regression model is estimated for the differenced unadjusted series (Y_{ij}), where the regression error (W_{ij}) is assumed to follow an ARMA model:[8]

$$\begin{aligned}&(1-\mathrm{B})^d(1-\mathrm{B}^s)^D Y_{ij} = (1-\mathrm{B})^d(1-\mathrm{B}^s)^D \sum_{k=1}^{n} \beta_k (x_{kij} - \bar{x}_{ki}) + W_{ij} \text{ and}\\ &\qquad \phi_p(\mathrm{B})\Phi_P(\mathrm{B}^s)W_{ij} = \theta_q(\mathrm{B})\Theta_Q(\mathrm{B}^s)a_{ij}\end{aligned} \qquad (3.1)$$

with

- i=1,…,4 for quarterly data (s=4) or i=1,…,12 for monthly data (s=12) and j for the year;
- the expression $(1-\mathrm{B})^d(1-\mathrm{B}^s)^D$ defining a non-seasonal differencing of order d and a seasonal differencing of order D using the backshift operator B (where $\mathrm{B}Y_{ij} = Y_{i-1j}$);
- $x_{kij} - \bar{x}_{ki}$ as the k-th regressor, which is given as the deviation of the value in month i of year j from its long-run average in month i. β_k denotes the respective regression coefficient;
- the polynomials of the ARMA model (line 2 of (3.1)), which are defined as follows: $\phi_p(\mathrm{B}) = (1-\phi_1\mathrm{B}-\phi_2\mathrm{B}^2-\ldots-\phi_p\mathrm{B}^p)$ is the non-seasonal (regular) auto-regressive (AR) operator to the p-th degree, $\Phi_P(\mathrm{B}^s) = (1-\Phi_1\mathrm{B}^s-\ldots-\Phi_P\mathrm{B}^{Ps})$ the seasonal AR operator to the P-th degree, $\theta_q(\mathrm{B}) = (1-\theta_1\mathrm{B}-\ldots-\theta_q\mathrm{B}^q)$ the non-seasonal moving average (MA) operator to the q-th degree and $\Theta_Q(\mathrm{B}^s) = (1-\Theta_1\mathrm{B}^s-\ldots-\Theta_Q\mathrm{B}^{Qs})$ the seasonal MA operator to the Q-th degree;

[8] For an account of ARIMA modelling, see Box, G.E.P. and G.M. Jenkins (1970). For more on integrating the regression analysis in ARIMA models, see Bell, W.R. (1992) and Census Bureau (2001), pp 15-22. For details of the quality of the estimation, see Chen, B.-C. and D.F. Findley (1993).

- a_t denotes the residuum or innovation which is uncorrelated in time with the other values and is identically normally-distributed (iid), with mean value 0 and a constant variance (white noise).

The differencing in (3.1) is applied if the error variable W_{ij} is not stationary. By differencing and/or transforming the unadjusted series, it is possible to obtain the required stationarity. The same differencing operations $(1-\mathrm{B})^d(1-\mathrm{B}^s)^D$ as were applied to the unadjusted series Y_{ij} are also then applied to the regression variables ($x_{kij}-\overline{x}_{ki}$). In the event that transformation is necessary, the unadjusted values Y_{ij} and the error variable W_{ij} are placeholders for transformed values in equation (3.1). For logarithmic transformation, they would denote the following, for example:[9] $Y_{ij}=\ln y_{ij}$ and accordingly $W_{ij}=\ln w_{ij}$ with y_{ij} as the non-logarithmic original data and w_{ij} as the non-logarithmic residuum.

A choice of predefined regression variables is available in the X-12-ARIMA application, such as calendar regressors with the number of weekdays per month or dummy variables for modelling outliers or series breaks.[10] It is also possible to input user-defined variables.

Here, we have selected the series indicating the deviations of working days from their respective monthly average as the explanatory variable for the impact of calendar variations on west German construction output. Specifically, this involves the use of two regressors: the number of working days in the months of January, February, March and November and the number of working days in the remaining months of the year (excluding December). This takes account of the fact that the effect of a working day in

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[9] In addition, note that in the event of a time series transformed using logarithms, the additive regression results have multiplicative effects on the unadjusted values (y_{ij}). If, for the sake of simplicity, the differencing is omitted (d=D=0), then $\ln y_{ij}=\sum_{k=1}^{n}\beta_k(x_{kij}-\overline{x}_{ki})+\ln w_{ij}$ is equivalent to $y_{ij}=\exp(\sum_{k=1}^{n}\beta_k(x_{kij}-\overline{x}_{ki}))\cdot w_{ij}$. The multiplicative effect on y_{ij} then results owing to the approximation $\exp(\sum_{k=1}^{n}\beta_k(x_{kij}-\overline{x}_{ki}))\approx 1+\sum_{k=1}^{n}\beta_k(x_{kij}-\overline{x}_{ki})$ for fairly small regression results (see Bell, W.R. (1992), p 137).

the colder months of the year is far less than at other points in the year. The allocation of the months to the two groups is based on a sample calculation for the individual months. Calendar effects of 2 to 3 index points per working day were estimated for the months January, February, March and November and an effect of 4 to 5 index points per working day for the remaining months (excluding December). Mainly as a result of production stoppages around Christmas, an additional working day in December does not have a perceptible effect.

In spite of the general preference for the additive approach discussed in section 3.1, there are problems with the results of additive calendar adjustment that do not occur in multiplicative decomposition. For example, the spectrum of the differenced and additive calendar and seasonally adjusted series, shown for the period January 1981 to November 2003 in figure 3.A, displays a significant peak at the most significant working-day frequency of 0.348 cycles per month.[11] In other words, calendar effects are still visible in the seasonally adjusted series notwithstanding the adjustment for working-day variation. Using the multiplicative approach, that is not the case (see figure 3.B).

In addition, there are difficulties with the economic interpretation of the additive model's results. For the end of the west German construction output series (January 2000 to November 2003), table 1 shows the unadjusted values alongside the calendar components[12] and the calendar adjusted values of the additive decomposition. The percentage calendar effect of an additional working day can be derived by expressing the unadjusted values in relation to the calendar adjusted values. This expression is then standardised by the number of working days[13]. At the end of the series, in particular,

[10] For more information, see Census Bureau (2001), pp 17-22.

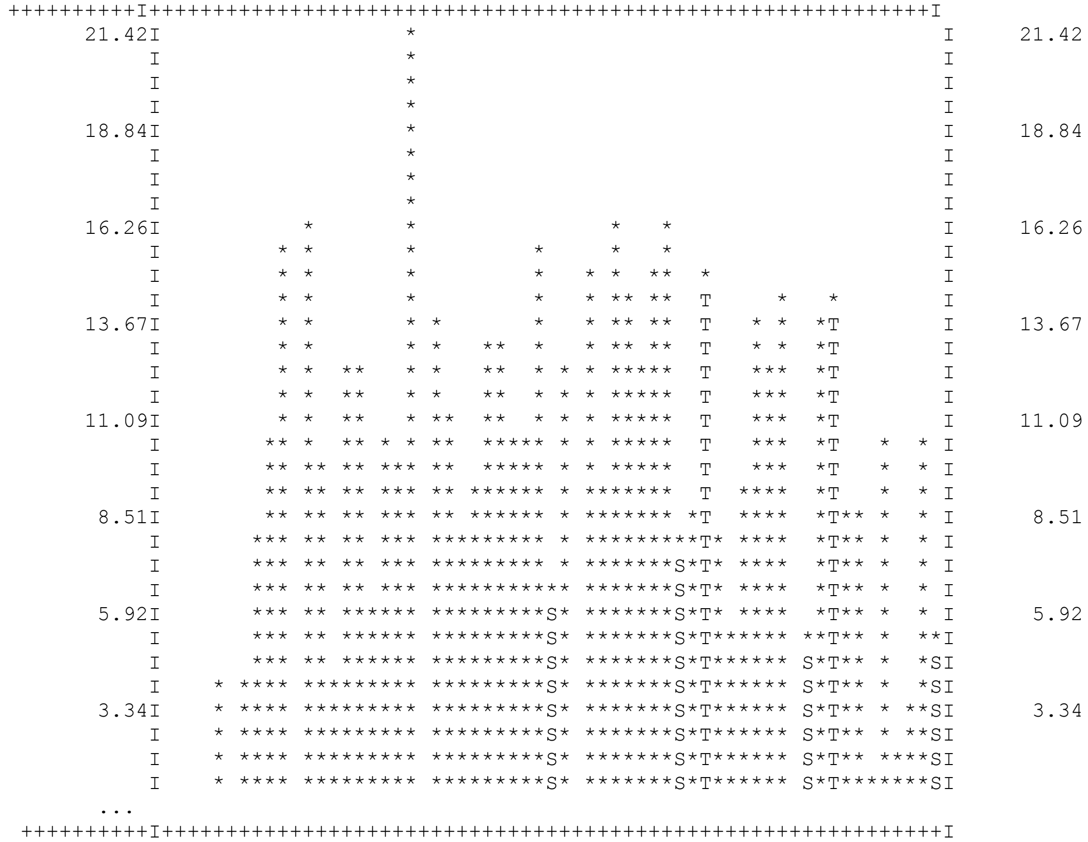
[11] See Soukup R.J. and D.F. Findley (1999).

[12] These were calculated using an estimated coefficient of 2.3 index points per additional working day for the months January-March plus November and 4.6 index points per additional working day for the months April-October. The coefficients were then multiplied with the regressors – ie the deviation of the number of working days from its monthly average.

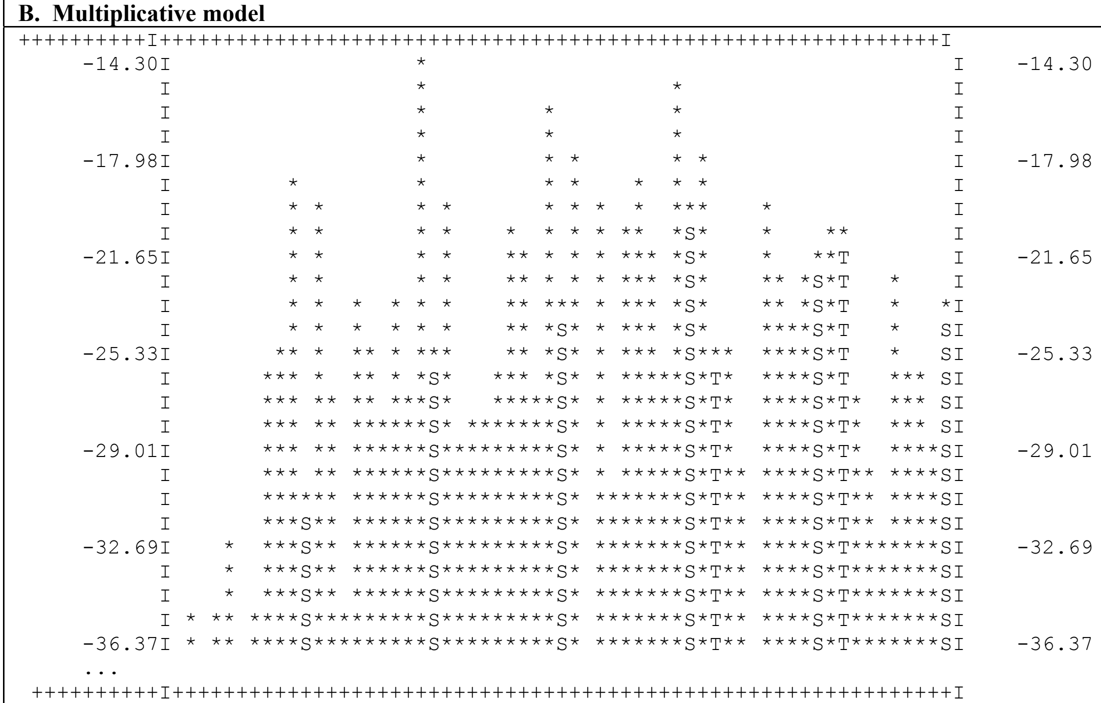
[13] These effects can also be derived (assuming, for the sake of simplicity, that k=1) as $\left[(Y_{ij}-\beta(x_{ij}-\bar{x}_i))-(Y_{ij}-\beta((x_{ij}+1)-\bar{x}_i))\right]/\left[(Y_{ij}-\beta(x_{ij}-\bar{x}_i))\right] = \beta/(Y_{ij}-\beta(x_{ij}-\bar{x}_i))$. With data that are available like in column (1) to (4) of table 1 this expression can be calculated as $\left[\beta(x_{ij}-\bar{x}_i)/(Y_{ij}-\beta(x_{ij}-\bar{x}_i))\right]/(x_{ij}-\bar{x}_i)$ which is equal to column (5).

Figure 3: Periodogram of the calendar and seasonally adjusted production index for the west German construction sector differenced against the preceding period
Spectrum estimated from January 1981 to November 2003

A. Additive model



B. Multiplicative model



S = Seasonal frequency, T = Working-day frequency

Table 1: Working-day effects in the production index for the west German construction sector

Time		Unadjusted values	Additive approach: Calendar component	Additive approach: Calendar adjusted values	Additive approach: Deviation of working days from monthly specific average	Additive approach: Calendar effect of an additional working day	Multiplicative approach: Calendar effect of an additional working day
		1995=100	Index points	1995=100	Number	in %	in %
Column		(1)	(2)	(3)=(1)-(2)	(4)	(5)[o]	(6)
2000	Jan	60,0	-1,2	61,2	-0,5042	3,9	3,3
	Feb	73,6	2,5	71,1	1,0917	3,2	3,3
	Mar	89,9	2,3	87,6	0,9917	2,6	3,3
	Apr	87,8	-8,5	96,3	-1,8333	4,8	4,6
	May	106,7	10,8	95,9	2,3333	4,8	4,6
	Jun	91,5	-3,5	95,0	-0,7500	4,9	4,6
	Jul	93,0	-5,8	98,8	-1,2500	4,7	4,6
	Aug	92,9	4,2	88,7	0,9000	5,3 *	4,6
	Sep	97,4	-2,1	99,5	-0,4583	4,6	4,6
	Oct	95,7	-3,7	99,4	-0,7917	4,7	4,6
	Nov	94,3	2,4	91,9	1,0542	2,5	3,3
	Dec	67,2	0,0	67,2	0,0000	0,0	0,0
2001	Jan	55,3	1,8	53,5	0,7958	4,2	3,3
	Feb	61,4	-0,7	62,1	-0,3083	3,7	3,3
	Mar	78,3	0,9	77,4	0,3917	3,0	3,3
	Apr	81,3	-3,9	85,2	-0,8333	5,5 *	4,6
	May	94,8	6,2	88,6	1,3333	5,2 *	4,6
	Jun	88,5	-3,5	92,0	-0,7500	5,1 *	4,6
	Jul	93,8	-1,2	95,0	-0,2500	5,1 *	4,6
	Aug	89,3	4,2	85,1	0,9000	5,5 *	4,6
	Sep	91,1	-6,8	97,9	-1,4583	4,8	4,6
	Oct	100,8	1,0	99,8	0,2083	4,8	4,6
	Nov	90,3	2,4	87,9	1,0542	2,6	3,3
	Dec	61,5	0,0	61,5	0,0000	0,0	0,0
2002	Jan	53,8	1,8	52,0	0,7958	4,3	3,3
	Feb	60,1	-0,7	60,8	-0,3083	3,7	3,3
	Mar	77,0	-3,7	80,7	-1,6083	2,9	3,3
	Apr	92,9	5,4	87,5	1,1667	5,3 *	4,6
	May	84,4	-2,2	86,6	-0,4667	5,4 *	4,6
	Jun	89,8	0,2	89,6	0,0500	4,5	4,6
	Jul	96,2	3,5	92,7	0,7500	5,0 *	4,6
	Aug	82,6	-0,5	83,1	-0,1000	6,0 *	4,6
	Sep	92,7	-2,1	94,8	-0,4583	4,8	4,6
	Oct	94,8	1,0	93,8	0,2083	5,1 *	4,6
	Nov	85,8	0,1	85,7	0,0542	2,2	3,3
	Dec	57,1	0,0	57,1	0,0000	0,0	0,0
2003	Jan	48,0	1,1	46,9	0,4958	4,7	3,3
	Feb	48,5	0,2	48,3	0,0917	4,5	3,3
	Mar	72,2	-2,3	74,5	-1,0083	3,1	3,3
	Apr	82,9	0,8	82,1	0,1667	5,8 *	4,6
	May	82,1	1,5	80,6	0,3333	5,6 *	4,6
	Jun	83,4	-3,5	86,9	-0,7500	5,4 *	4,6
	Jul	94,1	3,5	90,6	0,7500	5,2 *	4,6
	Aug	73,2	-5,1	78,3	-1,1000	5,9 *	4,6
	Sep	91,2	2,5	88,7	0,5417	5,2 *	4,6
	Oct	92,1	1,0	91,1	0,2083	5,3 *	4,6
	Nov	82,0	-0,6	82,6	-0,2458	3,0	3,3

o) (5)=[((1)/(3)-1)x100]/(4). See also footnote 13.
* Months with disproportionately large working-day effects.

with an average production index level of only around 80 index points (base: 1995 = 100), there are an increased number of cases where the impact of an additional working day is greater than 5%. At its peak, this effect reached 6% (August 2002).

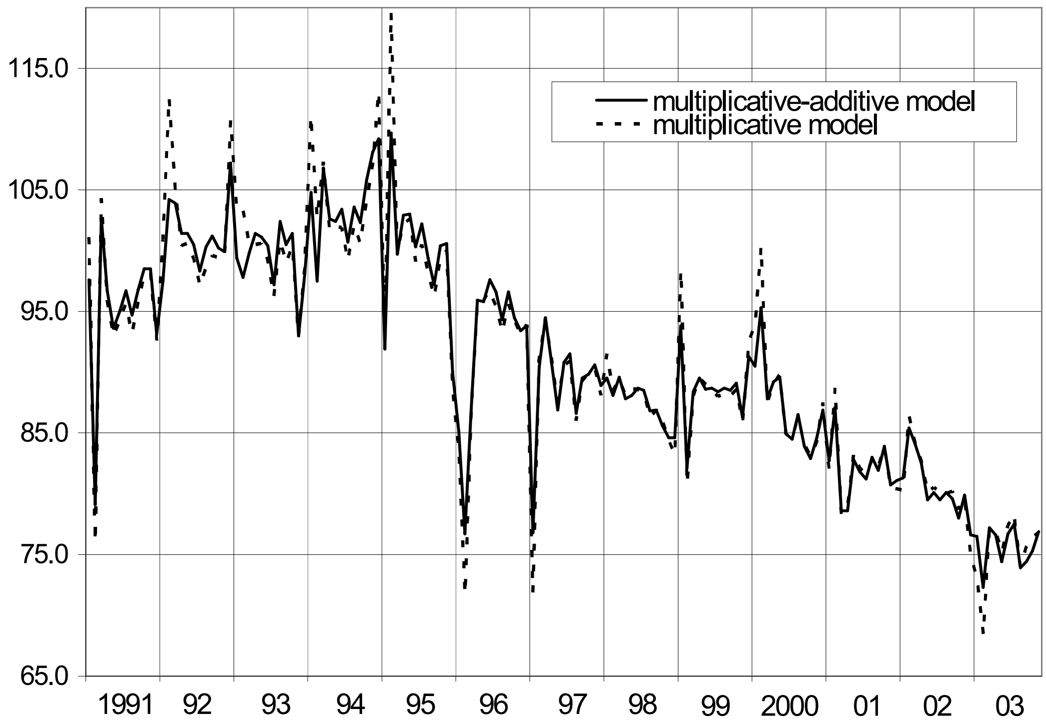
Meaningful economic interpretation of these figures is not possible because they exceed the limit for maximum calendar effects. To calculate this upper limit, it was assumed for the sake of simplicity that there were an average of 20 working days a month. If work is only done on weekdays (and not at weekends), 1/20th of monthly output is produced every working day, ie 5%. As some firms also work on weekends and on public holidays, however, meaningful economic interpretation is only possible for values between 0% and 5%. A disproportionately large working-day effect of more than 5% would imply that some of the output produced on weekdays was destroyed at weekends! However, such business practice is not rational and can therefore be ruled out. In a multiplicative approach, difficulties such as those experienced with additive decomposition do not arise (see the final column in table 1).

3.3 Estimating seasonal effects

As demonstrated in the previous section, the results of calendar adjustment based on logarithmic unadjusted values are preferable to those based on non-logarithmic unadjusted values. It is therefore worth considering whether the seasonal component should also be estimated using a multiplicative approach, even if this is not in line with the model preference determined in section 3.1. However, this causes problems, because the seasonally adjusted figures for west German construction output generated using a multiplicative approach have a much broader fluctuation range than those adjusted using the approach discussed in the next section: multiplicative calendar adjustment and additive seasonal adjustment (figure 4).

The following theoretical example will seek to illustrate that the wide dispersion of the results obtained using the multiplicative approach are linked with multiplicative seasonal factors. Let us assume that output in the winter months is usually only around half the annual average and, to keep things simple, that the latter equals 100 index points. With this information, it is possible to specify the seasonal components in the winter months: 0.5 in a multiplicative model and -50 index points using an additive approach. Were output in the current winter to fall very sharply to just 20 index points, the seasonally adjusted figures would be 40 index points using multiplicative decomposition and 70 using the additive model. Although the seasonally adjusted values would still be far below 100 index points using either approach, the effect of bad weather would be far more pronounced using the multiplicative approach. This shows that extra emphasis is placed on the weather-related variations when using a multiplicative model. The same kind of overemphasis would apply if the winter weather had been exceptionally mild and output unusually high. To prevent excessive distortion of the seasonally adjusted results it is better to use an additive model when estimating the seasonal fluctuations in the construction industry.

Figure 4: Seasonally adjusted production index for the west German construction industry
1995 = 100



3.4 The D(C+S+I) model

To summarise the previous sections, we recommend taking a multiplicative approach to calendar adjustment and an additive approach to seasonal adjustment for west German construction output.[14] The resulting multiplicative-additive model is

$$Y_t = D_t \cdot (C_t + S_t + I_t)\,. \tag{3.2}$$

The multiplicative calendar factors D_t are estimated using a REGARIMA model based on the logarithmically transformed unadjusted values. It is then possible to adjust the time series for working-day variation by dividing the unadjusted series by the calendar factors.

$$Y_t / D_t = C_t + S_t + I_t\,. \tag{3.3}$$

[14] Looking at alternative models, it is possible to apply pseudo-additive decomposition as available in the X-12-ARIMA application: $Y = C \cdot (S + I - 1)$ (see Census Bureau (2001), pp 153-155 and Findley, D.F., B.C. Monsell, W.R. Bell, M.C. Otto and B.-C. Chen (1998), pp 129-132.) However, because this model is defined without a calendar component, a relative working-day factor (D) has been added: $Y_t = C_t \cdot D_t \cdot (S_t + I_t - 1)$.
As with multiplicative-additive decomposition, it was possible to estimate relative calendar factors based on a logarithmic REGARIMA model. Relative seasonal factors were then estimated on the basis of the working-day adjusted series that had been calculated and, using the trend-cycle component, they were linked to the "seasonal difference" $C_t \cdot (S_t - 1)$. Applying this to the calendar adjusted values results in the following calendar and seasonally adjusted series: $Y_t / D_t - C_t \cdot (S_t - 1) = C_t \cdot I_t$.
In general, using pseudo-additive decomposition is recommended for seasonally adjusting time series with large, strongly fluctuating seasonal effects, because the seasonal difference in months which have very small values after adjustment for seasonal variation tend towards the negative trend value and thus, even given large irregularity, the seasonally adjusted series is forced towards the trend level. However, the fact that the seasonal difference is dependent on the level is also its Achilles' heel, since it reflects the problems with estimating the trend. At the beginning and end of the series, in particular, the seasonal difference is less informative, because, here, there are major difficulties associated with estimating the trend component. This is especially serious if the series's economic trend reaches a turning point (Meyer, N. (1997)). But precisely because the current-end of the series is significant for analysis of economic indicators, we will here not further pursue pseudo-additive decomposition as a means for obtaining seasonally adjusted figures. (For example, at the start of the series for west German construction output in January and February 1980, the calendar and seasonally adjusted values using the pseudo-additive approach are respectively 17% and 11% higher than for the multiplicative-additive model.)

The seasonal factor is estimated using additive decomposition of the working-day adjusted series. The series, adjusted for seasonal and working-day variation, is expressed as

$$Y_t / D_t - S_t = C_t + I_t \,. \tag{3.4}$$

4 Further sample uses of the D(C+S+I) model

The rationale applied to west German construction output can also be applied to other time series which are characterised by trend-cycle movements and sharply irregular seasonal variations. In the time series referred to in tables 2 and 3, an additive model was chosen based on the AICC criterion, but this model resulted in an over-adjustment for working-day variation. This could be avoided by using the D(C+S+I) model.

The German time series studied are production indices for construction, sub-divided into civil engineering work and general building work, and the production indices for capital goods and durable goods. Results for the period January 1991 to December 2005 are shown in table 2. In the case of foreign series (table 3), the analysis period depends on the data available. In addition, the ARIMA models used to model the series are stated, where an Airline model was estimated for all the German series and almost three-quarters of the foreign ones.

Table 2: Estimating working-day effects in different production indices for Germany
Period: Jan 1991 – Dec 2005, ARIMA model: (0 1 1)(0 1 1)

Time series	$AICC^{mult}$ - $AICC^{add}$	Additive approach: Coefficients in index points per working day (t-value in parenthesis)				Additive approach: Maximum calendar effect of an additional working day in %	Multiplicative approach: Maximum calendar effect of an additional working day in %
		Winter		Non-winter			
Civil engineering	109.4	1.72	(3.0)	5.52	(10.6)	6.6	4.8
Building work	50.5	1.97	(4.3)	5.03	(12.7)	6.6	4.6
Total construction	71.5	1.89	(4.0)	5.18	(12.1)	6.5	4.7
		December		January-November			
Capital goods	3.2	1.90	(5.4)	3.31	(23.0)	5.3	3.7
Durable goods	9.9	3.03	(5.9)	4.36	(19.9)	6.4	4.8

The tables only show results for time series for which it was decided (on the basis of the AICC test) to use additive decomposition. The differences between the AICC values for a multiplicative model (logarithmic transformation) and an additive model are stated in the table.

Table 3: Estimating working-day effects in different production indices for selected countries

Country Time series	Period	ARIMA model	$AICC^{mult}$ - $AICC^{add}$	Additive approach: Coefficient in index points per working day (t-value in parenthesis)		Additive approach: Maximum calendar effect of an additional working day in %	Multiplicative approach: Maximum calendar effect of an additional working day in %
Belgium							
Civil engineering	01/95 - 11/05	(0 1 1)(0 1 1)	17.0	3.23	(5.5)	9.1	4.2
Total construction	01/90 - 11/05	(0 1 1)(0 1 1)	37.5	3.04	(7.8)	7.2	3.5
Denmark							
Capital goods	01/90 - 11/05	(0 1 1)(0 1 1)	4.8	3.39	(12.6)	7.2	3.7
Durable goods	01/90 - 11/05	(0 1[1 2 4])(0 1 1)	3.4	3.14	(13.5)	10.1	3.8
Finland							
Civil engineering	01/95 - 10/05	(0 1 1)(0 1 1)	14.1	2.77	(7.2)	8.3	2.8
Durable goods	01/90 - 12/05	(0 1 1)(0 1 1)	37.6	3.28	(11.9)	23.2	3.9
France							
Building work	01/90 - 11/05	(0 1 2)(0 1 1)	9.7	2.55	(13.0)	5.9	2.6
Total construction	01/90 - 11/05	(0 1 1)(0 1 1)	17.7	2.78	(17.3)	5.1	2.9
Italy							
Capital goods	01/90 - 11/05	(0 1[1 3])(0 1 1)	103.8	3.50	(18.3)	16.0	3.7
Durable goods	01/90 - 11/05	(0 1 1)(0 1 1)	166.7	2.95	(13.9)	22.3	3.6
Austria							
Civil engineering	01/00 - 11/05	(0 1 1)(0 1 1)	6.4	3.87	(4.7)	9.6	3.9
Portugal							
Capital goods	01/95 - 12/05	(0 1 1)(0 1 1)	10.0	2.69	(8.6)	5.8	2.7
Durable goods	01/95 - 12/05	(1 1 1)(0 1 1)	9.2	3.23	(9.7)	6.6	3.5
Spain							
Durable goods	01/90 - 12/05	(0 1 1)(0 1 1)	73.1	3.62	(16.5)	22.6	4.0
Sweden							
Capital goods	01/90 - 11/05	(0 1 1)(0 1 1)	13.0	2.92	(14.3)	13.1	3.7

Source for foreign production series: Eurostat. The calendar regressors required for the analysis were provided by the ECB. These were based on data from national central banks pertaining to the number of working days from Monday to Friday (excluding public holidays) in each country.

The tables contain the estimated calendar coefficient β_k together with the corresponding t-value. It should be noted that to simplify the work required to adjust the foreign series for working-day effects, the analysis did not consider the possibility of monthly-specific calendar effects. By contrast, a distinction was drawn when adjusting the German series for working-day variation, for example between the two aforementioned regressors: a "winter regressor" with the number of working-days in cold-weather months (January, February, March, November and – unlike for west German construction output – December as well) and the number of working days in the remaining months of the year.[15] With values above 5 index points per working day, the estimated coefficient of this "non-winter regressor" is unusually high. This can be explained by the fact that the indices during months that favour construction output, ie April to October, were generally significantly above 100 between 1991 and 2002 (base: 2000 = 100) and only fell below that level from 2003 onwards.

Finally, for each series, the maximum calendar effect (in per cent) is given for a single working day, based on additive decomposition. The tables only contain cases where over-adjustment occurred, ie a working-day effect of more than 5% for an additional working day. For the purposes of comparison the (largest) calendar coefficient for a multiplicative approach is also shown, which does not imply any over-adjustment.

Consequently, the mixed model D(C+S+I) would be an interesting alternative to traditional approaches not just for German construction output, but also for adjusting numerous time series from various countries.

5 Adjusting for seasonal variation using the D(C+S+I) model with the X-12-ARIMA application

In comparison with purely additive or multiplicative decomposition, additional steps are required in the X-12-ARIMA application to adjust a time series for calendar effects and seasonal variations using a multiplicative-additive mixed model. Firstly, a meta file must be created with two SPC files:[16] one to estimate relative calendar factors

[15] As leap-year effects have little bearing on the German series analysed here, they were not shown separately.

[16] Census Bureau (2001), p 6.

and the other to estimate additive seasonal factors. In the first file, a REGARIMA model is estimated for the logarithmic unadjusted values and the calendar adjusted series is saved (table B1 in the X11 module).[17] This series is then imported into the second SPC file and is the data on which seasonal adjustment using the additive model is based. The following table is an example of streamlined programming.

It would be much easier for the user if the application would permit combining a REGARIMA model based on logarithmic unadjusted values with an additive X-11 component. The mixed model could then be processed in a single SPC file making the two-stage structure with a workaround using a meta file redundant.

6 Conclusion

Using the example of production in the west German construction sector, it has been demonstrated that an additive decomposition model, which was defined on the basis of the usual criteria, leads to an over-adjustment of calendar effects. This example can be generalised. In the case of time series

- for which an additive decomposition model is chosen
- with strongly marked and obviously fluctuating seasonal effects
- with movements in the trend-cycle component and
- with perceptible working-day effects

an over-adjustment of calendar effects often results. This can be remedied by a mixed multiplicative-additive model, which can be implemented in X-12-ARIMA. It is on this basis that the calendar and seasonally adjusted series is ultimately estimated: first, a working-day adjustment based on a multiplicative model and then an additive seasonal adjustment using the working-day adjusted values.

This method avoids the implausibilities which can arise during seasonal and calendar adjustment and makes the adjusted values more useful for short-term economic analysis.

[17] Entering “NOAPPLY=(AO,LS,TC)” in the REGRESSION mode ensures that outliers which may have been estimated in the REGARIMA model are not eliminated in table B1 containing data adjusted for working-day variation and (usually) for outliers (see Census Bureau (2001), p 100).

Table 4: SPC files for carrying out calendar and seasonal adjustment using a multiplicative-additive model

Stage 1: Estimating the relative calendar factors

```
SERIES
{
  NAME='OUTPUT CONSTRUCTION'
  FILE='C:\DATEN\X12ARIMA\X12A\BBK\UV61NA.SER'
}
TRANSFORM
{
  FUNCTION=log
}
REGRESSION
{
  USER=(WINTER NOWINTER)
  FILE='C:\DATEN\X12ARIMA\X12A\BBK\UV61NA.RGR'
  CENTERUSER=SEASONAL
  NOAPPLY=(AO,LS,TC)
}
OUTLIER
{
}
ARIMA
{
}
ESTIMATE
{
}
X11
{
  SAVE=(B1)
}
```

Stage 2: Estimating the additive seasonal factors

```
SERIES
{
  NAME='TD ADJUSTED SERIES'
  FILE='C:\DATEN\X12ARIMA\X12A\BBK\FILE1.b1'
  FORMAT='X12SAVE'
}
OUTLIER
{
}
ARIMA
{
}
ESTIMATE
{
}
X11
{
  MODE=ADD
  CALENDARSIGMA=ALL
  FINAL=USER
}
```

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