Common Trends and Common Cycles in the Euro-Zone
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Common Trends and Common Cycles in the Euro-Zone

(Preliminary version)

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Abstract: Among new issues raised by the creation of the European Monetary Union (EMU), the one concerning the existence of a common cycle among Member States has been seen as one of paramount importance. In this paper we investigate the degree of short-run and long-run co-movement in the Euro-zone economies by applying the Vahid and Engle (1993) methodology. Multivariate co-integration and common features (cycles) tests are performed in order to verify the hypotheses that Emu countries share common trends and common cycles. We found that the special case \( r + s = n \) occurs here.

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1. Introduction

Recent theoretical and empirical research proposed by Vahid and Engle (1993 and 1997), Engle and Kozicki (1993) and Issler and Vahid (2001) has revived interest in the co-movement of macroeconomic time series. In those papers, the authors extend results already achieved by Engle and Granger (1987) and Johansen (1988 and 1995) in the field of common trend identification (cointegration). They demonstrate that a similar approach can be applied to the stationary part of the series in a VAR system in order to identify common cyclical movements. As a consequence, they demonstrated that a joint application of the two methodologies could lead to a simultaneous decomposition of the series into common trend and cycle. By applying this methodology we investigate the degree of short-run and long-run co-movement among Eurozone economies. Multivariate co-integration and common features (cycles) tests are performed in order to verify the hypothesis that Emu countries share common trends and common cycles. Once this hypothesis is verified, relevant consequences can arise for applied economic and statistical analysis of the Eurozone. These might be applied in the construction of synthetic and leading indicators as well

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as in turning point detection of the business cycle. 
The paper is organised as follows: section 2 briefly presents the theoretical framework; section 3 gives a description of the data set we used; section 4 shows the empirical results and section 5 concludes.

2. Common Trends and Cycles: Theoretical Framework
In order to introduce the methodology proposed by Vahid and Engle, we consider first the n-dimensional VAR system of I(1) variables:

\[ y_t = \Pi_1 y_{t-1} + \Pi_2 y_{t-2} + \ldots + \Pi_p y_{t-p} + \epsilon_t \]

Following Engle and Granger (1987), if and only if the variables included in equation (1) are co-integrated, does the model admit a VECM representation of the form:

\[ \Delta y_t = \alpha \beta y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \epsilon_t \]

where, for ease of exposition, constant terms have been dropped, \( \beta \) and \( \alpha \) are full rank \( n \times r \) matrices, and \( r \) is the rank of the co-integration space. Cointegrating relations are often estimated using the Johansen procedure. Since \( \Delta y_t \) is assumed to be I(0), it has the Wold representation

\[ \Delta y_t = \epsilon_t + \sum_{i=1}^{\infty} C_i \epsilon_{t-i} = C(L)\epsilon_t \]

where \( C(L) = \Delta(I_n - \Phi(L)L)^{-1} \). Using the relationship

\[ C(L) = C(1) + \Delta C^*(L) \]

where \( C_j^* = -\sum_{l+j=1}^{\infty} C_l \), and in particular \( C_0^* = I_n - C(1) \), equation (3) can be written as

\[ \Delta y_t = C(1)\epsilon_t + \Delta C^*(L)\epsilon_t \]

Integrating, this becomes

\[ y_t = C(1)s_t + C^*(L)\epsilon_t \]

where \( s_t = \sum_{j=0}^{\infty} \epsilon_{t-j} \). If there are \( r \) cointegrating vectors, then \( C(1) \) is of reduced rank \( k = n - r \) and can be written as \( C(1) = \gamma \delta^\top \), where \( \gamma \) and \( \delta \) are both of rank \( k \). Thus, by defining

\[ \tau_t = \delta^\top s_t, \quad c_t = C^*(L)\epsilon_t \]

we have the Stock and Watson (1988) “common trends” representation

\[ y_t = \gamma \tau_t + c_t, \quad \tau_t = \tau_{t-1} + \delta^\top \epsilon_t \]

that expresses \( y_t \) as a linear combination of \( k \) random walks (the common trends \( \tau_t \)) plus some stationary ‘transitory’ components \( c_t \).

In the same way that common trends appear in \( y_t \) when \( C(1) \) is of reduced rank, common cycles appear if \( C^*(L) \) is of reduced rank, since \( c_t = C^*(L) \) is the cyclical component of \( y_t \). The presence of common cycles requires that there are linear combinations of the elements of \( y_t \) that do not contain these cyclical components, i.e. that there is a set of \( s \) linearly independent vectors, gathered together in the \( n \times s \) matrix \( \phi^\top \), such that

\[ \phi^\top c_t = \phi^\top C^*(L)\epsilon_t = 0 \]

in which case
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\[ \phi' y_t = \phi' \gamma_t \]

Such a matrix will exist if all the \( C_j^* \) have less than full rank and if \( \phi' C_j^* = 0 \) for all \( j \) (see Vahid and Engle, 1993, and Engle and Issler, 1995).

Following Vahid and Engle (1993), we estimate the following restricted reduced form system by two-stage least squares:

\[
\begin{bmatrix}
    I_s & \hat{\alpha}^* \\
    0 & \mathbf{I}_{n-s}
\end{bmatrix}
\begin{bmatrix}
    \Delta y_t \\
    \Delta Y_{t-1} \\
    \vdots \\
    \Delta Y_{t-p+1}
\end{bmatrix}
= 
\begin{bmatrix}
    0 \\
    \Gamma_1^* \\
    \cdots \\
    \Gamma_{p-1}^* \\
    \alpha^* y_{t-1}
\end{bmatrix}
+ v_t,
\]

where \( \Pi_j^* \) and \( \beta_j^* \) represent the partitions of \( \Pi_j \) and \( \beta_j \) corresponding to the bottom \( n-s \) reduced form VECM equations and \( s \) is the number of linearly independent co-feature vectors. The error term in equation (2) is given by:

\[
\begin{bmatrix}
    I_s & \hat{\alpha}^* \\
    0 & \mathbf{I}_{n-s}
\end{bmatrix}
\begin{bmatrix}
    \Delta y_t \\
    \Delta Y_{t-1} \\
    \vdots \\
    \Delta Y_{t-p+1}
\end{bmatrix}
= 
\begin{bmatrix}
    0 \\
    \Gamma_1^* \\
    \cdots \\
    \Gamma_{p-1}^* \\
    \alpha^* y_{t-1}
\end{bmatrix}
+ v_t.
\]

Inverting the coefficient matrix in equation (2) and multiplying through the right-hand \( \Delta Y_t \) side of equation (2) yields a reduced form VECM model that contains the common feature and co-integration information:

\[
\Delta Y_t = T'_{11} \Delta Y_{t,1} + \cdots + T'_{p,1} \Delta Y_{t,p+1} + \alpha^* \beta^* Y_{t,1} + \varepsilon_t.
\]

This reduced form representation then allows for efficiency gains due to common cycles. Representation (3) can be used in a multivariate decomposition of the series into trend and cycle components.

Testing for common cycle
The rank \( s \) of the matrix \( \phi \) can be determined by calculating the test statistic:

\[
C(s,0) = -(r - (p - 1)) \sum_{j=1}^s \ln(1 - \hat{\lambda}_j^2)
\]

where \( \hat{\lambda}_1^2, \ldots, \hat{\lambda}_s^2 \) are the \( s \) smallest estimated squared canonical correlations between \( \Delta Y_t \) and the set \( Y_{t,1} = (\Delta x_{t,1}, \ldots, \Delta x_{t,p+1}, e_{t,1}) \). By defining \( Y \) as the matrix of observations on \( \Delta Y_t \) and \( W_{t,1} \) as the matrix of observations on \( w_{t,1} \), \( \hat{\lambda}_1^2, \ldots, \hat{\lambda}_s^2 \) are obtained as the \( s \) smallest eigenvalues of \( (Y'Y)^{-1} Y' W_{t,1} (W_{t,1}' W_{t,1})^{-1} W_{t,1}' Y \). Under the null hypothesis that the rank of \( \phi \) is at least \( s \), this statistic has a \( \chi^2 \) distribution with \( s^2 + snp + sr - sn \) degrees of freedom (Vahid and Engle, 1993).

Vahid and Engle (1993) discuss what they call an “Important special case”. If the sum of the dimension of co-integration space and of the co-feature space is equal to the number of dependent variables \( (r+s=n) \) we can stack co-feature vectors and cointegrating vectors in a \((n \times n)\)matrix:
which has full rank. By inverting $A$, it is possible to recover the common trend and common cycle decomposition as:

$$y_t = A^{-1}Ay_t = \tilde{\alpha} - \alpha y_t + \tilde{\beta} - \beta z_t$$

3. The Data

In the following section we apply the methodology proposed by Vahid and Engle to macroeconomic data of the Euro-zone Member States. The data are quarterly observations on Gross domestic product of Euro-zone Member States, ranging from 1991Q1 to 2001Q2. Data are seasonally adjusted at constant prices. Although our first intention was to retain all 12 countries who joined the European Monetary Union (Emu), we had to reduce our sample to the following seven countries: Belgium, Germany, Spain, France, Italy, Netherlands and Austria. The choice was due in some cases to the lack of Quarterly National Accounts (Greece and Luxembourg), in another to the shortness of the series published (Portugal), and even due to the exceptional performance recorded by some countries (Ireland and Finland) which made their growth pattern completely different from the rest of the partners. All series were transformed by taking the logarithm.

4. Empirical analysis

We started our analysis by investigating the degree of integration of each series. In order to do that, we made use of the Augmented Dickey-Fuller, the Schmidt-Phillips and the Phillips-Perron Integration Tests. All three tests suggested that each series of ours is well approximated by an $I(1)$ process. Next we tested for the presence of serial correlation in the first differences of our variables. Using both Box-Pierce and Ljung-Box Statistics, we found evidence of serial correlation in all the series.

Figure 1 Logarithm of GDP, 1995=100
In the following stage of our analysis we focused our attention on the estimation of the VAR system. At the very beginning we retained in the system all the series available. However, the first results we reached following this strategy did not allow us to draw any clear conclusion on the structure of the model. In particular, we could not detect a co-integrating relationship among all Member States. Therefore, we decided to reduce our system by marginalising those countries whose behaviour is notably different from the majority of Member States (i.e. Ireland and Finland). In applying this reduction strategy, we were also guided by the results recently proposed by Blake et al. (2000) and Ladiray and Mazzi (2001). As a result, our final system includes the following five Member States: Belgium, Spain, Germany, France and Italy.

Following Vahid and Engle (1993) suggestions, a great deal of attention was devoted to the selection of the maximum lag ($p$) to be included in the system. The Akaike, Hannan-Quinn and Schwarz Information Criteria were all used. All the criteria coherently suggest a maximum lag of 4.

Conditional on these results, the presence of common trends was investigated. The likelihood-based co-integration tests in Johansen (1995) were used to investigate for cointegration in a vector error correction model (VECM) as in equation 2. Since 4 lags were retained in the unrestricted VAR system of the levels of the variables, we included here three lags. Table 1 shows the results obtained by applying both the trace test and the maximum eigenvalue test: as can be seen, both tests provide clear evidence that the rank of the matrix $\alpha\beta'$ of equation 2 is six at the 99% significance level.

Table 1 Johansen cointegration test

<table>
<thead>
<tr>
<th>Ho: rank=r</th>
<th>$-T\log(1-\mu)$</th>
<th>95%</th>
<th>$-T\sum \log(.)$</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>95.49 **</td>
<td>45.3</td>
<td>298.50 **</td>
<td>124.2</td>
</tr>
</tbody>
</table>
Therefore, conditional on the hypothesis that the rank is 6, we tested again the significance of the deterministic components previously included in the model. We keep in the VAR system an unrestricted constant term (included among the I(0) variables) and no deterministic trend.

Conditional on the estimates of the VECM model (results are not reported to save space), we tested for the presence of serial correlation common features in the Member States' GDP series using the canonical correlation-based tests proposed by Vahid and Engle (1993) and already described in Section 2. The test examines canonical correlation between $\Delta y_t$ and its relevant history $\hat{\Delta}y_t$, determined as the dependent variables in the estimated VECM representation of the system. The canonical correlations that are not significantly different from zero represent linear combinations of $\beta \Delta y_t$ that are uncorrelated with the past. As stated by Vahid and Engle, this can be viewed as evidence of common cycles. Ordering the squared canonical correlations from lowest to highest, the null hypothesis for the test is that the first $j$ correlations are zero but the $(j+1)^{st}$ is nonzero. The squared canonical correlation and the value of the test statistic for the dimension of the co-feature vector are reported in Table 2.

As can be seen, the data support the existence of one co-feature vector $s$ at the 5 per cent level of significance. It has to be noted that in our example the number of co-feature vectors $s$ and co-integration vectors $r$ sum up to the number of variables included in the model ($r+s=n$). Therefore we can decompose each series into permanent and transitory components as proposed by Vahid and Engle by using the multivariate Beveridge and Nelson decomposition. Transitory components of each of the seven series are shown in Figure 2.

Table 2 Cofeature test results

<table>
<thead>
<tr>
<th>Ho: rank=r</th>
<th>-$T\log(1-\mu)$</th>
<th>95%</th>
<th>-$T\sum \log(.)$</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>r &lt;= 1</td>
<td>65.50 **</td>
<td>39.4</td>
<td>203.00 **</td>
<td>94.2</td>
</tr>
<tr>
<td>r &lt;= 2</td>
<td>45.69 **</td>
<td>33.5</td>
<td>137.50 **</td>
<td>68.5</td>
</tr>
<tr>
<td>r &lt;= 3</td>
<td>39.84 **</td>
<td>27.1</td>
<td>91.82 **</td>
<td>47.2</td>
</tr>
<tr>
<td>r &lt;= 4</td>
<td>28.50 **</td>
<td>21.0</td>
<td>51.99 **</td>
<td>29.7</td>
</tr>
<tr>
<td>r &lt;= 5</td>
<td>20.29 **</td>
<td>14.1</td>
<td>23.48 **</td>
<td>15.4</td>
</tr>
<tr>
<td>r &lt;= 6</td>
<td>3.20</td>
<td>3.8</td>
<td>3.20</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 2 Cofeature test results
5. Conclusions
In this paper we examined the dynamics of Euro-zone economies by applying the methodology proposed by Engle, Kozicki and Vahid in order to test for the presence of Common Trends and Common Cycles. Our results suggest that Euro-area economies share both long and short run co-movements. Furthermore, we found that the dimension of the co-integration space and the co-feature space sum up to the number of variables retained in the model. This corresponds to the so-called “special case” discussed by Vahid and Engle (1993). This “special case” corresponds to the multivariate Beveridge and Nelson decomposition and can be used in a multivariate analysis to achieve an exact decomposition into permanent and transitory components for each of the considered Euro-zone country. The presence of a common trend and of six common cycles among the seven Member States retained in the present analysis seems to us particularly interesting. In fact, since our sample ranges from 1991Q1 to 2001Q4, our results suggest that during the period characterised by the various steps (the so called “convergence criteria”) toward the creation of EMU in 1999, European economies had already started moving together. Our results seem to confirm those already found by Blake et al. (2000) and Ladiray and Mazzi (2001).
Unfortunately, the lack of Quarterly data for Germany before 1991 does not enable us to apply the Vahid and Engle methodology to a longer period and to analyse whether acceleration in convergence occurred.
As suggested by Vahid and Engle, this result could be used to reduce the complexity of a multivariate system in the European framework. We believe our results can be considered encouraging for further research which considers the way different EMU economies respond to shocks. We believe that such a study could lead to positive results in analysing the Eurozone economy.
References

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