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Finite Approximations to Linear Filters and the Monitoring of Revisions in Seasonally Adjusted Series

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Finite Approximations to Linear Filters and the Monitoring of Revisions in Seasonally Adjusted Series

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Abstract

Statistical offices involved in the production of seasonally adjusted series face the problem of revising preliminary figures. The process of revising until historical estimators are obtained can last relatively long, and, in general, the number of periods needed to obtain a final estimator cannot be controlled by the analyst. The length of the revision period is closely related to the estimation method; we shall focus on the model-based signal extraction approach (see for example Box, Hillmer and Tiao, 1978, and Burman, 1980). In this paper, we show how infinite seasonal adjustment filters can be optimally approximated by finite ones, and we apply this result to the problem of controlling the length of the revision period. We also show how considering finite versions of the signal extraction filters improves the interpretation of the X11 filters in the model-based framework.

1 Introduction

Statistical offices involved in the production of seasonally adjusted series face the problem of revising preliminary figures. The process of revising until historical estimators are obtained can last relatively long, and, in general, the number of periods needed to obtain a final estimator cannot be controlled by the analyst. The length of the revision period is closely related to the estimation method; we shall focus on the model-based signal extraction approach (see for example Box, Hillmer and Tiao, 1978, and Burman, 1980). Optimal model-based decompositions via Wiener-Kolmogorov (WK) filtering generally involve infinite filters, and the number of periods needed to obtain a final estimator is related to the convergence properties of the filter: trivially, the faster the convergence, the shorter the revision period.

In this paper, we develop a procedure which allows practitioners to control the length of the revision period. It is based on finite approximations to linear filters. Given an observed series, a stochastic linear model describing its second moments, and the specification of the length of the revision period found acceptable, an optimal model-based decomposition is derived. We discuss this procedure on the basis of an application to two monthly economic time series.

Second, we bring some insights into the understanding of the links between the signal extraction and the X11 filters (see Dagum, 1988; Findley and al., 1998), which are still widely used by official statisticians. Such a question has been the subject of attention in the statistical literature: Cleveland and Tiao (1976) and Burridge and Wallis (1984) showed that there exist unobserved components models for which the X11's estimators are very close to the model-based ones. That question has also some applied interest. For instance, the Statistical Office of the European Community (EUROSTAT) currently uses both approaches for seasonally adjusting macroeconomic indicators through the implementations of the packages X12 (see Findley and al., 1998) and TRAMO-SEATS (see Gomez and Maravall, 1996), and in front of this duality it is important to understand the differences between the two methodologies. [EUROSTAT has developed since 1996 an in-depth comparison program]. We show that X11 filters are better interpreted as finite approximations to optimal WK filters, so that their major difference can be attributed to their respective dimension, i.e. finite versus infinite.

In section 3, we briefly review the analysis of revisions and introduce some tools used in the applications. Section 4 develops a procedure for approximating infinite filters by finite ones, and section 5 discusses its application to the case of monitoring revisions in the seasonal adjustment of two macroeconomic time series. In section 6, we investigate the links between X11's seasonal adjustment filters and the finite approximations to some signal extraction filters. We begin by introducing the overall framework in section 2.

2 Model-Based Signal Extraction Filters

We consider the problem of decomposing an observed time series into orthogonal unobserved components according to

$$x_t = s_t + n_t \tag{2.1}$$

where s_t is a seasonal component and n_t the nonseasonal part of the series. Both are assumed to be well described by stochastic linear processes of the type

$$\begin{aligned} \phi_s(B)\, s_t &= \theta_s(B)\, a_{st}, \\ \phi_n(B)\, n_t &= \theta_n(B)\, a_{nt}, \end{aligned} \tag{2.2}$$

where $\phi_\bullet(B)$ and $\theta_\bullet(B)$ denote finite polynomials in the lag operator B, having all roots on or outside the unit circle. The variables a_{st} and a_{nt} are independent white noise with variances V_s and V_n, respectively. The polynomials $\phi_s(B)$ and $\phi_n(B)$ are prime, while the MA polynomials $\theta_s(B)$ and $\theta_n(B)$ share no unit roots. Further, the polynomial $\theta_s(B)$ is assumed to be noninvertible, so that the spectrum of $\phi_s(B)s_t$ takes a 0-value at some frequency. In the terminology of Hillmer and Tiao (1982), the so-defined seasonal component is said *canonical.*

Equations (2.1) and (2.2) imply that the observed series x_t follows an ARIMA model of the type

$$\phi(B)\ x_t = \theta(B)\ a_t, \tag{2.3}$$

where a_t is a white noise with variance V_a. Without loss of generality, we set $V_a = 1$.

The polynomial $\phi(B)$ can be obtained as the product $\phi(B) = \phi_s(B)\phi_n(B)$. In the AMB approach, the roots of the polynomial $\phi(B)$ are allocated to the polynomials $\phi_s(B)$ and $\phi_n(B)$ according to the patterns that the components are expected to display. For instance, all positive roots of $\phi(B) = 0$ will be assigned to the trend component since they imply a spectral peak at the zero-frequency which reflects infinite period oscillations typical of a long term evolution. The canonical hypothesis on the seasonal component implies that for a given $\theta(B)$ polynomial, the decomposition (2.1)-(2.2) is unique (see Hillmer and Tiao 1982). Minimum mean square error estimators (MMSE) can be built as

$$\begin{aligned} \hat{n}_t &= \nu_n(B)x_t = \sum_{i=-\infty}^{\infty} \nu_{ni}x_{t+i} \\ &= V_n \frac{\theta_n(B)\theta_n(F)\phi_s(B)\phi_s(F)}{\theta(B)\theta(F)} x_t \end{aligned} \tag{2.4}$$

The filter $\nu_n(B)$ is known as the Wiener-Kolmogorov filter (see Whittle 1963). It has been shown to give finite error variance whether the observed series is stationary or not (see for example Bell 1984). The hypothesis that the polynomials $\theta_s(B)$, $\theta_n(B)$ do not share unit roots imply that $\theta(B)$ is invertible (see Maravall and Planas, 1998), and thus the filter $\nu_n(B)$ is infinite with converging weights ν_{ni}. Because of this convergence, (2.4) is valid for signal estimation for periods around the center of usual sample lengths. In finite samples, preliminary estimates must be computed for periods close to both ends of the sample. These preliminary estimates incorporate an estimation error, termed *revision error*, that we discuss in the next section.

3 Errors in Preliminary Estimates

The classical analysis of revisions has been developed by Pierce (1980). We briefly review it here, concentrating on preliminary estimates computed close to the end of the sample. Let $\hat{n}_{t|t+k}$ denote the preliminary estimate of n_t computed at time $t+k$, $k \geq 0$. Assuming that $X_{t+k} = \{x_1, \cdots, x_{t+k}\}$ is available, then

$$\begin{aligned} \hat{n}_{t|t+k} &= E(\hat{n}_t | X_{t+k}) \\ &= E(\sum_{i=-\infty}^{\infty} \nu_{ni} x_{t+i} | X_{t+k}) \end{aligned} \tag{3.1}$$

Let $\xi(B) = \cdots + \xi_{-n}B^n + \cdots + \xi_0 + \xi_1 F + \cdots + \xi_m F^m + \cdots$ denote the polynomial obtained as $\xi(B) = \nu_n(B)\theta(B)/\phi(B)$, assuming $\phi(B)$ satisfies the stationarity constraint. Expressing the estimator (2.4) in fonction of the innovations a_t, it is easily seen that $\hat{n}_t = \xi(B)a_t$, so that the revisions in the preliminary estimator of n_t computed at time $t+k$ are given by:

$$\hat{n}_t - \hat{n}_{t|t+k} = \sum_{i=k+1}^{\infty} \xi_i a_{t+i} \tag{3.2}$$

with variance:

$$V[\hat{n}_t - \hat{n}_{t|t+k}] = \sum_{i=k+1}^{\infty} \xi_i^2 \tag{3.3}$$

In Appendix, we show that expressions (3.2)-(3.3) are still valid in the nonstationary case of autoregressive unit roots. The scheme of the proof was given in Pierce (1980, p.104).

It can be seen from (2.4) and from the definition of the polynomial $\xi(B)$ that the weights ξ_i, $i > 0$, converge so that after a certain number of periods, say M, revisions become negligible: $\hat{n}_{t|t+M} \equiv \hat{n}_t$.

Since $\hat{n}_{t|t+k+i} - \hat{n}_{t|t+k+i-1} = \xi_{k+i} a_{t+k+i}$, successive revisions are independent and we get:

$$V[\hat{n}_t - \hat{n}_{t|t+k}] = \sum_{i=1}^{\infty} V[\hat{n}_{t|t+k+i} - \hat{n}_{t|t+k+i-1}] \tag{3.4}$$

In applications, we shall use this last expression when evaluating the total variance of empirical revisions, with k set to 0 since we shall focus on total revisions in concurrent estimates. Expression (3.4) will be evaluated on the $M + 1$ estimates $\hat{n}_{t|t+i}$, $i = 0, \cdots, M$, which will be derived using the updated samples X_{t+i}.

The convergence rate of the theoretical concurrent estimator can be evaluated using:

$$\begin{aligned} RC(m) &= 100 \frac{V[\hat{n}_{t|t+m} - \hat{n}_{t|t}]}{V[\hat{n}_t - \hat{n}_{t|t}]} \\ &= 100 \frac{\sum_{i=1}^{m} \xi_i^2}{\sum_{i=1}^{\infty} \xi_i^2} \end{aligned} \tag{3.5}$$

For the empirical analysis of the concurrent estimates convergence rate, we shall use

$$RC(m) = 100 \frac{\sum_{i=1}^{m} V[\hat{n}_{t|t+i} - \hat{n}_{t|t+i-1}]}{\sum_{i=1}^{M} V[\hat{n}_{t|t+i} - \hat{n}_{t|t+i-1}]}. \tag{3.6}$$

This analysis of revisions is still valid if the estimation is conducted with any linear time-invariant adjustment filter, say c(B), with the ξ_i's weights derived from the convolution $c(B)\psi(B)$. Since Geweke (1978) and Pierce (1980, p.102) showed that, given any filter, extending the series with minimum mean square error forecasts minimises the total revision error in preliminary estimates, we shall not modify the forecast function.

After convergence, the signal extraction procedure still yields an estimation error, called *final estimation error* and defined as $e_t = n_t - \hat{n}_t$. Assuming first observed series stationarity, a frequency domain expression for the variance of the final error in a estimator $\hat{n}_t = c(B)x_t$ is given by:

$$V[n_t - \hat{n}_t] = \frac{1}{\pi} \int_0^{\pi} g_n(e^{-iw})[1 - c(e^{-iw})]^2 + g_s(e^{-iw})c(e^{-iw})^2 dw \tag{3.7}$$

When $c(B)$ is the WK filter (2.4), the variance of the final estimation error turns out to be the variance of the ARMA process

$$\theta(B)z_t = \theta_n(B)\theta_s(B)b_t \tag{3.8}$$

with $V(b_t) = V_s V_n$. Bell's assumption A imply that (3.8) is still valid in the nonstationary case so that the final estimation error remains finite (see Bell, 1984).

In the process of seasonally adjusting time series, the number of periods needed for a concurrent estimate to become a final estimator can be relatively large. The theoretical convergence rate (3.5) helps in anticipating that delay, but practitioners have no direct control on the length of the revision period. According to (2.4), pure

AR models for observed series lead to signal extraction filters of relatively short length. An obvious possibility for reducing the length of the revision period could then be to concentrate on pure AR models for observed series, but such an approach would limit the flexibility of ARIMA modelling in describing economic time series properties. We choose to approximate infinite model-based filters with finite filters. The final estimator will then be slightly modified, in such a way that the convergence of preliminary estimates will be faster.

4 Finite Approximation to Model-Based Infinite Filters

Let $a(B)$ denote an infinite symmetric linear filter such that

$$a(B) = a_0 + \sum_{k=1}^{\infty} \frac{1}{2} a_k (B^k + F^k), \tag{4.1}$$

where the weights a_i are real, do not depend on time, and satisfy $\sum a_i = 1$, $a_i = a_{-i}$, and $\sum a_i^2 < \infty$. Symmetric filters are preferred because they induce no phase shift in output (see Priestley, 1981). Both WK and X11 historical filters share that property (see 2.4 and, for X11, Bell and Monsell, 1992). The frequency transfer function of $a(B)$ is given by $a(e^{-iw}) = a_0 + \sum_{k=1}^{\infty} a_k \cos kw$, for $w \in [0, \pi]$. We consider the problem of finding $b_m(B)$ defined as (4.1) with a given finite length m, such that $b_m(B)$ is the closest finite approximation to $a(B)$. For that concern, we use a distance measure discussed in Depoutot and Planas (1998); in our case

$$d(a, b_m) = \frac{1}{\pi} \int_0^{\pi} \mid a(e^{-iw}) - b_m(e^{-iw}) \mid^2 dw \tag{4.2}$$

Hence we are interested in finding $b_{m0}, \cdots, b_{mm}$ minimizing $d(a, b_m)$. That optimization problem is subject to several constraints which are related to the seasonal adjustment context. First, the resulting seasonal adjustment filter must locally preserve the mean of the input series; that is, $b_m(1) = 1$. Second, the power of the input series at the seasonal frequencies $2k\pi/s$, $k = 1, \cdots, s-1$, s denoting data periodicity, must be cancelled out. Thus $b_m(e^{-i2k\pi/s}) = 0$. For monthly series $s = 12$, so that this last equality defines 6 constraints $b_m(e^{-i2k\pi/12}) = 0$, $k = 1, \cdots, 6$. The problem can be written as :

$$
\begin{aligned}
\min \quad & {}_{b_{mk}, k=0,\cdots,m} d(a, b_m) \quad \text{with respect to} \\
& C.1 \quad b_m(1) = 1; \\
& C.2 \quad b_m(e^{-i2k\pi/12}) = 0, k = 1, \cdots, 6.
\end{aligned}
$$

Using the property that $\cos kw$, $k = 0, \cdots, \infty$ are linearly independent functions, it is easily seen that $\min d(a, b_m) \equiv \min \sum_{k=0}^{m} (a_k - b_{mk})^2$ (see Priestley, 1981). Hence the minimisation problem turns out to be:

$$
\begin{aligned}
min \quad & {}_{b_k, k=0,\cdots,m} \sum_{k=0}^{m} (a_k - b_{mk})^2 \quad \text{with respect to} \\
& C.1 \quad b_m(1) = 1; \\
& C.2 \quad b_m(e^{-i2k\pi/12}) = 0, k = 1, \cdots, 6.
\end{aligned}
\tag{4.3}
$$

Lagrange operators give straightforward solution to (4.3). Details are given in Appendix 2.

It is worth noticing that in this variance decomposition context, a third constraint does actually exist: since we want the variance of the observed series to be splitted at every frequency into orthogonal contributions, the frequency transfer function of $b_m(B)$ should never exceed 1. We shall see in the next section that the solution to

(4.3) does not necessarily satisfy that property. However, we found that imposing that third constraint distorts very much the filters' band-pass structure, in such a way that the solution is qualitatively not satisfying with respect to the objective of signal estimation. We thus decide to not impose that constraint (details are available on request). Notice that historical WK filters always have a frequency transfer function into $[0, 1]$, but all but one X11 historical adjustment filters have gains higher than 1 (see Bell and Monsell, 1992, and Planas, 1997).

We have designed an estimation procedure for seasonally adjusted series which respects any requisite about the lasting of revisions. However, controlling the length of the revision period implies a loss of accuracy in the final estimator. We believe that it is important to closely monitor that loss in order to avoid large deviations with respect to the optimal unconstrained final estimator. Two appropriate tool for measuring these deviations are given by the final estimator accuracy (3.7) and by the filter distance (4.2). We discuss that point in the next section on the basis of two applications.

Notice that since $b_m(B)$ is symmetric, the constraint C1 implies that $[1 - b_m(B)]$ has at least two unit roots at the zero frequency; similarly, C2 implies that $b_m(B)$ has at least two unit roots at the seasonal frequencies. For the two applications that we consider, these properties will be sufficient to insure that the integral (3.7) is well-defined and that the final estimation error obtained with constrained filters is still finite.

5 Monitoring the Revision Period

We apply the finite approximation procedure to the ARIMA-model-based decompositions of two monthly macroeconomic series. Our objective is to show how the convergence of the concurrent seasonally adjusted series towards the historical figure can be controlled, under a close monitoring of the deviations from the optimal unconstrained estimators.

The first series describes the French total production (FRTP) with 162 observations between 1985-1 and 1998-6. This series is plotted in figure 1 (in logs), where it can be seen that it displays a somewhat unstable seasonal pattern, the seasonal dips related to August decreasing in the last third of the sample. Fitting the airline model to the series log-transformation, maximum likelihood estimation yields:

$$\Delta\Delta_{12}x_t = \underset{(.09)}{(1-.62B)}\ \underset{(.12)}{(1-.28B^{12})}a_t \tag{5.1}$$

with residual standard deviation $V_a^{1/2} = .016$. Port-manteau correlation tests do not show any significant departure of the residuals properties from the white noise hypothesis: the Ljung-Box on the first 24 lags and the Box-Pierce tests on the first two seasonal lags take the values $Q_{24} = 24.92$ and $Q_{s2} = 1.23$ (see Ljung and Box, 1978, and Pierce, 1978). The third and fourth moments of the residuals seem in agreement with those of the normal distribution: the skewness and kurtosis statistics take values of .38 (.29) and 3.52 (.59). A further check of residual independence was performed by computing the Ljung-Box statistics on 24 squared residuals first autocorrelations (see McLeod and Li, 1983): the result, $Q_{24}(a_t^2) = 10.43$, does not suggest a significative nonlinear structure in the residuals.

We first consider the 12 concurrent seasonally adjusted estimates in the year 1992 ($t = 85, \cdots, 96$). Completing the sample from $t = 85$ until the last observation and updating the preliminary seasonally adjusted estimates $\hat{n}_{t|t+i}$, $i = 1, \cdots, 78$, provides us with a series of preliminary estimates for every month. We then use (3.4) to compute the empirical variance of the total revisions in the 12 concurrent seasonally adjusted figures. The results, shown in figure 2, are in agreement with the theoretical variance of the total revision error, .107. The most revised months are December, May and October, with variance .29, .17 and .14, respectively. We concentrate the analysis on these three months. Their empirical revisions convergence rates are displayed in figure 3 together with the theoretical convergence path, respectively computed as in (3.6) and in (3.5). It can be seen that observations occuring 12 and 24 months ahead are those implying the largest revisions. We consider the truncation lengths 36, 24, and 18; as we shall see, shorter lengths would not be appropriate.

Figure 4 displays the frequency transfer function of the historical WK filter (2.4) and of the approximations with length $m = 18, 24, 36$. As discussed in section 3, gains of finite filters can be higher than 1. Clearly, the larger the truncation point, the closer the approximations to the infinite WK filter. Figures 5a-5c show the seasonally adjusted series obtained with $b_m(B)$, $m = 36, 24, 18$, together with the WK estimator. For $m = 36$, both estimators are very close. Small discrepancies are slightly visible with $m = 24$, but they are more important with $m = 18$. These observations are confirmed by table 1 which displays the distance between WK and the truncated filters, the final estimation error variance (3.7) and the theoretical revision variance (3.3). It is seen that $d(\nu_n, b_m)$ lies between 10^{-4} for $m = 36$ and .0172 for $m = 18$, and that truncating the filters increases the final estimation error variance by 2% with $m = 36$ and by 10% with $m = 18$. Figure 6 shows that the approximations do not yield much larger variances of the total revisions in the concurrent estimates. Finally, figures 7a-7c show that the convergence rates of the preliminary figures is now completely under control, since the estimators get into their final figures as, respectively, 36, 24, and 18 observations are added.

In this first example, the truncation point $m = 18$ seems too high: it leads to a constrained filter which is a bit too far away from WK, a final estimation error too high relatively to the minimum, and as a consequence the seasonally adjusted series with $b_{18}(B)$ is too much different from the unconstrained seasonally adjusted series. The lengths $m = 24$ and $m = 36$ yield much more acceptable results.

Table 1

Finite Approx. vs WK Estimator

		FRTP	
Filter	$d(b_m, \nu_n)$	$V[n_t - \hat{n}_t]$	$V[\hat{n}_t - \hat{n}_{t\|t}]$
WK	—	.111	.107
$m = 36$	.0001	.113	.115
$m = 24$	.0013	.116	.131
$m = 18$	.0172	.124	.101
		DPCG	
Filter	$d(b_m, \nu_n)$	$V[n_t - \hat{n}_t]$	$V[\hat{n}_t - \hat{n}_{t\|t}]$
WK	—	.073	.073
$m = 60$	.0054	.087	.126
$m = 48$	.0114	.094	.150
$m = 36$	.0253	.109	.182

The second series describes the German production of capital goods (DPCG) between 1980-1 and 1997-8, that is a sample of 212 observations. That series is plotted in figure 8, where it can be seen that the most noticeable seasonal patterns are related the dips in August and December. As in the previous example, the airline model gives a satisfying description of the correlation structure of the log-tranformed series, with maximum likelihood paramaters estimates:

$$\Delta\Delta_{12} x_t = \underset{(.10)}{(1 - .51B)} \quad \underset{(.12)}{(1 - .73B^{12})} a_t \tag{5.2}$$

The residual standard deviation is $V_a^{1/2} = .023$. No residual correlation is significant since $Q_{24} = 24.85$ and $Q_{s2} = 4.77$. Furthermore, the skewness and kurtosis statistics, with respective values -.04 (.29) and 3.04 (.59), do not show any significant departure of the residual distribution for the normal distribution. No evidence of correlation could be found in the squared residuals: $Q_{24}(a_t^2) = 20.09$.

Let us first concentrate the analysis on the 12 concurrent seasonally adjusted estimates computed in the year 1987 ($t = 85, \cdots, 96$), which are updated until the last observation is available ($k = 1, \cdots, 127$). Figure 2 displays the variances (3.4) of the total empirical revisions for these 12 estimates. They are in agreement with the theoretical revision variance .073 that (3.3) yields. Since June, July and August are the more revised months, we focus the analysis on these 3 months. Figure 9 shows the empirical convergence rates (3.6) for their concurrent estimators, which are seen to behave actually better than what the theoretical convergence path suggests. Four years of new observations are anyway needed for a 90% convergence rate, five for a 95% convergence rate. Nearly all the revisions are due to the observations occuring every 12 periods beyond the date of the concurrent estimate. Given the slow convergence patterns, we decided to truncate the optimal WK filter at lengths 60, 48 and 36.

It can be seen on table 1 that the distances between the finite filters and WK lie between .0054 for $m = 60$ and .0253 for $m = 36$, with an intermediate value at .0114 for $m = 48$. With respect to the final error in the WK estimator, the loss in accuracy in final estimators lies between 20% for $m = 60$ and 50% with $m = 36$. The frequency transfer functions of the four filters are shown on figure 10: for $m = 60$ and $m = 48$, the discrepancies with WK are not very large. As expected, the seasonally adjusted series are very close to each other, only slight differences could be seen for the estimator obtained with $b_{48}(B)$; see figures 11a-11b. Regarding the estimator obtained with $m = 36$ and displayed on figure 11c, more discrepancies with respect to the unconstrained estimator are noticed. Figure 12 shows that the constraints about the convergence period did not result in much larger empirical revisions. Finally, figure 13 illustrates the improvement in the convergence of the concurrent estimates towards their final values.

In this second example, $m = 60$ and $m = 48$ are acceptable truncation values, while $m = 36$ is a bit too low and leads to relatively large deviations from the historical WK estimators. Again, care must be taken that imposing the lasting of revisions does not distort too much the decomposition. We can see that acceptable cutting points depend on the stochastic properties of the series. These two examples suggest a reasonable maximum filter distance of about .01 and a reasonable maximum loss in accuracy of order 20% with respect to the accuracy of the WK final estimator. Notice however that if we use $V[n_t - \hat{n}_t]$ to build a 5% confidence interval around the WK final estimator $\hat{n}_t$, then for the two examples the discrepancies observed when the WK filter is approximated by shorter ones are never significant.

6 Finite Model-Based Approximations to X11 Filters

The problem of giving a signal extraction interpretation to X11 filters has been the subject of discussions in the statistical literature. For instance, Burridge and Wallis (1984) derived ARIMA models for unobserved components leading to seasonal adjustment signal extraction filters very close to the X11 default adjustment filter. We show in this section that the X11 filters are actually closer to the finite approximations of the optimal signal extraction filters than to the unconstrained WK filter. Regarding historical estimation, the model specification proposed by Burridge and Wallis (1984) was:

$$\begin{aligned} (1 + B + \cdots + B^{11})s_t &= (1 + .71B^{12} + B^{24})a_{st} \\ \Delta^2 n_t &= (1 - 1.59B + .86B^2)a_{nt} \\ V_s/V_n &= .017 \end{aligned} \tag{6.1}$$

Denoting $\nu_X(B)$ the X11 default adjustment filter, this decomposition leads to the distance $d(\nu_n, \nu_X) = .0036$, which was a minimum in Burridge and Wallis' model specification search.

Using (6.1), we compute for $m = 24, \cdots, 100$ the filters $b_m(B)$ as optimal approximations to $\nu_n(B)$ according to (4.3). We then checked the distance between $b_m(B)$ and $\nu_X(B)$ for each value of m. The ratio r_m defined as

$$r_m = 100 \times \frac{d(b_m, \nu_X)}{d(\nu_n, \nu_X)} \tag{6.2}$$

is reported on figure 14 for $m = 24, \cdots, 100$. It shows that an improvement of order 40% in the approximation of the X11 default filter can be obtained by truncating the WK filter at length m=43. Figure 15 shows that this improvement is related to the capacity of truncated filters to have gains higher than 1. That result suggests that ad-hoc filters like the X11's ones are better interpreted as finite versions of some signal extraction filters. Moreover, if X11 filters are to be seen as finite approximations to infinite signal extraction filters, then these approximations could be further improved as they would not be the optimal ones.

In order to generalise this result, we have considered all the seasonal adjustment filters that recent releases of X11 like in particular X12 incorporates. The combination of 3×3, 3×5, 3×9, and 3×15 seasonal moving average lengths with Henderson trends of length 9, 13, 17, 23 offers in practice 16 possibilities. We have developed the previous experiment by considering all these possibilities. Instead of looking for the model leading to signal extraction as close as possible to the X11 filters, we restraint our search by focusing on the airline model specified as:

$$\Delta\Delta_{12} x_t = (1 + \theta_1 B)(1 + \theta_{12} B) a_t$$

and we compute the parameter values $(\theta_1^*, \theta_{12}^*)$ which minimise the distance between

X11 and the signal extraction seasonal adjustment filters. We then derive $b_m(B)$ as finite approximation to WK for m ranged between 24 and 120. Finally, we get m^*, the m-value which minimizes the ratio (6.2) at r_m^*.

For every X11 historical adjustment filter, table 2 reports (θ_1, θ_{12}), m^* and r_{m^*} eventually found, and the length of every X11 filter which is denoted m_X. First, it can be seen that airline model leads to signal extraction filters very close to the X11's ones, since $d(\nu_n, \nu_X)$ is always less than .015. The parameter θ_1 takes a very limited range of values, while the range of variation of θ_{12} is much larger: it is the parameter which controls the width of the power-vanishing band around the seasonal harmonics in the WK filter adjustment filter. The most striking result of this experiment is that in all the 16 cases, restricting the length of the Wiener-Kolmogorov filter decreases the distance approximation to the X11 filters. That reduction can be from 25% to 60% of the initial distance, with a remarkably stable ratio m^*/m_X around, roughly, .45. This experiment tends to show that the main difference between signal extraction and X11 filters is related to their respective lengths, i.e. infinite versus finite.

Table 2 Seasonal Adjustment Filters

WK Filters Closest to X11 Filters in Airline Model

X11 filters	Hend. MA	9-term	13-term	17-term	23-term
Seas. MA					
3×3	$(\theta_1^*, \theta_{12}^*)$	(-.599,-.400)	(-.597,-.384)	(-.594,-.375)	(-.589,-.364)
	$d(\nu_n), \nu_X)$	.008	.008	.007	.006
	m_X	71	73	75	78
	m^*	33	31	31	30
	r_{m^*}	74.85	66.14	60.62	54.57
3×5	$(\theta_1^*, \theta_{12}^*)$	(-.583,-.563)	(-.583,-.553)	(-.583,-.548)	(-.583,-.543)
	$d(\nu_n, \nu_X)$	.009	.009	.010	.010
	m_X	95	97	99	102
	m^*	45	44	43	42
	r_{m^*}	69.15	64.02	60.97	57.75
3×9	$(\theta_1^*, \theta_{12}^*)$	(-.583,-.732)	(-.583,-.728)	(-.583,-.726)	(-.583,-.723)
	$d(\nu_n, \nu_X)$	.012	.013	.013	.014
	m_X	143	145	147	150
	m^*	69	68	67	67
	r_{m^*}	56.61	53.22	51.65	50.51
3×15	$(\theta_1,^* \theta_{12}^*))$	(-.583,-.828)	(-.583,-.826)	(-.583,-.825)	(-.583,-.824)
	$d(\nu_n, \nu_X)$	.013	.014	.014	.015
	m_X	215	217	219	222
	m^*	105	104	103	103
	r_{m^*}	40.83	39.17	38.23	37.10

Conclusion

In this paper, we have built finite approximations to infinite filters and we have applied these approximations to the control of the revision period in the seasonal adjustment problem. Given a model for an observed series and any length of the revision period found acceptable, an optimal decomposition has been found. However, as truncating filters implies that the historical estimators eventually obtained deviate from the optimal WK ones, we believe that it is important to monitor these deviations and to select a truncation level which keeps them reasonably low. Trivially, there is a trade-off between closeness to the original estimator and the convergence time of the modified estimate.

We also have shown that X11 filters are better approximated by signal extraction filters with length contraints than by infinite ones. We have improved Burridge and Wallis' approximation, and we have shown that this result concerns every X11 filter. We have established that the main difference between X11 and signal extraction filters can be attributed to their respective length, a result which brings an important insight into the understanding of the X11 decompositions in the model-based framework.

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Appendix 1

We show that result (3.2) still holds if the AR polynomial $\phi(B)$ embodies d unit roots.

Let $x_* = (x_1, \cdots, x_d)'$ represent the process starting values. We shall consider that they are driven by a vector of polynomials $\delta(B)$; an explicit form of $\delta(B)$ is given in Bell (1984). Considering $[\psi(B)]^\ell = 1 + \psi_1 B + \cdots + \psi_\ell B^\ell$ the expansion $\theta(B)/\phi(B)$ truncated at the ℓ-th term, then, following Bell, an innovation representation of x_t is given by

$$x_t = \delta(B)' x_* + [\psi(B)]^{t-d-1} a_t. \tag{6.1}$$

Writing $\xi(B) = \cdots + \xi_{-n} B^n + \cdots + \xi_0 + \xi_1 F + \cdots + \xi_m F^m + \cdots$ the polynomial obtained as $\xi(B) = \nu_s(B)[\psi(B)]^{t-d-1}$, the signal estimator can be obtained as:

$$\hat{n}_t = \nu_n(B)\delta(B)' x_* + \sum_{i=-t+d+1}^{\infty} \xi_i a_{t+i} \tag{6.2}$$

A preliminary estimate of $\hat{n}_t$ computed at time $t+k$ is then given by

$$\begin{aligned} \hat{n}_{t|t+k} &= E(\hat{n}_t | X_{t+k}) \\ &= E(\nu_n(B) x_t | X_{t+k}) \\ &= \nu_n(B)\delta(B)' x_* + \sum_{i=-t+d+1}^{k} \xi_i a_{t+i} \end{aligned}$$

so that

$$\hat{n}_t - \hat{n}_{t|t+k} = \sum_{i=k+1}^{\infty} \xi_i a_{t+i}$$

which is the expected result. ∎.

Appendix 2

Solution to (4.3)

Let us denote $Q(b)=\sum_{k=0}^{m}(a_k-b_k)^2$, $w_{k\ell}=\cos 2\pi\ell k/12$, and $L(b)$ the expression defined by:

$$L(b)=Q(b)+\lambda(\sum_{k=0}^{m}b_k-1)+\sum_{\ell=1}^{6}\mu_\ell\sum_{k=0}^{m}b_k w_{k\ell}$$

where λ and μ_ℓ, $\ell=1,\cdots,6$ are the usual Lagrange operators. The solution, b^*, is such that:

$$\begin{aligned}\frac{dL(b^*)}{db_k} &= -2(a_k-b_k)+\lambda+\sum_{\ell=1}^{6}\mu_{k\ell}=0\\ \frac{dL(b^*)}{d\lambda} &= 0\\ \frac{dL(b^*)}{d\mu_\ell} &= 0 \quad \ell=1,\cdots,6.\end{aligned}$$

The operators $\lambda,\mu_\ell,\ell=1,\cdots,6$ can be derived by solving the system of 7 equations yielded by $\sum_{k=0}^{m}dL(b^*)/db_k=0$ and $\sum_{k=0}^{m}w_{k\ell}dL(b^*)/db_k=0$, $\ell=1,\cdots,6$. Inserting the solutions in

$$b_k=a_k-(1/2)(\lambda+\sum_{\ell=1}^{6}\mu_\ell w_{k\ell})$$

gives the coefficients of the frequency transfer function $b_m(e^{-iw})=\sum_{k=0}^{m}b_k^*\cos kw$ which solves (4.3).

Figure 1: Series FRTP (in logs)

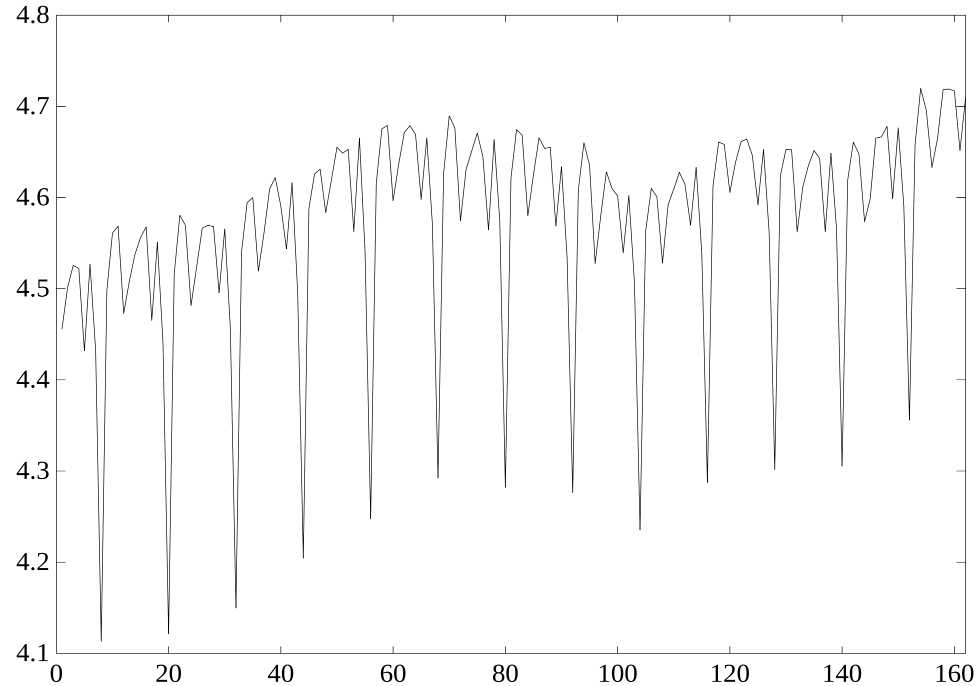


Figure 2: Empirical Variance of Total Revisions

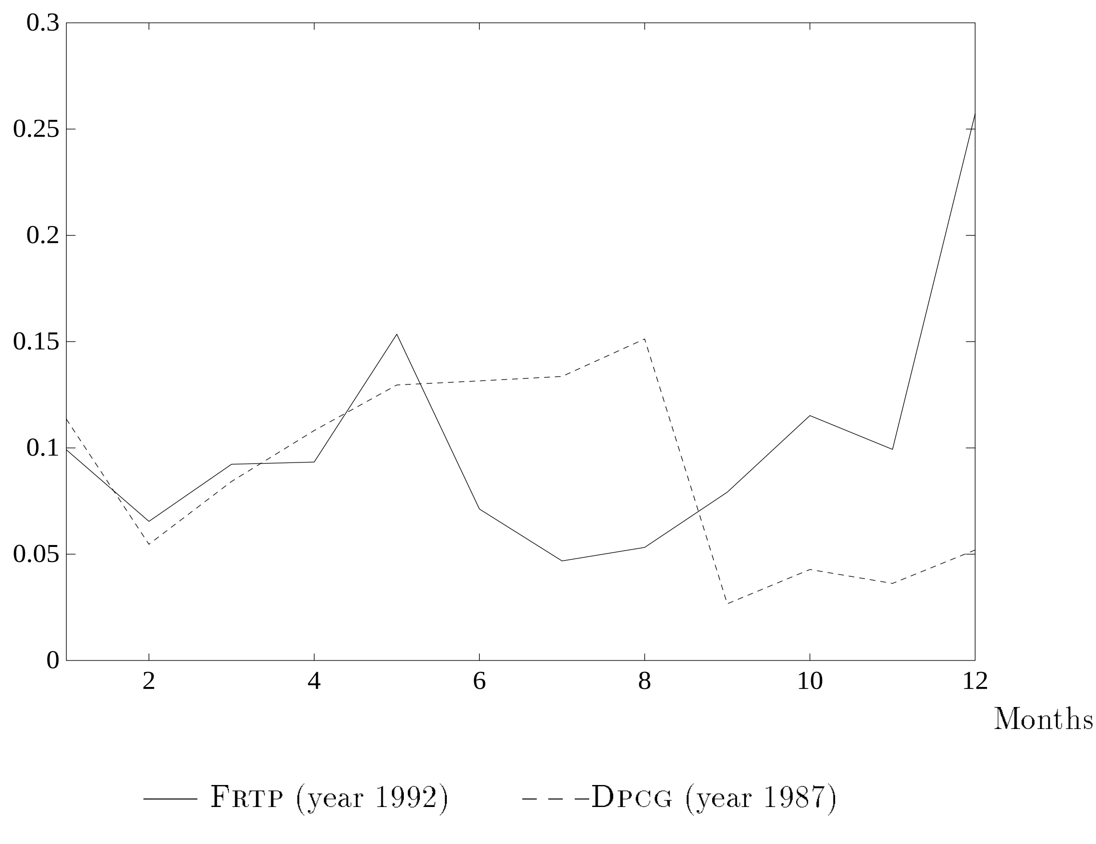
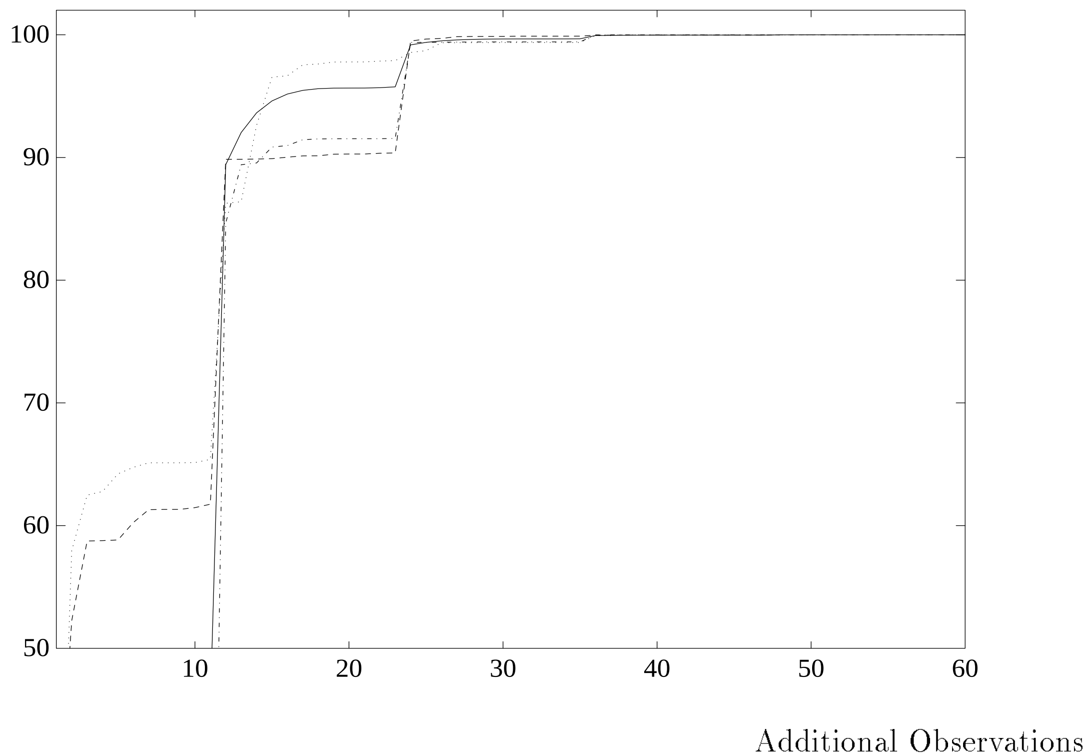
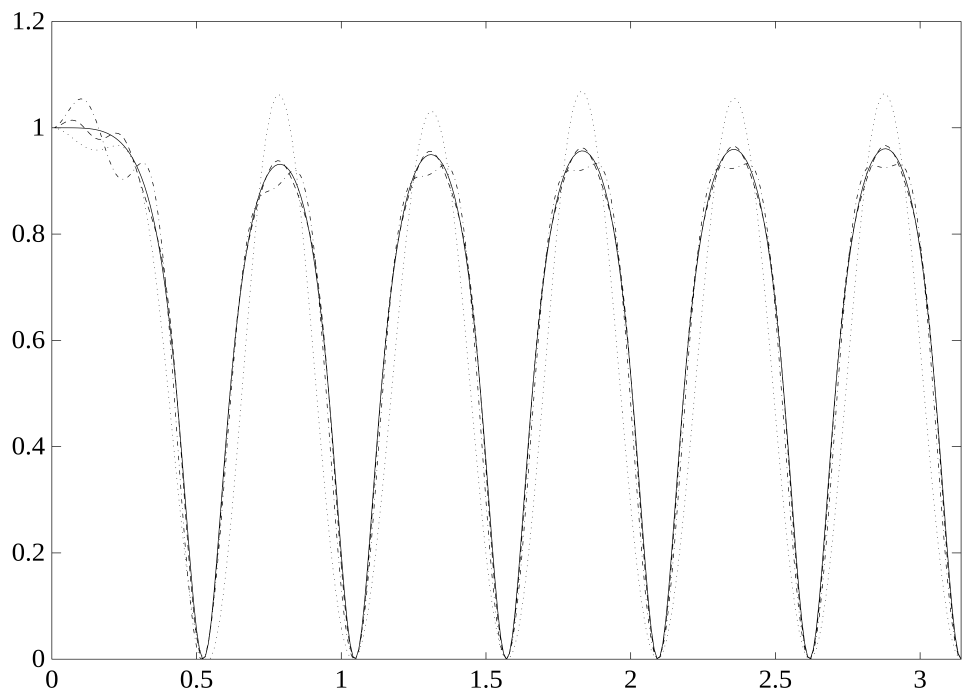


Figure 3: FRTP Rates of Convergence of Total Revisions



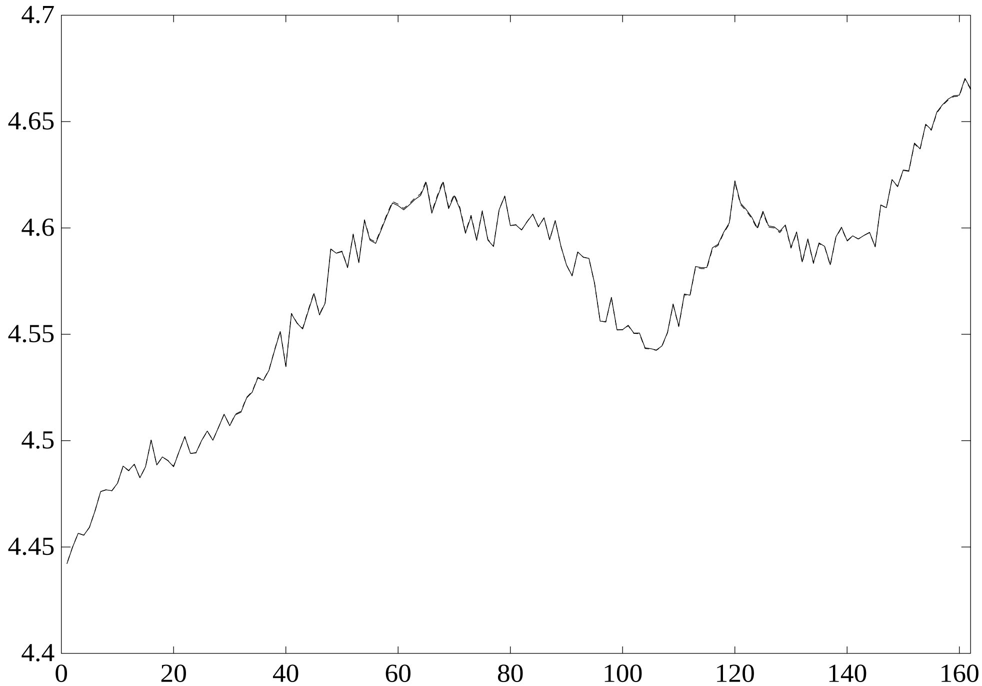
—— Theoretical   Empirical: - – – 5-1992   – · – 10-1992   · · · 12-1992

Figure 4: FRTP Seasonal Adjustment Filter



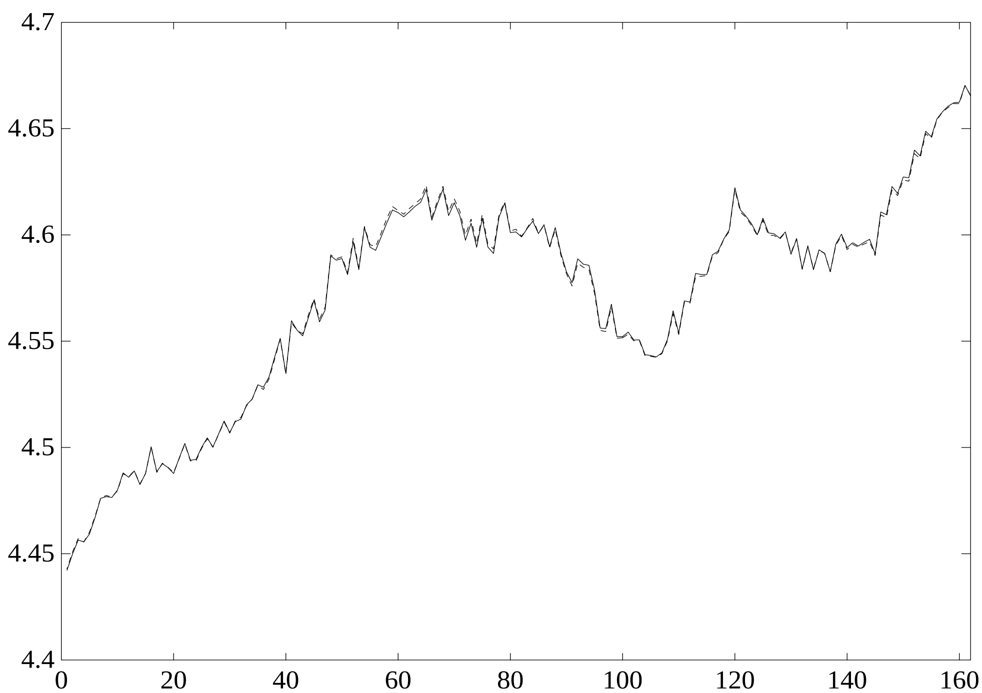
—— WK   Constraints: – – – $m = 36$ – · – $m = 24$ · · · $m = 18$

Figure 5a: FRTP Seasonally Adjusted Series



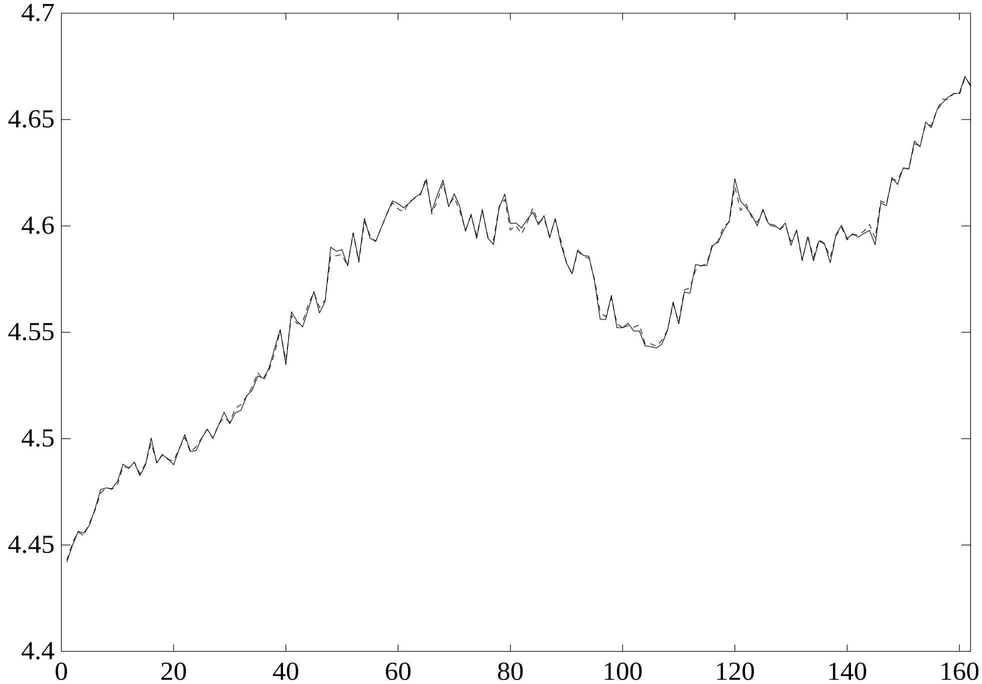
—— WK   - - - $m = 36$

Figure 5b: FRTP Seasonally Adjusted Series



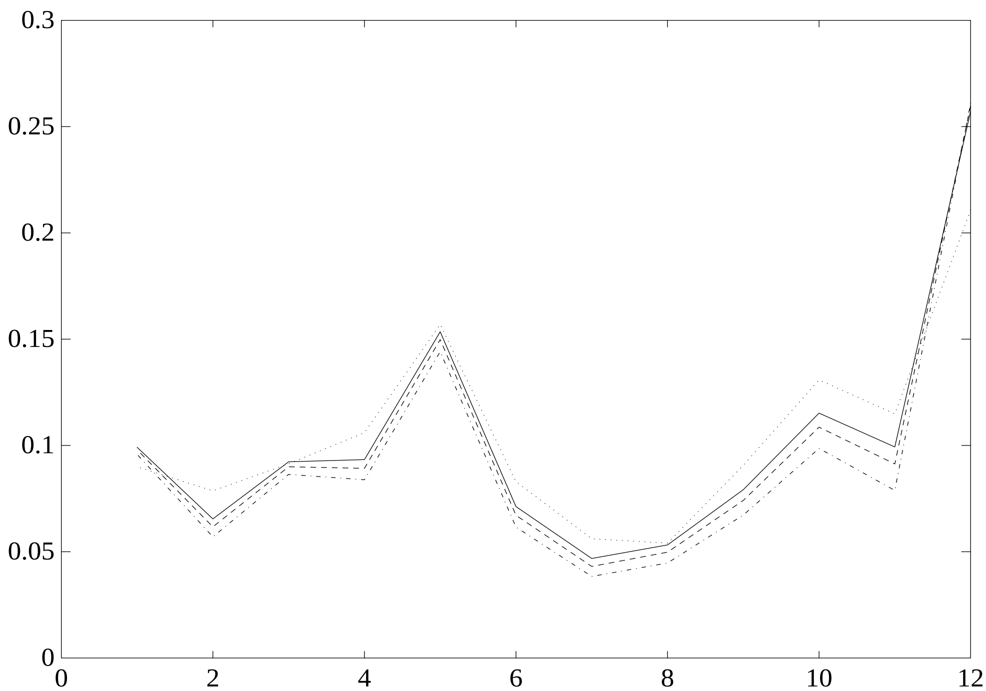
—— WK   - - - $m = 24$

Figure 5c: FRTP Seasonally Adjusted Series



—— WK   - - - $m = 18$

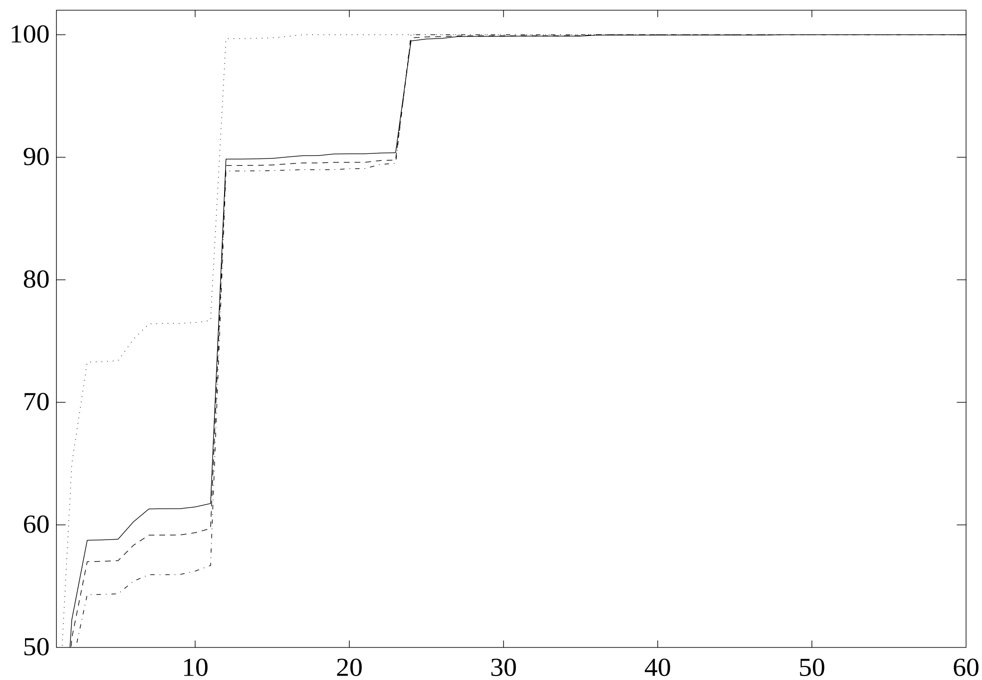
Figure 6: FRTP Empirical Variance of Total Revision



Month

—— WK   Constraints: - - - $m = 36$ - · - $m = 24$ · · · $m = 18$

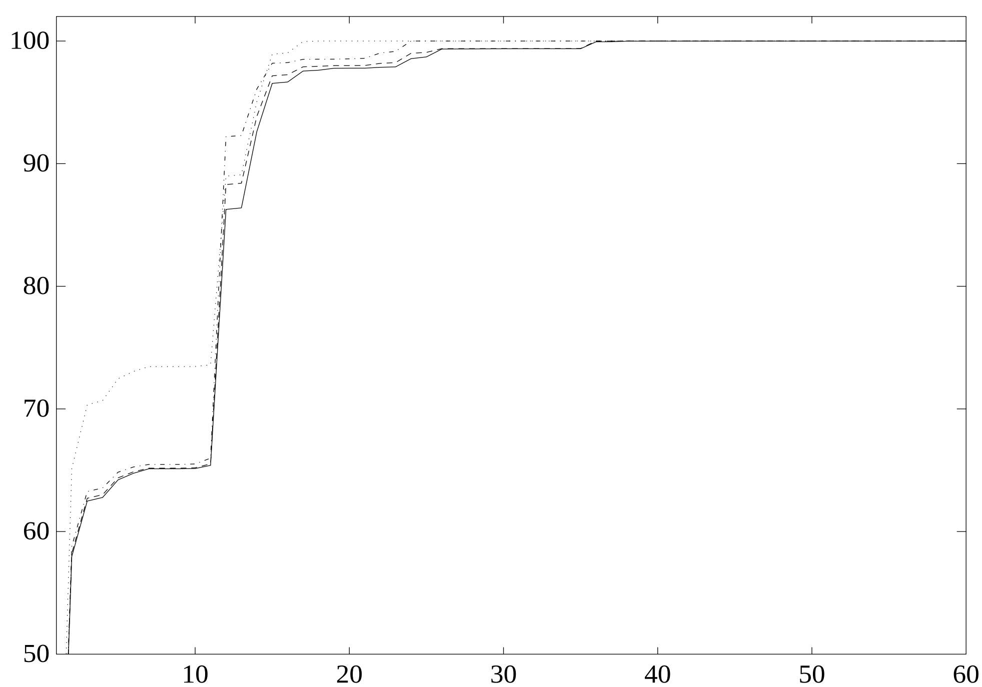
Figure 7a: FRTP **Empirical Rates of Convergence**



Additional Observations for 5-1992

—— WK Constraints: – – – $m = 36$ – · – $m = 24$ · · · $m = 18$

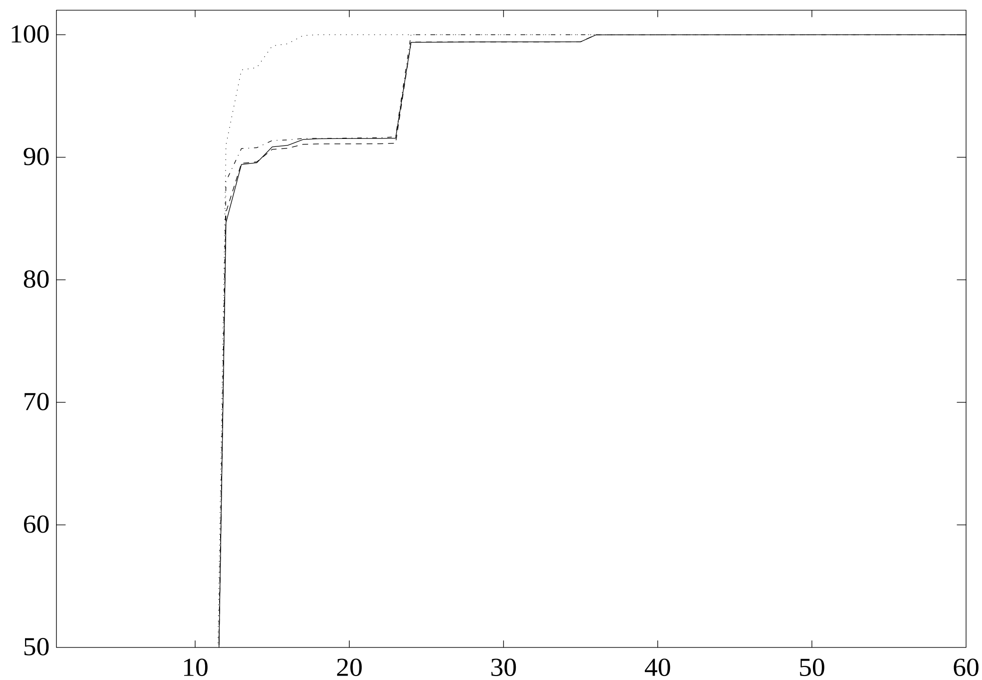
Figure 7b: FRTP **Empirical Rates of Convergence**



Additional Observations for 10-1992

—— WK Constraints: – – – $m = 36$ – · – $m = 24$ · · · $m = 18$

Figure 7c: FRTP Empirical Rates of Convergence



Additional Observations for 12-1992

—— WK   Constraints: $- - -\ m = 36$ $- \cdot -\ m = 24$ $\cdots\ m = 18$

Figure 8: Series DPCG (in logs)

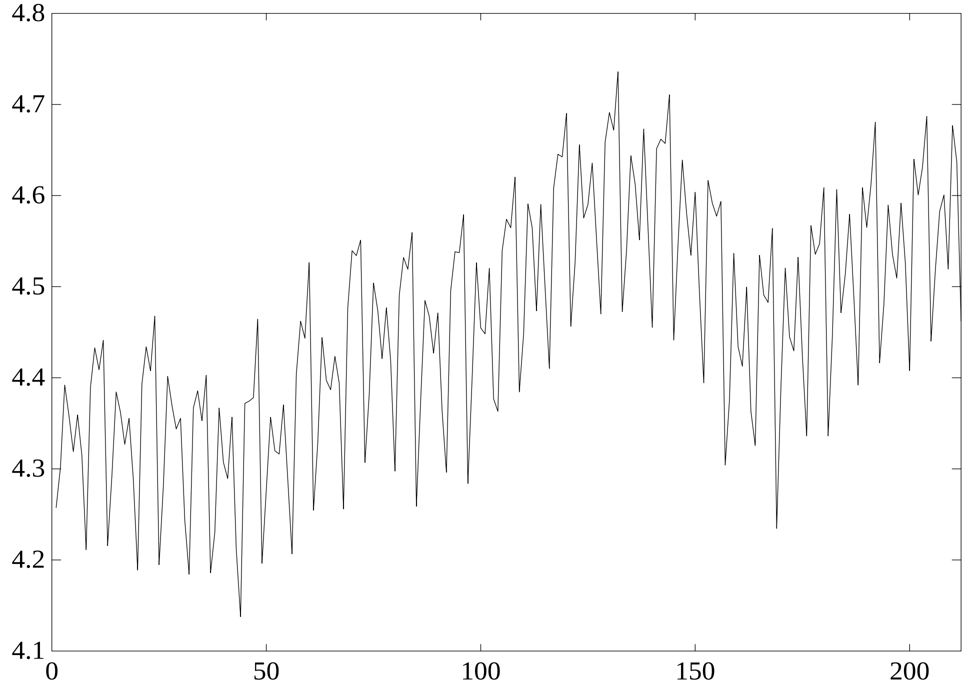


Figure 9: DPCG Rates of Convergence of Total Revisions

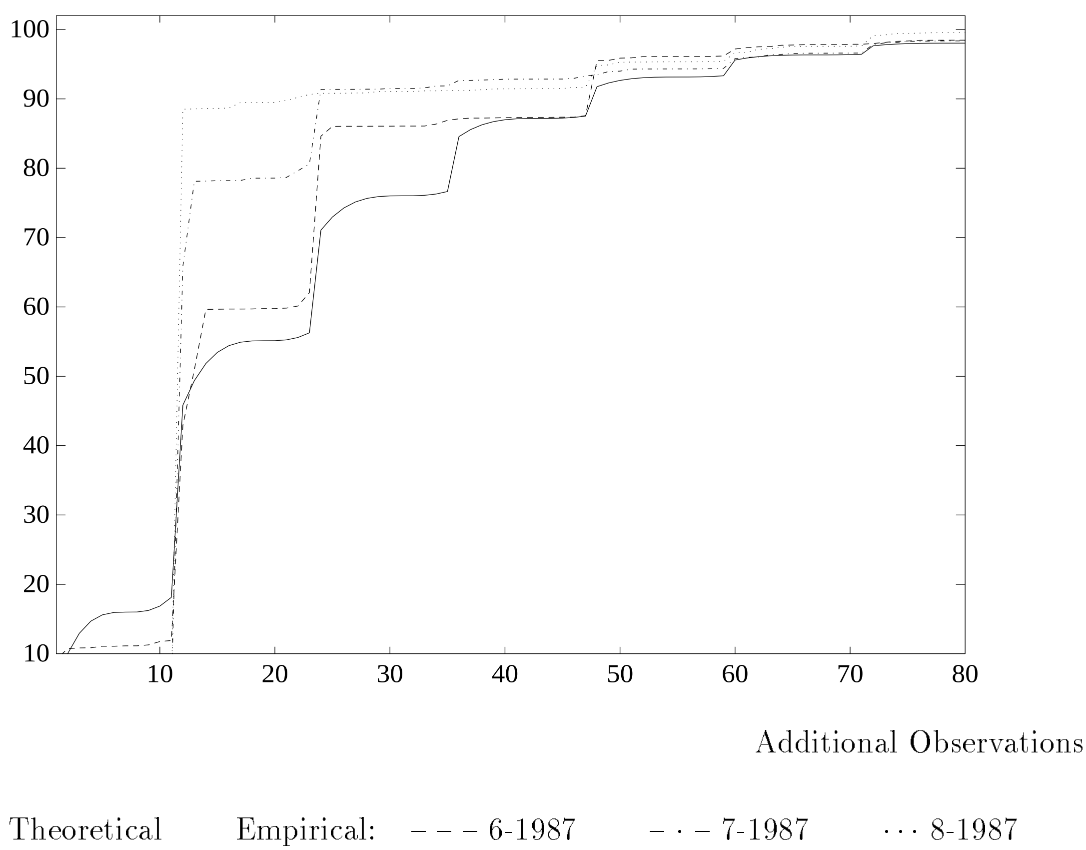


Figure 10: DPCG Seasonal Adjustment Filter

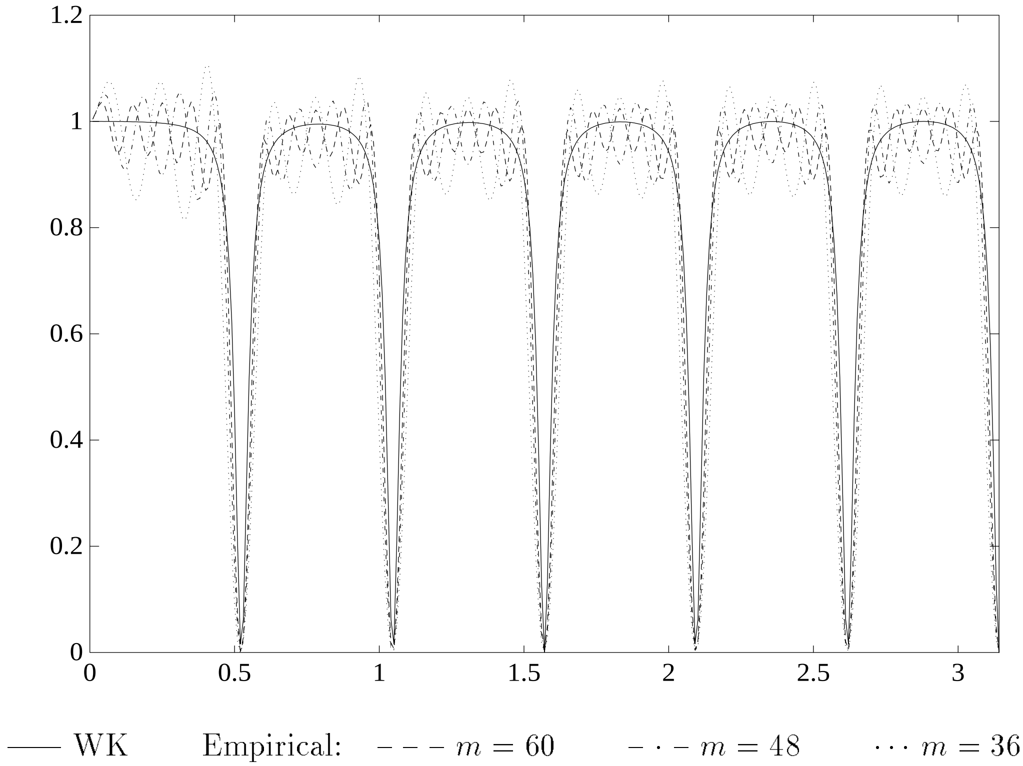


Figure 11a: DPCG Seasonally Adjusted Series

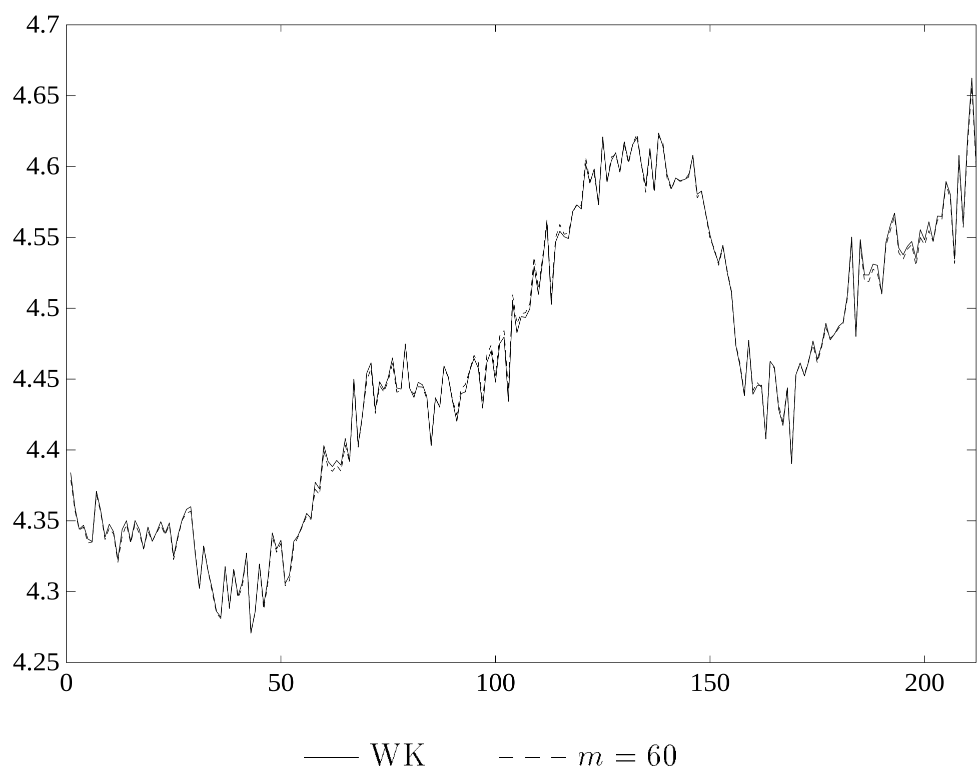


Figure 11b: Dpcg Seasonally Adjusted Series

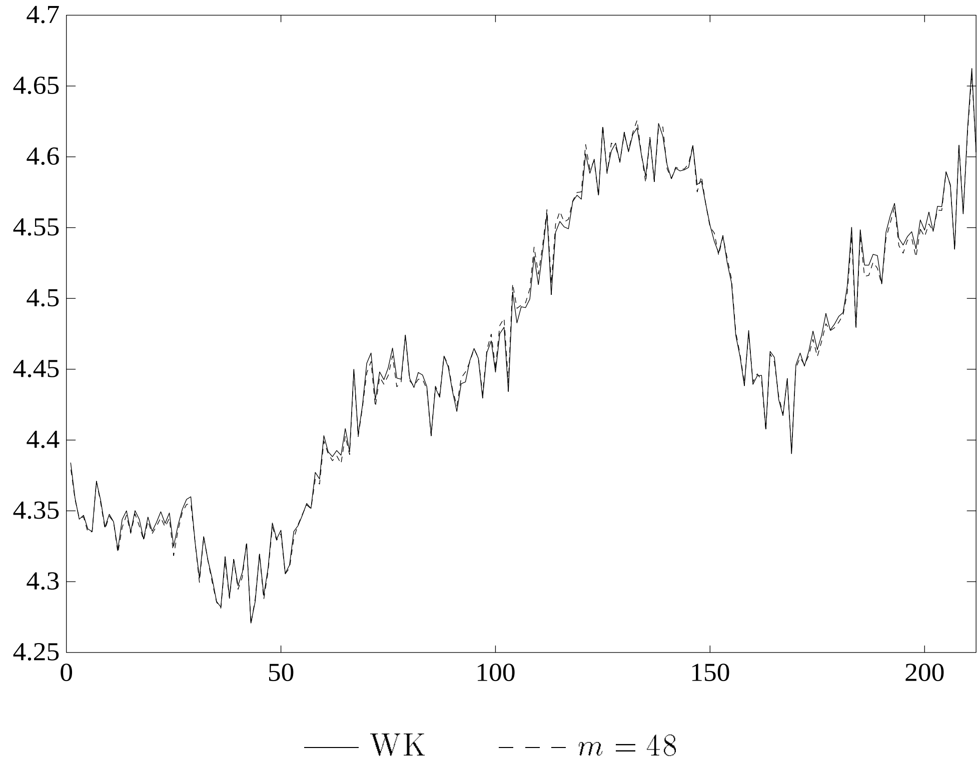


Figure 11c: Dpcg Seasonally Adjusted Series

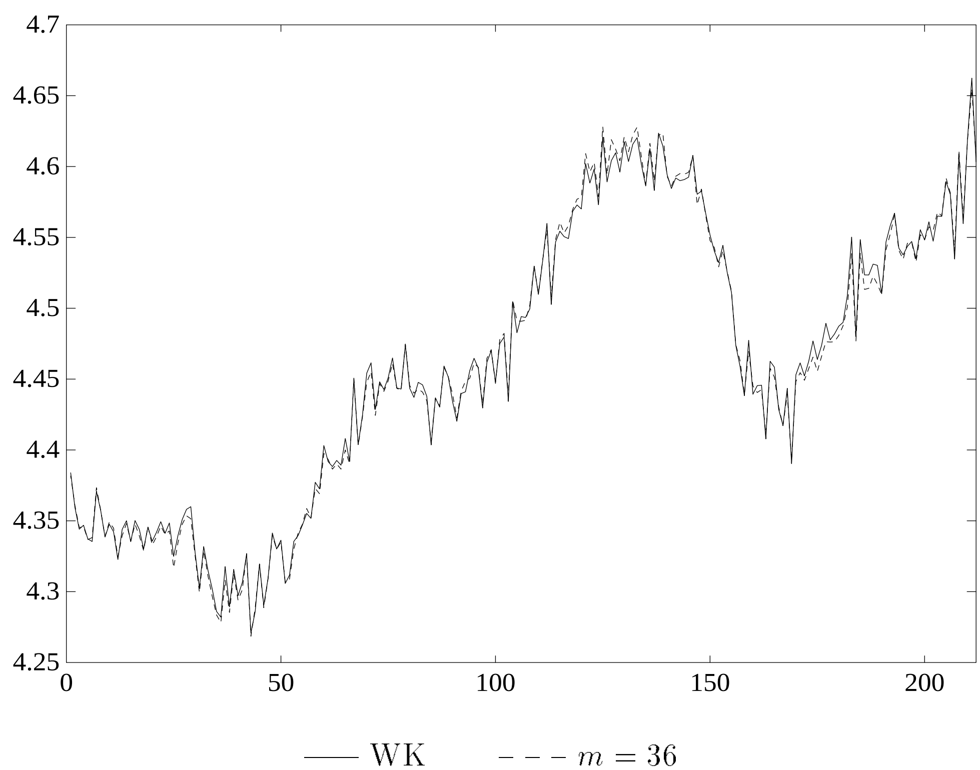
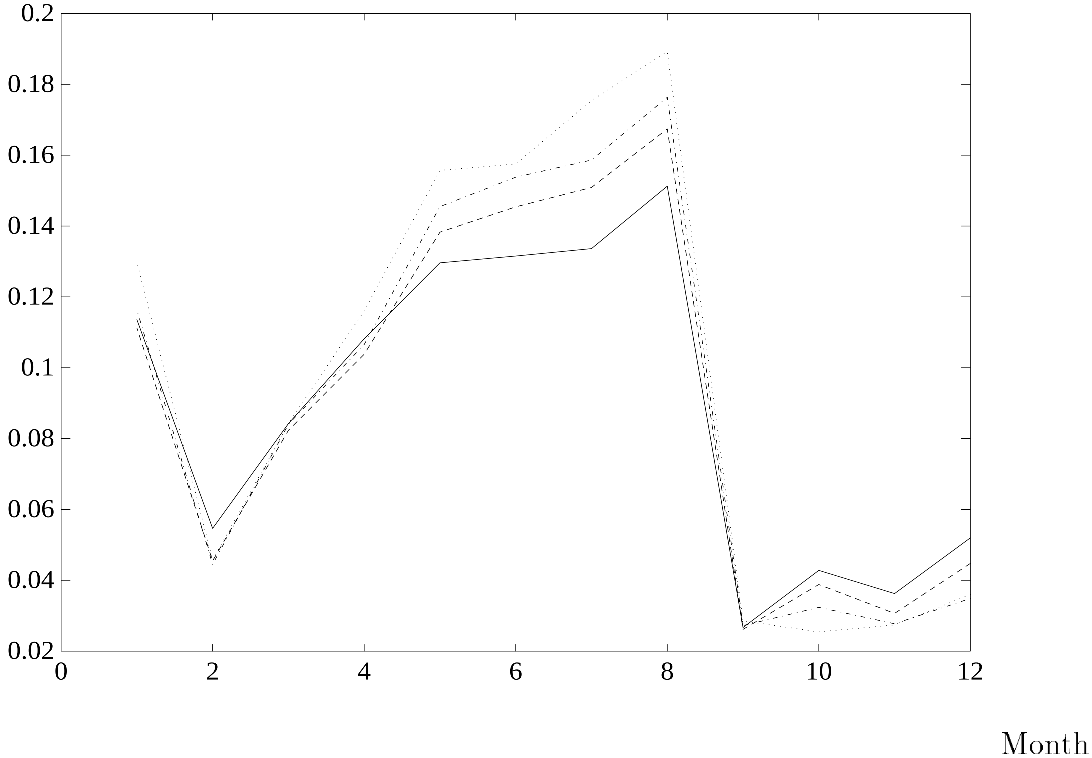
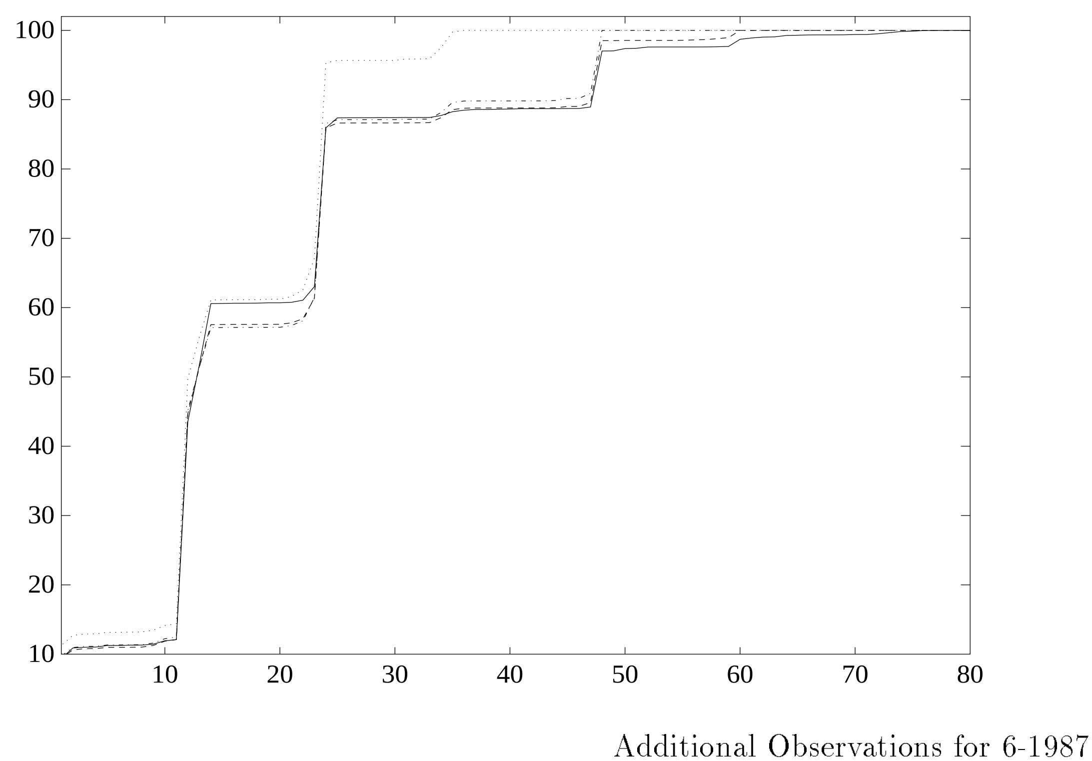


Figure 12: Dpcg Empirical Variance of Total Revision



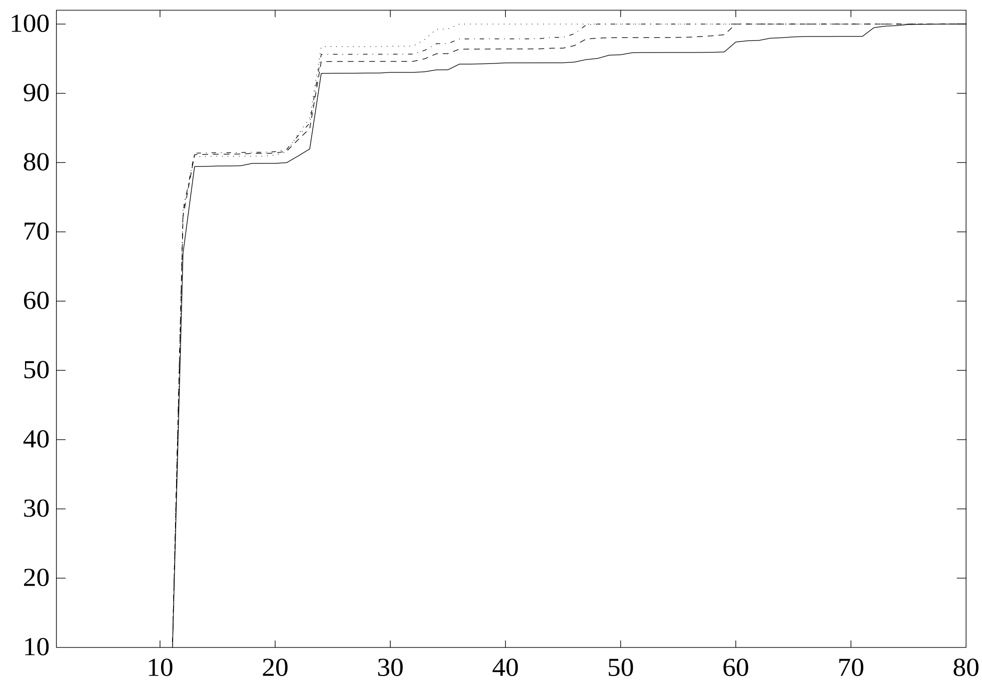
—— WK Constraints: $- - -\ m = 60$ $- \cdot -\ m = 48$ $\cdots\ m = 36$

Figure 13a: Dpcg Empirical Rates of Convergence



—— WK Constraints: $- - -\ m = 60$ $- \cdot -\ m = 48$ $\cdots\ m = 36$

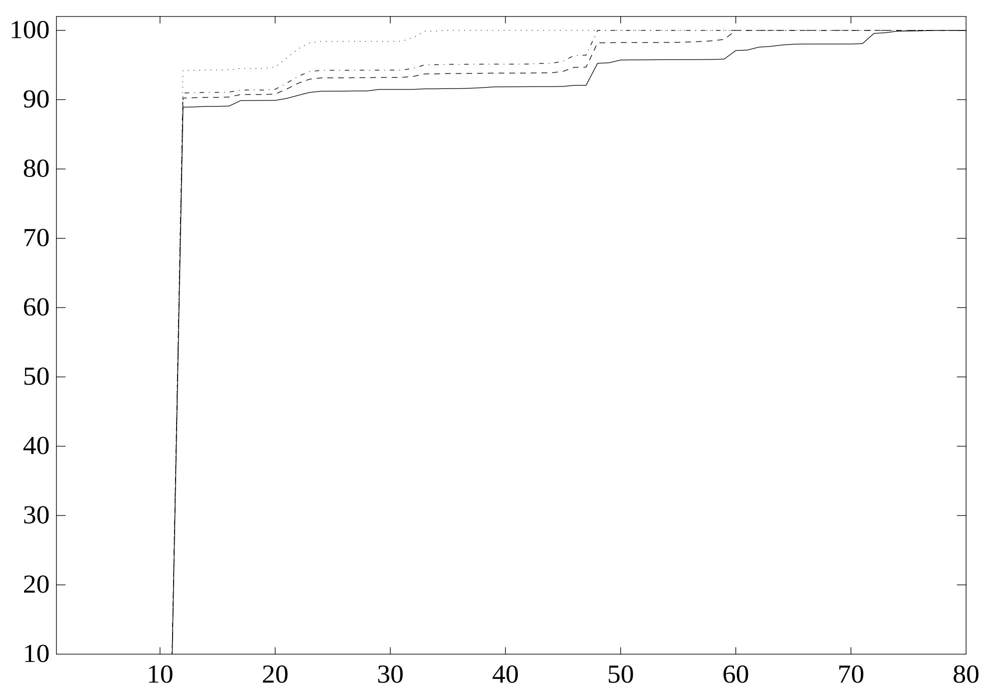
Figure 13b: Dpcg **Empirical Rates of Convergence**



Additional Observations for 7-1987

—— WK Constraints: $---\ m=60$ $-\cdot-\ m=48$ $\cdots\ m=36$

Figure 13c: Dpcg **Empirical Rates of Convergence**



Additional Observations for 8-1987

—— WK Constraints: $---\ m=60$ $-\cdot-\ m=48$ $\cdots\ m=36$

Figure 14: Ratio of distance between Adjustment Filters
100×distance finite WK-X11 / distance infinite WK-X11

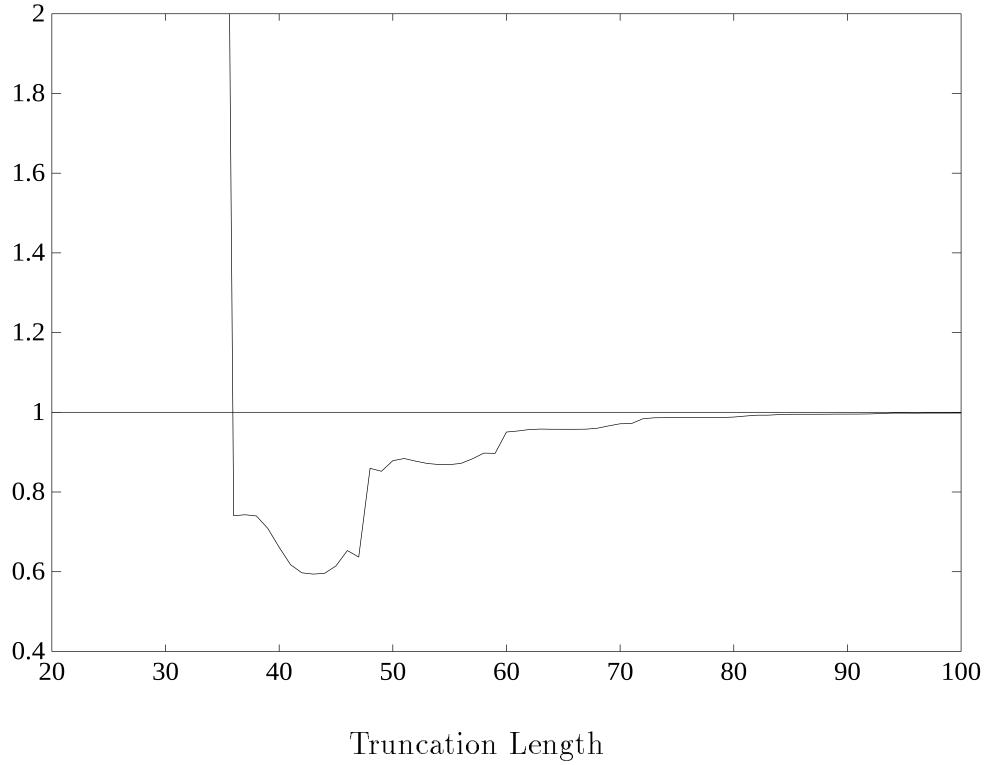


Figure 15: Frequency Transfer Function of Seasonal Adjustment Filters