Acknowledgements:

I am deeply grateful for their fruitful comments and contribution to the preparation of the Manual to Agustin Maravall (Banco de España) and Dominique Ladiray (INSEE).

Thanks are due to Jean Palate (National Bank of Belgium), Anna Ciammola (ISTAT), Faiz Alsuhail (Statistics Finland), Dario Buono (EUROSTAT), Alpay Koçak (Turkish Statistical Institute), Joerg Meier (Deutsche Bundesbank), Michael Richter (Deutsche Bundesbank), Kevin Moore (ONS) and Beata Rusek (NBP) for their valuable support in the preparation of this Manual.

I would like to thank the all the members of the Task Force on Seasonal Adjustment for their useful comments and helpful suggestions on various drafts of this document.

Disclaimer:

The Demetra+ User Manual is provided by Eurostat. This material:

- is information to assist new users of Demetra+ familiarise themselves with the interface and functionalities of the application of a general nature and is not intended to favour any method over another that have been incorporated into the application;

- is sometimes linked to further papers and documents over which Eurostat has no control and for which Eurostat assumes no responsibility;

- does not constitute professional or legal advice.

Demetra+ is designed to support the "ESS Guidelines on Seasonal Adjustment". While Demetra+ incorporates the seasonal adjustment methods of the U.S. Bureau of Census (X-12-ARIMA) and the Bank of Spain (TRAMO/SEATS), the "ESS Guidelines on Seasonal Adjustment" do not promote one method over another.

The paper presents the personal opinions of the author and does not necessarily reflect the official position of the institution with the author cooperate. All errors are author’s responsibility.

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Introduction

Seasonal adjustment (SA) is an important step of the official statistics business architecture and harmonisation of practices. Since the 1990s, Eurostat has been playing a leading role in the promotion, development and maintenance of an open source software solution for seasonal adjustment in line with established best practices. Developed by Eurostat, Demetra software was intended to provide a convenient and flexible tool for seasonal adjustment using TRAMO/SEATS\(^1\) and X-12-ARIMA\(^2\) methods.

In 2008 the European Statistical System (ESS) Guidelines on Seasonal Adjustment were endorsed by the Committee on Monetary, Financial and Balance of Payments statistics (CMFB) and the Statistical Programme Committee (SPC) as a framework for seasonal adjustment of Principal European Economic Indicators (PEEIs) and other ESS and ESCB economic indicators. The ESS Guidelines focus on two most commonly used seasonal adjustment methods, i.e. TRAMO/SEATS and X-12-ARIMA and present useful practical recommendations. Both methods are divided into two main parts. First one is called a pre-adjustment and removes the deterministic effects from the series by means of a regression model with ARIMA noises. Second one is the decomposition. TRAMO/SEATS and X-12-ARIMA use a very similar approach in the first part of the processing but they differ completely in the decomposition step. Therefore, the comparison of the results is often difficult, even for the modelling step. Moreover, their diagnostics focus on different aspects and their outputs take completely forms.

Eurostat faced a huge challenge of improving comparability of the results and diagnostics from both methods in Demetra, because this software was not flexible enough. Moreover, it wasn’t possible to implement all ESS Guidelines’ recommendations in full. The only effective long-term

\(^1\) TRAMO/SEATS is a model-based seasonal adjustment method developed by Victor GOMEZ and Agustin MARAVALL (Bank of Spain). It consists of two linked programs: TRAMO and SEATS. TRAMO ("Time Series Regression with ARIMA Noise, Missing Observations, and Outliers") performs estimation, forecasting, and interpolation of regression models with missing observations and ARIMA errors, in the presence of possibly several types of outliers. SEATS ("Signal Extraction in ARIMA Time Series") performs an ARIMA-based decomposition of an observed time series into unobserved components. Both programs are supported by Bank of Spain. More information on TRAMO/SEATS can be found in www.bde.es.

\(^2\) X-12-ARIMA is a seasonal adjustment program developed by the US Census Bureau. It includes all the capabilities of the X-11 program, which estimates trend and seasonal component using moving averages. X-12-ARIMA offers useful enhancements including: extension of the time series with forecasts and backcasts from ARIMA models prior to seasonal adjustment, adjustment for effects estimated with user-defined regressors, additional seasonal and trend filter options, alternative seasonal-trend-irregular decomposition, additional diagnostics of the quality and stability of the adjustments, extensive time series modelling and model selection capabilities for linear regression models with ARIMA errors. X-12-ARIMA is supported by the US Bureau of Census. More information on X-12-ARIMA can be found in http://www.census.gov.
solution was to create new seasonal adjustment software, namely Demetra+ which covers the recommendations of ESS Guidelines in this area.

Demetra+ has been developed by the National Bank of Belgium. The application seasonally adjusts large-scale sets of time series and provides user-friendly tools for checking the quality of the SA results. Demetra+ includes two seasonal adjustment methods: X-12-ARIMA and TRAMO/SEATS. As Demetra+’ IT solutions are utterly different from the old Demetra, all files created in old Demetra are not read by Demetra+. Old software is no more developed nor supported by Eurostat.

The Demetra+ User Manual aims to introduce the user to the main features of the Demetra+ software and to make the user able to take advantage of this powerful tool. This document presents an overview of the capabilities of the software and of its functionalities. Moreover, step by step descriptions how to perform a typical analysis are included. Therefore, the User Manual enables to reproduce the results with user’s own data. The guide shows the typical paths to follow and illustrates the user-friendliness of Demetra+. The reader is expected to have already acquired background knowledge about the concept of seasonal adjustment and is familiar with the X-12-ARIMA and the TRAMO/SEATS methods. However, a brief sketch of the X-12-ARIMA and the TRAMO/SEATS algorithms and concepts is presented. For readers interested in studying the seasonal adjustment methods in detail, bibliography is provided at the end of the Demetra+ User Manual.

It should be emphasized that Demetra+ makes use of the X-12-ARIMA and the TRAMO/SEATS algorithms, restricted with regard to their original implementations. For this reason there are some differences between the original programs and the programs implemented in Demetra+. The aim was to develop the software which enables the comparison of the result from TRAMO/SEATS and X-12-ARIMA. For this reason, e.g. the revision history and the sliding spans analysis are available in Demetra+ both for TRAMO/SEATS and X-12-ARIMA. On the contrary, some functionalities implemented in the original programs are missing in Demetra+ (e.g. in case of X-12-ARIMA Demetra+ does not allow to perform a pre-adjustment of the original series with "prior adjustment factors". Also the option to specify the ARIMA model \((p,d,q)(P,D,Q)\) without some lags in the regular part is not available for X-12-ARIMA in Demetra+\(^3\). In case of TRAMO/SEATS, Demetra+ does not display confidence intervals for seasonally adjusted series and does not separate the long term trend from the cycle).

The User Manual is divided into five parts.

Chapter 1 presents the general features of the software and installation procedure. It also includes a short note about Demetra+ application for Microsoft Excel.

In Chapter 2 the menu of the application is outlined. It informs how to work with the main panels of the application.

---

\(^3\) For example, the user cannot specify the model \((2,1,1)(0,1,1)\) without parameter AR(1).
Chapter 3 focuses on the workspace menu and statistical tools offered by Demetra+. Also some useful input/output options and are described here.

Chapter 4 describes how to define the seasonal adjustment for a single series and for the large sets of series. The X-12-ARIMA and the TRAMO/SEATS specifications are presented. In this part the results of seasonal adjustment as well as their interpretation are discussed. Some theoretical facets of seasonal adjustment using X-12-ARIMA and TRAMO/SEATS are also included.

Selected aspects of seasonal adjustment methods and technical issues are described in the Annex. they include descriptions of the theoretical models used by X-12-ARIMA and TRAMO/SEATS as well as

Instead of "X-12-ARIMA", "ARIMA" and "TRAMO/SEATS" Demetra+ uses notation "X12","Arima" and "TramoSeats", respectively. For this reason this notation is used further in this Manual.
1. Basic information

1.1. About Demetra+

The final release of Demetra+ contains Demetra+ itself (main graphical interface) and Excel add-ins: ColorAnalyser, Demetra+ XL and XL Functions. More information about Excel add-ins is available in next section: Demetra+ application for Microsoft Excel.

Demetra+ version 1.0.3.2114 uses the following core engines:

- TramoSeats dlls, dated 8/2009, and TramoSeats dlls, dated 1/2012;
- X12 dll (developed by the US Bureau of the Census, based on X-12-Arima version 0.3, dated 12/2010).

The most important results (including the complete RegArima model) directly come from the core engines. All the diagnostics are computed outside the core engines (see below).

One of the strategic choices of Demetra+ is to provide the common presentation/analysis tools for both TramoSeats and X12. Thus, the results from both methods can be easily compared. This implies that some diagnostics, statistics, auxiliary results, etc. are computed outside the core engines. Obviously, Demetra+ is highly influenced by the output of TramoSeats and of X12. Most analyses presented in Demetra+ are available in the core engines. However, the results with TramoSeats and X12 may slightly differ for a lot of reasons (different statistical/algorithmic choices, possible bugs). In any case the global messages of seasonal adjustment are (nearly) always similar.

Amongst the most important tools implemented in Demetra+, the following functionalities should be mentioned:

- likelihood (X12-like) / RegArima model (t-stat as in Tramo): RegArima model was recomputed in Demetra+ (X12, Tramo and "Stamp-like" solutions available in the framework);
- residuals analysis (Tramo-like, but based on another set of diagnostics);
- seasonality tests (X12-like);
- spectral analysis (X12 definition);
- Sliding spans (X12);
- Revision history;
- Wiener-Kolmogorov analysis (Seats-like).

Solutions implemented in Demetra+ lead to the flexible software. New features are easy to add to the software without modifying the core engine. One of the key features of Demetra+ is the possibility to use the underlying algorithms through a rich application programming interface.

---

4 STAMP is an econometric software, developed by Siem KOOPMAN, Andrew HARVEY, Jurgen DORNIK, and Neil SHEPARD, that uses the Kalman filter and related algorithms, for time series models with unobserved components such as trend, seasonal, cycle and irregular. More information about STAMP can be found in http://www.stamp-software.com/.
(API). This feature allows the integration of the routines in very different contexts as well as the building of new applications. The most important concepts (e.g. time series, seasonal adjustment) developed to encapsulate the core engines are common to both algorithms. The code for making basic seasonal adjustment is straightforward. However, it is possible to use the API to solve very tricky problems. A minimalist example is provided in the Annex (section 13A).

Amongst the peripheral services offered by Demetra+, the following ones should be stressed:

- Dynamic access to various "time series providers": Demetra+ provides modules to handle time series coming from different sources: Excel, databases (through ODBC), WEB services, files (TXT, TSW, USCB, XML, SDMX,...); the access is dynamic in the sense that time series are automatically refreshed by the software, which consults the providers to download new information. The model allows asynchronous treatment.
- Common XML formatting: the seasonal adjustment processing can be saved in XML files, which could be used to generate, for instance, WEB services around seasonal adjustment.

The software was designed to allow the new modules to be added without modifying the core application. The main features that can be enriched are listed below:

- time series providers: the users could add their own modules to download series coming from other databases;
- diagnostics on seasonal adjustment;
- output of SA processing.

As mentioned above, the API could be used to generate completely independent applications, but also to create more easily extensions to the current application.

Demetra+ is compatible with Windows XP, Windows Vista and Windows 7. Although Demetra+ is a 32 bits application, it also works with 64 bits version of operating system.

**1.2. Demetra+ application for Microsoft Excel**

The Demetra+ application for Microsoft Excel is attached to the stand alone Demetra+. The application consists of a set of Excel add-ins.

The aim of this tool is to provide, in the Microsoft Excel environment, a seasonal adjustment program inspired from the Demetra+ stand-alone application. The application is designated for efficient multiprocessing, hence information about quality is limited in comparison to Demetra+. Using Excel add-ins the user can easily and quickly calculate the seasonal adjustment for a whole set of time series in the frame of an Excel workbook with detailed results for each series in separate worksheets. Both TramoSeats and X12 methods are available.

The Demetra+ application for Microsoft Excel is delivered as the usual workbooks in two versions, one for Excel 2003 (Demetra+.xls) and one for Excel 2007 (Demetra+.xlsm). The workbook contains only the application code in VBA. The code structure is available for users.

Demetra+ application for Microsoft Excel consists of:

- **ColorAnalyser** (a tool to search outliers in an Excel worksheet containing time series);
- **Demetra+ XL** (a seasonal adjustment tool in the Microsoft Excel environment, inspired by the Demetra+, which can be used for multiprocessing);
- **XL Functions** (a set of Demetra+ Excel functions).

Manuals for applications are attached to the software. The picture below shows how to find them.

![Demetra+ User Interface](image)

**1.3. Uninstall previous version of Demetra+**

In order to remove previously installed Demetra+ version, the user should take the following steps:

- open the "Add/Remove Programs" function in the control panel;
- uninstall Demetra+ if listed;
- close the "Add/Remove Programs" function;
- delete the Demetra+ home directory;
- delete the program group/icons (if manually created).

**1.4. Installing Demetra+**

Demetra+ can be downloaded from [www.cros-portal.eu](http://www.cros-portal.eu). Execute the file "setup" and follow the instructions on the screen. Always take the default options, i.e. typical installation etc.

**1.5. Running Demetra+**

Start working with Demetra+, run the application via the newly installed Windows option under Programs, or start the Demetra.exe file directly from the Demetra sub-folder.
1.6. Closing Demetra+

In order to close the application, the user can select File/Exit from the main menu (See Chapter 2). The other way is to click on the close box in the upper right-hand corner of the Demetra+ window.

If you have created any unsaved work, Demetra+ will display a warning and provide you with the opportunity to save it. The exemplary message box is presented below.
2. Main application windows

2.1. Overview of the software

The main Demetra+ window, which is displayed after launching the program, is clearly divided into several panels.

The key parts of the application are:

- the browsers panel (left panel), which presents the available time series;
- the workspace panel (right panel), which shows information used or generated by the software;
- a central blank zone that contains actual analyses;
- two auxiliary panels at the bottom of the application; the left one (TSProperties) contains the current time series (from the browsers panel) and the right one (Logs) contains the record of events that occurred after launching the software.

Those areas will be described in the next paragraphs.

Panels can be moved, resized, superposed and closed\(^5\) depending on user’s needs. The presentation is saved between different sessions of Demetra+.

\(^5\) Closed panels can be re-opened through the main menu commands: Workspace \(\rightarrow\) View \(\rightarrow\)...
The application can contain multiple documents. Depending on the preferences, the user can present them in different tabs taking the full space (default) or in floating windows (choose this one to follow different steps). The item Window gives access to that functionality (see 3.5)\(^6\).

Time series can be dragged and dropped between windows (next section presents how to do it). This function is omnipresent in Demetra+, i.e. it is the usual way to move information between different components. The objects that can be moved (e.g. time series, collections of time series) can take different forms: nodes in trees, labels in lists, headers in tables, lines in charts etc.

When a drag and drop operation is initiated (which means that an object is indeed "moveable"), the cursor of the mouse changes to either a "no parking" sign or to a "+" sign. The second one indicates an acceptable drop zone.

Time series from Excel can easily be integrated in Demetra+\(^7\). The users can import their own data sets. The series must be formatted in Excel as follows:

- true dates in the first copied column;
- titles of the series in the corresponding cell of the first row;
- empty top-left cell [A1];
- empty cells in the data zone correspond to missing values (missing values can appear in the time series except the beginning and the end of the series).

This format corresponds with the format used by the Excel browser (which also requires the input zone to start at the beginning of the sheet [A1]). The exemplary file is presented below:

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</tr>
<tr>
<td>17</td>
<td>31-Mar-98</td>
<td>75621.69</td>
<td>183236.3</td>
</tr>
<tr>
<td>18</td>
<td>30-Apr-98</td>
<td>76561.04</td>
<td>185967.5</td>
</tr>
<tr>
<td>19</td>
<td>31-May-98</td>
<td>78720.63</td>
<td>191080.2</td>
</tr>
</tbody>
</table>

\(^6\) Refer to Chapter 3.5 for detailed description of arrangement of the windows.

\(^7\) Demetra+ is compatible with Excel 2003, Excel 2007 and Excel 2010.
Time series are identified by their names. Demetra+ derives some information (like data periodicity, starting and ending period) directly from the first column.

After they have been marked and copied in Excel, the data can be integrated in Demetra+ as follows:

- Select the XML panel in the browsers.
- Paste the data (they appear in the tree). This option doesn’t work if some files were previously opened via XML browsers. In this case, select the button New first and then Paste.
- Change the names of the series/collection in the tree if necessary (click twice on the item you would like to modify).
- Save the file (if need be).

2.2. Browsers

The browsers panel presents the series available in the software.

Different "time series providers" are considered: XML (specific schema), Excel, TSW⁸, USCB, TXT and ODBC.

The installation procedure has copied several files in different formats in the subfolders of "My Documents\Data". The method of opening Excel workbooks is presented below. The procedure is similar for the other providers, i.e.:

1. click on the Excel tab of the browsers panel;
2. click on the left button (see below);
3. choose an Excel workbook (for instance "INSEE.xlsx", see screen below).

---

⁸ TSW (TRAMO SEATS for Windows) is the seasonal adjustment software developed by the Bank of Spain. TSW can be downloaded from [http://www.bde.es/webbde/en/secciones/servicio/software/programas.html](http://www.bde.es/webbde/en/secciones/servicio/software/programas.html).
Final nodes of the trees represent time series and their parents represent collections of time series. Those nodes correspond with names of the spreadsheets. Different browsers show the data in trees that can be expanded by double-clicking their nodes (or single-clicking the "+/-" signs). The tree shows not only how the time series are organized in Excel workbook, but also how many series are in the whole workbook and in each particular spreadsheet.
Right click on any time series name opens the pop-up menu, which contains the following commands: **Add, Remove, Clear.**

- **Add** – opens new time series set from the Excel workbook.
- **Remove** – removes the workbook from the browser. The button is active only if the name of the workbook is marked. It is not possible to remove all workbooks at the same time.
- **Clear** – cleans the browsers.

If the user wants to put the workbook into cash memory one should activate the star next to the Excel workbook name. The list in the **Star** menu contains all workbooks, which are currently in the cash memory.
Using the Tool icon (see below) one can Remove marked item or Clear the window. The Simplify tree option collapses tree with opened branches.

Demetra+ reads files written for TSW (TRAMO-SEATS for Windows). The TSW folder can contain several levels of sub-folders with TSW files. They will appear in the tree navigator of the TSW provider. The series in a subfolder will be grouped in a collection called All series. The same idea was applied for USCB source.
2.3. TS Properties

TS Properties window (an abbreviation from Time Series Properties) can be used for examining the characteristics of individual raw series. This panel is strictly connected with Browsers. The window is presented in the picture below.

TS Properties window presents the basics statistics, chart and time series data. The function is launched by single clicking on the time series name in Browsers window. TS Properties provides also information about the name and source of the time series displayed in it.

2.4. Workspace

Workspace panel organizes all specifications as well as processings and variables defined by the user. The specification section contains a set of pre-specifications. The user can add new specifications by choosing Add New from the pop-up menu (right click on the name of the seasonal adjustment method). In Workspace panel the user can also define calendars and regression variables. The windows in which the user can define or change the seasonal adjustment parameters, calendars and regression variables will be described in Chapter 3.
Right click on any existing name opens the pop-up menu, which contains the following commands: **Open, Exclude, Delete, Clone, Active**.

- **Open** – opens the specification window with information on parameters. The user can’t change them. The same result is achieved by double click on the name of the specification.

- **Exclude** – removes the specification marked. It works only for specifications defined by the user.

- **Delete** – removes the specification marked. It works only for specifications defined by the user.

- **Clone** – creates a new specification, identical with the marked one. The parameters of the newly created specification can be edited by the user.

- **Active** – activates the marked specification. Time series will be seasonally adjusted using this specification.
In a similar way the user can add a new (or existing) specification in single processing and multi-processing sections. This can be achieved by right-clicking on the seasonal adjustment method.

2.5. Log

The Log window contains information about all bugs, warnings and other events that took place during session.

The user can also display messages which belong to a chosen category (like ERROR, EMERGENCY, etc.).
2.6. Results panel

The panel in the middle of the window is the place where Demetra+ displays various windows. More than one window can be displayed at the same time. Windows can overlap each other with the foremost window being in focus or active. The active windows have a darkened title bar.

The windows in the results panel can be arranged in many different ways, depending on the user’s needs (see 3.5). The example below shows one of the possible displays of this panel. The right part of the panel presents navigation tree while on the left the actual results are displayed.

The user can execute several seasonal adjustments and define some regression variables. The results are displayed in consecutive bookmarks, which allow the user to switch them over. On the picture below it is shown that three panels are opened - window containing seasonal adjustment results ("TramoSeatsDoc-1"), default calendar ("Default") and user defined variables ("Variables").
3. Application Menu

The menu of the application is situated at the very top of the main window. If the user moves the cursor to an entry in the main menu and click on the left mouse button, a drop-down menu will appear. Clicking on an entry in the drop-down menu selects the highlighted item. The functions available in the menu of the application are described in the paragraphs below.

3.1. Workspace menu

The Workspace menu offers the following functions:

- **New** – creates new workspace displayed in the right panel;
- **Open** – opens an existing project in a new window;
- **Save** – saves the project file named by the system under the name *Workspace_#number* that can be re-opened at a later point in time;
- **Save as** – saves the project file named by the user that can be re-opened at a later point in time;
- **View** – activates or deactivates the panels chosen by user (*Browsers, Workspace, Logs, TS Properties*);
- **Edit** – allows calendars and regression variables to be defined (this functionality is described further into this instruction);
- **Import** – allows calendars and regression variables to be imported from XML files (this functionality is described further into this instruction);
- **Recent Workspaces** – opens workspace recently saved by user;
- **Exit** – closes an open project.
3.1.1. Calendars

This functionality is helpful for detecting and estimating the calendar effects. Calendar effects are those parts of the movements in the time series that are caused by different number of the weekdays in calendar months (or quarters, respectively). They arise as the number of occurrences of each day of the week in month (quarter) differs from year to year. These differences cause regular effects in some series. In particular, such variation is caused by a leap year effect because of the extra day inserted into February every four years. As with seasonal effect, it is desirable to estimate and remove calendar effects from the time series.

The calendar effects can be divided into a seasonal part and a structural part. The seasonal part arises from the properties of the calendar that recur each year. Inter alia, the number of working days of months with 31 calendar days is on average larger than that of months with 30 calendar days. This effect is part of the seasonal pattern captured by the seasonal component (with the exception of leap year effects). The structural part of the calendar effect remains to be determined by the calendar adjustment. For example, the number of working days of the same month in different years varies from year to year. This is in line with item 1.3 of the ESS Guidelines on Seasonal Adjustment.

Both X12 and TramoSeats estimate calendar effects by adding regressors to the equation estimated in the pre-processing part (RegArima or Tramo, respectively). Regressors mentioned above are generated on the calendar basis.

The calendars of Demetra+ simply correspond to the usual trading days contrasts variables based on the Gregorian calendar, modified to take into account some specific holidays. Those holidays are handled as "Sundays" and the variables are properly adjusted to take into account long term mean effects.
Demetra+ considers three kinds of calendars:

- **National calendars**, identified by specific days;
- **Composite calendars**, defined as weighted sum of several national calendars;  
- **Chained calendars**, defined by two national calendars and a break date.

The calendars can be defined recursively. It is also possible to define calendar using user-defined regression variables (see 3.1.2).

The dialog box allows to define all types of the calendars described above. In the column on the right the number of calendars that have been already defined is shown.

If the user chooses the option **National calendars** the window presented below is displayed. The user can define new calendar (**Add** button) or modify existing one (by clicking on the calendar name in the left panel and modifying the country specific holidays). The list on the left contains all national calendars defined by the user. In the panel on the right the user could specify the successive parameters.

---

9 The user can also use default calendar to define composite calendar and chained calendar.
Next two pictures present how to define fixed holidays (by choosing the month from the list and specifying the appropriate day of the month). If the validity period hasn’t been specified, the regressor will be applied for whole time series span.

Demetra+ offers the list of pre-specified holidays presented on the picture below.
The descriptions of those pre-defined holidays are presented below.

<table>
<thead>
<tr>
<th>Holiday</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Year</td>
<td>Fixed holiday, falls on January, 1.</td>
</tr>
<tr>
<td>Ash Wednesday</td>
<td>Moving holiday, occurring 46 days before Easter.</td>
</tr>
<tr>
<td>Easter</td>
<td>Moving holiday, varies between March, 22 and April, 25.</td>
</tr>
<tr>
<td>Maundy Thursday</td>
<td>Moving holiday, falling on the Thursday before Easter.</td>
</tr>
<tr>
<td>Good Friday</td>
<td>Moving holiday, falling on the Friday before Easter.</td>
</tr>
<tr>
<td>Easter Monday</td>
<td>Moving holiday, falling on the day after Easter.</td>
</tr>
<tr>
<td>Ascension Day</td>
<td>Moving holiday, celebrated on Thursday, 40 days after Easter.</td>
</tr>
<tr>
<td>Pentecost</td>
<td>Moving holiday, celebrated 50 days after Easter Sunday.</td>
</tr>
<tr>
<td>Whit Monday</td>
<td>Moving holiday, falling on the day after Pentecost.</td>
</tr>
<tr>
<td>May Day</td>
<td>Fixed holiday, falls on May, 1.</td>
</tr>
<tr>
<td>Halloween</td>
<td>Fixed holiday, falls on October, 31.</td>
</tr>
<tr>
<td>All Saints Day</td>
<td>Fixed holiday, falls on November, 1.</td>
</tr>
<tr>
<td>Thanksgiving</td>
<td>Moving holiday, celebrated on the second Monday of October (Canada) or on the fourth Thursday of November (United States).</td>
</tr>
<tr>
<td>Christmas Day</td>
<td>Fixed holiday, falls on December, 25.</td>
</tr>
</tbody>
</table>

Calendars defined by the user are added to the workspace tree. The user can display, edit or add new calendar by clicking on Calendars in workspace tree and choosing appropriate option from the pop-up menu (for more details see 2.4 Workspace).

The data generated by each calendar can be viewed by a double click on the corresponding item in the workspace tree. The screen below presents the trading days variables that have been generated for default calendar.
The regression variables can be inspected for any frequency (monthly, bi-monthly, quadri-monthly, quarterly, half-monthly and yearly) and any (reasonable) time span through that window. The periodogram of marked series is also displayed.

**Variable type** option offers three different views:

- **Trading Days** – seven regression variables which correspond to differences in economic activity between all days of the week and leap year effect;

- **Working Days** – two regression variables which correspond to differences in economic activity between the working days (Monday to Friday) and non-working days (Saturday - Sunday) and the leap year effect;

- **None** – one regression variable which corresponds to the leap year effect.

This window can be used to analyze the data created by the calendar. Actually, Demetra+ enables the user to include/exclude the leap year effect from the seasonal adjustment model (see 4.2.4 and 4.3.3).

If the XML browser is selected, the series can be copied by drag and drop as it is shown in the picture below.
Once the calendar is displayed in the central panel, the calendar name is added to the menu toolbar. It contains two options:

**Specification** – displays the calendar specification window;

**Copy** – copies variables visible in the central panel. The series can be pastes into Excel.

Also the local menu can be used to copy and paste the series to other applications (e.g. Excel).
3.1.2. User-defined regression variables

User-defined regression variables are simply time series identified by their names. Those names will be used as identifier of the data while defining regression part of model (sub-chapter 4.2.5 presents how to incorporate the user-defined regression variables into the X12 specification, while sub-chapter 4.3.4 discusses the same issue for the TramoSeats method).

User-defined regression variables are used for measuring abnormalities and should not contain a seasonal pattern.

Demetra+ considers two kinds of user-defined regression variables:

- static variables, usually imported directly from external software (by drag/drop or copy/paste);
- dynamic variables, coming from files that have been opened with the browsers.

It should be emphasized that Demetra+ works on the assumption that a user-defined regressor is already in an appropriately centered form (i.e. the mean of each user-defined regressor is subtracted from the regressor or means for each calendar period (month or quarter) are subtracted from each of the user-defined regressor).

Static variables imported directly from external software (for instance Excel) must be formatted as it has been presented in section 2.1. To import them, select Edit item from the Workspace menu and then User variables (or double click the item User defined variables in the Workspace tree). Then drag and drop time series (or use the usual keys (ctrl-c and ctrl-v)) from already opened Excel file.
The observation values for static variables cannot be changed. The only way to update static series is to remove them from the list and to re-import them with the same names as previously.

Dynamic variables are imported to the *User-defined regression variables* panel by drag and drop series from a browser of the application.

Name of the series can be changed by selecting a series and clicking once again when it has been selected. The series selected from *User-defined regression variables* can be displayed in a small chart window by a double click on regressor name.

Dynamic variables are automatically updated each time the application is re-opened. Therefore, it is a convenient solution for creating the user-defined variables.
3.2. Tools menu

The Tools menu is divided into three parts:

- **Container** – tools for displaying data;
- **Tool Window** – charts and data transformation;
- **Options** – different windows, diagnostic and output options that can be set by the user.

Be advised that the current implementation is not able to detect recursive processing. An attempt to do so will generate a crash of Demetra+. The example of recursive processing is to select the series "D11" from the X12 window and drop the series "D11" into the same X12 window from which "D11" has been selected.

3.2.1. Container

Container includes helpful tools to display the data. The following items are available: **Chart, Grid, List** and **Growth Chart**.

At first, the user should choose at least one item from container list.
Then the user can drag any series or group of series from a given browser and drop it in a container.

The group cannot be marked using Ctrl button from the keyboard. One can add the series to chart or grid by dragging and dropping them one by one.

The series which appear in the results panel (X12 or TramoSeats) can be dragged and dropped to any other window of the Tools menu. It is also possible to drag and drop the results (e.g. table B2 from the X11 decomposition) on the item chosen from container (e.g. chart window).
When a container is active, its name is added to the menu toolbar.

The chart (or the growth chart) is automatically rescaled after adding a new series. Also the new item Chart (or Growth Chart, respectively) is added to menu toolbar. Putting numerous time series into one chart could make it confusing. In this case the user can click on one series which is then displayed in bold.
The right-button menu offers many useful options. Its content depends on the type of the container. For example, for the growth chart the following options are available:

**Copy** – copies raw series and allows to paste it e.g. into Excel. The function is active if the user clicks on the time series in the chart.

**Copy growth data** – copies m/m (or q/q) the growth rates of the marked time series and allows to paste it e.g. into Excel. The function is active if the user clicks on the time series in the chart.

**Remove** – removes time series from the chart. The function is active if the user clicks on the time series in the chart.

**Copy all** – copies all raw time series and allows to paste it into e.g. Excel.

**Copy all growth data** – copies m/m (or q/q) growth rates of the time series and allows to paste it e.g. into Excel.

**Remove all** – removes all time series from the chart.

**Paste** – pastes time series previously marked.

**Export** – settings for export the chart. The chart can be copied to clipboard and saved to file in BMP format.

**Print** – allows to print the graph, displays the print preview, and allows to set the printing options.

**Legend** – adds/removes legend from the chart.

**Kind** – displays m/m or (q/q) and y/y growth rates for all time series in the chart (*previous period* and *previous year* options respectively).

**Settings** – allows to adjust the chart to the user’s preferences (the user can change color scheme, change a line chart to the bar chart, show/hide vertical and horizontal axis, show/hide legend, show/hide title, modify title, change to log scale).
Grid option allows to display the chosen time series in one table.

List presents basic information about chosen time series. Apart from standard options that are available for charts, the local menu for List enables to mark series that have selected frequency.
3.2.2. Tool window

Tool window offers the following facilities: TS Properties, Chart, Growth Chart, Seasonal Chart, Spectral Analysis, Differencing and Connect to Browsers. The first three of these have been described in previous sections. The remaining ones are characterised below.

3.2.2.1. Seasonal chart

Seasonal charts present the final estimation of the seasonal-irregular component and final seasonal factors for each of the periods in a time series (months or quarters). To calculate them Demetra+ uses the active specification (the one which is marked in the Workspace menu).
The curves visible on the chart represent the final seasonal factors and the straight line represents the average for these values in each period. For more details see 4.4.2.1.1.

In right-button menu the standard options (Copy, Export, Print) are available.

### 3.2.2.2. Spectral analysis

Demetra+ offers two spectral estimators – periodogram and autoregressive spectral estimator\(^\text{10}\). After choosing one of them from Tools menu the empty window is displayed.

\(^{10}\) For more information see the Annex, section 9.
To calculate periodogram drag and drop a raw time series into the displayed window. A methodological note about spectral analysis is available at the end of the Manual (Annex, section 9A).

The auto-regressive spectrum can be generated in the same way.

For both periodogram and auto-regressive spectrum the right-button menu offers the standard options (Copy, Export, Print).

### 3.2.2.3. Differencing

*Differencing* window gives the access not only to the data (presented in chart and table) and spectral graphs but also to the ACF and PACF functions for selected time series. In order to obtain the output, the time series should be dragged from the list and dropped precisely into the *Name* box.

![Differencing Window](image)
Using the bookmarks on the right the user could switch to other functions like periodogram and auto-regressive spectrum (Periodogram bookmark), autocorrelation function (ac bookmark) and partial autocorrelation function (Pac bookmark). Description of autocorrelation function and partial autocorrelation function is given in Annex, section 15A.

Once the user changes the differencing orders (D – regular differencing order, BD – seasonal differencing order) or changes the time series, the results are updated automatically. Demetra+ automatically identifies D and BD parameters that generate stationary time series once the user drags/drops the time series. The user can change the D and BD parameters. Using Estimate button the user can restore the stationary time series.

For tabs: Periodogram, ac and pac the right-button menu offers the standard options (Copy, Export, Print). Additional functions are available for Data tab (e.g. Edit, Legend, Settings).

3.2.2.4. Connect to browsers

The Connect to browsers option creates the link between the content of the browsers and the active window. When no window is displayed in the main panel the option is unavailable.

To take advantage of this option first activate some window from the Tool menu. Then the Connect to browsers option can be marked.
When this option is enabled, Demetra+ automatically updates the content of the active window as the user switch from one time series to another one. For example, in the situation presented on the picture below, if the user clicks on the *Construction production index* series in the browsers, this series will be instantly displayed in the **TS chart** window.
3.2.3. Options

The window contains the default options used by Demetra+.

These initial settings can be modified by the user. The menu includes:

- setting for the workspace;
- default processing output;
- settings for the browsers;
• formatters for TXT and XML files;
• settings for presentation the diagnostic where the user can change the critical values and other parameters for diagnostic tests;
• outputs, where the folder that will contain the results is specified.

Those functions are discussed below.

**WorkSpace**

This node enables the user to switch on/off auto loading of the last workspace and to choose the colour for the active item in the *Workspace* panel. By default, the active item is blue. Here the change to red was made.

![WorkSpace node](image)

**Default SA processing output**

The user can decide which parts of the results will be presented after launching seasonal adjustment (SA) processing. To do it, for each SA method the user can show or hide the items from the list of results. By default, all items are displayed after executing SA processing. The picture below reveals that two diagnostics will not be visible in the SA results from TramoSeats.
Browsers

Demetra+ can load data from the following data sources:

- **Excel** (XCLPRVDR);
- **ODBC** (Open Database Connectivity – a standard software interface for accessing database management systems);
- **SDMX** (Statistical Data and Metadata eXchange – an ISO standard for exchanging and sharing statistical data and metadata among organizations);
- **TSW** (denotes "TRAMO-SEATS for Windows" – the seasonal adjustment software developed by the Bank of Spain\(^\text{11}\));
- **USCB** (denotes "X-12-ARIMA" – the seasonal adjustment software maintained by the U. S. Census Bureau\(^\text{12}\));
- **XML** (Extensible Markup Language designed to describe data);
- **TXT**.

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\(^\text{12}\) The software can be downloaded from [http://www.census.gov/srd/www/x12a/](http://www.census.gov/srd/www/x12a/).
With default settings the XML, Excel, TSW and USCB sources are available. The user can add/remove data sources with option isEnabled.

The order of the data sources visible in the Browsers window can be modified using the Position function (the source with the lowest, nonnegative position value is displayed first on the left in the Browsers panel).
**Formatters**

For the XML and TXT data sources Demetra+ offers formatting options like switching between vertical and horizontal presentation of the data, showing dates and titles of the series and using or not the first period for the date.

![Formatting Options](image1)

**Diagnostic**

This part includes information about the chosen significance level used by Demetra+ for an evaluation of the performed seasonal adjustment. The default settings for the tests, displayed in this section, can be changed by the user.

![Diagnostic Options](image2)
For the spectral analysis the following settings are also included: threshold value for identification of peaks, number of years (at the end of the series) considered in the spectral analysis, checking if the spectral peak appears on both SA series and irregular component.

**Outputs**

This section enables to specify which output items will be saved and folder in which Demetra+ saves the results. It is possible to save the results in formats TXT, XLS, CSV or send them to the database by ODBC.

**TXT**

With the TXT format the user can define the folder in which the results and the components will be saved.

**XLS**

In addition to the options available for the TXT format, the XLS format offers the user the opportunity to specify the layout.
If the user will set the option layout to *ByComponent*, the output will be generated in the following way:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>Unemployment rate</td>
<td>Dwellings competed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-01-1991</td>
<td>9916,368</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-02-1991</td>
<td>10498,27</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-03-1991</td>
<td>10747,64</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-04-1991</td>
<td>11737,07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-05-1991</td>
<td>12478,14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-06-1991</td>
<td>12440,96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>01-07-1991</td>
<td>11767,94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Components are placed in separate sheets

The option *OneSheet* will produce the following XLS file:

**ODBC**

After choosing the ODBC option, the user should specify database source name (DSN). Needless to say, this database should be previously created. The user defines the components that will be sent to the database.
### CSV

Using the CSV format it is possible to save a detailed output generated by the multi-processing. The CSV option produces a set of files, each of them containing a specific output. The content of the files is controlled by options: **Use default series**, **Use all series** and **Series**. The list of the settings and their descriptions is presented below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Presentation | Controls how the output, controlled by options: **Use default series**, **Use all series** and **Series**, is divided into separate files. Options:  
- HTable – the output series will be presented in the form of horizontal tables (time series in rows).  
- VTable – the output series will be presented in the form of vertical tables (time series in columns).  
- List – the output series will be presented in the form of vertical tables (time series in rows). Apart from that, for each time series each file contains in separate columns: the data frequency, the first year and of estimation span, the first period (month or quarter) of observation span and the number of observations. The files do not include dates. |
| Folder       | The consecutive files will be stored as follows:  
<folder>\{<workspace name>\}\<processing name>\_<code>.csv  
where:  
- <folder> is specified by the user or the temporary folder if unspecified;  
- <workspace name> is the workspace name (can be omitted);  
- <processing name> is the name of the multi-processing;  
- <code> is the symbol of the output. The list of available codes is given in Annex, section 14A.  
It should be noted that for multi-processing that doesn’t belong to a workspace, the [<workspace name>]<$processing name> sequence is replaced by "demetra". |
| Series       | List of series that will be included into set of output files. The list of available codes is given in Annex, section 14A. |
| Use all series | Options:  
- True – uses all available codes (see Annex, section 14A);  
- False – uses codes entered by the user into **Series** list. |
| Use default series | Options:  
- True – produces a set of files, containing: original series, trend-cycle, seasonally adjusted series, seasonal component, irregular component and the calendar effects;  
- False – produces a set of files, containing all available codes (if **Use all series** option is set to "True" or contains codes entered by the user into **Series** list **Use all series** option is automatically set to "False"). |

The series to be included into **Series** list must be introduced in the **String Collection Editor** (one code by row). The user can also use wildcards, in the usual way, to identify the series.
For example the following collection:

```
* _f
y?
```

will generate all the forecasts and the series "yc" and "yl".

**CSV matrix**

CSV matrix produces the CSV file containing information about model and quality diagnostic of the seasonal adjustment. The user may generate the list of default items or create own quality report. The list of the default items is given in Annex, section 14A.
3.3. X12 Doc

This item is displayed on the menu of the application when one of the X12 specifications is active and the X12Doc window is active (e.g. activated from Seasonal adjustment → Single analysis → New).

The list of available options is presented on the graph below.

- **Copy** – copies item chosen by the user (Results, Processing, Current specification, or Result spec). The item can be pasted into external software (e.g. Excel sheet, Txt file). **Copy Results** copies the content of the Main results table (see 4.4.2.1.1 and 4.4.2.2.1). The options: Copy Processing, Copy current specification, Copy Result spec create the respective items in the form of the xml file.

- **Paste** – pastes the item previously copied.

- **Add to workspace** – adds the single-processing to the workspace tree.

- **Refresh** – updates the processing. The option is available only for previously saved processings. To use this option, first add the processing to the workspace (option Add to the workspace, see above) and save the workspace (confirm saving the workspace while closing Demetra+). Then close Demetra+, update the time series (add/change the observations, but do not change neither the localisation of the file nor the file name) and
open Demetra+ once again. Double click on the previously saved processing and click on the Refresh option. Demetra+ automatically updates the results. The example is presented on the screen below. The user activated the X12Doc-1 single processing from the Workspace menu. Then the Refresh option is available.

- The option Current specification opens the specification that had been used for estimation of the single processing currently displayed in the central application panel. Therefore, the specification includes the initial settings (like the types of outliers that are considered by Demetra+ or the type of the transformation) but not the estimated effects and coefficients (compare with the option Specification from results). The user can modify the specification and validate the newly introduced changes using the Apply button (see example below). Demetra+ re-estimates the complete seasonal adjustment model automatically, so the results are updated immediately.

Detailed description of the X12 specifications is presented in Chapter 4.2.

- The option Specification from results works in the similar way as the Current specification. It is active when the processing has been executed. It displays the specification window that presents the final parameters estimated for the active single processing. Because of that the options presented in the window are fixed according to the results from the single processing. The example below presents the Result spec –X12Doc-1 window where the pre-specified outlier is introduced according to the results from X12Doc-1 window. The user can modify the specification and validate the newly introduced changes using the Apply button (see 4.5.2.3). Demetra+ re-estimates the complete seasonal adjustment model automatically, so the results are updated immediately.
The user is able to modify the specification that is currently used for processing and to see immediately the result of changes made. The specification could be edited through the main menu (TramoSeatsDocxxx / X12Docxxx → Specification...). It is possible to edit the specification used to generate the processing (current specification) or the specification that corresponds to the results (result specification). The specification is displayed in a dialog box, so the user can change any option and inspect its impact on the results.

The settings in the current specification dialog box can be changed e.g. in the following way:

- Activate previously generated output from X12;
- Select from menu X12DocDocxxx → Specification → Current Specification;
- Modify the span of the series in the Basic panel:
  - Click on the Basic item in the left panel of the specification dialog box;
  - Expand the "Selection type" node in the right panel;
  - Choose the "Excluding" selection type;
  - Write "12" in the "last" node;
- Press the Apply button.

The processing is computed on the series without the last 12 observations.
This feature is not available from *Workspace* menu. If the user changes the currently used specification by double clicking on its name in *Workspace*, current processing will not be recalculated.

The trading days regression variables can be suppressed by setting the "Trading days → Type" to "None" in the "Calendar effects" panel of the specification dialog box.

If the option has been used inappropriately used (calendar effect is present, but the user decided not to estimate them) the result will be clearly seen in spectral function computed in pre-processing part (see the picture above).
To refresh the report once more first close the software, update the series and open the software again.
3.4. TramoSeats Doc

This item is visible on the menu of the application if one of TramoSeats specifications is active and the TramoSeatsDoc window is active (activated from e.g. Seasonal adjustment → Single analysis → New).

This item offers the same options as the X12Doc (see 3.3). Detailed description of the TramoSeats specifications is presented in Chapter 4.3.
3.5. Window menu

*Window menu* offers the following functions:

- **Floating** – show additional information while keeping the user in the same window.
- **Tabbed** – arranges all windows in central zone as tabs.
- **Tile vertically** – arranges all windows in central zone vertically.
- **Tile horizontally** – arranges all windows in central zone vertically.
- **Skinning** – allows to custom graphical appearance of Demetra+.
- **Documents.**

List of windows currently displayed in the central panel. This list is dynamically updated when the user opens/closes some windows. On the example below four items are available. The active one is marked.

As an example, the following chart presents the windows arranged using **Tile horizontally** option.
Demetra+ offers six different skinning presented on the chart below.

Documents offers some additional options helpful for organising windows. The left panel contains the list of all windows currently displayed in central panel of Demetra+. On the right activate/close buttons and the presentation styles are available.
4. Seasonal adjustment

Demetra+ provides two methods of seasonal adjustment: TramoSeats and X12. For both methods a list of pre-defined specifications is available (using the naming conventions of TramoSeats). This list contains the most commonly used specification for seasonal adjustment. Pre-defined specifications correspond to the terminology used in TramoSeats and are described in the Annex (section 4A). The default specifications appear in the Workspace tree. The users are strongly recommended to start their analysis with one of those specifications (usually RSA4c or RSA5c for X12 and RSA4 or RSA5 for TramoSeats) and to change afterwards some of the options, if need be.

For more advanced users Demetra+ offers an opportunity to create the new specifications for seasonal adjustment and to add them to the list. This could be done by choosing the Seasonal adjustment item from the main menu and clicking the Specifications sub-menu. In the next step the user should make a choice between TramoSeats specification... and X12 specification.... Once the user has set all options in the Specifications dialog box, the new specification is automatically sent to the corresponding node of the Workspace. The new specification will be saved with the workspace for future use. It can be later used in the same way as any predefined specification.

---

The next section shows how to define and change the specifications. The description of X12 specifications is presented in 4.2 and a description of TramoSeats specifications is presented in 4.3.

Demetra+ is able to perform seasonal adjustment for a single time series as well as for the whole set of time series. The first option is called single processing (see 4.4) and is used for detailed analysis of the time series. The second option, called multi-processing (see 4.5), is a convenient tool for mass production of seasonally adjusted time series.

### 4.1. Defining and modifying a specification

Demetra+ offers few ways of defining new specification.

1. **by preliminary choice of the seasonal adjustment method**

   Specification can be defined by choosing TramoSeats specification or X12 specification from the main menu.

   ![Image of seasonal adjustment](image)

   Then the new specification window will be displayed with default settings. The user is allowed to change them. After clicking **OK** button the new specification is added to the list of the specifications in *Workspace* (TramoSeats or X12 list of the specifications depending on the initial choice).
2. by adding new specification

This option, activated from *Workspace* window, works in the same way as the previous one.

3. by adding existing specification

Add existing option, activated from *Workspace* window, enables to open previously saved specification.
By default, the window where Demetra+ saves the results is displayed.
4. by cloning specification

The new specification can be created directly in the Workspace window by clicking on any existing specification (pre-defined or previously created by the user) and choosing option Clone. New specification will be added to the list of the specifications. The pictures below illustrate this solution. First, the option Clone has been chosen for X12Spec-4.

![Diagram of specification creation process]

Then new specification (X12Spec-5) appears on the X12 specifications list.

![Diagram showing X12Spec-5 in the specifications list]

X12Spec-4 and X12Spec-5 are identical. The user can modify the settings of X12Spec-5 specification by double click on its name and changing parameters in the specification window and choosing OK button.
All specifications created by the user can be modified at any time, by double clicking on the specification name, changing the settings and saving them. The picture below presents the modification of the Selection type for TramoSeats Spec-1.
4.2. X12 specifications

The X12 specification is - to a very large extent - organized following the different individual specs of the original program (taking into account that peripheral specifications or specifications related to diagnostics are handled in a different way).

The different parts of the specification are presented in the order in which they are displayed in the graphical interface of Demetra+. Details on the links between each item and its corresponding X12 spec/argument are provided in the following paragraphs. The descriptions are based on the X-12-ARIMA Reference Manual, for an exact specification the user should refer to the documentation of the original X12 program\textsuperscript{14}.

4.2.1. General description

<table>
<thead>
<tr>
<th>Item</th>
<th>X12 spec file</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>series</td>
<td>General options for the processing.</td>
</tr>
<tr>
<td>Transformation</td>
<td>transform</td>
<td>Transformation of the original series.</td>
</tr>
<tr>
<td>Calendar effects</td>
<td>regression</td>
<td>Specification of the part of the regression related to calendar.</td>
</tr>
<tr>
<td>Regression</td>
<td>regression</td>
<td>Specification of the part of the regression which is not specifically related to calendar.</td>
</tr>
<tr>
<td>Automatic modeling</td>
<td>automdl</td>
<td>Automatic model identification.</td>
</tr>
<tr>
<td>Arima</td>
<td>arima</td>
<td>Arima modeling.</td>
</tr>
<tr>
<td>Outliers detection</td>
<td>outlier</td>
<td>Automatic detection of the outliers.</td>
</tr>
<tr>
<td>Estimation</td>
<td>estimate</td>
<td>Options on the estimation procedure of the RegArima model.</td>
</tr>
<tr>
<td>Decomposition (X11)</td>
<td>x11</td>
<td>X11 decomposition.</td>
</tr>
</tbody>
</table>

4.2.2. Basic

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-processing</td>
<td></td>
<td>Enable/disable the other individuals’ specs, except X11.</td>
</tr>
<tr>
<td>Series span → selection type</td>
<td></td>
<td>Span (data interval) of the available time series used for the processing. The span can be computed dynamically on the series (for instance &quot;Last 90 observations&quot;).</td>
</tr>
</tbody>
</table>

### 4.2.3. Transformation

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Transformation | transform function | Options:  
  - None – data are not transformed;  
  - Log – logarithms from original values are taken;  
  - Auto – Demetra+ tests which option: "None" or "Log" is better for the particular time series. |
| AIC Difference | transform aicdiff | Disabled when the transformation is not set to "Auto". |
| Adjust | transform adjust | The option is available when the transformation is set to "Log". Options:  
  - LeapYear – includes a contrast variable for leap-year;  
  - LengthofPeriod – includes length-of-month (or length-of-quarter) as a regression variable;  
  - None – does not include a regression variable that models the length of the period. |

\[ \text{AICC}_{\text{no log}} - \text{AICC}_{\text{log}} \geq \Delta_{\text{aicdiff}}, \]  
where: \( \text{AICC}_{\text{no log}} \) - the value of AICC from fitting the RegArima model to untransformed data, \( \text{AICC}_{\text{log}} \) - the value of AICC from fitting the RegArima model to log transformed data, \( \Delta_{\text{aicdiff}} \) - aicdiff parameter (with a default of -2). Description based on ONS (2007) ‘Guide to Seasonal Adjustment with X-12-ARIMA’.
4.2.4. Calendar effects

Owing to the definition presented in 3.1.1, calendar effects should be estimated and removed from the time series. In the RegArima model this is done by applying the calendar regressors which, for example, can be defined as the deviations of the number of working days in the particular months from their monthly-specific averages\textsuperscript{16}.

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Arguments</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>AICC Difference</td>
<td>regression</td>
<td>aicdiff</td>
<td>Demetra+ only considers pre-tests on regression variables related to calendar effects (trading days or moving holidays).</td>
</tr>
</tbody>
</table>
| Trading days Type | – | – | The user can choose between four ways of trading days estimation:  
- None – means that calendar effects will not be included in the regression.  
- Predefined – means that default Demetra+ calendar will be used.  
- Calendar – corresponds to the predefined trading days variables, modified to take into account specific holidays. It means that after choosing this option the user should |

\textsuperscript{16} In the US Census Bureau program X-12-ARIMA, the option "centeruser=seasonal" in the "regression" spec ensures that the calendar regressor is calculated as the difference from its monthly-specific average.
<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Individual spec</td>
<td>Argument</td>
<td></td>
</tr>
</tbody>
</table>
| specify the type of trading days effect (td1, td2, td6 or td7) and chose the calendar which will be used for estimation of the holidays.  
• UserDefined – used when the user wants to specify freely his own trading day variables. With this option calendar effect is captured only by regression variables chosen by user from the previously created list of user-defined variables (see 3.1.2). | | | |
| Trading days → Pretest | regression | aictest | Pretest the significance of the trading days regression variables using AICC statistics. |
| Trading days → Details → Trading days (option is available if Trading days ="Predefined" or "Calendar type") | regression | variables | Acceptable values:  
• Td – includes the six day-of-the week variables and the leap year effect;  
• td1Coef – includes the weekday-weekend contrast variable and a leap year effect;  
• tdNoLpYear – includes the six day-of-the week variables;  
• td1NoLpYear – includes the weekday-weekend contrast variable;  
• None – means that none calendar variables will not be included in the regression.  
Some options can be disabled when the Adjust option is used (see 4.2.3). |
| Trading days → Details → Length of period (option is available if Trading days ="Predefined" or "Calendar type") | regression | variables | Acceptable values:  
• LeapYear – includes a contrast variable for the leap-year;  
• LengthofPeriod – includes length-of-month (or length-of-quarter) as a regression variable.  
Can be disabled when the Adjust option is used (see 4.2.3) or with some trading days options. |
<p>| Trading days → Details → Holidays (option is available if Trading days = &quot;Calendar type&quot;) | – | – | When the user chooses the &quot;calendar&quot; type for the trading days, one must specify the corresponding holidays. It should be noted that such holidays must have been previously defined in a given calendar (see 3.1.1). |
| Trading days → | regression | user, | When the user chooses the &quot;userdefined&quot; |</p>
<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Details → Items</td>
<td></td>
<td>The option enables the user to consider the Easter effect in the RegArima model. The user can choose between:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• No – a correction for Easter effect is not performed;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Yes – the correction for Easter effect is considered. The inclusion of the Easter effect to the model depends on the choice made in the Pretest section.</td>
</tr>
<tr>
<td>Easter effect → IsEnabled</td>
<td></td>
<td>Pretest the significance of the Easter regression variables using AICC statistics. Trading days and holiday adjustments can be obtained from RegArima part or from irregular regression models. The user can choose between the following options:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Add – &quot;Easter&quot; is only added in the &quot;variables&quot; part of the regression spec. An automatic identification of the Easter length (1, 8 or 15 days) is executed;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Remove – &quot;Easter&quot; is added in the &quot;variables&quot; and in the aictest parts of the regression spec. The specified length of the Easter effect is used.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• None – &quot;Easter&quot; is only added in the &quot;variables&quot; of the regression spec. The length of the Easter effect specified by the user is used.</td>
</tr>
<tr>
<td>Pretest</td>
<td></td>
<td>Duration (length in days) of the Easter effect. The length of the Easter effect can range from 1 to 20 days. The Length option is visible when Pretest =&quot;None&quot; or Pretest =&quot;Remove&quot;.</td>
</tr>
<tr>
<td>Length</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example: predefined trading days

Example: calendar trading days

Example: user-defined trading days

This option is available if the user has created user-defined variables (see 3.1.2). Such variables can be used to create user-defined trading days.
First, user-defined regression variables must be specified (see 3.1.2)

Example: Easter effect
### 4.2.5. Regression

<table>
<thead>
<tr>
<th><strong>Item</strong></th>
<th><strong>X12</strong></th>
<th><strong>Comments</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Regression variables</strong></td>
<td></td>
<td>User-defined outliers are used if prior knowledge suggest that such effects exist at known time points(^{17}):</td>
</tr>
</tbody>
</table>
| Pre-specified outliers | regression | - Additive Outlier (AO) – additive, point outlier which occurred in a given date \(t_0\). It is modeled by regression variable:  
\[
AO_i^{t_0} = \begin{cases} 1 & \text{for } t = t_0 \\ 0 & \text{for } t \neq t_0 \end{cases};
\]
- Level shift (LS) – variable for a constant level shift beginning on the given date \(t_0\). It is modeled by regression variable:  
\[
LS_i^{t_0} = \begin{cases} -1 & \text{for } t < t_0 \\ 0 & \text{for } t \geq t_0 \end{cases};
\]
- Temporary change\(^{18}\) (TC) – variable for a temporary level change beginning on the given date \(t_0\). It is modeled by regression variable:  
\[
TC_i^{t_0} = \begin{cases} 0 & \text{for } t < t_0 \\ \alpha^{t-t_0} & \text{for } t \geq t_0 
\end{cases},
\]
where \(\alpha\) is a rate of decay back to the previous level \((0 < \alpha < 1)\). Seasonal outliers are not supported. Pre-specified outliers are simple forms of intervention variables. |
| Ramps | regression | Ramp effect means a linear increase or decrease in the level of the series over a specified time interval \(t_0\) to \(t_1\). It is modeled by regression variable:  
\[
RP_i^{(t_0,t_1)} = \begin{cases} -1 & \text{for } t \leq t_0 \\ (t-t_0)(t_1-t_0) - 1 & \text{for } t_0 \leq t < t_1 \\ 0 & \text{for } t \geq t_1 \end{cases}.
\]
All dates of the ramps must occur within the time series span. Ramps can overlap other |

---


\(^{18}\) In TramoSeats method this type of outlier is sometimes called as transitory change.
<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Individual</td>
<td>Argument</td>
</tr>
<tr>
<td></td>
<td>spec</td>
<td></td>
</tr>
<tr>
<td>Intervention</td>
<td></td>
<td></td>
</tr>
<tr>
<td>variables</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>User-defined</td>
<td></td>
<td></td>
</tr>
<tr>
<td>variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

\(^{19}\) See GOMEZ, V., MARAVALL, A. (1997).  
\(^{20}\) Dummy variable is the variable that takes the values 0 or 1 to indicate the absence or presence of some effect.  
\(^{21}\) The effect of this additional component is estimated by Tramo as separate regression effect.
<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Individual spec</th>
<th>Argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comments</td>
<td>period of the dependent series and the appropriate number of forecasts. For the other periods the variables are implicitly set to 0. If the forecasts are not provided, it will not alter the results of the seasonal adjustment but the forecasts of the final components will be unusable. The user can specify the structure of the lags. When regression variable Var is introduced with first lag $l_a$ and last lag $l_b$, Demetra+ estimates for this variable the following regression model: $Var_t = \beta_a x(t - l_a) + ... + \beta_b x(t - l_b)$. To estimate $Var_t = \beta_i x(t - l_i)$ the user should put first lag = last lag = 1. If one put first lag = 0 and last lag = 12 it means that in addition to instantaneous effect, the effect of variable Var is spread over one year. It has to be taken into account that Demetra+ is assuming that these regressors are already in an appropriately centered form, i.e. expressing the deviation from the monthly averages of the trading or working day numbers of each month. This form is necessary in order to achieve meaningful calendar component. Otherwise it is highly probable that the level effect occurs in the regression which leads to non-plausible results.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example: Pre-specified outliers

---

22 More details and examples can be found in MARAVALL, A. (2008).
Example: Ramps

Example: Intervention variables
Example: User-defined variables

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>IsEnabled</td>
<td>automdl</td>
<td>Enables automatic modeling of the specification.</td>
</tr>
<tr>
<td>Accept default</td>
<td>automdl</td>
<td>Controls weather the default model is chosen if the Ljung-Box Q statistics for these model residuals is acceptable.</td>
</tr>
<tr>
<td>Check Mu</td>
<td>automdl</td>
<td>Controls weather the automatic model selection procedure will check for the significance of a constant term.</td>
</tr>
<tr>
<td>Mixed</td>
<td>automdl</td>
<td>Controls weather Arima models with both AR and MA terms (seasonal or nonseasonal) will be considered in the automatic model identification procedure.</td>
</tr>
<tr>
<td>LjungBox limit</td>
<td>automdl</td>
<td>Acceptance criterion for confident intervals of the Ljung-Box Q statistic (see Annex, section 12A).</td>
</tr>
<tr>
<td>Balanced</td>
<td>automdl</td>
<td>Controls weather the automatic model procedure will have a preference for balanced model (i.e., models with similar AR and MA values).</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Individual spec</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>HR initial</td>
<td></td>
<td>automdl</td>
<td>hрині truthful</td>
<td>Controls weather Hannan-Rissanen estimation is done before Exact Maximum Likelihood Estimation to provide initial values.</td>
</tr>
<tr>
<td>Initial unit root</td>
<td></td>
<td>automdl</td>
<td>—</td>
<td>Threshold value for the initial unit root test in the automatic differencing procedure.</td>
</tr>
<tr>
<td>Final unit root</td>
<td></td>
<td>automdl</td>
<td>—</td>
<td>Threshold value for the final unit root test in the automatic differencing procedure. This value should be greater than one.</td>
</tr>
<tr>
<td>Cancelation limit</td>
<td></td>
<td>automdl</td>
<td>—</td>
<td>Cancellation limit for AR and MA roots.</td>
</tr>
<tr>
<td>ArmaLimit</td>
<td></td>
<td>automdl</td>
<td>armalimit</td>
<td>Threshold value for t-statistics of Arima coefficients used for final test of model parsimony.</td>
</tr>
<tr>
<td>ReduceCV</td>
<td></td>
<td>automdl</td>
<td>reducecv</td>
<td>The percentage by which the outlier critical value will be reduced when an identical model is found to have a Ljung-Box statistic with an unacceptable confidence coefficient (see Annex, section 12A).</td>
</tr>
<tr>
<td>Reduce SE</td>
<td></td>
<td>automdl</td>
<td>unavailable</td>
<td>Percentage reduction of SE.</td>
</tr>
<tr>
<td>Unit root limit</td>
<td></td>
<td>automdl</td>
<td>urfinal</td>
<td>Unit root limit for the final model. Should be &gt;1.</td>
</tr>
</tbody>
</table>

24 According to GÓMEZ, V., and MARAVALL, A. (2001), the Hannan-Rissanen method is a penalty function method based on BIC (Bayesian Information Criterion) where the estimates of Arma model parameters are computed by means of linear regressions. These estimates are computationally cheap and have similar properties to those obtained by Maximum Likelihood. See Annex (section 6A) for more details.

25 Refer to 4.4.2.1.2 for description of Maximum Likelihood method.

26 A unit root is an attribute of a statistical model of a time series whose autoregressive parameter is one.

27 Cancellation problem is presented in the Annex (section 8A).

28 See Annex 5A.
4.2.7. Arima

Options included in this section are active only if `IsEnabled` parameter from Automatic modelling section is set to false. In this window the user can manually specify the parameters of Arima model by setting P, D, Q, BP, BD, BQ values. The estimation of parameters values is iterative. For each autoregressive and moving average parameters the user can specify its initial value used in this estimation.

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td><code>regression</code></td>
<td>It is considered that the mean is part of the Arima model (it highly depends on the chosen model).</td>
</tr>
<tr>
<td>P, D, Q, BP, BD, BQ</td>
<td><code>arima</code></td>
<td>Parameters of Arima model ( {P,D,Q}(BP,BD,BQ) ), where:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• P – order of nonseasonal autoregressive polynomial;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• D – nonseasonal differencing order;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Q – order of nonseasonal moving average polynomial;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• BP – order of seasonal autoregressive polynomial;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• BD – seasonal differencing order;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• BQ – order of seasonal moving average polynomial.</td>
</tr>
<tr>
<td>theta, btheta, phi, bphi</td>
<td><code>arima</code></td>
<td>• theta – initial values(^{29}) of the parameters of the nonseasonal, autoregressive polynomial (P);</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• phi – initial values of the parameters of the nonseasonal, moving average polynomial (Q);</td>
</tr>
</tbody>
</table>

\(^{29}\) Initial values are described in the Annex (section 7A).
### Item | X12 | Comments
---|---|---
**Individual spec** | **Argument** | ・ btheta – initial values of the parameters of the seasonal autoregressive polynomial (BP);
・ bphi – initial values of the parameters of the seasonal moving average polynomial (BQ).

The user can choose the Arima model manually. In the example below Arima model (1,2,1)(0,1,1) has been specified.

![Parameter editor](image)

The value of each parameter can be estimated automatically by the program (using initial value if specified) or can be fixed by the user at initial value. In order to introduce fixed parameter value the user should click on the parameter name and then choose the button on the left-hand side, as it is shown on the picture above. Then, put the parameter value (using decimal point if necessary) and mark the *Fixed* option.

![Parameter editor](image)
If the initial value yields an unstable polynomial Demetra+ displays a warning.

![Unstable polynomial warning]

The example below shows the Arima (2,1,1)(0,1,1) model specified by the user. For phi(1), theta(1) and btheta(1) the user introduced the initial values. Moreover phi(1) and theta(1) values are fixed. It is not compulsory to specify initial values for the parameters. If the user changes the Arima parameters for an active processing, the model will be re-estimated and results will be updated after using Apply button (see 4.5.2.3).

In case of fixed parameters the standard errors, T-Stat and P-value are not computed.

---

30 The unstable estimate means that slight changes in input data lead to large changes in estimates. This situation can take place for a particular model because of the convergence of the seasonal component to a zero mean, and because of the correlation between the seasonal AR and MA parameters (MARAVALL, A. (2009)).
4.2.8. Outliers detection

Both X12 and TramoSeats detect outliers, which are defined as the abrupt changes that cannot be explained by the underlying normality of the Arima model. Three types of the outliers can be automatically detected:

- additive outlier (AO) which affects an isolated observation;
- level shifts (LS), which implies a step change in the mean level of the series;
- temporary (transitory) change (TC), which describes a temporary effect on the level of series after a certain point in time.  

<table>
<thead>
<tr>
<th>Item</th>
<th>X12 Individual spec</th>
<th>X12 Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>IsEnabled</td>
<td>outlier</td>
<td>–</td>
<td>Presence or not of the outlier individual specification.</td>
</tr>
<tr>
<td>Outliers detection span</td>
<td>outlier</td>
<td>span</td>
<td>Span used for the outlier detection. The span can be computed dynamically on the series (for instance “Excluding last 12 observations”).</td>
</tr>
<tr>
<td>Use default critical value</td>
<td>outlier</td>
<td>critical</td>
<td>The critical value is automatically determined. It depends on the number of observations considered in the outliers detection procedure. When Use default critical value is false, the procedure uses the critical value inputted in the Critical value item (see below). Otherwise, the default value is used (the first case corresponds to &quot;critical = xxx&quot;; the second corresponds to a specification without the critical argument). It should be noted that it is not possible to define separate critical value for each outlier type.</td>
</tr>
<tr>
<td>Critical value</td>
<td>outlier</td>
<td>critical</td>
<td>Critical value used in the outliers detection procedure.</td>
</tr>
<tr>
<td>AO</td>
<td>outlier</td>
<td>ao</td>
<td>Automatic identification of additive outliers.</td>
</tr>
<tr>
<td>LS</td>
<td>outlier</td>
<td>ls</td>
<td>Automatic identification of level shifts.</td>
</tr>
<tr>
<td>TC</td>
<td>outlier</td>
<td>tc</td>
<td>Automatic identification of transitory changes.</td>
</tr>
<tr>
<td>TC rate</td>
<td>outlier</td>
<td>tcrate</td>
<td>Rate of decay for transitory change outlier regressor.</td>
</tr>
<tr>
<td>Method</td>
<td>outlier</td>
<td>method</td>
<td>Determines how the program successively adds detected outliers to the model (could be add one by one (the outliers with the significant/insignificant t-statistic are added/removed at one time and the Arima model estimated and so on) or add all outliers.</td>
</tr>
</tbody>
</table>

### 4.2.9. Estimation

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Individual spec</td>
<td>Argument</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>together (all the significant/insignificant outliers are added/removed at once and the Arima model estimated and so on).</td>
</tr>
<tr>
<td>LS Run</td>
<td>outlier</td>
<td>lsrun</td>
<td>Compute t-statistic to test null hypotheses that each run of $n$ lsrun successive level shifts cancels to form a temporary level shift.</td>
</tr>
</tbody>
</table>

**Precision**

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Individual spec</td>
<td>Argument</td>
<td></td>
</tr>
<tr>
<td></td>
<td>estimate</td>
<td>tol</td>
<td>Precision used in the optimization procedure.</td>
</tr>
</tbody>
</table>
### 4.2.10. Decomposition (X11)

<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>x11</td>
<td>mode</td>
<td>Only multiplicative, additive or log-additive model is possible. Pseudo-additive mode is not supported. If Transformation is set to &quot;Log&quot; (see 4.2.3) Mode can be set into &quot;Multiplicative&quot; or &quot;LogAdditive&quot;. If the transformation is set to &quot;None&quot; Mode is automatically set to &quot;Additive&quot;. If the transformation is set to &quot;Auto&quot; Mode is automatically set to &quot;Undefined&quot;.</td>
</tr>
<tr>
<td>Use forecasts</td>
<td>forecast</td>
<td>maxlead</td>
<td>When UseForecasts is false, maxlead is set to 0 and the X11 procedure doesn't use any model-based forecasts. Otherwise, the forecasts of the RegArima model (default Airline model if the user doesn't use pre-processing - see Basic options) is used to extend the series.</td>
</tr>
<tr>
<td>LSigma</td>
<td>x11</td>
<td>sigmalim</td>
<td>First parameter of sigmalim – lower sigma boundary for the detection of the extreme values.</td>
</tr>
<tr>
<td>USigma</td>
<td>x11</td>
<td>sigmalim</td>
<td>Second parameter of sigmalim – upper sigma boundary for the detection of the extreme values.</td>
</tr>
</tbody>
</table>
| Seasonal filter| x11 | seasonalm | Specifies which seasonal moving average (i.e. seasonal filter) will be used to estimate the seasonal factors for the entire series. The following filters are available:
- Mixed – enables to assign different seasonal filters to a particular month or quarter using Details on seasonal filters option. Mixed option is available only after executing specifications using multi-processing seasonal adjustment (see description in section 4.5.2.3).
- \( S3 \times 1 - 3 \times 1 \) moving average.
- \( S3 \times 3 - 3 \times 3 \) moving average.
- \( S3 \times 5 - 3 \times 5 \) moving average.
- \( S3 \times 9 - 3 \times 9 \) moving average.
- \( S3 \times 15 - 3 \times 15 \) moving average.
- Stable – a single seasonal factor for each month. |

---

32 See Annex, section 4A for definition of the Airline model.
33 See the Annex (section 3A) for a description of moving averages.
<table>
<thead>
<tr>
<th>Item</th>
<th>X12</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>calendar period is generated by calculating the simple average of all the values for each period (taken after detrending and outlier adjustment).</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• X11default – $3 \times 3$ moving average is used to calculate the initial seasonal factors in each iteration and a $3 \times 5$ moving average to calculate the final seasonal factor.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Msr – automatic choice of seasonal filter. The seasonal filters can be selected for the entire series, or for a particular month or quarter.</td>
</tr>
<tr>
<td>Details on seasonal filters</td>
<td>x11</td>
<td>seasonalm</td>
<td>Period specific seasonal filters are offered as an option in X11 in order to account for seasonal heteroscedasticity (see Annex, section 3A). This option enables to assign different seasonal filters for each period. Option is active if Seasonal filter = &quot;Mixed&quot;. List of available options is the same as for Seasonal filter item (apart from &quot;Mixed&quot; option).</td>
</tr>
<tr>
<td>Automatic Henderson filter</td>
<td>x11</td>
<td>trendma</td>
<td>Automatic selection of the length of the Henderson filter is used when the corresponding item is selected. Otherwise, the length given by the user is used.</td>
</tr>
<tr>
<td>True 7 Term</td>
<td>x11</td>
<td>true7term</td>
<td>Specifies the end weights used for the seven term Henderson filter.</td>
</tr>
</tbody>
</table>
4.3. TramoSeats specifications

TramoSeats specification is based on the original program developed by Agustin Maravall and Victor Gomez (taking into account that peripheral specifications or specifications related to diagnostics are handled in a different way)\textsuperscript{35}.

The different parts of the specification are presented in order in which they are displayed in the graphical interface of Demetra+. Details on the links between each item and its corresponding X12 spec/argument are provided in next paragraphs. For an exact description of the different parameters, the user should refer to the documentation of the original TramoSeats program.

\textsuperscript{35} More information on TramoSeats can be found in www.bde.es.
## 4.3.1. General description

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats parameters</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core engine</td>
<td>–</td>
<td>TramoSeats core engine&lt;sup&gt;36&lt;/sup&gt;.</td>
</tr>
<tr>
<td>Transformation</td>
<td><em>Arima Model – Others</em></td>
<td>Transformation of the original series.</td>
</tr>
<tr>
<td>Calendar effects</td>
<td><em>Others – TradingDay/Easter Effect</em></td>
<td>Specification of the part of the regression related to calendar.</td>
</tr>
<tr>
<td>Regression</td>
<td><em>Others – ireg</em></td>
<td>Specification of the part of the regression which is not specifically related to the calendar.</td>
</tr>
<tr>
<td>Arima</td>
<td><em>Others – Arima dimension, parameters, fixed parameters</em></td>
<td>Arima modeling.</td>
</tr>
<tr>
<td>Outliers detection</td>
<td><em>Others – Outliers</em></td>
<td>Automatic outliers detection.</td>
</tr>
<tr>
<td>Estimation</td>
<td><em>Arima model – others</em></td>
<td>Options on the estimation procedure of the Seats model.</td>
</tr>
<tr>
<td>Decomposition (Seats)</td>
<td><em>Others – Seats parameters</em></td>
<td>Seats decomposition.</td>
</tr>
</tbody>
</table>

<sup>36</sup> Demetra+ v.1.0.3. includes two core engines for TramoSeats: the Tramo and the Seats engines dated 8/2009 and Tramo and Seats engines dated 26/1/2012. Therefore it is possible to choose between the previous and most current versions of TramoSeats. By default, the reference version remains the old one (dated 8/2009) as the last version of TramoSeats doesn’t work properly with user-defined regression variables and calendars. Also not all new features of TramoSeats (like the stochastic trading days) are available for the users of Demetra+. For these reasons the newest TramoSeats core engines are provided for exploratory purposes and for testing. They uses the improvements in the automatic model identification procedure of Tramo and in the search of decomposable models of Seats.
4.3.2. Transformation

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series span</td>
<td>Individual spec</td>
<td>Span used for the processing. The span can be computed dynamically on the series (for instance &quot;Last 90 observations&quot;).</td>
</tr>
<tr>
<td>Function</td>
<td>Arima model - Others</td>
<td>Transformation of data: logarithm, level (none) or log/level pretest.</td>
</tr>
<tr>
<td>Fct</td>
<td>Arima model - Others</td>
<td>Control of the bias in the log/level pretest (the function is active if Function=Auto); fct &gt; 1 favors levels, fct &lt; 1 favors logs.</td>
</tr>
</tbody>
</table>

The test for log-level specification used by TramoSeats is based on the maximum likelihood estimation of the parameter $\lambda$ in the Box-Cox transformations (which is a power transformations such that the transformed values of time series $y$ are a monotonic function of the observations, i.e.

$$y_i^\alpha = \begin{cases} 
(y_i^\alpha - 1)/\lambda, & \lambda \neq 0 \\
\log y_i^\alpha, & \lambda = 0
\end{cases}$$

The program first fits two Airline models (i.e. Arima $(0,1,1)(0,1,1)$) to the time series: one in logs ($\lambda = 0$), other without logs ($\lambda = 1$). The test compares the sum of squares of the model without logs with the sum of squares multiplied by the square of the geometric mean in the case of the model in logs. Logs are taken in case this last function is the maximum, GOMEZ, V., MARAVALL, A. (2009).
### 4.3.3. Calendar effects

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
</table>
| Trading days → Type | Others – TradingDay/Easter Effect | The user can choose between:  
- None – means that calendar effects will not be included in the RegArima model;  
- Predefined – means that the default calendar will be used;  
- Calendar – corresponds to the predefined trading days variables, modified to take into account specific holidays. It means that the user should specify the type of trading days effect (td1, td2, td6 or td7) and chose calendar which will be used for estimation of the holidays;  
- UserDefined – used when the user wants to specify freely his own trading day variables. With this option calendar effect is captured only by regression variables chosen by user from the previously created user-defined variables (see 3.1.2). |
| Trading days → Details → Trading days (option is available if Trading days = "Predefined") | regression variables | Acceptable values:  
- td1 – includes the weekday-weekend contrast variable;  
- td2 – includes the weekday-weekend contrast variable and the leap year effect;  
- td6 – includes six day-of-the-week variables; |
<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Individual spec</td>
<td>Argument</td>
</tr>
<tr>
<td>or &quot;Calendar type&quot;)</td>
<td></td>
<td>• td7 – includes six day-of-the week variables and the leap year effect.</td>
</tr>
<tr>
<td>Trading days → Pretest</td>
<td>Others – TradingDay/Easter Effect</td>
<td>itrad</td>
</tr>
<tr>
<td>Trading days → Details → Holidays (option is available if Trading days = &quot;Calendar type&quot;)</td>
<td>Others – ireg</td>
<td>regeff, iuser, ilong, nser</td>
</tr>
<tr>
<td>Trading days → Details → Items (option is available if Trading days = &quot;UserDefined&quot;)</td>
<td>Others – ireg</td>
<td>regeff, iuser, ilong, nser</td>
</tr>
<tr>
<td>Easter (IsEnabled)</td>
<td>Others – TradingDay/Easter Effect</td>
<td>ieast</td>
</tr>
<tr>
<td>Duration</td>
<td>Others – TradingDay/Easter Effect</td>
<td>idur</td>
</tr>
</tbody>
</table>

The current version of Demetra+ doesn’t allow the use of stock trading days. Pre-defined calendar day for the handling of Labor Day and of Thanksgiving are not available (see 3.1.1. for list of pre-defined holidays). Nevertheless the user is allowed to create any fix day regression variable.
Example: predefined trading days

![Predefined Trading Days](image1)

Example: calendar trading days

![Calendar Trading Days](image2)

Example: user-defined trading days

![User-Defined Trading Days](image3)
Example: Easter effect

4.3.4. Regression

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Argument</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-specified</td>
<td>Others –</td>
<td>ireg, iuser,</td>
<td>User-defined outliers are used if prior knowledge suggests that such</td>
</tr>
<tr>
<td>outliers</td>
<td>others</td>
<td>nser</td>
<td>effects exist at known time points(^{38}):</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Additive Outlier (AO) – additive, point outlier which occurred in a</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>given date (t_0). It is modeled by regression variable (AO_{t_0} = \begin{cases} 1 &amp; \text{for } t = t_0 \ 0 &amp; \text{for } t \neq t_0 \end{cases} );</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Level shift (LS) – variable for a constant level shift beginning on the</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>given date (t_0). It is modeled by regression variable (LS_{t_0} = \begin{cases} -1 &amp; \text{for } t &lt; t_0 \ 0 &amp; \text{for } t \geq t_0 \end{cases} );</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>• Temporary change(^{39})(TC) – a variable for a temporary level change</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>beginning on the given date (t_0). It is modeled by regression variable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(TC_{t_0} = \begin{cases} 0 &amp; \text{for } t &lt; t_0 \ \alpha^{t-t_0} &amp; \text{for } t \geq t_0 \end{cases} );</td>
</tr>
</tbody>
</table>


\(^{39}\) In TramoSeats method this type of outlier is sometimes called by transitory change.
<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item</td>
<td>TramoSeats</td>
<td>Comments</td>
</tr>
<tr>
<td>Ramps</td>
<td>Others – others</td>
<td>where ( \alpha ) is a rate of decay back to the previous level ((0 &lt; \alpha &lt; 1)). Seasonal outliers are not supported. Pre-specified outliers are simple forms of intervention variables.</td>
</tr>
</tbody>
</table>
| Intervention variables   | Others – others                 | Ramp effect means a linear increase or decrease in the level of the series over a specified time interval from \( t_0 \) to \( t_1 \). It is modeled by regression variable: \[
RP_t^{(i_{0:t})} = \begin{cases} 
-1 & \text{for } t \leq t_0 \\
(t-t_0)/(t_1-t_0) - 1 & \text{for } t_0 \leq t < t_1 \\
0 & \text{for } t \geq t_1 
\end{cases}
\]
All dates of the ramps must occur within the time series span. Ramps can overlap other ramps, additive and level shifts outliers. |
| User-defined variables   | Others – others                 | The intervention variables are special events known a-priori (strikes, devaluations, political events, and so on). Intervention variables are modeled as any possible sequence of ones and zeros, on which some operators may be applied. This option enables the user to define four types of intervention variables\(^{40}\):  
- Dummy variables\(^{41}\);  
- Any possible sequence of ones and zeros;  
- \( \frac{1}{(1-\delta B)} \) of any sequence of ones and zeros \((0 < \delta(Delta) \leq 1)\);  
- \( \frac{1}{(1-\delta_s B')} \) of any sequence of ones and zeros \((0 < \delta_s(DeltaS) \leq 1)\). |

---

\(^{40}\) See GOMEZ, V., MARAVALL, A. (1997).

\(^{41}\) Dummy variable is the variable that takes the values 0 or 1 to indicate the absence or presence of some effect.

\(^{42}\) The effect of this additional component is estimated by Tramo as separate regression effect.
The following rules should be obeyed:

- variables assigned to the trend should not contain seasonal patterns;
- variables assigned to the holiday should not contain trend (or level) patterns;
- variables assigned to the irregular should contain neither seasonal nor trend (or level) patterns.

The user-defined variables should cover the period of the dependent series and the appropriate number of forecasts. For the other periods the variables are implicitly set to 0. If the forecasts are not provided, it will not alter the results of the seasonal adjustment but the forecasts of the final components will be unusable.

The user can specify the structure of the lags. When regression variable \( Var \) is introduced with first lag \( l_a \) and last lag \( l_b \), Demetra+ estimates for this variable the following regression model:

\[
Var_t = \beta_1 x (t - l_1) + \ldots + \beta_b x (t - l_b).
\]

To estimate \( Var_t = \beta_1 x (t - l_1) \) the user should put first lag = last lag = 1. If one put first lag = 0 and last lag = 12 it means that in addition to instantaneous effect, the effect of variable \( Var \) is spread over one year.

It has to be taken into account that Demetra+ is assuming that these regressors are already in an appropriately centered form, i.e. expressing the deviation from the monthly averages of the trading or working day numbers of each month. This form is necessary in order to achieve meaningful calendar component. Otherwise it is highly probable that the level effect occurs in the regression which leads to non-plausible results.

More details and examples can be found in MARAVALL, A. (2008).
Example: Pre-specified outliers

Example: Ramps

Example: Intervention variables
### Example: User-defined variables

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>IsEnabled</td>
<td><em>Others – Automatic model identification</em></td>
<td>Enables automatic modeling of the specification.</td>
</tr>
<tr>
<td>Ub1</td>
<td><em>Arima model – Unit roots</em></td>
<td>Initial unit root limit in the automatic differencing procedure. Ub1 is advanced and rarely used option used in the detection of unit roots in Tramo. It is used as a threshold value to detect unit roots in the first step of the automatic identification of the differencing polynomial, which consists in the estimation of the Arma ((2,0,0)(1,0,0)) plus mean model.</td>
</tr>
<tr>
<td>Ub2</td>
<td><em>Arima model – Unit roots</em></td>
<td>Final unit root limit in the automatic differencing procedure. Ub2 is advanced and rarely used option used in the detection of unit roots in Tramo. It is used in the next steps of estimating procedure based on the estimation of the Arma ((1,d,1)(1,hd,0)) plus mean model.</td>
</tr>
<tr>
<td>Cancel</td>
<td><em>inic, idif</em></td>
<td>Cancellation limit for AR and MA roots⁴⁴.</td>
</tr>
<tr>
<td>Pcr</td>
<td><em>Others –</em></td>
<td>Ljung-Box Q statistic limit for the acceptance</td>
</tr>
</tbody>
</table>

⁴⁴ Cancellation issue is described in the Annex (section 8A).
### 4.3.6. Arima model

Options included in this section are active only if `IsEnabled` parameter from Automatic modelling section is set to false.

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Arima model – Others</td>
<td><em>immean</em></td>
</tr>
</tbody>
</table>
| P, D, Q, BP, BD, BQ | Arima model – Arima dimension | *P, D, Q, BP, BD, BQ, init* | Parameters of Arima model \((P,D,Q)(BP,BD,BQ)\), where:  
- \(P\) – order of nonseasonal autoregressive polynomial;  
- \(D\) – nonseasonal differencing order;  
- \(Q\) – order of nonseasonal moving average polynomial;  
- \(BP\) – order of seasonal autoregressive polynomial;  
- \(BD\) – seasonal differencing order;  
- \(BQ\) – order of seasonal moving average polynomial. |
<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>theta – initial values(^{45}) of the parameters of the nonseasonal, autoregressive polynomial (P); phi – initial values of the parameters of the nonseasonal, moving average polynomial (Q); btheta – initial values of the parameters of the seasonal autoregressive polynomial (BP); bphi – initial values of the parameters of the seasonal moving average polynomial (BQ).</td>
</tr>
<tr>
<td>theta, btheta, phi, bphi</td>
<td>Arima parameters, Arima fixed parameters</td>
<td>[th, jqr] [bth, jqs] [phi, jpr] [bphi, jps]</td>
</tr>
</tbody>
</table>

Imputation of initial values of parameters in Demetra+ is the same for TramoSeats and X12. For description refer to 3.2.7.

### 4.3.7. Outliers detection

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Presence or not of the outlier individual specification.</td>
</tr>
<tr>
<td>IsEnabled</td>
<td>Others – outliers</td>
<td>iatip</td>
</tr>
<tr>
<td>Outliers detection span</td>
<td>Others – outliers</td>
<td>int1, int2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Span used for the outlier detection. The span can be computed dynamically on the series (for instance &quot;Excluding last 12 observations&quot;).</td>
</tr>
<tr>
<td>Option</td>
<td>Others – outliers</td>
<td>aio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Describes the outliers considered in the automatic outliers detection. The following options are available:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• None – outliers are not detected automatically;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• AO_TC – only AO (additive outliers) and TC (transitory change) are automatically detected; AO_LS – only AO and LS (level shifts) are automatically detected;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• AO_TC_LS – all types of outliers.</td>
</tr>
<tr>
<td>Default critical value</td>
<td>Others – outliers</td>
<td>va</td>
</tr>
<tr>
<td></td>
<td></td>
<td>When &quot;Use default critical value&quot; is false, the procedure uses the critical value imposed by the user. Otherwise, the default is used (the first case corresponds to Critical value = &quot;xxx&quot;; the second corresponds to a specification without the Critical value argument). It should</td>
</tr>
</tbody>
</table>

\(^{45}\) Initial values are described in the Annex (section 7A).
### Item | TramoSeats | Comments
--- | --- | ---
| Individual spec | Argument | 
| Critical value | Others – outliers | va | Critical value used in the outliers detection procedure. The option is active when Default critical value is set to "True". |
| TC rate | Others – outliers | deltatc | Rate of decay for transitory change outlier regressor. |
| EML estimation | Others – outliers | imvx | True if exact likelihood estimation method\(^{46}\) is used, false if fast Hannan-Rissanen\(^{47}\) method is used. |

#### 4.3.8. Estimation

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual spec</td>
<td>Argument</td>
<td></td>
</tr>
<tr>
<td>EML estimation</td>
<td>Arima model – others</td>
<td>type</td>
</tr>
<tr>
<td>Precision</td>
<td>Arima model</td>
<td>tol</td>
</tr>
<tr>
<td>Udp</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

---

\(^{46}\) Refer to 4.4.2.1.2 for description of Maximum Likelihood method.

\(^{47}\) See the Annex, section 6A.
### 4.3.9. Decomposition (Seats)

<table>
<thead>
<tr>
<th>Item</th>
<th>TramoSeats</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force model</td>
<td>Seats</td>
<td>Force means that an approximation will be used when model does not accept an admissible decomposition.</td>
</tr>
<tr>
<td></td>
<td>parameters</td>
<td>noadmiss</td>
</tr>
<tr>
<td>MA unit root boundary</td>
<td>Seats</td>
<td>When the modulus of estimated root falls in the range ((x_l, 1)), it is set to 1 if it is root in AR polynomial. Alternatively it is set to (x_l) if root is in MA polynomial.</td>
</tr>
<tr>
<td></td>
<td>parameters</td>
<td>(x_l)</td>
</tr>
<tr>
<td>Trend boundary</td>
<td>Seats</td>
<td>Trend boundary is defined for the modulus of the AR root. If real positive root is equal or greater than that value, the AR root is integrated in the trend component. Below that value the root is integrated in the transitory component. Default parameter value is 0.5.</td>
</tr>
<tr>
<td></td>
<td>parameters</td>
<td>(rmod)</td>
</tr>
<tr>
<td>Seasonal tolerance</td>
<td>Seats</td>
<td>Tolerance (measured in degrees) to allocate AR roots into the seasonal component or the transitory component. Default parameter value is 2 (measured in degrees).</td>
</tr>
<tr>
<td></td>
<td>parameters</td>
<td>(\varepsilon_{phi})</td>
</tr>
</tbody>
</table>

The **MA unit root boundary**, **trend boundary** and **seasonal tolerance** parameters determine the allocation of AR roots to the components\(^{49}\).

---

\(^{48}\) See Annex, section 2A.

\(^{49}\) See Annex, section 2A.
4.4. Single processing

Demetra+ offers several ways to define seasonal adjustment of a single time series. A key question, which will determine the best way to proceed, concerns the specification that will be used to start the analysis. This chapter outlines possible ways to define the single processing and discuss the results of the processing.

4.4.1. Creation and modification of a single-processing

The first step to produce a fast seasonal adjustment is to create a processing. The user can take the existing specification or create a completely new specification. The first category includes pre-defined specifications and specifications previously defined and saved by the user. The second solution is to create a new specification for seasonal adjustment of particular time series. This can be done when a user wants to use in a frequent way a specification that is not available in the list of the predefined ones (for example if one wants to use calendar variables imported from external file or if one want to exclude some kinds of outliers). After creation of a new specification it can be added to the workspace.

Single processing can be launched in two different ways:

1. **by activating the specification or drag/drop the specification**

The user could activate the specification from the list displayed in the workspace panel before choosing the series. By default, RSA5c specification is ticked.

The procedure is as follows:
• select in the **Workspace** tree the specification you want to activate;
• open the local menu using the right click;
• choose the **Active** option from pop-up menu.

This specification, called active specification, will be used to generate the processing. It can be changed at any time.

If there is an active specification in the workspace panel, then, when the user double-clicks a series in a browser, the software follows the following logic:

• if some single-processing are opened (i.e. single-processing windows have been opened in the central panel), they are updated with the new series;
- when no unlocked single-processing is available, a new one is generated with the active specification.

The other option is to drag any specification from the workspace panel and drop it in the central panel of the application. A new single processing window will open automatically.
The data can be imported into specification window either by a double click on the series of the browsers or by dragging/dropping the series in the left panel of the single processing window.

2. **by the main menu**

Another method to define single seasonal adjustment is to use the **New** option from the main menu:

![Screenshot of Demetra+ interface showing the 'New' option in the main menu](image)

If there is an active specification (the one which is ticked in the **Workspace** tree) then Demetra+ will display a new single processing window with settings that correspond to the aforementioned specification.

![Screenshot of Demetra+ interface showing active specification](image)

Otherwise Demetra+ displays window in which user should choose one of the seasonal adjustment methods – TramoSeats or X12.
Once the choice is made Demetra+ displays the list of the available specifications (the content of the list depends on the method chosen). Click one specification from the list of specifications and confirm your choice using OK button.

Demetra+ displays the window with settings that correspond to the already chosen specification.
The last step is to drag the time series from the browser and drop it in this window. The seasonal adjustment process is started instantly and the output is displayed in the screen.

It is also possible to use the wizard to create a single processing. This function is also activated from the main menu:

In the first step the user should choose the series one wants to analyze, using the browser:
Then the methods can be selected:

After that, the user can choose the specification from the list displayed in the very top of the window, or create a new specification. In the example below the RSA0 specification will be used for seasonal adjustment. Obviously the user can define the new specification (New Spec option). The specification parameters depend on the method (TramoSeats or X12) chosen in the previous step. For X12 refer to 4.2. TramoSeats specification is described in 4.3.

Finally click the “Finishing” item and decide if you want add it into Workspace.
Once the results are displayed, the user is allowed to modify the specification that has been already applied. This facility can be useful if the quality of a given seasonal adjustment processing is not satisfactory. To take advantage of this tool, choose the *Current specification* from the main menu.

Then change the settings (for example, change Easter effect option from "No" to "Pretest") and confirm using *Apply* button. The seasonal adjustment results will be automatically refreshed.
4.4.2. Seasonal adjustment results - single processing

Once the active specification is chosen, the series that will be seasonally adjusted should be double clicked. The processing is immediately initiated, using the selected specification and the chosen series.

Results of seasonal adjustment are presented in the central panel.
The results contain a set of detailed panels organized in tree displayed on the left side of the output window. The user can go through them by selecting a node in the navigation tree of the processing.

Demetra+ presents several charts and tables with the results of seasonal adjustment and a set of measures of the quality of seasonal adjustment. The quality diagnostic implemented in original seasonal adjustment algorithms are different for each SA method. Moreover, their interpretation could be problematic for an unsophisticated user. For this reason, in Demetra+ the qualitative indicator was built-in. The values of the indicator are described in the following table:
Meaning of the quality indicator

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undefined</td>
<td>The quality is undefined because of unprocessed test, meaningless test, failure in the computation of the test, etc.</td>
</tr>
<tr>
<td>Error</td>
<td>There is a logical error in the results (for instance, it contains aberrant values or some numerical constraints are not fulfilled). The processing should be rejected.</td>
</tr>
<tr>
<td>Severe</td>
<td>There is no logical error in the results but they should not be accepted for serious quality reasons.</td>
</tr>
<tr>
<td>Bad</td>
<td>The quality of the results is bad, following a specific criterion, but there is no actual error and the results can be used.</td>
</tr>
<tr>
<td>Uncertain</td>
<td>The result of the test shows that the quality of the seasonal adjustment is uncertain.</td>
</tr>
<tr>
<td>Good</td>
<td>The result of the test is good from the aspect of the quality of seasonal adjustment.</td>
</tr>
</tbody>
</table>

Several qualitative indicators can be combined following the basic rules. Given a set of n diagnostics, the sum of the results is:

<table>
<thead>
<tr>
<th>Sum</th>
<th>Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undefined</td>
<td>All diagnostics are &quot;Undefined&quot;.</td>
</tr>
<tr>
<td>Error</td>
<td>There is at least 1 &quot;Error&quot;.</td>
</tr>
<tr>
<td>Severe</td>
<td>There is at least 1 &quot;Severe&quot; diagnostic but no error.</td>
</tr>
<tr>
<td>Bad</td>
<td>No &quot;Error&quot;, no &quot;Severe&quot; diagnostics; the average of the (defined) diagnostics (&quot;Bad&quot;=1, &quot;Uncertain&quot;=2, &quot;Good&quot;=3) is &lt; 1.5.</td>
</tr>
<tr>
<td>Uncertain</td>
<td>No &quot;Error&quot;, no &quot;Severe&quot; diagnostics; the average of the (defined) diagnostics (&quot;Bad&quot;=1, &quot;Uncertain&quot;=2, &quot;Good&quot;=3) is in [1.5, 2.5].</td>
</tr>
<tr>
<td>Good</td>
<td>No &quot;Error&quot;, no &quot;Severe&quot; diagnostics; the average of the (defined) diagnostics (&quot;Bad&quot;=1, &quot;Uncertain&quot;=2, &quot;Good&quot;=3) is ≥ 2.5.</td>
</tr>
</tbody>
</table>

According to the table, "Error" and "Severe" diagnostics are absorbent results.

The quality of each diagnostics (except for "Undefined" and "Error") can be parameterized by the user in Tools → Options → Diagnostic menu (see 3.2.3).

---

50 The model also contain a flag "Accepted", which simply means that the user decided to accept the results, no matter what are the different diagnostics.
4.4.2.1. **X12**

For X12 the basic output structure is as follows:

- **Main results;**
  - Charts;
  - Table;
  - S-I ratio;

- **Pre-processing (RegArima);**
  - Pre-adjustment series;
  - Arima;
  - Regressors;
  - Residuals;

- **Decomposition (X-11);**
  - A-Tables;
  - B-Tables;
  - C-Tables;
  - D-Tables;
  - E-tables;
  - Quality measures;

- **Diagnostics;**
  - Seasonality tests;
  - Spectral analysis;
  - Revisions history;
  - Sliding spans;
  - Model stability.

Basic information about X12 method is given in Annex (section 3A). Detailed description of the X12 results is presented below.
4.4.2.1.1. Main results

The Main results node includes basic information about pre-processing and the quality of the outcomes.

The first section summarises the results of pre-processing. The content of this panel depends on the specification used for processing and the results of the adjustment\(^{51}\).

**Pre-processing (RegArima)**

- Estimation span: [1-2000 : 2-2011]
- Series has been log-transformed
- No trading days effect
- Easter effect detected
- 3 outliers detected

The message "Series has been log-transformed" is displayed if a logarithmic transformation has been applied as a result of the test done by X12. Otherwise, information does not appear.

In case of pre-defined specifications X11, RSA1 and RSA3 no trading days effect is estimated. For RSA2c and RSA4c pre-defined specifications working days effect and the leap year effect are pretested and estimated if present. If working day effect is significant, pre-processing part includes information "Working days effect (1 regressor)". Message "Working days effect (2 regressors)" means that also the leap year effect has been estimated. For RSA5 trading days effect and the leap year effect are pretested. If the trading days effect has been detected, message "Trading days effect (6 regressors)" or "Trading days effect (7 regressors)" is displayed, depending whether the leap year effect has been detected or not. If Easter effect is statistically significant in series, "Easter effect detected" is displayed.

If X11 pre-defined specification is used or if any significant outliers have not been found under other specifications, information "No outliers found" is displayed. In this section only total number of detected outliers is visible. More information, i.e. type, location and coefficients of every outlier can be found in the node Pre-processing (RegArima).

---

\(^{51}\) For description of the pre-defined specifications see Annex, section 4A. User defined specifications are described in 4.1.
Second part of *Main Results* aims to inform the user about quality of the seasonal adjustment by reporting a summary of diagnostics. *Summary, Basic Checks, Visual spectral analysis, regarima residuals, residual seasonality, outliers and m-statistics* parts are described further.

**Diagnostics**

**Summary**

**Severe**

**Basic checks**

definition: Good (0.000)
annual totals: Good (0.001)

**Visual spectral analysis**

spectral aaa peaks: Good
spectral ttd peaks: Good

**regarima residuals**

normally: Good (0.910)
independence: Uncertain (0.071)
spectral ttd peaks: **Severe** (0.000)
spectral seas peaks: Uncertain (0.011)

**Residual seasonality**

on sa: Good (0.410)
on sa (last 3 years): Good (0.350)
on irregular: Good (0.821)

**Outliers**

total number of outliers: Good (0.022)

**m-statistics**

q: Good (0.584)
q without n2: Good (0.509)

In the *Charts* section the top panel presents the original series with forecasts, the final seasonally adjusted series, the final trend with forecasts and the final seasonal component with forecasts. The second panel shows the final irregular component and the final seasonal component with forecasts.
Table presents the original series with forecasts and forecast error, the final seasonally adjusted series, the final trend with forecasts, the final seasonal component with forecasts and the final irregular component in the following way:

<table>
<thead>
<tr>
<th></th>
<th>Original series</th>
<th>Final seasonally adjusted series</th>
<th>Final trend component</th>
<th>Final seasonal</th>
<th>Final irregular</th>
<th>C</th>
<th>Fr.</th>
<th>Final trend component (fore)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1991</td>
<td>8826</td>
<td>3929.93</td>
<td>10273.3</td>
<td>0.889635</td>
<td>0.9657</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-1991</td>
<td>8239</td>
<td>10504.2</td>
<td>10736.7</td>
<td>0.784352</td>
<td>0.578344</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-1991</td>
<td>7173</td>
<td>10791.4</td>
<td>11152</td>
<td>0.684398</td>
<td>0.967982</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-1991</td>
<td>8586</td>
<td>11744</td>
<td>11955</td>
<td>0.737099</td>
<td>1.01635</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-1991</td>
<td>8724</td>
<td>12473.7</td>
<td>11656.5</td>
<td>0.659352</td>
<td>1.05206</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6-1991</td>
<td>11785</td>
<td>1243.4</td>
<td>11962</td>
<td>0.948577</td>
<td>1.0395</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7-1991</td>
<td>11350</td>
<td>11759.4</td>
<td>1180.0</td>
<td>0.888527</td>
<td>0.789783</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8-1991</td>
<td>8618</td>
<td>10648.2</td>
<td>11687.5</td>
<td>0.808036</td>
<td>0.511081</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9-1991</td>
<td>10244</td>
<td>12247.4</td>
<td>11483.3</td>
<td>0.823499</td>
<td>1.06554</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19</td>
<td>10712</td>
<td>10754.9</td>
<td>11309.7</td>
<td>0.993241</td>
<td>0.936396</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-19</td>
<td>12685</td>
<td>11165.2</td>
<td>11153.9</td>
<td>1.13701</td>
<td>1.00101</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12-19</td>
<td>30993</td>
<td>11762.6</td>
<td>10945.1</td>
<td>2.63208</td>
<td>1.07469</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table can be copied to Excel by dragging and dropping the top left corner cell to Excel sheet.
The user can also use the option **Copy all** from the local menu to copy all table and paste it into Excel file.

The **S-I ratio** chart presents the final estimation of the seasonal-irregular (S-I) component and final seasonal factors for each of the period in time series (months or quarters). It is calculated as the ratio of the original series to the estimated trend, thus it presents an estimate of the detrended series. Blue curves represent the final seasonal factors and the red straight lines represent the mean seasonal factor for each period. The **S-I ratio** (dots) presented on the chart is modified for extreme values. S-I ratio values are derived from table D9 of X12 results (see Annex, section 3A).
Final seasonal factors (blue curves) are calculated by applying moving average to the $S-I$ ratio values and displayed in table D10\textsuperscript{52}.

Using local menu data can be copied from the picture and pasted to the other software.

The user can enlarge a specific period in the $S-I$ ratio chart by clicking in its zone.

The chosen period is displayed in a resizable pop-up window (drag the right bottom corner).

\textsuperscript{52} For more details refer to LADIRAY, D., and QUENNEVILLE, B. (1999).
The $S-I$ ratio chart is a useful diagnostic tool. This chart supports detection of the seasonal breaks. These would show up as an abrupt change to the level of the $S-I$ ratios for a given period. A seasonal break could distort the estimation of the seasonal component and because of that it should be appropriately modeled\textsuperscript{53}.

The $S-I$ ratio chart also reveals the periods with more statistical variability than typical periods (i.e. typical variability for specific time series). If the $S-I$ ratio seem to be very erratic, the seasonal factors will be erratic too. The seasonality is expected to be relatively stable, so in case of high variability of seasonal component the user should choose a longer moving average for its estimation.

Changes in seasonality over time are acceptable unless there is a noticeable change from below to above the overall mean (or vice versa). The problem is illustrated with the chart below. It presents $S-I$ ratios for time series for which the additive model decomposition have been assumed (see Annex, section 3A). The $S-I$ ratios for majority of periods are highly unstable. For some of them (e.g. $S-I$ ratios for July, August, September) the impact of seasonality on time series changes from positive to negative. On the contrary, the values of the seasonal component for April gradually change from negative to positive. It means that for April in the beginning of the sample the seasonally adjusted data are higher than raw series while in the end of the period the seasonally adjusted data are lower than raw series (for additive decomposition $Y_t = SA_t + S_t$).

Generally, the overall mean of the seasonal factors (i.e. mean value of all seasonal factors) should be close to 0 in case of additive model and 1 in case of multiplicative model\textsuperscript{54}.

### 4.4.2.1.2. Pre-processing

The first part of the pre-processing output includes information about data. The notation of estimation span varies according to the frequencies (for example, the span [2-1993 : 10-2006] represents monthly time series and the span [II-1994 : I-2011] represents quarterly time series), number of observations actually used in the model, number of parameters in the model, data transformation, correction for leap years) and various information criteria calculated for the model. Number of effective observations is the number of observations used to estimate the

\textsuperscript{54} Actually, the sum over a year of the seasonal component should be small in relative terms (but not necessary in absolute terms).
model, i.e. the number of observations of regular and seasonal differenced series. Number of estimated parameters is the sum of regular and seasonal parameters for both AR and MA, mean effect, trading/working days effect, outliers, regressors and standard error of model.

In the pre-processing part the model is estimated by Exact Maximum Likelihood Estimation method. Standard error of the regression (ML estimate) is the standard error of the regression from Maximum Likelihood Estimation. Demetra+ displays a maximized value of Likelihood function after iterations processed in Exact Maximum Likelihood Estimation. This value is used by model selection criteria: AIC, AICC, BIC, BIC (Tramo definition) and Hannan-Quinn. Those criteria are used in seasonal adjustment procedures for the selection of the proper Arima model. The model with the smaller value of the model selection criteria is preferred.

The charts below present an exemplary output.

**Data transformation**


**Model adequation**

Number of effective observations = 108
Number of estimated parameters = 11

Log likelihood = -329.7818
Standard error of the regression (ML estimate) = 5.07962
AIC = 681.5635
AICC = 684.3135
BIC = 711.0670
BIC (Tramo definition) = 3.0840
Hannan-Quinn = 693.5261

Next the estimated model parameters, their standard errors, t-statistics and corresponding p-values are displayed. Demetra+ uses the following notation:

- d: regular difference order;
- D: Seasonal difference order;
- Phi(p): regular AR parameter in p\textsuperscript{th} – order;
- Th(q): regular MA parameter in q\textsuperscript{th} – order;

55 Maximum Likelihood Estimation is a statistical method for estimating the coefficients of a model. This method determines the parameters that maximize the probability (likelihood) of the sample data.

56 The likelihood function is the joint probability (density) function of observable random variables. It is viewed as the function of the parameters given the realized random variables. Therefore, this function measures the probability of observing the particular set of dependent variable values that occur in the sample.

57 AIC, AICC, BIC and Hannan-Quinn criteria are used by X12 while BIC (Tramo definition) by TramoSeats. Information criteria formulas are given in the Annex, section 5A.
• BPhi(P): seasonal AR parameter in $P^{th}$ – order;
• BTh(Q): seasonal MA parameter in $Q^{th}$ – order.

In the example below the Arima model $(0,1,0)(0,1,1)$ was chosen, which means that only one seasonal moving average parameter was calculated. The $p$-value indicates that the regressor is significant.\(^{58}\)

**ARIMA model $(0,1,0)(0,1,1)$**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BTh(1)</td>
<td>-0.3376</td>
<td>0.0882</td>
<td>-3.83</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

For fixed Arima parameters (see 4.1.3.4) Demetra+ shows only the values of the parameters. From the example below it is clear that the user has chosen manually Arima model $(0,1,1)(0,1,1)$ with fixed parameter Th(1).

**ARIMA model $(0,1,1)(0,1,1)$**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Th(1)</td>
<td>-0.7220</td>
<td>0.0489</td>
<td>-17.62</td>
<td>0.0000</td>
</tr>
<tr>
<td>BTh(1)</td>
<td>-0.8613</td>
<td>0.0489</td>
<td>-17.62</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

If Arima model contains a constant term (detected automatically or introduced by the user), estimated value and related statistics are reported.

**Mean effect**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mu</td>
<td>-0.0007</td>
<td>0.0001</td>
<td>-6.50</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Demetra+ presents estimated values of coefficients of one or six regressors depending on the type of calendar effect specification. Joint F-test value is reported under the estimated values if six regressors specification are chosen.

If a leap year regressor has been used in the model specification, the value of the estimated leap year coefficient is also reported with its standard error, t-statistics and the corresponding $p$-value. If option "UserDefined" in calendar effect has been chosen, Demetra+ displays "User-defined trading days" section with variables and theirs estimation results (the values of the parameters, standard errors, t-statistics and corresponding $p$-values) and joint F-test result.

---

\(^{58}\) The variable is called statistically significant if it is so extreme that such a result would be expected to arise simply by chance only in rare circumstances (with probability equal to $p$-value). Generally, the regressor is thought to be significant if $p$-value is lower than 5%.
In the example below the RSA5c specification has been used and trading days effects have been detected. From the table below it can be noticed, that the regressor for Saturday influences time series in the opposite direction to the other trading days regressors. In spite of the fact that some trading days regressors are insignificant on 5% significance level, the outcome of the join F-test indicates that the trading days regressors are jointly significant (F-test statistics is lower than 5%).

### Calendar effects

#### Trading days

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>1.56913</td>
<td>0.659382</td>
<td>2.38</td>
<td>0.0191</td>
</tr>
<tr>
<td>Tuesday</td>
<td>-2.00054</td>
<td>0.649465</td>
<td>-3.08</td>
<td>0.0027</td>
</tr>
<tr>
<td>Wednesday</td>
<td>1.47783</td>
<td>0.678829</td>
<td>2.18</td>
<td>0.0319</td>
</tr>
<tr>
<td>Thursday</td>
<td>0.581356</td>
<td>0.659519</td>
<td>0.88</td>
<td>0.3802</td>
</tr>
<tr>
<td>Friday</td>
<td>0.250281</td>
<td>0.632428</td>
<td>0.40</td>
<td>0.6931</td>
</tr>
<tr>
<td>Saturday</td>
<td>-0.629103</td>
<td>0.623534</td>
<td>-1.32</td>
<td>0.1902</td>
</tr>
<tr>
<td>Sunday (derived)</td>
<td>-1.04896</td>
<td>0.659838</td>
<td>-1.59</td>
<td>0.1151</td>
</tr>
</tbody>
</table>

Join F-Test on trading days: $F = 4.1350$ [P-Value = 0.0010]

If Easter effect is estimated, the following table will be displayed in the output. It is clear, that in the case presented below, Easter has a positive, significant effect on the time series.

#### Easter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easter[15]</td>
<td>4.45711</td>
<td>1.30909</td>
<td>3.40</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

The p-value suggests that the leap year effect is insignificant.

#### Leap year

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>leap year</td>
<td>0.331051</td>
<td>2.03402</td>
<td>0.10</td>
<td>0.8710</td>
</tr>
</tbody>
</table>

Demetra+ presents also the results of detection of the outliers. The table includes the type of outlier, its time point/date, the value of the parameter and significance.

#### Detected outliers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AO[4-2002]</td>
<td>18.7046</td>
<td>3.41647</td>
<td>5.47</td>
<td>0.0000</td>
</tr>
<tr>
<td>LS[1-2007]</td>
<td>38.5629</td>
<td>4.60194</td>
<td>8.38</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
If the user adds a ramp regressor to the model specification, range of ramp variable, estimated value of coefficient and related statistics are shown in appropriate section.

**Ramps**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>rp:2010-05-30-2010-06-30</td>
<td>-0.0104</td>
<td>0.0121</td>
<td>-0.86</td>
<td>0.3908</td>
</tr>
</tbody>
</table>

If a user adds the intervention variables to the model specification, estimated value of coefficient and related statistics are shown under "Intervention Variables" table.

User-defined variables are displayed in section "Other variables ".

**Other variables**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std error</th>
<th>T-Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var_1</td>
<td>-0.00635619</td>
<td>0.0358048</td>
<td>-0.18</td>
<td>0.8593</td>
</tr>
</tbody>
</table>

Demetra+ also reports a list of missing observations. Demetra+ applies AO approach to the missing observations\(^{59}\).

**Missings**

<table>
<thead>
<tr>
<th>Position</th>
<th>Value</th>
<th>Std error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999-8</td>
<td>-39,4125</td>
<td>3,37574</td>
</tr>
<tr>
<td>1999-11</td>
<td>-52,4028</td>
<td>3,37041</td>
</tr>
</tbody>
</table>

**Pre-adjustment series**

The table presented in this section contains series estimated by RegArima part. The contents of the table depend on the effects estimated by RegArima. The following items can appear here:

- **Interpolated series** – series interpolated for the missing observations (if any);
- **Linearized series** – all deterministic effect-adjusted series;
- **Series corrected for the calendar effect** – series corrected for all calendar effects (also user-defined variables assigned to calendar component);
- **Deterministic component** – all deterministic effects such as outliers, ramps, calendars etc.;
- **Calendar effect** – total calendar effect, i.e. joint effect of moving holidays, trading day and Easter effects;

\(^{59}\) Missing observations are interpolated by treating them as Additive Outliers.
• **Moving holidays effect** – the same (provisionally) as **Easter effect**;
• **Trading day effect** – automatically detected or user-entered trading-day effects, i.e. pre-defined calendar effect, user-defined calendar and user-defined regressors in calendar module;
• **Easter effect** – automatically detected or user-entered Easter effect;
• **Outliers effect on the irregular component** – additive and transitory change outliers;
• **Outliers effect on the trend component** – level shift effects;
• **Total outliers effect** – the sum of the outliers effects on trend and irregular components;
• **Separate regression effect** – user-defined variable effect assigned to none of components;
• **Regression effect on the trend component** – ramps, intervention variables for which Delta≠0 and DeltaS=0 and user-defined variable effects assigned to trend component;
• **Regression effect on the seasonal component** – intervention variables for which DeltaS≠0 and user-defined variable assigned to holiday;
• **Regression effect on the irregular component** – user-defined variables effects assigned to irregular component;
• **Regression effect on the seasonally adjusted series** – the sum of the regression effects on the trend and irregular components, and separate regression effects;
• **Total regression effect** – the sum of the regression effects on the trend, seasonal component, irregular component, and separate regression effects.

**Arima**

This section demonstrates a theoretical spectrum of the stationary and non-stationary model. The blue line represents the Arima model identified by RegArima. In the bottom part the Arima model used by RegArima is presented using symbolic notation \((P, D, Q)(PB, DB, QB)\). Estimated coefficients of parameters (regular and seasonal AR and MA) are shown in closed form (i.e. using the backshift operator \(B\)). For each regular AR root the argument and modulus are given. The calculation method is presented in Annex, section 2A.

---

60 If both Delta≠0 and DeltaS≠0, intervention variable is automatically assigned to seasonal component.
61 See Annex, section 2A.
62 Backshift operator \(B\) is defined as: \(B y_t = y_{t-1}\). It is used to denote lagged series.
X12 model \( (2,1,0)(0,1,1) \)

**Polynomials**

- regular AR: 1 + 0.55363 B + 0.5126 B^2
- seasonal AR: 1
- regular MA: 1
- seasonal MA: 1 - 0.5535 S

**Regular AR inverse roots**

- argument=-1.968, modulus=0.716
- argument=1.968, modulus=0.716

Standard **Copy/Print/Export** options are available for this chart.

**Regressors**

This section presents all deterministic regressors used by RegArima part, including trading days variables, the leap year effect, outliers, the Easter effect, ramps, intervention variables and user-defined variables.
Residuals

Residuals from the model are presented in the graph and in the table.

Analysis of the residuals consists of several tests, which are described in the Annex (section 12A). It is divided into two sections: summary and details.

Summary statistics are presented in the following tables:

<table>
<thead>
<tr>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Normality of the residuals</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.9131</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.9051</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.0000</td>
</tr>
<tr>
<td>Normality</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
For each test the corresponding p-value is reported. A p-value is the probability of obtaining a test statistic at least as extreme as the one that was actually observed. Green p-value means "Good", yellow means "Uncertain" and red means "Bad". In the example above, for tests one to three the null hypothesis has been accepted (p-values higher than 5%). It means that it can be assumed that residuals are independent and random. They are approximately normally distributed. Durbin-Watson statistics indicates no autocorrelation in the residuals.

The p-value marked in red indicates that the null hypothesis has been rejected. Linearity of the residuals test provides an evidence of autocorrelation in residuals. A linear structure is left in the residuals.

Apart from section 0-Statistics, all detail sections correspond to the appropriate tables from Summary diagnostics.
1 - Distribution

Mean

<table>
<thead>
<tr>
<th>Value</th>
<th>Standard deviation</th>
<th>T-Stat</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1246</td>
<td>1.1401</td>
<td>0.1093</td>
<td>0.9131</td>
</tr>
</tbody>
</table>

Normality tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
<th>P-Value</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skewness</td>
<td>0.0194</td>
<td>0.9061</td>
<td>Normal(0.00 0.15)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.2720</td>
<td>0.0000</td>
<td>Normal(3.00 0.33)</td>
</tr>
<tr>
<td>Joint-test</td>
<td>35.0230</td>
<td>0.0000</td>
<td>ChiZ(Z)</td>
</tr>
</tbody>
</table>

2 - Independence tests

Ljung-Box and Box-Pierce tests on residuals:

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelation</th>
<th>Standard deviation</th>
<th>Ljung-Box test</th>
<th>P-Value</th>
<th>Box-Pierce test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.0078</td>
<td>0.0070</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0081</td>
<td>0.0070</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0056</td>
<td>0.0070</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.0100</td>
<td>0.0070</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0658</td>
<td>0.0070</td>
<td>1.5384</td>
<td>0.2074</td>
<td>1.5387</td>
<td>0.2148</td>
</tr>
<tr>
<td>6</td>
<td>-0.0673</td>
<td>0.0070</td>
<td>2.0407</td>
<td>0.2679</td>
<td>2.5479</td>
<td>0.2797</td>
</tr>
<tr>
<td>7</td>
<td>0.0236</td>
<td>0.0070</td>
<td>2.7711</td>
<td>0.4263</td>
<td>2.8725</td>
<td>0.4449</td>
</tr>
<tr>
<td>8</td>
<td>0.1289</td>
<td>0.0070</td>
<td>6.6676</td>
<td>0.1545</td>
<td>6.7866</td>
<td>0.1725</td>
</tr>
<tr>
<td>9</td>
<td>0.1200</td>
<td>0.0070</td>
<td>10.0566</td>
<td>0.0720</td>
<td>9.5667</td>
<td>0.0878</td>
</tr>
<tr>
<td>10</td>
<td>0.0135</td>
<td>0.0070</td>
<td>10.1020</td>
<td>0.1204</td>
<td>9.0296</td>
<td>0.1411</td>
</tr>
<tr>
<td>11</td>
<td>0.0537</td>
<td>0.0070</td>
<td>10.2666</td>
<td>0.1422</td>
<td>10.7176</td>
<td>0.1737</td>
</tr>
<tr>
<td>12</td>
<td>0.0002</td>
<td>0.0070</td>
<td>10.1046</td>
<td>0.2141</td>
<td>10.5716</td>
<td>0.2995</td>
</tr>
<tr>
<td>13</td>
<td>-0.0989</td>
<td>0.0070</td>
<td>13.1245</td>
<td>0.1568</td>
<td>12.4525</td>
<td>0.1999</td>
</tr>
<tr>
<td>14</td>
<td>0.0541</td>
<td>0.0070</td>
<td>15.4148</td>
<td>0.2014</td>
<td>15.7117</td>
<td>0.2302</td>
</tr>
<tr>
<td>15</td>
<td>0.0114</td>
<td>0.0070</td>
<td>13.4462</td>
<td>0.2852</td>
<td>12.7465</td>
<td>0.3106</td>
</tr>
<tr>
<td>16</td>
<td>-0.0966</td>
<td>0.0070</td>
<td>14.5269</td>
<td>0.2604</td>
<td>13.7289</td>
<td>0.3103</td>
</tr>
<tr>
<td>17</td>
<td>-0.0256</td>
<td>0.0070</td>
<td>15.2697</td>
<td>0.2944</td>
<td>14.5521</td>
<td>0.3495</td>
</tr>
<tr>
<td>18</td>
<td>-0.1677</td>
<td>0.0070</td>
<td>22.1290</td>
<td>0.0751</td>
<td>20.6200</td>
<td>0.1118</td>
</tr>
<tr>
<td>19</td>
<td>0.0541</td>
<td>0.0070</td>
<td>22.8455</td>
<td>0.0875</td>
<td>21.2721</td>
<td>0.1284</td>
</tr>
<tr>
<td>20</td>
<td>0.0566</td>
<td>0.0070</td>
<td>22.8534</td>
<td>0.1177</td>
<td>21.2791</td>
<td>0.1680</td>
</tr>
<tr>
<td>21</td>
<td>-0.0126</td>
<td>0.0070</td>
<td>22.9851</td>
<td>0.1527</td>
<td>21.4163</td>
<td>0.2125</td>
</tr>
<tr>
<td>22</td>
<td>0.0455</td>
<td>0.0070</td>
<td>23.5103</td>
<td>0.1717</td>
<td>21.6632</td>
<td>0.2351</td>
</tr>
<tr>
<td>23</td>
<td>-0.1290</td>
<td>0.0070</td>
<td>27.7059</td>
<td>0.0803</td>
<td>25.5740</td>
<td>0.1425</td>
</tr>
</tbody>
</table>

Ljung-Box and Box-Pierce tests on seasonal residuals:

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelation</th>
<th>Standard deviation</th>
<th>Ljung-Box test</th>
<th>P-Value</th>
<th>Box-Pierce test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.0637</td>
<td>0.0070</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>-0.1290</td>
<td>0.0070</td>
<td>4.0862</td>
<td>0.0272</td>
<td>4.3528</td>
<td>0.0399</td>
</tr>
<tr>
<td>56</td>
<td>0.0516</td>
<td>0.0070</td>
<td>5.1476</td>
<td>0.0762</td>
<td>4.5750</td>
<td>0.1015</td>
</tr>
</tbody>
</table>
Demetra+ also shows a distribution of the residuals. In this section autocorrelation and partial autocorrelation functions as well as histogram graphics of residuals estimated from RegArima model are presented. For description of autocorrelation function and partial autocorrelation function see Annex, section 15A.
4.4.2.1.3. Decomposition

This part includes tables with results from consecutive iterations of X-11 algorithm and quality measures.

**Tables**

In this section key tables from the X-11 procedure are available. Some tables produced by the original X-11 algorithm are omitted. As an example the view of B20 table is presented below:
A full list of the X11 tables displayed by Demetra+ can be found in the Annex (section 3A).

**Quality measures**

This section presents the seasonal and trend moving average filters used to estimate the seasonal factors and the final trend-cycle. Demetra+ selects the filters automatically, taking into account the global moving seasonality ratio, which is computed on preliminary estimates of the irregular component and of the seasonal. For description of moving averages (seasonal and Henderson) filters see the Annex (section 3A).

**Final filters**

**Trend filter:** 13-term Henderson moving average  
**Seasonal filter:** 5 x 5 moving average

If the different seasonal filters are used for each period (see sub-chapter 4.2.10), then Demetra+ displays in this section information:

**Final filters**

**Trend filter:** 0-term Henderson moving average

The true orders of seasonal and trend filters can be checked in specification panel (to check the seasonal filter the user should expand *Details on seasonal filter* item. Trend filter order is given in *Henderson filter row*).
Next section presents the relative contribution of the components to the stationary portion of the variance in the raw time series (see description of the table in Annex, section 3A). The following notation has been used:

- $I$ – irregular component;
- $C$ – trend-cycle;
- $S$ – seasonal component;
- $P$ – preliminary factors (regressors estimated by RegArima);
- $TD \& H$ – calendar component;
- $Total$ – total changes in raw time series.

For the time series presented below the highest contribution to changes in the raw time series it has the seasonal component (73.93). The decomposition presented in the table below uses the approximate raw series rather than the exact one. The value in last column is close to 100, which indicate that the approximate values were close to the exact ones.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>$P$</th>
<th>TD$&amp;H$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>3.62</td>
<td>12.85</td>
<td>73.93</td>
<td>1.53</td>
<td>6.95</td>
</tr>
</tbody>
</table>

The M statistics are used to assess the quality of seasonal adjustment\(^{63}\). These statistics vary between 0 and 3 but only values smaller than 1 are acceptable.

---

\(^{63}\) For the definitions of the M statistics see LADIRAY D., QUENNEVILLE B. (1999).
M1 measures the contribution of the irregular component to the total variance. M2, which is very similar to M1, is calculated on the basis of the contribution of the irregular component to the stationary portion of the variance. Statistic M3 compares the irregular to the trend-cycle taken from a preliminary estimate of the seasonally adjusted series. If this ratio is too large, it is difficult to separate the two components from each other. Statistic M4 tests the randomness of the irregular component. The statistic M5 is used to compare the significance of changes in trend with that in the irregular. Statistic M6 checks the S-I (seasonal - irregular components ratio). If annual changes in the irregular component are too small in relation to the annual changes in the seasonal component, the $3 \times 5$ seasonal filter used for the estimation of the seasonal component is not flexible enough to follow the seasonal movement. It should be underlined that statistic M6 is calculated only if this filter has been applied in the model. Statistic M7 is the combined test for the presence of identifiable seasonality. The test compares the relative contribution of stable and moving seasonality. Statistics M8 to M11 measure if the movement due to short-term quasi-random variations and movement due to long term changes are not changing too much over the years. If the changes are too strong then the seasonal factors could be erroneous.

Q statistic is a composite indicator calculated from M statistics.

$$Q = \frac{10M1 + 11M2 + 10M3 + 8M4 + 11M5 + 10M6 + 18M7 + 7M8 + 7M9 + 4M10 + 4M11}{100}$$

Q without M2 (also called Q2) is the Q statistic without the M2 statistics. If time series does not cover at least 6 years, statistics M8, M9, M10 and M11 cannot be calculated. In this case the Q statistics is computed as:

---

64 See the Annex (section 12A).
The model has a satisfactory quality if \( Q \) statistic is less than 1.

**Results of the test**

<table>
<thead>
<tr>
<th>Threshold value</th>
<th>Diagnostic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 2 )</td>
<td>Severe</td>
</tr>
<tr>
<td>( [1,2[ )</td>
<td>Bad</td>
</tr>
<tr>
<td>(&lt; 1 )</td>
<td>Good</td>
</tr>
</tbody>
</table>

### 4.4.2.1.4. Diagnostics

The *Diagnostic* panel contains detailed information on the seasonal adjustment process.
Main results

Main results are presented in the first chart.

A description of tests from the Diagnostic panel is presented below. For each test the table with default settings has been attached. The threshold values displayed in the tables can be changed by the user in Tools → Options (see 3.2.3).

- Basic checks

The first section includes two quality diagnostics: definition and annual totals.

  - Definition

This test is inspecting some basic relationships between different components of the time series. The following components are used in formulas that are tested65:

65 The names mentioned in the document appear in the graphical interface of Demetra+. The corresponding codes are used in the CSV output. For compatibility issues with previous versions, they have
<table>
<thead>
<tr>
<th>Name</th>
<th>Code</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>y(_f)</td>
<td>Original series</td>
</tr>
<tr>
<td>Yc</td>
<td>yc(_f)</td>
<td>Interpolated time series (i.e. original time series with missing values replaced by their estimates)</td>
</tr>
<tr>
<td>T</td>
<td>tl(_f)</td>
<td>Trend (without regression effect)</td>
</tr>
<tr>
<td>S</td>
<td>sl(_f)</td>
<td>Seasonal (without regression effect)</td>
</tr>
<tr>
<td>I</td>
<td>il</td>
<td>Irregular (without regression effect)</td>
</tr>
<tr>
<td>SA</td>
<td>sal</td>
<td>Seasonally adjusted series (without regression effect)</td>
</tr>
<tr>
<td>SI</td>
<td></td>
<td>S-I ratio</td>
</tr>
<tr>
<td>TDE</td>
<td>td(_f)</td>
<td>Trading days (or Working days) effect</td>
</tr>
<tr>
<td>MHE</td>
<td>mh(_f)</td>
<td>Moving holidays effect</td>
</tr>
<tr>
<td>EE</td>
<td></td>
<td>Easter effect</td>
</tr>
<tr>
<td>OMHE</td>
<td></td>
<td>Other moving holidays effect</td>
</tr>
<tr>
<td>CAL</td>
<td>cal(_f)</td>
<td>Calendar effect</td>
</tr>
<tr>
<td>OTOT</td>
<td>out_t, out_s, out_i</td>
<td>Outliers effect</td>
</tr>
<tr>
<td>REGTOT</td>
<td>reg(_f), reg_y(_f), reg_sa(_f), reg_tl(_t), reg_s(_f), reg_i(_f)</td>
<td>Other regression effect</td>
</tr>
<tr>
<td>DET</td>
<td>det(_f), det_y(_f), det_sa(_f), det_tl(_t), det_s(_f), det_i(_f)</td>
<td>Deterministic effect</td>
</tr>
<tr>
<td>C</td>
<td>t(_f), s(_f), i, sa</td>
<td>Components, including deterministic effect</td>
</tr>
<tr>
<td>Ycal</td>
<td>ycal</td>
<td>Calendar adjusted series</td>
</tr>
<tr>
<td>Yl</td>
<td>yl</td>
<td>Linearized series</td>
</tr>
</tbody>
</table>

For those components in additive case the following relationships should be true:

- \( MHE = EE + OMHE \) (1);
- \( CAL = TDE + MHE \) (2);
- \( OTOT = OT + OS + OI \) (3);
- \( REGTOT = REGT + REGS + REGI + REGY \) (4);
- \( REGSA = REGT + REGI \) (5);

not been aligned on the names. For some series, it is possible to generate the forecasts (computed on 1 year); the corresponding code is defined by adding the ".f" suffix (for example, y becomes y_f).
• $\text{DET} = \text{CAL} + \text{OTOT} + \text{REGTOT}$ (6);
• $\text{CT} = \text{T} + \text{OT} + \text{REGT}$ (7);
• $\text{CS} = \text{S} + \text{CAL} + \text{OS} + \text{REGS}$ (8);
• $\text{CI} = \text{I} + \text{OI} + \text{REGI}$ (9);
• $\text{CSA} = Y_c - \text{CS} = \text{CT} + \text{CI} + \text{REGY}$ (10);
• $Y_c = \text{CT} + \text{CS} + \text{CI} + \text{REGY} = \text{T} + \text{S} + \text{I} + \text{DET}$ (11);
• $Y_i = Y_c - \text{DET} = \text{T} + \text{S} + \text{I}$ (12);
• $\text{SA} = Y_i - \text{S} = \text{T} + \text{I}$ (13);
• $S_i = Y_i - T = S + I$ (14);

A multiplicative model\(^{66}\) is obtained in the same way by replacing the operations "+" and "-" by "*" and "/" respectively.

The Definition test verifies that all the definition constraints are well respected. The maximum of the absolute differences is computed for the different equations and related to the Euclidean norm of the initial series (Q).

**Results of the test**

<table>
<thead>
<tr>
<th>$Q$</th>
<th>Diagnostic</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0.0000001</td>
<td>Error</td>
</tr>
<tr>
<td>&lt;= 0.000001</td>
<td>Good</td>
</tr>
</tbody>
</table>

**Annual totals**

The test compares the annual totals of the original series and those of the seasonally adjusted series. The maximum of their absolute differences is computed and related to the Euclidean norm of the initial series.

**Results of the test**

<table>
<thead>
<tr>
<th>$Q$</th>
<th>Diagnostic</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0.5</td>
<td>Error</td>
</tr>
<tr>
<td>[0.1, 0.5]</td>
<td>Severe</td>
</tr>
<tr>
<td>[0.05, 0.1]</td>
<td>Bad</td>
</tr>
<tr>
<td>[0.01, 0.05]</td>
<td>Uncertain</td>
</tr>
</tbody>
</table>

\(^{66}\) See definition in the Annex, section 3A.
• **Visual spectral analysis**

Demetra+ identifies spectral peaks in seasonal and trading days components using an empirical criterion of "visual significance". For more information see the Annex (section 9A).

• **RegArima residuals diagnostics**

Several tests are computed on the residuals of the RegArima model. The definition of the residuals, which is identical for both methods (RegArima and Tramo), slightly differs from those of the original programs. However, their global messages are nearly always very similar.

  o **Normality test**

The joint normality test (which combines skewness and kurtosis tests) is the Doornik-Hansen test (see the Annex, section 12A), which is distributed as $\chi^2$.

*Results of the test*

<table>
<thead>
<tr>
<th>$Pr(\chi^2 &gt; val)$</th>
<th>Demetra+ default setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.01</td>
<td>Bad</td>
</tr>
<tr>
<td>[0.01, 0.1]</td>
<td>Uncertain</td>
</tr>
<tr>
<td>≥0.1</td>
<td>Good</td>
</tr>
</tbody>
</table>

  o **Independence test**

The independence test is the Ljung-Box test (see the Annex, section 12A), which is distributed as $\chi^2(k - np)$, where $k$ depends on the frequency of the series (24 for monthly series, 8 for quarterly series, $4 \times freq$ for other frequencies, where $freq$ is a frequency of the time series) and $np$ is the number of hyper-parameters of the model (number of parameters in the Arima model).

*Results of the test*

<table>
<thead>
<tr>
<th>$Pr(\chi^2(k - np) &gt; val)$</th>
<th>Demetra+ default setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.01</td>
<td>Bad</td>
</tr>
<tr>
<td>[0.01, 0.1]</td>
<td>Uncertain</td>
</tr>
<tr>
<td>≥0.1</td>
<td>Good</td>
</tr>
</tbody>
</table>

67 In future versions of Demetra+, it will be possible to choose the definition of the residuals that is used in the tests and displayed in the graphical interface. Obviously, the choice is more a question for purists.
- **Spectral test**

Demetra+ checks the presence of the trading days and seasonal peaks in the residuals using the test based on the periodogram of the residuals. The periodogram is computed at the so-called Fourier frequencies. Under the hypothesis of Gaussian white noise of the residuals, it is possible to derive simple test on the periodogram, around specific (groups of) frequencies. The exact definition of the test is described in the Annex (section 12A).

### Results of the test

<table>
<thead>
<tr>
<th>$P(\text{stat}&gt;\text{val})$</th>
<th>Demetra+ default setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.001</td>
<td>Severe</td>
</tr>
<tr>
<td>[0.001, 0.01[</td>
<td>Bad</td>
</tr>
<tr>
<td>[0.01, 0.1[</td>
<td>Uncertain</td>
</tr>
<tr>
<td>≥0.1</td>
<td>Good</td>
</tr>
</tbody>
</table>

- **Residual seasonality diagnostics**

The residual seasonality diagnostics implemented in Demetra+ correspond to the set of tests developed for X12. One of them is F-test on stable seasonality (see the Annex, section 12A), which is computed on the differences of the seasonally adjusted series (component CSA, see above) and on the irregular component (component CI, see above).

In order to extract the trend from the monthly time series, a first order difference of lag three is applied (a first order difference of lag one in the other cases)\(^{68}\). For the seasonally adjusted series the presence of residual seasonality is tested. Test is performed twice: on the complete time span and on the last 3 years span.

### Results of the test

<table>
<thead>
<tr>
<th>$Pr(F&gt;\text{val})$</th>
<th>Demetra+ default setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.01</td>
<td>Severe</td>
</tr>
<tr>
<td>[0.01, 0.05[</td>
<td>Bad</td>
</tr>
<tr>
<td>[0.05, 0.1[</td>
<td>Uncertain</td>
</tr>
<tr>
<td>≥0.1</td>
<td>Good</td>
</tr>
</tbody>
</table>

- **Number of outliers**

A high number of outliers indicates that there is a problem related to a weak stability of the process or of the reliability of the data is low. If the high number of outliers has been detected

(above 3%, according to the table), the chosen Arima model cannot be fitted all of the observations.

**Results of the test**

<table>
<thead>
<tr>
<th>Threshold value</th>
<th>Demetra+ default setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥0.1</td>
<td>Severe</td>
</tr>
<tr>
<td>[0.05, 0.1]</td>
<td>Bad</td>
</tr>
<tr>
<td>[0.03, 0.05]</td>
<td>Uncertain</td>
</tr>
<tr>
<td>&lt;0.3</td>
<td>Good</td>
</tr>
</tbody>
</table>

- **M-statistics**

  For the test results refer to 4.4.2.1.3.

**Seasonality tests**

- **Non-parametric tests**
  - **Friedman test**

  The seasonal component includes the intra-year variation that is repeated each year (stable seasonality) or evolving from year to year (moving seasonality). To determine if stable seasonality is present in a series, Demetra+ computes the Friedman test using the seasons (months or quarters) as the factors on the preliminary estimation of the unmodified S-I component.
A high test statistics and low significance level indicates that a significant amount of variation in the S-I ratios is due to months (or quarters, respectively), which in turn is evidence of seasonality. If the p-value is lower than 0.1% the null hypothesis of no seasonal effect is rejected. Conversely, a small value of the F-test and high significance level (close to 1.0) is evidence that variation due to months or quarters could be due random error and the null hypothesis of no month/quarter effect is not rejected.

In the example above p-value is 0.0000 so the null hypothesis is rejected and it could be assumed that seasonality is present.

- **Kruskal-Wallis test**

The second test for stable seasonality provided by Demetra+ is the Kruskal-Wallis test.

- **Test for the presence of seasonality assuming stability**

Test for the presence of seasonality assuming stability uses the following decomposition of the variance: $S^2 = S_A^2 + S_R^2$,

where:

$$S^2 = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_{ij} - \bar{X})^2$$ – the total sum of squares,

$$S_A^2 = \sum_{j=1}^{k} n_j (\bar{X}_j - \bar{X})^2$$ – variance of the averages, due to seasonality,
$$S^2_R = \sum_{j=1}^k \sum_{i=1}^{n_j} (X_{ij} - \bar{X}_j)^2$$ – the residual sum of squares.

The test statistics is calculated as:

$$F_S = \frac{S^2_A}{S^2_R} \sim F(k-1, n-k),$$

where $k - 1$ and $n - k$ are degrees of freedom.

The example is shown below:

**Test for the presence of seasonality assuming stability**

<table>
<thead>
<tr>
<th>Between months</th>
<th>Sum of squares</th>
<th>degrees of freedom</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>0.0544</td>
<td>179</td>
<td>0.0003</td>
</tr>
<tr>
<td>Total</td>
<td>0.4075</td>
<td>190</td>
<td>0.0021</td>
</tr>
</tbody>
</table>

Value: 105.6788  
Distribution: F-stat with 11 degrees of freedom in the numerator and 179 degrees of freedom in the denominator  
P-Value: 0.0000  
Seasonality present at the 1 per cent level

The test statistic was calculated in the following way:

$$F_S = \frac{0.3532}{0.0544} \sim F(11,179).$$

The p-value is 0.0000 so the null hypothesis is rejected and it could be assumed that the seasonality in time series is significant.

- **Evolutive seasonality test**

The test verifies if seasonality is stable over years. The test value placed below indicates no evidence of moving seasonality at the 20 per cent level.
The test value was computed in a following way:

\[
F_S = \frac{0,0021}{0,0305} = 0,76.
\]

- **Combined seasonality test**

Combined seasonality test uses Kruskal-Wallis test, test for the presence of seasonality assuming stability and evaluative seasonality test for detecting the presence of identifiable seasonality. For the time series analyzed in this section combined seasonality test seasonality has been identified.

- **Residual seasonality test**

Residual seasonality test is F-test computed on seasonally adjusted series on the complete time span and on the last 3 years span. It is the same test as test for the presence of seasonality assuming stability, described above. Demetra+ displays here F-statistic and conclusion drowned from its value (*No evidence of residual seasonality...*). P-values calculated for this test are given in Diagnostic → Residual seasonality diagnostics part.

---

70 Because sum of squares displayed by Demetra+ are rounded rather than exact, the result of computation made by the user manually is not the same as one obtained by Demetra+. 
Spectral analysis

Demetra+ provides spectral plots to alert the presence of remaining seasonal and trading day effects. The graphics are available for residuals, irregular component and seasonally adjusted time series. Standard Copy/Print/Export options are available for these charts.

Two spectrum estimators are implemented: periodogram and auto-regressive spectrum\(^{71}\). Seasonal frequencies are marked as grey, vertical lines, while violet lines correspond to trading-days frequencies. The X-axis shows the different frequencies. The periodicity of phenomenon at frequency \( f \) is \( \frac{2\pi}{f} \). It means that for monthly time series the seasonal frequencies are:

\[
\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}
\]

(which are equivalent to 1, 2,... cycles per year i.e. in the case of a monthly series, the frequency \( \frac{\pi}{3} \) corresponds to a periodicity of 6 months (2 cycles per year are completed)). For the quarterly series there are two seasonal frequencies: \( \frac{\pi}{2} \) (one cycle per year) and \( \pi \) (two cycles per year). Peak at the zero frequency always corresponds to the trend component of the series. For more detail about spectral analysis refer to the Annex (section 9A).

\(^{71}\)The theoretical motivation for the choice of spectral estimator is provided by SOKUP, R., and FINDLEY, D. (1999).
At seasonal and trading days frequencies, a peak in the residuals indicates the need for a better fitting model. In particular, peaks at the seasonal frequencies are caused by inadequate filters chosen for decomposition. Peaks at the trading days frequencies could occur due to inappropriate regression variables used in the model or the significant change of the calendar effect because the calendar effect cannot be modeled by fixed regression effect on the whole time series span.

A peak in the spectrum of the seasonally adjusted series or irregulars reveals inadequacy of the seasonal adjustment filters for the time interval used for spectrum estimation. In this case different model specification or data span length should be considered.

If significant peaks (P-value >0.05) have been detected for residuals, Demetra+ displays a yellow line that denotes the .005 significance level. As a result, the peaks above this limit are considered to be significant at the .005 significance level\(^2\).

\(^2\) This indicator appears only if the series is a white noise (residuals) as the periodogram is then distributed as a Chi2.
Revision histories

Revision histories illustrate the changes in the seasonally adjusted series and trend series which take place as new observations are added to the end of the original time series. The illustrated difference is between the initial estimate (marked by a blue circle) and the latest estimate (red line). A revision is defined as a difference between those two values. As a rule, smaller revisions are better. The grey broken line is the considered series (trend, SA) as it was 1 year before (the main revisions take place after 1 year).

The revision history is useful for comparing results from competing models. When the user defines two seasonal adjustment models for one time series and both these models are acceptable, then revision history can be used for choosing the better model in terms of revisions. However, it is not actual statistical test but a supplementary descriptive analysis.

More detailed description is available in the Annex (section 10A).
If the user clicks on a blue circle which represents the initial estimation for period $t_n$, an auxiliary window will appear. The figure shows the successive estimations (computed on $[t_0,...,t_n]$, $[t_0,...,t_{n+1}]...[t_0,...,t_T]$) of the considered series for the period $t_n$. From this figure the user can evaluate how the seasonally adjusted observations have changed from initial to final estimation. The analogous graph is available for trend analysis.

By looking at the vertical axis the user could judge the size of the revision. In the figure above the revisions are about 6% (108 - 102 = 106). The figure size can be enlarged by dragging the bottom-right corner.

By default, for revision analysis purpose only the parameters of the model are re-estimated. It is also possible to make a complete re-estimation or a re-identification of the outliers. That option can be changed through the local menu of the revision history node (left panel), at the expense of the speed of the processing and for results that are usually very similar.
In the revisions history panels a complete overview of the different revisions for a given time span can be obtained by selecting with the mouse (just like for zooming) the considered periods. The successive estimations are displayed in a separate pop-up window.

Standard Copy/Print/Export options are available for Revision history charts.
One can also get all the revisions for a specific period by clicking on the point that corresponds to the first estimate for that period. The results of these pop-up windows can be copied or dragged and dropped to other software (e.g. Excel).
The history analysis plot is accompanied by information about the relative difference between initial and final estimation for the last four years. For the additive decomposition absolute revisions are used, for multiplicative decomposition relative differences are considered. Values, which absolute value are larger (in absolute term) than 2 times the root mean squared error of the (absolute or relative) revisions, are marked in red and provide information about the instability of the outcome. Information about mean relative difference between initial and final estimation over period displayed in table is also provided. As relative difference can be positive as well as negative, mean value is not very informative for detecting the possible bias. Magnitude of varying revisions is measured by root mean square error (RMSE). RMSE has the same units as the mean.

### Relative differences

<table>
<thead>
<tr>
<th></th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>-0.818</td>
<td>-1.986</td>
<td>-6.272</td>
<td>0.460</td>
<td></td>
</tr>
<tr>
<td>February</td>
<td>-3.354</td>
<td>-2.070</td>
<td>-0.220</td>
<td>0.724</td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>-1.383</td>
<td>-1.660</td>
<td>0.516</td>
<td>0.567</td>
<td></td>
</tr>
<tr>
<td>April</td>
<td>-1.622</td>
<td>-2.772</td>
<td>0.807</td>
<td>0.833</td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>-0.810</td>
<td>-2.465</td>
<td>0.660</td>
<td>0.775</td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>3.596</td>
<td>-0.118</td>
<td>-1.393</td>
<td>0.718</td>
<td></td>
</tr>
<tr>
<td>July</td>
<td>8.366</td>
<td>-1.159</td>
<td>-1.190</td>
<td>-0.320</td>
<td></td>
</tr>
<tr>
<td>August</td>
<td>1.367</td>
<td>-0.122</td>
<td>3.015</td>
<td>1.244</td>
<td></td>
</tr>
<tr>
<td>September</td>
<td>2.640</td>
<td>4.056</td>
<td>0.959</td>
<td>0.827</td>
<td></td>
</tr>
<tr>
<td>October</td>
<td>-0.090</td>
<td>0.345</td>
<td>-2.964</td>
<td>0.282</td>
<td></td>
</tr>
<tr>
<td>November</td>
<td>-0.252</td>
<td>-0.020</td>
<td>1.335</td>
<td>0.668</td>
<td></td>
</tr>
<tr>
<td>December</td>
<td>-0.559</td>
<td>-0.540</td>
<td>-0.632</td>
<td>0.509</td>
<td></td>
</tr>
</tbody>
</table>

### Sliding spans

It is expected that seasonally adjusted data are stable, which means that removing or adding data points at either end of the series does not change the SA results very much. The sliding spans
analysis checks the stability of seasonal adjustment outcome. It is also used to detect the timing significant changes in the time original time series. Such changes include seasonal brakes and large number of outliers and fast moving seasonality. The sliding spans analysis is particularly useful in case of seasonal breaks\(^{73}\), large number of outliers and fast moving seasonality\(^{74}\).

A span is a range of data between two dates. The sliding spans are series of two, three or four, depending on the length of the original time series overlapping spans. The program sets up a maximum of 4 spans, the length of each span is always 8 years. The spans start in 1 year intervals. The sliding spans analysis stands for the comparison of the correlated seasonal adjustments of a given observation, obtained by applying the adjustment procedure to a sequence of three or four overlapping spans of data, all of which contain this observation\(^{75}\). Each period (month or quarter), that is common to more than one span, is examined to see if its seasonal adjustments or some related quantities vary more than a specified amount across the spans. The summary of the sliding spans analysis is presented below. It contains information about spans, results of the seasonality tests for each span and means of seasonal factors for each month in each span. For the description of the seasonality tests see the Annex (section 12A).

\[\text{Sliding spans summary}\]

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{Time spans} & Span 1 & Span 2 & Span 3 & Span 4 \\
\hline
\textbf{Tests for seasonality} & & & & \\
\hline
Seasonality & 73 & 74 & 75 & 75 \\
\hline
\textbf{Means of seasonal factors} & & & & \\
\hline
\textbf{January} & 76 & 76 & 76 & 76 \\
\hline
\textbf{February} & 77 & 77 & 77 & 77 \\
\hline
\textbf{March} & 78 & 78 & 78 & 78 \\
\hline
\textbf{April} & 79 & 79 & 79 & 79 \\
\hline
\textbf{May} & 80 & 80 & 80 & 80 \\
\hline
\textbf{June} & 81 & 81 & 81 & 81 \\
\hline
\textbf{July} & 82 & 82 & 82 & 82 \\
\hline
\textbf{August} & 83 & 83 & 83 & 83 \\
\hline
\textbf{September} & 84 & 84 & 84 & 84 \\
\hline
\textbf{October} & 85 & 85 & 85 & 85 \\
\hline
\textbf{November} & 86 & 86 & 86 & 86 \\
\hline
\textbf{December} & 87 & 87 & 87 & 87 \\
\hline
\end{tabular}
\end{table}

\(^{73}\) A seasonal break is defined as a sudden and sustained change in the seasonal pattern of a series. The presence of this event is reflected in SI ratio. A seasonal breaks are unwanted feature of time series as the moving averages used by X12 are designed to deal with series which have a smoothly evolving ‘deterministic’ seasonal component plus an irregular component with stable variance.

\(^{74}\) The following casus are mentioned in 'Guide to Seasonal Adjustment', (2007). Fast moving seasonality means that the seasonal pattern displays rapidly evolving fashion from year to year.

\(^{75}\) The procedure of withdrawing spans from time series is described in FINDLEY, D., MONSELL, B. C., SHULMAN, H. B., and PUGH, M. G. (1990) as follows: "To obtain sliding spans for a given series, an initial span is selected whose length depends on the seasonal adjustment filters being used. A second span is obtained from this one by deleting the earliest year of data and appending the year of data following the last year in the span. A third span is obtained from the second in this manner, and a fourth from the third, data permitting. This is done in such a way that the last span contains the most recent data".
The seasonal and the trading day panels compare the (relative) changes of the levels of those components. The SA changes panel is related to period-to-period percentage changes. When an additive decomposition is used, the sliding spans analysis uses absolute differences. The threshold value to detect abnormal values is set to 3% of the testing statistics (see the Annex, section 11A).

Detailed results of the sliding spans analysis is conducted separately for seasonal component, trading days effect and SA series (changes). The description of these results is the same for each part of the series. Below, explanation of the output for the sliding spans analysis for seasonal component is presented. The user should be aware that an unstable estimate of a seasonal factor for a given month can give rise to unstable estimates of the two associated month-to-month changes. Because of that, in majority of cases, more months are flagged for unreliable month-to-month changes than for unreliable seasonal factors.

The first panel shows the sliding spans statistic obtained for each period. This statistic calculates the maximum percentage difference in the seasonal factors for period (month or quarter) \( t \). The estimation of seasonal component is regarded as unstable if statistic is greater than 3%. The exact statistics formula is given in the Annex (section 11A).

The next panel presents the cumulative frequency distribution of the sliding spans statistics (months or quarters) using frequency polygon. On the horizontal axis values of the sliding spans statistics are shown, while vertical axis presents the frequency (in percentages) of each class interval. The example below shows distribution where the first label on the X-axis is 0.0025. This represents an interval extending from 0 to 0.005. This interval has a frequency 25%, which means that 25% the sliding spans statistics are in this interval.

---

77 In frequency polygon data presented on the horizontal axis are grouped into class intervals.
According to the FINDLEY, D., MONSELL, B. C., SHULMAN, H. B., and PUGH, M. G. (1990), the results of seasonal adjustment are stable if the percentage of unstable (abnormal) seasonal factors is less than 15% of total number of observations. Empirical surveys support the view that adjustments with more than 25% of the months (or quarters) flagged for unstable seasonal factor estimates are not acceptable\textsuperscript{78}. Therefore, the user should check the total frequency of the intervals between 0.03 and 1.

The last panel contains detailed information about the percentage of values for which the sliding spans condition is not fulfilled. In the example presented below 4.3% of values has been marked by the sliding spans diagnostic as abnormal. Moreover, Demetra+ provides information about number of breakdowns of unstable factors and average maximum percent differences grouped by month (or quarter) and by year. It gives idea weather observations with unreliable adjustment cluster in certain calendar periods and whether their sliding spans statistics barely or substantially exceed the threshold value. The table below presents that two sliding spans statistic calculated for January have been above 3% and average maximum percentage difference across spans for this period was 1.8.

A large number of unstable estimates revealed by the sliding spans analysis supports an idea of changing the model specification. The example of such a situation is presented below. Because of the large share of moving seasonality (for all spans test statistic is above 4), the test for presence of identifiable seasonality failed (see the Annex, section 12A for description of the tests).
In this section **Copy/Print/Export** options are available only for graphs that present the sliding spans statistics.
Model stability

The diagnostics output window provides some purely descriptive features to analyze the stability of some parts of the model, like trading days, Easter and Arima. Model stability analysis calculates Arima parameters and coefficients of the regressors for different periods and visualizes these results on the graphics. The parameters of the model chosen for the complete time span are computed on a moving window. Therefore, the number of estimations of the parameters depends on the length of the series. The length of the moving window is always 8 years, so if the length of the series is e.g. 12 years then only five estimations will be calculated. The points displayed on the figure correspond to the successive estimations.

The figures are useful for assessing about the stability of the model parameters. On the picture below the results of model stability diagnostic for trading days, Easter and Arima model are shown.

- trading days parameters stability
• Easter effect parameter stability

Small deviations from the mean parameter value are preferable. Taking into account a scale on the vertical axis, the most significant differences between the values of the RegArima model parameters occur in Arima part.

• Arima parameters stability
The user can enlarge the charts from *Model stability* section for one parameter, by clicking in its zone. The details are displayed in a resizable pop-up window (drag the right bottom corner to change the size of the chart).

Standard **Print/Export** options are available.
4.4.2.2. TramoSeats

The basic output structure is as follows:

- Main results;
  - Charts;
  - Table;
  - S-I ratio;

- Pre-processing (Tramo);
  - Pre-adjustment series;
  - Arima;
  - Regressors;
  - Residuals;

- Decomposition (Seats);
  - Stochastic series;
  - Model-based tests;
  - WK analysis;

- Diagnostics;
  - Seasonality tests;
  - Spectral analysis;
  - Revisions history;
  - Sliding spans;
  - Model stability.

TramoSeats method and related concepts are presented in the Annex (section 2A). Detailed description of the seasonal adjustment outcome is presented below. For those features that are very similar to the X12, appropriate descriptions and drawings are omitted. The user can find those in Seasonal adjustment results for X12. In this section only issues specific for TramoSeats will be discussed in detail.
4.4.2.2.1. Main results

Basic information about seasonal adjustment and the quality of the outcomes are divided into three parts: **Pre-processing (Tramo)**, **Decomposition** and **Diagnostics**.

The first part contains results from Tramo. Information "Series has been log-transformed" is displayed if logarithmic transformation has been applied as a result of specification test done by Tramo. Otherwise, information does not appear.

In case of pre-defined specifications RSA0, RSA1 and RSA3 specifications trading days effect is not estimated. For RSA2 and RSA4 pre-defined specifications specifications the working days effect and the leap year effect are pretested and estimated. If the working day effect is significant, pre-processing part includes information "Working days effect (1 regressor)". "Working days effect (2 regressors)" means that also the leap year effect is significant. For RSA5 pre-defined specification the trading days effect and the leap year effect are pretested. If the trading days effect has been detected "Trading days effect (6 regressors)" or "Trading days effect (7 regressors)" is displayed, depending whether the leap year effect has been estimated or not. When the Easter effect is statistically significant in the series, "Easter effect detected" is displayed.

If RSA0 pre-defined specification is used or any significant outliers have not been found under other specifications, information "No outliers found" is displayed. In this section only total number of detected outliers is visible. More information, i.e. the type, date and coefficients of outliers are specified in node "Pre-processing (Tramo)".

**Pre-processing (Tramo)**

- **Estimation span [1-1999 : 2-2011]**
- **Series has been log-transformed**
- **No trading days effects**
- **No easter effect**
- **3 outliers detected**

The second part of the Main results presents the variance of the white noise innovation for each component extracted by Seats procedure from the observed time series $x_t$. Observed time series $x_t$ follows an Arima model of the type $\delta(B)x_t = \psi(B)\epsilon_t$, where $\epsilon_t$ is a white-noise variable with variance $V(\epsilon)$. The residuals $\epsilon_t$ from this model are also called "innovations" because they are

79 For definition of innovations refer to the Annex, section 2A.
the new (unpredictable at $t-1$) part of $x_t$. They are estimators of the one-period-ahead forecast error of the observed series $x_t$.

Seats decomposes a time series into several orthogonal components. The main components are trend-cycle, seasonal, transitory and irregular. For additive decomposition original time series can be presented as a sum of the components:

$$x_t = \sum_{i=1}^{k} x_{it}.$$ 

Each component follows the general Arima model:

$$\delta_i(B)x_{it} = \psi_i(B)a_{it},$$

where:

$i$ - trend, seasonal, transitory or irregular components, respectively,

$a_{it} \sim WN(0, V(a_i))$ - assumed an i.i.d. white-noise innovation of the $i$-th component; it is also an estimator of the 1-period-ahead forecast error of the $i$-th component.

$$\psi_i(B) = \frac{\theta_i(B)}{\varphi_i(B)}.$$ 

The polynomials $\theta_i(B), \varphi_i(B)$ and $\delta_i(B)$ are of finite order. A white-noise variable is normally, identically and independently distributed, with a zero-mean and variance of the component innovation (the variance of the 1-period ahead forecast error of the component) $V(a_i)$. Two different components don’t share the same unit autoregressive roots.

The components can be also expressed in compact form:

$$\phi_i(B)x_{it} = \theta_i(B)a_{it},$$

**Orthogonality** means that behavior of each component is uncorrelated with other components. In particular, causes of seasonal fluctuations are uncorrelated with causes of long term evolution of the series.

**The trend-cycle will also be referred to as the "trend component".**

**It is assumed that irregular component is a white noise variable, which means that it follows Arima (0,0,0)(0,0,0) model.**
where:

\( \phi_i(B) \) is a product of the stationary and the non-stationary autoregressive polynomials.

Seats decomposition fulfills the canonical property, that is it maximizes the variance of the irregular component providing trend, seasonal and transitory components as stable as possible (in accordance with the models)\(^{83}\).

For each component the value of innovation variance is represented through the ratio of the component innovation variance \( V(a_i) \) to the component-Arima-model to variance of the series innovation \( V(a) \)\(^{84}\):

\[ k_i = \frac{\text{Var}(a_i)}{\text{Var}(a)}, \]

where \( k_i \) represents the ratio of component innovation variance to the series innovation variance\(^{85}\). The variance of the irregular component is maximized while the variance of other components are minimized considering the rule of canonical decomposition.

An example of the output is presented below.

![Decomposition]

If some components have not been extracted by Seats from related time series (e.g. transitory component) they are not displayed in this section.

The *Diagnostics* includes the most important statistics which informs the user about the quality of the seasonal adjustment by reporting a summary of diagnostics. *Summary, Basic Checks, Visual spectral analysis, regarima residuals, residual seasonality and outliers* parts have been already described in 4.3.2.1. Seats diagnostics is characteristic only for TramoSeats method and shows results of the tests of assumptions made by Seats. In particular, *seas variance* and *irregular variance* show the probability value of a test to check whether the variance of estimators of the

---

\(^{83}\) In order to identify the components Seats assumes that components are orthogonal to each other and each component except for the irregular one is clean of noise. This is called the canonical property, and implies that no additive white noise can be extracted from a component that is not the irregular one.


seasonal component and of the irregular component, respectively, is close to the variance of their actual estimates. The third test, \textit{seas/irr cross-correlation}, checks the theoretical cross-correlation (between estimators) and empirical cross-correlation (between empirical estimates).

\textbf{seats}  
- \textit{seas variance}: Good (0.732)  
- \textit{irregular variance}: Good (0.254)  
- \textit{seas/irr cross-correlation}: Bad (0.031)

For each of three tests three different results are possible. "Bad" means that the test statistics is significant at 1% level, "Uncertain" means that the test statistics is significant at 5% level and "Good" means that the test statistics is not significant at 5% level. Uncertain or bad results for \textit{seas variance} may evidence over/under adjustment.

Additional information is available in three subsections: \textit{Charts}, \textit{Table} and \textit{S-I ratio}. In \textit{Charts} section the user will find:

- the original series with forecasts;
- the final seasonally adjusted series;
- the final trend with forecasts;
- the final seasonal component with forecasts;
- the final irregular component;
- the final seasonal component with forecasts.

The same time series are presented in \textit{Table} section. The final estimation of the seasonal-irregular component and final seasonal factors are presented in the \textit{S-I ratio} chart.
4.4.2.2.2. Pre-processing (Tramo)

Pre-processing section is organized in the similar way in TramoSeats and X12. For details refer to RegArima description (4.3.2.1.2). Major differences between methods concern mostly Arima section. Be aware that at Demetra+ version 1.0.3 doesn’t include all Tramo diagnostic, e.g. the out-of-sample forecasting test.

Pre-adjustment series

The table presented in this section includes:

- series corrected by Tramo, i.e.:
  - interpolated series;
  - linearized series;
  - series corrected for calendar effects (if calendar effects are not specified, series corrected for calendar effects is the same as interpolated series);

- deterministic effects detected and estimated by Tramo, i.e.:
  - deterministic component;
  - calendar effects;
  - trading days effect;
  - moving holidays effect;
  - outliers effect on trend component;

For particular time series the pre-processing table results includes only those deterministic effects that have been detected during estimation.
- outliers effect on irregular component;
- total outliers effect;
- regression effect on seasonally adjusted series;
- regression effect on the trend component;
- regression effect on the irregular component;
- regression effect on the seasonal component;
- separate regression effect;  
- total regression effect.

### Arima

Arima section shows the theoretical pseudo-spectrum of the Arima model estimated on the series. The blue line represents Arima model identified by Tramo. If this model has been changed by Seats, a second line in magenta, corresponding to the new Arima model, is overlapped. In the bottom part the Arima model used by RegArima is presented using symbolic notation \((P,D,Q)(PB, DB, QB)\). Estimated coefficients of parameters (regular and seasonal AR and MA) are shown in closed form (i.e. using the backshift operator \(B\)).

---

87 Regression effect in a separate component is an effect of user-defined variable that hasn’t been assigned to any component of the time series.

88 Backshift operator \(B\) is defined as: \(B^t_y = y_{t-1}\). It is used to denote lagged series.
In this part for each regular AR root (i.e. the solution of the characteristic equation, see the Annex, section 2A) the argument and modulus are also reported (if present, i.e. if \( P > 0 \)). The regular AR roots will generate a stationary contribution to the components. rules used by Seats to assign the roots to the components are given in the Annex, section 2A.

The example below presents information about the Arima model identified by Tramo. Two complex roots of AR polynomial have been assigned to the seasonal component (Demetra+ always highlights such AR roots). The last root is real and positive with high module value so it has been assigned to the trend.
If the Arima model has been changed by Seats, Demetra+ displays appropriate information\textsuperscript{89}.

\textbf{Tramo model [3,1,1](0,1,1)}

\textbf{Polynomials}

- regular AR: 1 - 0.33002 B - 0.12418 B^2 - 0.27291 B^3
- seasonal AR: 1
- regular MA: 1 - 0.39433 B
- seasonal MA: 1 - 0.27336 S

\textbf{Regular AR inverse roots}

- argument=2.050, modulus=0.566 (seasonal frequency)
- argument=2.050, modulus=0.566 (seasonal frequency)
- argument=0.000, modulus=0.952

\textsuperscript{89} See Annex, section 2A for further information.
**Tramo model \([3,1,0][1,0,0]\)**

**Polynomials**

- regular AR: \(1 + 0.10248 B + 0.093918 B^2 - 0.49099 B^3\)
- seasonal AR: \(1 - 0.69516 S\)
- regular MA: 1
- seasonal MA: 1

**Regular AR inverse roots**

- argument=2.090, modulus=0.827 (seasonal frequency)
- argument=2.090, modulus=0.827 (seasonal frequency)
- argument=0.000, modulus=0.718

**Seats model \([3,1,0][0,1,1]\)**

**Polynomials**

- regular AR: \(1 + 0.11962 B + 0.094067 B^2 - 0.50213 B^3\)
- seasonal AR: 1
- regular MA: 1
- seasonal MA: 1 - 0.30078 S

**Regular AR inverse roots**

- argument=2.097, modulus=0.835 (seasonal frequency)
- argument=2.097, modulus=0.835 (seasonal frequency)
- argument=0.000, modulus=0.720
**Tremo model** \((3,1,1)(1,0,0)\)

**Polynomials**

- regular AR: \(1 + 0.32091 B + 0.11212 B^2 - 0.29417 B^3\)
- seasonal AR: \(1 - 0.6732 S\)
- regular MA: \(1 - 0.00046 B\)
- seasonal MA: \(1\)

**Regular AR inverse roots**

- argument=2.174, modulus=0.747 (td frequency)
- argument=-2.174, modulus=0.747 (td frequency)
- argument=0.000, modulus=0.527

**Seat model** \((3,1,1)(0,1,1)\)

**Polynomials**

- regular AR: \(1 + 0.36963 B + 0.15127 B^2 - 0.23365 B^3\)
- seasonal AR: \(1\)
- regular MA: \(1 - 0.00 B\)
- seasonal MA: \(1 - 0.20204 S\)

**Regular AR inverse roots**

- argument=2.175, modulus=0.721 (td frequency)
- argument=-2.175, modulus=0.721 (td frequency)
- argument=0.000, modulus=0.450

Standard **Copy/Print/Export** options are available for this chart.
Information presented in the *Arima* panel corresponds with the *Decomposition (Seats)* panel.

**Regressors and Residuals**

*Regressors* section presents all deterministic regressors used in Tramo part, including trading days variables, the leap year effect, the Easter effect, outliers, ramps, intervention variables and other user-defined variables.

In the next part the residuals, which are obtained after estimation of Arima model in Tramo, are presented both in the graph and the table. Analysis of the residuals consists of several tests and presentation of the residuals distribution.

### 4.4.2.2.3. Decomposition

Seats receives from Tramo the "linearized" series (original series corrected for the deterministic effects and missing observations). The decomposition made by Seats assumes that all components in time series - trend, seasonal and irregular - are orthogonal and can be expressed by Arima model\(^9\). Identification of the components requires that only irregular components

\(^9\) It is assumed that irregular component is a white noise variable, which means that it follows ARIMA \((0,0,0)(0,0,0)\) model.
include noise. Each model is presented in closed form (i.e. using the backshift operator B). In the main page of *Decomposition (Seats)* the following items are presented:

- Model - Arima model for the series;
- Trend - Arima model for the trend component of the series;
- Seasonal - Arima model for the seasonal component of the series;
- Transitory - Arima model for the transitory component of the series;
- Irregular - Arima model for the irregular component of the series.

The trend component captures the low-frequency variation of the series and displays a spectral peak at frequency 0. On the contrary, the seasonal component picks up the spectral peaks at seasonal frequencies and the irregular component captures white noise behavior. Transitory component, which is additional component estimated by Seats for some time series, can be seen as a noise-free, detrended and seasonally adjusted time series. This component captures highly transitory variation that is not white noise and should not be assigned to the seasonal component or trend component. It will capture spectral peaks at frequencies that are neither zero nor seasonal.\(^91\) The model for transitory component is stationary Arma model, with low-order MA components (order \(Q - P\), when \(Q > P\))\(^92\) and AR roots with small moduli that should not be included in the trend component or seasonal component.

The example of time series decomposition calculated by Seats is presented below. It can be seen that overall autoregressive polynomial has been factorized into polynomials assigned to the components according to the roots' frequencies. As an example, the model for trend is:

\[
(1 - 2B + B^2) \cdot (1) \cdot x_{\text{trend},t} = (1 + 0,059791B - 0,94021B^2) \cdot a_{\text{trend},t}\] with innovation variance 0,1454.

---


\(^{92}\) \(Q\) is an order of MA process, while \(P\) is an order of AR process.
Stochastic series

This part presents the table containing the following series produced by Seats:

- seasonally adjusted series;
- trend;
- seasonal component;
- irregular component (contains transitory component, if any).

Trend and seasonal component are extended with two years of forecasts. These forecasts are presented in the separate columns.
Model-based tests

Model-based tests concentrate on distribution of components, theoretical estimators and empirical estimates (stationary transformation). This node is divided into three sections.

- Variance

In this section the variances of the component innovations are displayed (variance of the component innovation, ("Component"), theoretical variances of the stationary transformation of the estimated components and empirical variances of the stationary transformation of the estimated components ("Estimate")) are displayed (see also section 4.3.2.2.1.).

Seats identifies the components assuming that except for irregular they are clean of noise. It implies that the variance of irregular is maximized on the contrary the trend-cycle and seasonal component are as stable as possible. In this section Demetra+ presents a table which compares the variance of the stationary transformation of the components innovation (second column) with variance of their theoretical estimators ("Estimator") and variance of their empirical (actually obtained) estimate ("Estimate").

<table>
<thead>
<tr>
<th>Component</th>
<th>Estimator</th>
<th>Estimate</th>
<th>PValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>trend</td>
<td>0.0490</td>
<td>0.0024</td>
<td>0.0021</td>
</tr>
<tr>
<td>sa</td>
<td>2.1384</td>
<td>1.9304</td>
<td>1.9814</td>
</tr>
<tr>
<td>seasonal</td>
<td>0.1730</td>
<td>0.0105</td>
<td>0.0081</td>
</tr>
<tr>
<td>transitory</td>
<td>0.5150</td>
<td>0.2604</td>
<td>0.2327</td>
</tr>
<tr>
<td>irregular</td>
<td>0.1366</td>
<td>0.0604</td>
<td>0.0581</td>
</tr>
</tbody>
</table>

It follows from properties of MMSE (Minimum Mean Square Error) estimation for the estimator that this estimator will always underestimate the component (estimators always have smaller variance than components). The size of underestimation depends on the particular model. The underestimation will be relatively large when the variance of the component is relatively small. It

---

93 The variance of the component innovation is the variance resulting from Arima model for this component.

means that, for example, the trend estimator always has a smaller variance than trend component and the ratio of the two variances get further away from one as the trend becomes more stable. Therefore, the more stochastic the trend is, the less will its variance be underestimated. On the other hand, the variation of a very stable trend will be extremely underestimated. It means that the trend estimator provides more stable trend than the one implied by the theoretical model.

For all components it is expected that:

- $\text{Var(Component)} > \text{Var(Estimator)}$;
- $\text{Var(Estimator)}$ is close to $\text{Var(Estimate)}$.

If for a given component, $\text{Var(Estimator)} >> \text{Var(Estimate)}$, then this component is underestimated. On the contrary, $\text{Var(Estimator)} << \text{Var(Estimate)}$, indicates the overestimation of the component.

In the last column of the table, p-values of the second over/under estimation tests are provided. Green p-value means "Good", yellow means "Uncertain" and red means "Bad".

If $\text{Var(Estimator)} > \text{Var(Estimate)}$ for a particular component then:

- p-values in red indicate strong underestimation of the component variance;
- p-values in yellow indicate mild underestimation of the component variance;
- p-values in green indicate no underestimation of the component variance.

If $\text{Var(Estimator)} < \text{Var(Estimate)}$ for a particular component then:

- p-values in red indicate strong overestimation of the component variance;
- p-values in yellow indicate mild overestimation of the component variance;
- p-values in green indicate no overestimation of the component variance.

98 From TramoSeats structure it can be shown that estimator will always underestimate the component. The amount of the underestimation depends on the particular model, as a rule, the relative underestimation will be large when the variance of the component is relatively small, MARAVALL, A. (1995).
99 The theoretical variance (Estimator) should be similar to the estimate actually obtained (Estimate). Large differences between the theoretical and empirical values would indicate misspecification of the overall model, MARAVALL, A. (1995).
• **Autocorrelation function**

The autocorrelation function (ACF)\(^{100}\) is a basic tool in the time domain analysis of a time series. For each component Demetra+ exhibits autocorrelations of stationary transformation of components, estimators and sample estimates. They are calculated from the first lag up to the seasonal lag. If the model is correct, the empirical estimate of the autocorrelation function should be close to the theoretical estimator autocorrelation function. For \(i\)th component the discrepancy between ACF function of the components and of the estimator can be substantial for small values of innovation variance \(Var(a_i)\). The more stable a component is the larger will be this discrepancy\(^{101}\).

For ACF functions Demetra+ presents the following tables:

### trend

<table>
<thead>
<tr>
<th>Lag</th>
<th>Component</th>
<th>Estimator</th>
<th>Estimate</th>
<th>PValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0002</td>
<td>0.4431</td>
<td>0.3811</td>
<td>0.1288</td>
</tr>
<tr>
<td>2</td>
<td>-0.4969</td>
<td>-0.1863</td>
<td>-0.2069</td>
<td>0.7770</td>
</tr>
<tr>
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<td>-0.0565</td>
<td>0.0621</td>
</tr>
<tr>
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<td>-0.1191</td>
<td>-0.0793</td>
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</tr>
<tr>
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<td>-0.0972</td>
<td>-0.2186</td>
<td>0.1521</td>
</tr>
<tr>
<td>6</td>
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<td>-0.0650</td>
<td>-0.6955</td>
<td>0.0940</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
<td>-0.0453</td>
<td>-0.0049</td>
<td>0.6417</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>-0.0278</td>
<td>-0.4589</td>
<td>0.7231</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>-0.0692</td>
<td>-0.0182</td>
<td>0.9101</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>-0.0016</td>
<td>-0.0077</td>
<td>0.9440</td>
</tr>
<tr>
<td>11</td>
<td>0.0000</td>
<td>-0.0687</td>
<td>-0.0782</td>
<td>0.9114</td>
</tr>
<tr>
<td>12</td>
<td>0.0000</td>
<td>-0.1275</td>
<td>0.0334</td>
<td>0.0359</td>
</tr>
</tbody>
</table>

### sa

<table>
<thead>
<tr>
<th>Lag</th>
<th>Component</th>
<th>Estimator</th>
<th>Estimate</th>
<th>PValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.4566</td>
<td>-0.4585</td>
<td>-0.4479</td>
<td>0.8110</td>
</tr>
<tr>
<td>2</td>
<td>-0.1685</td>
<td>-0.1575</td>
<td>-0.2381</td>
<td>0.3170</td>
</tr>
<tr>
<td>3</td>
<td>0.1171</td>
<td>0.1160</td>
<td>0.2280</td>
<td>0.1489</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0451</td>
<td>0.5541</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.1633</td>
<td>0.0551</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0430</td>
<td>0.5673</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0933</td>
<td>0.2642</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0995</td>
<td>0.2303</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
<td>-0.0134</td>
<td>0.0009</td>
<td>0.6032</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
<td>0.0182</td>
<td>0.1396</td>
<td>0.1425</td>
</tr>
<tr>
<td>11</td>
<td>0.0000</td>
<td>0.0529</td>
<td>-0.1771</td>
<td>0.0054</td>
</tr>
<tr>
<td>12</td>
<td>0.0000</td>
<td>-0.1154</td>
<td>0.0180</td>
<td>0.0726</td>
</tr>
</tbody>
</table>

---

100 See Annex, section 15A.

For each table the P-values of the test are given in the last column. The user should check whether the empirical estimates agree with the model, i.e. if their ACF functions are close to those of the model for estimator. Special attention should be given to first and/or seasonal order autocorrelation\textsuperscript{102}.

The coefficients of the autocorrelation function of the irregular component are always null ("Component" column) as a theoretical model for irregular component ($u_t$) is a white noise\textsuperscript{103}. However the irregular estimator follows the Arma model:

\[
\hat{u}_t = k_u \frac{1}{\psi(B)\psi(F)} x_t,
\]

where:

\textsuperscript{102} MARAVALL, A. (2000).

\textsuperscript{103} MARAVALL, A. (1987).
Estimator $\hat{u}_t$ is expressed as a linear function of present and future innovations. Therefore it is autocorrelated ("Estimator" column contains non-zero values).

### Meaning of the p-value for autocorrelation tests

<table>
<thead>
<tr>
<th>Value</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>no evidence for autocorrelation</td>
</tr>
<tr>
<td>Uncertain</td>
<td>a mild evidence for autocorrelation</td>
</tr>
<tr>
<td>Bad</td>
<td>strong evidence for autocorrelation</td>
</tr>
</tbody>
</table>

It should be stressed that this test gives no information about the direction of autocorrelation.

Comparison of the theoretical MMSE estimators with the estimates actually calculated can be used as a diagnostic tool. The closeness between estimators and estimates points towards validation of the results\(^{104}\).

- **Cross-correlation function**

The decomposition made by Seats assumes orthogonal components. To test this assumption, Demetra+ presents a table that contains cross-correlations between the stationary transformations of the components, the estimators and the actual estimates (theoretical components are uncorrelated). A table containing these correlations is presented (they refer to: trend and seasonal, trend and irregular, seasonal and irregular and, if the transitory is present, trend and transitory, seasonal and transitory, irregular and transitory.

Although components of the time series are assumed to be uncorrelated, their estimators can be correlated as estimator variance will always underestimate the component variance. MMSE estimator implies correlation between the estimators of the components\(^{105}\). For this reason correlations between the stationary transformations of the estimators and of the estimates actually obtained should be checked\(^{106}\).

The last column (PValue) in the table below displays the results of the test for no correlations between components. The outcome of the test is signalized by the color of the p-value (see table above). In the example below, PValues are green, which indicates that all discrepancies in the

---


correlations are negligible, so, for each component, estimator and estimate provide similar results.

**Cross-correlation**

<table>
<thead>
<tr>
<th>Component</th>
<th>Estimator</th>
<th>Estimate</th>
<th>PValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>trend/seasonal</td>
<td>-0.1513</td>
<td>-0.2073</td>
<td>0.5748</td>
</tr>
<tr>
<td>trend/transitory</td>
<td>-0.4470</td>
<td>-0.3754</td>
<td>0.4813</td>
</tr>
<tr>
<td>seasonal/transitory</td>
<td>0.0606</td>
<td>0.0138</td>
<td>0.6115</td>
</tr>
<tr>
<td>trend/irregular</td>
<td>-0.1025</td>
<td>-0.0494</td>
<td>0.4341</td>
</tr>
<tr>
<td>seasonal/irregular</td>
<td>0.0586</td>
<td>0.0268</td>
<td>0.4827</td>
</tr>
<tr>
<td>transitory/irregular</td>
<td>0.6499</td>
<td>0.8751</td>
<td>0.4228</td>
</tr>
</tbody>
</table>

It is expected that the theoretical cross-correlations between the component estimators will be close to their sample estimates\(^\text{107}\).

**Wiener-Kolmogorow analysis**

Wiener-Kolmogorow analysis concerns results obtained by Seats and concentrates on\(^\text{108}\):

- Components (spectrum, ACF);
- Final estimators (spectrum, square gain function, WK filters, ACF, PsiE-weights);
- Preliminary estimators (Frequency response (square gain function, phase effect), WK filter, ACF);


• Revision analysis (total error, revision error).

This section presents various graphs concerning components of time series. As a rule, dark blue color indicates seasonally adjusted time series, navy blue indicates seasonal component, red indicates trend-cycle, green indicates transitory and pink indicates irregular component.

• Components

This section presents the (pseudo)spectra of the particular components. The sum of the components spectra should be equal to the spectrum for the observed time series, which is presented in the Pre-processing (Tramo) part (if the Tramo model has been accepted by Seats, the figure displays one spectra, otherwise spectra of the model chosen by Seats is visible). A seasonally adjusted series spectra (dark blue) is sum of trend-cycle component spectra (red), transitory component spectra (green), if present, and irregular component spectra (pink).

Since \( a_i \) generates the stochastic variability in the \( i-th \) component, small values of \( V(a_i) \) are associated with stable component, large values of \( V(a_i) \) with unstable component. The spectrum of the \( i-th \) component is proportional to \( V(a_i) \). Further, stable trend and seasonal components are those with thin spectral peaks while unstable ones are characterized by wide spectral peaks. For monthly time series there are six seasonal frequencies, \( \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi \), while for quarterly data there are two seasonal frequencies: \( \frac{\pi}{2}, \pi \). Spectrum for seasonal component has peaks around these frequencies.

On the figure below a standard spectral trend-cycle (red), seasonal (light blue), transitory (green) and irregular (pink) decomposition is displayed. The peak in the transitory component is an evidence of trading days effect.
Second panel shows ACF function of stationary components. They are theoretical values (i.e. they are not computed on the data).

For both graphs Copy/Copy model/Print/Export options are available.
The `Copy model` option enables to copy basic information about models for each component (innovation variance, AR, MA and differencing orders, the estimated values of the coefficients). The example is shown below.

<table>
<thead>
<tr>
<th>Trend-Cycle</th>
<th>Innovation variance: 0.053214661</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1,049791 - 0,95022</td>
</tr>
<tr>
<td>MA</td>
<td>1,013409 - 0.986592</td>
</tr>
<tr>
<td>Seasonally adjusted</td>
<td>Innovation variance: 0.3905031101</td>
</tr>
<tr>
<td>AR</td>
<td>1,0301499 - 0.362904</td>
</tr>
<tr>
<td>D</td>
<td>1,013409 - 0.986592</td>
</tr>
<tr>
<td>MA</td>
<td>1,013409 - 0.986592</td>
</tr>
<tr>
<td>Seasonal</td>
<td>Innovation variance: 0.089451056</td>
</tr>
<tr>
<td>AR</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1,457698 - 1,14489 - 1,116565 - 1,100626 - 0,851992</td>
</tr>
<tr>
<td>MA</td>
<td>0,633349 - 0,428592 - 0,279669</td>
</tr>
<tr>
<td>Transitory</td>
<td>Innovation variance: 0.003574362</td>
</tr>
<tr>
<td>AR</td>
<td>1,0301499 - 0.362904</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>MA</td>
<td>1,067609 - 0.30359</td>
</tr>
<tr>
<td>Irregular</td>
<td>Innovation variance: 0.152976052</td>
</tr>
<tr>
<td>AR</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>MA</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Final estimators**

- Main results
- Pre-processing (Tiamo)
- Decomposition (Seatsa)
  - Stochastic series
  - Model-based tests
  - WK analysis
    - Components
      - Final estimators
      - Preliminary estimators
      - Errors analysis
- Diagnostics
TramoSeats uses filters to compute the values of different time series components. The convergence of these symmetric filters requires that for each observation the past and future observations exist. Obviously, they are not available at the beginning and end of the time series. Hence, one needs to extend the time series at both ends (calculate forecasts and backcasts) to be able to use the filter. This is done by using the Arima model, which has been chosen in the Tramo phase of seasonal adjustment. Then, Seats applies the filter to extended series (CLEVELAND, W. P., and TIAO, C. G., 1976).

Regarding to the importance of final (or historical) estimators derived applying the WK filters (that are bi-infinite and symmetric filters), Demetra+ presents several graphics showing their properties (see the Annex, section 2A). The corresponding graphs for components and for final estimators of the components vary, as components and final estimators follow different models. For example, the seasonal component follows the model: \( \phi_s (B)s_t = \theta_s (B)a_{ts} \), while MMSE estimator of seasonal component follows model: \( \hat{\phi}_s (B) \hat{s}_t = \hat{\theta}_s (B)\hat{\alpha}_s(F)\hat{a}_t \), where \( \hat{\alpha}_s(F) \) is a difference between the theoretical component and the estimator\(^{109}\). These graphics are listed below. For each graph Copy/Print/Export options are available.

- **Spectrum of final estimators**

The shape of the spectrum of the final estimators is shown in the first graph. Spectrum of estimator of the seasonal component is obtained by multiplying squared gain of the filter by spectrum of the linearized series\(^{110}\).

From the example below it is clear that these spectra are similar to those of the components, although estimator spectra show spectral zeros at the frequencies where the other components have spectral peaks. Estimator adapts to the structure of the analyzed series, i.e., the width of the spectral holes in seasonally adjusted series (dark blue line) depends on the width of the seasonal peaks in the seasonal component estimator spectrum (navy blue lines)\(^{111}\).

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110 See the Annex, section 2A.
**Square gain of components filter**

The squared gain controls the extent to which a movement of particular amplitude at a frequency $\omega$ is delivered to the output series\(^{112}\). It determines how the variance of the series contributes to the variance of the component for the different frequencies. In other words, it specifies which frequencies will contribute to the signal (that is, it filters the spectrum of the series by frequencies)\(^{113}\). If squared gain is zero in band $[\omega_1, \omega_2]$ it means that the output series is free of movements in this range of frequencies\(^{114}\). On the contrary, if for some $\omega$ square gain is 1, then all variation is passed on to the component estimator.

The figure below points out that seasonal frequencies are assigned to the seasonal component while the seasonally adjusted series captures the variance of the non-seasonal part of the series. As a consequence, it is expected that seasonal component estimator captures only the seasonal frequencies, so its peaks assume unitary values at the latter frequencies. On the contrary, estimator of the nonseasonal part of the time series is expected to eliminate seasonal frequencies, leaving unmodified non-seasonal frequencies. Therefore, squared gain of seasonally adjusted data should be nearly zero for seasonal frequencies.

The squared gain depends on the model for the time series. In the next two figures, squared gains derived from two different models are represented. In the first graph, the squared gain of the seasonal adjustment filter shows large troughs to suppress highly stochastic seasonal component.

---


\(^{113}\) Squared gain definition is given in the Annex, section 2A.

- **WK filter**

Wiener-Kolmogorow filter $V_i(B, F)$ shows the weights that have been applied to the original series $x_t$ to extract the $i-th$ component $x_{it}$ in the following way (see the Annex, section 2A for description of the WK filter):

$$x_{it}^* = V_i(B, F) x_{t,i}$$

where:

$$V(B, F) = v_0 + \sum_{j=1}^{\infty} v_j (B^j + F^j).$$
Since WK filters are symmetric and centered, it is also convergent which enable to approximate infinite number of realization $x_i$ by finite number of terms (from the graph below it could be noticed that $j = 36$, so the WK filter includes $36 + 1 + 36 = 73$ terms). In order to apply filter to all observations from $x_i$, original time series is extended with forecasts and backcasts using Arima model. As new observation (i.e. observation for period $t + 1$) is available, forecast for period $t + 1$ is replaced by this new observation and all forecasts for periods $g > t + 1$ are updated. It means that in the end of time series estimator of the component is preliminary and is a subject of revisions, while in the central periods estimator can be treated as final (also called "historical" estimator)\textsuperscript{115}.

Since WK filters are symmetric and convergent, they are valid for computing the estimators in the central periods of the sample. The following graph demonstrates these features. The chart shows weights that are applied to the each observation for each component (weights applied to seasonally adjusted series are dark blue, to trend-cycle are red, to transitory component are green, and to irregular component are pink).

\textsuperscript{115} MARAVALL, A. (2011).
- **Auto Correlation Function**

The window *ACF (stationary)* displays the autocorrelation functions of the final estimators of the stationary components. The following graph represents an example.

![Graph showing autocorrelation function](image)

- **PsiE-weights**

PsiE-weights (ψ) are a different representation of the final estimator, i.e. this representation shows estimator as a filter applied to the innovation $a_t$, rather than on the series $x_t$. For each component figure below presents how the contribution of total innovation to component estimator $\hat{x}_t$ varies in time (the size of this contribution is shown in Y-axis). For observations $t \geq 0$ (X-axis) PsiE-weights show the effect of starting conditions, present and past innovations in series, while for observations $t < 0$ they present the effect of future innovations. It can be seen that they are non-convergent in the past (they are convergent when series $x_t$ is stationary). On the contrary, the effect of future innovations is a zero-mean and convergent process. PsiE-weights are important to analyze convergence of estimators and revision errors.

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116 See the Annex, section 15A.

117 See the Annex, section 2A. For further details see MARAVALL, A. (2008).
In this part different types of graphics are presented showing properties of preliminary estimators (estimated by WK filter) of each theoretical component. Preliminary estimators are obtained by replacing observations not yet available with forecasts and extending series with backcasts. Both forecasts and backcasts are obtained from Arima model. Then filter is applied to the extended series. By default, lag is set by default to zero, so the semi-infinite concurrent estimators \( \hat{x}_{t+j} \) are considered. User can set a different lag, from 1 up to 60, and therefore consider semi-infinite preliminary estimators \( \hat{x}_{t+j} \).

118 See the Annex, section 2A.
In this part different types of graphics, which show properties of preliminary estimators (estimated by WK filter) of each theoretical components, are presented. For each graph Copy/Copy model/Print/Export options are available. The graphs include:

- "Frequency response" window contains two graphics, i.e. the squared gain function and the phase effect.

Squared gain of preliminary estimators filter determines which frequencies will contribute to the component (that is, it filters the spectrum of the series by frequencies (see aforementioned description of the squared gain).

The phase effect graphics shows the phase delay in concurrent estimator of the seasonally adjusted series (or trend-cycle). It means that the phase effect indicates how frequency components are shifted in time by the filter, so it measures the difference in detecting turning points between original and seasonally adjusted data (or trend-cycle) in terms of period (month or quarter). The phase effect function is calculated separately for seasonally adjusted series (red line) and trend-cycle (blue line). As a rule, phase effect has a positive value, which means that seasonally adjusted series (and trend-cycle) shows turning points later than original time series. This delay is undesired featured of seasonally adjusted time series and is regarded as a drawback by statisticians who use seasonally adjusted data for modelling and forecasting. For this reason it is expected that phase delay is zero (or nearly zero). The phase effect is measured in number of periods (vertical axis). Horizontal axis presents range of frequencies of cyclical interests. Frequencies close to 0 indicate long-term trend, while \( \frac{\pi}{12} \) is a 2-year cycle. Hence, for monthly time series presented below, in seasonally adjusted data estimator induces high phase delay for the long-term and
short term cycle (approximately 3 months), while for the 2-year cycle phase delay is a bit smaller (2 months).

WK filters. The use of forecasts by preliminary component estimator will imply one-sided (asymmetric) WK filter (see aforementioned description of the WK filter), and because of that, will be adversely affected by phase effect.

The graph below presents WK filter weights for lag=12 (i.e. for the observation $x_{t-12}$ where $t$ is the last available observation. It can be noticed that in this case WK filter uses
both observations $x_{i-12}, i = 1, 2, ..., 48$ and observations $x_{i+1}, i = 1, 2, ..., 12$ to calculate preliminary component estimator).

- ACF (stationary) represents autocorrelation functions of the preliminary estimators of the stationary components. For the preliminary estimators of the stationary components Demetra+ calculates ACF for lag 0 to 60. The preliminary estimators imply the use of asymmetric filters, while when lag=0 the preliminary estimator is the concurrent one and it is obtained with a one-sided filter. The ACF profiles of preliminary estimators when lag=60 (preliminary estimators approach the final ones), i.e. $x_{60+60}$, are very close to the profiles of the ACF of final estimators. When the lag approaches 0, they differ more.
- **Errors analysis**

An error analysis is performed in last WK analysis node. Formulas for estimation errors are included in the Annex (section 2A). For each graph Copy/Print/Export options are available.

- **Total estimation errors**

This panel reports the standard deviation of the total estimation error in the estimator of the components (trend-cycle, seasonally adjusted series, seasonal and irregular) for the last 36 observed periods of a time series. The total estimation error is the largest for the last observation (concurrent estimators (lag = 0)) and it decreases (for preliminary estimators) until it reaches a constant value. This constant value is a standard deviation of the historic estimator. In the graph presented below total estimation error for historic estimator is around 0.4.
This graph is useful for judgement how many periods it takes for a new observation to no longer significantly affect the estimate.

Revision error is the difference between preliminary and final estimator. For each component the graph shows the percentage reduction in the standard error of the revision after additional periods (up to 36 observations). Comparisons are made with concurrent estimators. This graph gives information about the time needed by the concurrent estimators to converge to the final ones. As stressed in Maravall (1995), large revisions are associated to the highly stochastic components and converge fast, while smaller revisions are implied by the very stable components and converge slowly.

The graph below shows how, starting with concurrent estimation, the percentage of the variance of the concurrent estimator future revision will be removed from the successive preliminary estimators as new observations become available. The X-axis presents the number of additional periods observed, and the Y-axis shows the percent of the revision variance that has been removed after including the number of observations presented in the X-axis in the WK filter.

For the particular time series presented on the graph below, after one year of additional data (12 observations) the percentage reduction in the standard error of the trend revision was approximately 80% (60% for seasonally adjusted series). The trend estimator converges faster than that of the seasonally adjusted series because trend component was stochastic while seasonal component was rather stable\textsuperscript{119}. After 3 years (36 observations) all estimators have practically converged (estimators are close to 100%).

\textsuperscript{119} MARAVALL, A. (1996).
4.4.2.2.4. Diagnostics

For TramoSeats Demetra+ calculates the following statistics:

- Friedman test;
- Kruskal-Wallis test;
- Test for the presence of seasonality assuming stability;
- Evaluative seasonal test;
- Residual seasonality test;
- Combined seasonality test.

In the Diagnostic section the user will find also Spectral analysis, Revisions history, Sliding spans and Model stability. For details refer to the description of the seasonal adjustment results for X12 and to the Annex (section 3A). Description of the results and options available in Diagnostic section are presented in 4.4.2.1.4.
4.5. Multi-processing

Multi-processing specification is designed for quick and efficient seasonal adjustment of large data sets. Multi-processing specifications that mix different seasonal adjustment methods and different specifications are enabled. The software provides two different ways to perform multi-processing. The first solution is based on the "active" specification; in this solution, the series that are subject of a multi-processing are automatically associated with the "active" specification. The second solution consists in using a wizard, which allows the users to associate series and specifications step by step. Both functions are activated from the main menu.

4.5.1. Defining a multi-processing

1. Creation of a new multi-processing

This option opens the following window.

The user should first activate the specification and then drag and drop the time series from the Browsers panel into this window. We recall that the active specification (see 4.4.1) can be selected in the workspace through a local menu; it can be either a pre-defined specification or a user-defined one. If there is no active specification in the Workspace panel, the user is unable to drag and drop time series into specification window.

The user can change initial choice of the active specification and choose other specification for next set of series. This option enables to launch the seasonal adjustment for one time series using
different specifications in order to compare the results. The picture below presents multi-
processing in which four different specifications \{TS[RSA3], TS[RSA5], X12[RSA3], X12[RSA5c]\} has
been used.

The multi-processing is launched by means of the Run command under the SAProcessing-1 main
menu item.

The user can also launch the seasonal adjustment of the time series by clicking on its name on the
list in the multi-processing window ("Series" column).

2. **Creation of a multi-processing via wizard**

When the user activates the wizard, the empty window is displayed. The wizard guides the user
through the construction of the associations "series-specifications". It also gives him the
possibility to define and to use specifications that don’t belong to the workspace.
Consecutive steps are similar to those which were described in single seasonal adjustment part. However, there are two main differences.

First of all, in the first panel the user can choose more than one time series and drop them into Selection window.

Then, the user should decide which seasonal adjustment method - X12 or TramoSeats - will be used (Choose the method panel). After that the user can choose existing specification or create new specification as it was shown in 4.1 and 4.2.

Next, in the Add items panel Demetra+ presents time series which will be added to the list of items in the multi-processing. Add items part is not about adding time series to the regression part of the pre-adjustment model but aims to present the user the list of time series which have
been chosen in the first step. It is not possible to add new time series to the multi-processing here.

![Multi-processing definition wizard]

At the last stage of the wizard ("Finishing") the user can modify the name of the multi-processing (SAProcessing-xx, default); one can also add the multi-processing to the workspace, for future re-use and the user can decide if the execution is automatically started (the default) when the wizard is closed.

![Multi-processing definition wizard]

It should be mentioned that the user can go back to the first step of the wizard at any time, if one wants to add other series with other specifications.
4.5.2. Seasonal adjustment results - multi-processing

4.5.2.1. Generalities

The outcome of the multi-processing is presented in the window which contains three panels.

The first panel - *Processing* - gives an overview of the processing of each series and more especially of the diagnostics computed by Demetra+ on its seasonal adjustment. Some warnings can also be put forward, for short series, non-decomposable models (Seats) or when the differenced series doesn’t show seasonal peaks. Information on those warnings is displayed by a tooltip on the series. The user can sort the multi-processing by clicking a column header. The example is shown below:

By clicking on the name of the time series a summary of the tests results is displayed in the right panel. For the description of those tests refer to Chapter 4.3.2.1. At the bottom of the *Processing* panel the graph of final seasonally adjusted series and raw series is displayed.
The Summary panel gives general information on the results of the adjustment. The report is organised into sections corresponding to SA methods and frequencies (monthly and quarterly). The example below shows that TramoSeats method has been chosen for four time series. Three of them have been logarithmically transformed. The list of the Arima models shows the model parameters used in time series set. There were 28 outliers detected, the majority of which were additive outliers. Calendar effects haven’t been detected for any of the time series seasonally adjusted using TramoSeats method.
Last section – *Matrix view* panel – provides information similar to the matrix output of TSW (TramoSeats for the Windows program).

The summary information is divided into five folds available in the right side of the panel:

- **Main** – contains main statistical properties of the Arima model used in pre-processing step;
- **Calendar** – presents calendar specification results;
- **Outliers** – outlier structure of each series and coefficients of Arima model and their significance levels;
- **Arima** – the values of the parameters and their t-stat values;
- **Tests** – p-values of different tests computed on the residuals and with other information (annual discrepancies between raw and adjusted data, spectral visual peaks).

Main *Matrix view* panel is presented below:
The matrices can be copied by the usual keys combination (Ctrl+C), and used in other software, e.g. Excel.

### 4.5.2.2. Multi-processing menu

Multi-processing menu offers the following options:

**Run** – runs the defined multi-processing seasonal adjustment.

**Update reports** – updates the processing after changes in seasonal adjustment specifications.

**Refresh** – refreshing a processing with new data.

**Edit** – allows to add the new times series to the list (using multi-processing wizard) and pasting previously removed time series again in the list. Last three edit options: **Cut**, **Copy** and **Delete** are active if the time series was marked on the list (see description below).

**Priority** – indicator that can be used to mark series that require more or less attention. Priorities take values from 0 to 10. Demetra+ computes them automatically, based on the average of the (logged) series. The user can chose the method of computation (log-based or level based).

**Save** – saves the processing.

**Generate output** – offers a set of output formats (TXT, XLS, ODBC, CSV, CSV matrix), the choice of the folder that will contain the results (in the example below the file will be saved on disk C:\Documents and Settings...) and the content of the exported file (see 3.2.3).

**Add to workspace** – adds the multi-processing to the workspace tree.

**Initial order** – displays times series on the list in initial order. The option restores the initial order if the list has been sorted by given column (e.g. by quality or method).
After defining a multi-processing the user should execute the estimation using Run option. After that it is possible to activate the Generate output option. The Save option is inactive as soon as the user adds (the Add option) the processing to the Workspace panel. Once the output was created, the user can save the multi-processing. The appropriate item will appear in the workspace tree.

The user can add new time series to the multi-processing, using Edit → Add items option.

Option Edit → Paste enables to add to the existing multi-processing a new time series directly from external source (e.g. Excel). Before choosing this option the user should copy the time series (data, name of the time series and dates). Otherwise the following message is displayed:
4.5.2.3. Detailed results and modification of the specification

For each time series that belong to the multi-processing specification Demetra+ offers the access to the complete description of the results by a double click on the name of the time series. This option is available for both Processing and Matrix view panels. The user is allowed to modify the specification by changing the options in the left part of the window. This option could be useful in case the quality of a specific processing is low and the user wishes to modify some options to get a better result.

As an example, the following panel shows how to change the pre-specified outliers.

When the new options are chosen, the user should click on Apply button to launch the seasonal adjustment with modified settings.
The user can save the new settings and results using **Save** button. The multi-processing will contain then the modified specification for that series. Otherwise, the user can come back to the previous settings using **Restore** button.

![Save and Restore buttons](image)

It is not necessary to close the *Detailed results* window to get information on another series; that window is updated by a simple click on another series of the multi-processing view.

Creation of the separate single-processing from a multi-processing document is also possible by dragging the corresponding item from the series column to the central panel of Demetra+.

![Detail results window](image)

Demetra+ allows the user to accept the models for which the quality wasn’t satisfactory. If the user clicks on the **Accept** option (local menu), Demetra+ changes the message displayed in *Quality* column into *Accepted*.

![Accept option](image)
For X12 method it is possible to assign different seasonal filter to each period using option *Mixed* in specification window. It is done in a two-step procedure. First, the time series should be seasonally adjusted using the same seasonal filter for every period. Once seasonal adjustment has been executed, the user is able to modify settings for seasonal filter and change the filter that will be used for estimating seasonal component for each period. To do it **Seasonal filter** should be set to *Mixed*, then the user chooses **Details on seasonal filter**. Finally, the user should specify seasonal filter for each period.
This option could be useful if for some periods the seasonal pattern changes faster/slower than for the others. The evaluation can be made using S-I ratio chart.

If for the particular time series the multi-processing hasn’t been executed yet, option Mixed is not available, as Demetra+ needs information about time series frequency. In such case Demetra+ displays the following warning:

The Mixed option is unavailable for single-processing.
4.5.2.4. Assigning priority to the series

Priorities are simple indicators (from 0 to 10) that users can use to mark series that require more or less attention. The software is able to compute automatically priorities based on the average of the (logged) series. By default priority is not calculated.

The user can calculate it automatically by choosing one of the Priority options available in SAProcessing menu.

Priorities will be added to the SAProcessing output window.
For particular time series the user can modify *Priority* value manually by clicking on the time series name in **SAProcessing** window and choosing *Priority* value from the list (from 1 to 10).
4.5.3. Period-to-period data production

Multi-processing is designed for regular production (month-to-month or quarter-to-quarter) of the seasonally adjusted data. This process consists of several steps. To start with, the multi-processing should be created, add to workspace and saved. Then, while new observations are available, the multi-processing can be refreshed. The final step is to generate the output. Those steps are discussed in the next sections.

When the multi-processing had been created, the user should add it to the workspace and then saved it using the options from multi-processing menu. Then one can use this multi-processing for regular data production (month-to-month or quarter-to-quarter).

The multi-processing defined for period-to-period data production should use the data from the browsers, i.e.:

In this case Demetra+ saves the location of the file from which the data come from.

If the variables in multi-processing come directly from external source (e.g. they are copied from Excel and pasted directly into SAProcessing window), it won’t be possible to update the processing. Such variables are static, so their location is not saved by Demetra+. Data can be copied from Excel and pasted into Demetra+ multiprocessing window using Copy/Paste options. The variables will not be added into the Browsers panel (see picture below).
4.5.3.1. Saving the workspace

By default, the multi-processings generated through the so-called "short-ways" are not put in the current workspace. To be able to save and to refresh them, the user must first add them to the workspace. That can be done, for instance, through the main menu "SAProcessingXXX → Add to Workspace".

The user still has to save the workspace, using the usual menu command Save from the Workspace menu.
When Demetra+ is re-opened, it will automatically open at the last used workspace. The software also maintains a list of the most recently used workspaces, which can be easily accessed.

A saved item of a workspace can be opened by a double click on it or by its local menu. It is then showed in its previous state.
4.5.3.2. Refreshing of the processing

When the multi-processing had been created, added it to the workspace and saved, it can be used for regular data production. This process should be conducted in the following way:

1. Update the time series in the external file or source from which the variables come from (e.g. update the file "data.xls" with the new observations but don’t change neither the file name nor its location).

2. Start Demetra+.

3. Choose the multi-processing from the workspace tree by double-clicking on it.

4. Choose in which way you would like to refresh the results.

5. Confirm that you want to refresh the data.

120 Description of the refreshing options is presented further in this section. For clarification of the revision policies refer to the ‘Guidelines on Seasonal Adjustment’, (2007).
6. Choose the option **Generate output** from the menu.

7. Choose the output options (see 3.2.3) and click **OK**.

8. Demetra+ creates the file with the output in the location specified in the **Folder**. When neither location nor the file name have been changed since previous execution, the old
version of the file (e.g. filed created in the previous period) will be replaced by the new version.

Detailed aspects of saving the results in external files are discussed in section 4.5.3.3.

Demetra+ proposes several options to refresh the results:

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current adjustment (partial)</td>
<td>Re-estimation of the coefficients of the RegArima model.</td>
</tr>
<tr>
<td>Partial concurrent adjustment</td>
<td>Only the Arima model parameters are refreshed. The order of the Arima (p,d,q)(P,D,Q) is unchanged.</td>
</tr>
<tr>
<td>→ Parameters</td>
<td></td>
</tr>
<tr>
<td>Partial concurrent adjustment</td>
<td>The outliers and the Arima model parameters are re-estimated.</td>
</tr>
<tr>
<td>→ All outliers (+ params)</td>
<td></td>
</tr>
<tr>
<td>Partial concurrent adjustment</td>
<td>The outliers on the last year and the Arima model parameters are re-estimated.</td>
</tr>
<tr>
<td>→ Last outliers (+params)</td>
<td></td>
</tr>
<tr>
<td>Partial concurrent adjustment</td>
<td>The RegArima model is completely re-estimated.</td>
</tr>
<tr>
<td>→ Arima and outliers (+params)</td>
<td></td>
</tr>
<tr>
<td>Concurrent adjustment</td>
<td>The RegArima model and filters are completely re-identified and the respective parameters and factors are re-estimated.</td>
</tr>
</tbody>
</table>

When the refresh option has been selected, Demetra+ automatically goes to the suitable time series provider(s) to ask for the updated observations; the new estimations are done on these series (using the previous models, modified by the chosen option).

The example below presents results obtained by applying option "Last outliers (+params)". Outliers are divided into two sections: pre-defined outliers (outliers detected during penultimate execution of the multi-processing) and detected outliers (outliers identified in span \([t_{s-k},t_{s+n}]\), where \(s\) is the number of last available observations during penultimate execution of the multi-processing, \(n\) is the number of observations added to the revised time series, \(k\) is the frequency of the time series (for quarterly series \(k = 4\), for monthly time series \(k = 12\)).
In majority of cases multi-processing is defined by choosing a rather general specification with numerous "free" options that will be used for the series. This specification is called the "reference specification".

If the results are not acceptable for some series, the user will modify the reference specification to achieve a better adjustment (for example by forcing the use of calendar variables). In such a case, the reference for the considered series becomes the specification that has been manually improved.

When a series is processed, its estimation produces a fully identified specification, which is called a "point specification" (in the sense that it corresponds to a unique model). For each series of a multi-processing, the software stores the reference specification and the point specification (in an XML file).

When the user wants to refresh a processing, one has to define for the updated series the specifications (called estimation specification) that will be used. Following the refreshing option, Demetra+ removes some constraints of the point specification, in the limits of the reference specification. For example, when the "All outliers" option is selected, any automatically identified outliers is removed and the automatic outliers identification option of the reference specification is used. If that reference specification doesn't allow automatic outliers identification, the estimation specification will not allow either. Without such an approach, it would be difficult to define exactly the specification that should be used when a processing is refreshed.
The list of the constraints that are removed from the point specifications following the refreshing options is presented below (the second column should be interpreted as a cumulative list).

<table>
<thead>
<tr>
<th>Option</th>
<th>Removed constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current adjustment (partial)</td>
<td>Re-estimation of the coefficients of the regression model.</td>
</tr>
<tr>
<td>Partial concurrent adjustment → Parameters</td>
<td>Re-estimation of the parameters of the RegArima model.</td>
</tr>
<tr>
<td>Partial concurrent adjustment → Last outliers (+parameters)</td>
<td>Re-estimation of the outliers of the last year only.</td>
</tr>
<tr>
<td>Partial concurrent adjustment → All outliers (+parameters)</td>
<td>Re-estimation of all the outliers.</td>
</tr>
<tr>
<td>Partial concurrent adjustment → Arima and outliers (+parameters)</td>
<td>Re-estimation of the whole RegArima model.</td>
</tr>
<tr>
<td>Concurrent adjustment</td>
<td>The reference spec is used.</td>
</tr>
</tbody>
</table>

Considering the way Demetra+ works, it is clear that the reference specification should be chosen as general as possible. Otherwise, the refreshing options could be to a large extent useless.

The user can change the specification of a series at any time. What they change manually is actually the reference specification, which is used for the current processing, but also for future re-estimations.

Considering the design of Demetra+, the convenient way for processing many series in a recurrent production context might be as follows:

1. Chosen a large specification that will be applied on the set of series.
2. Modify the specifications that produce bad results. Try to minimize the restrictions you impose on the specification.
3. Save the processing, which is the basis for all the next steps.
4. During one year, refresh for each new period the processing without changing too much the model (typically, use the "Parameters" option). Modify with caution any unacceptable results that could be generated by the new data (new outliers...).
5. After one year, carry out a more serious revision (for instance "All outliers").
4.5.3.3. Sending the results to external devices

When the multi-processing is created, it is possible to generate several outputs (Excel workbook, CSV files...), through the main menu command: \textit{SAProcessingXXX $\rightarrow$ Generate output} or \textit{(TSProcessingXXX $\rightarrow$ Generate output)}. It should be noted that Excel and CSV outputs will be put in the temporary folder if their target folders are not specified.

The user is expected to choose the output format by marking the appropriate box in left-hand side of the \textit{Output} window. The settings which are displayed in the other part of the window come from \textit{Tool $\rightarrow$ Options} menu. All changes in those settings should be done in the \textit{Tool $\rightarrow$ Options} menu. If the user changes the settings (e.g. output folder) in the \textit{SAProcessingXXX $\rightarrow$ Generate output} window (or \textit{TSProcessingXXX $\rightarrow$ Generate output}), it will not have any effect on the output content.

To the multi-processing which doesn't belong to a workspace, Demetra+ assigns default name ("demetra"). If multi-processing is saved in the workspace the name of the multi-processing is used.
Annex

1A  Least squares estimation by means of the QR decomposition

We consider the regression model:

\[ y = X\beta + \varepsilon. \]

The least squares problem consists in minimizing the quantity \( \| X\beta - y \|_2^2 \).

Provided that the regression variables are independent, it is possible to find an orthogonal matrix \( Q \), so that \( Q \cdot X = \begin{pmatrix} R \\ 0 \end{pmatrix} \) where \( R \) is upper triangular.

We have now to minimize:

\[ \| QX\beta - Qy \|_2^2 = \begin{pmatrix} R \\ 0 \end{pmatrix} \beta - Qy \|_2^2 = \| R\beta - a \|_2^2 + b \|_2^2, \]

where \( (Qy)_{1\ldots n-1} = a \) and \( (Qy)_{n-1} = b \).

The minimum of the previous norm is obtained by setting \( \beta = R^{-1}a \). In that case, \( \| R\beta - a \|_2^2 = 0 \).

The residuals obtained by that procedure are then \( b \), as defined above.

It should be noted that the QR factorization is not unique, and that the final residuals also depend on the order of the regression variables (the columns of \( X \)).
2A  **TramoSeats method**

TramoSeats is a model-based seasonal adjustment method developed by Victor Gomez and Agustin Maravall (Bank of Spain). It consists of two linked programs: Tramo and Seats. Tramo ("Time Series Regression with Arima Noise, Missing Observations, and Outliers") performs estimation, forecasting, and interpolation of regression models with missing observations and Arima errors, in the presence of possibly several types of outliers. Seats ("Signal Extraction in Arima Time Series") performs an Arima-based decomposition of an observed time series into unobserved components. The advantage of TramoSeats method is that, when a large number of series is adjusted, the automatic procedure is reliable and the outcomes are satisfactory. Information about TramoSeats method the user find below derives directly from papers by GÓMEZ, V., and MARAVALL, A. More information about TramoSeats method, TramoSeats software (DOS version and TSW – Tramo Seats Windows software) and its documentation as well as papers on methodology and application of the programs, can be found in [www.bde.es](http://www.bde.es) in the dedicated section (Services → Professionals → Statistical and Econometric Software).

**Pre-processing in Tramo**

The program Tramo fits the following regression model to the original time series:

\[ z_t = y_t \beta + x_t, \]

where:

\[ \beta = (\beta_1, ..., \beta_n) \] – vector of regression coefficients;

\[ y_t = (y_{t_1}, ..., y_{t_m}) \] – \( n \) regression variables (trading days variables, the leap year effect, outliers, the Easter effect, ramps, intervention variables, user-defined variables);

\[ x_t \] – term that follows the general Arima process: \( \phi(B) \delta(B) y_t = \theta(B) a_t \);

where:

\[ B \] is the backshift operator\(^{121} \);

\( \phi(B), \theta(B) \) and \( \delta(B) \) are finite polynomials in \( B \);

\( a_t \) is a white-noise variable with constant variance.

In general the polynomials have the multiplicative structure:

\[^{121}\] Backshift operator \( B \) is defined as: \( B y_t = y_{t-1} \). It is used to denote lagged series.
\( \phi(B) \) – a stationary autoregressive (AR) polynomial in \( B \), which is a product of stationary regular AR polynomial in \( B \) and stationary seasonal polynomial in \( B^S \):

\[
\phi(B) = \phi_p(B) \Phi_{ps}(B^S) = (1 + \phi_1 B + \ldots + \phi_p B^p)(1 + \Phi_1 B^S + \ldots + \Phi_{ps} B^{pS}),
\]

where:

- \( p \) – the number of regular autoregressive terms;
- \( p_S \) – the number of seasonal autoregressive terms;
- \( S \) – the number of observations per year (frequency of observation);

\( \theta(B) \) – an invertible moving average (MA) polynomial in \( B \) which is a product of invertible regular MA polynomial in \( B \) and invertible seasonal MA polynomial in \( B^S \):

\[
\theta(B) = \theta_q(B) \Theta_{qs}(B^S) = (1 + \theta_1 B + \ldots + \theta_q B^q)(1 + \Theta_1 B^S + \ldots + \Theta_{qs} B^{qS}),
\]

where:

- \( q \) – the number of regular autoregressive terms;
- \( q_S \) – the number of seasonal autoregressive terms;
- \( S \) – the number of observations per year (frequency of observation);

\( \delta(B) \) – non-stationary autoregressive (AR) polynomial in \( B \) (unit roots):

\[
\delta(B) = (1 - B)^d (1 - B^S)^{d_S},
\]

where:

- \( d \) – regular differencing order;
- \( d_S \) – seasonal differencing order.

Demetra+ uses the following notation:

- \( p = P \);
- \( q = Q \);
- \( d = D \);
- \( p_S = BP \);
- \( q_S = BQ \);
Therefore, information about parameters of Arima model is usually presented as Arima \((P, D, Q)(BP, BP, BQ)\).

Parameters of the Arima model are estimated using the Exact Maximum Likelihood Estimation.

**Signal extraction in Seats**

- **Estimation procedure**

The model based signal extraction procedure consist in estimation of the seasonally adjusted time series by means of the Wiener-Kolmogorow filter as the Minimum Mean Square Error estimators using UCArima (unobserved component Arima) model. Seats decomposes a series \(x_t\) received from Tramo into components \(x_{it} : x_t = \sum_{i=1}^{k} x_{it}\).

Each component follows the general Arima model:

\[
d_i(B)x_{it} = \psi_i(B)a_{it},
\]

where:

- \(i\) – trend, seasonal, transitory or irregular components\(^{122}\), respectively;
- \(a_{it}\) – a white-noise variable;
- the polynomial \(\psi_i(B) = \frac{\theta_i(B)}{\phi_i(B)}\).

The polynomials \(\theta_i(B), \phi_i(B)\) and \(\delta_i(B)\) are of finite order. A white-noise variable is normally, identically and independently distributed and has a zero-mean and variance \(V(a_i)\). Time series \(x_t\) also follows Arima model of the type \(\delta_i(B)x_t = \psi_i(B)a_{it}\), where \(a_i\) is a white-noise variable with variance \(V(a)\), also called the innovation at \(t\).

In an unobserved-components model, residuals \(a_i\) are disturbances associated with the unobserved components. These disturbances are functions of the innovations in the series and are called "pseudo-innovations". Demetra+ uses term "innovations" to refer to "pseudo-innovations"\(^{123}\).

\(^{122}\) For the irregular component it is an Arima \((0,0,0)(0,0,0)\) model.

In order to decompose the series $x_t$ into components Seats factorize the full AR polynomial

$$\Phi(B) = \phi(B)\delta(B) = \phi_p(B)\Phi_{ps}(B^S)(1 - B)^d(1 - B^S)^{ds}$$

in the following way\(^{124}\):

$$\Phi(B) = \Phi_{\text{trend-cycle}}(B)\Phi_{\text{seasonal}}(B)\Phi_{\text{transitory}}(B),$$

where:

$$\Phi_{\text{trend-cycle}}(B)$$ – trend-cycle roots;

$$\Phi_{\text{seasonal}}(B)$$ – seasonal roots;

$$\Phi_{\text{transitory}}(B)$$ – transitory roots.

Such decomposition requires calculation of the $\Phi(B)$ solutions.

In principle, the solutions to the equation $x_t + \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} = 0$ can be found by replacing $x_t$ by $r^i$ i.e. $r^p + \phi_1 r^{p-1} + \ldots + \phi_p r + \phi_p = 0$.

The equation above is so called characteristic equation which has real and complex roots. Complex number is represented as $z = a + bi$, which is equivalent to $z = r(\cos \phi + i \sin \phi)$, where:

- $a$ and $b$ are real numbers;
- $i$ – the imaginary unit ($i^2 = -1$);
- $r$ – a modulus of $z$, $r = |z| = \sqrt{a^2 + b^2}$;
- $\phi$ – argument (frequency).

If roots are complex, they are always the pairs of complex conjugates.

The roots are assigned to the components according to their associated modulus and frequency:

- Roots of $(1 - B)^d$ are assigned to trend component.
- Roots of $(1 - B^S)^{ds} = ((1 - B)(1 + B + \ldots + B^{r-1}))^{ds}$ are assigned to trend component (root of $(1 - B)$) and to the seasonal component (roots of $(1 + B + \ldots + B^{r-1})$).
- Real positive inverse roots\(^{125}\) of $\phi_p(z)$ that are greater than $k$ are assigned to the trend component, otherwise they are assigned to the transitory component, where $k$ is the threshold value controlled by $rmod$.

---

\(^{124}\) MARAVALL, A. (2008b).

\(^{125}\) Roots of $\phi_p(z)$, where $z = B^{-1}$. 
parameter\textsuperscript{126}. Real negative inverse roots of $\phi_p(B)$ are assigned to the seasonal component. Complex roots are assigned to the seasonal component if the argument (frequency) is close to the seasonal frequency (closeness is controlled by $\text{epsphi}$ parameter\textsuperscript{127}). Otherwise they are assigned to the transitory component. For $\Phi_{p_s}(B^s)$, if $\phi \geq k$ ($\phi$ is a real positive root of $(-BP)^{\frac{1}{S}}$), then AR root of $(1 - \phi B)$ is assigned to the trend component. Otherwise it is assigned to the transitory component. Also when $Q > P$, the Seats decomposition yields a pure MA $(Q - P)$ component (hence transitory). In this case the transitory component will appear even when there is no AR factor allocated to it. These roots may represent trading-day effect or cycle of transitory component to be extracted in Seats. Frequency of AR roots is useful for detecting a stochastic trading days\textsuperscript{128} effect or stationary seasonality.

Once the roots are assigned to the components, $x_t$ can be expressed as:

\[
\theta(B) a_t = \frac{\theta_{\text{trend}}(B)}{\delta_{\text{trend}}(B)\phi_{\text{trend}}(B)} a_{\text{trend},t} + \frac{\theta_{\text{seasonal}}(B)}{\delta_{\text{seasonal}}(B)\phi_{\text{seasonal}}(B)} a_{\text{seasonal},t} +
\]

\[
\frac{\theta_{\text{transitory}}(B)}{\delta_{\text{transitory}}(B)\phi_{\text{transitory}}(B)} a_{\text{transitory},t} + u_t,
\]

where:

$u_t$ – white noise.

If the all components spectra are non-negative for all frequencies, the decomposition is called admissible\textsuperscript{129} decomposition.

\textsuperscript{126} See 4.3.9.

\textsuperscript{127} See 4.3.9.

\textsuperscript{128} A stochastic trading day component is always modeled as a stationary ARMA(2,2), where the AR contains the roots close to the TD frequency, and the MA(2) is obtained from the model decomposition (MARAVALL, A. (2011)). This component, estimated by Seats, is not displayed by the current version of Demetra+.

\textsuperscript{129} If the Arima model selected by Tramo leads to non-admissible decomposition, Seats changes the Arima model and uses the modified one to decompose the series. There are also other, rare situations when the Arima model chosen by Tramo is changed by Seats. It happens when, for example, Arima models generate unstable seasonality or produce a senseless decomposition. Such examples are discussed by MARAVALL, A. (2009). In cases when Arima model is changed by Seats, the forecasts of raw time series and linearized series are calculated using Arima model selected by Tramo, while the decomposition and the components’ forecasts are based on Arima model changed by Seats.
The pseudo-spectrum\textsuperscript{130} of $x_i$ is defined as the Fourier transform of $\frac{\psi_i(B) \psi_i(F)}{\delta_i(B) \delta_i(F)}$ and is denoted by $g_i(\omega)$, where $\omega$ is a frequency argument and $F$ is a forward operator, for which $F = B^{-1}$.

For particular realization of $X_T = [x_1, x_2, \ldots, x_T]$, where $T$ is the last observed period, Seats aims to obtain for each component the estimator $\hat{x}_{i,t}$ such that $E\left[(x_{i,t} - \hat{x}_{i,t})^2 \mid X_T\right]$ is minimized (it is Minimum Mean Square Error (MMSE) estimator). Under the joint normality assumption $\hat{x}_{i,t}$ is also equal to the conditional expectation $E(x_{i,t} \mid X_T)$, so it can be presented as a linear function of the elements in $X_T$:

$$\hat{x}_{i,t} = \ldots + \nu_k x_{i,-k} + \ldots + \nu_0 x_t + \ldots + \nu_k x_{i+k} + \ldots$$

When $T \to \infty$ the estimator becomes final (historical) estimator. In practice it is achieved for large $k$. When $T - k < t < T$, $\hat{x}_{i,t}$ yields a preliminary estimator and for $t > T$, a forecast.

The joint distribution of the stationary transformation of the components and of their MMSE estimators (i.e., variances, autocorrelations and cross-correlations) is used for model diagnostic.

For each component the estimate $\hat{x}_{i,t}$ is obtained by applying Wiener-Kolmogorow ($\nu_i(B, F)$) filter on $x_i$:

$$\hat{x}_{i,t} = \nu_i(B, F) x_i,$$

where:

$$\nu_i(B, F) = u_0 + \sum_{j=1}^{\infty} \nu_j (B^j + F^j).$$

which is symmetric and convergent. It can thus be truncated replacing $\infty$ with $L$.

In practice, $L$ typically expands between 3 and 5 years. Hence, when $T > 2L + 1$, where $T$ is the last observed period, final estimators can be assumed for the central observations of the series\textsuperscript{131}. Symmetric and centered filter allow to avoid the phase effect. The filter can be expand in the following way:

\textsuperscript{130} Term pseudo-spectrum is used for non-stationary time series, while term spectrum – for stationary time series.

\textsuperscript{131} MARAVALL, (1998).
When $T - L < t < T$, observations at the end of time series, that are necessary to calculate $x_t^*$ are not available yet, so the filter cannot be applied. Because of that these future values are replaced by their optimal forecasts from the Arima model on $x_t$. The estimator that uses such forecasted values is called a preliminary estimator. As the forecasts are linear functions of present and past observations of $x_t$, the preliminary estimator of $x_t^*$ obtained with the forecasts will be a truncated filter applied to the $x_t$. Similarly, for $1 < t < L$ the estimator will use backcasts, and hence yields preliminary estimators for the starting period. This truncated filter will be neither centered, nor symmetric. As a result, the phase effect occurs\textsuperscript{132}.

- \textbf{Wiener-Kolmogorow filter and ACGF function}

For the two component model ($s_t$ – seasonal component, $n_t$ – non-seasonal component) in the frequency domain Wiener-Kolmogorow filter ($v(B, F)$) that provide the final estimator of $s(t)$ is expressed as the ratio of the $s(t)$ and $x(t)$ pseudo-spectra:

$$
\hat{v}(\omega) = \frac{g_s(\omega)}{g_x(\omega)}.
$$

The spectrum of the estimator of the seasonal component is expressed as:

$$
g_s(\omega) = \left[ \frac{g_s(\omega)}{g_x(\omega)} \right]^2 g_x(\omega).
$$

From this equation it is clear that the squared gain of the filter determines how the variance of the series contributes to the variance of the seasonal component for the different frequencies.

In time domain the ratio of pseudo-spectra are replaced by the ratio of pseudo-autocovariance generating function (p-ACGF)\textsuperscript{133}:

\textsuperscript{132} KAISER, R., MARAVALL, A. (2000).

\textsuperscript{133} The ACGF is well defined for the stationary time series, i.e. ACGF of \( \delta_s(B)s_s \) is \( \frac{\theta_s(B)\theta(F)}{\phi_s(B)\phi_s(F)}V(a_s) \); \( \delta_f(B) \) and \( \delta(B) \) contain differencing operators that make, respectively, \( s_t \) and \( x_t \) stationary. Thus, for \( s_t \) the pseudo-ACGF is calculated as: \( \frac{\theta_s(B)\theta_s(F)}{\delta_s(B)\delta_s(F)\phi_s(B)\phi_s(F)}V(a_s) \).
\[ V(B, F) = k_s \frac{\gamma_s(B, F)}{\gamma(B, F)}, \]

where:

\[ \gamma_s(B, F) = \frac{\theta_s(B)\theta_s(F)}{\phi_s(B)\delta_s(B)\varphi_s(F)\delta_s(F)} V(a_s) \text{ is ACGF of } s_i; \]

\[ \gamma(B, F) = \frac{\theta(B)\theta(F)}{\phi(B)\delta(B)\varphi(F)\delta(F)} V(a) \text{ is ACGF of } x_i; \]

\[ k_s = \frac{V(a_s)}{V(a)}. \]

Thus, Wiener-Kolmogorow filter for seasonal component \( s_i \) is expressed as:

\[ v_s(B, F) = k_s \frac{\theta_s(B)\theta_s(F)\varphi_n(B)\varphi_s(F)}{\theta(B)\theta(F)}. \]

Let \( g_\cdot(\dot{\lambda}) \) denotes a pseudo-spectrum. One can define\(^{134} \):

\[ g_n(\omega) = \left[ \frac{\hat{g}_n(\omega)}{g_n(\omega)} \right]^2, \text{ were: } g_n(\omega) = \left[ \frac{\hat{g}_n(\omega)}{g_n(\omega)} \right]^2 g_s(\omega) = \left[ \frac{\hat{g}_n(\omega)}{g_n(\omega)} \right] g_n(\omega), \text{ so } g_n(\omega) < g_n(\omega) \]

and \( g_\cdot(\omega) = \left[ \frac{\hat{g}_\cdot(\omega)}{g_\cdot(\omega)} \right] g_\cdot(\omega), \) so \( g_\cdot(\omega) < g_\cdot(\omega). \)

Then the expression: \( g_\cdot(\omega) - \left[ \frac{g_n(\omega) + g_\cdot(\omega)}{g_n(\omega) + g_\cdot(\omega)} \right] \) is the cross-spectrum (in time domain it is cross-covariance function). As \( g_\cdot(\omega) - \left[ \frac{g_n(\omega) + g_\cdot(\omega)}{g_n(\omega) + g_\cdot(\omega)} \right] > 0, \) MMSE yields correlated estimators. Nevertheless, if at least one non-stationary component exists, cross-correlations estimated by TramoSeats will tend to zero as cross-covariances between estimators of the components are finite.

The ACGF for the stationary transformation of component $\delta_s(B)s_i$ that follows the model $\delta_s(B)s_i = \psi_s(B)a_s$ is $\gamma_s = \frac{\theta_s(B)\theta_x(F)}{\phi_s(B)\phi_x(F)} V(a_s)$. Replacing $x_i$ with its Arima expression in $s_i = \nu(B,F)x_i$, the final estimator can be expressed in terms of the innovations in original series:\(^{135}\):

$$\delta_s(B)s_i = k_s \frac{\theta_s(B)\theta_x(F)\phi_n(F)\delta_n(F)}{\phi_s(B)\theta(F)} a_i .$$

ACGF of theoretical final estimator is calculated as:

$$\gamma_s = \frac{\alpha(B)\alpha(F)}{\beta(B)\beta(F)} k_s ,$$

where:

$$\alpha(B) = \theta_s(B)\phi_n(B)\delta_n(B) ;$$

$$\alpha(F) = \theta_x(F)\phi_n(F)\delta_n(F) ;$$

$$\beta(B) = \phi_s(B)\theta(B) ;$$

$$\beta(F) = \phi_x(F)\theta(F) ;$$

$$k_s = \frac{V(a_i)^2}{V(a_i)} .$$

ACGF can be also presented as a following function:

$$\gamma_0 + \sum_{j=1}^{\infty} \gamma_j (B^j + F^j) ,$$

where:

$$\gamma_j - \text{covariance between observations separated by lag } j .$$

The spectrum can be obtained from ACGF function by applying Fourier Transform:\(^{136}\).

- **PsiE-weights**

Estimator of seasonal component is calculated as $s_i = \nu_s(B,F)x_i$. By replacing $x_i = \frac{\theta(B)}{\gamma(B)\delta(B)} a_i$, a seasonal factor can be expressed as:

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\[ s_t = v_t(B,F) \frac{\theta(B)}{\gamma(B) \delta(B)} a_t. \]

Let \( \xi_t(B,F) = v_t(B,F) \frac{\theta(B)}{\gamma(B) \delta(B)} \),

i.e. \( \xi_t(B,F) = ... + \xi_j B^j + ... + \xi_0 F + ... + \xi_{-j} F^j + ... \), which are called PsiE-weights. Therefore, the PsiE-weights are obtained from the Wiener-Kolmogorov filter by multiplying by \( \frac{\theta(B)}{\gamma(B) \delta(B)} \).

Hence, PsiE-weights can be divided into two components: first one: \( ... + \xi_j B^j + ... + \xi_0 F + ...\) applies to prior and concurrent innovations, second one: \( \xi_{-j} F^j + ... \) applies to future (i.e. posterior to \( t \)) innovations. \( \xi_j \) determines contribution of \( a_{t-j} \) to \( s_t \) while \( \xi_{-j} \) determines contribution of \( a_{t+j} \) to \( s_t \).

For \( j \geq 0 \) PsiE-weight \( j \) determines contribution of total innovation from period \( T - j \) to component estimator \( x_t \). For \( j < 0 \) PsiE-weight \( j \) determines contribution of total innovation from period \( T + j \) to component estimator \( x_t \). It is assumed that \( T > 2L + 1 \).

Finally, the estimator of the seasonal component can be expressed as
\[ s_t = \xi_t(B)^- a_t^- + \xi_t(F)^+ a_{t+1}. \]

\( \xi_t(B)^- a_t^- \) is an effect of starting conditions, present and past innovations in series while \( \xi_t(F)^+ a_{t+1} \) is an effect of future innovations which is a zero-mean, convergent, one-sided (stationary) MA process.

- Errors analysis

For each \( i-th \) component total error in the preliminary estimator \( d_{it+k} \) is expressed as:
\[ d_{it+k} = m_{it} - m_{it+k}, \]
where:
\( m_{it} \) - \( i-th \) component;
$m_{\hat{m}t+k}$ - the estimator of $m_t$ when the last observation is $x_{t+k}$ ($x_t$ is a time series).

$d_{\hat{m}t+k}$ can be presented as the sum of the final estimation error ($e_t$) and the revision error ($r_{\hat{m}t+k}$), i.e.:

$$d_{\hat{m}t+k} = m_{\hat{m}t} - m_{\hat{m}t+k} = (m_{\hat{m}t} - m_t) + (m_t - m_{\hat{m}t+k}) = e_t + r_{\hat{m}t+k}.$$  

The final estimation error ($e_t$) and the revision error ($r_{\hat{m}t+k}$) are orthogonal\textsuperscript{137}.

\textsuperscript{137} MARAVALL, A. (2000).
3A X12

X12 is seasonal adjustment software developed by the United States Census Bureau. The program originates from X11 algorithm and is the result of a long tradition of non-parametric smoothing based on moving averages.

Moving averages, which are weighted averages of a moving span of a time series (see hereafter), have two important defaults:

- They are not resistant and might be deeply impacted by outliers;
- The smoothing of the ends of the series cannot be done but with asymmetric moving averages which introduce phase-shifts and delays in the detection of turning points.

These drawbacks adversely affect the X11 output and stimulate the development of this method. The last United States Census Bureau program is X12 that includes all the capabilities of X11 and a pre-processing program RegArima. Therefore, X12 is divided in two parts (see Figure 1).

- In a first step, RegArima (linear regression model with Arima time series errors) models are used to clean the series from non-linearities, mainly outliers and trading-day effects. A global Arima model is adjusted to the series in order to compute the forecasts.
- In a second step, an enhanced version of the X11 algorithm is used to compute the trend-cycle, the seasonal component and the irregular.

A detailed presentation of X12 can be found in FINDLEY, D. et al (1998) and in the documentation of the software. A complete description of the X11 method is available in LADIRAY, D. and QUENNEVILLE, B. (2001). Flow diagram for seasonal adjustment with X12 is shown below.

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Moving averages

A moving average of coefficients \( \{ \theta_i \} \), noted \( M \{ \theta_i \} \) or simply \( M \), is defined as:

\[
M(X_t) = \sum_{k=-p}^{+f} \theta_k X_{t+k}.
\]

The value at instant \( t \) of the series is therefore replaced by a weighted average of \( p \) "past" values of the series, the current value, and \( f \) "future" values of the series. The quantity \( p + f + 1 \) is called the moving average order. When \( p \) is equal to \( f \), that is, when the number of points in the past is the same as the number of points in the future, the moving average is said to be centred.

If, in addition, \( \theta_{-k} = \theta_k \) for any \( k \), the moving average \( M \) is said to be symmetric.

Generally, with a moving average of order \( p + f + 1 \) calculated for instant \( t \) with points \( p \) in the past and points \( f \) in the future, it will be impossible to smooth out the first \( p \) values and the last \( f \) values of the series.

In the X11 method, symmetric moving averages play an important role as they do not introduce any phase-shift in the smoothed series. But, to avoid losing information at the series ends, they are either supplemented by ad hoc asymmetric moving averages or applied on the series completed by forecasts.

One of the simplest moving averages is the symmetric moving average of order \( P = 2p + 1 \) where all the weights are equal to \( \frac{1}{P} \).

In the estimation of the seasonal component, X12 uses \( P \times Q \) composite moving averages, obtained by composing a simple moving average of order \( P \), whose coefficients are all equal to \( \frac{1}{P} \), and a simple moving average of order \( Q \), whose coefficients are all equal to \( \frac{1}{Q} \).

In the estimation of the trend cycle, X12 uses Henderson moving averages which have been chosen for their smoothing properties. The coefficients of Henderson moving average of order \( 2p + 1 \) may be calculated using the formula:

\[
\theta_i = \frac{315 [(n-1)^2 - i^2] [n^2 - i^2] [(n+1)^2 - i^2] [3n^2 - 16 - 11i^2]}{8n(n^2 - 1)(4n^2 - 1)(4n^2 - 9)(4n^2 - 25)},
\]

where: \( n = p + 2 \).
Components and decomposition models

X12 allows to perform the decomposition and seasonal adjustment of monthly and quarterly series. The main components, each representing the impact of certain types of events on the time series, are:

- The trend-cycle \( (TC_i) \), that captures long-term and medium-term behaviour;
- The seasonal component \( (S_i) \) representing intra-year fluctuations, monthly or quarterly, that are repeated more or less regularly year after year;
- The irregular component \( (I_i) \), combining all the other more or less erratic fluctuations not covered by the previous components.

The trend-cycle is theoretically divided in 2 sub-components:

- The trend representing the long-term evolution of the series;
- The cycle, that represents the smooth, almost periodic movement around the trend, revealing a succession of phases of growth and recession.

X12 does not separate these two components: the series are generally too short for both components to be easily estimated. Consequently, hereafter X12 estimates the trend-cycle component.

Apart from the three main components, the following effects may appear at one time or another in the decomposition:

- The outlier component \( (O_i) \), itself composed of several kind of atypical values: additive outliers, transitory changes, level shifts, ramps and seasonal outliers\(^{140}\);
- A trading-days component \( (TD_i) \), that measures the impact on the series of the everyday composition of the month or quarter\(^{141}\);
- A component measuring the effect of the Easter holiday \( (E_i) \)\(^{142}\).

These three effects are usually detected and estimated in the RegArima part of the process.

The X12 method considers four decomposition models:

- The additive model: \( X_i = TC_i + S_i + TD_i + E_i + O_i + I_i \) ;

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\(^{140}\) See 4.2.5.

\(^{141}\) See 4.2.4.

\(^{142}\) See 4.2.4.
The multiplicative model: $X_t = TC_t \times S_t \times TD_t \times E_t \times O_t \times I_t$.

The log additive model:

$$X_t = \log(TC_t) + \log(S_t) + \log(TD_t) + \log(E_t) + \log(O_t) + \log(I_t);$$

The pseudo-additive model: $X_t = TC_t(S_t + TD_t + E_t + O_t + I_t - 1)$.

The additive and log-additive models are used also by TramoSeats, while the multiplicative and pseudo-additive models are implemented only in X12.

**The basic algorithm of the X11 method**

The X11 method is based on an iterative principle of estimation of the different components using appropriate moving averages at each step of the algorithm. The successive results are saved in the special tables. The list of the X11 tables displayed in Demetra+ is included in the end of this section.

The basic algorithm of the X11 method will be presented for monthly time series $X_t$ that is assumed to be decomposable into trend-cycle, seasonality and irregular component according to an additive model: $X_t = TC_t + S_t + I_t$.

A simple seasonal adjustment algorithm can be thought of in eight steps:

1. **Estimation of Trend-Cycle by 2x12 moving average:** $TC_t^{(1)} = M_{2\times12}(X_t)$.
   
   The moving average used here is a 2x12 moving average, of coefficients $\frac{1}{24}\{1,2,2,2,2,2,2,2,2,2,2,2,1\}$, that retains linear trends, eliminates order-12 constant seasonality and minimizes the variance of the irregular component.

2. **Estimation of the Seasonal-Irregular component:** $(S_t + I_t)^{(1)} = X_t - TC_t^{(1)}$.

3. **Estimation of the Seasonal component by 3x3 moving average over each month:**
   
   $$S_t^{(1)} = M_{3\times3}\left[(S_t + I_t)^{(1)}\right].$$
   
   The moving average used here is a 3x3 moving average over 5 terms, of coefficients $\frac{1}{9^2}2\frac{3}{9}2\frac{1}{9}$. This kind of moving average retains linear trends. The seasonal factors are then normalized such that the seasonal effects over the whole 12-month period are approximately cancelled out.

   $$S_t^{(1)} = S_t^{(1)} - M_{2\times12}(S_t^{(1)}).$$
4. **Estimation of the seasonally adjusted series**: \( SA_t^{(1)} = (TC_t + I_t)^{(1)} = X_t - S_t^{(1)} \).

This first estimation of the seasonally adjusted series must, by construction, contain less seasonality. The X11 method again executes the algorithm presented above, changing the moving averages to take this property into account.

5. **Estimation of Trend-Cycle by 13-term Henderson moving average**: \( TC_t^{(2)} = H_{13} \left( SA_t^{(1)} \right) \).

Henderson moving averages, while they do not have special properties in terms of eliminating seasonality (limited or none at this stage), have a very good smoothing power and retain a locally polynomial trend of degree 2 and preserves a locally polynomial trend of degree 3.

6. **Estimation of the Seasonal-Irregular component**: \( (S_t + I_t)^{(2)} = X_t - TC_t^{(2)} \).

7. **Estimation of the Seasonal component by 3x5 moving average over each month**: \( S_t^{(2)} = MM_{3x5} \left( (S_t + I_t)^{(2)} \right) \).

The whole difficulty lies, then, in the choice of the moving averages used for the estimation of the trend-cycle in steps 1 and 5 on the one hand, and for the estimation of the seasonal component in steps 3 and 5.

8. **Estimation of the seasonally adjusted series**: \( SA_t^{(2)} = (TC_t + I_t)^{(2)} = X_t - S_t^{(2)} \).

The iterative principle of X11

To evaluate the different components of the series, while taking into account the possible presence of extreme observations, X11 will proceed iteratively: estimation of components, search for disruptive effects in the irregular component, estimation of components over a corrected series, search for disruptive effects in the irregular component, and so on.

The Census X11 program presents 4 processing stages (A, B, C, and D), plus 3 stages, E, F, and G, that propose statistics and charts and are not part of the decomposition per se. On stages B, C and D the basic algorithm is used.
• **Part A: Pre-adjustments**

This part, which is not obligatory, corresponds in X12 to the first cleaning of the series done using the RegArima facilities: detection and estimation of outliers and trading-day effects, forecasts and backcasts of the series. Based on these results, the program calculates prior adjustment factors that are applied to the raw series. The series thus corrected, Table B1 of the printouts, then proceeds to part B.

• **Part B: First automatic correction of the series**

This stage consists in a first estimation and downweighting of the extreme observations and, if requested, a first estimation of the trading-day effects. This stage is performed by applying the basic algorithm detailed earlier. These operations lead to Table B20, adjustment values for extreme observations, used to correct the unadjusted series and result in the series from Table C1.

• **Part C: Second automatic correction of the series**

Still applying basic algorithm once again, this part leads to a more precise estimation of replacement values of the extreme observations (Table C20). The series, finally "cleaned up", is shown in Table D1 of the printouts.

• **Part D: Seasonal adjustment**

This part, at which our basic algorithm is applied for the last time, is that of the seasonal adjustment per se, as it leads to final estimates:

- of the seasonal component (Table D10);
- of the seasonally adjusted series (Table D11);
- of the trend-cycle component (Table D12);
- of the irregular component (Table D13).

• **Part E: Components modified for large extreme values**

Parts E includes:

- Components modified for large extreme values;
- Comparison the annual totals of the raw time series and seasonally adjusted time series;
- Changes in the final seasonally adjusted series;
- Changes in the final trend-cycle;
- Robust estimation of the final seasonally adjusted series.

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143 This is a general estimation procedure used by Census Bureau. Demetra+ do not calculate backcasts for X12.
The results from part E are used in part F to calculate the quality measures.

- **Part F: Seasonal adjustment quality measures**

  Part F contains statistics for judging the quality of the seasonal adjustment. Demetra+ presents selected output for part F, i.e.:
  - M and Q statistics;
  - The relative contribution of components (irregular (I), trend-cycle (C), seasonal (S), preliminary factors (P), trading days and holidays effects (TD&H) to the variance of the stationary part of the original time series.

- **Part G: Graphics**

  Part G presents spectra estimated for:
  - Raw time series adjusted a priori (Table B1);
  - Seasonally adjusted time series modified for large extreme values (Table E2);
  - Final irregular component adjusted for large extreme values (Table E3).

Originally, graphics were displayed in character mode. In Demetra+, these graphics are replaced favourably by the usual office-based graphics software.

**The Henderson moving average and the trend-cycle estimation**

In iteration B (Table B7), iteration C (Table C7) and iteration D (Table D7 and Table D12) the trend-cycle component is extracted from an estimate of the seasonally adjusted series using the Henderson moving averages. The length of the Henderson filter is chosen automatically by X12 in two-step procedure.

It is possible to specify the length of the Henderson moving average to be used. X12 provides an automatic choice between a 9-term, a 13-term or a 23-term moving average. The automatic choice of the order of the moving average is based on the value of an indicator called $\frac{I}{C}$ ratio which measures the significance of the irregular component in the series; the greater its significance, the higher the order of the moving average selected. Moreover, X12 makes it possible to choose manually any odd-numbered Henderson moving average. The procedure used in each part is very similar; the only differences are the number of options available and the treatment of the observations in the both ends of the series. The procedure below is applied for monthly time series.

In order to calculate $\frac{I}{C}$ ratio a first decomposition of the SA series (seasonally adjusted) is computed using a 13-term Henderson moving average.

For both the $C$ (trend-cycle) and $I$ (irregular) components, the average of the absolute values for monthly growth rates (multiplicative model) or for monthly growth (additive model) are
computed. They are denoted $\bar{C}$ and $\bar{I}$, receptively, where $\bar{C} = \frac{1}{n-1} \sum_{t=2}^{n} |C_t - C_{t-1}|$ and $\bar{I} = \frac{1}{n-1} \sum_{t=2}^{n} |I_t - I_{t-1}|$.

Then the value of $\bar{I}/\bar{C}$ ratio is checked and:

- If the ratio is smaller than 1, a 9-term Henderson moving average is selected;
- Otherwise, a 13-term Henderson moving average is selected.

Then the trend-cycle is computed by applying selected Henderson filter to the seasonally adjusted series from Table B6. The observations at the beginning and at the end of the time series that cannot be computed by means of symmetric Henderson filters are estimated by ad hoc asymmetric moving averages.

In iterations C and D:

- If the ratio is smaller than 1, a 9-term Henderson moving average is selected;
- If the ratio is greater than 3.5, a 23-term Henderson moving average is selected.
- Otherwise, a 13-term Henderson moving average is selected.

The trend-cycle is computed by applying selected Henderson filter to the seasonally adjusted series from Table C6, Table D7 or Table D12, accordingly. At the both ends of the series, where a central Henderson filter cannot be applied, the asymmetric ends weights for the 7 term Henderson filter are used.

**Choosing the composite moving averages when estimating the seasonal component**

In iteration D, Table D10 shows an estimate of the seasonal factors implemented on the basis of the modified SI (Seasonal-Irregular) factors estimated in Tables D4 and D9bis. This component will have to be smoothed to estimate the seasonal component; depending on the importance of the irregular in the Seasonal-Irregular component, we will have to use moving averages of varying length as in the estimate of the Trend/Cycle where the I/C ratio was used to select the length of the Henderson moving average. The estimation includes several steps.

**Step 1: Estimating the irregular and seasonal components**

An estimate of the seasonal component is obtained by smoothing, month by month and therefore column by column, Table D9bis using a simple 7-term moving average, i.e. of coefficients $\left\{ \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7} \right\}$. In order not to lose three points at the beginning and end of each column, all columns are completed arbitrarily. Let us assume that the column that corresponds to
the month is composed of \( N \) values \( \{x_1, x_2, x_3, \ldots, x_{N-1}, x_N\} \). It will be transformed into a series \( \{x_{-2}, x_{-1}, x_0, x_1, x_2, x_3, \ldots, x_{N-1}, x_N, x_{N+1}, x_{N+2}, x_{N+3}\} \) with:

\[
x_{-2} = x_{-1} = x_0 = \frac{x_1 + x_2 + x_3}{3} \quad \text{and} \quad x_{N+1} = x_{N+2} = x_{N+3} = \frac{x_N + x_{N-1} + x_{N-2}}{3}.
\]

We then have the required estimates: \( S = M_\gamma (D9\text{bis}) \) and \( I = D9\text{bis} - S \).

**Step 2: Calculating the "Moving Seasonality Ratios"**

For each \( i - th \) month we will be looking at the mean annual changes for each component by calculating:

\[
S_i = \frac{1}{N_i - 1} \sum_{j=2}^{N_i} |S_{ij} - S_{i,j-1}|, \quad I_i = \frac{1}{N_i - 1} \sum_{j=2}^{N_i} |I_{ij} - I_{i,j-1}|,
\]

where \( N_i \) refers to the number of months \( i \) in the data, and the moving seasonality ratio of month \( i \): \( RSM_i = \frac{I_i}{S_i} \). These ratios are published in Table D9A.

These ratios are used to measure the significance of any "noise" in the Seasonal-Irregular component. The idea is to obtain, for each month, an indicator capable of selecting the appropriate moving average for the removal of any noise and providing a good estimate of the seasonal factor. The higher the ratio, the more "chaotic" the series, and the greater the order of the moving average should be used. As for the rest, the default program selects the same moving average to smooth each monthly series, but we have here the elements needed to select a moving average for each month.

**Step 3: Calculating the overall Moving Seasonality Ratio**

An overall Moving Seasonality Ratio is calculated as follows:

\[
MSR = \frac{\sum_i N_i \ast \overline{I}_i}{\sum_i N_i \ast \overline{S}_i}.
\]

**Step 4: Selecting a moving average and estimating the seasonal component**

Depending on the value of the ratio, the program automatically selects a moving average that is applied, column by column (i.e. month by month) to the Seasonal/Irregular component in Table D8 modified, for extreme values, using values in Table D9.

The default selection procedure of a moving average is based on the Moving Seasonality Ratio in a following way (see figure below):
If this ratio occurs within zone A (MSR < 2.5), a $3 \times 3$ moving average is used; if it occurs within zone C (3.5 < MSR < 5.5), a $3 \times 5$ moving average is selected; if it occurs within zone E (MSR > 6.5), a $3 \times 9$ moving average is used;

If the MSR occurs within zone B or D, one year of observations is removed from the end of the series, and the MSR is recalculated. If the ratio again occurs within zones B or D, we start over again, removing a maximum of five years of observations. If this does not work, i.e. if we are again within zones B or D, a $3 \times 5$ moving average is selected.

The chosen symmetric moving average corresponds, as the case may be, to 5 ($3 \times 3$), 7 ($3 \times 5$) or 11 ($3 \times 9$) terms, and therefore does not provide an estimate for the values of seasonal factors in the first 2 (or 3 or 5) and the last 2 (or 3 or 5) years. These are then calculated using ad hoc asymmetric moving averages.


**Identification and replacement of extreme values**

In order to improve the estimation of Seasonal-Irregular component X11 adjust it for extreme values in six steps procedure.

**Step 1: Estimating the seasonal component**

The seasonal component is estimated by smoothing Seasonal-Irregular component separately for each period using $3 \times 3$ moving average, i.e.:
Step 2: Normalizing the seasonal factors

The preliminary seasonal factors are normalized in such way that for one year their average is equal to zero (additive model) or to unity (multiplicative model).

Step 3: Estimating the irregular component

The initial normalized seasonal factors are removed from the Seasonal-Irregular component to provide an estimate of the irregular component.

Step 4: Calculating a moving standard deviation

Next a moving standard deviation of the irregular component is calculated at five-years intervals. Each standard deviation is associated with the central year used to calculate it. The values in central year that in absolute terms deviate from average by more than 2.5 standard deviations are marked as extreme values and assigned zero weight. After excluding the extreme values the moving standard deviation is calculated once again.

Step 5: Detecting extreme values and weighting the irregular

Using the following rule a weight is assigned to each value of irregular component:

- Values which are more than 2.5 standard deviation away (in absolute terms) from the 0 (additive) or 1 (multiplicative) are assigned zero weight;
- Values which are less than 1.5 standard deviation away (in absolute terms) from the 0 (additive) or 1 (multiplicative) are assigned full weight (equal to one);
- Values which are lie between 1.5 and 2.5 standard deviation away (in absolute terms) from the 0 (additive) or 1 (multiplicative) are assigned a weight that varies linearly between 0 and 1 depending on their position.

Step 6: Adjusting extreme values of the seasonal-irregular component

A values which assigned weights are less than 1 are regarded as extreme. Those values are replaced by a weighted average of five values:

- The value itself with its weight;
- The two preceding values, for the same period, having full weight;
The next two values, for the same period, having full weight.

This general algorithm is used with some modification in parts B and C for detection and replacement of extreme values.

X12 detects and removes extreme values in RegArima part. However, if there is a seasonal heteroscedasticity in a time series (i.e. the variance of the irregular component is different in different calendar months; an example for this effect is the weather and snow-dependent output of the construction sector in Germany during the winter time or changes in Christmas allowances in Germany and resulting from this a transformation in retail trade turnover before Christmas), the Arima model is not on its own able to cope with this characteristic. The practical consequence is given by the detection of additional extreme values by X11. This may be not appropriate if the seasonal heteroscedasticity is produced by political interventions or other influences. Arima models assume a constant variance and are therefore not by themselves able to cope with this problem. Choosing longer (in the case of diverging weather conditions in the winter time for the construction sector) or shorter filters (in the case of a changing pattern of retail trade turnover in the Christmas time) may be reasonable in such cases. It may even be sensible to take into account the possibility of period-specific (e.g. month-specific) standard deviations using the calendarsigma ="all" option in X-11 part of the specification in order to detect extreme values (see 4.2.9).

**Relative contribution of components to changes in the raw series**

One of the seasonal adjustment quality measures calculated in part F is the relative contribution of the components to the stationary portion of the variance in the raw time series. The values presented by Demetra+ in Quality measures section is calculated in the following way:

First, for additive decomposition, for each component the average absolute changes over several periods\(^{144}\) are calculated as\(^{145}\):

\[
Component_d = \frac{1}{n - d} \sum_{t=d+1}^{n} \left| Table_t - Table_{t-d} \right|
\]

where:

\(d\) – time lag in periods (from monthly time series \(d\) varies from 1;

\(^{144}\) Average percentage change without regard to sign over several periods (for monthly time series: 1 to 12 periods, for quarterly time series: 1 to 4 periods).

\(^{145}\) For multiplicative decomposition the following formula is used:

\[
Component_d = \frac{1}{n - d} \sum_{t=d+1}^{n} \left( \frac{Table_t}{Table_{t-d}} - 1 \right)
\]
Component – the name of component from list below;

Table – the name of table the from table below.

<table>
<thead>
<tr>
<th>Table</th>
<th>Component</th>
<th>Series</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>O</td>
<td>Original series</td>
</tr>
<tr>
<td>D12</td>
<td>C</td>
<td>Final trend-cycle</td>
</tr>
<tr>
<td>D13</td>
<td>I</td>
<td>Final irregular component</td>
</tr>
<tr>
<td>D10</td>
<td>S</td>
<td>Final seasonal component</td>
</tr>
<tr>
<td>A2</td>
<td>P</td>
<td>Final preliminary adjustment factors</td>
</tr>
<tr>
<td>C18</td>
<td>TD&amp;H</td>
<td>Final trading-days factors</td>
</tr>
</tbody>
</table>

Assuming that the components are independent, the following relation is true:

\[ O^2 = C^2 + S^2 + I^2 + P^2 + TD \& H^2. \]

In order to simplify the analysis, the approximation can be replaced by the following equation:

\[ O^2' = C^2 + S^2 + I^2 + P^2 + TD \& H^2. \]

Then the relative contribution of each component presented by Demetra+ is calculated as the appropriate ratios, respectively: 

\[ 100 \times \frac{I^2}{O^2}, \quad 100 \times \frac{C^2}{O^2}, \quad 100 \times \frac{S^2}{O^2}, \quad 100 \times \frac{P^2}{O^2}, \quad 100 \times \frac{TD \& H^2}{O^2}. \]

The quality of the approximation is measured by the ratio \[ 100 \times \frac{O^2}{O^2'}, \] which is presented in last column of the table displayed by Demetra+.

The example is shown below:

<table>
<thead>
<tr>
<th>I</th>
<th>C</th>
<th>S</th>
<th>P</th>
<th>TD&amp;H</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.18</td>
<td>1.51</td>
<td>97.44</td>
<td>0.38</td>
<td>1.41</td>
<td>100.91</td>
</tr>
</tbody>
</table>
**X12 tables**

Part A – Preliminary Estimation of Extreme Values and Calendar Effects. This part includes prior modifications to the original data made in the RegArima part:

- Table A1 – Original series;
- Table A1a – Forecast of Original Series;
- Table A2 – Leap year effect;
- Table A6 – Trading Day effect (1 or 6 variables);
- Table A7 – Easter effect;
- Table A8 – Total Outlier Effect;
- Table A8ao – Additive outlier effect;
- Table A8ls – Level shift effect;
- Table A8tc – Transitory effect;

Part B – Preliminary Estimation of Time Series Components:

- Table B1 – Original series after adjustment by the RegArima model;
- Table B2 – Unmodified Trend-Cycle (preliminary estimation using composite moving average);
- Table B3 – Unmodified Seasonal-Irregular Component (preliminary estimation);
- Table B4 – Replacement Values for Extreme S-I Values;
- Table B5 – Seasonal Component;
- Table B6 – Seasonally Adjusted Series;
- Table B7 – Trend-Cycle (estimation using Henderson moving average);
- Table B8 – Unmodified Seasonal-Irregular Component;
- Table B9 – Replacement Values for Extreme S-I Values;
- Table B10 – Seasonal Component;
- Table B11 – Seasonally Adjusted Series;
- Table B13 – Irregular Component;
• Table B17 – Preliminary Weights for the Irregular;
• Table B20 – Adjustment Values for Extreme Irregulars.

Part C – Final Estimation of Extreme Values And Calendar Effects:
• Table C1 – Modified Raw Series;
• Table C2 – Trend-Cycle (preliminary estimation using composite moving average);
• Table C4 – Modified Seasonal-Irregular Component;
• Table C5 – Seasonal Component;
• Table C6 – Seasonally Adjusted Series;
• Table C7 – Trend-Cycle (estimation using Henderson moving average);
• Table C9 – Seasonal-Irregular Component;
• Table C10 – Seasonal Component;
• Table C11 – Seasonally Adjusted Series;
• Table C13 – Irregular Component;
• Table C20 – Adjustment Values for Extreme Irregulars.

Part D – Final Estimation of the Different Components:
• Table D1 – Modified Raw Series;
• Table D2 – Trend-Cycle (preliminary estimation using composite moving average);
• Table D4 – Modified Seasonal-Irregular Component;
• Table D5 – Seasonal Component;
• Table D6 – Seasonally Adjusted Series;
• Table D7 – Trend-Cycle (estimation using Henderson moving average);
• Table D8 – Unmodified Seasonal-Irregular Component;
• Table D9 – Replacement Values for Extreme S-I Values;
• Table D10 – Final Seasonal Factors;
• Table D10A – Forecast of Final Seasonal Factors;
• Table D11 – Final Seasonally Adjusted Series;
• Table D11A – Forecast of Final Seasonally Adjusted Series;
• Table D12 – Final Trend-Cycle (estimation using Henderson moving average);
• Table D12A – Forecast of Final Trend Component;
• Table D13 – Final Irregular Component;
• Table D13U – Irregular component (excluded outliers effect);
• Table D16 – Seasonal and Calendar Effects;
• Table D16A – Forecast of Seasonal and Calendar Component;
• Table D18 – Combined Calendar Effects Factors.

Part E – Components Modified for Large Extreme Values:

• Table E1 – Raw Series Modified for Large Extreme Values;
• Table D2 – SA Series Modified for Large Extreme Values;
• Table E3 – Final Irregular Component Adjusted for Large Extreme Values;
• Table E11 – Robust Estimation of the Final SA Series.
4A Specifications

<table>
<thead>
<tr>
<th>SA Method</th>
<th>Name</th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TramoSeats</strong></td>
<td>RSA0</td>
<td>Level, Airline model.</td>
</tr>
<tr>
<td>RSA1</td>
<td>Log/level, outliers detection, Airline model.</td>
<td></td>
</tr>
<tr>
<td>RSA2</td>
<td>Log/level, working days, Easter, outliers detection, Airline model.</td>
<td></td>
</tr>
<tr>
<td>RSA3</td>
<td>Log/level, outliers detection, automatic model identification.</td>
<td></td>
</tr>
<tr>
<td>RSA4</td>
<td>Log/level, working days, Easter, outliers detection, automatic model identification.</td>
<td></td>
</tr>
<tr>
<td>RSA5</td>
<td>Log/level, trading days, Easter, outliers detection, automatic model identification.</td>
<td></td>
</tr>
<tr>
<td><strong>X12</strong></td>
<td>X11</td>
<td>No pre-processing.</td>
</tr>
<tr>
<td>RSA1</td>
<td>Log/level, outliers detection, Airline model.</td>
<td></td>
</tr>
<tr>
<td>RSA2c</td>
<td>Log/level, working days, Easter, outliers detection, Airline model, pre-adjustment for leap-year if logarithmic transformation has been used.</td>
<td></td>
</tr>
<tr>
<td>RSA3</td>
<td>Log/level, outliers detection, automatic model identification.</td>
<td></td>
</tr>
<tr>
<td>RSA4c</td>
<td>Log/level, working days, Easter, outliers detection, automatic model identification, pre-adjustment for leap-year if logarithmic transformation has been used.</td>
<td></td>
</tr>
<tr>
<td>RSA5</td>
<td>Log/level, trading days, Easter, outliers detection, automatic model identification, pre-adjustment for leap-year if logarithmic transformation has been used.</td>
<td></td>
</tr>
</tbody>
</table>

**Explanations for settings:**

- **Level** – no transformation is performed;
- **Log/level** – Demetra+ tests for the log/level specification;
- **Working days** – a pretest is made for the presence of the working day effect by using one parameter specification (working vs. non-working days);
- **Trading days** – a pretest is made for the presence of the trading day effect by using six parameters specification (for working days, the day of week: Monday,...,Fridays specified);
- **Easter** – the program tests for the necessity of a correction for Easter effect in the original series;
- **Outliers detection** – Demetra+ automatically detects all types of outliers including: AO (additive outliers), LS (level shifts), TC (transitory changes) using default critical values;
- **Airline model** – the Arima model (0,1,1)(0,1,1). The Airline model is used as a default model in several TramoSeats and X12 specifications because it has been shown in many studies that this model is appropriate for many real seasonal monthly or quarterly time series.
series. Moreover, the Airline model approximates well many other models and provides an excellent "benchmark" model\(^\text{146}\);

- Automatic model identification – Demetra+ automatically identifies and estimates the best Arima model.

\(^{146}\) MARAVALL, A. (2009).
5A Model selection criteria

Model selection criteria are statistical tools for selecting the optimal order of the Arima-model. The basic idea behind all these criteria is to obtain much explanatory power (measured by the value of the likelihood function) with only a few parameters. The model selection criteria "penalize" for using many parameters and "rewards" for a high value of the likelihood function. Some of the most known information criteria are: Akaike Information Criteria (AIC), Corrected Akaike Information Criteria (AICC), Hannan-Quinn Information Criteria (HannanQuinn) and Schwarz-Bayes information criterion (BIC). For each model selection criteria the model with smaller value is preferred.

The formulas for model selection criteria are:

\[ AIC_N = -2L_N + 2n_p, \]
\[ AICC_N = -2L_N + 2n_p \left(1 - \frac{n_p + 1}{N} \right)^{-1}, \]
\[ HannanQuinn_N = -2L_N + 2n_p \log \log N, \]
\[ BIC_N = -2L_N + n_p \log N. \]

where:

- \( N \) – number of observations in time series;
- \( n_p \) – number of estimate parameters;
- \( L_N \) – loglikelihood function.

To choose Arima model parameters Tramo uses \( BIC_N \) criteria with some constrains aimed at increasing the parsimony and favoring balanced models. It should be noted that BIC criterion imposes a greater penalty term than does AIC. Parsimonious models are those which have a great deal of explanatory power using a relatively small number of parameters. Balanced models are models with similar AR and MA values\(^{147}\).

\(^{147}\) GOMEZ, V., MARAVALL, A. (1997).
6A Hannan-Rissanen algorithm

Hannan-Rissanen algorithm\textsuperscript{148} is a two-step procedure for the selection of appropriate orders for the autoregressive and moving average parameters of the Arima model. In a first step a high-order AR\((m)\), where \( m > \text{max}(p, q) \) model is fitted to the time series \( X_t \). Then the residuals \( \hat{a}_k \) from this model are used to provide estimates of innovations in Arma model \( \epsilon_t \):

\[
\epsilon_t = X_t - \sum_{k=1}^{m} \hat{a}_k X_{t-k}.
\]

In the second step the parameters \( p \) and \( q \) of the Arma model are estimated using at least squares linear regression of \( X_t \) onto \( X_{t-1}, \ldots, X_{t-p}, \epsilon_{t-1}, \ldots, \epsilon_{t-q} \) for combination of values \( p \) and \( q \). Finally, Hannan-Rissanen algorithm selects a pair of \( p \) and \( q \) values for which \( \log \delta^2_{p,q} + \frac{(p+q) \log T}{T} \) is the smallest.

The advantage of Hannan-Rissanen algorithm is a speed of computation in comparison with exact likelihood estimation.

Tramo uses Hannan-Rissanen algorithm in the automatic outlier detection step, while RegArima uses this algorithm to provide initial values when generating likelihood statistics for identifying the Arma orders.

7A Initial values for Arima model estimation

The default choice of initial parameter values in X12 is 0.1 for all AR and MA parameters. For majority of time series this default value seems to be appropriate. Introducing better initial values (as might be obtained, e.g., by first fitting the model using conditional likelihood) could slightly speed up convergence. Users are allowed to introduce manually initial values for AR and MA parameters that are then used to start the iterative likelihood maximization. This is rarely necessary, and in general not recommended. A possible exception to this occurs if initial estimates that are likely to be extremely accurate are already available, such as when one is re-estimating a model with a small amount of new data added to a time series. However, the main reason for specifying initial parameter values is to deal with convergence problems that may arise in difficult estimation situations.\(^{149}\)

8A Cancellation of AR and MA factors

A cancellation problem consists in cancelling some factors on both sides of the Arima model. This problem concerns mixed Arima \((P, D, Q)(BP, BP, BQ)\) models (i.e. \(p > 0\) and \(q > 0\), or \(P > 0\) and \(Q > 0\)). For example, cancellation problem occurs with Arima \((1, 1)\) model, 
\[(1 - \phi B)z_t = (1 - \theta B)a_t\]
when \(\phi = \theta\) as then model is simply form: \(z_t = a_t\). Such model causes problems with convergence of the nonlinear estimation. For this reason X12 and the TramoSeats programs check cancellation problem by computing zeros of the AR and MA polynomials. As cancellation does not need to be exact, the cancellation limit can be provided by the user\(^{150}\).

9A Spectral analysis

Definition of the periodogram

The periodogram of the series \{y_t\}_{t=1}^{T} is computed as follows:

1. The \( y_t \) is standardized, i.e.:
   
   \[ y = \frac{\sum_{t=1}^{T} y_t}{n}, \]

   \[ s^2 = \frac{\sum_{t=1}^{T} (y_t - y)^2}{n}, \]

   \[ z_t = \frac{(y_t - y)}{s}. \]

2. The periodogram is computed on the standardized \( z_t \):

   \[ I_{n,c}(\lambda) = \frac{2}{n} (C_{n,c}^2(\lambda) + S_{n,c}^2(\lambda)), \]

   where:

   \[ C_{n,c}(\lambda) = \sum_{t=1}^{n} \cos(\lambda t) z_t \text{ and } S_{n,c}(\lambda) = \sum_{t=1}^{n} \sin(\lambda t) z_t. \]

Periodogram at the Fourier frequencies

The Fourier frequencies are defined by:

\[ \lambda_j = \frac{2\pi j}{n}, 0 < j \leq \left[ \frac{n}{2} \right]. \]

If \( z_t \) are iid \( N(0,1) \), it is easy to see that the corresponding quantities \( I_{n,c}(\lambda_j) \) are iid \( \chi^2(2) \).

We have indeed that:

\[ \sum_{j=1}^{n} e^{\mu(\lambda_j - \lambda_k)} = \begin{cases} n & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases}, \]

and
\[ \sum_{j=1}^{n} \cos^2(\lambda, t) = \sum_{j=1}^{n} \sin^2(\lambda, t) = \frac{n}{2}, \]

so that \( \sqrt{\frac{2}{n}} C_{n,z}(\lambda_j) \) and \( \sqrt{\frac{2}{n}} S_{n,z}(\lambda_k) \) are uncorrelated \( N(0,1) \) random variables.

**Test on the periodogram**

Under the hypothesis that \( z_i \) is a Gaussian white noise, and considering subset \( J \) of Fourier frequencies, we have:

\[
\Pr \left( \max_{j \in J} I_{n,z}(\lambda_j) \leq \alpha \right) = \left[ 1 - e^{-\frac{\alpha^2}{2}} \right]^{\#J}.
\]

If we consider the sets of Fourier frequencies on or near the trading days frequencies on one side and on or near the seasonal frequencies on the other side, we can use the above formula as rough test regarding the absence of trading days/seasonal effects in the considered series.

The software considers the Fourier frequencies which are on or near the following frequencies (the nearest is chosen, or two if they are equidistant):

<table>
<thead>
<tr>
<th>Annual frequency</th>
<th>Seasonal</th>
<th>Trading days</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>(2\pi/12, 4\pi/12, 6\pi/12, 8\pi/12, 12\pi/12)</td>
<td>d, 2.714</td>
</tr>
<tr>
<td>6</td>
<td>(2\pi/6, 4\pi/6)</td>
<td>d</td>
</tr>
<tr>
<td>4</td>
<td>(2\pi/4)</td>
<td>d, 1.292, 1.850, 2.128</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>d</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>d</td>
</tr>
</tbody>
</table>

where \( d \) is computed as follows:

when \( s \) is the frequency of the series, then \( n = \frac{365.25}{s} \) and \( d = \frac{2\pi}{7 \cdot (n \text{ modulo } 7)} \).
**Autoregressive spectrum**

Autoregressive spectrum estimator is defined as follows\(^{151}\):

\[
\hat{s}(\hat{\lambda}) = 10 \log_{10} \left( \frac{\hat{\delta}_m^2}{2\pi \left( \sum_{j=1}^{m} \hat{\phi}_j e^{i2\pi\hat{\lambda}j} \right)^2} \right),
\]

where:
- \(\hat{\lambda}\) – frequency, \(0 \leq \hat{\lambda} \leq 0.5\);
- \(\hat{\delta}_m^2\) – the sample variance of the residuals;
- \(\hat{\phi}_j\) – coefficients from regression \(x_i - \bar{x}\) on \(x_{i-j} - \bar{x}\), \(1 \leq j \leq m\).

**Visual spectral analysis**

Criterion of "visual significance" is based on the range \(s_{\text{max}}^{\hat{\lambda}} - s_{\text{min}}^{\hat{\lambda}}\) of the \(\hat{s}(\hat{\lambda})\) values, where:

\[
s_{\text{max}}^{\hat{\lambda}} = \max_k \hat{s}(\hat{\lambda}_k);
\]

\[
s_{\text{min}}^{\hat{\lambda}} = \min_k \hat{s}(\hat{\lambda}_k);
\]

\(\hat{s}(\hat{\lambda}_k) - k-th\) value of autoregressive spectrum estimator.

The particular value is considered to be visually significant if \(\hat{s}(\hat{\lambda}_k)\) at a trading day or at a seasonal frequency \(\hat{\lambda}_k\) (other than the seasonal frequency \(\hat{\lambda}_{60} = 0.5\)) is above the median of the plotted values of \(\hat{s}(\hat{\lambda}_k)\) and is larger than both neighboring values \(\hat{s}(\hat{\lambda}_{k-1})\) and \(\hat{s}(\hat{\lambda}_{k+1})\) by at least 6/52 times the range \(s_{\text{max}} - s_{\text{min}}\).

For a given series \( Y_{t, 0 \leq t < T} \), which may contain missing values, the periodogram is computed as follows:

In the first step, the series is standardized:

\[
    z_t = \frac{Y_t - \bar{Y}}{\sigma(Y)},
\]

In a second step we compute at the so-called Fourier frequencies \( (\omega = \frac{2\pi}{T}, 0 \leq i < \frac{T + 1}{2}) \), which are the values of the periodogram:

\[
    \frac{2}{N} \left| \sum_{t=0, \text{defined}}^{T} z_t e^{i\omega t} \right|,
\]

where \( N \) is the number of non-missing values.

Under the white noise assumption, the values of the periodogram should be asymptotically distributed as a Chi-square with 2 degrees of freedom.

The default frequency \( td \) for trading days is computed as follows (for series of quarterly series):

\[
    td = \frac{2\pi}{7} \left( n - 7 \cdot \left\lfloor \frac{n}{7} \right\rfloor \right),
\]

where:

\[
    n = \frac{365.25}{q}, q = 4.
\]

Other frequencies correspond to trading days frequencies:

- For monthly series, 2.714 (default = 2.188);
- For quarterly series, 1.292, 1.850, 2.128 (default = 0.280).
10A Revision histories

Revisions are calculated as a difference between the first (earliest) adjustment of an observation computed when that observation is the final period of the time series (concurrent adjustment, denotes as $A_{t_0}$) and a later adjustment based on all data span (most recent adjustment, denotes as $A_{t_N}$).

In case of multiplicative decomposition the revision history of the seasonal adjustment from time $N_0$ to $N_1$ is a sequence of $R_{tN}$ calculated in a following way$^{152}$:

$$R_{tN}^A = 100 \times \frac{A_{tN} - A_{t_0}}{A_{t_0}}.$$

The revision history of the trend is calculated in a similar way:

$$R_{tN}^T = 100 \times \frac{T_{tN} - T_{t_0}}{T_{t_0}}.$$

With additive decomposition $R_{tN}^A$ is calculated in the same way if all values $A_{t_0}$ have the same sign$^{153}$. Otherwise differences are calculated as:

$$R_{tN}^A = A_{tN} - A_{t_0}.$$

The analogous quantities are calculated for final trend.

---


11A Sliding spans

Each period (month or quarter) which belongs to more than one span is examined to see if its seasonal adjustments vary more than a specified amount across the spans\(^{154}\). For multiplicative decomposition seasonal factor is regarded to be unreliable if the following condition is fulfilled:

\[
SS_t = \frac{\max_{k \in N_t} S_t(k) - \min_{k \in N_t} S_t(k)}{\min_{k \in N_t} S_t(k)} > 0.03,
\]

For additive decomposition Demetra+ uses the rule below for checking the reliability of the seasonal factor:

\[
SS_t = \frac{\max_{k \in N_t} S_t(k) - \min_{k \in N_t} S_t(k)}{\sqrt{\frac{\sum y_t^2}{n}}} > 0.03.
\]

where:

- \(S_t(k)\) – the seasonal factor estimated from span \(k\) for month \(t\);
- \(N_t = \{k : \text{period } t \text{ is in the } k\text{-th span}\}\);
- \(y\) – original series;
- \(n\) – number of observations in original series.

For seasonally and trading days adjusted series the following statistic is being calculated:

\[
\max_j A^j - \min_j A^j \over \min_j A^j,
\]

where the index \(j\) ranges over all spans containing month \(t\).

The value $A_t$ is considered to be unreliable if it is higher than 0.03. If both period $t$ and $t-1$ belong to at least two spans, the seasonally adjusted period-to-period percentage changes
\[
\left(\frac{100(A_t - A_{t-1})}{A_{t-1}}\right)
\]
are marked as unstable if\textsuperscript{155}:
\[
\max_j \frac{A^j_t}{A^j_{t-1}} \cdot \min_j \frac{A^j_t}{A^j_{t-1}} > 0.03,
\]
where:

- $A_t(k)$ – the seasonally (or trading day) adjusted value from span $k$ for month $t$;
- $N1(t) = \{k : \text{period } t \text{ and } t-1 \text{ are in the } k\text{-th span}\}$;

The index $j$ ranges over all spans containing month $t$.

\textsuperscript{155} 'X-12-ARIMA Reference Manual' (2007).
12A Tests

**Doornik-Hansen test**

Doornik-Hansen test is defined as follows: let $s$ = skweness, $k$ = kurtosis of the $n$ (non-missing) residuals.

We make the following transformations:

*Transformation of the skewness (D'Agostino):*

$$
\beta = \frac{3(n^2 + 27n - 70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)},
$$

$$
\omega^2 = -1 + \sqrt{2(\beta-1)},
$$

$$
\delta = \frac{1}{\sqrt{0.5 \log \omega^2}},
$$

$$
y = s \sqrt{\frac{(\omega^2 - 1)(n+1)(n+3)}{12(n-2)}},
$$

$$
z_1 = \delta \log(y + \sqrt{y^2 - 1}).
$$

*Transformation of the kurtosis (Wilson-Hilferty):*

$$
\delta = (n-3)(n+1)(n^2 + 15n - 4),
$$

$$
a = \frac{(n-2)(n+5)(n+7)(n^2 + 27n - 70)}{6\delta},
$$

$$
c = \frac{(n-7)(n+5)(n+7)(n^2 + 2n - 5)}{6\delta},
$$

$$
l = \frac{(n+5)(n+7)(n^3 + 37n^2 + 11n - 313)}{12\delta},
$$

$$
\alpha = a + c \cdot s \cdot s,
$$

$$
\chi = 2l(k - 1 - s^2),
$$

$$
z_2 = (\sqrt{9\alpha}) \left( \frac{1}{9\alpha} - 1 + \sqrt{\frac{\chi}{2\alpha}} \right).$$
Then the Doornik-Hansen test statistic is defined as the sum of squared transformations of the skweness and kurtosis. Asymptotically the test statistic follows a chi-square distribution, i.e.:

\[ DH = z_1^2 + z_2^2 - \chi^2(2). \]

**Ljung-Box test**

Ljung-Box test is defined as follows:

Let \( \rho_j \) the sample autocorrelation at rank \( j \) of the \( n \) residuals. The Ljung-Box statistics is:

\[ LB(k) = n \cdot (n - 2) \sum_{j=1}^{k} \frac{\rho_j^2}{n - j}. \]

If the residuals are random, it will be distributed as \( \chi^2(k - np) \), where \( np \) is the number of hyper-parameters of the model from which the residuals are derived.

**Wald-Wolfowitz test (Runs test)**

Wald-Wolfowitz test is test for randomness of the residuals. Generally, it examines the hypothesis that a series of numbers is random. For data centred around the mean the test calculates the number and length of runs. A run is defined as a set of sequential values that are either all above or below the mean. An up run is a sequence of numbers each of which is above the mean; a down run is a sequence of numbers each of which is below the mean.

The test checks if the number of up and down runs are distributed equally in time. Both too many runs and too few runs are unlikely a real random sequence. The null hypothesis is that the values of the series has been independently drawn from the same distribution. The test also verify the hypothesis that the length of runs is random.

**Seasonality tests**

This section presents the set of seasonality tests calculated by Demetra+. Detailed description of these tests and testing procedure is available in LADIRAY D. and QUENNEVILLE B. (1999).

- **Non-parametric tests**
  - **Friedman test (stable seasonality test)**

Friedman test is a non-parametric method for testing that samples are drawn from the same population or from populations with equal medians. In the regression equation the significance of
the month (or quarter) effect is tested. Friedman test requires no distributional assumptions. It uses the rankings of the observations.

Seasonal adjustment procedures use Friedman test for checking the presence of seasonality. Friedman test is called a stable seasonality test. This test uses preliminary estimation of the unmodified Seasonal-Irregular component\(^{156}\) (for X12 this time series is shown in Table B3) from which \(k\) samples are derived (\(k = 12\) for monthly series and \(k = 4\) for quarterly series) of size \(n_1, n_2, \ldots, n_k\) respectively. Each \(k\) corresponds to a different level of seasonality. It is assumed that seasonality affect only the means of the distribution and not their variance. Assuming that each sample is derived from a random variable \(X_j\) following the normal distribution with mean \(m_j\) and standard deviation \(\sigma\) the null hypothesis:

\[
H_0: \quad m_1 = m_2 = \ldots = m_k
\]

is tested against:

\[
H_1: \quad m_p \neq m_q \text{ for the least one pair } (p, q).
\]

The test uses the following decomposition of the variance:

\[
\sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{x})^2 = \sum_{j=1}^{k} n_j (x_{*j} - \bar{x*})^2 + \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - x_{*j})^2,
\]

where \(\bar{x}_{*j}\) is the average of \(j\)-th sample.

The total variance is therefore broken down into a variance of the averages due to seasonality and a residual seasonality.

The test statistics is calculated as:

\[
F_S = \frac{\sum_{j=1}^{k} n_j (x_{*j} - \bar{x*})^2}{k-1} \sim F(k-1, n-k),
\]

\[
\sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - x_{*j})^2
\]

\[
n - k
\]

where \(k-1\) and \(n-k\) are degrees of freedom.

The number of observations in preliminary estimation of the unmodified Seasonal-Irregular is lower than in final estimation of the unmodified Seasonal-Irregular component. Because of that the number of degrees of freedom in stable seasonality test is lower than number of degrees of

\(^{156}\) Unmodified Seasonal-Irregular component is the Seasonal-Irregular factors with the extreme values.
freedom in test for the presence of seasonality assuming stability (see 4.4.3) (e.g. X12 uses centered moving average of order 12 to calculate the preliminary estimation of trend-cycle. As a result the first six and last six points in the series are not computed at this stage of calculation. Preliminary estimation of trend-cycle is then used for calculation the preliminary estimation of the unmodified Seasonal-Irregular).

If the null hypothesis of no stable seasonality is not rejected at the 0.10% significance level ($P_s \geq 0.001$), then the series is considered to be non-seasonal.

- **Kruskal-Wallis test**

Kruskal-Wallis test is a non-parametric test used for comparing samples from two or more groups. The null hypothesis states that all months (or quarters, respectively) have the same mean.

The test is calculated for the final estimation of the unmodified Seasonal-Irregular component from which $k$ samples $A_j$ are derived ($k = 12$ for monthly series and $k = 4$ for quarterly series) of size $n_1, n_2, ... n_k$ respectively.

The test is based on the statistic:

$$ W = \frac{12}{n(n+1)} \sum_{j=1}^{k} \frac{S_j^2}{n_j} - 3(n+1), $$

where $S_j$ is the sum of the ranks of the observations from the sample $A_j$ within the whole sample of $n = \sum_{j=1}^{k} n_j$ observations.

Under the null hypothesis the test statistic follows a chi-square distribution with $k - 1$ degrees of freedom.

- **Test for the presence of seasonality assuming stability**

The test statistics and testing hypothesis are the same as for Friedman stable seasonality test. The test statistics is calculated for final estimation of the unmodified Seasonal-Irregular Component (in case of X12 this series is presented in Table D8).
• **Evolutive seasonality test (Moving seasonality test)**

The test is based on a two-way analysis of variance model. The model uses the values from complete years only. For the Seasonal-Irregular component it uses one of the following models, depending on the type of the decomposition:

Multiplicative: \[ SI_{ij} - 1 = X_{ij} = b_i + m_j + e_{ij}, \]

Additive: \[ SI_{ij} = X_{ij} = b_i + m_j + e_{ij}, \]

where:

- \(m_j\) refers to the monthly or quarterly effect for \(j\)-th period, \(j = (1,\ldots,k)\) where \(k = 12\) for monthly series and \(k = 4\) for quarterly series;
- \(b_j\) refers to the annual effect \(i\) \((i = 1,\ldots,N)\) where \(N\) is the number of complete years;
- \(e_{ij}\) represents the residual effect.

The test is based on the decomposition \(S^2 = S_B^2 + S_R^2 + S_A^2\) where:

\[ S^2 = \sum_{j=1}^{k} \sum_{i=1}^{N} (X_{ij} - \bar{X})^2 \text{ – the total sum of squares,} \]

\[ S_A^2 = N \sum_{j=1}^{k} (X_{ij} - \bar{X})^2 \text{ – the inter-month (inter-quarter, respectively) sum of squares, which mainly measures the magnitude of the seasonality,} \]

\[ S_B^2 = k \sum_{j=1}^{N} (X_{ij} - \bar{X})^2 \text{ – the inter-year sum of squares, which mainly measures the year-to-year movement of seasonality,} \]

\[ S_R^2 = \sum_{i=1}^{N} \sum_{j=1}^{k} (X_{ij} - \bar{X}_{ij} = \bar{X}_{ij} + \bar{X}_{ij}^*)^2 \text{ – the residual sum of squares.} \]

The null hypothesis \(H_0\) is that \(b_1 = b_2 = \ldots = b_N\), which means that there is no change in seasonality over the years.

This hypothesis is verified by the following test statistics:

\[ F_M = \frac{\frac{S_B^2}{(n-1)}}{\frac{S_R^2}{(n-1)(k-1)}}, \]
which follows the $F$-distribution with $k - 1$ and $n - k$ degrees of freedom.

- **Test for presence of identifiable seasonality**

  This test combines the values of the F-statistic (of parametric test for stable seasonality) and the values of the moving seasonality test, which was described above.

  The test statistic is:

  $$ T = \left( \frac{7}{F_S} + \frac{3F_M}{F_S} \right)^{0.5} $$

  where $F_S$ is a stable seasonality test statistic and $F_M$ is moving seasonality test statistic. The test checks if the stable seasonality is not dominated by moving seasonality. In such case the seasonality is regarded as identifiable. The detailed description of the test is available in LOTHIAN, J., and MORRY, M. (1978).

- **Combined seasonality test**

  This test combines the Kruskal-Wallis test along with test for the presence of seasonality assuming stability, evaluative seasonality test for detecting the presence of identifiable seasonality. All those tests are calculated using the final unmodified S-I component. The main purpose of the combined seasonality test is to check whether the seasonality of the series is identifiable. For example, identification of the seasonal pattern is problematic if the process is dominated by highly moving seasonality\(^{157}\). The testing procedure is shown below:

Test for the presence of stable seasonality at 0.1% level ($F_S$)

$H_0$ not rejected $\rightarrow$ Test for the presence of moving seasonality at the 5% level ($F_M$)

$H_0$ not rejected $\rightarrow$ Test for the presence of identifiable seasonality

$T = \left(\frac{7}{F_S} + \frac{3F_M}{F_S}\right)^{0.5}$

Failure if $T \geq 1$

$H_0$ not rejected $\rightarrow$ No identifiable seasonality present

$H_0$ rejected $\rightarrow$ Test for the presence of identifiable seasonality

$T = \left(\frac{7}{F_S} + \frac{3F_M}{F_S}\right)^{0.5}$

Failure if $\frac{7}{F_S} \geq 1$ or $\frac{3F_M}{F_S} \geq 1$

$H_0$ rejected $\rightarrow$ Non-parametric Kruskal-Wallis test at the 0.1% level

$H_0$ not rejected $\rightarrow$ Probably no identifiable seasonality present

$H_0$ rejected $\rightarrow$ Identifiable seasonality present

13A Code to generate simple seasonal adjustments (C#)

(Some namespaces have been removed to simplify the reading)

```csharp
// creates a new time series
// parameters: frequency/first year/first period (0-based)/array of doubles/copy
data (uses the current array or creates a copy)
TSData s = new TSData(12, 1967, 0, g_prodind, false);

// basic processing

// tramo-seats specification. RSA5 (full automatic)
tr amoSeats.Specification ts_spec = TramoSeats.Specification.RSA5;

// launches tramo-seats core engine
TramoSeats.Monitor ts_monitor = new TramoSeats.Monitor();
// executes the processing
TramoSeats.TramoSeatsResults ts_rslts = ts_monitor.Process(s, ts_spec);

// x12 specification. equivalent RSA5 (full automatic)

// launches tramo-seats core engine
X12.Monitor x_monitor = new X12.Monitor();
// executes the processing
X12.X12Results x_rslts = x_monitor.Process(s, x_spec);

// seasonally adjusted series
TSData ts_sa = ts_rslts.Series(SAComponentType.CSA);
TSData x_sa = x_rslts.Series(SAComponentType.CSA);

// computes differences between both results...
TSData diff = ts_sa - x_sa;

// computes statistics on the differences...
DescriptiveStatistics stats = new DescriptiveStatistics(diff.Values);

double max = stats.Max, min = stats.Min, rmse = Math.Sqrt(stats.SumSquare / diff.Length);

// more advanced uses (computed "on the fly")
Periodogram periodogram = new Periodogram(x_rslts.X11Results.DTables["D8"].Values);

// roots of the moving average polynomial of the arima model used by Seats
Complex[] roots = ts_rslts.Seats.SArima.MA.Roots();
```
### 14A The content of the output in CSV files

The output files produced by Demetra+ in simple CSV files may include:

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>original time series</td>
</tr>
<tr>
<td>yc</td>
<td>interpolated time series (i.e. original time series with missing values replaced by their estimates)</td>
</tr>
<tr>
<td>yl</td>
<td>original time series without regression effect (linearized series)</td>
</tr>
<tr>
<td>yc</td>
<td>calendar adjusted series</td>
</tr>
<tr>
<td>y_f</td>
<td>the forecasts of the original time series</td>
</tr>
<tr>
<td>t</td>
<td>trend component</td>
</tr>
<tr>
<td>t_f</td>
<td>forecasts of the trend component</td>
</tr>
<tr>
<td>sa</td>
<td>seasonally adjusted series</td>
</tr>
<tr>
<td>s</td>
<td>seasonal component</td>
</tr>
<tr>
<td>s_f</td>
<td>the forecasts of the seasonal component</td>
</tr>
<tr>
<td>i</td>
<td>irregular component</td>
</tr>
<tr>
<td>tl</td>
<td>trend component without regression effect</td>
</tr>
<tr>
<td>sal</td>
<td>seasonally adjusted series without regression effect</td>
</tr>
<tr>
<td>sl</td>
<td>seasonal component without regression effect</td>
</tr>
<tr>
<td>il</td>
<td>irregular component without regression effect</td>
</tr>
<tr>
<td>cal</td>
<td>calendar effect</td>
</tr>
<tr>
<td>td</td>
<td>trading days (or working days) effect</td>
</tr>
<tr>
<td>mh</td>
<td>moving holiday effect</td>
</tr>
<tr>
<td>det</td>
<td>deterministic effects (i.e. sum of calendar effect, outliers effect and other regression effects)</td>
</tr>
<tr>
<td>det_y</td>
<td>deterministic effect on the original time series</td>
</tr>
<tr>
<td>det_sa</td>
<td>deterministic effect on the seasonally adjusted time series</td>
</tr>
<tr>
<td>det_t</td>
<td>deterministic effect on the trend component</td>
</tr>
<tr>
<td>det_s</td>
<td>deterministic effect on the seasonal component</td>
</tr>
<tr>
<td>det_i</td>
<td>deterministic effect on the irregular component</td>
</tr>
<tr>
<td>reg</td>
<td>total regression effect</td>
</tr>
<tr>
<td>reg_y</td>
<td>regression effect on the original time series</td>
</tr>
<tr>
<td>reg_t</td>
<td>regression effect on the trend component</td>
</tr>
<tr>
<td>reg_sa</td>
<td>regression effect on the seasonally adjusted time series</td>
</tr>
<tr>
<td>reg_s</td>
<td>regression effect on the seasonal component</td>
</tr>
<tr>
<td>reg_i</td>
<td>regression effect on the irregular component</td>
</tr>
<tr>
<td>out</td>
<td>total outliers effect</td>
</tr>
<tr>
<td>out_t</td>
<td>outliers effect on the trend component</td>
</tr>
<tr>
<td>Item</td>
<td>Description</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>out_s</td>
<td>outliers effect on the seasonal component</td>
</tr>
<tr>
<td>out_i</td>
<td>outliers effect on the irregular component</td>
</tr>
<tr>
<td>cal_f</td>
<td>the forecasts of the calendar component</td>
</tr>
<tr>
<td>td_f</td>
<td>the forecasts of the trading days (or working days) effect</td>
</tr>
<tr>
<td>mh_f</td>
<td>the forecasts of the moving holiday effect</td>
</tr>
<tr>
<td>det_f</td>
<td>the forecasts of the deterministic effect</td>
</tr>
<tr>
<td>det_y_f</td>
<td>the forecasts of the deterministic effect on the original time series</td>
</tr>
<tr>
<td>det_sa_f</td>
<td>the forecasts of the deterministic effect on the seasonally adjusted time series</td>
</tr>
<tr>
<td>det_t_f</td>
<td>the forecasts of the deterministic effect on the trend component</td>
</tr>
<tr>
<td>det_s_f</td>
<td>the forecasts of the deterministic effect on the seasonal component</td>
</tr>
<tr>
<td>det_i_f</td>
<td>the forecasts of the deterministic effect on the irregular component</td>
</tr>
<tr>
<td>reg_f</td>
<td>the forecasts of the regression effect</td>
</tr>
<tr>
<td>reg_y_f</td>
<td>the forecasts of the regression effect on the original time series</td>
</tr>
<tr>
<td>reg_t_f</td>
<td>the forecasts of the regression effect on the trend component</td>
</tr>
<tr>
<td>reg_sa_f</td>
<td>the forecasts of the regression effect on the seasonally adjusted time series</td>
</tr>
<tr>
<td>reg_s_f</td>
<td>the forecasts of the regression effect on the seasonal component</td>
</tr>
<tr>
<td>reg_i_f</td>
<td>the forecasts of the regression effect on the irregular component</td>
</tr>
</tbody>
</table>

The output files produced by Demetra+ as CSV matrix may include:

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>series</td>
<td>series name</td>
</tr>
<tr>
<td>span.start</td>
<td>the date of the first observation of the time series</td>
</tr>
<tr>
<td>span.end</td>
<td>the date of the last observation of the time series</td>
</tr>
<tr>
<td>span.n</td>
<td>the number of observations of the time series</td>
</tr>
<tr>
<td>likelihood.bic</td>
<td>the value of Bayesian information criteria for the model estimated by RegArima model</td>
</tr>
<tr>
<td>residuals.stderr</td>
<td>standard error of the residuals from RegArima model</td>
</tr>
<tr>
<td>residuals.skewness</td>
<td>skewness of the residuals from RegArima model</td>
</tr>
<tr>
<td>residuals.kurtosis</td>
<td>kurtosis of the residuals from RegArima model</td>
</tr>
<tr>
<td>residuals.lb</td>
<td>the outcome of the Ljung-Box test of autocorrelation performed on the residuals</td>
</tr>
<tr>
<td>residuals.lb2</td>
<td>the outcome of the Ljung-Box test of autocorrelation performed on the squared residuals</td>
</tr>
<tr>
<td>residuals.seaslb</td>
<td>the outcome of the Ljung-Box test of autocorrelation performed on the residuals (autocorrelation between seasonal lags)</td>
</tr>
<tr>
<td>mstatistics.m1</td>
<td>quality measure M1</td>
</tr>
<tr>
<td>mstatistics.m2</td>
<td>quality measure M2</td>
</tr>
<tr>
<td>Item</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>mstatistics.m3</td>
<td>quality measure M3</td>
</tr>
<tr>
<td>mstatistics.m4</td>
<td>quality measure M4</td>
</tr>
<tr>
<td>mstatistics.m5</td>
<td>quality measure M5</td>
</tr>
<tr>
<td>mstatistics.m6</td>
<td>quality measure M6</td>
</tr>
<tr>
<td>mstatistics.m7</td>
<td>quality measure M7</td>
</tr>
<tr>
<td>mstatistics.m8</td>
<td>quality measure M8</td>
</tr>
<tr>
<td>mstatistics.m9</td>
<td>quality measure M9</td>
</tr>
<tr>
<td>mstatistics.m10</td>
<td>quality measure M10</td>
</tr>
<tr>
<td>mstatistics.m11</td>
<td>quality measure M11</td>
</tr>
<tr>
<td>mstatistics.q</td>
<td>quality measure Q</td>
</tr>
<tr>
<td>mstatistics.q-m2</td>
<td>quality measure Q without M2</td>
</tr>
<tr>
<td>diagnostics.quality</td>
<td>Summary test result</td>
</tr>
<tr>
<td>diagnostics.basic</td>
<td>basic checks: Definition test - result and statistics</td>
</tr>
<tr>
<td>checks.definition:2</td>
<td></td>
</tr>
<tr>
<td>diagnostics.basic</td>
<td>basic checks: Annual totals test - result and statistics</td>
</tr>
<tr>
<td>checks.annual totals:2</td>
<td></td>
</tr>
<tr>
<td>diagnostics.visual spectral</td>
<td>the result of visual spectral analysis of seasonal peaks in residuals</td>
</tr>
<tr>
<td>analysis.spectral seas peaks</td>
<td>from RegArima model</td>
</tr>
<tr>
<td>diagnostics.visual spectral</td>
<td>the result of visual spectral analysis of trading days peaks in residuals</td>
</tr>
<tr>
<td>analysis.spectral td peaks</td>
<td>from RegArima model</td>
</tr>
<tr>
<td>diagnostics.regarima</td>
<td>test of normality of the residuals from RegArima model - result and</td>
</tr>
<tr>
<td>residuals.normality:2</td>
<td>statistics</td>
</tr>
<tr>
<td>diagnostics.regarima</td>
<td>test of normality of the residuals from RegArima model - result and</td>
</tr>
<tr>
<td>residuals.independence:2</td>
<td>statistics</td>
</tr>
<tr>
<td>diagnostics.regarima</td>
<td>the result and test statistic of visual spectral analysis of trading days</td>
</tr>
<tr>
<td>residuals.spectral td peaks:2</td>
<td>peaks in residuals from RegArima model</td>
</tr>
<tr>
<td>diagnostics.regarima</td>
<td>the result and test statistic of visual spectral analysis of seasonal</td>
</tr>
<tr>
<td>residuals.spectral seas peaks:2</td>
<td>peaks in residuals from RegArima model</td>
</tr>
<tr>
<td>diagnostics.residual</td>
<td>the result and test statistic of residual seasonality test for residuals</td>
</tr>
<tr>
<td>seasonality.on sa:2</td>
<td>from RegArima model</td>
</tr>
<tr>
<td>diagnostics.residual</td>
<td>the result and test statistic of residual seasonality test for residuals</td>
</tr>
<tr>
<td>seasonality.on sa (last 3</td>
<td>(last 3 years only) from RegArima model</td>
</tr>
<tr>
<td>years):2</td>
<td></td>
</tr>
<tr>
<td>diagnostics.residual</td>
<td>the result and test statistic of residual seasonality test for irregular</td>
</tr>
<tr>
<td>seasonality.on irregular:2</td>
<td>component</td>
</tr>
<tr>
<td>diagnostics.seats.seas</td>
<td>the result and test statistic of the test that checks if variance of the</td>
</tr>
<tr>
<td>variance:2</td>
<td>estimators of the seasonal component is close to the variance of its</td>
</tr>
<tr>
<td></td>
<td>actual estimates</td>
</tr>
<tr>
<td>Item</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
</tr>
<tr>
<td>diagnostics.seats.irregular variance:2</td>
<td>the result and test statistic of the test that checks if variance of the estimators of the irregular component is close to the variance of its actual estimates</td>
</tr>
<tr>
<td>diagnostics.seats.seas/irr cross-correlation:2</td>
<td>the result and test statistic of the test that checks the theoretical crosscorrelation (between estimators) and empirical cross-correlation (between empirical estimates)</td>
</tr>
<tr>
<td>log</td>
<td>informs whether the series has been log-transformed (1) or not (0)</td>
</tr>
<tr>
<td>arima.mean</td>
<td>informs whether the mean (a constant term) is part of the Arima model (1) or not (0)</td>
</tr>
<tr>
<td>arima.p</td>
<td>the order of nonseasonal autoregressive polynomial in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.d</td>
<td>the order of nonseasonal differencing in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.q</td>
<td>the order of nonseasonal moving average polynomial in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.bp</td>
<td>the order of seasonal autoregressive polynomial in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.bd</td>
<td>the order of seasonal differencing in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.bq</td>
<td>the order of seasonal moving average polynomial in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.phi(1)</td>
<td>the estimated value of (\phi(1)) coefficient (coefficient assigned to the first lag in nonseasonal autoregressive polynomial) in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.phi(2)</td>
<td>the estimated value of (\phi(2)) coefficient (coefficient assigned to the second lag in nonseasonal autoregressive polynomial) in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.phi(3)</td>
<td>the estimated value of (\phi(3)) coefficient (coefficient assigned to the third lag in nonseasonal autoregressive polynomial) in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.phi(4)</td>
<td>the estimated value of (\phi(4)) coefficient (coefficient assigned to the fourth lag in nonseasonal autoregressive polynomial) in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>arima.th(1)</td>
<td>the estimated value of (\theta(1)) coefficient (coefficient assigned to the first lag in nonseasonal moving average polynomial) in Arima model ((p, d, q)(BP, BD, BQ))</td>
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<tr>
<td>arima.th(2)</td>
<td>the estimated value of (\theta(2)) coefficient (coefficient assigned to the second lag in nonseasonal moving average polynomial) in Arima model ((p, d, q)(BP, BD, BQ))</td>
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<td>arima.th(3)</td>
<td>the estimated value of (\theta(3)) coefficient (coefficient assigned to the third lag in nonseasonal moving average polynomial) in Arima model ((p, d, q)(BP, BD, BQ))</td>
</tr>
<tr>
<td>Item</td>
<td>Description</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>arima.th(4)</td>
<td>the estimated value of th(4) coefficient (coefficient assigned to the fourth lag in nonseasonal moving average polynomial) in Arima model ( (p, d, q)(BP, BD, BQ) )</td>
</tr>
<tr>
<td>arima.bphi(1)</td>
<td>the estimated value of phi(1) coefficient (coefficient assigned to the first lag in seasonal autoregressive polynomial) in Arima model ( (p, d, q)(BP, BD, BQ) )</td>
</tr>
<tr>
<td>arima.bth(1)</td>
<td>the estimated value of th(1) coefficient (coefficient assigned to the first lag in seasonal moving average polynomial) in Arima model ( (p, d, q)(BP, BD, BQ) )</td>
</tr>
<tr>
<td>calendar.lp</td>
<td>informs whether the series has been corrected for leap year (1) or not (0)</td>
</tr>
<tr>
<td>calendar.td(1):2</td>
<td>value and the T-Stat for the first trading days regressor (Monday)</td>
</tr>
<tr>
<td>calendar.td(2):2</td>
<td>value and the T-Stat for the second trading days regressor (Tuesday)</td>
</tr>
<tr>
<td>calendar.td(3):2</td>
<td>value and the T-Stat for the third trading days regressor (Wednesday)</td>
</tr>
<tr>
<td>calendar.td(4):2</td>
<td>value and the T-Stat for the fourth trading days regressor (Thursday)</td>
</tr>
<tr>
<td>calendar.td(5):2</td>
<td>value and the T-Stat for the fifth trading days regressor (Friday)</td>
</tr>
<tr>
<td>calendar.td(6):2</td>
<td>value and the T-Stat for the sixth trading days regressor (Saturday)</td>
</tr>
<tr>
<td>calendar.td(7):2</td>
<td>value and the T-Stat for the seventh trading days regressor (Sunday)</td>
</tr>
<tr>
<td>calendar.easter:2</td>
<td>length, value and the T-Stat for the Easter effect</td>
</tr>
<tr>
<td>outliers.out(n):3</td>
<td>location, value and T-Stat for the n-th outlier</td>
</tr>
</tbody>
</table>
15A Autocorrelation function and partial autocorrelation function

**Autocorrelation function**

Correlation is a measure of the strength and the direction of a linear relationship between two variables. For time series the correlation can refer to the relation between its observations, e.g. between current observation and observation lagged by given number of units. In this case all observations come from one variable, so similarity between a given time series and a $k$-lagged version of itself over successive time intervals is called autocorrelation.

The autocorrelation coefficient at lag $k$ is defined as:

$$
\rho(k) = \frac{\sum_{t=k+1}^{n} (x_t - \bar{x})}{\sum_{t=1}^{n} (x_t - \bar{x})^2}
$$

where:

- $x_t$ – time series;
- $n$ – total number of observations;
- $\bar{x}$ – mean of the time series.

The set of autocorrelation coefficients ($k$) arranged as a function of $k$ is autocorrelation function (ACF). The graphical or numerical representation of ACF is called autocorrelogram. The autocorrelogram presents not only autocorrelation coefficients but also the confidence intervals. If the autocorrelation coefficient is in the confidence interval, it is regarded as statistically insignificant. Therefore, the user should focus on the values which absolute value is greater than confidence interval. In Demetra+ the confidence interval is indicated by two green, horizontal, dotted lines.
The autocorrelation plot is a tool for identification and estimation of the Arima model. It can show e.g. whether the series is stationary or not.

**Partial autocorrelation function**

The partial autocorrelation is a tool for identification and estimation of the Arima model. It is defined as the amount of correlation between two variables which is not explained by their mutual correlations with a given set of other variables.

Partial autocorrelation at lag $k$ is defined as the autocorrelation between $x_t$ and $x_{t-k}$ that is not accounted for by lags 1 through $k-1$ which means that correlations with all the elements within the lag $k$ are removed. Following the definition, partial autocorrelation for lag of 1 is equivalent to autocorrelation.

Partial autocorrelation function (PACF) is the set of partial autocorrelation coefficients ($k$) arranged as a function of $k$. This function can be used to detect the presence of autoregressive process in time series and identify the order of this process. Theoretically, the number of significant lags determines the order of autoregressive process.

The PACF graph presents partial autocorrelation coefficients and the confidence intervals. If the partial autocorrelation coefficient is in the confidence interval, it is regarded as statistically insignificant. Therefore, the user should focus on the values which absolute value is greater that confidence interval. In Demetra+ the confidence interval is indicated by two green, horizontal, dotted lines.
REFERENCES


