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Dating business cycle: a methodological contribution with an application to the Euro Area

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Dating Business Cycles: a Methodological Contribution with an Application to the Euro Area

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Abstract

This paper proposes a dating algorithm based on an appropriately defined Markov chain that enforces alternation of peaks and troughs, and duration constraints concerning the phases and the full cycle. The algorithm, which implements Harding and Pagan's non-parametric dating methodology, allows to assess the uncertainty of the estimated turning points due to filtering and can be used to construct indices of business cycle diffusion, aiming at assessing how spread are cyclical movements throughout the economy. Its adaptation to the notion of a deviation cycle and the imposition of depth constraints are also discussed. We illustrate the algorithm with reference to the issue of dating the Euro area business cycle and analyzing its characteristics, both from the classical and the growth cycle perspectives.

JEL Classification: E32, E39

Keywords: Euro area, Diffusion indices, Markov chains, Filtering

1 Introduction

The business cycle can be defined as a broadly-based movement of economic variables in a sequentially oscillatory manner. The term ’cycle’ is a misnomer to the extent to which it suggests a regular periodicity; one of its features is that the length and depth (duration and amplitude) of the cycle seems to vary. Indeed one of the current preoccupations of US business cycle experts (e.g., Stock and Watson, 2002) is to explain the apparent lengthening of the cycle there in recent history.

There are several reasons for taking an interest in the cycle. The evolution of the cycle carries with it an evolution in variables of considerable consequence for policy-makers: indeed, policy-makers are commonly depicted as endeavouring to reduce the extent of fluctuations by exercising stabilization policy. A closely related interest has been in the use of business cycle evidence in the context of optimal currency area theory and its indication for the optimality of monetary union. Other things equal, business cycle symmetry is a positive indicator for monetary union as it indicates that a single monetary policy will be broadly appropriate for all participants in the monetary union. On the other side, an asymmetry of business cycle experience is usually treated as a negative indicator for participation in monetary union.

The literature recognizes two broad definitions of the cycle, the so-called classical cycle and the growth or deviation cycle. The difference between the two is conceptually simple: in the case of the deviation cycle, turning points are defined with respect to deviations of the rate of growth of GDP from an appropriately defined trend rate of growth. There is a large technical literature which is concerned with the best method of extracting a trend from the data, and it turns out that the method adopted may carry quite important implications for the subsequent dating of the turning points. The classical cycle, by contrast, selects its turning points on the basis of an absolute decline (or rise) in the value of GDP.

In early post-war decades, especially in Western Europe, growth was relatively persistent and absolute declines in output were comparatively rare; the growth cycle then seemed to be of more analytical value especially as inflexions in the rate of growth of output could reasonably be related to fluctuations in the levels of employment and unemployment. In more recent decades, however, there have been a number of instances of absolute decline in output, and popular description at any rate has focussed more on the classical cycle (for example there is a widespread impression that a recession defines itself as two consecutive quarters of absolute decline). In addition, the concern mentioned above that de-trending methods can affect the information content of the series in unwanted ways, has reinforced the case for examining the classical cycle.

In this paper we propose a dating algorithm based on the theory of Markov chains that enforces alternation of peaks and troughs and duration constraints, and discuss how it is adapted to a variety of problems. Our approach is essentially non-parametric and adopts Pagan's dating methodology, (Harding and Pagan, 2001, Pagan, 2002), which in turn relates to the Bry-Boschan (1971) dating procedure In our approach we extend these procedures to compute the probability of a phase switch, to introduce depth or amplitude restrictions, and to construct diffusion indices. Our methodology is presented in section 2.

The empirical part of the paper reports on four analyses that we carried out with the support of the dating methodology: the first concerns the characterisation of the aggregate Eurozone business cycle (section 3); in section 4 we turn our attention to the country-specific cycles for the main Eurozone countries, for which we can obtain consistent output series. The third seeks to employ higher frequency data, namely monthly industrial production data (section 5); the advantage of concentrating on this series is that its higher frequency should enable a more precise dating of the cycle whilst it is already known that the most cyclically-sensitive component of GDP is in fact industrial production. Finally, in section 6 we illustrate how our dating algorithm can be employed to construct indices of business cycle diffusion, thereby providing a multivariate assessment of the cycle. Section 7 summarises and concludes.

2 The Dating Algorithm

This section lays out the methodology employed in the paper for dating the Euro area classical and growth cycle. We start off by proposing an algorithm based on an appropriately defined Markov chain that automatically enforces the alternation of peaks and troughs and the minimum duration constraints concerning the full cycle and its phases. For the sake of exposition the dating algorithm will be illustrated with reference to the quarterly case; details concerning the monthly case are deferred to paragraph 2.3.

The dating algorithm is described in separate paragraphs, the first concerning the definition of the underlying Markov chain, and the second dealing with the scoring of its transition probabilities. We then discuss enhancements that cope with the nature of the deviation cycle and amplitude restrictions.

2.1 The underlying Markov Chain

At any time t the economy can be in either of two mutually esclusive states or *phases*: expansion, denoted E_t, or recession, denoted R_t. We adopt the convention that a peak terminates an expansion, whereas a trough terminates a recession. For the imposition of minimum duration constraint and to enforce the alternation of peaks and troughs, it is useful to distinguish turning points within these basic states, by posing:

$$\begin{aligned} \mathsf{E}_t &\equiv \begin{cases} \mathsf{EC}_t & \text{Expansion Continuation} \\ \mathsf{P}_t & \text{Peak} \end{cases} \\ \mathsf{R}_t &\equiv \begin{cases} \mathsf{RC}_t & \text{Recession Continuation} \\ \mathsf{T}_t & \text{Trough} \end{cases} \end{aligned}$$

¿From EC_t we can make a transition to P_{t+1} or continue the expansion, $(\mathsf{EC}_t \to \mathsf{EC}_{t+1})$, but not viceversa, since only $\mathsf{P}_t \to \mathsf{RC}_{t+1}$ is admissible. Analogously, from RC_t we can visit either RC_{t+1} or T_{t+1}, but from T_t we move to EC_{t+1} with probability 1.

Denoting by $p_{EP} = P(\mathsf{P}_{t+1}|\mathsf{EC}_t)$ the probability of making a transition to a peak within an expansionary phase, $p_{EE} = P(\mathsf{EC}_{t+1}|\mathsf{EC}_t) = 1 - p_{EP}$, and analogously $p_{RT} = P(\mathsf{T}_{t+1}|\mathsf{RC}_t)$, and $p_{RR} = P(\mathsf{RC}_{t+1}|\mathsf{RC}_t) = 1 - p_{RT}$, we define a first order Markov chain (MC) with four states, denoted S_t, with transition matrix:

	EC_{t+1}	P_{t+1}	RC_{t+1}	T_{t+1}
EC_t	p_{EE}	p_{EP}	0	0
P_t	0	0	1	0
RC_t	0	0	p_{RR}	p_{RT}
T_t	1	0	0	0

The dating rules impose ties on the minimum duration of a phase, which amounts to two quarters, and this is automatically enforced in the quarterly case by our four states characterisation (as a matter of fact $\{\mathsf{EC}_{t-1}, \mathsf{P}_t\}$, complies with this requirement, since both events belong to the expansionary phase; similarly a trough cannot occur immediately after a peak), and on the minimum duration of a full cycle. The latter is defined in terms of peak-to-peak or trough-to-trough patterns and amounts to five quarters, as a direct transposition of the original (monthly) Bry and Boschan rule to the quarterly case. In imposing this rule, it must be remembered that T (or P) cannot be counted both as the end of recession (expansion) and as the beginning of expansion (recession): thus for instance, the pattern $\{\mathsf{T}_{t-4}, \mathsf{EC}_{t-3}, \mathsf{P}_{t-2}, \mathsf{RC}_{t-1}, \mathsf{T}_t\}$ is not admissible as a full cycle. The minimum duration constraints are important for the characterisation of the chain, determining the order of the MC and the number of admissible states.

The tie on the full cycle duration yields a 5th order MC that can be converted to a first order one by combining elements of the original chain, S_t. The states of the derived MC are defined by the collection:

$$S_t^* = \{S_{t-4}, S_{t-3}, S_{t-2}, S_{t-1}, S_t\}.$$

The ties however reduce the number of states to 24. These are listed in table 1: the first column labels the states and the second spells out how they are formed by combining the elementary states of the original MC. The last two columns indicate the states to which a transition is admissible (two at most) and the associated transition probability. The transition matrix is thus immediately derived from the above table. It should be noticed that all the states ending with a peak or a trough must visit certain states with probability one.

[TABLE 1 about here]

The two parameters $p_{_{EP}}$ $p_{_{RT}}$ uniquely specify the Markov chain. In the next paragraph we show how these are computed with the support of a time series or a stochastic process. As a matter of fact, the dating algorithm is completed by establishing rules for scoring the transition probabilities. It is perhaps useful at this point to discuss briefly how the features of the MC depend on those two elementary transition probabilities; this can be done by deriving the ergodic probabilities of expansions and recessions and those of peaks and troughs, which are easily computed from the ergodic probabilities of the 5th order MC, by marginalising previous states.

The following table provides the ergodic expansion probabilites for different values of $p_{_{RT}}$ and $p_{_{EP}}$:

	$p_{_{RT}}$			
$p_{_{EP}}$	0.05	0.15	0.25	0.35
0.05	0.50	0.73	0.81	0.85
0.15	0.27	0.50	0.61	0.67
0.25	0.19	0.39	0.50	0.57
0.35	0.15	0.33	0.43	0.50

The ergodic recession probabilities concerned are obtained by transposing the table. When $p_{_{RT}} = p_{_{EP}}$, these probabilities always equal $1/2$.

The ergodic probabilities of a peak are presented in the table below. It should be noticed that the table is symmetric, a fact that underlies a major implication of the chain, namely that the probability of a peak is equal to that of a trough.

	p_{RT}			
p_{EP}	0.05	0.15	0.25	0.35
0.05	0.02	0.03	0.04	0.04
0.15	0.03	0.06	0.08	0.08
0.25	0.04	0.08	0.10	0.11
0.35	0.04	0.08	0.11	0.12

Also, these probabilities do not change sensibly as either p_{RT} or p_{EP} or both are increased: this stems from the minimum duration constraints that limit the number of turning points.

The numbers reported in the two tables establish that if conditional probability of terminating an expansion and a recession are respectively 5% and 25%, an average of 81 periods out of 100 will be spent in expansion and 8 turning points will be experienced (four full cycles in 25 years, in the quarterly case).

2.2 Scoring the transition probabilities

As seen previously, the characterisation of the phases of the business cycle and the duration constraints define an underlying MC that is fully specified once the core parameters, p_{RT} and p_{EP} are known. These can be estimated by maximum likelihood techniques from an observed time series in a model based framework, if it is assumed that the latter is a realisation of a stochastic process that is dependent upon the state of the economy as represented by the chain. This idea is at the foundation of the class of Markov-Switching models, that postulate that the growth rate and/or the innovation variance and/or the transmission mechanism vary according to recessions and expansions.

In this paper we adopt the alternative strategy of scoring the two parameters according to patterns in the series, y_t. Hence we follow Harding and Pagan's non-parametric approach; see Harding and Pagan (2003,a), for a comparison with the parametric one, and the interesting exchange with Hamilton (2003, Harding and Pagan, 2003,b) it has originated.

In particular, we will concentrate on the BBQ rule by Harding and Pagan (2001), according to which an expansion termination sequence, ETS_t, and a recession terminating sequence, RTS_t, are defined respectively as follows:

$$\begin{aligned}\mathsf{ETS}_t &= \{(\Delta y_{t+1} < 0) \cap (\Delta_2 y_{t+2} < 0)\} \\ \mathsf{RTS}_t &= \{(\Delta y_{t+1} > 0) \cap (\Delta_2 y_{t+2} > 0)\} \end{aligned} \tag{1}$$

The former defines a candidate point for a peak, which terminates the expansion, whereas the latter defines a candidate for a trough. Here Δ is the backward difference operator, $\Delta y_t = y_t - y_{t-1}$.

The joint distribution of the sequences $\{\mathsf{ETS}_t, \mathsf{RTS}_t, t = 1, \ldots, T\}$ depends on the stochastic process generating the available series and is usually analytically intractable, due to the presence of serial correlation and the nature of the termination sequences, which are not mutually exclusive. As regards the latter, denoting by $\overline{\mathsf{ETS}}_t$ the complementary event of ETS_t, $\overline{\mathsf{RTS}}_t$ that of RTS_t, and defining $\mathcal{P}_t^{(ETS)} = P(\mathsf{ETS}_t), \mathcal{P}_t^{(RTS)} = P(\mathsf{RTS}_t)$, the joint probability distribution of the possible occurrences at time t is provided by the following table:

	ETS_t	$\overline{\mathsf{ETS}}_t$	$Marginal$
RTS_t	0	$\mathcal{P}_t^{(RTS)}$	$\mathcal{P}_t^{(RTS)}$
$\overline{\mathsf{ETS}}_t$	$\mathcal{P}_t^{(ETS)}$	$1 - \mathcal{P}_t^{(ETS)} - \mathcal{P}_t^{(RTS)}$	$1 - \mathcal{P}_t^{(RTS)}$
$Marginal$	$\mathcal{P}_t^{(ETS)}$	$1 - \mathcal{P}_t^{(ETS)}$	1

whence it can be seen that ETS_t and RTS_t cannot both be true at the same time.

Serial correlation complicates the computation of $\mathcal{P}_t^{(ETS)}$ and $\mathcal{P}_t^{(RTS)}$, since the terminating sequences are not independent of their past; furthermore, it must be stressed that the BBQ rule induces autocorrelation itself, that is even if $\Delta y_t \sim \mathrm{NID}(\mu, \sigma^2)$, e.g. y_t is a random walk, $\{\mathsf{ETS}_t, \mathsf{RTS}_t, t = 1, \ldots, T\}$ will be autocorrelated. Therefore it seems that the only way to go about the characterisation of the business cycle for a particular stochastic process is stochastic simulation.

Let us return to the non parametric scoring of the transition probabilities according to the available time series. If at time t the chain S_t^* is in any of the expansionary states for which a transition to a peak is possible and an expansion terminating sequence occurs at time $t+1$, i.e ETS_{t+1} is true, then we move to a new state S_{t+1}^*, such that $S_{t+1} = \mathsf{P}_{t+1}$ and the previous four elementary states are common to the last four in S_t^*.

It is useful at this point to classify the states of S_t^* by defining the sets:

$\mathcal{S}_{EP} = \{S_3^*, S_4^*, S_5^*, S_9^*, S_{19}^*, S_{20}^*, S_{22}^*\}$ defines the set of states featuring an expansionary state at time t ($S_t = \mathsf{EC}_t$) and that are available for a transition to a peak.

$\mathcal{S}_{P} = \{S_6^*, S_{10}^*, S_{21}^*, S_{23}^*\}$ defines the set of states featuring a peak at time t ($S_t = \mathsf{P}_t$).

$\mathcal{S}_{RT} = \{S_1^*, S_7^*, S_8^*, S_{11}^*, S_{12}^*, S_{14}^*, S_{17}^*\}$ defines the set of states featuring a recessionary state at time t ($S_t = \mathsf{RC}_t$) and that are available for a transition to a trough.

$\mathcal{S}_{T} = \{S_2^*, S_{13}^*, S_{15}^*, S_{18}^*\}$ defines the set of states featuring a trough at time t ($S_t = \mathsf{T}_t$).

The set of expansionary states, $\mathcal{S}_{E}$, results from the union of the sets $\mathcal{S}_{EP}, \mathcal{S}_{P}$ and S_{16}^*; in symbols: $\mathcal{S}_{E} = \mathcal{S}_{EP} \cup \mathcal{S}_{P} \cup S_{16}^*$

The set of recessionary states, $\mathcal{S}_R$, is the union of $\mathcal{S}_{RT}, \mathcal{S}_{T}$ and S_{24}^*: $\mathcal{S}_{R} = \mathcal{S}_{RT} \cup \mathcal{S}_{T} \cup S_{24}^*$

The scoring rules are then formalised in the following algorithm:

If $\{S_t^* = s_{EP}, s_{EP} \in \mathcal{S}_{EP}\}$ *and* ETS_{t+1} *is true, then* $\{S_{t+1}^* = s_{P}, s_{P} \in \mathcal{S}_{P}\}$. *Hence, the transition probability* p_{EP} *is computed as:*

$$\begin{aligned} p_{EP} &= P\left(\{S_t^* = s_{EP}, s_{EP} \in \mathcal{S}_{EP}\} \cap \mathsf{ETS}_{t+1}\right) \\ &= \mathrm{I}(\mathsf{ETS}_{t+1}) \sum_{s_{EP} \in \mathcal{S}_{EP}} P\left(S_t^* = s_{EP}\right), \end{aligned} \tag{2}$$

where $\mathrm{I}(\cdot)$ *is the indicator function. Else, if* ETS_{t+1} *is false then the expansion is continued, that is* $S_{t+1}^* = s_{EP}, s_{EP} \in \mathcal{S}_{EP}$; *the associated transition probability is* $p_{EE} = 1 - p_{EP}$.

Else, if $\{S_t^* = s_{RT}, s_{RT} \in \mathcal{S}_{RT}\}$ *and* RTS_{t+1} *is* true, *then* $\{S_{t+1}^* = s_{T}, s_{T} \in \mathcal{S}_{T}\}$. *Hence, the transition probability* p_{RT} *is computed as:*

$$\begin{aligned} p_{RT} &= P\left(\{S_t^* = s_{RT}, s_{RT} \in \mathcal{S}_{RT}\} \cap \mathsf{RTS}_{t+1}\right) \\ &= \mathrm{I}(\mathsf{RTS}_{t+1}) \sum_{s_{RT} \in \mathcal{S}_{RT}} P\left(S_t^* = s_{RT}\right), \end{aligned} \tag{3}$$

Else, if RTS_{t+1} *is false, then the recession is continued, that is* $S_{t+1}^* = s_{RT}, s_{RT} \in \mathcal{S}_{RT}$; *the associated transition probability is* $p_{RR} = 1 - p_{RT}$ ∎

The case when ETS_{t+1} and RTS_{t+1} are both false is implicitly covered by the above dating rule. Probabilistic dating based on a maintained stochastic process replaces the indicator function, $\mathrm{I}(\cdot)$, with the probability of the terminating sequences, $\mathcal{P}_{t+1}^{(ETS)}, \mathcal{P}_{t+1}^{(RTS)}$.

Let now $\mathcal{F}_t$ denote the collection of $\mathrm{I}(\mathsf{ETS}_j), \mathrm{I}(\mathsf{RTS}_j), j = 1, 2, \ldots, t$, and let $P(S_t^* | \mathcal{F}_t)$ denote the probability of being in any particular state at time t conditional on this information set. Assuming that this probability is known we can compute recursively the probability of the chain at subsequent times by the following filter:

i. Given the availability of $P(S_t^*|\mathcal{F}_t)$ at time t, let us denote by π_t^* the $m \times 1$ vector containing them, with $m = 24$ in the quarterly case. Define the two $m \times 1$ selection vectors v_{EP}, with ones corresponding to the elements of $\mathcal{S}_{EP}$ and zero otherwise, and v_{RT}, with ones corresponding to the elements of $\mathcal{S}_{RT}$ and zero otherwise.

ii. Compute the transition probabilities of the chain according to (2) and (3), that is $p_{EP} = \mathrm{I}(\mathsf{ETS}_{t+1})v'_{EP}\pi_t^*$, $p_{RT} = \mathrm{I}(\mathsf{RTS}_{t+1})v'_{RT}\pi_t^*$, $p_{EE} = 1 - p_{EP}$, $p_{RR} = 1 - p_{RT}$ and insert them in the transition matrix of the chain, hereby denoted by $\mathcal{T}$.

iii. Compute the probabilities $P(S_{t+1}^*|\mathcal{F}_{t+1})$ belonging to the vector π_{t+1}^* as

$$\pi_{t+1}^* = \mathcal{T}'\pi_t^*.$$

The algorithm is initialised by assigning values to π_1^*: if one knows that at the beginning of the sample we are in expansion, $\pi_1^* \propto v_E$, where v_E is the selection vector corresponding to $\mathcal{S}_E$, whereas if we know that the system was in recession, $\pi_1^* \propto v_R$, where v_R selects the elements of $\mathcal{S}_R$. Otherwise, we can learn from the first observations about the initial probability vector, and in the case these are ambiguous use a uniform prior, which amounts to setting the elements of π_1^* equal to $1/m$.

The algorithm recursively produces $P(S_t^*|\mathcal{F}_t)$, for all $t = 1, \ldots, T$, and hence, marginalising previous states $S_{t-j}, j = 1, 2, 3, 4$, the probabilities of each elementary event, $P(S_t|\mathcal{F}_t)$, and $P(\mathsf{E}_t|\mathcal{F}_t) = P(\mathsf{EC}_t|\mathcal{F}_t) + P(\mathsf{P}_t|\mathcal{F}_t)$, $P(\mathsf{R}_t|\mathcal{F}_t) = P(\mathsf{RC}_t|\mathcal{F}_t) + P(\mathsf{T}_t|\mathcal{F}_t)$, can be obtained. For instance,

$$P(\mathsf{E}_t|\mathcal{F}_t) = \sum_{s_E \in \mathcal{S}_E} P\left(S_t^* = s_E\right).$$

2.3 Dating monthly time series

The dating algorithm is readily adapted to the monthly frequency. The minimum durations are respectively 6 months for each phase and 15 months for full cycles. This yields a 15th order MC that can be represented as a first order MC with $m = 122$ states. As far as the scoring of the transition probabilities is concerned, the terminating sequences are defined as follows:

$$\begin{aligned}
\mathsf{ETS}_t &= \{\textstyle\bigcap_{j=1}^{5}(\Delta_j y_{t+j} < 0)\} \\
\mathsf{RTS}_t &= \{\textstyle\bigcap_{j=1}^{5}(\Delta_j y_{t+j} > 0)\}
\end{aligned}$$

where $\Delta_j = 1 - L^j$.

2.4 Dating unobserved components and filtering

In real applications it is usually the case that we date the business cycle on a signal extracted from a time series, rather then on the original series itself. For instance, all the series considered in this paper are seasonally adjusted.

If the unobserved component, here denoted by ς_t, arises from a model-based signal extraction technique, then, apart from the obvious option of dating the sequence $\tilde{\varsigma}_{t|T}$, which denotes some inference (usually the expectation) on the signal conditional on the full available sample, we can score the transition probabilities using the probability of the terminating sequences $\mathcal{P}_{t+1}^{(ETS)}$ and $\mathcal{P}_{t+1}^{(RTS)}$, referred to ς_t, rather than the indicator function. These probabilities can be estimated via the simulation smoother proposed by de Jong and Shephard (1995), which repeatedly draws simulated samples from the posterior distribution $\tilde{\varsigma}_t^{(i)} \sim \varsigma_t | y_1, \ldots, y_T$, so that repeating the draws a sufficient number of times we can get Monte Carlo estimates of different aspects of the marginal and joint distribution of the terminating sequences. The virtue of this strategy is that we can be aware of the uncertainty surrounding the estimated turning points.

There are other plausible reasons for dating unobserved components rather than the original series itself: the first is to render the dating procedure more resistant to outlier contamination; the second is to censor variability that is not relevant to the analysis of business cycle fluctuations, such as high frequency noise. The need especially arises with reference to monthly industrial production, which displays relevant high frequency components even after a working days adjustment. This motivated us to employ in specific cases low pass filters defined on the basis of the popular Hodrick and Prescott (1997, HP henceforth) filter. The latter is the minimum mean square estimator of the signal for the model:

$$\begin{array}{rcl} y_t & = & \varsigma_t + \epsilon_t, \qquad\qquad t = 1, 2, \ldots, T, \\ \Delta^2 \varsigma_t & \sim & \mathrm{NID}(0, \sigma_\varsigma^2), \quad \epsilon_t \sim \mathrm{NID}(0, \lambda\sigma_\varsigma^2), \end{array}$$

where λ is the smoothness parameter. Using frequency domain arguments it can be shown that the HP filter can be interpreted as a low-pass filter (Gómez, 2001) with implicit cut-off frequency, ω_c, that is related to the smoothness parameter λ by: $\omega_c = \arccos(1 - 0.5\lambda^{-1/2})$. As such, the HP filter with parameter λ will significantly reduce the amplitude of high frequency components, characterised by a periodicity smaller than $p = 2\pi/\omega_c$; the latter is the period in time units corresponding to the cut-off frequency. If we let s denote the number of observations in a year, in the sequel we shall write HP(p/s) to denote a low-pass filter that retains to a large extent those components with period greater than p/s years. For instance, HP(1.25) aims at dampening all the fluctuations with a periodicity less than the minimum cycle duration, i.e. five quarters or 15 months.

2.5 Dating the deviation cycle

Dating the deviation cycle raises a very controversial measurement issues, with respect to which we take a rather eclectic view. This led us to experiment various nonparametric and parametric measures, among which the Baxter and King (BK, 1999) filter, the HP detrended series, and an HP band-pass filter for business cycle extraction designed from the difference of two HP detrending filters, the first with cut-off frequency $\omega_c = 2\pi/(1.25s)$, corresponding to a period of 1.25 years and the second for $\omega_c = 2\pi/(8s)$, corresponding to a period of 8 years. The corresponding measure aims at retaining to a given extent those fluctuations whose period is comprised between these two thresholds.

The dating algorithm needs also to be adapted to the notion of a deviation cycle. For instance, we want to avoid identifying a peak when output is below its trend level. We can impose that is so by requiring that an expansion must have brought output above trend. Therefore, for a zero mean deviation cycle, we may want to redefine the terminating sequences as follows:

$$\begin{aligned} \mathsf{ETS}_t &= \{(y_t > 0) \cap (\Delta y_{t+1} < 0) \cap (\Delta_2 y_{t+2} < 0)\} \\ \mathsf{RTS}_t &= \{(y_t < 0) \cap (\Delta y_{t+1} > 0) \cap (\Delta_2 y_{t+2} > 0)\} \end{aligned} \tag{4}$$

The algorithm scores the cycle in real time; thus, nothing prevents that, within a period in which $y_t < 0$, the first local minimum is flagged as a trough and that this is above the global minimum. A solution would be to run the algorithm on the reversed series, but this strategy is effective only if just two minima occur within that period. Our experience is that multiple minima are a likely occurrence, and thus our preferred alternative strategy works out as follows:

- Run the dating algorithm on the cumulated y_t series, $c(y)_t$. The turning points detected by this procedure correspond to the crossing of the zero line. For instance a peak in $c(y)_t$ coincides with the latest $y_t > 0$; all subsequent values will be below zero until a trough is found, which is the last point such that $y_t < 0$. Minimum duration constraints continue to operate, although they relate to successive crossing of zero.

- Locate the maximum of y_t between a trough and a peak, which corresponds to the global peak of y_t; locate the minimum of y_t between a peak and the next trough, which corresponds to the global trough of y_t.

2.6 Depth (amplitude) restrictions

Amplitude restrictions aim at isolating major fluctuations, thereby robustifying the dating process. They inevitably introduce a judgemental element, but they enforce *depth*, that together with *diffusion* and *duration*, makes up the three "D"s, that represent key features for the qualification of economic fluctuations as business cycles.

The algorithms presented above can be readily modified to enhance depth or amplitude restrictions on the definition of expansion- and recession- terminating sequences.

Given a threshold $c > 0$, for the quarterly case we can define:

$$\begin{aligned} \mathsf{ETS}_t &= \{(\Delta y_{t+1} < -c) \cap (\Delta_2 y_{t+2} < -2c)\} \\ \mathsf{RTS}_t &= \{(\Delta y_{t+1} > c) \cap (\Delta_2 y_{t+2} > 2c)\} \end{aligned}$$

In the classical case the amplitude constraints need not be symmetric, due to the fact that expansions are longer but less steep, which suggests that that c could vary according to the phase.

An alternative strategy is to employ signal extraction techniques, e.g. low-pass filters, to isolate the most relevant fluctuations. While this strategy comes at the cost of abandoning sharp turning point identification, because the probability of peaks and troughs is smeared onto adjacent sample points, it certainly offers an alternative way of making the dating algorithm more resistant to outliers and high frequency dynamics, that do not pertain to the business cycle.

3 The aggregate Euro area cycle

This section analyses aggregate time series data available for the Eurozone, both from the classical and deviation cycle perspectives. The emphasis is on Euro area real GDP, but we also consider its decomposition into expenditure components and the labour market[1].

Any study of the Eurozone economy faces a problem of data availability. The Eurozone only came into being on the 1st of January 1999, and the study of business cycles needs a larger sample than three-and-a-half years. To extend the data back in time encounters the problem of aggregation when exchange rates are prone to change: in these circumstances there is no "perfect" method of aggregation. We have employed, for the most part, the data that have been constructed for the ECB's

[1]All the computations in the paper were performed using the object oriented matrix programming language Ox 3.0 by Doornik (2001), and the library of state space function SsfPack 2.3 by Koopman et al. (1999).

Area-wide model (AWM, Fagan, Henry and Mestre, 2001), conducting a check against the series produced by Beyer, Doornik and Hendry (BDH, 2001).

3.1 Classical cycle

Our classical business cycle chronology is presented compactly in figure 1 with reference to two alternative GDP measures: the “AWM series” and the ”BDH” series. The former has a longer sample period (1970-2001) than the latter (1980-2001) and is able to reveal an additional cycle. Otherwise, the three cycles identified in the shorter data period overlap almost exactly, the only difference being in the location of the last trough, which is anticipated by one quarter if one takes the BDH measure, and the three decades from 1970 comprise four cycles altogether. The chronology of turning points, not surprisingly, is also exactly as in Harding and Pagan (2001). Figure 1 also presents the expansion/recession classification based on GDP growth rates.

Table 2 displays some descriptive statistics measuring some basic business cycle features for a set of Eurozone time series available from the ECB’s Area-wide model database. Those pertaining to GDP (denoted YER in the table) highlight a notable asymmetry between the average length of expansions and recessions, the former being much longer (28 quarters) than the latter (3 quarters), which is to be expected of classical cycles in a growing economy. The probabilities of being in one or other phase reflect the relative values of these phase lengths (about 90% versus 10%). The amplitudes of the expansion periods are also much bigger than those of the recession periods. Expansions last longer, and are steeper than recessions, which are quite brief and yet more gently sloped.

Table 2 also displays comparable information for a number of other series - notably the national accounts categories pertaining to private and government consumption (PCR and GCR), fixed capital formation (ITR), imports and exports of goods and srvices, and net exports (MTR, XTR, Net Imp) and inventory change (SCR), together with employment (LNN), productivity (LPROD), unemployment (URX) and unit labour costs (ULC). Standard theory would suggest that investment and inventories are likely to be the most cyclical components of GDP, and this expectation is borne out in the data: more cycles are identified, the recession and expansion probabilities are more nearly equal and the steepness of the phases is more nearly equal. It is not surprising perhaps to find, on the other hand, that the cyclical behaviour of private consumption is much in line with that of GDP as a whole, whilst government consumption is the smoothest component of all. Exports and imports of goods and services, and even more, the net of the two, seem to be highly cylical in their behaviour. Employment and unemployment exhibit more cycles than GDP, which might seem surprising.

[Figure 1 about here]

3.2 Deviation Cycles

The deviation or growth cycle typically represents an unobserved component and we have experimented with various methods and filters to extract it, both in the model-based and the nonparametric frameworks. In the sequel we shall denote the deviation cycle by ψ_t.

Figure 2 presents several measures of the deviation cycle in the Euro area GDP, with the associated turning points detected by the dating algorithm outlined in paragraph 2.5, with restrictions on the size of the fluctuations that will be discussed shortly. The first measure is the Baxter and King cycle, that is available for the central part of the sample excluding the first and last 12 quarters; the second, displayed on the upper right panel, is the HP band-pass cycle, that results from subtracting the HP trend with smoothing parameter $\lambda = 0.52$, which defines a low-pass filter dampening the fluctuations with a period smaller than 5 quarters (1.25 years), from the HP trend with smoothing parameter $\lambda = 677$, which in turn defines a low-pass filter cutting off the fluctuations with a period smaller than 8 years. The resulting component retains to a given extent the fluctuations with a period between 5 quarters and 8 years, and in this respect produces estimates of the cycle that are comparable to, although slightly noisier than the BK cycle, without suffering from unavailability of the end of sample estimates.

The bottom panels display parametric measures of the output gap derived respectively from a bivariate model of GDP and CPI inflation and a multivariate model based on total factor productivity, labour force participation rates, the unemployment rate, capacity utilisation and CPI inflation, implementing the production function approach, see Proietti, Musso and Westermann (2002) for details. The notion of an output gap is more specialised than the deviation cycle in output, since it provides a measure of inflationary pressures. This poses a new issue to the dating of the gaps: one possibility is to score $\psi_t > 0$ and $\psi_t < 0$, as the interest lies in dating periods in which the inflationary pressures are positive or negative. However, as the evidence reported in Proietti, Musso and Westermann (2002) clearly points out, and in line with the suggestions by Harding and Pagan, it is the change effect associated to $\Delta\psi_t$ that is more relevant than the level effect exerted by the output gap, which brings us back to the problem of dating expansions and recessions in the level of ψ_t. We also notice in passing that the scoring of the gap according to whether it is positive or negative is a by-product of the dating algorithm.

Figure 2 shows a broad agreement in the identified turning points: the 74.1 and 80.1 peaks are common to the four representations. The location of the start of the 90s recession is more uncertain since there are two neighbouring local maxima at the beginning of 1990 and 1992, which is featured by the expenditure components and the GDP of individual countries. Also the beginning of the '80s expansion is scored differently by the different methods.

[Figure 2 about here]

As stated above, the dating algorithm featured restrictions on the amplitude of fluctuations: in its first stage, by which change of sign in ψ_t are identified by running the usual dating algorithm on the cumulated cycle, amending the definition of the expansions and terminating sequences as in paragraph 2.6 by setting $c = 0.005$, which amounts to censoring fluctuations around zero with amplitude less than 0.5% of total GDP.

Table 3 presents some characteristics of the HP band-pass deviation cycles, this time not using any censoring rule on the amplitude of the fluctuations. This results in a relatively large number of turning points, and affects the duration and the amplitude statistics. A stylised fact that is however robust to the choice of censoring rules is that the average amplitude of recessions and expansions is about the same, as implied by the symmetry of the cyclical model or signal extraction filter. Table 3 confirms that investment (ITR) is one of the most cyclically variable expenditure component of GDP, featuring an average amplitude of 5% for both phases. Employment and unemployment are now less cyclical than GDP.

[Tables 2 and 3 about here]

4 Country specific cycles

Our analysis of country specific cycles focuses on two data sets, the first relating to GDP at constant prices for five countries, Germany, France, Italy, UK and the USA, starting from 1970 and available from various sources, among them the OECD Main Economic Indicators and the US Bureau of Economic Analysis. The German series, made available by the IFO, has been seasonally adjusted, corrected for working days and for the level shift due to reunification, using the basic structural model with regression effects (Harvey, 1989). The Eurozone series is used for comparison. The second set is produced by Eurostat and provides a highly comparable set of statistics about real GDP based on the new system of national accounts (ESA95), but for a shorter sample.

Figure 3 presents the turning points of the classical BC for the Euro Area, Germany, France, Italy, UK and the USA, identified using the HP(1.25) filtered series on the the first data set. We recall that this is a low-pass filter dampening the fluctuations with a period less than five quarters, which strictly do not pertain to the business cycle.

[Figure 3 about here]

We next address the issue of synchronisation and concordance among the country specific classical business cycles. The dating algorithm, applied to the HP(1.25) filtered GDP series furnishes the indicator variables of the state of the economy, R_t and E_t (recession and expansion, respectively). The index of concordance between the classical BC for the individual countries and the Euro Area aggregate cycle, I_{ij}, is simply the percentage of time units spent in the same phase, also known as the simple matching similarity coefficient. The mean corrected concordance index, is $I^*_{ij} = I_{ij} - \bar{I}_{ij}$, where $\bar{I}_{ij}$ is the estimate of the expected value of the index under the assumption of independence, which represents the fraction to be expected if there were no relationship between the cycle in the two countries. Finally, dividing I^*_{ij} by its asymptotic standard error estimated nonparametrically using a Newey-West estimator, we get the standardized index, which is reported in Table 4 and can be interpreted as a t-statistic for the null hypothesis of independence of the cycles (see the Appendix for details).

[Table 4 about here]

Looking at the Euro area as a whole, the concordance is lowest with the UK, though still statistically different from zero, highest with the countries within the Euro area, as expected, and intermediate with the US. Germany, France and Italy are also the group of countries with the highest cross concordances. The highest concordance for the UK is with the US.

Harding and Pagan (2001, 2002, Pagan, 2002) also propose to regress the recession indicator for one country on the same indicator for another country, and evaluate BC independence using the t-statistic for the significance of the parameters, computed using HAC standard errors. The results of running such a regression, reported in table 5, now suggest that the UK cycle is even independent of that of the EA, Germany and France, whereas there is a significant association with Italy and the US; independence across the Euro area countries is strongly rejected.

[Table 5 about here]

The analysis of the Eurostat series, not reported here for brevity, is useful in pinpointing an additional peak that was not identified from the other Eurozone series considered before, taking place in the second quarter of 2001. This is mainly due to Germany, but is also anticipated in the series for Finland, Belgium, the Netherlands and Austria.

As far as the deviation cycles are concerned, the standardised concordance index (table 6) and the robust test for cycle independence (table 7), computed on the HP bandpass cycles, largely confirm the previous outcome: there is a high degree of synchronisation within the Euro area, with the lowest concordance for the US and intermediate for the UK; in all cases the hypothesis of business cycle independence is rejected.

[Tables 6 and 7 about here]

5 Monthly indicators

This section focuses on the analysis of business cycles in monthly industrial production series for most European countries and the U.S. The series, seasonally adjusted, are drawn from the OECD Main Economic Indicators and cover a sample period that differs for the individual countries, but is usually rather long.

We started our investigation with the identification of outlying observations; the strategy was to fit a structural time series model (Harvey, 1989) and to add pulse intervention variables one at a time in correspondence with the sample observation that were characterised by an irregular auxiliary residual greater than 4 in absolute value; see Harvey and Koopman (1992) for the definition of auxiliary residuals and their use for outlier and structural break detection.

Despite the outlier correction, too many turning points were identified due to the presence of high frequency components that may result from intrinsic volatility, underadjustment of working days variation and other events such as moving festivals and strikes. This problem may be tackled either by setting up amplitude restrictions or by smoothing the series.

Figure 4 illustrates the impact of using different filters on dating the classical business cycle. The plots on the left hand side refer to the signal ς_t, introduced in section 2.4, produced by HP(1.25), which is tailored to dampen out the fluctuations with a period less than 15 months, whereas those on the right refer to the smoother signal extracted by HP(4), that, broadly speaking, passes the fluctuations with period greater that four years (48 months). We employ the simulation smoother to estimate the probability of recessions and of a turning point; as expected HP(1.25) produces a relatively large number of cycles. However, some stylised facts are common. The turning points are not particularly sharp as their probabilities are smeared over adjacent data points, but the recession and expansion probabilities are rather sharp. This is due to the uncertainty surrounding the signal estimates in an environment where noise contamination is high. The time series pattern of the probabilities highlights some interesting features, among which it is remarkable that expansion and recession probabilities behave asymmetrically (expansion termination is usually quicker than recession termination). Moreover, when average growth is reduced, as occurs in France in the second half of the 70s, the probability of recession is higher. For Germany, France and Italy the plots display a high degree of synchronisation.

As for concordance analysis, table 8 reports the standardized concordance index for all the available chronologies. The coefficients are in general all rather high for the countries in the Euro area, whose business cycles turn out to be also not independent from those of the UK and US. Similar results, not reported here on account of space constraints,are obtained for the HP band-pass deviation cycles.

[Figure 4 and table 8 about here]

6 Diffusion and Multivariate Business Cycle Assessment

An index of business cycle diffusion measures the percentage of economic time series in a certain state, e.g. recession. It typically aims at assessing on a 0-1 continuous scale how business cycle movements are spread throughout the economy, by looking at several phenomena that have know nature, eg. coincident or leading.

There are two ways in which diffusion indexes can be constructed. The first amounts to scoring each individual time series and then taking the cross-sectional average:

$$D_t = \frac{1}{N}\sum_{i=1}^{N} S_{it}, \qquad t = 1, \dots, T$$

where S_{it} takes value 1 in recessions and zero otherwise, and N is the cross-sectional dimension.

It can be worth weighting the series according to their economic relevance and/or their proved efficacy in signalling recessionary events. If a system of (possibly time-varying) weights w_{it} is available then

$$D_t = \sum_{i=1}^{N} w_{it} S_{it}, \quad t = 1, \dots, T, \quad \sum_i w_{it} = 1.$$

The underlying model is that the aggregate index, D_t, is a finite mixture of a Markov process with two states, the mixture probabilities being given by w_{it}. Suppose that the individual time series are the components of an aggregate $y_t = \sum_i w_i y_{it}$ and that we score recessions according to the calculus rule, that is $S_t = \mathrm{I}(\Delta y_t < 0)$, where $\mathrm{I}(\cdot)$ is the indicator function, then

$$E(S_t) = P(\sum_i w_i \Delta y_{it} < 0) > E(D_t) = \sum_{i=1}^{N} w_{it} E(S_{it}),$$

so that the diffusion index does not measure the probability of a recession in the aggregate series; rather it measures the proportion of the aggregate that is in a recession.

The second method to compute diffusion indexes exploits is based upon scoring the transition probabilities of the Markov chain using the probability attached to expansion and recession terminating sequences, determined according to the following rules:

$$\mathcal{P}_t^{(ETS)} = \sum_{i=1}^{N} w_{it} \mathrm{I}(\mathsf{ETS}_{it}), \quad \mathcal{P}_t^{(RTS)} = \sum_{i=1}^{N} w_{it} \mathrm{I}(\mathsf{RTS}_{it}).$$

Recalling that an expansion (recession) terminating sequence defines a candidate point for a peak (trough), the dating rules set out in paragraph 2.2 imply that the transition probabilities depend on the sum of the weights of the series that are in those two terminating sequences. Again, the underlying assumption is that the aggregate ETS_t is a finite mixture of cross-sectional ETS_{it} and the dating algorithm furnishes probabilities that must be interpreted as $P(D_t = 1)$, not as $P(S_t = 1)$.

Assessing the diffusion of the business cycle in the Euro area requires the evaluation of sector and country specific data, and many disaggregated time series, but given our data availability for the present being we consider three sets of data that can be used to produce a multivariate assessment of the classical cycle in the Euro area: the first is made up of the 5 expenditure components of GDP (private consumption, government consumption, fixed capital formation, net exports and variation in stocks) considered in section 3.1. The set of weights is immediately available from the GDP shares. The second set considers total factor productivity, as measured by Solow's residual using the time-averaged labour share $\alpha = 0.35$ and a constant returns to scale Cobb-Douglas technology, total employment, and capital. This yields another decomposition of log output into components whose weights are proportional to the Cobb-Douglas weights. The third set consists of the 12 industrial production series for the Euro area countries; the weights were obtained from the total Gross Value added at basic prices for the year 2001, available from the individual countries account (except for Greece, Luxembourg and Ireland, for which it was interpolated from total GDP estimates).

Figure 5 presents the diffusion indices emerging from the three distinct sets. The plot reveals the following: the diffusion of recessions is higher for industrial production and there is a tendency to peaking with a short lead, usually one quarter. A recessionary pattern that is idiosyncratic to the industrial sector can be found in 1987. Industrial production and the variables in the production function approach (PFA) signal entry to a recessionary state in 1990 and 1991 respectively, whereas the index based on expenditure components peaks in 1992. For the PFA index an important contribution is made by labour, which peaks before GDP. It is also worth noticing that the three indices behave asymmetrically along the time axis; this feature stems from the fact that the proportion of time series entering a recession is larger than that leaving it, which explains the positively skewed pattern.

The example also illustrates that weighting is a crucial issue: if we were to combine the three diffusion indexes into an aggregate one by simple averaging, then we would presumably overstate the diffusion, due to the influence of the industrial production diffusion index, that dominates the others.

[Figure 5 about here]

7 Conclusions

This paper has proposed a new dating algorithm that automatically enforces alternation of turning points and duration constraints; it has shown how it is adapted to the two main definitions of the business cycle and illustrated its main uses with reference to the Eurozone business cycle and the cycles of the main constituent economies.

A number of topics for further research are suggested by the preliminary identification of the Euro area business cycle we have made. One is to examine further the issue of synchronicity or coherence between cycles. In future research one would expect to be able to track movements in the coherence of the cyclical experience of the Eurozone economies, whether in the direction of greater convergence or not. Other topics can easily be suggested: thus, following identification of the cycle, one would hope to be able to build leading indicators; and to be able to explain the main determinants of cyclical experience and its evolution over time. And, with better high frequency data, it would become possible to data the cycle more accurately.

Appendix: the standardised concordance index

Given a panel of binary indicators of the state of the economy, $S_{it}, t = 1, \ldots, T, i = 1, \ldots, N$, available for N countries, a measure of business cycle concordance between the pair of countries i and j is the simple matching similarity coefficient:

$$I_{ij} = \frac{1}{T} \sum_{t=1}^{T} \left[S_{it} S_{jt} + (1 - S_{it})(1 - S_{jt}) \right].$$

Let $\bar{S}_i = T^{-1} \sum_t S_{it}$ denote the estimated probability of being in state 1 (e.g. recession); then, under the assumption that S_{it} and S_{jt} are independent the estimate of the expected value of the concordance index is $2\bar{S}_i \bar{S}_j = 1 - \bar{S}_i - \bar{S}_j$. Subtracting this from I_{ij} gives the mean corrected concordance index (Harding and Pagan, 2001, 2002):

$$I_{ij}^* = 2 \frac{1}{T} \sum_{t=1}^{T} (S_{it} - \bar{S}_i)(S_{jt} - \bar{S}_j).$$

The asymptotic test proposed in the paper is based on a standardised concordance index. For this purpose we need to divide I_{ij}^* by a consistent estimate of the standard error of I_{ij}^* under the null of independence. Now, under the null

$$\begin{aligned} \text{Var}(I_{ij}) &= \tfrac{4}{T^2} \mathsf{E} \left[\textstyle\sum_{t=1}^{T} (S_{it} - E(S_{it})(S_{jt} - E(S_{jt})) \right]^2 \\ &= \tfrac{4}{T} \left[\gamma_i(0)\gamma_j(0) + 2 \textstyle\sum_{\tau=1}^{T-1} \tfrac{T-\tau}{T} \gamma_i(\tau)\gamma_j(\tau) \right] \end{aligned}$$

where $\gamma_i(0) = \mathsf{E}[(S_{it} - E(S_{it}))(S_{i,t-\tau} - E(S_{it}))]$.

Hence,

$$T^{1/2} I_{ij}^* \to \text{N}(0, 4\sigma^2), \quad \sigma^2 = \gamma_i(0)\gamma_j(0) + 2 \sum_{\tau=1}^{\infty} \gamma_i(\tau)\gamma_j(\tau),$$

and a consistent estimate of σ^2 is

$$\hat{\sigma}^2 = \hat{\gamma}_i(0)\hat{\gamma}_j(0) + 2 \sum_{\tau=1}^{l} \left(1 - \frac{\tau}{T}\right) \hat{\gamma}_i(\tau)\hat{\gamma}_j(\tau),$$

where l is the truncation parameter.

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Table 1: Description of the Markov chain generated by the quarterly dating rules.

States	$S^*_t = \{S_{t-4}, S_{t-3}, S_{t-2}, S_{t-1}, S_t\}$					States S^*_{t+1} that can visited			
S^*_t	S_{t-4}	S_{t-3}	S_{t-2}	S_{t-1}	S_t	S^*_{t+1}	Trans. Prob.	S^*_{t+1}	Trans. Prob.
S^*_1	P	RC	RC	RC	RC	S^*_{17}	p_{RR}	S^*_{18}	p_{RT}
S^*_2	P	RC	RC	RC	T	S^*_{19}	1		
S^*_3	P	RC	RC	T	EC	S^*_{20}	p_{EE}	S^*_{21}	p_{EP}
S^*_4	P	RC	T	EC	EC	S^*_{22}	p_{EE}	S^*_{23}	p_{EP}
S^*_5	T	EC	EC	EC	EC	S^*_9	p_{EE}	S^*_{10}	p_{EP}
S^*_6	T	EC	EC	EC	P	S^*_{11}	1		
S^*_7	T	EC	EC	P	RC	S^*_{12}	p_{RR}	S^*_{13}	p_{RT}
S^*_8	T	EC	P	RC	RC	S^*_{14}	p_{RR}	S^*_{15}	p_{RT}
S^*_9	EC	EC	EC	EC	EC	S^*_9	p_{EE}	S^*_{10}	p_{EP}
S^*_{10}	EC	EC	EC	EC	P	S^*_{11}	1		
S^*_{11}	EC	EC	EC	P	RC	S^*_{12}	p_{RR}	S^*_{13}	p_{RT}
S^*_{12}	EC	EC	P	RC	RC	S^*_{14}	p_{RR}	S^*_{15}	p_{RT}
S^*_{13}	EC	EC	P	RC	T	S^*_{16}	1		
S^*_{14}	EC	P	RC	RC	RC	S^*_1	p_{RR}	S^*_2	p_{RT}
S^*_{15}	EC	P	RC	RC	T	S^*_3	1		
S^*_{16}	EC	P	RC	T	EC	S^*_4	1		
S^*_{17}	RC	RC	RC	RC	RC	S^*_{17}	p_{RR}	S^*_{18}	p_{RT}
S^*_{18}	RC	RC	RC	RC	T	S^*_{19}	1		
S^*_{19}	RC	RC	RC	T	EC	S^*_{20}	p_{EE}	S^*_{21}	p_{EP}
S^*_{20}	RC	RC	T	EC	EC	S^*_{22}	p_{EE}	S^*_{23}	p_{EP}
S^*_{21}	RC	RC	T	EC	P	S^*_{24}	1		
S^*_{22}	RC	T	EC	EC	EC	S^*_5	p_{EE}	S^*_6	p_{EP}
S^*_{23}	RC	T	EC	EC	P	S^*_7	1		
S^*_{24}	RC	T	EC	P	RC	S^*_8	1		

Table 2: Classical BC dating of Euro Area time series: summary statistics.

	YER	PCR	ITR	MTR	XTR	GCR	LNN	LPROD	ULC	URX	SCR	Net Exp.
Number of cycles P-P	4.0000	4.0000	7.0000	5.0000	8.0000	2.0000	7.0000	4.0000	3.0000	7.0000	16.0000	13.0000
Number of cycles T-T	4.0000	4.0000	7.0000	5.0000	8.0000	2.0000	7.0000	4.0000	3.0000	6.0000	16.0000	13.0000
Average Expansion Prob.	0.9032	0.9355	0.7339	0.8629	0.8548	0.9677	0.6935	0.9113	0.9274	0.6613	0.5968	0.4719
Average Recession Prob.	0.0968	0.0645	0.2661	0.1371	0.1452	0.0323	0.3065	0.0887	0.0726	0.3387	0.4032	0.5281
Average Duration of Exp.	28.0000	29.0000	13.0000	21.4000	13.2500	60.0000	12.2857	28.2500	38.3333	11.7143	4.6250	4.6369
Average Duration of Rec.	3.0000	2.0000	4.7143	3.4000	2.2500	2.0000	5.4286	2.7500	3.0000	7.0000	3.1250	5.1975
Average Amplitude of Exp.	0.2117	0.2121	0.1398	0.3852	0.2441	0.3872	0.0297	0.1762	0.6118	0.3067	1.5200	0.7682
Average Amplitude of Rec.	-0.0143	-0.0072	-0.0519	-0.0591	-0.0261	-0.0023	-0.0129	-0.0082	-0.0064	-0.0960	-1.4997	-0.5531
Steepness of expansions	0.0076	0.0073	0.0108	0.0180	0.0184	0.0065	0.0024	0.0062	0.0160	0.0262	0.3287	0.1657
Steepness of recessions	-0.0048	-0.0036	-0.0110	-0.0174	-0.0116	-0.0012	-0.0024	-0.0030	-0.0021	-0.0137	-0.4799	-0.1064

Average phase duration is the number of periods (here, quarters) which an expansion, on average, contains. Amplitude is a measure of the depth of a cycle, that is the cumulative increase, during the expansion phase, of the measure of economic activity in question, whilst average amplitude is simply the average value over all expansion phases of this quantity. Dividing amplitude by duration gives a measure of steepness. For a sample of complete cycles the probability of being in expansion is simply the ratio of average expansion duration to the sum of average expansion and recession durations. See Harding and Pagan (2001) for more details.

Table 3: Deviation Cycles dating of Euro Area time series: summary statistics.

	YER	PCR	ITR	MTR	XTR	GCR	LNN	LPROD	ULC	URX	SCR	Net Exp.
Number of cycles P-P	10.0000	7.0000	6.0000	9.0000	8.0000	7.0000	5.0000	10.0000	7.0000	6.0000	10.0000	9.0000
Number of cycles T-T	9.0000	6.0000	5.0000	9.0000	9.0000	7.0000	5.0000	9.0000	7.0000	6.0000	11.0000	8.0000
Average Expansion Prob.	0.6290	0.6532	0.6048	0.6129	0.5565	0.6129	0.6048	0.5323	0.4839	0.4032	0.5081	0.5081
Average Recession Prob.	0.3710	0.3468	0.3952	0.3871	0.4435	0.3871	0.3952	0.4677	0.5161	0.5968	0.4919	0.4919
Average Duration of Exp.	7.8000	11.5714	12.5000	8.4444	8.6250	10.8571	15.0000	6.6000	8.5714	8.3333	6.3000	7.0000
Average Duration of Rec.	5.1111	7.1667	9.8000	5.3333	6.1111	6.8571	9.8000	6.4444	9.1429	12.3333	5.5455	7.6250
Average Amplitude of Exp.	0.0159	0.0142	0.0503	0.0548	0.0592	0.0138	0.0140	0.0137	0.0252	0.1183	1.6075	0.7576
Average Amplitude of Rec.	-0.0168	-0.0157	-0.0550	-0.0512	-0.0492	-0.0118	-0.0127	-0.0151	-0.0249	-0.1317	-1.5157	-0.8823
Steepness of expansions	0.0020	0.0012	0.0040	0.0065	0.0069	0.0013	0.0009	0.0021	0.0029	0.0142	0.2552	0.1082
Steepness of recessions	-0.0033	-0.0022	-0.0056	-0.0096	-0.0081	-0.0017	-0.0013	-0.0023	-0.0027	-0.0107	-0.2733	-0.1157

See footnote to table 2.

Table 4: Classical BC: Standardised Concordance Index.

	EA	D	UK	F	I	US
EA	-	7.15	2.48	6.29	6.35	3.40
D	7.15	-	1.93	5.41	5.43	4.43
UK	2.48	1.93	-	3.00	2.33	3.50
F	6.29	5.41	3.00	-	4.59	1.92
I	6.35	5.43	2.33	4.59	-	3.20
US	3.40	4.43	3.50	1.92	3.20	-

Table 5: Test for BC independence using HAC standard errors (Newey-West estimator with truncation parameter equal to 5).

	EA	D	UK	F	I	US
EA	-	52.52	1.80	4.41	12.15	2.62
D	7.85	-	1.53	3.07	6.73	3.33
UK	1.87	1.66	-	1.90	2.37	4.25
F	10.47	9.02	1.89	-	5.49	1.51
I	4.86	4.79	1.82	2.94	-	2.57
US	3.02	4.02	2.65	1.52	3.70	-

Table 6: Deviation cycles: Standardised Concordance Index.

	EA	D	UK	F	I	US
EA	-	4.83	3.42	4.71	5.77	2.75
D	4.83	-	2.95	2.66	3.48	2.53
UK	3.42	2.95	-	2.07	2.33	2.26
F	4.71	2.66	2.07	-	3.67	2.47
I	5.77	3.48	2.33	3.67	-	1.90
US	2.75	2.53	2.26	2.47	1.90	-

Table 7: Test for deviation cycle independence using HAC standard errors (Newey-West estimator with truncation parameter equal to 5).

	EA	D	UK	F	I	US
EA	-	15.27	4.96	12.93	11.12	4.45
D	8.89	-	2.06	4.53	4.49	2.75
UK	3.68	2.14	-	5.39	3.33	6.56
F	8.38	4.68	4.91	-	4.81	2.87
I	13.02	6.30	5.55	5.32	-	3.22
US	3.60	3.78	4.27	2.28	2.79	-

Table 8: Industrial Production - Classical BC: Standardised Concordance Index.

A	-	3.94	2.52	2.99	4.61	5.70	3.24	2.28	3.08	4.11	1.10	2.03	4.02	4.47	1.16	2.24
B	3.94	-	2.29	1.72	4.48	3.87	1.17	1.21	3.77	3.74	1.45	1.77	3.62	2.15	2.36	1.26
DK	2.52	2.29	-	1.62	2.53	4.22	1.86	2.85	2.92	3.28	0.52	2.07	2.32	2.17	2.80	2.16
FIN	2.99	1.72	1.62	-	3.19	2.69	2.24	3.05	3.30	3.19	3.79	1.44	4.82	3.35	2.90	3.07
F	4.61	4.48	2.53	3.19	-	4.44	4.03	2.77	5.04	4.35	2.66	2.87	4.38	5.02	2.57	2.33
D	5.70	3.87	4.22	2.69	4.44	-	2.61	3.88	3.25	4.98	2.17	3.06	3.49	3.47	3.71	3.22
GR	3.24	1.17	1.86	2.24	4.03	2.61	-	0.14	2.34	1.39	0.46	1.41	1.95	3.31	0.61	1.49
IRL	2.28	1.21	2.85	3.05	2.77	3.88	0.14	-	3.11	3.58	2.09	2.83	3.09	3.59	3.73	4.54
I	3.08	3.77	2.92	3.30	5.04	3.25	2.34	3.11	-	5.79	2.90	1.81	3.54	4.11	2.49	2.91
NL	4.11	3.74	3.28	3.19	4.35	4.98	1.39	3.58	5.79	-	2.70	2.10	4.15	3.83	2.77	3.60
N	1.10	1.45	0.52	3.79	2.66	2.17	0.46	2.09	2.90	2.70	-	-0.19	3.39	2.08	2.99	2.32
P	2.03	1.77	2.07	1.44	2.87	3.06	1.41	2.83	1.81	2.10	-0.19	-	1.18	1.99	2.34	1.25
S	4.02	3.62	2.32	4.82	4.38	3.49	1.95	3.09	3.54	4.15	3.39	1.18	-	3.78	2.72	1.96
E	4.47	2.15	2.17	3.35	5.02	3.47	3.31	3.59	4.11	3.83	2.08	1.99	3.78	-	1.80	3.26
UK	1.16	2.36	2.80	2.90	2.57	3.71	0.61	3.73	2.49	2.77	2.99	2.34	2.72	1.80	-	3.84
US	2.24	1.26	2.16	3.07	2.33	3.22	1.49	4.54	2.91	3.60	2.32	1.25	1.96	3.26	3.84	-

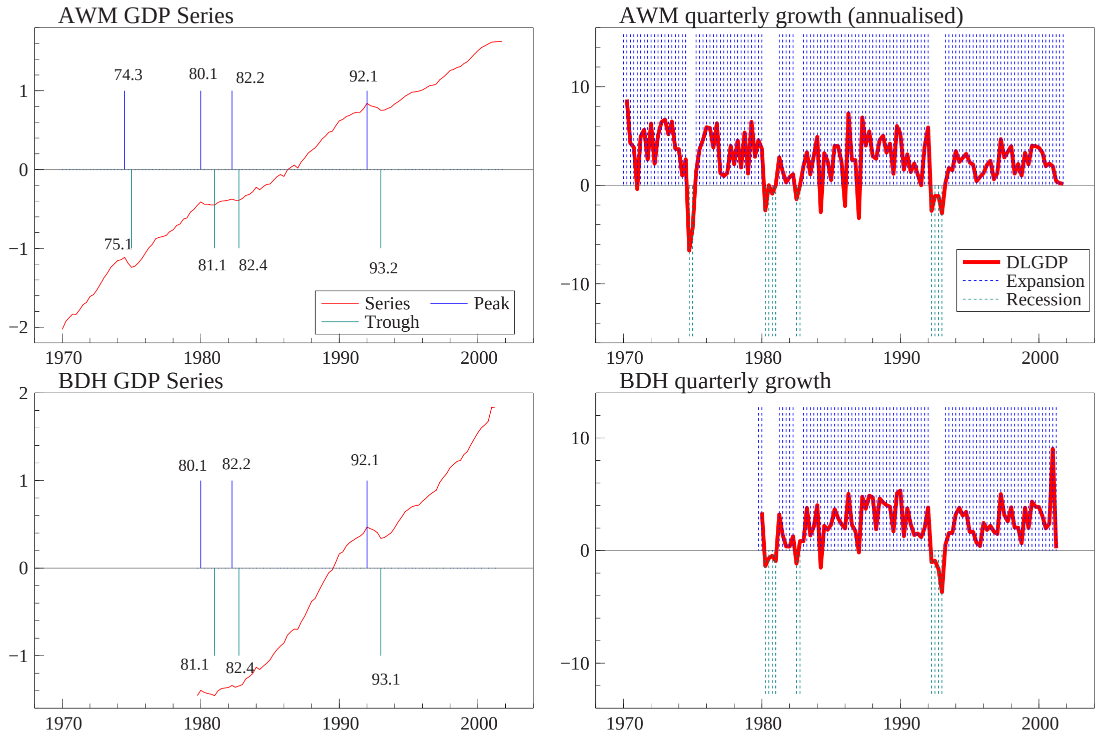

Figure 1: Classical cycle turning points, expansions and recessions, in the Euro area quarterly real GDP (seasonally adjusted, logarithms); ECB series and Beyer, Doornik and Hendry (2000) estimates.

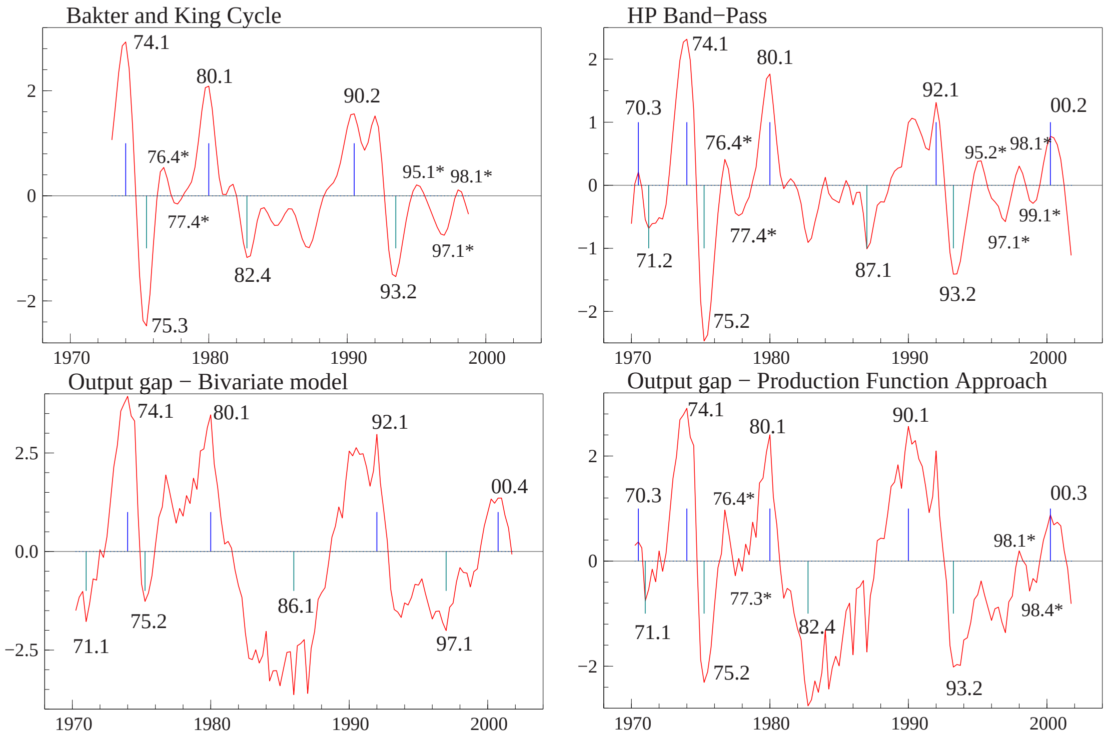

Figure 2: Turning points for four alternative measures of the Euro Area deviation cycle. An asterisk (*) denotes a turning point that was censored according to amplitude considerations (see text for details).

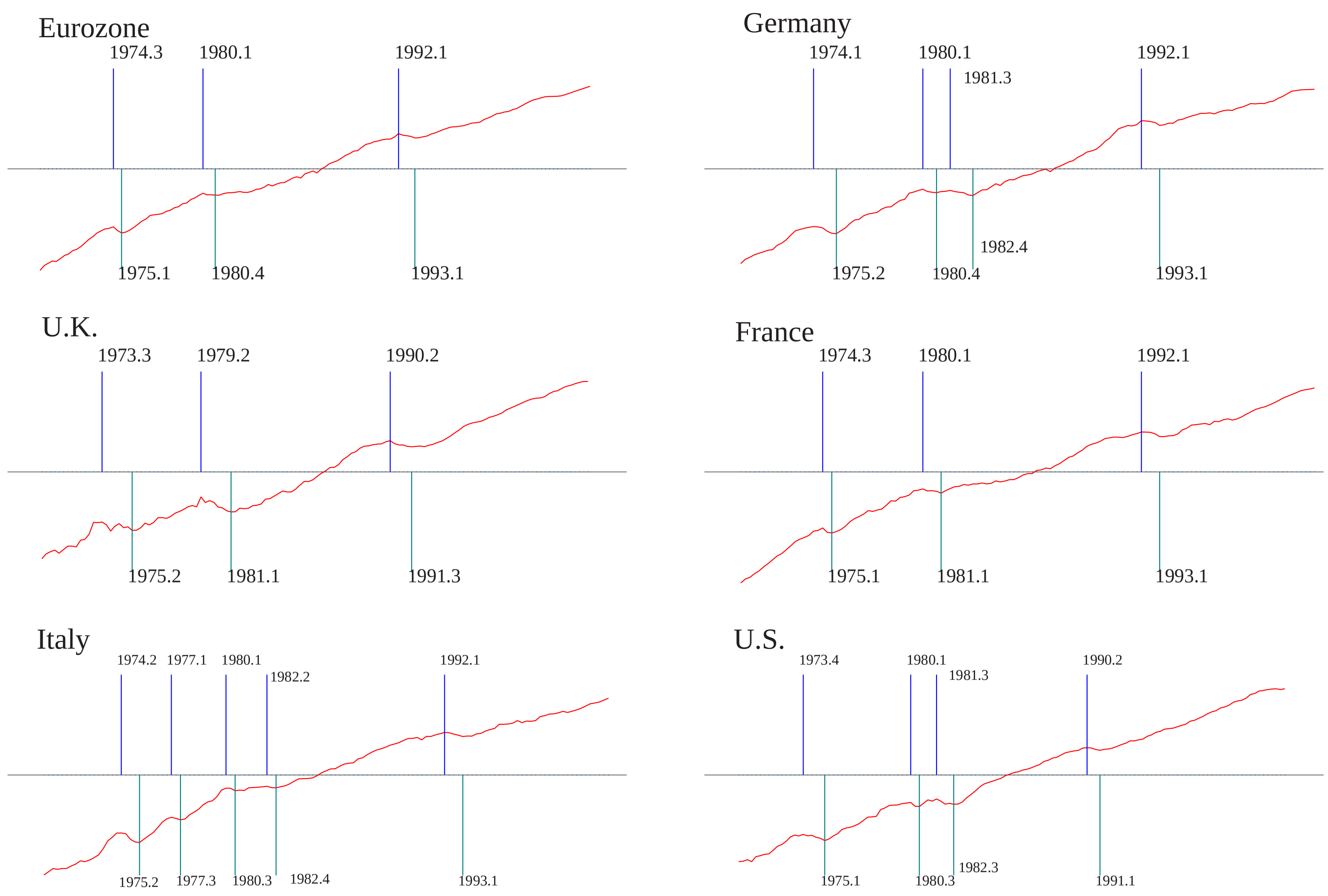

Figure 3: Classical cycle turning points for EA, Germany, France, Italy, UK and the USA, based on HP(1.25) filtered quarterly real GDP.

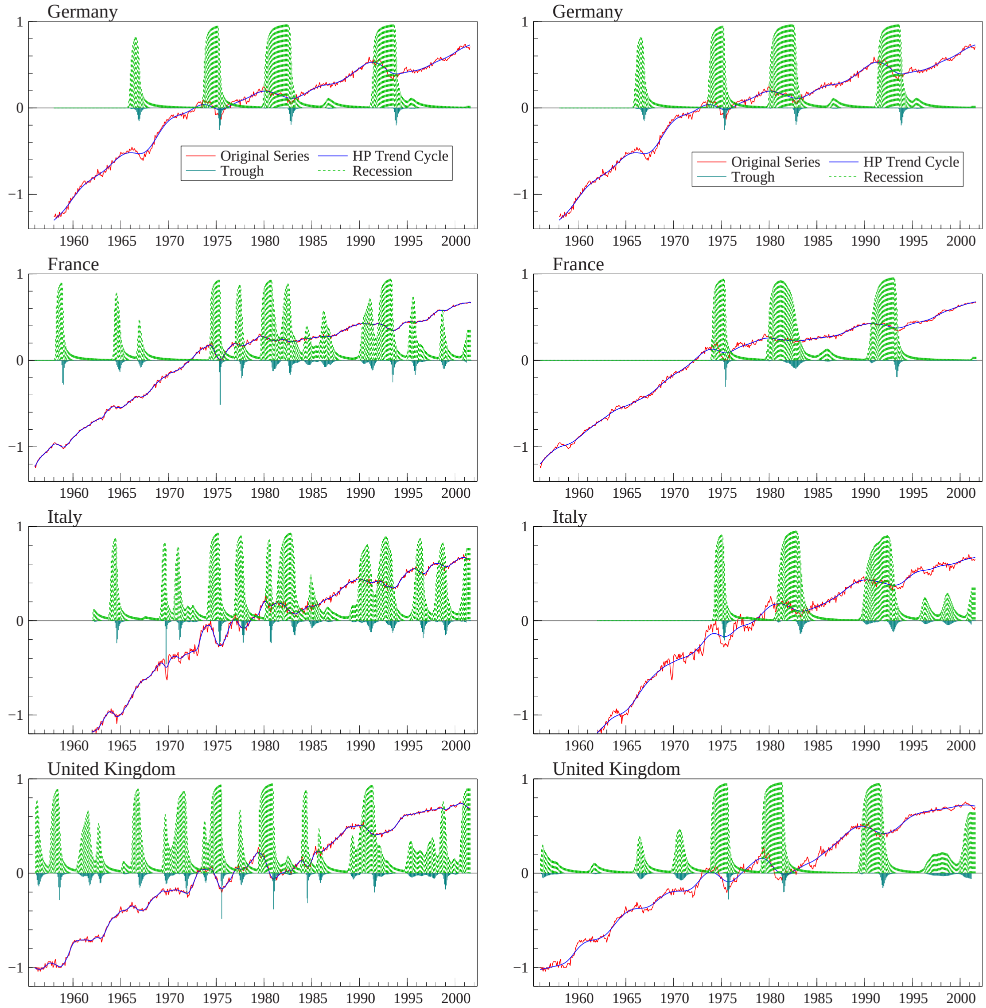

Figure 4: Classical BC dating of monthly industrial production based on HP(1.25) (left) and HP(4) (right); recession and turning points (reverse scale) probabilities.

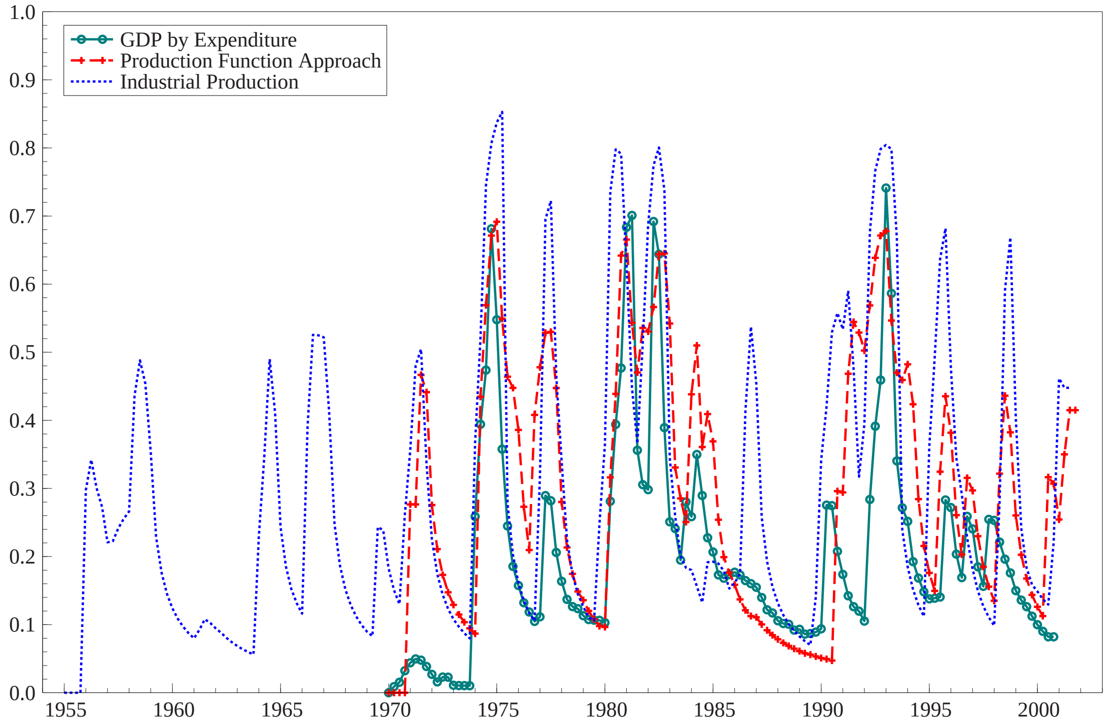

Figure 5: Three sets of diffusion indices for classical business cycles in the Euro area.