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An Exact Algorithm For Computing The Model-Based Variance of Revisions in ARIMA-Model--Based Seasonal Adjustment

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TABLE OF CONTENTS

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1 Introduction

Some units of the Statistical Office of the European Community (EUROSTAT) use the ARIMA-model-based approach (see for example Box, Hillmer and Tiao, 1978;

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Burman, 1980) for seasonally adjusting macroeconomic indicators through the implementations of the program TRAMO-SEATS (see Gomez and Maravall, 1996). Like every method involving double-sided moving average filters, as new observations become available, seasonally adjusted figures close to the end of the sample need to be revised. An appealing feature of the model-based decompositions is that the variance of the revision process is can be obtained (see Maravall 1996). That measure has great applied interest since it enables practitioners first to anticipate the incoming change in the recent seasonally adjusted figures, and second to perform a validation check of the new data when the revision is recorded. That measure is made available in the program SEATS. Its computation generally involves a polynomial inversion and typically implies a truncation (see Maravall, 1999). Because of the truncation, the resulting measure can be seen as approximated. Of course the higher the truncation level the more accurate the approximation. However, when seasonal roots of the inverted polynomial are close to the unit circle in modulus, large truncation levels may be required for the approximation to be reasonable. Besides, the higher the truncation level, the longer the computing time. This note develops a simple algorithm for the exact computation of the model-based variance of the revision process. The procedure does not require any polynomial inversion. We use the result to check the accuracy of the approximation used in the program SEATS.

2 Problem specification

Let an observed time series be made up of orthogonal unobserved components according to

$$x_t = s_t + n_t \tag{2.1}$$

where s_t is a seasonal component and n_t the nonseasonal part of the series. Both are assumed to be well described by stochastic linear processes of the type

$$\phi_s(B)\; s_t = \theta_s(B)\; a_{st},$$

$$\phi_n(B)\; n_t = \theta_n(B)\; a_{nt}, \tag{2.2}$$

where $\phi_\bullet(B)$ and $\theta_\bullet(B)$ denote finite polynomials in the lag operator B, having all roots on or outside the unit circle. The variables a_{st} and a_{nt} are independent white noise with variances V_s and V_n, respectively. The polynomials $\phi_s(B)$ and $\phi_n(B)$ are prime, while the MA polynomials $\theta_s(B)$ and $\theta_n(B)$ share no unit roots. Further, the polynomial $\theta_s(B)$ is assumed to be noninvertible, so that the spectrum of $\phi_s(B)s_t$ takes a 0-value at some frequency. In the terminology of Hillmer and Tiao (1982), the so-defined seasonal component is said *canonical.*

Equations (2.1) and (2.2) imply that the observed series x_t follows an ARIMA model of the type

$$\phi(B)\; x_t = \theta(B)\; a_t, \tag{2.3}$$

where a_t is a white noise with variance V_a. Without loss of generality, we set $V_a = 1$.

The polynomial $\phi(B)$ can be obtained as the product $\phi(B) = \phi_s(B)\phi_n(B)$. The canonical hypothesis on the seasonal component implies that for a given $\theta(B)$ polynomial, the decomposition (2.1)-(2.2) is unique (see Hillmer and Tiao 1982). Minimum mean square error estimators (MMSE) can be built as

$$\begin{aligned} \hat{n}_t &= \nu_n(B)x_t = \sum_{i=-\infty}^{\infty} \nu_{ni}x_{t+i} \\ &= V_n \frac{\theta_n(B)\theta_n(F)\phi_s(B)\phi_s(F)}{\theta(B)\theta(F)} x_t \end{aligned} \tag{2.4}$$

The filter $\nu_n(B)$ is known as the Wiener-Kolmogorov filter (see Whittle 1963). The hypothesis that the polynomials $\theta_s(B)$, $\theta_n(B)$ do not share unit roots imply that $\theta(B)$ is invertible (see Maravall and Planas, 1998), and thus the filter $\nu_n(B)$ is infinite with converging weigths ν_{ni}. Hence (2.4) is valid for signal estimation for periods around the center of usual sample lengths. In finite samples, preliminary estimates must be computed for periods close to the sample ends. I focus on the concurrent estimate case which is by far the most interesting one. Unknown future observations are replaced by their forecasts, and as new observations become available, correcting for the forecast error yields revisions in the preliminary estimates.

The process followed by the revision errors can be described as follows. Let $\hat{n}_{t|t+k}$ denote the preliminary estimate of n_t computed at time $t+k$, $k \geq 0$. Assuming that $X_{t+k} = \{x_1, \cdots, x_{t+k}\}$ is available, then

$$\begin{aligned} \hat{n}_{t|t+k} &= E(\hat{n}_t | X_{t+k}) \\ &= E(\sum_{i=-\infty}^{\infty} \nu_{ni} x_{t+i} | X_{t+k}) \end{aligned} \tag{2.5}$$

Let $\xi(B) = \cdots + \xi_{-n}B^n + \cdots + \xi_0 + \xi_1 F + \cdots + \xi_m F^m + \cdots$ denote the polynomial obtained as

$$\phi(B)\xi(B) = \nu_n(B)\theta(B)$$

Expressing the estimator (2.4) in fonction of the innovations a_t, it is easily seen that $\hat{n}_t = \xi(B)a_t$, so that the conditional expectation in (3.1) reduces to:

$$\hat{n}_{t|t+k} = \sum_{-\infty}^{i=k} \xi_i a_{t+i}$$

The revisions in the preliminary estimator of n_t computed at time $t+k$ are given by:

$$\hat{n}_t - \hat{n}_{t|t+k} = \sum_{i=k+1}^{\infty} \xi_i a_{t+i} \tag{2.6}$$

with variance:

$$V[\hat{n}_t - \hat{n}_{t|t+k}] = \sum_{i=1}^{\infty} \xi_{k+i}^2 \tag{2.7}$$

It can be seen from (2.4) and from the definition of the polynomial $\xi(B)$ that the weigths ξ_i, $i > 0$, converge so that after a certain number of periods, say M, revisions become negligible: $\hat{n}_{t|t+M} \equiv \hat{n}_t$. The program SEATS uses $M = 600$, a safe value as it is very large. This is however an approximation, and in the next section an exact solution is proposed.

3 Exact computation of the revision variance

Given (2.1) and (2.4) the polynomial $\xi(B)$ verifies:

$$\xi(B) = V_n \frac{\theta_n(B)}{\phi_n(B)} \frac{\theta_n(F)\phi_s(F)}{\theta(F)} \tag{3.1}$$

By splitting the denominator of that last expression, $\xi(B)$ can be re-written as:

$$\xi(B) = \frac{N_B(B)}{\phi_n(B)} + \frac{N_F(F)}{\theta(F)} \tag{3.2}$$

where $N_F(F)$ is such that $N_F(F) = n_1 F + ... + n_Q F^Q$. Let p_n, p_s, q_n and q denote the respective orders of the polynomials $\phi_n(B)$, $\phi_s(B)$, $\theta_n(B)$ and $\theta_n(B)$. The order of the polynomial $N_F(F)$ is such that $Q = p_s + q_n$. The revisions are seen to follow the process

$$\theta(F)r_t = N_F(F)b_t \tag{3.3}$$

In order to obtain the exact revision variance it is enough to obtain the polynomial $N_F(F)$ and to compute the variance of the process (3.3) using standard time series techniques.

Let $c(B)$ denote the polynomial convolution $c(B,F) = V_n\theta_n(B)\theta_n(F)\phi_s(F) = c_{-q_n}B^{q_n} + \cdots + C_1F + \cdots + c_{q_n+p_s}F^{q_n+p_s}$. From (3.2) it is easily seen that

$$N_B(B)\theta(F) + \phi_n(B)N_F(F) = c(B,F) \tag{3.4}$$

Equating coefficients of $F^{q_n+p_s}, ..., F, B^0, B, \cdots, B^{p_n}$,

$$\begin{aligned}
N_{Fq_n+p_s} &= c_{q_n+p_s} \\
N_{Fq_n+p_s-1} + N_{Fq_n+p_s}\phi_{n1} &= c_{q_n+p_s-1} \\
&\dots \\
N_{Fq} + \phi_{s1}N_{Fq+1} + \cdots + \phi_{np_s}N_{Fq_n} + \theta_q N_{B0} &= c_q \\
N_{Fq-1} + \phi_{s1}N_{Fq} + \cdots + \phi_{np_s}N_{Fq_n-1} + \theta_q N_{B1} + \theta_{q-1}N_{B2} &= c_{q-1} \\
&\dots \\
\phi_{s1}N_{F1} + \cdots + \phi_{np_n}N_{Fp_n} + N_{B0} + \theta_1 N_{B1} + ... + \theta_q N_{Bq} &= c_0 \\
&\dots \\
\phi_{np_n}N_{F1} + \theta_{p_n+1}\theta_1 N_{B1} + ... + \theta_q N_{Bq+p_n} &= c_{-p_n} \\
&\dots \\
N_{B-q_n} &= c_{-q_n}
\end{aligned} \tag{3.5}$$

The system (3.5) is made up of $q_n + p_n + q_s + 1$ equations which identify the $q_n +$

$p_n + q_s + 1$ unknowns in $N_B(B)$ and in $N_F(F)$. Inserting the polynomial $N_F(F)$ in (3.3) enables to find out the revision variance.

4 Evaluation of SEATS approximation

The SEATS approximation can then evaluated using the exact algorithm. The evaluation is conducted on the airline model specified as:

$$(1 - B)(1 - B^{12})x_t = (1 + \theta_1 B)(1 + \theta_{12} B^{12})a_t$$

over a range of values for the moving average parameters. The result is displayed on Table 1 below.

Table 1

Model-Based Variance of Revision Error

θ_{12} \ θ_1		-.2	-.4	-.6	-.8	-.9	-.95	-.98
-.5	SEATS	.150	.127	.102	.072	.038	.020	.007
	EXACT	.150	.127	.102	.072	.038	.020	.007
-.6	SEATS	.119	.103	.084	.061	.033	.017	.006
	EXACT	.119	.103	.084	.061	.034	.018	.007
-.7	SEATS	.100	.090	.076	.057	.032	.017	.006
	EXACT	.100	.090	.076	.057	.033	.017	.007
-.8	SEATS	.089	.085	.076	.059	.034	.018	.006
	EXACT	.089	.085	.076	.059	.035	.019	.008
-.9	SEATS	.088	.087	.079	.062	.036	.019	.007
	EXACT	.089	.087	.079	.063	.037	.0205	.009
-.95	SEATS	.089	.089	.081	.064	.038	.020	.007
	EXACT	.089	.089	.082	.065	.039	.021	.010
-.98	SEATS	.090	.091	.083	.066	.039	.021	.007
	EXACT	.090	.091	.084	.067	.040	.022	.010

Two messages can be get from that investigation:

1. The SEATS approximation is in general quite accurate, in particular when the MA polynomial do not lie close to the unit circle;

2. Close to the non-invertibility region, a slight loss in accuracy in the SEATS approximation can be noticed: the revision error variance comes out as under-evaluated.