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VALIDATING A PRELIMINARY ESTIMATE WHEN TEMPORALLY DISAGGREGATING A TIME SERIES

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This work deals with the problem of obtaining an appropriate preliminary estimate of an unobserved high frequency (say monthly) time series. This problem arises when temporally disaggregating an observed series of low frequency (say quarterly) data. The preliminary estimated series should not only satisfy some basic criteria deduced from subject matter considerations, but it has to be valid in data. Thus, a test statistic is suggested for empirically validating the preliminary series. A real situation is described in which carrying out the monthly disaggregation of Mexico's Gross Domestic Product required a careful analysis of the preliminary estimate employed. A detailed exposition of the problem found as well as its possible causes and solutions is presented here.

KEYWORDS: ARIMA models, Compatibility testing, Seasonality, Stationarity.

JEL CLASSIFICATION: C13, C22, C53.

1 Introduction

In Mexico, Real Gross Domestic Product (GDP) is calculated from basic sources of information in a quarterly basis. This task is done by Mexico's official statistical agency called National Institute of Statistics, Geography and Informatics (INEGI). Nevertheless, in order to carry out a more timely analysis of the economic situation, the need still exists of producing an estimate of GDP with higher frequency. A step in the direction of responding to that requirement was given in 1993, when INEGI started producing an index that could be used as a monthly indicator of GDP. With these data already available, it was possible to get a monthly estimate of GDP through the application of a temporal disaggregation procedure. Such an application is described in Guerrero (2003), where a new disaggregation method was devised. Since experts working at INEGI deemed reasonable the results of this method, for the sample period January 1993 – December 1999, it was adopted for routine application in year 2000. Since then, monthly disaggregation of the quarterly GDP figures has been carried out for internal use at INEGI. However, in year 2002 some evidence appeared against the stationarity assumption of a series employed by the method, casting some doubts about the validity of the disaggregated data.

This paper describes the temporal disaggregation method in a general setting in Section 2, emphasizing its underlying assumptions. One main ingredient of this method is a preliminary estimate, which in practice is usually obtained by combining information from related variables, also called indicators. Selection of appropriate related variables is based primarily on subject matter considerations, and some criteria for choosing those variables are presented in Section 3. Section 4 is dedicated to the empirical validation of the preliminary estimate and a test statistic is recommended for that purpose. An application of the method and the test

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statistic to the Mexican data is presented in Section 5. The problem that gave rise to this work is also described in that section. The main part of this study is Section 6, which is devoted to the search for possible causes of the problem detected and their corresponding solutions. Then, a feasible solution is derived and applied to disaggregate Mexico's GDP. The most important results of this new application are shown and compared with the results obtained with the standard method. Section 7 concludes with some final comments.

2 Some results on temporal disaggregation

This section is devoted to present some basic notation and results that will be employed in the sequel. To start, let $\{Z_t;\ t = 1,..., mn\}$ be a high frequency (say monthly) unobserved time series which we are interested in. The number of low frequency periods (say quarters) is denoted by $n > 1$, while $m > 2$ is the intraperiod frequency (e.g. $m = 3$ when we want to disaggregate a quarterly series into a monthly series). Here, we shall assume that the following unobserved component representation is valid

$$Z_t = W_t + S_t \quad \text{for} \quad t = 1,..., mn \tag{1}$$

where $\{W_t;\ t = 1,..., mn\}$ is a series of preliminary estimates of $\{Z_t\}$ and $\{S_t;\ t = 1,..., mn\}$ is an unobserved zero-mean stationary process. We call $\{W_t\}$ a preliminary estimate because it is believed to behave essentially as $\{Z_t\}$, except for some discrepancies that might be considered unimportant in some practical applications. Therefore we should pay special attention to the procedure employed for obtaining $\{W_t\}$, which should not only have a sound subject matter foundation, but has to be consistent with the formulation proposed in (1) as well. Hence, no evidence of nonstationarity may be allowed in the series $\{S_t\}$.

Rather than observing $\{Z_t\}$, we usually observe an aggregated series $\{Y_i;\ i = 1,..., n\}$ which is given by

$$Y_i = \sum_{j=1}^{m} c_j Z_{m(i-1)+j} \quad \text{for } i = 1,..., n \tag{2}$$

where the coefficients $c_1,..., c_m$ are known constants that depend on the type of aggregation involved. For example, when the Y's are aggregated flows (as it happens when disaggregating a series of Gross Domestic Product, GDP, values), $c_j = 1/m$ for $j = 1,..., m$. In general we shall use the vector of coefficients $\mathbf{c} = (c_1,..., c_m)'$ with the prime symbol denoting transpose. Then, by calling $\mathbf{Y} = (Y_1,..., Y_n)'$ and $\mathbf{Z} = (Z_1,..., Z_{mn})'$, as well as $C = I_n \otimes \mathbf{c}'$, with I_n the n×n identity matrix and $\otimes$ the Kronecker product, we get the linear expression

$$\mathbf{Y} = C\mathbf{Z} \tag{3}$$

that summarizes how the whole set of aggregated data relates to the disaggregated one.

Expression (3) together with (1), written as

$$\mathbf{Z} = \mathbf{W} + \mathbf{S} \tag{4}$$

with $\mathbf{W} = (W_1,..., W_{mn})'$, and $\mathbf{S} = (S_1,..., S_{mn})'$, can be used to get the Minimum Mean Square Error Linear Estimator (MMSELE) of $\mathbf{Z}$, given all the available data. That is, if we let Var($\mathbf{S}$) $= \sigma_S^2 \Omega$ be the variance-covariance matrix of $\mathbf{S}$, then the MMSELE of $\mathbf{Z}$, given $\mathbf{W}$ and $\mathbf{Y}$, is given by

$$\hat{\mathbf{Z}} = \mathbf{W} + \Omega C'(C\Omega C')^{-1}(\mathbf{Y} - C\mathbf{W}) \tag{5}$$

with Mean Square Error (MSE) matrix

$$\mathrm{MSE}(\hat{\mathbf{Z}}) = \sigma_S^2 [I_{mn} - \Omega C'(C\Omega C')^{-1}C]\Omega. \tag{6}$$

For a proof of a similar result, see Guerrero (2003). Some of the most important procedures in use nowadays in many official statistical agencies, such as those of Chow and Lin (1971) or Denton (1971) can be related to this result.

3 The preliminary estimate

The idea of using a preliminary estimate can be traced back to Friedman (1962), who suggested using a linear combination of several related variables, say $X_1,..., X_G$, where $G \geq 1$. Such variables are chosen in such a way that their intraperiod movements be "highly" correlated with the intraperiod movements of $\{Z_t\}$. Therefore, the preliminary estimate is given by

$$W_t = \mathbf{X}_t'\hat{\boldsymbol{\beta}} \quad \text{for } t = 1,..., mn \tag{7}$$

where the vector of estimated coefficients is usually obtained from the available data, that is, from the vector of aggregated values $\mathbf{Y}$ and the matrix of disaggregated variables $X = (\mathbf{X}_1,..., \mathbf{X}_{mn})'$ that are related to $\mathbf{Z}$.

3.1 Obtaining the preliminary estimate

To obtain the estimated coefficients $\hat{\boldsymbol{\beta}}$ we postulate, as did Chow and Lin (1971), the following linear regression model for the disaggregated data

$$\mathbf{Z} = X\boldsymbol{\beta} + \mathbf{u} \tag{8}$$

where $\mathbf{u} = (u_1,..., u_{mn})'$ is a zero-mean random error vector with $\text{Var}(\mathbf{u}) = V$. Then, it follows that

$$\mathbf{Y} = C\mathbf{Z} = CX\boldsymbol{\beta} + C\,\mathbf{u} = X^a\boldsymbol{\beta} + \mathbf{u}^a \tag{9}$$

with

$$X^a = CX = (I_n \otimes \mathbf{c}')\,(\mathbf{X}_1,..., \mathbf{X}_{mn})' = (\mathbf{c}'\mathbf{X}_1,..., \mathbf{c}'\mathbf{X}_{mn})' \tag{10}$$

and

$$\mathbf{u}^a = C\mathbf{u} = (I_n \otimes \mathbf{c}')\,(u_1,..., u_{mn})' = (\mathbf{c}'u_1,..., \mathbf{c}'u_{mn})' \tag{11}$$

so that $E(\mathbf{u}^a) = \mathbf{0}_n$ and $\text{Var}(\mathbf{u}^a) = (I_n \otimes \mathbf{c}')V(I_n \otimes \mathbf{c}')' = V^a$. Therefore $\boldsymbol{\beta}$ can be estimated by Ordinary Least Squares on the assumption that $V = \sigma_u^2 I_{mn}$, in which case $V = \sigma_u^2\,\mathbf{c'c}\,I_n$ and $\hat{\boldsymbol{\beta}}$ will be the Best Linear Unbiased Estimator. When V is not a diagonal matrix, which is the most likely situation, $\hat{\boldsymbol{\beta}}$ will not be the most efficient estimator, although it will remain unbiased.

Some criteria useful to select the related variables were put forward in Guerrero (2003). A given variable X_g can be considered as a potentially useful related variable to Z when:

(i) it admits an adequate economic interpretation,
(ii) it is believed to be linearly correlated with Z,
(iii) its historical record $\{X_{g,t}\}$ runs from $t = 1,..., mn$, and it will be observed for $t > mn$,
(iv) its observed values are released timely, and
(v) its statistical quality, in the sense of the measurement method, is good and does not change with time.

In particular, when subject matter knowledge of Z indicates that a structural break or a seasonal effect must be present, we should force its presence in W through an appropriate dummy variable X_g.

3.2 Empirical validation of the estimate

It should be stressed that because Z and W are linked through the linear relationship (1), the MMSELE (5) makes use only of the first two moments of the distribution for Z, given W. Then, since

$$\mathrm{C}\mathbf{Z} - \mathrm{C}\mathbf{W} = \mathrm{C}\mathbf{S} \tag{12}$$

it follows that this variable is also Normally distributed with

$$E(\mathrm{C}\mathbf{Z} - \mathrm{C}\mathbf{W} \mid \mathbf{W}) = \mathbf{0}_n \text{ and } Var(\mathrm{C}\mathbf{Z} - \mathrm{C}\mathbf{W} \mid \mathbf{W}) = \sigma_S^2 C\Omega C'. \tag{13}$$

Now, in order to test for compatibility between preliminary estimate and true disaggregated series, we should postulate the following null hypothesis

$$H_0: E(\mathrm{C}\mathbf{Z} \mid \mathbf{W}) = \mathrm{C}\mathbf{W} \tag{14}$$

for which a test statistic becomes

$$K = (\mathbf{Y} - \mathrm{C}\mathbf{W})'(C\Omega C')^{-1}(\mathbf{Y} - \mathrm{C}\mathbf{W}) / \sigma_S^2 \tag{15}$$

where **Y** is used as observed value of C**Z.** The distribution of this statistics is Chi-square with n degrees of freedom. This is an omnibus test whose power against specific alternatives will be low, but its usefulness can be appreciated when trying to detect gross incompatibilities between the two different sources of information providing **Y** and **W**.

Since in practice $\sigma_S^2\Omega$ must be estimated from the observed data, we are forced to use a variant of K, which is given by

$$K^* = (\mathbf{Y} - \mathrm{C}\mathbf{W})'(C\hat{\Omega} C')^{-1}(\mathbf{Y} - \mathrm{C}\mathbf{W}) / \hat{\sigma}_S^2 \tag{16}$$

whose asymptotic distribution is again χ_n^2, on the assumption that $\hat{\sigma}_S^2\hat{\Omega}$ is a consistent estimator of $\sigma_S^2\Omega$. We could also compare K*/n against an F distribution with n and mn-p degrees of freedom, when $\hat{\sigma}_S^2\hat{\Omega}$ comes from a series $\{S_t\}$ to which a model with p parameters is fitted. Of course, when mn is large, using either distribution (χ_n^2 or $F_{n,mn-p}$) will make no big difference in the conclusion. The same type of argument was used by Box and Tiao (1976) when justifying a test statistic similar to K*, although in a different context. Besides, those authors reported the results of a small simulation study they carried out to see how well the test statistic performed empirically. Their results indicate that the estimation errors tend to increase the mean value of the Chi-square statistic approximately by a factor of 1+p/(mn), which is evidently negligible when mn is large. Therefore, the suggestion is to carry out the test and only when the calculated statistic satisfies

$$K^*_{calc} < \chi_n^2(\alpha) \tag{17}$$

we should not reject the compatibility hypothesis, at the α significance level, where $\chi_n^2(\alpha)$ denotes the α-percent point of the χ_n^2 distribution.

4 An application to Mexico's GDP

In Mexico, Real GDP is measured directly in a quarterly basis, but the need still exists of obtaining an estimated monthly GDP series. To that end, INEGI produces a monthly index called IGAE (Global Index of Economic Activity) that provides a timely proxy variable to GDP. This variable differs from GDP, besides of being expressed as an index ((Base 1993 = 100), basically because it has less coverage, but both variables are calculated with the same methodology. The criteria exposed in Section 3 for selecting a related variable are clearly satisfied by IGAE, therefore it was considered an adequate auxiliary variable to generate the monthly preliminary series $\{W_t\}$ for $Z_t = GDP_t$, where t runs from 1 up to 132 (January 1993 up to December 2003). The data on IGAE used in this work are presented in the Appendix.

In order to get the preliminary series, a Simple Linear Regression model was fitted to the quarterly GDP data from 1993:I to 2003:IV, i.e. for i = 1,..., 44, using as independent variable the monthly data on IGAE aggregated to the quarter, called $IGAE^a$. The results from this fit are as follows (standard errors in parenthesis)

$$GDP_i = \underset{(9\ 487\ 146.14)}{3\ 889\ 783.35} + \underset{(82\ 116.06)}{11\ 516\ 944.30} \times IGAE^a_i \tag{18}$$

with $\bar{R}^2 = 0.998$, $\hat{\sigma}_a = 6\ 663\ 836.10$ and Durbin-Watson = 2.01

These results show a strong linear relationship between GDP and $IGAE^a$, while the Durbin-Watson statistic shows no evidence of misspecification. Even though the intercept is not significantly different from zero, it was decided to leave it in the model in order to prevent possible biases when estimating the preliminary series. Therefore, the monthly preliminary series from January 1993 through December 2003 was generated with the following expression

$$W_t = 3\ 889\ 783.35 + 11\ 516\ 944.30 \times IGAE_t \tag{19}$$

Such a series was then aggregated to the quarter in order to produce the series $\{W^a_i;\ i = 1,..., 44\}$ as well as the series of differences given by

$$D_i = GDP_i - W^a_i \tag{20}$$

and shown in Figure 1.

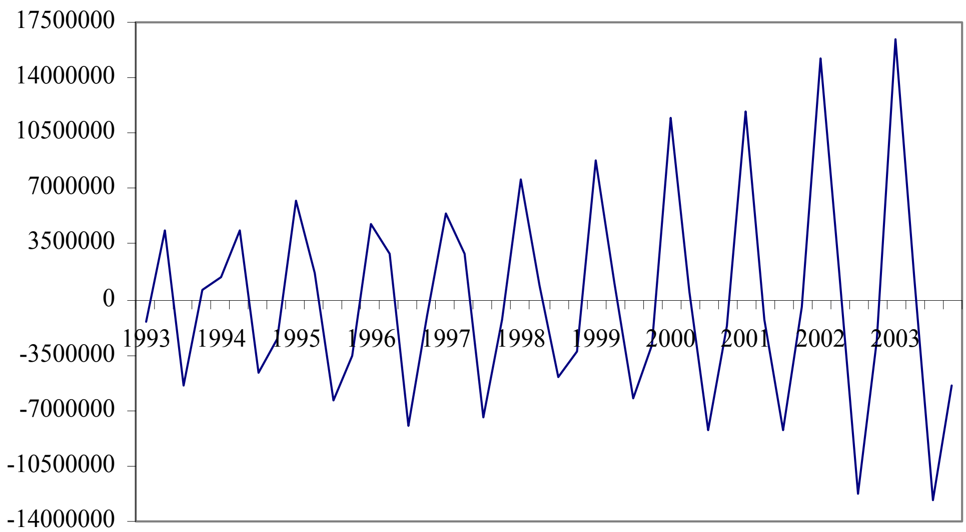

Figure 1: Time series plot of the series of differences $\{D_i\}$.

A descriptive summary of $\{D_i\}$ follows:

Sample mean = 0.00001, sample standard deviation = 6 585 893.8142
t-statistic (H_0: $\mu = 0$) = 0.00, p-value = 1.00
Skewness (H_0: Sk = 0) = 0.50, p-value = 0.19
Kurtosis (H_0: Ku = 0) = 0.28, p-value = 0.72

Hence, if $\{D_i\}$ were a stationary series, it could reasonably be considered Normally distributed with zero-mean. However, as Figure 1 allows us to see, the seasonal behavior of this series increases with time. This fact is corroborated numerically by looking at the sample Auto-Correlation Function (ACF) and sample Partial Auto-Correlation Function (PACF) of Table 1 where a strong autocorrelation structure is evident.

Table 1: Sample ACF and PACF of series $\{D_i\}$.

Lag	ACF					
1 to 6	-0.0135	-0.8344	-0.0457	0.8205	-0.0075	-0.7196
7 to 12	-0.0079	0.6581	-0.0379	-0.6006	0.0389	0.5747
	PACF					
1 to 6	-0.0135	-0.8347	-0.2565	0.3682	-0.1997	0.0483
7 to 12	0.0499	-0.1345	-0.0845	-0.0691	-0.0632	0.0835
	Standard error of ACF and PACF on the assumption of white noise: 0.1508					

A model found useful to fit the series of differences with data up to 1999:IV is a purely Seasonal Auto-Regressive model of order one and seasonality period 4, $SAR(1)_4$. Such a model produced the following results

$$\underset{(0.0516)}{(1-1.0895\,L^4)}D_i = \hat{\varepsilon}_i \quad \text{with} \quad \hat{\sigma}_\varepsilon = 1\,935\,248.80 \tag{21}$$

and Ljung-Box statistic Q(9)=15.17, whose p-value is 0.09

This model cannot be considered adequate because its AR coefficient is greater than one implying nonstationarity. This result should be expected because in Figure 1 we see that the seasonal amplitude has been growing steadily, particularly since the beginning of year 2000. Now, by assumption, the series of differences $\{S_t\}$ appearing in (1) has to be stationary. Then, since $\{S_t\}$ and $\{D_i\}$ are related by the same aggregation procedure that links $\{Z_t\}$ with $\{Y_i\}$, it should be clear that nonstationarity of $\{D_i\}$ implies that of $\{S_t\}$. This must be true because, as shown by Engel (1984), the temporal aggregation of a stationary Auto-Regressive and Moving Average (ARMA) model yields another stationary ARMA model.

Even though (21) indicates that an assumption of the disaggregation procedure has been violated, we could apply the procedure and obtain a disaggregated series $\{\tilde{Z}_t\}$ which will fulfill the temporal aggregation restrictions. However, we should be aware that such an estimated series would not be optimal in a statistical sense. In fact, we will not be able to estimate the MSE matrix for the estimated series and therefore no measure of uncertainty will be available. In the present situation, the disaggregated series was obtained by means of an application of the method proposed in Guerrero (2003). The compatibility statistic was calculated, yielding the value K^*_{calc} = 193.50 which, when compared with a Chi-square distribution with 44 degrees of freedom leads to the conclusion that the preliminary series is not a valid preliminary estimate of the true monthly GDP. Such a conclusion was then interpreted as evidence that either the auxiliary variable employed, IGAE, is no longer sufficient for generating the preliminary series, or the regression model was in fault, or both.

5 Looking for possible causes and solutions of the problem

The possibilities that were taken into consideration arose from the following basic arguments. (a) The 1995 economic crisis was so strong that affected with different intensities the quarterly GDP and its monthly proxy index, IGAE. Therefore, by shortening the sample period, say from 1996 onwards, the economic crisis effect could be avoided. (b) The linear regression model is somehow misspecified because IGAE is designed to measure the month/month relative changes in economic activity. Therefore, a procedure different from linear regression should be used when generating the preliminary series using IGAE as related variable. (c) The simple linear regression model should include some other auxiliary variables that may serve as complement to IGAE. This argument implies that IGAE has lost some of its explanatory power of GDP. Therefore, the index should be revised to extend its coverage of some important economic activities and its base year (1993) should be updated.

5.1 Entertaining the possible solutions

The first possibility was tried out by using data from 1996 to 2003, but the results were very similar to those obtained previously with the full sample period (covering data from 1993 to 2003). That is, the aggregated series of differences showed again an increasing seasonal pattern and the $SAR(1)_4$ model had an estimated parameter equal to 1.1081 (0.0487), leading to the conclusion that such a series was nonstationary. A similar exercise was carried out with data from 1997 to 2003 and yielded essentially the same results as before.

The second possibility was then entertained by applying the monthly relative structure (in percentage terms) of IGAE to the yearly GDP (obtained as average of its quarterly values). For instance, IGAE averages 100 for 1993, while the average of GDP for that year is 1 155 132 189, so that multiplying the monthly IGAE values of that year by the factor 11 551 321.89 we get the preliminary monthly values for GDP corresponding to 1993. The preliminary values for the remaining years were obtained in similar fashion. Then the preliminary series was aggregated to the quarter and its differences with respect to GDP were calculated. This alternative series of aggregated differences is shown in Figure 2, where we can appreciate the same dynamic behavior as that shown by the "old" series in Figure 1. In fact, the visual impression of Figures 1 and 2 is very similar.

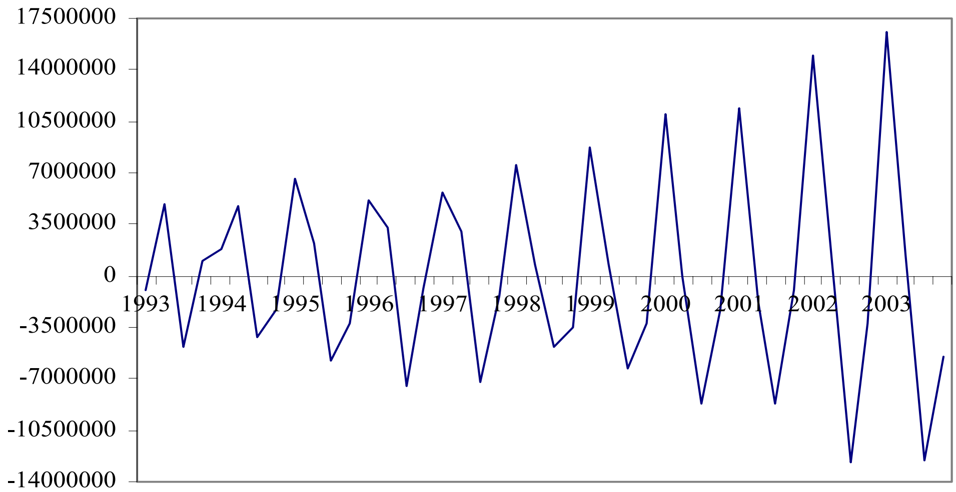

Figure 2: Series of aggregated differences obtained with the alternative procedure.

The values of the aggregated preliminary series, as well as their differences with respect to GDP are shown in Table 2 for some selected quarters. There we see that the series corresponding to the two procedures differ from each other, but the numerical differences are so small that the results may be considered equivalent. Nevertheless, by applying a linear regression model we know that a proper and statistically sound procedure has been employed, while the other procedure will require further justification. As a conclusion from this exercise, we may say that the growing seasonal pattern is not attributable to the use of a linear regression model.

Table 2: Aggregated preliminary series and differences obtained by linear regression and by the alternative procedure that keeps the relative structure (selected quarters).

		Linear regression		Alternative method	
Year	Quarter	W^a	D	W^a	D
1993	I	1 149 710 273	-1 447 689.53	1 149 260 169	-997 585.68
	II	1 154 529 725	4 424 107.00	1 154 073 300	4 880 532.32
	III	1 119 555 426	-5 450 013.11	1 118 995 201	-4 889 787.92
	IV	1 198 541 433	665 493.05	1 198 200 085	1 006 841.27
1994	I	1 173 694 008	1 381 344.22	1 173 271 670	1 803 682.15
	II	1 219 947 081	4 415 061.99	1 219 669 634	4 692 509.12
	III	1 170 046 777	-4 582 631.94	1 169 652 244	-4 188 098.61
	IV	1 264 204 078	-2 408 304.11	1 264 103 867	-2 308 092.66
1995	I	1 163 618 169	6 255 272.57	1 163 211 246	6 662 195.53
	II	1 110 146 340	1 638 662.08	1 109 536 421	2 248 581.00
	III	1 078 208 013	-6 391 689.70	1 077 539 448	-5 723 125.28
	IV	1 176 474 597	-3 590 606.26	1 176 071 642	-3 187 651.25
…	…	...	...	...	...
2002	I	1 420 688 727	15 239 174.68	1 421 007 126	14 920 775.80
	II	1 513 630 779	1 619 794.35	1 514 264 930	985 642.68
	III	1 466 113 766	-12 203 872.10	1 466 596 408	-12 686 513.80
	IV	1 526 090 737	-2 570 266.36	1 526 740 376	-3 219 904.62
2003	I	1 455 901 855	16 390 112.17	1 455 709 257	16 582 710.30
	II	1 515 585 680	1 384 030.15	1 515 557 177	1 412 532.84
	III	1 475 479 574	-12 685 695.95	1 475 337 835	-12 543 957.10
	IV	1 559 179 353	-5 390 253.33	1 559 240 386	-5 451 285.96

5.2 A feasible solution

The third possibility was then called for, considering only as a feasible solution that of including some additional variables deemed to be related to the monthly GDP. Of course, increasing IGAE's coverage or updating its base year was beyond the scope of this work. A regression model with seasonal indicator variables, together with IGAE aggregated to the quarter, was then employed. The resulting estimated model with data covering the full sample period became

$$\underset{}{GDP_i} = \underset{(4842815.54)}{10119758.75D1_i} + \underset{(4968167.61)}{3971319.18D2_i} - \underset{(4838738.85)}{5502812.92D3_i} - \underset{(5131488.92)}{15269.52D4_i} + \underset{(42152.45)}{11532364.53IGAE^a_i}$$

$$\text{with } \overline{R}^2 = 0.9995,\ \hat{\sigma}_a = 3\,323\,762.30 \text{ and Durbin-Watson} = 2.00 \qquad (22)$$

where D1 through D4 serve to capture the deterministic seasonal effects in quarters I through IV, respectively. Their associated estimated coefficients have t-ratios amounting to 2.09, 0.80, -1.14 and –0.02, so that only the first quarter has a significant seasonal effect. This effect is essentially due to the fact that IGAE has less coverage than GDP, particularly for the primary sector, and it is during the first quarter when there is more agricultural production. The strong linear relationship between GDP and $IGAE^a$ becomes evident again by looking at the corresponding t-ratio, which now reaches the value 274. The same indication is given by the adjusted coefficient of determination, which for all practical purposes is equal to 1. The residual standard deviation shows a substantial gain in precision with respect to that of model (18), since the ratio of residual standard deviations becomes 3 323 762.3/6 663 836.1 = 0.5.

Even though not all the seasonal indicators have significant effects, all of them were retained in the model in order to maintain the interpretation of their respective coefficients as (deterministic) seasonal effects.

In order to generate the monthly preliminary series for the whole period (January 1993 – December 2003) a set of monthly indicators denoted by d1 through d12, for months 1 = January up to 12 = December, will be employed. These monthly indicators are related to the quarterly ones as indicated in Section 3. That is, we should define the variables related to Z as $X_{1,t} = d1_t, \ldots, X_{12,t} = d12_t$ and aggregate them in accordance to expression (10). By so doing we obtain, for instance,

$$X^a_{1,i} = (d1_{3(i-1)+1} + d1_{3(i-1)+2} + d1_{3(i-1)+3})/3 \quad \text{for } i = 1,\ldots, 44 \tag{23}$$

so that

$$D1_i = X^a_{1,i} + X^a_{2,i} + X^a_{3i} = (d1_{3(i-1)+1} + d2_{3(i-1)+2} + d3_{3(i-1)+3})/3 \tag{24}$$

since $d1_{3(i-1)+2} = d1_{3(i-1)+3} = 0$, $d2_{3(i-1)+1} = d2_{3(i-1)+3} = 0$ and $d3_{3(i-1)+1} = d3_{3(i-1)+2} = 0$ for all i. In a similar fashion, it follows that

$$\begin{aligned} D2_i &= (d4_{3(i-1)+1} + d5_{3(i-1)+2} + d6_{3(i-1)+3})/3 \\ D3_i &= (d7_{3(i-1)+1} + d8_{3(i-1)+2} + d9_{3(i-1)+3})/3 \\ D4_i &= (d10_{3(i-1)+1} + d11_{3(i-1)+2} + d12_{3(i-1)+3})/3. \end{aligned} \tag{25}$$

Then, since the model for the (aggregated) quarterly GDP has the form

$$Y_i = \beta_1(X^a_{1,i} + X^a_{2,i} + X^a_{3,i}) + \ldots + \beta_4(X^a_{10,i} + X^a_{11,i} + X^a_{12,i}) + \beta_5\, IGAE^a_i + \varepsilon^a_i \tag{26}$$

the monthly preliminary series must be given by

$$\begin{aligned} W_t = {} & 10\,119\,758.75 \times (d1_t+d2_t+d3_t) + 3\,971\,319.18 \times (d4_t+d5_t+d6_t) - 5\,502\,812.92 \times \\ & (d7_t+d8_t+d9_t) - 115\,269.52 \times (d10_t+d11_t+d12_t) + 11\,532\,364.53 \times IGAE_t. \end{aligned} \tag{27}$$

The values so obtained were then aggregated to the quarter and served to produce a new series of aggregated differences $\{D_i\}$. As a summary of this series, we have the following figures, as well as the sample FAC and PACF shown in Table 3.

Sample mean = 0.00000, sample standard deviation=3 165 395.9131
t-statistic (H_0: $\mu = 0$) = 0.00, p-value = 1.00
Skewness (H_0: Sk = 0) = -0.21, p-value = 0.58
Kurtosis (H_0: Ku = 0) =1.69, p-value = 0.04

Table 3: Sample ACF and PACF of the new series of differences $\{D_i\}$.

Lag	ACF					
1 to 6	-0.1291	-0.4844	-0.3191	0.7466	0.1224	-0.3945
7 to 12	-0.3735	0.4320	0.2232	-0.1615	-0.3442	0.2585
	PACF					
1 to 6	-0.1291	-0.5096	-0.6584	0.4445	0.11949	-0.02717
7 to 12	0.1199	-0.1736	-0.2248	0.1712	0.0462	0.2546
	Standard error of ACF and PACF on the assumption of white noise: 0.1508					

Thus, as before, if the series were stationary it could reasonably be considered Normally distributed with zero-mean. The dynamic behavior of the series can be seen in Figure 3, where we can also appreciate the fit achieved by a $SAR(2)_2$ model and the corresponding standardized residuals. Besides, the actual figures of the aggregated preliminary series, together with the other quarterly series employed by the disaggregation procedure, appear in Table 4.

Table 4: Quarterly series employed by the disaggregation method with the feasible solution.

Year	Quarter	GDP	IGAE[a]	W[a]	D = GDP - W[a]
1993	I	1 148 262 583	99.49	1 157 474 406	-9 211 822.93
	II	1 158 953 832	99.91	1 156 151 872	2 801 960.30
	III	1 114 105 413	96.87	1 111 656 613	2 448 799.91
	IV	1 199 206 926	103.73	1 196 135 919	3 071 007.10
1994	I	1 175 075 352	101.57	1 181 490 253	-6 414 901.40
	II	1 224 362 143	105.59	1 221 656 816	2 705 326.90
	III	1 165 464 145	101.26	1 162 215 568	3 248 577.38
	IV	1 261 795 774	109.43	1 261 886 481	-90 706.88
1995	I	1 169 873 442	100.70	1 171 400 924	-1 527 482.35
	II	1 111 785 002	96.05	1 111 709 061	75 941.03
	III	1 071 816 323	93.28	1 070 253 839	1 562 484.06
	IV	1 172 883 991	101.81	1 174 039 538	-1 155 546.56
1996	I	1 170 629 352	100.90	1 173 715 964	-3 086 611.52
	II	1 183 799 944	102.20	1 182 599 165	1 200 778.83
	III	1 148 180 991	100.05	1 148 315 582	-134 591.33
	IV	1 256 342 084	108.83	1 254 984 541	1 357 543.19
1997	I	1 224 440 456	105.51	1 226 869 499	-2 429 043.22
	II	1 283 060 307	110.82	1 281 981 087	1 079 219.98
	III	1 234 131 769	107.47	1 233 830 909	300 860.49
	IV	1 340 087 631	116.13	1 339 171 844	915 787.18
1998	I	1 316 480 543	113.32	1 316 918 334	-437 791.07
	II	1 338 329 244	115.79	1 339 276 788	-947 544.43
	III	1 299 073 202	112.88	1 296 291 129	2 782 072.72
	IV	1 376 299 514	119.45	1 377 421 660	-1 122 146.27
1999	I	1 343 372 356	115.54	1 342 539 698	832 658.34
	II	1 383 309 782	119.68	1 384 159 775	-849 992.68
	III	1 354 865 950	117.84	1 353 498 563	1 367 386.55
	IV	1 448 472 132	125.68	1 449 219 639	-747 507.26
2000	I	1 442 746 972	123.94	1 439 473 360	3 273 612.30
	II	1 484 631 770	128.53	1 486 215 049	-1 583 278.66
	III	1 449 947 983	126.27	1 450 734 351	-786 368.16
	IV	1 516 141 790	131.45	1 515 846 739	295 051.23
2001	I	1 470 996 482	126.36	1 467 323 338	3 673 144.39
	II	1 487 382 824	128.91	1 490 660 536	-3 277 712.15
	III	1 431 419 274	124.66	1 432 175988	-756 714.13
	IV	1 495 828 815	129.58	1 494 281 380	1 547 435.21
2002	I	1 435 927 902	123.02	1 428 815 678	7 112 223.70
	II	1 515 250 573	131.09	1 515 733 732	-483 158.73
	III	1 453 909 894	126.96	1 458 678 966	-4 769 071.80
	IV	1 523 520 471	132.17	1 524 123 785	-603 313.53
2003	I	1 472 291 967	126.08	1 464 075 953	8 216 013.76
	II	1 516 969 710	131.26	1 517 691 250	-721 540.38
	III	1 462 793 878	127.78	1 468 057 314	-5 263 435.70
	IV	1 553 789 100	135.04	1 557 256 703	-3 467 603.40

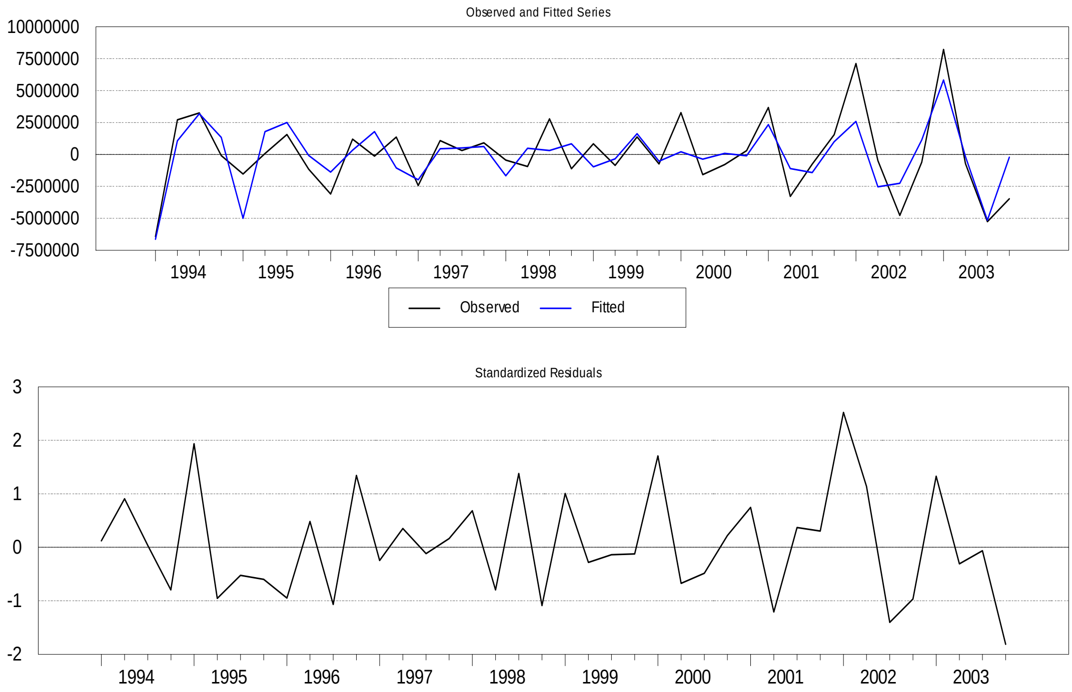

Figure 3: Series of aggregated differences resulting from the feasible solution, and fitted values produced by the $SAR(2)_2$ model.

The $SAR(2)_2$ model fitted to the series of aggregated differences produced the following results

$$\underset{(0.0155)}{(1+0.2470\,L^2}\ \underset{(0.1110)}{-\,0.6542\,L^4)}D_i=\hat{\varepsilon}_i \quad \text{with} \quad \hat{\sigma}_\varepsilon = 1\ 794\ 463.57 \tag{28}$$

and Ljung - Box statistic $Q(8) = 9.11$, whose p-value is 0.33

The roots of the characteristic equation of this model are $x_1 = 1.4395$, $x_2 = -1.0619$, $x_3 = 1.0305i$ and $x_4 = -1.0305i$, so that they all are outside the unit circle. Therefore, the series of aggregated differences behaves properly as a stationary process and we can proceed to apply the disaggregation procedure proposed by Guerrero (2003). First, the series has to be filtered with the $SAR(2)_2$ model in order to obtain a deseasonalized series of aggregated differences, that is

$$FD_i = D_i + 0.2470\,D_{i-2} - 0.6542\,D_{i-4} \quad \text{for } i = 5, 6, \ldots, 44. \tag{29}$$

Then we must apply Wei and Stram's (1990) method to find the nonseasonal structure of the model for the monthly series of differences, while its seasonal structure is given by the SAR polynomial

$$\hat{\Phi}(B) = 1 + 0.2470\,B^6 - 0.6542\,B^{12}. \tag{30}$$

By looking at the sample ACF and PACF of $\{FD_i\}$ provided in Table 5, we see that there is no significant autocorrelation structure, so that we can safely conclude that the AR and MA polynomials are of orders $P = Q = 0$, implying that the corresponding monthly filtered series $\{FS_t\}$ must have polynomials of orders $p = 0$ and $q = 1$.

Table 5: Sample ACF and PACF of series {FDi}.

Lag	ACF					
1 to 6	-0.2569	-0.0239	-0.1786	0.1689	0.0620	-0.1807
7 to 12	-0.1274	0.0141	0.1136	-0.0489	-0.1407	0.1760
	PACF					
1 to 6	-0.2569	-0.0963	-0.2270	0.0599	0.1080	-0.1679
7 to 12	-0.1928	-0.1032	-0.0160	-0.0492	-0.1416	0.1230
	Standard error of ACF and PACF on the assumption of white noise: 0.1508					

The next step consists of estimating the parameters of the model for $\{FS_t\}$. Thus we must recognize that the aggregation is such that

$$FD_i = \frac{1}{3}(1+B+B^2)FS_{3i} \tag{31}$$

hence, the autocovariances are related by the following equations

$$\gamma_{FD}(0) = \frac{1}{9}(1+B+B^2)^2\gamma_{FS}(2) = \frac{1}{9}\left[\gamma_{FS}(-2)+2\gamma_{FS}(-1)+3\gamma_{FS}(0)+2\gamma_{FS}(1)+\gamma_{FS}(2)\right]$$

and (32)

$$\gamma_{FD}(1) = \frac{1}{9}(1+B+B^2)^2\gamma_{FS}(5) = \frac{1}{9}\left[\gamma_{FS}(1)+2\gamma_{FS}(2)+3\gamma_{FS}(3)+2\gamma_{FS}(4)+\gamma_{FS}(5)\right]$$

where it is assumed that $\gamma_{FD}(k) = 0 = \gamma_{FS}(k)$ for k ≠ -1, 0, and 1. Thus, we get the system of equations

$$\begin{pmatrix}\gamma_{FD}(0)\\ \gamma_{FD}(1)\end{pmatrix} = \begin{pmatrix}3/9 & 4/9\\ 0 & 1/9\end{pmatrix}\begin{pmatrix}\gamma_{FS}(0)\\ \gamma_{FS}(1)\end{pmatrix}. \tag{33}$$

From the series $\{FD_i\}$ we get the estimated values $\hat{\gamma}_{FD}(0)$ = 3 128 172 723 319.36 and $\hat{\gamma}_{FD}(1)$= -803 728 925 416.98 so that (33) produces

$$\hat{\gamma}_{FS}(0) = 19\,029\,265\,274\,961.80 \text{ and } \hat{\gamma}_{FS}(1) = -7\,233\,560\,328\,752.81 \tag{34}$$

Therefore, since the model for $\{FS_t\}$ is of the form

$$FS_t = (1+\theta B)e_t, \tag{35}$$

we know that $\gamma_{FS}(0) = (1+\theta^2)\sigma_e^2$ and $\gamma_{FS}(1) = \theta\sigma_e^2$. Hence $\hat{\theta}$ can be obtained by solving

$$\hat{\gamma}_{FS}(1) - \hat{\gamma}_{FS}(0)\hat{\theta} + \hat{\gamma}_{FS}(1)\hat{\theta}^2 = 0, \tag{36}$$

which yields

$$\hat{\theta} = \frac{\hat{\gamma}_{FS}(0)}{2\hat{\gamma}_{FS}(1)} \pm \sqrt{[\hat{\gamma}_{FS}(0)/2\hat{\gamma}_{FS}(1)]^2 - 1}. \tag{37}$$

Then, by plugging in the values (34) we obtain $\hat{\theta}_1$ = -0.4609 and $\hat{\theta}_2$ = -2.1698, so that $\hat{\theta}$ = $\hat{\theta}_1$ is chosen to ensure invertibility of the model. Thus, the series of differences $\{S_t\}$ is modeled by

$$(1+0.2470B^6-0.6542B^{12})S_t = (1-0.4609B)e_t, \tag{38}$$

with $\hat{\sigma}_e^2 = \hat{\gamma}_{FS}(1)/\hat{\theta}$ = 15 695 558 069 526.7

Since this model is of the form $(1-\Phi_1 B^6 - \Phi_2 B^{12})S_t = (1+\theta B)e_t$, the weights of its pure MA representation $S_t = \psi_S(B)e_t$ can be obtained by equating coefficients of powers of B in $(1+\psi_{S,1}B+\psi_{S,2}B^2+...)(1-\Phi_1 B^6 - \Phi_2 B^{12}) = (1+\theta B)e_t$, thus producing

$$\psi_{S,0} = 1,\ \psi_{S,1} = \theta,\ \psi_{S,6} = \Phi_1,\ \psi_{S,7} = \theta\Phi_1,\ \text{and for } i = 2, 3,\ldots$$

$$\psi_{S,6i} = \Phi_1 \psi_{S,6(i-1)} + \Phi_2 \psi_{S,6(i-2)},\ \psi_{S,6i+1} = \theta\psi_{S,6i},\ \psi_{S,i} = 0 \text{ otherwise.} \tag{39}$$

Moreover, a correction for nonconstant variance can be performed by modifying the diagonal of the matrix $\Psi_S \Psi'_S$ where Ψ_S is the lower triangular matrix containing the MA weights (see Guerrero, 2003). In this way, we force the variance–covariance matrix of $\{S_t\}$ to take on the steady state values, that is

$$\mathrm{Var}(S_t)/\sigma_e^2 = (1+\theta^2)/(1-\Phi_1^2-\Phi_2^2). \tag{40}$$

5.3 Disaggregation results

Once the model for $\{S_t\}$ has been obtained, the GDP series can be disaggregated and the results obtained from the direct procedure are shown both in Figure 4 and in the Appendix. In particular, Figure 4 only presents the results for the five most recent years, together with their 95% prediction bands and the preliminary series employed.

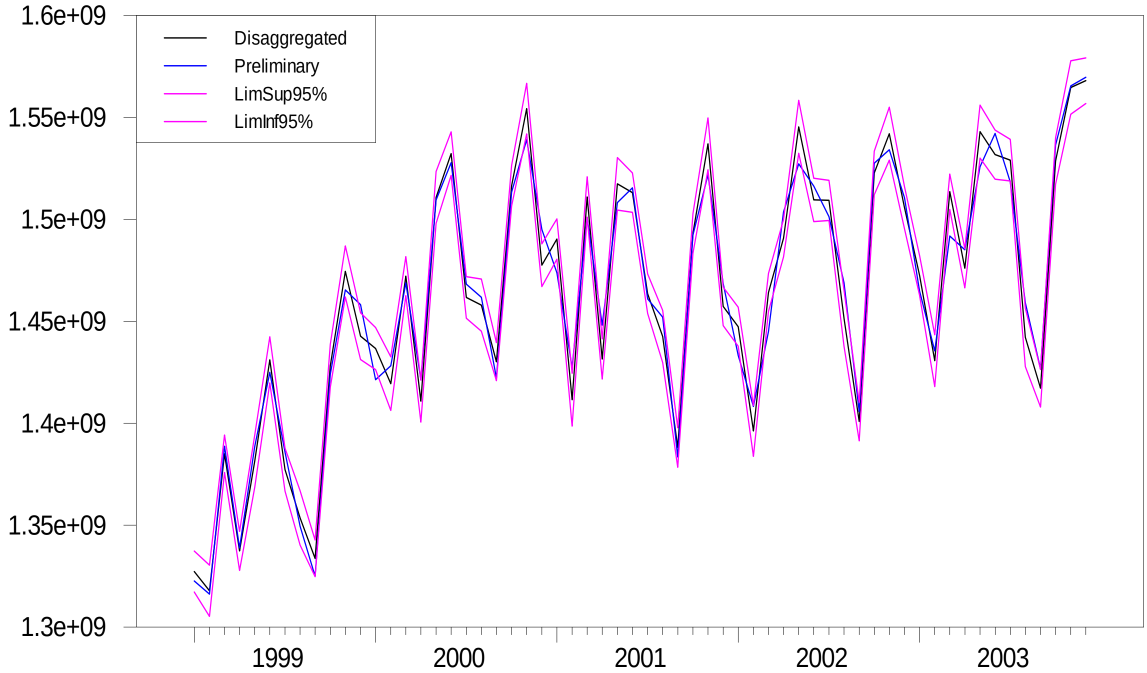

Figure 4: Monthly disaggregation results of Mexico's GDP.

The present application of the disaggregation procedure included calculation of the compatibility statistic $K^*_{calc} = 48.17$ which, when compared with a Chi-square distribution with 44 degrees of freedom leads to the conclusion that we should not reject the compatibility hypothesis at the 31% significance level. Now, by looking at Tables A1, A2 and A3 in the Appendix, we can observe that the old disaggregated series (whose preliminary series was obtained with IGAE as the only related variable) has basically the same dynamic behavior as the new disaggregated series. They differ from each other mainly in the first quarter of every year and in the most recent years. Besides, the old disaggregated series tends to get closer to the new preliminary estimates than the new disaggregated series. This fact points out in the direction that the new preliminary series works almost as well as the old disaggregated series, except for not fulfilling the temporal restrictions. Moreover, it should be clear that the new disaggregated series is an improvement over the old one by taking into account the dynamic structure of the differences better, besides of providing a statistically more efficient estimate of the true series.

The disaggregation method employed can be used to obtain the disaggregated figures in a recursive manner, as indicated in Guerrero (2003). Thus, the recursive application of this method yielded the results shown in Table 6 for quarters 2004:I through 2004:III. This table also shows the K* recursive statistics for testing compatibility between the preliminary monthly estimates and the corresponding GDP datum of the quarter. In this application, the statistic lends ample support to the preliminary estimates for the first and third quarters, but it may cast some doubt about the appropriateness of the estimates for the second quarter. This result is considered as an indication that the feasible solution is only a provisional way to tackle the problem, but a more comprehensive solution is still required.

Table 6: Monthly (recursively) disaggregated GDP for year 2004.

Month/ Quarter	IGAE	Preliminary estimate	Disaggregated series	K^*_{calc} (p-value)
Jan	129.0	1 499 054 000	1 501 035 976	
Feb	127.8	1 485 454 000	1 485 611 970	
Mar	135.6	1 574 833 000	1 594 962 542	
2004:I	---	---	1 527 203 496	0.70 (0.404)
Apr	133.5	1 543 094 000	1 535 527 091	
May	136.2	1 573 786 000	1 583 376 533	
Jun	140.3	1 621 484 000	1 611 407 339	
2004:II	---	---	1 576 770 321	3.18 (0.075)
Jul	136.6	1 568 858 000	1 577 605 344	
Aug	133.8	1 536 984 000	1 524 782 160	
Sep	129.7	1 489 481 000	1 480 149 282	
2004:III	---	---	1 527 512 262	1.10 (0.294)

6 Conclusions

This study has shown the importance of working with an appropriate preliminary estimate of the true unobserved high-frequency series. The case study has lead to the finding that IGAE, the monthly index usually employed as proxy of Mexico's GDP, needs (i) to extend its coverage of some important economic activities and (ii) it has to be updated, because its base year no longer reflects the country's current economic conditions. In the meantime, a feasible and simple solution was applied to produce a statistically reasonable preliminary estimate.

Such a solution is very simple indeed, since it consists of using seasonal indicators as related variables, together with IGAE, in the linear equation used to generate the preliminary series. The basic lesson that we can learn from this work is that we should validate the adequacy of the preliminary estimate empirically, not just based on subject matter considerations. To do that we can employ a compatibility statistic, as the one used here, to verify the underlying statistical assumptions of the procedure employed to disaggregate the series. Therefore, when using a disaggregation method routinely, we should maintain a routine monitoring scheme as well, in order to validate the assumptions with the data at hand.

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Appendix. Monthly disaggregation results of Mexico's real GDP (Millions of pesos at 1993 value)

Table A1: Monthly series. January 1993 - December 1995.

Year	Month	IGAE	Preliminary	Disaggregated (new)	Standard error	Disaggregated (old)
1993						
	Jan	97.57	1 135 285 514	1 124 937 756	5 126 939	1 126 103 360
	Feb	97.95	1 139 771 315	1 132 373 675	5 556 370	1 130 583 163
	Mar	102.95	1 197 366 389	1 187 476 318	5 101 920	1 188 101 225
	Apr	97.74	1 131 187 093	1 136 188 418	5 078 596	1 134 022 434
	May	100.23	1 159 866 591	1 161 430 662	5 549 380	1 162 663 585
	Jun	101.75	1 177 401 931	1 179 242 416	5 078 998	1 180 175 477
	Jul	100.17	1 149 713 879	1 152 103 430	5 076 567	1 152 111 791
	Aug	95.54	1 096 351 432	1 098 394 816	5 580 783	1 098 820 697
	Sep	94.90	1 088 904 529	1 091 817 993	5 071 225	1 091 383 751
	Oct	101.80	1 173 898 177	1 175 813 365	5 066 186	1 176 998 919
	Nov	103.45	1 192 953 905	1 195 972 562	5 579 044	1 196 029 166
	Dec	105.93	1 221 555 675	1 225 834 850	5 063 322	1 224 592 693
1994						
	Jan	100.35	1 167 446 843	1 159 989 780	5 107 221	1 161 050 719
	Feb	99.74	1 160 414 617	1 156 114 639	5 824 143	1 154 027 896
	Mar	104.62	1 216 609 300	1 209 121 637	5 075 635	1 210 147 440
	Apr	103.18	1 193 837 811	1 198 903 602	5 053 636	1 196 580 335
	May	104.81	1 212 731 104	1 213 933 449	5 816 129	1 215 448 365
	Jun	108.77	1 258 401 534	1 260 249 378	5 053 467	1 261 057 729
	Jul	101.67	1 167 041 789	1 169 646 141	5 050 216	1 170 283 913
	Aug	102.39	1 175 263 666	1 177 632 278	5 868 143	1 178 494 797
	Sep	99.71	1 144 341 247	1 149 114 016	5 038 995	1 147 613 724
	Oct	108.18	1 247 412 130	1 246 060 975	5 024 874	1 247 340 777
	Nov	110.60	1 275 307 456	1 276 680 070	5 865 937	1 275 198 805
	Dec	109.52	1 262 939 856	1 262 646 277	5 017 345	1 262 847 741
1995						
	Jan	103.38	1 202 279 723	1 201 611 468	5 035 340	1 200 710 951
	Feb	97.90	1 139 166 566	1 137 335 492	5 989 538	1 137 682 185
	Mar	100.82	1 172 756 484	1 170 673 366	5 002 047	1 171 227 189
	Apr	93.81	1 085 839 060	1 085 367 977	4 981 138	1 085 949 592
	May	96.22	1 113 623 305	1 112 379 140	5 987 341	1 113 696 686
	Jun	98.13	1 135 664 818	1 137 607 889	4 994 502	1 135 708 727
	Jul	93.21	1 069 390 246	1 067 124 956	5 045 263	1 070 953 884
	Aug	95.13	1 091 560 654	1 091 998 366	6 022 506	1 093 094 647
	Sep	91.51	1 049 810 618	1 056 325 646	5 029 279	1 051 400 437
	Oct	98.95	1 141 069 312	1 136 564 294	5 042 536	1 139 957 851
	Nov	101.93	1 175 327 882	1 177 237 978	5 992 160	1 174 170 612
	Dec	104.56	1 205 721 420	1 204 849 701	4 993 786	1 204 523 511

Table A2: Monthly series. January 1996 - December 1998.

Year	Month	IGAE	Preliminary	Disaggregated (new)	Standard error	Disaggregated (old)
1996						
	Jan	100.58	1 170 078 675	1 165 986 077	5 113 854	1 166 996 927
	Feb	100.64	1 170 683 322	1 164 694 120	5 970 242	1 167 600 765
	Mar	101.48	1 180 385 894	1 181 207 860	4 987 850	1 177 290 364
	Apr	99.01	1 145 775 693	1 146 711 282	4 727 278	1 147 025 709
	May	103.15	1 193 490 949	1 200 813 398	6 324 808	1 194 677 164
	Jun	104.45	1 208 530 854	1 203 875 152	5 508 038	1 209 696 959
	Jul	102.66	1 178 396 604	1 189 496 080	5 650 225	1 178 221 790
	Aug	100.35	1 151 746 791	1 150 077 781	6 353 657	1 151 607 612
	Sep	97.14	1 114 803 352	1 104 969 112	4 881 780	1 114 713 570
	Oct	106.91	1 232 803 175	1 231 552 691	5 067 368	1 234 190 377
	Nov	109.92	1 267 548 826	1 258 458 915	6 152 979	1 268 889 570
	Dec	109.67	1 264 601 621	1 279 014 647	5 519 228	1 265 946 305
1997						
	Jan	106.14	1 234 144 800	1 218 883 653	5 300 056	1 231 706 029
	Feb	104.99	1 220 901 943	1 230 818 223	6 447 107	1 218 480 878
	Mar	105.39	1 225 561 755	1 223 619 492	5 022 304	1 223 134 460
	Apr	109.20	1 263 309 183	1 272 630 658	5 121 387	1 264 413 369
	May	111.00	1 284 023 356	1 281 128 223	6 276 292	1 285 099 846
	Jun	112.26	1 298 610 722	1 295 422 040	5 042 393	1 299 667 705
	Jul	109.53	1 257 610 208	1 262 009 431	5 089 297	1 257 879 273
	Aug	107.05	1 228 987 904	1 232 561 227	6 252 699	1 229 295 239
	Sep	105.82	1 214 894 615	1 207 824 648	5 021 630	1 215 220 795
	Oct	116.55	1 343 948 388	1 345 501 898	4 964 145	1 344 857 788
	Nov	116.25	1 340 512 273	1 331 724 084	6 229 559	1 341 426 268
	Dec	115.60	1 333 054 870	1 343 036 911	4 925 831	1 333 978 837
1998						
	Jan	111.61	1 297 240 685	1 289 191 095	4 918 127	1 296 829 205
	Feb	111.59	1 297 049 792	1 308 938 243	6 278 908	1 296 638 568
	Mar	116.74	1 356 464 526	1 351 312 291	4 938 412	1 355 973 856
	Apr	112.70	1 303 616 681	1 310 044 023	4 802 777	1 302 716 819
	May	116.10	1 342 904 012	1 331 981 920	6 516 223	1 341 951 617
	Jun	118.57	1 371 309 673	1 372 961 788	5 204 579	1 370 319 296
	Jul	115.48	1 326 281 330	1 322 580 837	5 074 331	1 329 023 303
	Aug	112.28	1 289 324 549	1 297 793 781	6 677 778	1 292 115 937
	Sep	110.89	1 273 267 508	1 276 844 988	4 815 846	1 276 080 367
	Oct	118.60	1 367 583 586	1 370 752 251	5 260 249	1 366 474 595
	Nov	119.79	1 381 393 057	1 379 943 538	6 424 473	1 380 265 602
	Dec	119.96	1 383 288 337	1 378 202 753	5 324 324	1 382 158 346

Table A3: Monthly series. January 1999 - December 2001.

Year	Month	IGAE	Preliminary	Disaggregated (new)	Standard error	Disaggregated (old)
1999						
	Jan	113.81	1 322 644 185	1 327 253 257	5 124 814	1 323 503 446
	Feb	113.26	1 316 218 709	1 317 837 058	6 404 775	1 317 086 562
	Mar	119.54	1 388 756 198	1 385 026 752	4 691 957	1 389 527 059
	Apr	115.76	1 338 906 531	1 337 385 489	4 896 295	1 338 117 048
	May	120.07	1 388 605 843	1 381 467 445	6 533 685	1 387 749 905
	Jun	123.22	1 424 966 950	1 431 076 412	5 770 061	1 424 062 392
	Jul	120.65	1 385 925 958	1 377 231 248	5 368 183	1 387 249 985
	Aug	117.50	1 349 534 602	1 353 628 778	6 799 680	1 350 907 288
	Sep	115.37	1 325 035 131	1 333 737 825	4 576 559	1 326 440 577
	Oct	123.49	1 424 024 388	1 428 203 926	5 383 701	1 423 310 570
	Nov	127.08	1 465 435 149	1 474 468 761	6 381 614	1 464 665 959
	Dec	126.45	1 458 199 381	1 442 743 709	5 846 519	1 457 439 868
2000						
	Jan	122.37	1 421 344 481	1 436 666 489	5 268 248	1 424 642 333
	Feb	122.97	1 428 220 645	1 419 438 882	6 704 646	1 431 509 304
	Mar	126.49	1 468 854 953	1 472 135 545	4 873 461	1 472 089 279
	Apr	122.91	1 421 364 747	1 410 819 841	5 240 807	1 419 868 180
	May	130.55	1 509 507 797	1 510 788 239	6 487 557	1 507 893 373
	Jun	132.13	1 527 772 603	1 532 287 230	5 441 132	1 526 133 756
	Jul	127.79	1 468 231 820	1 461 687 048	5 210 973	1 467 422 056
	Aug	127.23	1 461 767 583	1 457 944 812	6 517 271	1 460 966 462
	Sep	123.80	1 422 203 651	1 430 212 089	4 765 346	1 421 455 432
	Oct	131.24	1 513 366 952	1 516 515 677	5 065 189	1 513 665 319
	Nov	133.47	1 539 139 248	1 554 338 373	6 338 863	1 539 403 155
	Dec	129.65	1 495 034 016	1 477 571 320	5 350 559	1 495 356 897
2001						
	Jan	126.92	1 473 787 434	1 490 370 048	5 044 882	1 477 451 936
	Feb	122.89	1 427 357 196	1 411 570 675	6 597 928	1 431 083 780
	Mar	129.26	1 500 825 382	1 511 048 723	5 046 663	1 504 453 730
	Apr	125.23	1 448 165 767	1 431 526 623	5 021 689	1 444 944 876
	May	130.44	1 508 296 081	1 517 457 603	6 560 247	1 504 994 788
	Jun	131.07	1 515 519 761	1 513 164 246	4 928 623	1 512 208 809
	Jul	127.17	1 461 028 940	1 463 565 953	4 955 334	1 460 233 646
	Aug	126.39	1 452 049 588	1 442 559 880	6 528 128	1 451 266 300
	Sep	120.44	1 383 449 436	1 388 131 989	4 942 028	1 382 757 876
	Oct	129.41	1 492 328 713	1 493 099 783	4 948 762	1 493 878 759
	Nov	131.98	1 521 938 194	1 537 076 595	6 473 943	1 523 448 648
	Dec	127.35	1 468 577 233	1 457 310 067	4 818 892	1 470 159 037

Table A4: Monthly series. January 2002 - December 2003.

Year	Month	IGAE	Preliminary	Disaggregated (new)	Standard error	Disaggregated (old)
2002						
	Jan	123.40	1 433 212 085	1 447 330 448	4 879 246	1 440 318 430
	Feb	121.23	1 408 237 992	1 396 263 680	6 369 239	1 415 377 731
	Mar	124.42	1 444 996 958	1 464 189 578	4 748 761	1 452 087 546
	Apr	130.04	1 503 608 440	1 490 766 337	4 693 164	1 503 141 495
	May	132.09	1 527 269 698	1 545 409 484	6 622 230	1 526 771 115
	Jun	131.14	1 516 323 056	1 509 575 898	5 444 066	1 515 839 110
	Jul	130.67	1 501 374 423	1 509 358 663	5 026 794	1 496 548 262
	Aug	127.85	1 468 894 057	1 451 497 197	7 010 289	1 464 111 327
	Sep	122.37	1 405 768 418	1 400 873 822	4 821 140	1 401 070 094
	Oct	132.47	1 527 583 662	1 522 784 453	5 441 160	1 526 975 723
	Nov	133.04	1 534 162 606	1 542 038 988	6 639 414	1 533 545 870
	Dec	131.00	1 510 625 085	1 505 737 972	5 254 919	1 510 039 821
2003						
	Jan	126.15	1 464 943 286	1 472 611 135	4 982 166	1 473 158 140
	Feb	123.60	1 435 463 879	1 430 702 038	6 485 971	1 443 718 150
	Mar	128.48	1 491 820 695	1 513 562 728	4 418 514	1 499 999 611
	Apr	128.44	1 485 133 233	1 476 066 149	4 916 781	1 484 455 227
	May	131.96	1 525 792 311	1 543 065 229	6 617 713	1 525 059 938
	Jun	133.38	1 542 148 208	1 531 777 752	6 143 245	1 541 393 965
	Jul	132.14	1 518 354 234	1 529 054 155	5 212 510	1 513 023 544
	Aug	127.00	1 459 112 085	1 442 150 149	7 304 499	1 453 860 610
	Sep	124.19	1 426 705 623	1 417 177 330	4 717 787	1 421 497 479
	Oct	133.25	1 536 529 924	1 528 666 197	5 818 219	1 533 090 034
	Nov	135.76	1 565 512 407	1 564 676 416	6 682 371	1 562 033 765
	Dec	136.12	1 569 727 780	1 568 024 687	5 693 403	1 566 243 500