Calibration for Nonresponse Treatment: in One or Two Steps?

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Calibration for Nonresponse

- Nonresponse causes bias and increased variance of estimators
- If auxiliary information is available, calibration is a possibility in order to "compensate" for nonresponse
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• Some important references:


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- Population $U$ of size $N$
- Sample $s$ of size $n_s$
- Response $r$ of size $n_r$

- Target: Population total $\sum_U y_k$
- General form of estimator: $\sum_r w_k y_k$, where $w_k$ is the calibration weight
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• Two main components of calibration:
• The underlying distance measure (here assumed to be the “standard”)
• The calibration equation: $\sum_r w_k x_k = X$, where $X$ is the (linear) calibration constraint.
• Example: $X = \sum_U x_k$, or $X = \sum_S d_k x_k$, where $d_k = 1/\pi_k$ (the sampling weight)
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• Auxiliary information at two levels:
  • Population level: ”star” vector $x^*$
  • Sample level: ”moon” vector $x^o$

• The star-vector values are assumed known for all $k \in U$
• The moon-vector values are assumed known for $k \in s$
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• Observe that the calibration constraint $X$ can contain either “star-information”, “moon-information or both.
• Calibration can now be performed in one or two steps.
• One step: all auxiliary information is used simultaneously
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Two-steps (two possibilities):

- "Bottom-up": First the moon-information is used for a calibration from $r$ to $s$ and then we use the star-info (or both types of info) for a calibration from $r$ to $U$.

- "Top-down": First a calibration from $s$ to $U$ using the star-information and then a calibration from $r$ to $s$ using the moon-info (or both types of info)
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• Questions:
  • Calibration in one or two steps?
  • Should we "reuse" information from the first step in the second step?
  • For the two-step situation: bottom-up or top-down?
  • Simplification: using sample instead of population information, what effect has that?
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• \( y_k \) is expenditure on administration and maintenance

• Division into four groups according to size, yielding the moon-vector, \( x^o_k \) consisting of indicator variables
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• $x^*_k = (1, x_k)$, where $x_k$ is the square root of Revenue advances
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The simulation study:

- Sample size $n_s = 300$
- Response probability $\theta_k = 1 - \exp(-0.0318x_k)$: increasing exponential response distribution
- Here this leads to the average response probability 0.86.
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- 10,000 simulated samples according to simple random sampling
- Each response set created by 300 independent Bernoulli trials, each with probability $\theta_k$ of success
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Measures of performance of estimators:
• Empirical first and second moments, yield estimates of:
• Bias
• Variance
• Mean squared error (MSE)
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• As a benchmark the estimator $N\bar{y}$ was also considered. As expected the simulation shows this estimator to be inferior to the other choices.
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• Simulation results:
• The MSE is much lower when calibrating on the star-information at the population level, instead of calibrating at the sample level. The bias though, is smaller for the latter case.
• Comparing bottom-up with top-down, bottom-up estimators yield slightly less biased estimators with similar variance.
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• For the bottom-up approach, using the moon information AGAIN in the second step, leads to a slight decrease in bias and similar variance.

• Direct calibration produces estimates with slightly higher bias than the two-step procedures, but with similar variance.
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"This is not the end,..."