WORKING PAPERS AND STUDIES

COPYRIGHT

Luxembourg: Office for Official Publications of the European Communities, 2003

ISBN 92-894-5341-9
ISSN 1725-4825
Cat. No. KS-AN-03-013-EN-N

2003 EDITION

Business cycles analysis and related software applications

EUROPEAN COMMISSION

EURO INDICATORS

Europe Direct is a service to help you find answers to your questions about the European Union

New freephone number:
00 800 6 7 8 9 10 11

A great deal of additional information on the European Union is available on the Internet.
It can be accessed through the Europa server (http://europa.eu.int).

Luxembourg: Office for Official Publications of the European Communities, 2003

ISBN 92-894-5341-9
ISSN 1725-4825

TABLE OF CONTENTS

Business cycles analysis and related software applications

Gian Luigi MAZZI* and Marco SCOCCO**

*Eurostat, Unit A6
Statistical Indicators for Euro-zone Business Cycle Analysis
Jean Monnet Building, L-2920 Luxembourg
e-mail: gianluigi.mazzi@cec.eu.int

**2SDA 17, rue des Bains L-1219 Luxembourg
e-mail: marco.scocco@2sda.lu

Website: http://europa.eu.int/comm/euroindicators

Abstract: Despite the importance of business cycle analysis, currently there is no specific software containing all the techniques that could be used for the purposes of diagnosis of the cyclical behavior of the economy. In this paper we present the most widely used techniques for business cycle analysis along with a software review linking together theoretical and computational problems. We focus on two main views of the business cycle, the classical and the growth cycle views. Moreover we present the main applications based on aggregated data for the Eurozone. A final table provides useful web links where programs to replicate our results can be downloaded.

Keywords: Business cycles; Turning points; Markov Switching; Software applications.

Business cycles analysis and related software applications

Gian Luigi MAZZI* and Marco SCOCCO**

*Eurostat, Unit A6
Statistical Indicators for Euro-zone Business Cycle Analysis
Jean Monnet Building, L-2920 Luxembourg
e-mail: gianluigi.mazzi@cec.eu.int

**2SDA 17, rue des Bains L-1219 Luxembourg
e-mail: marco.scocco@2sda.lu

Website: http://europa.eu.int/comm/euroindicators

1 Introduction

In modern macroeconomics the analysis of growth and fluctuations represent two of the main topics. The behavior of the economy is described by an interaction between growth and cycles. A correct understanding of those phenomena is essential for a correct analysis of economic momentum and for anticipating future movements. It is also quite clear that the attention of analysts is more oriented to long-term growth in the so-called "development economics" whereas the need for reducing economic fluctuations is the first objective for the mature market economies.
In the same way, long term policy measures (structural policies) have economic growth as a principal objective. By contrast, short-term policies (stabilization policies) have the reduction of economic fluctuations as a principal objective.
Stabilization policies were firstly defined by Tinbergen[20]. More recently Mundell[21], Sargent[22], Lucas[23] and others aim to minimize the variance of some relevant macroeconomic variables around their trend which is often viewed as the steady-state position of the economy. In order to build correct and efficient stabilization policies it is necessary to have a good understanding of the cyclical situation of the economy as well as of its future perspectives. This is why business cycle analysis is now considered as an essential instrument not only for identifying the state of the economy but, and much more, to supply policy makers with the relevant information on which they can build their measures to ensure economic stability.
From the end of the second world war a large variety of techniques and tools for analyzing business cycle behavior of the economy have been proposed. Various approaches are often based on quite different assumption and theories so that their results and interpretations can sometimes be complex. Moreover, for the practice of business cycle analysis is not really easy to link different methodology proposed in a variety of scientific papers and computer tools which can permit their daily use.
The aim of this paper is to present the most widely used techniques for business cycle analysis associated with a detailed software review linking together theoretical, empirical and computational problems and aspects. We focus on two main approaches to describe business fluctuations: the classical one developed by NBER for the US economy and the growth cycle approach derived from the concept of permanent and transitory components in economic variables. Clearly, our survey does not intend to be exhaustive but only to supply a useful didactic instrument for people who are interested in business cycle analysis.
The paper is organized as follows: section 2 presents the different views of the business cycles, section 3 introduces the problem of statistical methods and tools availability, section 4 presents growth cycles and the different ways for trend estimation and detrending, section 5 presents parametric and

non-parametric approaches to turning points detection; in section 6 some applications to Eurozone IPI[1] are presented, in section 7 we summarize the software available for business cycle analysis and in section 8 we present some conclusions and recommendations.

2 Alternative assessments of the economic fluctuations

The measurement and the analysis of economic cycles have been a "holy grail" of economic research for many years. The first studies dated from the end of XIX, but only around 1930 was there a first important rationalization of the matter by Schumpeter[24]. The modern theory of economic cycles is usually referred to as the pioneering study of Burns and Mitchell[1] who introduced the concept of the business cycle.
In the economic literature we can identify two kinds of approaches to analyzing business cycle fluctuations: the classical business cycle and the growth cycle. They differ in the way the economic fluctuations are measured: the first one is based on trend-cycle figures and the second one on detrended data.
Following Pagan[2], classical cycles are:

"...hills and valley in a plot of the levels of the series...representing the general level of economic activity."

Growth cycles are defined as fluctuations in the stationary part of the relevant economic variables. Growth cycles relate short-term movements of aggregate activity to the long-run term ones.
Other alternative views of the economic, even if less relevant in terms of the literature available as well as of the empirical studies, the recovery cycle and the growth rate cycle. The following subsection briefly describes such alternative approaches.

2.1 The classical approach

This view of business cycles relates to the monumental work of A.F. Burns and W.C. Mitchell[1]; the two authors define business cycles in the following way:

"*Business cycles are a type of fluctuations found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many*

[1]Plotted in Figure 2.

economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary form more than one year to ten or twelve years; they are not divisible into shorter cycles of similar characters with amplitudes approximating their own."

The Burns and Mitchell classical definition of the business cycles still forms the basis of the work by the NBER[2]. The methodologies proposed by the two authors were refined and extended by Victor Zarnowitz[8], G.H. Moore[27] and Anirvan Banerji[10] at the Economic Cycles Research Institute(ECRI) in New York to obtain a characterization of economic fluctuations for a number of developed countries.
Within each economy the ECRI approach distinguishes between cycles in growth, employment, inflation and foreign trade; even cycles in the major sectors of the domestic economy are addressed with the aim of producing a system of leading indicators for monitoring critical aspects of the economy.
As defined by Burns and Mitchell the business cycle is a pattern seen in a series taken to represent the aggregate economic activity of a nation. As pointed out by Harding and Pagan[3] such a statement lacks precision on two counts; it does not say how one can measure aggregate economic activity and it does not indicate how one is to describe the patterns in it.
As for the first question the two authors use a wide range of time series to date the business cycle as no single reliable monthly or quarterly measure of the aggregate economic activity was available to them. The pattern in each single series is then described by identifying the turning points.
These turning points define the "specific cycles" in each series, from these specific cycles a single sequence of turning points is distilled and constitutes the "reference chronology" which *is* the business cycle of a country. Two points emerge from this brief description:

1. Burns and Mitchell do not distinguish between business cycles and economic growth.
2. To characterize the business cycle of a nation it is sufficient to determine the turning points in the series representative of aggregate economic activity.

It is self evident from[1] that the two authors never tried to isolate the trend from the series under investigation. The Burns and Mitchell approach focuses on the "cycles relatives"; once the turning points in the reference series are detected, a value of 100 is attributed to the observations which

[2]The dating committee of the NBER determines the official business cycles chronology for the United States.

correspond to these turning points; all other values are compared with 100. This procedure while removing the inter-cycle trend leaves the intra-cycle trend unaltered.
The second point above mentioned shifts our attention to the different methodologies that the current literature provides for turning points detection. A review of these methodologies as well as of the software routines currently available for these purposes is presented in section 5.1 and 5.2.

2.2 The growth cycle view

It is necessary to point out that even if the main attention of the NBER has historically been focused on classical cycles, the growth cycles have consistently also been studied.
Impressed by the fact that countries in western Europe, whose capital stock was destroyed during WW2, did not have a classical recession in the post war period, the NBER researchers thought it was interesting to study the growth cycles for these countries. The most relevant results of these studies are due to the work of of Mintz[6][7].
As Mintz[6] points out, to distinguish between classical expansions and recessions is useful, but when one of the two phases doesn't occur frequently, a new definition of business cycles phases is required. According to the empirical evidence the suggestion of Mintz was to keep the analysis in the same framework developed by Burns and Mitchell by simply changing their definition and introducing the concept that the series assumed to measure aggregate economic activity must be considered in a deviation from trend form.
Before dealing with the relevant issue of trend and cycle decomposition it is useful to introduce some empirical evidence about the relationship between growth cycles and classical business cycles. This evidence mainly comes from the US experience; a valuable description of the same can be found in Zarnowitz[8] and in Niemira[9].

1. Growth cycle highs tend to lead the corresponding peaks in the classical cycles[3];

2. Growth cycles tend to be more symmetric in duration and amplitude with respect to the classical cycles. These are characterized by a strong asymmetry in duration[4];

3. Growth cycles manifest a strong connection with inflationary cycles. Empirical evidence for the U.S. suggests that a change in the rate of

[3]In the business cycles terminology the words high and low are usually referred to the growth cycles, peaks and troughs to the classical cycles.

[4]Expansionary periods are considerably longer than recessionary periods ones.

growth for the economy precedes a change in prices by an average of seven months.

2.3 Other views of business cycle

Finally it can be of interest to recall that classical and growth cycles are not the only ways to describe economic fluctuations. There are at least two other ways to do it: growth rate cycle and recovery cycles.
By the late 1980s, the use of growth rate cycles for measurement of series which manifested few actual cyclical declines but did show cyclical slowdown[5] was introduced.
The growth rate cycle is based on the ratio of the latest monthly figure to the average of the preceding twelve months. This approach can be viewed as an alternative, even if quite raw, way to obtain the detrended series. Turning points could be identified by the same procedures described in section 5.1 and 5.2. The growth rate cycle displays some interesting features that render it particularly suitable for real time monitoring and forecasting:

1. It avoids the problem of trend estimation;

2. The cyclical turns in the broad measures of aggregate economic activity in the form of output, income, employment and sales cluster together;

Recovery cycles are an important addition to the measure of the duration of classical business cycles. The duration of business cycles is measured as the period from peak over trough to the recovery date, the date where the level of the previous peak is reached.

2.4 Some conclusions

Despite the fact that in some cases the classical and the growth cycle views of business cycle have been presented as alternative ways to analyze economic fluctuations, in reality they can be used as a complementary analytical instrument. As mentioned above peaks in growth cycles tend to lead those observed by the classical approach. This characteristic is not a consequence of a bias in one of the two views but it indicates that the two series of peaks measure different economic situations. Peaks in the growth cycle represent a slow down and an acceleration of the economic activities whereas in the

[5] See Banerji and Hiris[10] for an exhaustive treatment of the argument

classical view, they respectively describe trend inflection and recovery. By putting together the four elements described above we obtain a complete definition of the different states of the economy during a complete cycle. Moreover, empirical evidence shows that anticipating peaks in the growth cycle is easier than in the cyclical Burns and Mitchell approach. In other words decelerations and accelerations of economies are easier anticipated than trend inflections and recoveries. Finally, it is useful to point out that, as Mintz observed, decelerations and accelerations may even occur without trend inflections and recoveries.

3 Statistical methods and tools availability

The different views proposed in section 2 are related to some statistical methods for the analysis of business cycles. We decided to concentrate our attention on two major categories: detrending methods and turning points detection methods. The first one is strongly related to growth cycles analysis described in section 2.2 whereas the second one can be applied to all different views of business cycles. As it can clearly be shown, the classical view doesn't require the use of special statistical techniques with the exception of normal smoothing ones(i.e. moving averages) to eliminate the irregular component. In this paper we decided not to analyze with particular attention the remaining views proposed in section 2.3.

Both detrending and turning points detection methods can be broadly classified in two main approaches following their statistical foundation: parametric and non-parametric.

A non-parametric method extracts a signal from a time series by taking a weighted moving average over its observations. A parametric method involves the specification of a statistical model, the estimation of a set of parameters, and the application of a signal extraction algorithm.

3.1 Tools availability

Despite the importance and the relevance of business cycle analysis, currently we observe that there is not a specific software containing all the techniques that could be used for the purposes of diagnosis of the cyclical behavior of the economy.

The BUSY software currently developed in a research program financed by the European commission, should in principle cover this gap even if it is not yet available. Different techniques are available on a wide range of software sometimes proposed by the official developers but most commonly by researchers and academics. For these reasons different techniques available in the same environment are not necessarily coherent in terms of data management and characteristics of the output and for the same reasons, business cycle analysis is frequently carried on by using different software with a large human intervention. In our survey we decided to concentrate our attention on the main econometric packages such as Rats, E-views and Pc-Give and on the most used matrix programming languages like Gauss, Matlab, Ox and SAS.

4 Filtering techniques for trend extraction

A crucial problem in modern empirical business cycle research is the detrending of macroeconomic time series to compute the stylized facts[6] of the growth cycles.
For a long time the dominant approach to modelling these time series was to view them as a sum of a deterministic trend and stochastic deviations treated as the residual "cyclical" component[7].
Since Nelson and Plosser[11] it has been common to regard macroeconomic time series as being difference stationary instead of trend-stationary.
A trend stationary series can be represented in the following way:

$$y_t = \sigma_0 + \sigma_1 t + c_t \tag{1}$$

$$c_t = \alpha c_{t-1} + \epsilon_t \tag{2}$$

under the stationarity condition $|\alpha| < 1$.
The above equations assert that the variable y_t is subject to a constant growth trend and that the deviations from the trend follow a stationary $AR(1)$ process. The actual values are seen to fluctuate around the trend with no obvious tendency for the amplitude of the fluctuations to increase or decrease. The steady increase in the mean level renders the series non-stationary but its first difference is stationary:

$$\Delta y_t = \sigma_1 + \Delta c_t \tag{3}$$

as Δc_t is stationary since c_t is stationary.
If the parameter α in the equation(2) is one, meaning the autoregressive part of the relation(1) has a unit root, we have a series that strays away from the linear trend.

[6]The moments of the detrended series.
[7]For instance a linear time trend fitted to the log of a macroeconomic series representing the output.

The first difference has the form:

$$\Delta y_t = \sigma_1 + \epsilon_t \tag{4}$$

and it is stationary, being a constant plus white noise.
We refer to the y variable in this case as Difference Stationary(DS).
Figure 1 in section 6 displays a TS and a DS series generated by simulation.
Even if the two cases mentioned seem to be quite similar there is a fundamental difference between TS series and DS series involving the reaction of the system to innovations or shocks ϵ_t. These have a transient effect in TS processes and a permanent effect in DS case[8].
Of course it is unlikely that any type of linear deterministic trend can persist over a long time as this would imply that economic growth is not influenced by structural and technical changes as well as by wars and financial crisis. Trends are variable because of interactions with shorter fluctuations and possibly structural breaks.
A large number of statistical methods have been proposed for isolating the permanent and transitory components in macroeconomic time series, these can be classified mainly as non-parametric and parametric.
We present below a review of the most commonly used techniques for signal extraction, the Hodrick and Prescott filter and the Baxter and King filter are classified as non-parametric according to the above mentioned definition, the Unobserved component model and the Beveridge and Nelson decomposition as parametric.

4.1 The Hodrick and Prescott filter

The Hodrick and Prescott filter is the most widely known and commonly used univariate method for the estimation of the trend component of a time series. It is largely used in scientific papers as well as by international organizations like the IMF and the OECD. In the European Union it is used by the Economic and Financial Affairs Directorate and in the Economic Directorate of the European Central Bank.

4.1.1 Description

The application of the Hodrick and Prescott filter extracts from a series y_t the growth component g_t . The estimation of g_t is obtained through the minimization of the sum of squares of the transitory component subject to a penalty for the variation in the second differences in the growth component. That is g_t is the solution to the following minimization problem:

[8]Johnston and DiNardo[12] provide an exhaustive treatment of the problem.

$$\min_{[g_t]_{t=1}^T} \sum_{t=1}^{T} [(y_t - g_t)^2 + \lambda[(g_{t+1} - g_t) - (g_t - g_{t-1})]^2] \quad (5)$$

where λ is a penalty parameter which is closely related to the smoothness of the estimated trend. The larger λ is, the smoother is the result.
It is easy to understand that for this filter the role of the smoothing parameter λ is crucial. In their original paper Hodrick and Prescott recommended some values for λ. These are the following:

- $\lambda = 100$ for annual data;
- $\lambda = 1600$ for quarterly data;
- $\lambda = 14400$ for monthly data.

The proposed values for λ can be derived from the formula $100 * f^2$ where f is the frequency of the observations (f=1, 4, 12 for annual,quarterly and monthly data).
The effect of the filter depends on the properties of y_t; when it is a stationary series Pedersen[17] derives the optimal estimator of the λ parameter; as many economic time series are assumed to have a unit root, a judgement must be used to choose the value of λ.

4.1.2 Tools

As above mentioned the HP filter is a smoothing technique commonly used to estimate the long-term trend of a series. Its recent widespread use is probably due to its simple estimation procedure that rends it extremely simple to implement from a practical point of view.
The HP filter is included in a built-in command in E-views, Pc-Give, Microfit and as specific program or procedure in Gauss, Matlab, Rats and SAS. Figure 3 in section 6 plots the HP(λ = 14400) trend for the Eurozone IPI obtained by the algebra function in Pc-Give, Figure 4 displays the corresponding cyclical component.

4.2 The Baxter and King filter

An ideal band pass filter is used to isolate the components of a time series that lie within a fixed range of frequencies, say ω_1 and ω_2. To implement a similar filter in the time domain an infinite order moving average is required and so an infinite number of observations.

4.2.1 Description

Baxter and King[13] proposed a band-pass filter that passes through components of the time series with fluctuations between 6 and 32 quarters and removes the components of higher and lower frequencies.
The Baxter and King filter is a linear filter that approximates the ideal band-pass filter above described via a *finite* terms moving average. It assumes the following form:

$$BK(L) = \sum_{j=-k}^{k} a_j L^j \tag{6}$$

The moving average was defined to be finite as it involves only $2k+1$ terms. The "bands" that define the range of periodicities to be extracted must be chosen a priori and economic theory can support this decision. In particular, as mentioned above, Baxter and King opted for a $6-32$ quarters band. They refer to Burns and Mitchell's definition of the business cycles in support of their choice[9].
It is useful to recall that from an historical point of view, for the United States, very few cycles are shorter than two or longer than eight years[10].
But NBER research always required that business cycles involve absolute declines in economic activity, whereas Baxter and King, by attempting to obtain a statistical trend-cycle decompositions obtain estimates of growth cycles and not business cycles.
The approximation proposed by Baxter and King[13] has been shown to fail to filter out the desired components when y_t is nonstationary. An improved approximation of the ideal filter was proposed by Christiano and Fitzgerald[25]; the two authors noted that the optimal approximation to the ideal band pass filter requires knowing the true DGP of y_t and derive the optimal approximation when the series is $I(1)$. This choice produce good results for standard macroeconomic time series.

4.2.2 Tools

Recently the BK filter was extensively used in macroeconomic research and its ability to isolate cyclical components of a time series investigated. The original code implementing the filter was developed in Matlab is available at Marianne Baxter Home page at Boston University; a similar code was developed by Mark Watson and used to characterize the US economy in "Business

[9]See section 2.

[10]According to the historical NBER chronology for the United States, which goes back to 1796(monthly after 1854), only four of the 45 peak to peak cycles exceeded eight years or 100 months and only two were shorter than two years or 20 months. The more recent peak to peak cycle lasted 128 months and was officially determined by the decision of the NBER to date the end of current expansionary phase at March 2001.

cycles fluctuations in US macroeconomic time series"; Estima web page contains a Rats procedure for the same purpose[11].
When using the Baxter and King filter, k observations are lost at the beginning and the end of the sample period, according to the required degree of approximation to the ideal filter.
To avoid the lost of observations, it is possible to forecast and backcast the series before applying the filter so as to use the complete moving average. All the programs presented in this section operate in such a way. It is trivial to note as the reliability of forecasts over such a long period is quite low. Moreover if this forecast is made with univariate ARIMA models it is well known that they will not correctly capture turning points. An alternative procedure is the one adopted by Dominique Ladiray[28] in producing the Macro SAS to extract the band passed cyclical component. It consists in reducing progressively the length of moving average as less data are available. Figure 5 in section 6 presents the band-passed Eurozone IPI obtained by exploiting a Rats procedure; by comparing this graphic with the one in Figure 6 where a decreasing span filter was applied we note the two are quite similar.

4.3 The Beveridge and Nelson decomposition

Beveridge and Nelson[19] were the first to propose a model based decomposition method of an integrated time series, which provides a convenient way to estimate its permanent and its transitory components.

4.3.1 Description

Following the two authors, any $ARIMA(p, 1, q)$ process can be represented as the sum of a stochastic trend plus a stationary component, where a stochastic trend is defined to be a random walk, possibly with drift.
For an $ARIMA(0, 1, 1)$ model the decomposition can be easily obtained; suppose Δy_t is a $MA(1)$ process so that $\Delta y_t = e_t + be_{t-1}$, where e_t is white noise and $|b| < 1$.

$$y_t = y_t - 1 + e_t + be_{t-1} \tag{7}$$

Solving equation(7) recursively and assuming $y_0 = e_0 = 0$, we obtain:

$$y_t = \sum_{j=1}^{t} e_j + b \sum_{j=1}^{t-1} e_j \tag{8}$$

[11] See Table 3 in section 7.

and therefore:

$$y_t = (1+b)\sum_{j=1}^{t} e_t - be_t \tag{9}$$

Equation(9) gives the decomposition of the series y_t into the permanent and the transitory components, which are given by, respectively:

$$g_t = (1+b)\sum_{j=1}^{t} e_t \qquad and \qquad d_t = -be_t \tag{10}$$

An alternative expression for the trend component is:

$$g_t = g_{t-1} + (1+b)e_t \tag{11}$$

Evidently g_t is a random walk without drift and d_t is a stationary process. Some general remarks on the Beveridge and Nelson decomposition:

- No a priori definition of the characteristics of the cyclical component is required by the Beveridge and Nelson approach. All the parameters involved in the decomposition are estimated from the available dataset so the decomposition does not depend upon external parameters linked, for example, to the smoothness of the trend or to the frequency range of the cyclical component.

- The estimated permanent and transitory components depend heavily on the particular ARIMA model which is fitted to the data. It is well known that the identification of an ARIMA model is quite a subjective issue. Beveridge and Nelson is thus characterized by a certain degree of arbitrariness and different decompositions can be consistent with the same dataset.

- The transitory component is defined as a cumulative sum of shocks so the cyclical pattern is generated by a diffusion mechanism like the one described by Frisch. This implies that the cycle obtained using the Beveridge and Nelson decomposition can significantly differ with respect to the ones obtained using mechanical approaches, such as the Hodrick and Prescott and the Baxter and King filter.

4.3.2 Tools

As the Beveridge and Nelson decomposition is a model-based approach, it can be implemented by exploiting the ARIMA procedures available in all

the most widely used econometric and statistical packages.
Specific codes are available at Estima web page for Rats; Gauss and SAS offers similar programs. Figure 7 in section 6 plots the Beveridge and Nelson (0,1,1) decomposition obtained by exploiting Rats. For the reasons explained in the methodological description it appears evident that the cyclical component isolated in such a way can not be clearly identified by its turning points.

4.4 The Unobserved components decomposition of Harvey

The unobserved components approach refers mainly to the work of Harvey[14]. The main idea is that each economic time series y_t is the result of some unobserved components:

$$y_t = g_t + c_t + \epsilon_t \tag{12}$$

where the intended interpretation of the components is as follows:

g_t = trend component
c_t = cyclical component
ϵ_t = unsystematic component following the Gaussian process $\epsilon_t \sim NID(0, {\sigma^2}_\epsilon)$

4.4.1 Description

The key hypothesis is that all the components are generally stochastic and generated by independent processes. So the Harvey decomposition is based upon the hypothesis that trend and cycle have a separate dynamic structure. The general model for the trend component g_t proposed by Harvey is the so-called local linear trend model:

$$g_t = g_{t-1} + \beta_{t-1} + \eta_t \tag{13}$$

$$\beta_t = \beta_{t-1} + \zeta_t \tag{14}$$

The component g_t is usually referred to as the level of the trend, while the β_t is interpreted as its slope; η_t and ζ_t are orthogonal white noise so that $Cov(\eta_t, \zeta_t) = 0 \ \forall t, s$. A variety of interesting sub-models are nested within the local linear model by imposing suitable restrictions on the variance parameters σ^2_ϵ, σ^2_η and σ^2_ζ.
When $\sigma^2_\eta = \sigma^2_\zeta = 0$, the trend g_t is purely deterministic having this formulation:

$$g_t = g_{t-1} + \beta_t = g_0 + \beta_t \tag{15}$$

when only $\sigma_\zeta^2 = 0$, g_t is a random walk with drift of the form:

$$g_t = g_{t-1} + \beta_t + \eta_t \tag{16}$$

From Koopman, Harvey, Doornik & Shepard[15] we present Table 1 specifying both the name and the appropriate restriction of the various models: Given a particular level and trend specification, the cyclical component can

Level	σ_ϵ	σ_η	
constant term	*	0	
local level	*	*	
random walk	0	*	
Trend	σ_ϵ	σ_η	σ_ζ
deterministic trend	*	0	0
level with fixed slope	*	*	0
random walk with fixed drift	0	*	0
local linear trend	*	*	*
smooth trend	*	0	*

Table 1: Restrictions of the various models

be modelled to capture a cycle that may be present in the observed series.

4.4.2 Tools

Given the variety of possible specifications, it is difficult to address the reader to a specific package to estimate unobserved component models. When working with time series it is important to have generic programming tools which offer complete flexibility. Such a tool is offered by Stamp developed by Harvey and Koopman and by its free counterpart Ssfpack by Koopman. Stamp is a program designed to model and forecast time series basing on structural time series models; the main advantage of Stamp is its user-friendliness, the hard word is done by the program, leaving the user free to concentrate on formulating models.
SsfPack is a library of C routines based on state space form and the Kalman filter. In general terms it is able to replicate almost all the models which can be specified in Stamp but it is based on Ox and it requires some programming skills to operate properly. SsfPack is free of charge for workers in university and government with no commercial purposes.
An interesting Gauss application is presented in Kim and Nelson[18], it relates the decomposition of Real GNP to random walk (stochastic trend)

component and stationary (cyclical) component.
Another interesting series of Gauss programs accompanies the paper[26]. The authors compare the two techniques and determine under which conditions they produce the same trend-cycle decomposition.

5 Turning points detection

As we pointed out in section 1 it is possible to fully understand the characteristics of the business cycle of a nation by determining the turning points in the series assumed to be representative of the overall economic activity. Turning points in the levels of the series describe the classical business cycles, the ones in the series measured as deviation from trend, the growth cycles.
In the following subsection we present the most widely used techniques to detect turning points in business cycle. These techniques can be mainly classified as parametric or non-parametric.
Parametric procedures refer to the seminal work of Hamilton[4]. By applying a Markov Switching model to the quarterly growth rates of the US GDP, Hamilton develops an optimal dating rule for the inferred smoothed probabilities and obtained an almost perfect fit to the NBER official dating for the US economy.
Non-parametric techniques refer to the NBER tradition and in particular to the work of Bry and Boschan[5] who developed an algorithm to detect cyclical turning points in a time series.

5.1 Detecting turning points using a computer algorithm

In this subsection we deal with the problem of turning points detection using computer algorithms. These algorithms detect the cyclical turning points mainly by identifying the local max and min in the selected series. Firstly we refer to the Bry and Boschan algorithm.
Bry and Boschan translate the NBER turning points detection method into a set of simple decision rules:

1. Peaks and Troughs must alternate.
2. Each phase (from peak to trough or trough to peak) must have a duration of at least six months.
3. A cycle (from Peak to Peak or from Trough to Trough) must have a duration of at least 15 months.
4. Turning points within six months of the beginning or end of the series are eliminated as are peaks or troughs within 24 months of the beginning or end of the series if any of the points after or before are higher (or lower) than the peak (trough).

Table 2 presents this method in step by step form.
The procedure refers to the Burns and Mitchell methodology by determining

tentative peaks and troughs in a highly smoothed series and subsequently refine the dating of turning points based on less smoothed series.
The original series is smoothed firstly by a 12-months moving average, then by a Spencer curve[12] and finally by a lower order moving average depending on the MCD[13].
The Bry and Boschan algorithm is based on seasonally adjusted monthly time series.
When the data is measured at a quarterly frequency an analogue to the BB algorithm would be the algorithm developed by Harding and Pagan[3]. The two authors treats the differenced series Δy_t as a measure of the derivative of y_t with respect to t; this leads to the use of the sequence $\Delta y_t > 0, \Delta y_{t+1} < 0$ as signalling a peak. Some "censoring rules" to ensure that peaks and troughs alternate and the single phases and cycles respect a minimum duration are incorporated.

5.1.1 Tools

The original algorithm developed by Bry and Boschan is actually available for Gauss thanks to M. Watson. The quarterly algorithm developed by Harding and Pagan is also available for Gauss. We tested the ability of Mark Watson's program to detect cyclical turning points by applying it to the monthly industrial production for the Eurozone (1985:01-2002:02). The algorithm was applied to the series in the levels and the detrended series as well.
The resulting turning points can be observed in Figures 4, 5, 6 and 8 of Section 6. Notably the algorithm detects more turning points in the detrended series than it does in the levels. This is natural as economic slowdown and recovery are more frequent than classical recessions and expansions.

[12]The Spencer curve is a 15-month moving average with negative weights at the extremes and positive and higher weights closer to the center.
[13]The months of cyclical dominance is defined as the number of months needed for the average change in the irregular component to become smaller than the average change in the trend component. Once the MCD scalar has been obtained for e series it can be used as the length of a moving average that removes the irregular component for the same.

I Determination of extremes and substitution of values.
II Determination of cycles in 12-month moving average (extremes replaced). A. Identification of points higher (or lower) than 5 months on either side. B. Enforcement of alternation of turns by selecting highest of multiple peaks (or lowest of multiple troughs).
III Determination of corresponding turns in Spencer curve (extremes replaced). A. Identification of highest (or lowest) value within 5 months of selected turn in 12-month moving average. B. Enforcement of minimum cycle duration of 15 months by eliminating lower peaks and higher troughs of shorter cycles.
IV Determination of corresponding turns in short-term moving average of 3 to 6 months, depending on MCD (months of cyclical dominance). A. Identification of highest (or lowest) value within 5 months of selected turn in Spencer curve.
V. Determination of turning points in unsmoothed series. A. Identification of highest (or lowest) value within 4 months, or MCD term, whichever is larger, of selected turn in short-term moving average. B. Elimination of turns within 6 months of beginning and end of series. C. Elimination of peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end. D. Elimination of cycles whose duration is less than 15 months. E. Elimination of phases whose duration is less than 5 months.
VI. Statement of final turning points.

Table 2: The Bry and Boschan computer algorithm.

5.2 Detecting turning points with Markov switching models

An increasingly popular way to detect turning points in time series is by specifying a statistical model for the data and then using the estimated parameters of the model to obtain a turning points chronology.
Probably the best known member of this class of models is the one developed by Hamilton[4]. Hamilton specified a Markov switching model for the quarterly growth rate of US GDP in this way:

$$\Delta y_t = \mu_{s_t} + \sum_{i=1}^{4} [\Delta y_{t-i} - \mu_{s_{t-i}}] + \epsilon_t \tag{17}$$

with:

$$S_t = 0, 1 \tag{18}$$

The model is locally linear[14] but globally non-linear. This nonlinearity is forced by the presence of the latent variable S_t. This variable is assumed to follow a first order Markov chain. An interesting result of the obtained estimation is the possibility to compute the smoothed and filtered probabilities which can be used to segment the time series into expansionary and recessionary periods. To achieve this goal the optimal filter of Hamilton and Kim's smoother can be used.
The presentation of the full methodology developed by Hamilton exceeds the aims of this paper but we would like to emphasize two points:

1. Even if the first difference is applied to the data to obtain stationarity, Hamilton obtained a classical business cycle chronology.
2. Hamilton's model is not *per se* a dating rule as a rule to detect turning points must be specified on the inferred probabilities.[15]

As for the dating rule, Hamilton opted for a 0.5 rule meaning when the probability of a recession crosses the 0.5 level, a change in the phase of the business cycle is assumed.
A natural generalization of the Hamilton model is the Markov-switching vector autoregressive(MS-VAR) model characterizing the business cycles as common regime shifts in the stochastic process of economic growth in different nonstationary time series. Here the K-dimensional time series $\Delta y_t = (\Delta y_{1t}, ..., \Delta y_{kt})$ t=1,...,T, is generated by a vector autoregression of order p:

$$\Delta y_t - \mu_{s_t} = A_1(\Delta y_{t-1} - \mu_{s_{t-1}}) + \cdots + A_p(\Delta y_{t-p} - \mu_{s_{t-p}}) + \epsilon_t \qquad (19)$$

Recently Krolzig[16] extended the basic multivariate methodology above mentioned by considering Markov-switching cointegrated vector autoregressive processes; in this extension the idea of Markovian regime shifts proposed by Hamilton, is applied to multiple cointegrated time series.

5.2.1 Tools

A number of different programs for several packages are available to estimate MS models. James Hamilton's home page provides Gauss programs for this purpose; Rats procedures and Matlab programs are available as well.
A remarkable set of programs was developed by Kim and accompanies the book of Kim and Nelson[18].

[14] Linear conditional on the state of the economy.
[15] On the smoothed or filtered depending on the amount of information the researcher wants to use

However, the most flexible way to estimate MS models is by Ox and the MSVAR package developed by Hans Martin Krolzig. The package allows for a wide range of models specifications and can be freely used for research purposes. Table 3 in section 7 presents the web sites where these applications are available.
Figure 9 in section 6 presents the smoothed probabilities obtained by specifying Hamilton's model for the monthly growth rate of the Eurozone industrial production by using MSVAR. The related turning points are obtained by applying a 0.5 rule. It appears natural to compare these turning points with the ones of Figure 8; the turning points detected by the two methods appear quite similar, the main difference regards the peak at 1992:03 which is detected by the non-parametric method at 1991:11.
Hamilton's model determines a further turning point, a trough at 2002:02, and considers the actual recessionary phase as concluded. This result must not be emphasized as the probabilities of a recession cross the 0.5 level only for the last observation available at the moment; for this data observation smoothed and filtered probabilities are the same so further observations are needed.

6 Applications

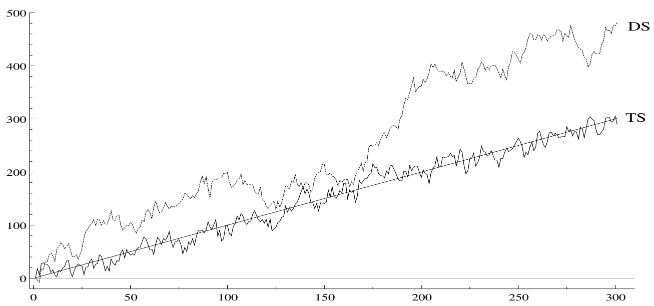

Figure 1: A trend stationary and a difference stationary series.

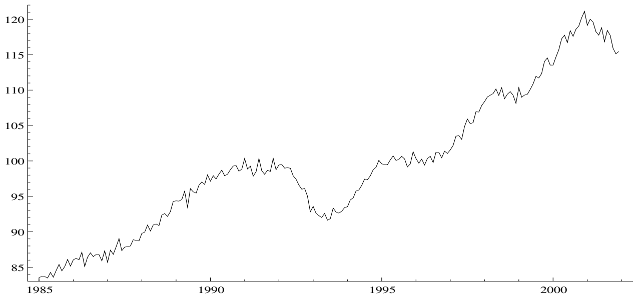

Figure 2: Eurozone IPI, 1985:01-2002:02.

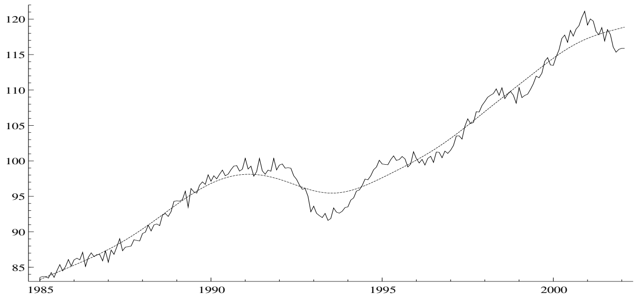

Figure 3: Eurozone IPI and HP(14400) trend

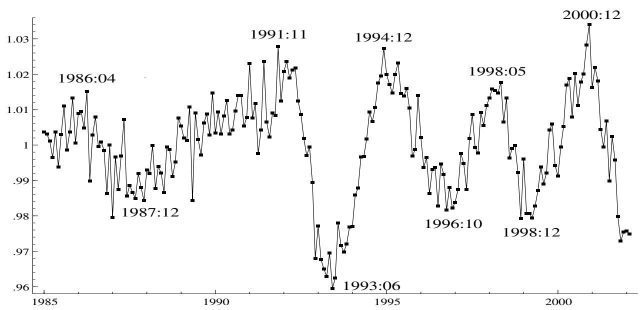

Figure 4: HP(14400) filtered Eurozone IPI

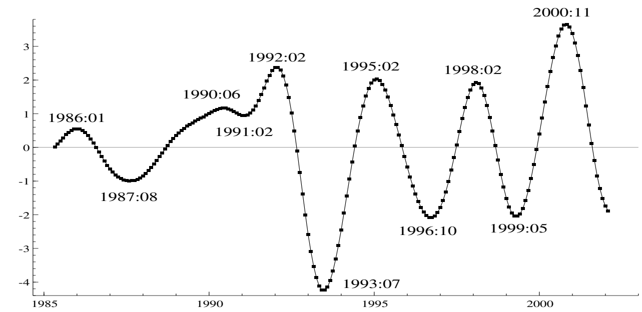

Figure 5: Band-passed 18-96 Eurozone IPI

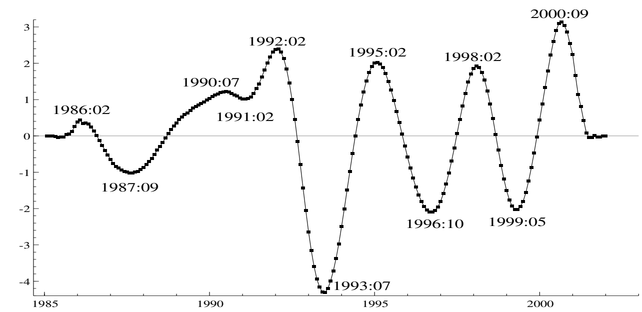

Figure 6: Band-passed 18-96 with decreasing span filter Eurozone IPI

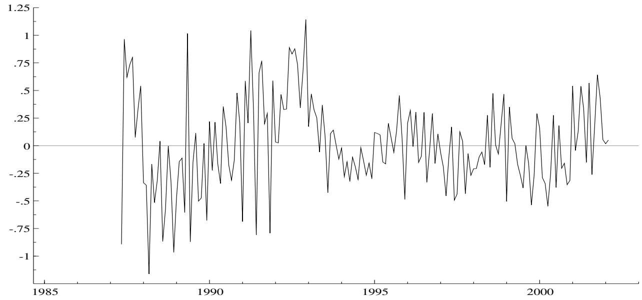

Figure 7: Beveridge and Nelson(0,1,1) decomposition, Eurozone IPI

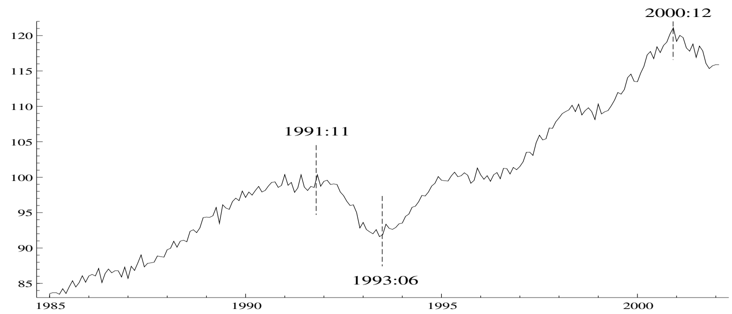

Figure 8: Turning points detected by the BB procedure on Eurozone IPI.

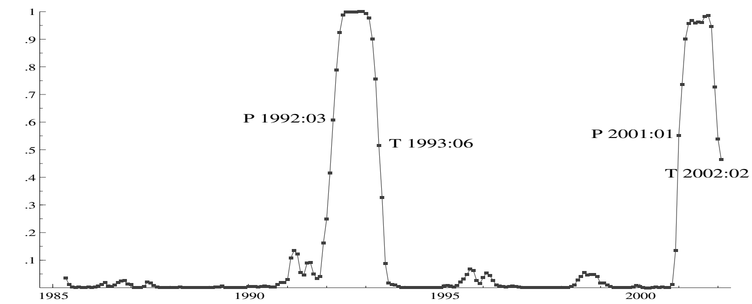

Figure 9: Smoothed probabilities of a recessionary phase, Eurozone IPI, P=peak T=trough.

7 Software review for business cycles analysis

Signal extraction technique	*Package*	*available at:*
Hodrick and Prescott filter	Gauss, Rats E-views, Matlab Microfit, Pc-Give SAS	 ideas.uqam.ca/QMRBC/codes.html http://www.unige.ch/ses/ sococ/mirage/welcome.html
Baxter and King filter	Matlab Gauss Rats SAS	http://people.bu.edu/mbaxter/ www.wws.princeton.edu/~mwatson www.estima.com See above
Christiano and Fitzgerald optimal filter	Eviews, Gauss Rats, Matlab	www.clev.frb.org/research/ economists/fitzgerald /tjfbandpass.html
Unobserved component	Stamp ssfpack Gauss	http://stamp-software.com/ http://www.ssfpack.com http://wuecon.wustl.edu/~morley/
Beveridge and Nelson decomposition	Gauss Rats SAS	http://wuecon.wustl.edu/~morley/ www.estima.com See above
Markov switching models	Gauss Ox-MSVAR Rats	http://weber.ucsd.edu/ jhamilto/software.htm www.econ.washington.edu/user/cnelson http://www.economics.ox.ac.uk/ research/hendry/krolzig/default.htm www.estima.com
Bry and Boschan algorithm	Gauss	www.wws.princeton.edu/ /~mwatson

Table 3: Business cycles software availability

8 Conclusions

In this paper we presented a review of the main techniques for business cycle analysis as well as the software available for the same purpose. What emerges from this analysis is at the moment there is not a specific software containing all the techniques required for a complete analysis of the economic fluctuations phenomena. Various applications are available but they cover a large variety of packages.
Two solutions for this problem can be presented:

1. The development of such a specific software; in this sense the BUSY project seems to be really promising.

2. The integration of the different software for business cycle analysis;

The second point mentioned raises the problem of the creation of a common interface among the different packages, this is the real challenge for the future.

References

[1] Burns, A.F. and Mitchell W.C. (1946), *Measuring business cycles*, New York, National Bureau of Economic Research.

[2] Pagan, A. (1997),*Toward an understanding of some business cycle characteristics*, Australian Economic Review, 30,1-15.

[3] Harding, D. and Pagan, A. (2001), *Dissecting the cycle: A methodological investigation*, journal of monetary economics, Forthcoming.

[4] Hamilton, J. (1989), *A new approach to the andlysis of non-stationary time seires and the business cycle*, Econometrica 57, 357-384.

[5] Bry, G. and Boschan, C. (1971), *Cyclical analysis of time series: selected procedures and computer programs*, technical paper 20, New York: Columbia University Press for NBER.

[6] Mintz, I. (1969), *Dating postwar business cycles, Methods and their applications to Western Germany, 1950-1967*, Nber Occasional paper 107, New York.

[7] Mintz, I. (1972), *Dating American Growth Cycles*, in V.Zarnowitz: Business cycles today, NBER, New York, 39-88.

[8] Zarnowitz V. (1992), *Business cycles: Theory, History, Indicators and Forecasting*, University of Chicago Press for the NBER.

[9] Niemira M.P. (1993), *Forecasting financial and economic cycles*, John Wiley and Sons.

[10] Banerji A. and Hiris L., *A multidimensional framework for measuring business cycles*, ECRI research paper, available www.businesscycle.com.

[11] Nelson, C.R. and Plosser, C.I. (1982), *Trend and random walks in macroeconomic time series*, Journal of Monetary Economics 10,139-62.

[12] Johnston J. and DiNardo J., *Econometric methods*, Fourth Edition, 220-23, McGraw-Hill International Editions.

[13] Baxter, M. and King, T. (1999), *Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series*, Rview of Economic and Statistifcs, Vol.81(4), 575-93.

[14] Harvey, A.C. (1985), *Trends and Cycles in macroeconomic time series*, Journal of Business and Economic Statistics 3,216-30.

[15] Koopman, S.J., Harvey, A.C., Doornik, J.A. &Shephard, N. (1999), *Stamp 6.0*, Timberlake Consultants.

[16] Krolzig, H.M. (1996), *Statistical analysis of cointegrated VAR processes with markovian regime shifts*, Discussion paper 25/1996, Institute for Economics and Statistics University of Oxford.

[17] Pedersen, T.M. (2000), *The Hodrick and Prescott filter, the Slutzky effect and the distortionary effect of filters*, Journal of Economic dynamic and control.

[18] Kim, C.J. and Nelson, C.R. (1998), *State-Space models with regime switching:calssical and Gibbs sampling approach with applications*, MIT Press.

[19] Beveridge, S. and Nelson, C.R. (1981), *A new approach to the decomposition of economic time series into permanent and transitory components with particular attention to measurement of the "business cycle"*, journal of monetary economics 7, 154-174.

[20] Tinbergen, J. (1940), *Statistical Testing of Business-Cycle Theories*, Geneva: League of Nations. Vol. I: A Method and its application to Investment Activity.

[21] Mundell, R.A. (1962), *The appropriate use of Monetary and Fiscal policy for internal and external stability.*

[22] Sargent, Thomas J. (1986),*Rational Expectations and Inflation.*

[23] Lucas, R. E. (1976), *Econometric Policy Evaluation: A Critique*, Carnegie Rochester Conference Series on Public Policy, 1, 19-46.

[24] Schumpeter, J. A. (1939),*Business Cycles*, New York, McGraw-Hill.

[25] Christiano L.J. and Fitzgerald, T.J. (1999), *The band pass filter*, NBER Workijng Paper 7257.

[26] Morley, J. Nelson, C.R. and Zivot, E. (2001), *Why are Beveridge-Nelson and Unobserved-Component Decompositions of GDP So Different?*, Review of Economics and Statistics.

[27] Zarnowitz, V. and Moore, G. (1982), *Sequential signals of recession and recovery*, Journal of Business, January, 57-85.

[28] Ladiray, D. (2001), *DIVERSES MACROS SAS: Analyse Exploratoire des Donnes, Analyse des Sries Temporelles*, available at: www.unige.ch/ses/sococ/mirage/welcome.html.