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Stability Analysis in ARMA and Unobserved Component Models

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Stability Analysis in ARMA and unobserved component models

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ABSTRACT: The chronology of cycle phases may be obtained from the estimation of the cyclical components in Unobserved Component Models (UCM). When instabilities are present in the coefficients of the cycle equation, the chronology obtained may be spurious. These instabilities will be transmitted to the coefficients of the ARMA reduced form models of the stationary observable variables under study. Therefore, the ARMA model, or the cyclical equation of the UCM, should be tested for instabilities before any use of the estimated cyclical component or the ARMA model is made. In this paper, we test for parameter instability in the ARMA and Unobserved Component Models of the Gross Domestic Product (GDP) and the Industrial Production Index (IPI) of several European countries, making use of a recursive Wald type statistic applicable to linear models. To do so, for the case of ARMA models we first linearise the model and, for the case of UCM, we first obtain the Kalman filtered cyclical component. After these initial steps, the recursive statistic may be applied. The empirical size and the power of the resulting statistic are presented for simple models. The final results show that the null hypothesis of constant coefficients cannot be rejected in most of the models studied. An important exception is that of the IPI series computed for the Eurozone. Slight differences in the implied dates of the business cycle are found in the cyclical components uncorrected and corrected for parameter instability.

Keywords: Recursive Wald statistic, ARMA models, Trend Cycle Decomposition, Parameter Instability

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1. Introduction

Since the definition of business cycle given by Burns and Mitchell (1946), many analytical approaches have been used to detect, and eventually to date, the different phases of the business cycle. Some very popular rules are roughly coherent with Burns and Mitchell's definition (e.g.: two consecutive declines in quarterly GDP to locate a recession) but the complexity of business cycle dynamics and the desire for a richer phase characterization compel to design more elaborate tools. With this purpose, time domain and frequency domain filters are widely used to extract unobserved components useful for business cycle analysis. The risk of spurious components due to the automatic implementation of these methodologies has been extensively analyzed (Nelson and Kang, 1981; Nelson, 1988; Harvey and Jaeger, 1993; Cogley and Nason, 1995). Additionally, the literature has noted the subjectivity in trend cycle decompositions due to the *a priori* considerations related, for example, to the degree of smoothness of the trend component or the interval of frequencies selected as specifically cyclical (García Ferrer and Queralt, 1998).

Further considerations arise when trend cycle decomposition involves the estimation of parameters. In such case, parameter instability could lead to several problems mentioned in business cycle analysis. In this context, testing for parameter instability can be necessary to avoid spurious components or inappropriate business cycle chronologies. Stock and Watson (1996) studied the stability of univariate and bivariate autoregressive models associated to macroeconomic US time series and found instabilities in many of the models considered. So, it could happen that models for European GDPs and IPIs may have also non constant parameters.

When UCM are employed, the ARMA models directly obtained from the stationary observed variables and those derived from the UCM (i.e.: the constrained reduced form) should be compatible (Nelson, 1988; Watson, 1986; and Harvey, 1989). In practice, however, the unconstrained ARMA models seem to differ significantly from the constrained ones. A possible explanation for these differences could be that the ARMA or the UCM may have non constant parameters. It is well known that if a time series has different autocorrelation functions (acf) in different subsamples, when analysing the full sample, we will obtain an acf which will be a combination of the different acf's. In this case, the identification via the full sample acf will point towards incorrect models that will be a mixing of the different models in each subsample. Therefore, it seems important to test for parameter stability in the context of UCM, specifically in trend-cycle decompositions, as well as for the ARMA models associated to the stationary observed time series. If breaks are present but ignored, the interpretation and the chronology of the cycle phases, as well as the policy implications, may be misleading.

Statistics based on recursive estimations for testing the existence of at least one break in the parameters (with unknown location) are available. Barnejee, Lumsdaine and Stock (1992) focus on linear models, while Andrews (1993) generalizes to non-linear models making use of the GMM estimators, and refers to the optimal properties of the test in comparison to other alternatives.

For non linear models, Andrews' tests may be quite involved and more burdensome than for linear models[1]. Some apparently simple models, like ARMA, are non linear, and therefore the tests devised for linear models to detect if a break has taken place in any of the parameters, are not directly applicable. We will show how a Wald type statistic for testing the existence of at least one break in linear models may be applicable both to ARMA and to UC models by following two step procedures which slightly differ when applied to ARMA and to UC models.

For ARMA models the two step procedure starts from the null hypothesis that the entertained ARMA model is well specified and that their coefficients are constant (del Hoyo and Llorente, 2000). Therefore, using Maximum Likelihood Estimation, consistent estimates of the model parameters can be obtained. Next, by substituting the perturbations by their consistent estimates in the original ARMA model, we can obtain an asymptotic equivalent model, which is linear and allows for the use of the recursive Wald test available for linear models. As a result of the linearisation process, the critical points of the empirical distribution of the derived statistic (under the null of constant coefficients) must be computed for different sample sizes and different location points in the parameter space to know how the empirical critical points differ from the "true" ones. The power of the resulting test will also be studied for some simple cases.

For UC models, the unobserved cyclical components are usually modelled as stationary AR(p) equations. For these models, under the null of constant coefficients, it is also possible to show that if in the equation for the cyclical component, the unobserved component is substituted by its Kalman's filtered counterpart, the resulting equation, apart from some heteroskedasticity, will provide a valid linear equation where we can apply simple recursive tests for parameter instability.

This paper applies a recursive Wald type statistic to detect parameter instabilities in the estimation of the cyclical equation of trend cycle decompositions, and in the implied ARMA models for the UCM. We consider two decomposition approaches, first, the classical trend plus cycle decomposition (e.g. Clark 1987) and second, the cyclical trend decomposition (Harvey, 1985, 1989). The series under study are the GDPs and the IPIs of several European countries.

In the rest of the paper we proceed as follows. Section 2 briefly presents the recursive Wald statistic for testing the existence of at least one break in the parameters of a linear model. Section 3 presents the two step procedures to apply the Wald type statistic to ARMA and UC models. Section 4 deals with the estimated models for several European GDPs and IPIs series. When the recursive test detects substantial parameter instability we model it with intervention analysis. Finally, Section 5 concludes.

[1] For an α symmetric trimming, the number of evaluations to compute the Andrew's statistic for a non-linear model is $T(1-2\alpha)$, with T the sample size.

2. Recursive Wald Test for parameter instability in Linear Models

The distribution of the recursive statistics considering the possibility of at least one break with unknown *a priori* date, while not conventional is known (see Banerjee, Lumsdaine and Stock (1992) for linear models and Andrews (1993) for non linear models). For linear models, it is a simple matter to compute recursive statistics using subsamples of increasing size $1 < t_{min} \leq t \leq t_{max} < T$. The recursive estimates derived from linear models allow an easy computation of the recursive statistics to detect break points. Also, by plotting the recursive estimates of the coefficients, it can be obtained valuable information on the stability of the coefficients and, in many cases, on the nature of the intervention model to be used to achieve constancy.

Assume that the observations are generated by the following model:

$$y_t = X_{t-1}ß_t + e_t \quad ; \quad t = 2,3,\cdots T$$

and also that the model is well specified with $ß_t = ß$ a *(kx1)* vector of constants. The perturbations[2] (e_t) are assumed to be a martingale difference sequence with respect to the σ-fields generated by $\{\varepsilon_{t-1}, X_{t-1}, \varepsilon_{t-2}, X_{t-2}, \cdots\}$, where X_t is a *(1xk)* vector of regressors. The regressors are constant and/or I(0) with $E(X_t^{'}X_t) = \Sigma_X$. Usually, by defining $? = \frac{t}{T}$, $0 \leq \lambda_{min} \leq \lambda \leq \lambda_{max} \leq 1$, we limit the analysis of distributions to the interval $[0,1]$. It is also assumed that $V_T(\lambda) = T^{-1}\sum_{t=2}^{[T\lambda]} X_t^{'}X_t \xrightarrow{P} \lambda\Sigma_X$ uniformly in λ for $\lambda \in [0,1]$; $E(\varepsilon_t^2) = \sigma^2 \ \forall\, t$; and $v_T(\lambda) = \frac{1}{\sqrt{T}}\sum_{t=2}^{[T\lambda]} X_t^{'}\varepsilon_t \Rightarrow \sigma\Sigma_X^{1/2}W_k(\lambda)$ where $\Sigma_X = \Sigma_X^{1/2}\Sigma_X^{1/2}$, $W_k(\lambda)$ is a *k*-dimensional vector of independent Wiener or Brownian motion process, and $\Rightarrow$ denotes weak convergence on $D[0,1]$. X_{t-1} can include lagged dependent variables as long as they are I(0) under the null (see Stock (1994)). Finally, $[\bullet]$ is the integer part of the value inside brackets. The recursive OLS coefficients can be written as random elements of $D[0,1]$:

$$\beta(\lambda) = \left(\sum_{t=2}^{[T\lambda]} X_{t-1}^{'}X_{t-1}\right)^{-1}\left(\sum_{t=2}^{[T\lambda]} X_{t-1}^{'}y_t\right) \ ; \ 0 \leq \lambda_{min} \leq \lambda_{max} \leq 1 \qquad (2.1)$$

Notice that $\beta(1)$ is the vector with the full sample OLS estimates.

The Wald type statistic used in this paper to test $H_0: R\beta(\lambda) = r$, where R is a non stochastic matrix of rank *m*, $r = R\,\beta(1)$, and *m* the number of coefficients to be tested, is:

[2] ε_t can be conditionally (on lagged ε_t and X_t) homoskedastic and the results do not change.

$$F_T(\lambda)=\frac{(R\beta(\lambda)-r)'\left(R\left(\sum_{2}^{[T\lambda_{max}]}X'_{t-1}X_{t-1}\right)^{-1}R'\right)^{-1}(R\beta(\lambda)-r)}{m\sigma^2(\lambda)} \qquad (2.2)$$

where $\sigma^2(\lambda)$ is the recursive estimate of the residual variance.

The asymptotic behaviour of this statistic is derived applying the Functional Central Limit Theorem and the Central Mapping Theorem. The form of the final distributions will depend on R and r. In what follows, the subscripts and superscripts refer to the particular null hypothesis to be tested. In particular, if we want to test for stability along the sample with respect to $m \leq k$ of the final estimates $\beta(1)$, we represent the statistic by $F(\lambda)^m_{\beta(1)}$. Following Stock (1994), it may be shown that:

$$F_T(\lambda)^m_{\beta(1)} \Rightarrow \frac{B_k(\lambda)'\,\Sigma_X^{-1/2}R'\left(R\Sigma_X^{-1}R'\right)^{-1}R\Sigma_X^{-1/2}B_k(\lambda)}{m\,\lambda} \qquad (2.3)$$

where $B_k(\lambda)=W_k(\lambda)-\lambda W_k(1)$ is a k-dimensional Brownian bridge. Therefore, for testing all the coefficients recursively along the sample against the full sample estimators, i.e. $R=I_k$ and $r=\beta(1)$, we will obtain that:

$$F_T(\lambda)^k_{\beta(1)} \Rightarrow \frac{B_k(\lambda)'\,B_k(\lambda)}{k\,\lambda} \qquad (2.4)$$

By defining $\tilde{F}_T(\lambda)^k_{\beta(1)}=\frac{k}{(1-\lambda)}F_T(\lambda)^k_{\beta(1)} \Rightarrow \frac{B_k(\lambda)'\,B_k(\lambda)}{\lambda(1-\lambda)}$, the Andrews (1993) statistic is obtained. The statistic to be tabulated is:

$$\max_{\lambda_{min}\leq\lambda\leq\lambda_{max}} F_T(\lambda)^k_{\beta(1)} \Rightarrow \sup_{\lambda_{min}\leq\lambda\leq\lambda_{max}} \frac{B_k(\lambda)'\,B_k(\lambda)}{k\lambda} \qquad (2.5)$$

It is easy to show that for the case of testing only $m \leq k$ coefficients, the statistic is:

$$\max_{\lambda_{min}\leq\lambda\leq\lambda_{max}} F_T(\lambda)^m_{\beta(1)} \Rightarrow \sup_{\lambda_{min}\leq\lambda\leq\lambda_{max}} \frac{B_m(\lambda)'\,B_m(\lambda)}{m\,\lambda} \qquad (2.6)$$

In Table 2.1, the critical values for the size of the statistic are presented. They have been computed using a 15 percent trimming as it is usual in this kind of work. The approximate critical values for this sequential statistic, that we call $\sup\ F(\lambda)^m_{\beta(1)}$, were computed as the sup values of the functionals of Brownian processes using 10000 replications with T=3600. These critical values are directly applicable for asymptotic sample sizes. For other sample sizes the critical values should be adjusted.

Table 2.1: **Critical values for the statistic** $\sup\ F(\lambda)^k_{\beta(1)}$

Percentile	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
0.75	2.8088	2.2295	1.9666	1.7759	1.6550	1.5695	1.5045	1.4613	1.4172	1.3967
0.80	3.1724	2.4475	2.1194	1.9199	1.7796	1.6717	1.5942	1.5407	1.5004	1.4611
0.85	3.5573	2.7013	2.3164	2.0755	1.9261	1.7984	1.7126	1.6537	1.5882	1.5469
0.90	4.4013	3.1631	2.6265	2.3128	2.1511	1.9921	1.8881	1.8210	1.7485	1.6998
0.95	5.5368	3.7825	3.0944	2.6896	2.4585	2.2710	2.1316	2.0234	1.9591	1.9019
0.99	7.9646	5.1483	4.2098	3.6146	3.1835	2.9418	2.6524	2.5430	2.3886	2.2765

Entries are the sup values of the functionals of Brownian processes. All the critical values have been computed by 10000 Monte Carlo replications and T=3600. Sup $F(\lambda)^k_{\beta(1)}$ is the sup Wald type statistic to test the recursive estimations against the full sample estimations. The recursive statistic has been computed with symmetric 15% trimming.

3. A Sequential Test for parameter instability in ARMA and UC models

3.1. ARMA models

The application of (2.5) or (2.6) to AR(p) models is straightforward, but for ARMA models the moving average part induces nonlinearities that prevent the direct use of this sequential test. An easy solution[3] to this problem makes use of a two-step procedure to obtain linearity. The first stage assumes the null hypothesis of correct specification of the model and, in particular, that the model coefficients are constant. Under this null, consistent estimates of the model parameters and perturbations can be obtained. Next, a pseudolinear regression model is obtained[4] by substituting the unknown perturbations by their consistent estimates. Finally, once the model is linear, the recursive Wald test for the detection of breaks may be applied.

Without loss of generality, let y_t follow an ARMA(1,1) model:

$$y_t = \mu + \phi_1 y_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \qquad (3.1)$$

If e_t is a consistent estimation of ε_t, and $\nu_t = \varepsilon_t - e_t$, then:

$$y_t = \mu + \phi_1 y_{t-1} + \theta_1 e_{t-1} + (\varepsilon_t + \theta_1 \nu_{t-1}) = \mu + \phi_1 y_{t-1} + \theta_1 e_{t-1} + (\varepsilon_t + o_p(1))$$

since $v_t = o_p(1)$, then:

$$y_t = \mu + \phi_1 y_{t-1} + \theta_1 e_{t-1} + \varepsilon_t \qquad (3.2)$$

Model (3.2) is linear and asymptotically equivalent to model (3.1). Moreover, it can be shown (Del Hoyo, Llorente and Rivero 2003), that the asymptotic distribution of the statistic (2.5) applied to model (3.2), where the unknown perturbation ε_{t-1} has been substituted by its consistent estimation e_{t-1}, converges to the same distribution of the statistic (2.5) applied to model (3.1). This convergence to the same asymptotic distribution is a particular case of a more general result valid for ARMA models in which the lagged unknown perturbations are substituted by consistent estimates. In proving this result the same hypothesis assumed in obtaining (2.4) plus an additional one are stated. This new assumption states that if T is the full sample size, then for each T, the estimated parameters are uniformly consistent in T. That is, if X_{t-1}^T is the consistent estimate of the regressors X_{t-1}, given the sample size T, then:

$$\max_{t=2,\cdots T} \left| X_{t-1}^T - X_{t-1} \right| \xrightarrow[T\to\infty]{P} 0$$

Under these conditions, we can apply the recursive Wald type test to (3.2).

However, using the recursive test in (3.2) may distort the size and power of the test. In particular, while the critical points will coincide asymptotically, the approximation to the "true" ones in Table 2.1 will depend on the particular location in the parameter space as well as the actual sample size considered in estimating the

[3] Other solutions also applicable to more complex models, i.e. Transfer Function models, may be seen in Del Hoyo and Llorente, 2000.

[4] We have estimated the ARMA models using the armax.m sentence from Matlab 6.5p and obtained the empirical distributions of (2.6) under the null and alternative hypothesis to compute the size and power presented in Tables B.1 to B.8.

coefficients[5]. To illustrate this, we can compare the critical points of the sup test when applied for ARMA models. To facilitate comparisons we have simulated AR(1), MA(1) and ARMA(1,1) models, whose size and power are presented in Tables B.1 to B.8. As can be seen, the size of the two-step statistic is quite good but the power depends on the sample size as well as on the distance between the null and the alternative hypothesis.

3.2. UC models

In unobserved component models a two-step procedure may also be used to decide whether the parameters are constant in a similar way as in ARMA models. The procedure first estimates the model parameters consistently, under the null hypothesis of no structural change using the whole sample. Then, by applying the Kalman filter estimable linear equations for the cyclical components are obtained. Once they have been written as linear estimable equations, it will be possible to test for stability of the parameters conditional on the first stage consistent estimates.

We employ two UC models: the Trend plus Cycle Model and the Cyclical Trend Model. The Trend plus Cycle model decomposes y_t (commonly the logarithmic transformed of the series in levels) as:

$$y_t=T_t+C_t+e_t \qquad (3.3a)$$

where T_t is the trend component, C_t is the cyclical component, and e_t is the noise series. A commonly used decomposition consists on a trend "*viewed as a nonstationary stochastic process, generally a random walk with drift*", and a cycle viewed as "*a stationary process, generally an autoregression*" (Nelson, 1988). Following this specification:

$$T_t=\mu+ T_{t-1}+\varepsilon_{1t} \qquad (3.3b)$$

$$\phi_p(L)C_t=\varepsilon_{2t} \qquad (3.3c)$$

where μ is the drift[6] and $\varepsilon_{1t} \sim$ iid $N(0,\sigma_1^2)$. C_t is the autoregressive cyclical component, which follows $\phi_p(L)C_t=\varepsilon_{2t}$, with $\varepsilon_{2t} \sim$ iid $N(0,\sigma_2^2)$ and the roots of $\phi_p(L)$ lying outside the unit circle[7]. Additional assumptions in the decomposition (3.1a) to (3.1c) are the following orthogonality conditions between components: $E(e_t\ \varepsilon_{1s})=0$, $E(e_t\ \varepsilon_{2s})=0$, and $E(\varepsilon_{1t}\varepsilon_{2s})=0$ for all pair (t,s) [8].

The specification (3.3a) to (3.3c) can be modified to allow for a non-constant drift (Clark, 1987; Harvey, 1985; Young, 1994). For example, specifying a random walk for the drift, we obtain the local linear trend model:

$$T_t=\mu_t+T_{t-1}+\varepsilon_{1t} \qquad (3.3b')$$

$$\mu_t =\mu_{t-1} + \varepsilon_{3t} \qquad (3.3d)$$

where $\varepsilon_{3t} \sim$ iid $N(0,\sigma_3^2)$ and it is uncorrelated with e_t, ε_{1t} and ε_{2t}.

---

[5] When the coefficients are close to the non-stationary region or the sample size is not very large, better size and power may be obtained if more efficient initial conditions are used to estimate the perturbations, i.e. backforecasting.

[6] In some models we will include this term in the cyclical equation

[7] The method may be extended to the autoregressive coefficients of a stationary ARMA model.

[8] Orthogonality restrictions are also necessary for identification (Nelson, 1988; Harvey, 1989).

In empirical estimations (Harvey 1985, Clark, 1987) it is often found that σ_3^2 is very small, reducing the random walk drift component to a constant drift component. The decomposition (3.3a) to (3.3c) is generally considered as the standard Trend plus Cycle decomposition[9].

The second decomposition we consider is the Cyclical Trend model (Harvey, 1985). The main difference with respect to the Trend plus Cycle decomposition is that the trend component is assumed to emerge from the accumulation of the cyclical variation, so in (3.3a) the cyclical component must be eliminated. Consequently, in the Cyclical Trend decomposition we have that

$$y_t = T_t^* + e_t \qquad (3.4a)$$

$$T_t^* = \mu + T_{t-1}^* + C_{t-1}^* + e_{1t} \qquad (3.4b)$$

$$\psi_{p*}(L)C_t^* = \varepsilon_{2t}^* \qquad (3.4c)$$

where we employ the asterisk to denote different components with respect to the Trend plus Cycle decomposition[10]. Stationarity conditions[11] on $\psi_{p*}(L)$ and orthogonality restrictions also apply.

In both decompositions, a smooth trend component is obtained by assuming that $\sigma_1^2 = 0$. In the estimated models, we impose this restriction given our interest on cyclical behaviour. In the Trend plus Cycle model this restriction implies a linear deterministic trend.

The State Space representations of the Trend plus Cycle and the Cyclical Trend decompositions allow Maximum Likelihood Estimation of the parameters and the estimation of the filtered components $T_{t/t}$ and $C_{t/t}$. The smoothed trend and cycle components $T_{t/T}$ and $C_{t/T}$ are obtained conditional on full sample information by means of a Fixed Interval Smoothing algorithm. The smoothed components may be used to date business cycle phases, as we shall do later.

The State Space form (Hamilton, 1994) is

$$\begin{aligned} \xi_t &= \delta + F\xi_{t-1} + \varepsilon_t \\ y_t &= H\xi_t + e_t \end{aligned} \qquad (3.5)$$

[9] We have added the noise term e_t for comparative purposes with the Cyclical Trend model that we also consider. In our empirical applications, we have also tried a random walk drift component, obtaining in most cases that σ_3^2=0. When this was not the case (in 3 of the 13 series considered), the drift component μ_t followed a cyclical pattern and, instead, the estimated cyclical parameters of $\phi_p(L)$ were nearly zero.

[10] To simplify notation, we assume the series noise e_t, the trend noise ε_{1t}, and the drift μ remain the same in both decompositions, (3.3) and (3.4).

[11] In Trend plus Cycle decompositions stationary restrictions must be imposed on $\phi_p(L)$ by bounding the parameter search space. In its absence, the search algorithm tends to violate the stationarity conditions. When imposing these restrictions the estimated roots are close to unity.

Without loss of generality and for notational convenience, let us assume $\psi_{p*}(L) = \phi_p(L)$ and p=2. Then, in the Trend plus Cycle decomposition $\xi_t = \begin{bmatrix} T_t \\ C_t \\ C_{t-1} \end{bmatrix}$, $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix}$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ 0 \end{bmatrix}$, and $H = [1 \quad 1 \quad 0]$. While in the Cyclical Trend decomposition, $\xi_t = \begin{bmatrix} T_t^* \\ C_{t-1}^* \\ C_{t-2}^* \end{bmatrix}$, $F = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix}$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2,t-1}^* \\ 0 \end{bmatrix}$ and $H = [1 \quad 0 \quad 0]$.

Finally, for both decompositions, $\delta = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$.

Under both decompositions we are interested in analyzing whether the parameters in the cycle equation are constant. If the cyclical component C_t were observable the solution would be straightforward. The problem here is that C_t (or C_{t-1}^* in the Cyclical Trend decomposition) is unobserved and must be estimated firstly. In the Appendix A it is shown a solution by means of the Kalman filter:

$$C_{t|t} = \phi_1 C_{t-1|t-1} + \phi_2 C_{t-2|t-2} + s_C k_t \eta_{t|t-1} \qquad (3.6)$$

where the unobserved cycle component C_t is substituted by its filtered estimation $C_{t|t}$; $s_C = [0 \quad 1 \quad 0]$ is a selection vector; k_t is related to the Kalman gain and is defined in the Appendix A; and $\eta_{t|t-1} = y_t - y_{t|t-1}$ is the prediction error. In Appendix A, it is also shown that the error term $\zeta_t = s_C k_t \eta_{t|t-1}$ is heteroskedastic but uncorrelated. So the only remaining problem to obtain efficient OLS estimations is to correct the heteroskedasticity. To do so, if $\sigma^2_{\zeta_t}$ is the variance of ζ_t, define:

$$\tilde{C}_{t/t} = \sigma^{-1}_{\zeta_t} C_{t/t}$$

and the final equation to be estimated recursively turns out to be:

$$\tilde{C}_{t/t} = \phi_1 \tilde{C}_{t-1|t-1} + \phi_2 \tilde{C}_{t-2|t-2} + \omega_t \qquad (3.7)$$

with $\omega_t = \sigma^{-1}_{\zeta_t} \zeta_t$. This final equation is ready to be recursively estimated and tested for the existence of at least one break in the parameters along the sample. This is so because it can be shown (Del Hoyo, Llorente y Rivero, 2003) that the recursive statistics (2.5) or (2.6) applied to (3.3c) or (3.4c), with p=2, and to (3.7) have the same asymptotic distributions. This result can be proved assuming that we can obtain consistent estimates, uniformly in T, for the model parameters $(\mu, \phi_1, \phi_2, \sigma_e^2, \sigma_1^2, \sigma_2^2)$ and assuming that the largest eigenvalue of the transition matrix F, is unity; then, if $\tilde{C}^T_{t|t}$ is the filtered cyclical component corrected for heteroskedasticity obtained with the Kalman filter, being T the full sample size:

$$\max_{t=2,\cdots T} \left| \tilde{C}^T_{t|t} - \tilde{C}_{t|t} \right| \xrightarrow[T\to\infty]{P} 0$$

which is a similar situation to the case of estimating the unknown regressors $(\varepsilon_{t-1},\cdots,\varepsilon_{t-p})$.

Again, the size and power of the statistic will depend of the sample size and on the particular region of the parametric space under study, in particular, for parameter values near the non-stationary boundary like the obtained in Trend plus Cycle decomposition.

4. Testing for parameter stability for selected European GDPs and IPIs

In this section we first follow the procedure discussed above to detect parameter instabilities in the cyclical components of several European GDPs and IPIs, and then in their implied ARMA models. We start by estimating UCM for the two decompositions analyzed in the previous section. The estimation results and the sup Wald type statistic (2.6) for parameter stability in the autoregressive polynomials are presented in Tables 4.1 and 4.2. The estimated parameters correspond to a constant drift, the variance of the cyclical component and the autoregressive parameters. In the Appendix C, the graphs of some smoothed cyclical components have been plotted[12].

As indicated, once we dispose of the cyclical filtered components with the heteroskedasticity correction presented in the previous section, it is possible to calculate the $\text{Sup}\ F(\lambda)^{k}_{\beta(1)}$ to test for the stability of the autoregressive parameters. To test for parameter stability, the values presented in Tables 4.1 and 4.2 should not be compared directly with those of Table 2.1 because the latter are asymptotic and we dispose of reduced sample sizes. Instead, under the null hypothesis of no structural change, it is convenient to calculate the empirical distributions of statistic (2.5) for the size and parameters values shown in Tables 4.1 and 4.2. This has been done for 1000 replications in each one of the estimated models. The critical values are tabulated in the Tables B.9 and B.10 of the Appendix.

By comparing the calculated $\text{Sup}\ F(\lambda)^{k}_{\beta(1)}$ for each of the models with the tabulated values, we deduce that we cannot reject the null hypothesis of no structural change in most of the cases[13]. There are few exceptions like the Spanish GDP, and the Eurozone IPI in the Trend plus Cycle model, and, again, Spanish GDP and Spanish and Eurozone IPIs in the Cyclical Trend model[14].

[12] When necessary, intervention analysis was performed modifying appropriately the UCM in order to avoid the possible distorting effects of outliers on stability analysis. Their presence is evident in some of the IPIs.

[13] The graphs of the recursive estimates of the autoregressive coefficients show some instability but not statistically significant according to the $\text{Sup}\ F(\lambda)^{k}_{\beta(1)}$ statistic.

[14] According to Tables B.9 and B.10 the rejection levels of the null hypothesis of no structural change for the five cases mentioned are at least of 10% . The information provided by these tests should be completed with power analysis as we indicated in relation to ARMA models.

If instability in the parameters of the UCM is confirmed, their implied reduced form may also be affected by parameter variation. As mentioned in Section 2, ARMA models are non linear and the sequential tests must be performed after obtaining consistent estimates of both the parameters and the perturbations.

From (3.3) and (3.4), the reduced forms corresponding to the Trend plus Cycle and the Cyclical Trend decompositions can be obtained. When no common factors are present in the AR and MA parts, the reduced form of the first decomposition is a restricted ARIMA(p,1,p+1):

$$\phi_p(L)\Delta y_t = \phi_p(1)\mu + \phi_p(L)\varepsilon_{1t} + \Delta\varepsilon_{2t} + \phi_p(L)\Delta e_t$$

Applying the restriction that the variance of the trend component is zero ($\sigma_1^2 = 0$) this expression simplifies to

$$\phi_p(L)\Delta y_t = \phi_p(1)\mu + \Delta\varepsilon_{2t} + \phi_p(L)\Delta e_t \qquad (4.1)$$

The reduced form of the Cyclical Trend decomposition (3.4) corresponds to the restricted ARIMA(p^*,1,p^*+1)

$$\psi_{p^*}(L)\Delta y_t = \psi_{p^*}(1)\mu + \psi_{p^*}(L)\varepsilon_{1t} + \varepsilon^*_{2,t-1} + \psi_{p^*}(L)\Delta e_t$$

and when $\sigma_1^2 = 0$,

$$\psi_{p^*}(L)\Delta y_t = \psi_{p^*}(1)\mu + \varepsilon^*_{2,t-1} + \psi_{p^*}(L)\Delta e_t \qquad (4.2)$$

The ARMA models of Table 4.3 have been estimated with the series in deviations from their means given our interest in the autoregressive parameters. The estimations have been performed with a general parametric specification roughly coherent with the two UCM analysed. Again, the sup statistics in Table 4.3 must be evaluated by comparing their values with those presented in Table B.11. As for UC models, the null hypothesis of no structural change could not be rejected for most of the cases[15] (except for Spanish GDP and Euro4 GDP). The discrepancies in the rejection of the null hypothesis of parameter stability in the UCM and the compatible ARMA models for the analysed series can be attributed to the low power of the test for small sample sizes and the proximity between the null and the alternative hypothesis as we indicated in Section 3.1.

In absence of a power analysis of the $\text{Sup } F(\lambda)^k_{\beta(1)}$ test in UCM, visual inspection of the recursive parameters may offer complementary information to the sup statistic. We have plotted the recursive estimates of the autoregressive parameters of some of the UCM in the Appendix C (Graph C.5). These graphs also contain additional information about how to model parameter instability. The recession periods[16] in these graphs are shaded and the correspondence between recessions and changes in the autoregressive parameters is clear. Although the parameter variations are not substantial in most of the cases, as indicated by the $\text{Sup } F(\lambda)^k_{\beta(1)}$, some stability gains would be possible if we model the most important changes in a convenient way (i.e. by means of regime switching or with intervention analysis).

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[15] According to Table B.11, the rejection levels are less than 10%.

[16] The criteria used to date recession periods are exposed in the Tables B.12 and B.13 of the Appendix B.

Table 4.1: Estimation results of the Trend plus Cycle model (3.3)

	Sup $F(\lambda)^{k}_{\beta(1)}$	μ	σ_2^2	ϕ_1	ϕ_2	ϕ_3
UK GDP 70.I–03.I	0.8188	0.5539 (0.0140)	0.1351 (0.0530)	1.4263 (0.0524)	-0.1133 (0.0071)	-0.3527 (0.0397)
Germany GDP 70.I–03.I	1.7743	0.5721 (0.0161)	0.2231 (0.0570)	1.2644 (0.0095)	0.0371 (0.0062)	-0.3336 (0.0007)
France GDP 70.I–03.I	0.3021	0.5998 (0.0521)	0.1441 (0.0283)	1.6864 (0.0112)	-0.9591 (0.0119)	0.2727 (0.0005)
Italy GDP 70.I–03.I	0.4537	0.6041 (0.0412)	0.3722 (0.0829)	1.3924 (0.0162)	-0.3985 (0.0099)	
Spain GDP 80.I–03.I	2.7475	0.6499 (0.0403)	0.0659 (0.0192)	1.8874 (0.0253)	-1.0855 (0.0221)	0.1898 (0.0044)
Eurozone GDP 91.I–03.I	0.1543	0.4386 (0.0327)	0.0421 (0.0203)	1.8155 (0.0958)	-0.9547 (0.0912)	0.1138 (0.0115)
Euro-4 GDP(*) 80.I–03.I	1.0560	0.4938 (0.0248)	0.0681 (0.0189)	1.7146 (0.0239)	-0.8669 (0.0150)	0.1367 (0.0024)
UK IPI 70.01-03.04	1.0347	0.1112 (0.0058)	0.0408 (0.0125)	1.8723 (0.0359)	-0.8820 (0.0353)	
Germany IPI 70.01-03.04	1.4030	0.1281 (0.0061)	0.0809 (0.0168)	1.5651 (0.0452)	-0.3797 (0.1108)	-0.1965 (0.0626)
France IPI 72.01-03.04	0.4009	0.1316 (0.0059)	0.0448 (0.0149)	1.8619 (0.0560)	-0.8811 (0.0666)	0.0098 (0.0118)
Italy IPI 70.01-03.03	0.9306	0.1626 (0.0080)	0.1173 (0.0374)	1.6338 (0.0358)	-0.4501 (0.0319)	-0.1978 (0.0050)
Spain IPI 75.01-03.04	1.9140	0.1616 (0.0107)	0.2217 (0.0473)	1.4129 (0.0115)	-0.4242 (0.0052)	
Eurozone IPI 85.01-03.03	6.7900	0.1627 (0.0088)	0.0330 (0.0089)	1.2236 (0.0023)	0.4109 (0.0018)	-0.6466 (0.0008)

Each series corresponds to y_t=100*log(GDP_t) or y_t=100*log(IPI_t).
The standard deviations are present in parenthesis.
(*) Sum of the GDPs of Germany, France, Italy and Spain in Euros.

Table 4.2: Estimation results of the Cyclical Trend model (3.4)

	Sup $F(\lambda)^{k}_{\beta(1)}$	μ	σ^2_2	ϕ_1	ϕ_2	ϕ_3
UK GDP 70.I–03.I	0.5865	0.5722 (0.0961)	0.0118 (0.0093)	1.6565 (0.1489)	-0.7571 (0.1262)	
Germany GDP 70.I–03.I	1.6039	0.5475 (0.0860)	0.0086 (0.0088)	1.9948 (0.4336)	-1.4301 (0.7433)	0.3430 (0.3702)
France GDP 70.I–03.I	0.3649	0.5820 (0.1081)	0.0019 (0.0006)	2.4138 (0.0481)	-2.1851 (0.0562)	0.7336 (0.0387)
Italy GDP 70.I–03.I	2.2946	0.5958 (0.0781)	0.0473 (0.0149)	1.4710 (0.0380)	-0.7503 (0.0085)	
Spain GDP 80.I–03.I	3.1467	0.6083 (0.1414)	0.0138 (0.0063)	0.8507 (0.0092)	0.6816 (0.0531)	-0.5496 (0.0403)
Eurozone GDP 91.I–03.I	0.0566	0.4439 (0.0884)	0.0068 (0.0077)	1.8202 (0.3971)	-1.2216 (0.5015)	0.2625 (0.1587)
Euro-4 GDP(*) 80.I–03.I	1.3916	0.4777 (0.0906)	0.0042 (0.0038)	1.9713 (0.3320)	-1.4369 (0.5528)	0.3936 (0.2704)
UK IPI 70.01-03.04	0.9850	0.0937 (0.0603)	0.0041 (0.0015)	1.7090 (0.0306)	-0.7651 (0.0255)	
Germany IPI 70.01-03.04	1.1794	0.1311 (0.0503)	0.0037 (0.0011)	1.0200 (0.0605)	0.6436 (0.1299)	-0.7256 (0.0709)
France IPI 72.01-03.04	1.6951	0.1291 (0.0451)	0.0039 (0.0012)	1.1227 (0.0076)	0.4461 (0.0060)	-0.6434 (0.0013)
Italy IPI 70.01-03.03	0.9040	0.1502 (0.0525)	0.0042 (0.0014)	0.9707 (0.0056)	0.8274 (0.0126)	-0.8655 (0.0144)
Spain IPI 75.01-03.04	4.0036	0.1595 (0.0499)	0.0007 (0.0004)	1.8469 (0.0593)	-0.8770 (0.0541)	
Eurozone IPI 85.01-03.03	5.1546	0.1525 (0.0538)	0.0017 (0.0008)	0.8964 (0.0065)	0.9139 (0.0656)	-0.8654 (0.0634)

Each series corresponds to y_t=100*log(GDP_t) or y_t=100*log(IPI_t).
The standard deviations are present in parenthesis.
(*) Sum of the GDPs of Germany, France, Italy and Spain in Euros.

Table 4.3: Estimation results of the ARMA Models

	Sup $F(\lambda)^{k}_{\beta(1)}$	σ^2_a	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
UK GDP	0.3103	0.7391	0.0374	-0.2046	-0.3764	0.3002	-0.2397	-0.2031
70.I–03.I			(0.2034)	(0.1845)	(0.1460)	(0.2259)	(0.2452)	(0.1897)
Germany GDP	1.8873	0.7724	-0.2856	0.6570	-0.5353	-0.2559	0.7489	-0.3772
70.I–03.I			(0.3850)	(0.1430)	(0.3373)	(0.4203)	(0.1368)	(0.3914)
France GDP	0.2067	0.2593	1.0177	-0.3609	-0.5578	1.3747	0.1697	-0.2629
70.I–03.I			(0.2028)	(0.2889)	(0.1554)	(0.2340)	(0.3913)	(0.2223)
Italy GDP	1.3783	0.6072	-1.4996	0.8708		-1.1842	0.5211	0.1285
70.I–03.I			(0.1244)	(0.1243)		(0.1577)	(0.1546)	(0.1142)
Spain GDP	2.8948	0.3141	0.4979	-0.5608	-0.4564	0.6230	-0.3551	
80.I–03.I			(0.1873)	(0.1420)	(0.1033)	(0.2147)	(0.2154)	
Eurozone GDP	0.1757		-0.5338	0.6429	-0.3528	-0.0322	0.8150	
91.I–03.I		0.1726	(0.3247)	(0.3452)	(0.2465)	(0.2723)	(0.2340)	
Euro-4 GDP(*)	3.0227	0.2396	-1.5863	1.4694	-0.5552	-1.4324	1.3667	-0.3412
80.I–03.I			(0.3139)	(0.3386)	(0.2627)	(0.3500)	(0.3885)	(0.3493)
UK IPI	1.7404	1.1289	-0.1114	-0.0400		-0.2027	-0.0392	0.1277
70.01-03.04			(0.0608)	(0.0568)		(0.0805)	(0.0778)	(0.0526)
Germany IPI	0.6239	1.8633	0.0841	-0.0431	0.0413	-0.2359	-0.0566	0.1552
70.01- 03.04			(0.1164)	(0.1192)	(0.0942)	(0.1273)	(0.1144)	(0.0922)
France IPI	1.6829	1.3863	-0.0898	-0.1210	-0.3096	-0.3667	0.0183	-0.1869
72.01-03.04			(0.1633)	(0.1562)	(0.1334)	(0.1735)	(0.1521)	(0.1315)
Italy IPI	1.0075	5.0048	0.0589	0.1555	0.1665	-0.3868	0.1797	0.1666
70.01-03.03			(0.0896)	(0.0863)	(0.0746)	(0.1041)	(0.1016)	(0.0819)
Spain IPI	1.5211	2.8882	0.4145	0.2874		-0.1047	0.2000	-0.0856
75.01-03.04			(0.2538)	(0.2273)		(0.2621)	(0.2085)	(0.1201)
Eurozone IPI	1.5087	0.4801	-0.1968	-0.5982	0.0191	-0.5785	-0.6704	0.6706
85.01-03.03			(0.0913)	(0.0659)	(0.0908)	(0.0747)	(0.0690)	(0.0746)

Each series corresponds to $y_t=100*\Delta\log(GDP_t)$ or $y_t=100*\Delta\log(IPI_t)$ in deviations from the mean.
The standard deviations are present in parenthesis. The signs of the autoregressive parameters follow the notational convention of the estimation command armax.m of Matlab, so, to compare with those of Tables 4.1 and 4.2 must be changed.
(*) Sum of the GDPs of Germany, France, Italy and Spain in Euros.

5. Some illustrative examples of parameter instability

The computed values of the $\mathrm{Sup}\, F(\lambda)^{k}_{\beta(1)}$ for UCM and ARMA reduced forms do not reject the null hypothesis of constant coefficients in most of the series modelled. Nevertheless, Spanish GDP and the Industrial Production Index of the Eurozone show high values for this statistic both in Trend plus Cycle and Cyclical Trend decompositions. The graphs of the recursive coefficients (Graph C.5 in the Appendix C) seem to confirm this result.

To achieve parameter stability we have several modelling strategies that consider parameter variation as Markov Switching models or SETAR models. Intervention analysis can also be useful in some cases. For illustrative purposes we have modelled the Eurozone IPI with a simple intervention model that allows for variation in the autoregressive parameters from 1991.11 until the end of the sample. This date coincides with the beginning of a recession period located using the smoothed cyclical component of the Eurozone IPI. For the case of the Spanish GDP, the second sample period begins after 1991.I in which the graph of $\Delta \log(y_t)$ clearly shows a different behaviour. The $\mathrm{Sup}\, F(\lambda)^{k}_{\beta(1)}$ statistic in the ARMA model confirms parameter instabilities.

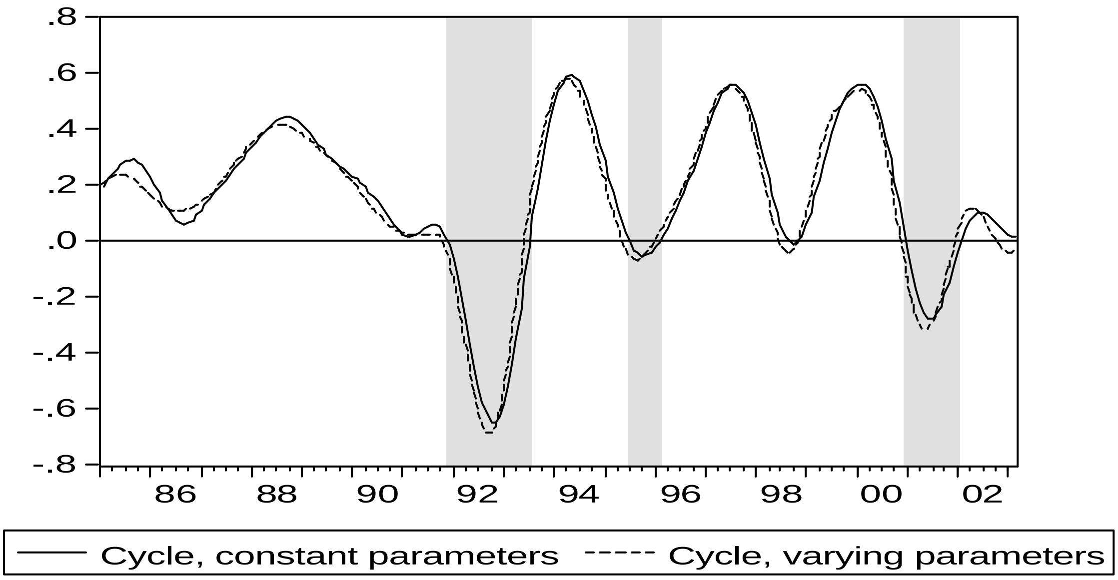
Table 5.1: Estimation results with varying autoregressive parameters of the Eurozone IPI and the Spanish GDP

	μ	σ^2_2	ϕ_1	ϕ_2	ϕ_3
Eurozone IPI	0.1525	0.0017	0.8964	0.9139	-0.8654
85.01-03.03	(0.0538)	(0.0008)	(0.0065)	(0.0656)	(0.0634)
85.01-91.10			0.3016	0.6579	0.0402
	0.1598$^{(*)}$	0.0038$^{(*)}$	(0.0058)	(0.0026)	(0.0030)
	(0.0689)	(0.0014)			
91.11-03.03			0.8894	0.8690	-0.8213
			(0.0253)	(0.0739)	(0.0516)
Spain GDP	0.6083	0.0138	0.8507	0.6816	-0.5496
80.I–03.I	(0.1414)	(0.0063)	(0.0092)	(0.0531)	(0.0403)
80.I-90.IV			0.8946	0.4440	-0.3526
	0.5976$^{(*)}$	0.0142$^{(*)}$	(0.0872)	(0.1199)	(0.0731)
	(0.3813)	(0.0063)			
91.I-03.I			1.0846	0.2583	-0.3680
			(0.2792)	(0.4860)	(0.2621)

Standard deviations in parenthesis.
$^{(*)}$ These parameters have been assumed constant in both subperiods.

Table 5.1 shows that, in both cases, the parameter variations seems substantial. The plotted filtered components of the Spanish GDP and the Eurozone IPI obtained from these new estimations show only slight differences, but the smoothed component of the Eurozone IPI (Graph 5.1) experiments enough variation to change the dating obtained when the parameter variation in the coefficients is not modelled.

Graph 5.1: Smoothed cyclical components of the Eurozone IPI with and without parameters variation modelled (Trend Cycle decomposition (3.4))(*)



(*)Shadowed areas: recession periods from the constant parameter UC model.

In the case of the Eurozone IPI, the changes in the dating of the business cycle anticipate at least one month the inflexion points (beginnings and endings of recessions) and also signal a possible recession period at the end of the sample. The confirmation of this period as really recessive depends on the data available in the following months (the sample period ends in 2003.03) due to the conditioning of the smoothed component on future information[17].

6. Conclusions

When using UCM, the dating of the cycle phases departs from the estimation of the cyclical unobserved components. When the UCM specified to estimate these components have parameter instability, the estimated cyclical components may be misleading. If instabilities are present in the UCM parameters these may be translated to the parameters of the ARMA model of the corresponding observed stationary variable. To detect the presence of instabilities in the UCM and ARMA models, first we have obtained linear models and then computed a recursive Wald type test. Since the results are asymptotic, the combined effect of samples of moderate size and parameters values near nonstationarity may distort the size and power of the recursive Wald tests. To illustrate this point, the empirical size and power of the test have been presented for some selected examples, showing reasonable values.

[17] The data later available confirm a good portion of the year 2003 as recessive.

When the recursive statistics have been computed for the UC and ARMA models of selected GDPs and IPIs, we have found stability in most of the models analysed. The main exceptions are Spain's GDP and the Eurozone IPI. Once corrected for the instabilities, the chronology of the cycle phases of the Eurozone IPI, as given by the smoothed cyclical component, shows small changes that indicate a systematic anticipation for all the phases when comparing to the chronology obtained with the uncorrected cyclical component.

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Appendix A: Heteroskedasticity correction in the cyclical equation

A general State Space form (Hamilton, 1994) is

$$\xi_t = \delta + F\xi_{t-1} + \varepsilon_t$$
$$y_t = H\xi_t + e_t$$

Without loss of generality and for notational convenience, let us assume $\psi_{p*}(L) = \phi_p(L)$ and p=2. Then, in the Trend plus Cycle decomposition (3.3), $\xi_t = \begin{bmatrix} T_t \\ C_t \\ C_{t-1} \end{bmatrix}$, $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix}$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ 0 \end{bmatrix}$ and $H = [1 \quad 1 \quad 0]$. While in the Cyclical Trend decomposition (3.4), $\xi_t = \begin{bmatrix} T_t^* \\ C_{t-1}^* \\ C_{t-2}^* \end{bmatrix}$, $F = \begin{bmatrix} 1 & 1 & 0 \\ 0 & \phi_1 & \phi_2 \\ 0 & 1 & 0 \end{bmatrix}$, $\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2,t-1}^* \\ 0 \end{bmatrix}$ and $H = [1 \quad 0 \quad 0]$. Finally, in both decompositions, $\delta = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$.

Under both decompositions we are interested in analyzing whether the parameters in the cycle equation are constant. If the cyclical component C_t were observable the solution would be straightforward by applying the sup statistic. The problem here is that C_t (or C_{t-1}^* in the Cyclical Trend decomposition without any substantial change in what follows) is unobserved and must be estimated firstly. Let us see a solution by means of the Kalman filter.

Define $\xi_{t|t} = \hat{E}(\xi_t|_t)$, $\xi_{t|t-1} = \hat{E}(\xi_t|_{t-1})$ and $y_{t|t-1} = \hat{E}(y_t|_{t-1})$, where $\hat{E}(\bullet|t)$ means the linear projection on information set *t*. Define also the variance-covariance matrices $P_{t|t} = E(\xi_t - \xi_{t|t})(\xi_t - \xi_{t|t})'$; $P_{t|t-1} = E(\xi_t - \xi_{t|t-1})(\xi_t - \xi_{t|t-1})'$ and $\sigma^2_{\eta_{t|t-1}} = E(y_t - y_{t|t-1})^2$. Then, the Kalman filter prediction equations are:

$$\xi_{t|t-1} = \delta + F\,\xi_{t-1|t-1}$$
$$P_{t|t-1} = FP_{t-1|t-1}F' + \Sigma_\varepsilon$$
$$y_{t|t-1} = H\xi_{t|t-1}$$
$$\eta_{t|t-1} = y_t - y_{t|t-1}$$
$$\sigma^2_{\eta_{t|t-1}} = HP_{t|t-1}H' + \sigma_e^2$$

while the updating equations are:

$$\xi_{t|t} = \xi_{t|t-1} + k_t \eta_{t|t-1}$$

where the matrix k_t is:

$$k_t = P_{t|t-1}H'(HP_{t|t-1}H' + \sigma_e^2)^{-1}$$

and:

$$P_{t|t} = P_{t|t-1} + P_{t|t-1}H'(HP_{t|t-1}H'+\sigma_e^2)^{-1}HP_{t|t-1}$$
$$y_{t|t} = H\xi_{t|t}$$
$$\eta_{t|t} = y_t - y_{t|t}$$
$$\sigma^2_{\eta_{t|t}} = HP_{t|t}H' + \sigma_e^2$$

Notice that k_t is not the Kalman gain, although the two are related.

The forecasting errors are:

$$\alpha_{t|t} = \xi_t - \xi_{t|t}$$
$$\alpha_{t|t-1} = \xi_t - \xi_{t|t-1}$$

where $\xi_{t|t}$, $\xi_{t|t-1}$, $y_{t|t}$ and $y_{t|t-1}$ may be obtained from the Kalman filter.

Define the selection vector $s_C = [0 \quad 1 \quad 0]$. When substituting the unobserved cycle component C_t by its estimation $C_{t|t}$ and similarly with its lags, the resulting equation is estimable. Substituting $C_t = C_{t|t} + s_C\alpha_{t|t}$, in the state equation, we easily obtain:

$$C_{t|t} = \phi_1 C_{t-1|t-1} + \phi_2\, C_{t-2|t-2} + \zeta_t$$

with:

$$\zeta_t = s_C(\varepsilon_t - \alpha_{t|t} + \phi_1\alpha_{t-1|t-1} + \phi_2\alpha_{t-2|t-2})$$

Now, $\alpha_{t|t}$ is:

$$\alpha_{t|t} = (\delta + F\xi_{t-1} + \varepsilon_t) - \left(\xi_{t|t-1} + k_t\eta_{t|t-1}\right) = F(\xi_{t-1} - \xi_{t-1|t-1}) + \varepsilon_t - \left(k_t\eta_{t|t-1}\right)$$

Simplifying:

$$\alpha_{t|t} = F\alpha_{t-1|t-1} + \varepsilon_t - \left(k_t\eta_{t|t-1}\right)$$

and selecting the second element for the cyclical component

$$s_C\alpha_{t|t} = \alpha^C_{t|t} = s_C F\alpha_{t-1|t-1} + s_C\varepsilon_t - s_C\left(k_t\eta_{t|t-1}\right)$$
$$\alpha^C_{t|t} = [0 \quad \phi_1 \quad \phi_2]\begin{bmatrix}\alpha^T_{t-1|t-1}\\ \alpha^C_{t-1|t-1}\\ \alpha^C_{t-2|t-2}\end{bmatrix} + \varepsilon_{2t} - s_C k_t\eta_{t|t-1}$$

or

$$\alpha^C_{t|t} = \phi_1\alpha^C_{t-1|t-1} + \phi_1\alpha^C_{t-2|t-2} + \varepsilon_{2t} - s_C k_t\eta_{t|t-1}$$

Therefore, the perturbation of the estimable equation for the cyclical component simplifies to:

$$\zeta_t = s_C k_t\eta_{t|t-1}$$

and the estimable equation turns out to be:

$$C_{t|t} = \phi_1 C_{t-1|t-1} + \phi_2\, C_{t-2|t-2} + s_C k_t\eta_{t|t-1}$$

Now, under the null, the prediction error $\eta_{t|t-1}$, is uncorrelated with $\xi_{t-1|t-1}$ and $\xi_{t-2|t-2}$, because the latter are linear combinations of the information sets up to *t-1* and *t-2*, say ψ_{t-1} and ψ_{t-2}; and $\eta_{t|t-1}$ is orthogonal to ψ_{t-1} and ψ_{t-2}. Therefore, $C_{t-1|t-1}$ and $C_{t-2|t-2} \perp \eta_{t|t-1}$. Moreover, while ς_t is heteroskedastic because

$$\sigma^2_{\varsigma_t} = s_C k_t \left[HP_{t|t-1}H' + \sigma^2_e\right] k'_t s'_C$$

it is not correlated, because $E(\eta_{t|t-1}\eta'_{s|s-1}) = 0$, $t \neq s$ (Harvey, 1989, p. 112). Therefore, $\zeta_t = s_C k_t \eta_{t|t-1}$ is also uncorrelated. So the only remaining problem to obtain efficient OLS estimations is to correct for the heteroskedasticity. To do so, given that $\sigma^2_{\varsigma_t} = s_C k_t \left[HP_{t|t-1}H' + \sigma^2_e\right] k'_t s'_C$, we define

$$\sigma_{\varsigma_t} = \sqrt{s_C k_t \left[HP_{t|t-1}H' + \sigma^2_e\right] k'_t s'_C}$$

and denoting

$$\tilde{C}_{t/t} = \sigma^{-1}_{\zeta_t} C_{t/t}$$

the final equation to be estimated recursively will be:

$$\tilde{C}_{t/t} = \phi_1 \tilde{C}_{t-1|t-1} + \phi_2 \tilde{C}_{t-2|t-2} + \omega_t$$

with $\omega_t = \sigma^{-1}_{\varsigma_t} \zeta_t$. This final equation is ready to be recursively estimated and tested for the existence of at least one break along the sample.

Appendix B

Table B.1: Critical values for the $\sup F(\lambda)^{k}_{\beta(1)}$ **of an AR(1) model**

	$\phi = -\mathbf{0.5}$				$\phi = -\mathbf{7}$		
Percentile	T=100	T=250	T=500	Percentile	T=100	T=250	T=500
0.75	2.70	2.67	2.88	**0.75**	2.77	2.68	2.73
0.80	3.05	3.15	3.22	**0.80**	3.15	3.03	3.21
0.85	3.58	3.72	3.72	**0.85**	3.68	3.66	3.70
0.90	4.42	4.33	4.23	**0.90**	4.50	4.37	4.34
0.95	5.73	5.49	5.38	**0.95**	6.03	5.30	5.35
0.99	11.17	8.50	7.77	**0.99**	9.85	7.58	8.07
	$\phi = -\mathbf{0.9}$				$\phi = -\mathbf{0.95}$		
Percentile	T=100	T=250	T=500	Percentile	T=100	T=250	T=500
0.75	2.72	2.70	2.77	**0.75**	2.54	2.53	2.70
0.80	3.06	3.09	3.10	**0.80**	2.97	2.84	3.11
0.85	3.61	3.55	3.65	**0.85**	3.44	3.29	3.44
0.90	4.26	4.31	4.27	**0.90**	4.09	3.97	4.02
0.95	5.66	5.47	5.46	**0.95**	5.48	5.71	5.37
0.99	9.23	7.95	8.71	**0.99**	9.64	7.92	9.31

The critical values have been computed by 1000 Monte Carlo replications and different T's. $\sup F(\lambda)^{k}_{\beta(1)}$ is the Wald type statistic to test the recursive estimations against the full sample estimations. The recursive statistic has been computed with symmetric 15% trimming. Models have been simulated and estimated using the filter.m and armax.m sentences from Matlab 6.5p.

Table B.2: Power of the $\sup F(\lambda)^{k}_{\beta(1)}$ **for an AR(1) model with varying** ϕ

Critical values (*k=1*)	$\mathbf{T_1}$**=50** **T=100**	$\mathbf{T_1}$**=75** **T=150**	$\mathbf{T_1}$**=100** **T=200**	$\mathbf{T_1}$**=250** **T=500**	$\mathbf{T_1}$**=500** **T=1000**
	Case 1: $\mathbf{1 \le t \le T_1}$, ϕ **= -0.5;** $\mathbf{T_1+1 \le t \le T}$, ϕ **= -0.7**				
4.40	24.9	31.4	37.0	67.9	94.5
	Case 2: $\mathbf{1 \le t \le T_1}$, ϕ **= -0.5;** $\mathbf{T_1+1 \le t \le T}$, ϕ **= -0.9**				
4.40	77.8	91.5	97.2	100.0	100.0
	Case 3: $\mathbf{1 \le t \le T_1}$, ϕ **= -0.5;** $\mathbf{T_1+1 \le t \le T}$, ϕ **= -0.95**				
4.40	89.4	97.1	99.2	100.0	100.0

Entries are the percent rejections based on 10% critical values from Table 2.1. The coefficients are assumed to experiment a change in the middle of the sample. Numerical calculations are computed by Monte Carlo simulations based on 1000 replications. *k* is the number of regressors. The recursive statistic has been computed with symmetric 15% trimming. Models have been simulated and estimated using the filter.m and armax.m sentences from Matlab 6.5p.

Table B.3: Critical values for the $\sup F(\lambda)^{k}_{\beta(1)}$ of a MA(1) model

q = **0.5**				q = **0.7**			
Percentile	T=100	T=250	T=500	Percentile	T=100	T=250	T=500
0.75	2.68	2.81	2.77	**0.75**	2.76	2.73	2.77
0.80	3.08	3.16	3.09	**0.80**	3.11	3.07	3.06
0.85	3.51	3.65	3.51	**0.85**	3.60	3.54	3.50
0.90	4.48	4.53	4.22	**0.90**	4.34	4.25	4.10
0.95	5.80	5.72	5.50	**0.95**	5.58	5.62	4.93
0.99	10.74	8.11	8.44	**0.99**	9.58	9.07	7.85
q = **0.9**				q = **0.95**			
Percentile	T=100	T=250	T=500	Percentile	T=100	T=250	T=500
0.75	2.76	2.73	2.77	**0.75**	2.38	2.62	2.59
0.80	3.11	3.07	3.06	**0.80**	2.76	2.98	2.92
0.85	3.60	3.54	3.50	**0.85**	3.44	3.41	3.34
0.90	4.34	4.25	4.10	**0.90**	4.20	4.26	3.91
0.95	5.58	5.62	4.93	**0.95**	5.81	5.78	5.00
0.99	9.58	9.07	7.85	**0.99**	9.60	9.38	7.73

See footnote of Table B.1.

Table B.4: Power of the $\sup F(\lambda)^{k}_{\beta(1)}$ for a pseudo-MA(1) model with varying q

Critical values (*k=1*)	**T_1=50 T=100**	**T_1=75 T=150**	**T_1=100 T=200**	**T_1=250 T=500**	**T_1=500 T=1000**
	Case 1: $1 \le t \le T_1$, q = 0.5; $T_1+1 \le t \le T$, q = 0.7				
4.40	17.4	20.1	25.5	43.6	73.9
	Case 2: $1 \le t \le T_1$, q = 0.5; $T_1+1 \le t \le T$, q = 0.9				
4.40	28.1	43.5	53.3	90.0	99.7
	Case 3: $1 \le t \le T_1$, q = 0.5; $T_1+1 \le t \le T$, q = 0.95				
4.40	34.5	46.5	60.1	94.6	100.0

See footnote of Table B.2.

Table B.5: Critical values for the $\sup F(\lambda)^{k}_{\beta(1)}$ **of an ARMA(1,1) model**

f = -**0.5** **and** q = **0.5**				f = -**0.7** **and** q = **0.5**			
Percentile	T=100	T=200	T=500	Percentile	T=100	T=200	T=500
0.75	2.11	2.09	2.13	**0.75**	2.06	2.24	2.22
0.80	2.36	2.31	2.36	**0.80**	2.40	2.46	2.42
0.85	2.63	2.56	2.66	**0.85**	2.80	2.65	2.72
0.90	3.05	3.00	3.14	**0.90**	3.13	3.00	3.12
0.95	3.93	3.53	3.78	**0.95**	4.06	3.65	3.96
0.99	5.85	5.22	5.11	**0.99**	5.67	5.08	5.28
f = -**0.9** **and** q = **0.5**				f = -**0.95** **and** q = **0.5**			
Percentile	T=100	T=200	T=500	Percentile	T=100	T=200	T=500
0.75	1.96	2.19	2.27	**0.75**	1.92	2.28	2.51
0.80	2.27	2.45	2.50	**0.80**	2.14	2.56	2.77
0.85	2.62	2.76	2.80	**0.85**	2.43	2.87	3.08
0.90	3.18	3.19	3.18	**0.90**	2.91	3.23	3.47
0.95	4.31	3.80	3.83	**0.95**	3.69	4.11	4.25
0.99	6.17	5.56	5.45	**0.99**	6.29	6.45	5.81

See footnote of Table B.1.

Table B.6: Power of the $\sup F(\lambda)^{k}_{\beta(1)}$ **for a pseudo-ARMA(1,1) model with varying** f

Critical values ($k=2$)	T_1=50 T=100	T_1=75 T=150	T_1=100 T=200	T_1=250 T=500	T_1=500 T=1000
Case 1: 1 £ t £ T_1, f =**- 0.5,** q = **0.5; T_1+1 £ t £ T,** f = **-0.7,** q = **0.5**					
3.16	20.6	26.4	31.4	60.5	88.6
Case 2: 1 £ t £ T_1, f = **-0.5,** q = **0.5; T_1+1 £ t £ T,** f = **-0.9,** q = **0.5**					
3.16	63.5	82.6	92.6	100.0	100.0
Case 3: 1 £ t £ T_1, f = **-0.5,** q = **0.5; T_1+1 £ t £ T,** f = **-0.95,** q = **0.5**					
3.16	80.9	93.6	97.9	100.0	100.0

See footnote of Table B.2.

Table B.7: Critical values for the $\sup F(\lambda)^{k}_{\beta(1)}$ **of an ARMA(1,1) model**

f = **-0.5 and** q = **0.5**				f = **-0.5 and** q = **0.7**			
Percentile	T=100	T=250	T=500	Percentile	T=100	T=250	T=500
0.75	2.11	2.09	2.13	**0.75**	2.14	2.11	2.22
0.80	2.36	2.31	2.36	**0.80**	2.39	2.36	2.44
0.85	2.63	2.56	2.66	**0.85**	2.73	2.66	2.69
0.90	3.05	3.00	3.14	**0.90**	3.19	3.06	3.01
0.95	3.93	3.53	3.78	**0.95**	3.88	3.80	3.89
0.99	5.85	5.22	5.11	**0.99**	5.85	5.46	5.26
f = **-0.5 and** q = **0.9**				f = **-0.5 and** q = **0.95**			
Percentile	T=100	T=250	T=500	Percentile	T=100	T=250	T=500
0.75	2.03	2.14	2.25	**0.75**	2.09	2.19	2.13
0.80	2.24	2.35	2.47	**0.80**	2.34	2.41	2.36
0.85	2.59	2.67	2.83	**0.85**	2.68	2.71	2.67
0.90	3.06	3.17	3.24	**0.90**	3.15	3.04	2.97
0.95	3.77	3.94	3.93	**0.95**	4.07	3.90	3.60
0.99	6.15	5.72	5.74	**0.99**	6.97	5.86	4.85

See footnote of Table B.1.

Table B.8: Power of the $\sup F(\lambda)^{k}_{\beta(1)}$ **for a pseudo-ARMA(1,1) model with varying** q

Critical values (*k=2*)	**T_1=50 T=100**	**T_1=75 T=150**	**T_1=100 T=200**	**T_1=250 T=500**	**T_1=500 T=1000**
Case 1: 1 £ t £ T_1, f **=- 0.5,** q **= 0.5; T_1+1 £ t £ T,** f **= -0.5,** q **= 0.7**					
3.16	14.8	16.7	19.5	39.8	69.3
Case 2: 1 £ t £ T_1, f **= -0.5,** q **= 0.5; T_1+1 £ t £ T,** f **= -0.5,** q **= 0.9**					
3.16	25.8	33.9	44.4	88.6	99.7
Case 3: 1 £ t £ T_1, f **= -0.5,** q **= 0.5; T_1+1 £ t £ T,** f **= -0.5,** q **= 0.95**					
3.16	27.2	39.2	51.3	92.6	99.9

See footnote of Table B.2.

Table B.9: Empirical distributions of the $\sup F(\lambda)^{k}_{\beta(1)}$
for the Trend plus Cycle UCM of Table 4.1

Percentile	UK GDP T=133	Germany GDP T=133	France GDP T=133	Italy GDP T=133	Spain GDP T=93	Eurozone GDP T=49	Euro-4 GDP(*) T=93
0.75	2.1573	2.0285	1.8904	2.1973	1.9863	2.2498	2.0402
0.80	2.3573	2.2393	2.0543	2.5176	2.2057	2.6302	2.2575
0.85	2.7107	2.5555	2.2855	2.8616	2.4943	3.1114	2.6118
0.90	3.1980	3.0191	2.6262	3.4396	2.7598	3.6726	2.9809
0.95	4.0500	3.6118	3.1463	4.2454	3.6534	5.5359	3.7116
0.99	5.6730	5.7750	4.6026	6.1257	6.6353	9.5509	6.8035

	UK IPI T=400	Germany IPI T=400	France IPI T=376	Italy IPI T=399	Spain IPI T=340	Eurozone IPI T=219
0.75	2.9362	1.9112	2.0003	2.0334	2.3089	2.4280
0.80	3.1452	2.1070	2.2308	2.2676	2.5674	2.7878
0.85	3.4659	2.3253	2.5279	2.5422	2.8222	3.1505
0.90	3.9546	2.6835	2.8745	2.7713	3.3334	3.6647
0.95	5.0148	3.2499	3.6335	3.4135	3.9264	4.4763
0.99	7.3014	4.7132	5.8579	4.7325	5.5893	6.7788

The critical values have been computed by 1000 Monte Carlo replications. $\sup F(\lambda)^{k}_{\beta(1)}$ is the Wald type statistic to test the recursive estimations against the full sample estimations. The recursive statistic has been computed with symmetric 15% trimming.
(*) Sum of the GDPs of Germany, France, Italy and Spain in Euros.

Table B.10: Empirical distributions of the $\sup F(\lambda)^{k}_{\beta(1)}$
for the Cyclical Trend UCM of Table 4.2

Percentile	UK GDP T=133	Germany GDP T=133	France GDP T=133	Italy GDP T=133	Spain GDP T=93	Eurozone GDP T=49	Euro-4 GDP(*) T=93
0.75	2.4122	1.9862	2.2962	2.7558	2.0485	2.2593	2.0880
0.80	2.7289	2.1964	2.5266	3.1537	2.3155	2.7002	2.4497
0.85	3.2126	2.4541	2.8024	3.5608	2.6174	3.1337	2.8234
0.90	3.8133	2.9081	3.2454	4.4057	3.0040	3.8276	3.4050
0.95	4.8675	3.8784	4.0639	5.6218	3.6709	5.3221	4.1919
0.99	7.8998	6.0793	8.2065	9.0936	7.0053	11.1172	6.2328

	UK IPI T=400	Germany IPI T=400	France IPI T=376	Italy IPI T=399	Spain IPI T=340	Eurozone IPI T=219
0.75	2.2319	2.0747	2.1147	2.5265	2.4084	2.1033
0.80	2.4644	2.3090	2.3442	2.8585	2.7057	2.3378
0.85	2.7604	2.5715	2.6259	3.2468	3.0649	2.7052
0.90	3.2133	2.8981	3.0025	3.8819	3.6833	3.3183
0.95	3.8870	3.5990	4.0310	5.0490	4.6788	4.1845
0.99	6.3655	5.1800	6.4884	8.6995	7.1584	6.3569

See footnote of Table B.9.

Table B.11: Empirical distributions of the $\sup F(\lambda)^{k}_{\beta(1)}$ for the ARMA models of Table 4.3

Percentile	UK GDP T=133	Germany GDP T=133	France GDP T=133	Italy GDP T=133	Spain GDP T=93	Eurozone GDP T=49	Euro-4 GDP(*) T=93
0.75	1.5938	1.6826	1.6809	1.7436	1.7412	2.5129	1.8028
0.80	1.7505	1.8461	1.7954	1.9011	1.8778	2.9415	1.9947
0.85	1.9138	1.9521	1.9594	2.1139	2.0653	3.4948	2.2151
0.90	2.0867	2.2143	2.2711	2.3671	2.4137	4.4967	2.4728
0.95	2.5170	2.5882	2.7432	2.8080	2.8867	7.3732	3.1284
0.99	3.5059	3.5799	4.3039	3.7015	4.2177	21.1274	5.3628

	UK IPI T=400	Germany IPI T=400	France IPI T=376	Italy IPI T=399	Spain IPI T=340	Eurozone IPI T=219
0.75	1.6160	1.6419	1.5919	1.5585	1.6177	1.5483
0.80	1.7749	1.7724	1.7161	1.6802	1.7409	1.6656
0.85	1.9227	1.9307	1.8499	1.8525	1.9308	1.7990
0.90	2.1395	2.0900	2.1328	2.0342	2.1526	1.9985
0.95	2.5852	2.3983	2.4091	2.3721	2.4820	2.3589
0.99	3.6221	3.1301	3.0539	3.1331	3.3124	3.1595

The critical values have been computed by 1000 Monte Carlo replications. $\sup F(\lambda)^{k}_{\beta(1)}$ is the Wald type statistic to test the recursive estimations against the full sample estimations. The recursive statistic has been computed with symmetric 15% trimming.
(*) Sum of the GDPs of Germany, France, Italy and Spain in Euros.

Table B.12: Business cycle chronology (recession periods) for several European GDPs

UK GDP 70.I–03.I	Germany GDP 70.I–03.I	France GDP 70.I–03.I	Italy GDP 70.I–03.I	Spain GDP 80.I–03.I	Eurozone GDP 91.I–03.I	Euro-4 GDP(*) 80.I–03.I
73.III-74.I	74.III-75.I	74.III-75.I	74.III-75.II			
			77.I-77.III			
79.IV-81.I	81III-82.IV	80.I-80.IV	82.I-82.III	80.I-80.III		80.I-80.IV
84.I-84.III	85.III-86.I					
90.II–92.II	91.I-91.III					
	92.I - 93.II	92.III-93.II	92.I-93.I	92.III-93.I	92.I-93.I	92.I-93.II
	01.I-01.IV		01.I-01.III			
	02.III-					

The criterion to date a recession period considers that a peak (the beginning of a recession) is located in quarter *t* when $\{\Delta y_t > 0, \Delta y_{t+1} < 0, \Delta y_{t+2} < 0\}$; and a trough (the end of a recession) is located in quarter *t* when $\{\Delta y_{t-1} < 0, \Delta y_t < 0, \Delta y_{t+1} > 0\}$.

(*) Sum of the GDPs of Germany, France, Italy and Spain in Euros.

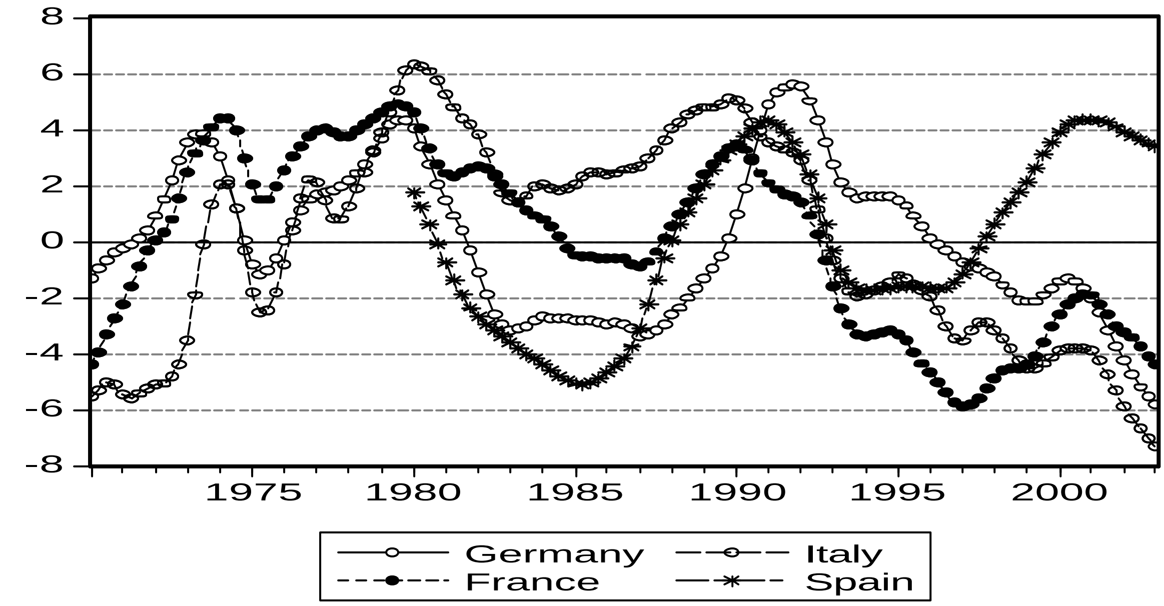
Table B.13: Business cycle chronology (recession periods) for several European IPIs

UK IPI 70.01-03.04	Germany IPI 70.01- 03.04	France IPI 72.01-03.04	Italy IPI 70.01-03.03	Spain IPI 75.01-03.04	Eurozone IPI 85.01-03.03
70.10-71.11			70.04-71.01		
74.05-75.08	73.10-75.05	74.04-75.06	74.02-75.06		
		76.12-77.10	76.12-77.12		
79.07-81.02	79.12-82.12	79.10-81.02 81.11-82.12	80.05-83.05	80.03-82.03	
83.11-84.08	86.05-87.01	86.03-86.09			
90.03-91.09		90.07-91.05	89.12-91.04	89.11-91.02	
	91.07-93.07	92.01-93.08	92.01-93.07	91.10-93.05	91.11-93.07
	95.01-95.12	95.04-95.11	95.08-96.09	95.05-96.04	95.06-96.02
98.06-98.12	< 6 months	< 6 months	98.03-99.02		< 6 months
00.09-	00.12-02.02	01.03-02.04	00.11-02.08	00.08-02.01	00.12-02.01

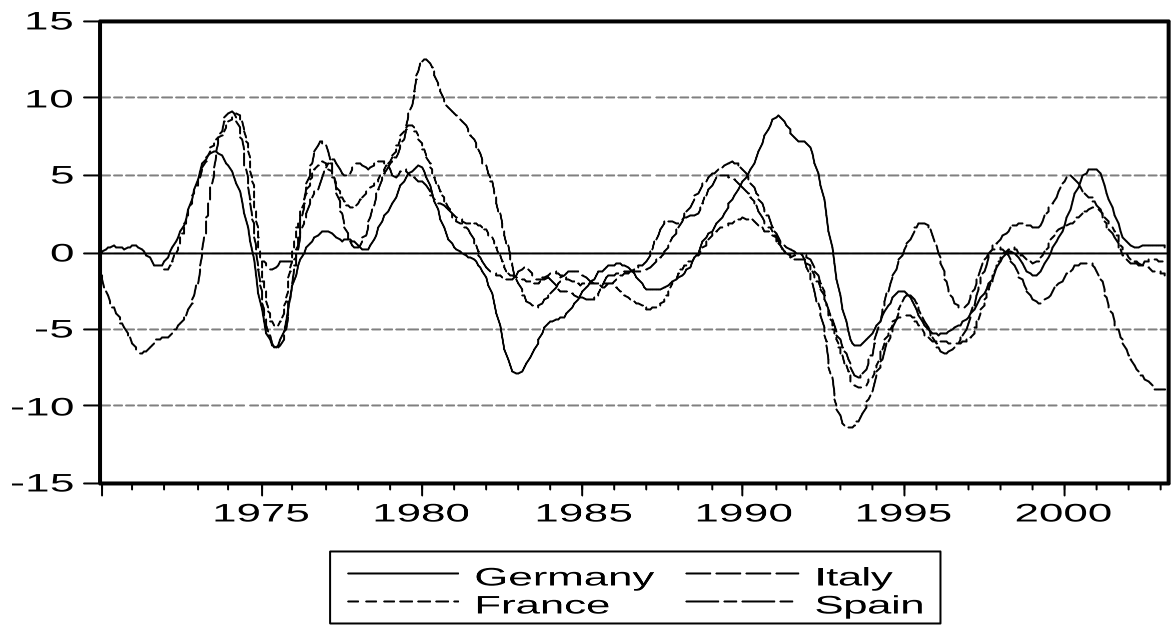
The criterion to date a recession period in the Indexes of Industrial Production is the translation to monthly data of the conventional criterion applied to GDP. Because of the IPIs series are very noisy we have employed as a cyclical signal the smoothed cyclical component $C_{t/T}$ of the estimated Cyclical Trend UCM of Table 4.2. So, we have consider that a peak (the beginning of a recession) is located in month *t* when $\{C_{t/T} > 0, C_{t+1/T} < 0, ..., C_{t+6/T} < 0\}$; and a trough (the end of a recession) is located in month *t* when $\{C_{t-5/T} < 0, ..., C_{t/T} < 0, C_{t+1/T} > 0\}$.

Appendix C

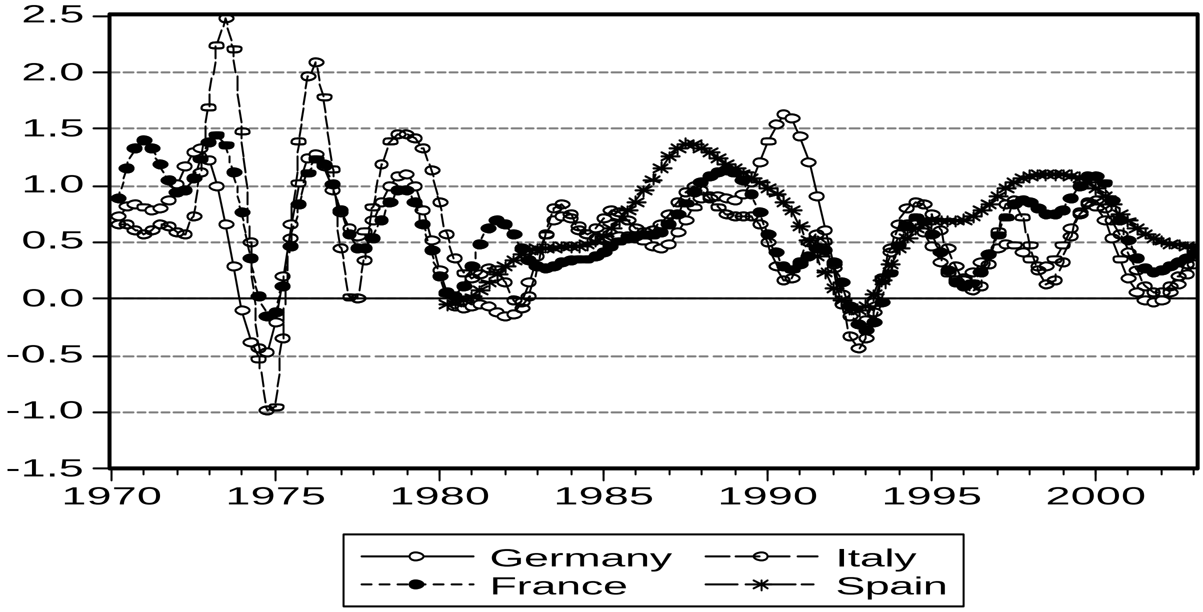
Graph C.1: Smoothed cyclical components of some of the GDPs for the Trend plus Cycle decomposition (3.3)



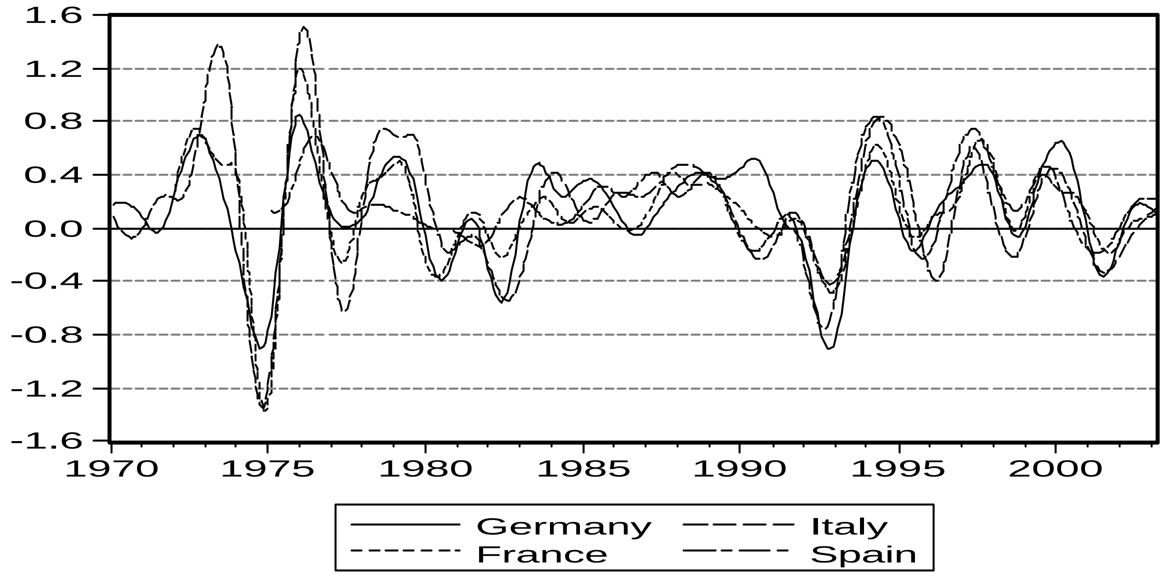
Graph C.2: Smoothed cyclical components of some of the IPIs for the Trend plus Cycle decomposition (3.3)



Graph C.3: Smoothed cyclical components of some of the GDPs for the Cyclical Trend decomposition (3.4)



Graph C.4: Smoothed cyclical components of some of the IPIs for the Cyclical Trend decomposition (3.4)

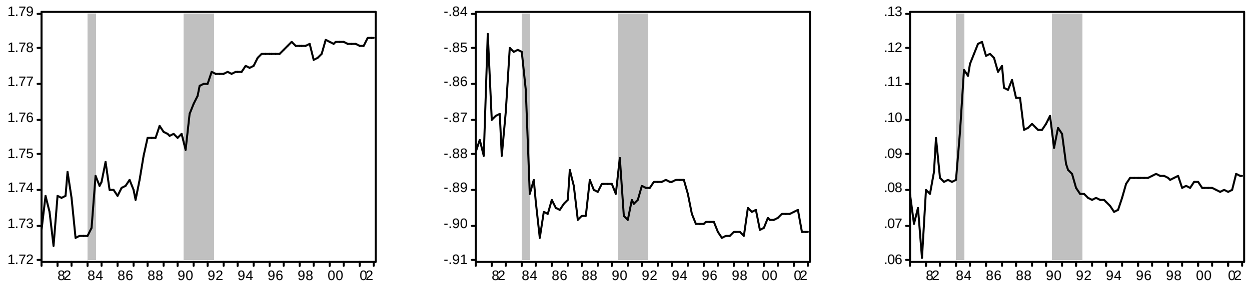


Graph C.5: Recursive coefficients of some of the UC models of Tables 4.1 and 4.2

Shadowed areas: recession periods according criteria of Tables B.12 and B.13

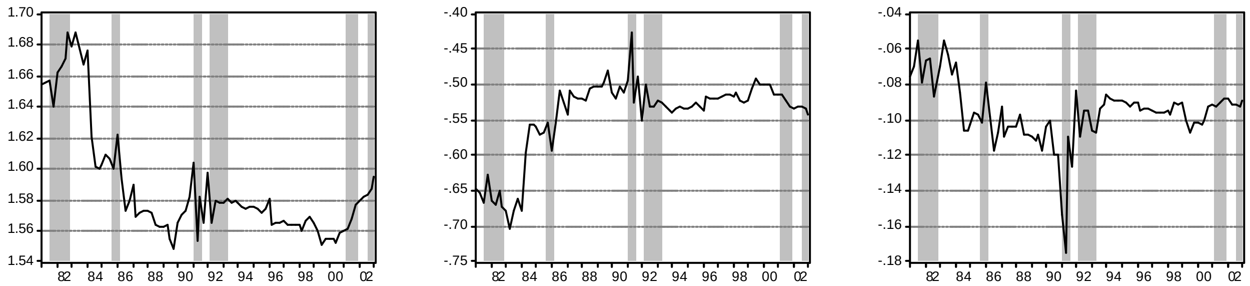
UK GDP, Trend plus Cycle decomposition (3.3)

f_1 f_2 f_3



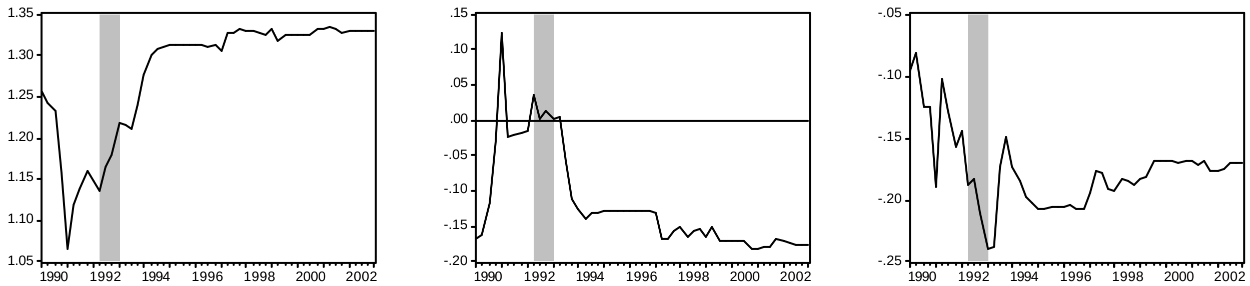
Germany GDP, Trend plus Cycle decomposition (3.3)

f_1 f_2 f_3



Spain GDP, Cyclical Trend decomposition (3.4)

f_1 f_2 f_3



Eurozone IPI, Cyclical Trend decomposition (3.4)

f_1 f_2 f_3