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Matthieu LEMOINE*

In order to assess whether the UK can enter the Euro-zone, the first test proposed by the Chancellor of the Exchequer, which is based on the Optimum Currency Area Theory (Mundell, 1961), concerns the convergence of British and Euro-zone business cycles. To measure adjustment costs caused by potential asymmetric shocks, authors generally estimate business cycles over some length of time; they then compute the correlation coefficient between the cycles of two countries. As an alternative, we suggest here to combine in a unique model cyclical and convergence dynamics. A non-linear analytical framework is developed in this paper for the joint estimation of UK and Euro-zone cycles. It combines Hodrick-Prescott filters with time-varying parameter models. This method exhibits a current high convergence level, even if economic monetary policy divergence is taken into account.

Key-words: (A)symmetric shocks, business cycles, multivariate filters, structural time series models, stochastic variance models.

JEL codes: C32, E32, F42

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Introduction

In 1997, the Chancellor of the Exchequer rejected the entry of the United Kingdom in the Euro-zone, although the nominal criteria defined in the Maastricht treaty were respected. Indeed, he preferred to assess the benefit for the UK to join the euro-zone with five tests concerning the cyclical convergence, the flexibility of the economy in the case of asymmetrical shocks, the effects of the Euro on British investment and on financial services and, more generally, the benefit for long-term growth and employment (HM Treasury, 2003). The first test about the cyclical convergence is a key criterion, which influences the overall assessment of the five economic tests. In particular, in the case of a lack of cyclical convergence, the labour market flexibility might have an strong role in the UK, for adopting the Euro.

In order to analyse the convergence of cycles, many approaches are possible. For example, four approaches have been followed by the HM Treasury (2003): analysing how close have been the main macro-economic variables in the recent period (interest rates, GDP, consumption, labour, inflation and exchange rates); assessing the degree of cyclical convergence during the last decades, to measure the risk of cyclical divergence in the future (Artis and Zhang, 1995), considering structural factors like the commercial balance structure, the oil production asymmetry or house markets differences; assessing endogenous factors like the trade effects associated with the integration in a currency area (Rose, 1999); evaluating the appropriate exchange rate level for entry.

This paper proposes a new model and its estimation methodology, which are useful for studying the historical convergence of British and Euro-zone cycles (second approach). Even if many cyclical models have been proposed, the cyclical convergence is generally studied with ad hoc indicators applied on estimated cycles. As far as we know, few models have been developed for studying the cyclical convergence dynamics. Common cycle models have been proposed by Vahid and Engle (1993), but we think they do not allow to study dynamics of the convergence. More recently, Luginbuhl and Koopman (2002) modelled the convergence dynamics with logistic functions. But such a mechanism does not allow convergence and divergence movements to occur successively. This article proposes a new bivariate cyclical model, where a correlation coefficient follows an auto-regressive dynamics. Trend and cycles
are modelled with Hodrick-Prescott filters in an unobserved component framework. The auto-regressive process allows some reversible dynamics. As this model is non-linear, a specific procedure is also proposed for the estimation.

The cyclical convergence model is first applied to British and aggregated Euro-zone GDP series from 1970 to 2002. The analysis of historical divergences might provide indications about the first test for the UK entry in the Euro-zone. We are aware that such an approach has a major shortcoming: the historical information about convergence might be useless about the future. Indeed, some idiosyncratic shocks that will not occur again in the future might have implied convergence or divergence movement during the sample period. We hope to limit such a problem, by considering three decades.

A second application is proposed, which aims to take into account the endogenous effect of the UK entry in the Euro-zone. As the UK was not in a European currency area from 1970 to 2002, cyclical divergences might be partly explained during this period by monetary policy divergences. But the future convergence process might change, if the UK enter the Euro-zone. Thus, we try to imagine what would have been the convergence if a monetary union had occurred in Europe thirty years ago.

**Modelling cyclical convergence**

**Trend-cycle decompositions and Hodrick-Prescott filters**

For decomposing a time series $y_t$ into a trend $T_t$ and a cycle $C_t$, no method satisfies all criteria. However, despite of known criticisms\(^1\), Hodrick-Prescott filters (HP) became the most popular methodology, because of their operational simplicity. In Hodrick and Prescott (1997), the original algorithm solved the following optimisation programme:

$$
\text{Min} \left( \sum_{i=2}^{T} C_i^2 + \lambda \sum_{i=2}^{T} (\Delta^2 T_i)^2 \right) \quad \text{with} \quad y_t = T_t + C_t
$$

This filter simultaneously minimises two opposite criteria using a weight $\lambda$: the cycle should not be to volatile and the trend should be smooth enough. The filter is equivalent to a band-pass filter, which identifies the trend as a low frequency component, using a cut-off parameter

---

\(^1\) See, for example, Cogley and Nason (1995).
related to \( \lambda \) (see King and Rebelo, 1993). The weight \( \lambda \) is generally fixed to 1600 for quarterly series, which corresponds to a cycle with a period of around 8 years.

The filter is also equivalent to a structural time series model. The structural time series model splits every series \( Y_t \) into independent terms representing trend \( T_t \) and cycle \( C_t \). More precisely, we have the decomposition (for a non-seasonal series, which will be the standard case in this paper):

\[
Y_t = T_t + C_t, \quad t = 1, \ldots, T
\]

\[
T_t = T_{t-1} + \beta_{t-1}
\]

\[
\beta_t = \beta_{t-1} + \zeta_t,
\]

\[
C_t = \kappa_t, \quad \text{with } \sigma^2_{\zeta} / \sigma^2_{\zeta} = \lambda
\]

where \( \zeta_t \) and \( \kappa_t \) are orthogonal white noises with variances \( \sigma^2_{\zeta} \) and \( \sigma^2_{\kappa} \), related by a constraint on their ratio.

Clark (1987) calls this trend *Slowly Moving Smooth Trend*. This trend gives, as indicated by its name, a smooth curve similar to those obtained with the methods used by business cycles analysts. The cycle is a stationary process, which implicitly takes the cyclical behaviour of this component into account.

In the case of the United Kingdom, Harvey (1985) has proposed interesting extensions, for better taking into account some properties of the trend and the cycle. In the previous state-space model, the trend can be modelled as a general local level trend and the cycle autoregressive dynamics might be explicitly specified. As our article focuses on convergence and not directly on cycles, we kept our model in the simplest form, associated to the HP filter.
**Bivariate models**

Following Harvey (1989), we present the bivariate extension of HP filters in a state-space framework. The bivariate case will be sufficient for the analysis carried on in this article.

The bivariate models have a form similar to univariate models. $y_t$ is now an $(2 \times I)$ vector of observations that depends on unobserved components that are also $(2 \times I)$.

$$
\begin{align*}
    y_{it} &= T_{it} + C_{it} \\
    T_{it} &= T_{i,t-1} + \beta_{i,t-1} \\
    \beta_{it} &= \beta_{i,t-1} + \zeta_{it} \\
    C_{it} &= \kappa_{it}
\end{align*}
$$

where $i$ is the index of the two variables.

In this equation, the $(2 \times I)$ disturbance vectors $\zeta_{it}$, $\kappa_{it}$ are bivariate normal disturbances $N(0, \Sigma)$, which are mutually uncorrelated in all time periods and have $(2 \times 2)$ covariance matrices $\Sigma_{\zeta}$ and $\Sigma_{\kappa}$. Models of this kind are called *Seemingly Unrelated Time Series Equations*. Explanatory and intervention variables may be included in the multivariate versions of these models.

In a common factors model, the components are driven by the same disturbance series. The covariance matrices of the relevant disturbances are less than full rank. The interest of common factors is the possible interesting interpretation but also more efficient inference. Common cycles may be introduced by using a $(n \times I)$ loading vector $\Theta$ and the common cycle vector $C^{o_i}$, yield the following form:

$$
\begin{align*}
    y_{1t} &= T_{1t} + C^{o_i} \\
    y_{2t} &= T_{2t} + \theta_i C^{o_i}
\end{align*}
$$

Not only the single common cycle induces similar cyclical features between variables, but the innovations of the cycle are the same for all the variables. In short, their cycles are perfectly synchronised. The only difference is in the strength of the cycles: each cycle is proportional to the others. Common cycles embody much stronger restrictions than similar cycles.
Convergence model

A traditional approach, initiated by Vahid and Engle (1993), has studied the convergence of cycles in a group of countries, by testing the presence of common components in VECM models. But this test only shows if the convergence had already occurred before the sample period, i.e. often before the sixties! Moreover, VECM models provide transitory components which do not show enough persistence, a property which is generally expected from a cycle. Luginbuhl and Koopman (2002) have proposed a multivariate unobserved components model that tries to face these two problems: cycles are modelled as Harvey cycles; the convergence is associated with a progressive reduction of the correlation matrix rank of the cycles. But this reduction is adjusted with logistic functions, which do not allow stochastic transitory divergences. Indeed, each turning point is an opportunity of divergence, e.g. some countries might have begun their recoveries, while others remain in recession. Such recurrent divergences require a stochastic model of correlation coefficients, for example an auto-regressive model.

Thus, we decide here to also use the multivariate unobserved components framework and to change the convergence mechanism. We consider that the correlation might evolve as a stationary stochastic process, e.g. an auto-regressive process. Such a time-varying correlation might be modelled as a time-varying coefficient in a regression of a cycle on the other one\(^2\). However, as we focus our analysis on the convergence of the two cycles, following the parsimony principle, we simplify the bivariate trend-cycle model. Trend-cycle decompositions are specified as HP filter ones, with smooth trends and constrained variance ratios (see Harvey, 1985). The convergence trend-cycle model is written as follows:

\[
\begin{align*}
X_t &= T_{X,t} + C_{X,t} \\
Y_t &= T_{Y,t} + C_{Y,t} \\
\Delta^2 T_{i,t} &= \zeta_{i,t} & \text{for } i = X \text{ or } Y \\
C_{X,t} &= a_t C_{Y,t} + u_t \\
a_t &= \mu + \alpha(L)a_{t-1} + v_t \\
C_{X,t} &= \kappa_t
\end{align*}
\]

with \(\zeta_{X,t}, \zeta_{Y,t}, u_t, v_t\) and \(\kappa_t\) white independent Gaussian noises, with standard errors \(\sigma_{\zeta}, \sigma_{\zeta}, \sigma_u, \sigma_v\) and \(\sigma_{\kappa}\).

\(^2\) Boone (1997) used such time-varying regressions for measuring convergence of supply and demand shocks, in a SVAR model. The idea came from an application on exchange rates (Hall et al., 1992). But SVAR models
The model assumes some usual HP constraints:

\[ \frac{\sigma^2}{\sigma^2_{\xi,y}} = 1600 \quad (8) \]

\[ \frac{\text{Var}(C_{i,j})}{\sigma^2_{\xi,i}} = 1600 \quad (9) \]

Taking into account the cyclical interaction modelled by equation (5), the constraint (9) can be rewritten:

\[ \frac{\text{Var}(C_{i,j})}{\sigma^2_{\xi,i}} = \frac{\text{Var}(a_i, C_{i,j}) + \sigma^2_u}{\sigma^2_{\xi,i}} = 1600 \]

with,

\[ \text{Var}(a_i, C_{i,j}) = E(a_i^2, C_{i,j}^2) - E(a_i, C_{i,j})^2 = E(a_i^2). E(C_{i,j}^2) = (\text{Var}(a_i) + E(a_i)^2)\text{Var}(C_{i,j}) \]

hence,

\[ \sigma^2_u = 1600 \left[ \frac{\sigma^2_{\xi,i} - (\text{Var}(a_i) + E(a_i)^2)\sigma^2_{\xi,y}}{\sigma^2_{\xi,y}} \right] \quad (9') \]

The difficulty for estimating such a model comes from the non-linear relation in equation (5), where two state variables are multiplied by each other. Because of the correlation evolution we want to model, the model cannot be written in a linear form. But the 5th equation can be approximated with a Taylor development\(^3\) of order 1 around estimates \(\hat{a}_i\) and \(\hat{C}_{i,j}\):\(^3\)

\[ C_{i,j} = \hat{a}_i \hat{C}_{i,j} + \hat{a}_i (C_{i,j} - \hat{C}_{i,j}) + (a_i - \hat{a}_i)\hat{C}_{i,j} + u_i \]

\[ C_{i,j} + \hat{a}_i \hat{C}_{i,j} = \hat{a}_i C_{i,j} + a_i \hat{C}_{i,j} + u_i \quad (5') \]

With equation (5') instead of (5), the model can be estimated with a standard Kalman filter and an EM algorithm. \(\hat{a}_i\) and \(\hat{C}_{i,j}\) are initialised with HP estimates and then updated with Kalman filters outputs. The estimation is iterated until the estimates \(\hat{a}_i\) and \(\hat{C}_{i,j}\) are stabilised.

---

3 For estimating non-linear models, Durbin and Koopman (2001) proposed such a method. However, they also recommend improving the estimation with importance sampling techniques. This improvement is let for further research.
Application results

In this section, we use models outlined above, in order to study the convergence of British and euro-zone cycles. After a short presentation of data sources, of estimation software and of usual trend-cycle decompositions, links are detailed between the United Kingdom (UK) cycle and the aggregated euro-zone (EU). In a last section, we investigate what would have been the convergence if United Kingdom had followed the German monetary policy.

Data sources and estimation software

GDP time series come from Eurostat and have been retropolated to 1970 with the OECD Business Sector Database (BSDB). Series are expressed in 1995 euros, have been seasonally adjusted and have quarterly frequency. Short-term real interest rates come from OECD and are available from 1970 to 2002. Estimation of all models have been proceeded using algorithms and routines written on Eviews.

Trend-cycle decompositions of UK and Euro-zone output series

We consider trend-cycle decompositions of the United Kingdom and of the euro-zone. Fluctuations are wider in the United Kingdom than in the Euro-zone (figure 1). Recession were synchronised in the seventies with both oil shocks. Then, the Euro-zone slowdown occurred later at the beginning of the nineties because of the German re-unification. Finally, after the high growth at the end of the nineties, both areas went through a slowdown in 2001. The convergence seems to have been reinforced at the end of the nineties.

Concerning the trend slopes, i.e. the long-term growth rates, after a catching-up phase towards the United States, with growth rates above 4%, a major slowdown follows the oil shocks. In the nineties, trend slopes become divergent: the American-British potential growth increase due to the “new economy” does not occur in the euro-zone. Does this divergence also imply a cyclical divergence?
**Cyclical convergence of the UK and the Euro-zone**

Let us consider estimation results of the convergence model applied to British and Euro-zone GDP series. Except the constant $\mu$, parameter estimates are all significant (table 1). To analyse the convergence between both cycles, a Convergence Model Indicator (CMI) has been built with $a(t)$ and compared to a simple *ad hoc* moving average indicator (MAI). The MAI is computed at each quarter $t$ as a correlation coefficient of both cycles on a three years centered window\(^4\). The CMI comes from the time-varying coefficient $a(t)$ estimated with the convergence model. This coefficient has been transformed into a correlation coefficient, using local standard deviations (computed on three years window). We consider the three years moving average\(^5\) of this correlation coefficient, which has also been truncated to be kept in the $[-1;1]$ interval.

The CMI is smoother than the MAI, but follows similar evolutions. We distinguish two synchronisation periods (with persistent values above 0.4): during the seventies (oil shocks)

\(^4\) As this correlation coefficient is computed with 24 values, its is significantly different from zero, when its value is above 0.4 or below –0.4.

\(^5\) Moving averages provide results, which can be compared with MAI. Moreover, they smooth divergences of short cycles, which have a period of 3 years. See Bentoglio, Fayolle and Lemoine (2001) for details about these short cycles.
and at the end of the nineties (a common slowdown followed the explosion of new markets). In contrast, we observe many independence periods and a noticeable divergence period at the end of the eighties, when the UK entered with the US into recession, contrary to continental Europe (with persistent values below –0.4). If we come back to the issue raised by the British government about entry in the euro-zone, this indicator show positive results concerning convergence in the recent years.

Figure 2: Moving average and time-varying correlation (UK,EU)

Table 1: Parameters of the cyclical convergence model; applied to British and Euro-zone GDP series

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend innovations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{eu}}$</td>
<td>0.05</td>
<td>0.00</td>
<td>17.19</td>
</tr>
<tr>
<td>$\sigma_{\text{uk}}$</td>
<td>0.04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>British cycle residual</td>
<td>$\sigma_{b}$</td>
<td>0.86</td>
<td>0.04</td>
</tr>
<tr>
<td>Time-varying coefficient</td>
<td>$\sigma_{c}$</td>
<td>0.26</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.02</td>
<td>0.03</td>
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<tr>
<td></td>
<td>$\alpha_1$</td>
<td>1.54</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2$</td>
<td>-0.66</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Note 1: $\sigma_{\text{eu}}$, $\sigma_{\text{uk}}$ and $\sigma_{b}$ have been multiplied by 100.

Note 2: Except $\sigma_{\text{uk}}$, all parameters estimates and standard errors have been computed with a maximum likelihood algorithm. $\sigma_{\text{uk}}$ has been computed with $\sigma_{c}$, $\mu$, $\alpha_1$, $\alpha_2$ and $\sigma_{\text{eu}}$. Indeed, as shown in the theoretical section, trend innovations of $X_t$, here the British GDP, have a standard error related to time-varying coefficient parameters and to the variance of $Y_t$, trend innovations.
Cyclical convergence, with a single monetary policy

A second application is proposed, which aims to take into account the endogenous effect of the UK entry in the Euro-zone. We now wonder what influence has had monetary policy independence on the convergence, analysed in the last section. As the Euro-zone monetary policy did not exist before 1999, the analysis is restrained to the cyclical convergence between the UK and the major euro-zone country, Germany. Indeed, Euro-zone and German cycle have been highly correlated\(^6\) in the last thirty years (see figure 3).

**Figure 3: German and Euro-Zone cycles**

![Graph showing German and Euro-Zone cycles](image)

In order to take into account monetary policy divergences, the British GDP is adjusted to the values it would have reached if the German monetary policy had been followed in the UK. This correction is applied on GDP growth rates, by computing monetary impulses\(^7\) with short-term real interest rates (figure 4). However, such an adjustment is too simplistic for two reasons. First, in a virtual currency area, exchange rates would have been fixed and this would require another adjustment. Second, the whole policy-mix would have changed and other

---

\(^6\) The correlation coefficient between Euro-zone and German cycles is equal to 92%, on the period 1970-2002.

\(^7\) Monetary impulses are defined as variations of short-term real interest rates. Following Le Bihan and Lerais (1997), we consider that a monetary impulse has 3 months later an elasticity of 0.2.
fiscal policies would have been followed. We left for further research the estimation in a convergence model of these general endogenous effects.

Figure 4: Short-term real interest rates in the United Kingdom and in Germany

![Graph showing short-term real interest rates in the United Kingdom and in Germany.]

Figure 5: UK cycles, below British or German monetary policies

![Graph showing UK cycles, below British or German monetary policies.]

In % of GDP
British cycles would have been different, if the German monetary policy had been followed in the United Kingdom (figure 5). Estimated parameters have not significantly changed (table 2). But two new points appear about the British-German synchronisation (figure 6): the adjusted indicator (CMI1) is higher at the beginning of the eighties and at the beginning of the nineties than the CMI indicator. First, at the beginning of the eighties, the British Central Bank kept short-term real interest rates above 8%, while the German one brought them to around 5%. This British choice has generated a strong slowdown and a divergence with the German recovery. On the contrary, during the next recession, because of the future Maastricht treaty, Germany kept the rates high until the end of 1992, while the UK ones began to fall in 1990. This reactive policy allowed a quick recovery in the UK, during the German persistent slowdown. At the end of the nineties, monetary policy become convergent and confirm the synchronisation movement showed in the previous section.

However, in spite of the policy adjustment, divergence periods remain: the CMI1 indicator falls to -1 in 1989 and to 0 in 1998. A strong divergence occurred at the end of the eighties: the British cycle has known its turning point two years before the Euro-zone. Moreover, even if the British Central Bank had followed the Bundes Bank during the nineties, both zones could not avoid the asymmetric influence of the Asian crisis in 1998. In conclusion, if co-ordinated monetary policies had been followed, the convergence would have been generally high, e.g. last years, but asymmetric shocks would remain and imply cyclical divergences. All Euro members assume this risk of asymmetric shocks, which increases on the other hand the importance of fiscal co-ordination improvement.
Figure 6: Synchronisation of German and British cycles, below one single (CMI1) or two independent monetary policies (CMI)

Table 2: Parameters of the cyclical convergence model, applied to German and corrected British GDP series

<table>
<thead>
<tr>
<th>Trend innovations</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend innovations</td>
<td>$\sigma_{ge}$</td>
<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{uk}$</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>British cycle residual</td>
<td>$\sigma_0$</td>
<td>0.90</td>
<td>0.04</td>
</tr>
<tr>
<td>Time-varying coefficient</td>
<td>$\sigma_\nu$</td>
<td>0.28</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>$\mu$</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>1.48</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2$</td>
<td>-0.61</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Note 1: $\sigma_{ge}$, $\sigma_{uk}$ and $\sigma_0$ have been multiplied by 100.

Note 2: Except $\sigma_{uk}$, all parameters estimates and standard errors have been computed with a maximum likelihood algorithm. $\sigma_{uk}$ has been computed with $\sigma_\nu$, $\mu$, $\alpha_1$, $\alpha_2$ and $\sigma_{ge}$. Indeed, as shown in the theoretical section, trend innovations of $X_t$, here the British GDP, have a standard error related to time-varying coefficient parameters and to the variance of $Y_t$ trend innovations.
Conclusion

A model has been proposed, that integrates models of cycles and a dynamic model of the convergence between cycles. An estimation procedure has been developed for this non-linear model. The application on the convergence between British and Euro-zone cycles has validated the methodology, showing its coherence with ad hoc moving average indicators. Both indicators indicate a growing convergence during the nineties. Taking care of possible divergence of monetary policies, it has appeared that this growing convergence had begun earlier, but was perturbed by the pro-cyclical policy followed by Germany after the reunification, and by the asymmetric influence of the Asian crisis. Hence, although the convergence test seems to have been validated recently, the risk of asymmetric shocks remain and the entry of the United Kingdom in the Euro-zone would require an improved fiscal co-ordination.

Even if our convergence model has proved to be an interesting starting point for assessing in a probabilistic framework the dynamic convergence of business cycles, some improvements have been let for future research. The estimation procedure of such a non-linear model could become more precise with importance sampling techniques. The model might integrate more realistic auto-regressive models for the cycle and we could try to explain better the convergence by integrating exogenous variables, like exchange rates and fiscal indicators, in the convergence mechanism.


