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Saving in an Ageing Society with Public Pensions: implications from lifecycle analysis

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Abstract

This paper studies saving in an economy where decline in fertility to a permanently lower level and increasing longevity are changing the age structure permanently and where the public pension system helps to smooth consumption over a lifetime of working and retirement. A simple overlapping generations (OLG) model is used for simulations, with the emphasis on the transition path. It is shown that, under plausible assumptions, the effect of population ageing on the capital to income ratio is positive and also that the saving rate increases in the first two to three decades. This first positive effect on the saving rate is highlighted and contrasted with results in previous literature. It is also shown that moving from a pure PAYG pension system to partial funding of mandatory pensions affects saving positively and has implications for intergenerational equity.

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1. Introduction

The purpose of this paper is to study saving and capital accumulation in an economy where population is ageing due to a decline in fertility to a permanently lower level and a gradual increase in longevity (given the below reproduction level fertility rates and the projected increase in longevity over the next 50 years or so in Europe and elsewhere). Private agents are assumed to work and then retire at a certain age (which can change). Public pension system provides benefits financed from pension contributions collected from wages. People optimise their lifetime consumption path, taking into account the consumption needs of the household when their children stay at home and the fall in their disposable income when they retire. They may receive and leave behind bequests.

The literature on the effect of population ageing on saving and capital stock is extensive but inconclusive: even the sign of the effect is often ambiguous. Here, we try to find explanations for the diverging results and to understand the issues better.

Starting with the simplest possible OLG model, the effect of a permanent decline in fertility on saving must be negative as the rate of growth of the economy declines. Comparing steady states, a lower saving rate supports a given capital to income ratio if the rate of growth determined by fertility declines. This result is often generalised for the transition path, and thus the change is understood to take place monotonically. Alternatively, this result is derived from a large-scale dynamic general equilibrium model. A notable example of this can be found in Börsch-Supan et al. (2006) where various dynamic simulations are reported with the common feature that saving rates decline steadily under population ageing. It is probably fair to say that most authors take it as a relatively robust result that in the long run lower fertility is associated with a lower saving rate, and for many economists the long run starts now, meaning that the saving rate will decline steadily.

However, even in a simple OLG model the increase in longevity, assuming that retirement age increases less than proportionally to adult longevity, must be positive as people have to save more to cover their consumption needs during the extended time in retirement. Thus, the combined effect of fertility decline and longevity increase is ambiguous.

These elementary statements can also be expressed by the simple equation for saving, capital stock and growth in *steady states*:

$$S_t / Y_t = (K_t / Y_t) * g_t / (1 + g_t),$$

i.e. the saving rate S_t / Y_t, (S_t for saving, Y_t for income) is equal to the growth rate of the economy (g_t) times the capital to income ratio (K_t / Y_t), divided by $(1 + g_t)$ in discrete time analysis. The growth rate is affected negatively by the fall in fertility (via the number of workers), but the capital to income ratio may well increase with longevity. We shall see below that *ultimately* (i.e. when the new steady state is reached)*, under plausible assumptions, the combined effect of population ageing on the saving rate is slightly negative, but that, at least equally importantly, the capital to income ratio increases.

If the capital to income ratio in the new steady state is then higher than initially, it may be that over the *transition* period the saving rate first increases and decreases only later, say, after three decades. This swing may well be more relevant for policy and forecasting than the

ultimate decline emerging only in 80 years or so, i.e. after the lifetime of the representative individual, or even later if the pension system rules are changed during the process, as they probably will be. This first positive effect of population ageing on saving is the primary object of this paper. It will be shown that this is implied by plausible assumptions under an OLG model, a result that contrasts with many previous studies. The transition path will be shown to depend decisively on the alternative assumptions regarding peoples' expectations.

The rest of the paper is organised as follows: in section 2 we review the literature, with special attention on various structural factors relevant for the basic OLG model for the steady states, and make preliminary remarks on dynamic models for demographic transition. In section 3 we set up and calibrate a basic steady state OLG model for stable population age structures, and in section 4 we construct a simple dynamic model to look into population ageing, this being understood as the *transition* from one steady state to another. In section 5 we assess further the results of previous literature on the dynamic effects, especially those derived from large-scale multi-period simulation models (e.g. Börsch-Supan et al., 2006, and Fehr et al., 2006), and compare them to the results from the model in this paper. Section 6 concludes.

2. Selective review of the literature

Samuelson (1958) introduced the basic elements of both a PAYG public pension system and a fully funded system (which could be either voluntary or mandatory by law) with a model for overlapping generations (OLG) of lifecycle consumers with given endowments.[1] Diamond (1965) extended it by introducing production and an endogenous interest rate into the model. This laid the basis for the extensive literature where agents have finite life-times and, in many applications, are assumed not to leave bequests. Diamond derived results for the effect of government debt, showing for example that debt reduces total saving and thereby the permanent (steady state) capital stock. This reduces welfare in an economy which operates below the Golden Rule point, i.e. the interest rate exceeds the rate of growth (decomposed into the rate of growth and productivity of labour).

Aaron (1966) studied the case where the rate of growth of the economy is higher than the rate of interest, i.e. the economy is dynamically inefficient due to excessive saving and capital stock. In this case current consumption can be increased without reducing the consumption of future generations by the "introduction of some social insurance pensions on a pay-as-you-go basis [as this] will improve the welfare position of each person". He showed that establishing a pure PAYG pension system had the same effect as increasing government debt in the Diamond model. Since then, the negative effect on saving of either public debt or PAYG pensions has been reiterated by many, e.g. Feldstein (1974) and Feldstein and Liebman (2002), in the context of a dynamically efficient economy: saving and capital stock and therefore welfare are permanently reduced.[2]

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[1] See Weil (2008) for the jubilee of this 50 year-old paper, which has become one of the most cited in economics.

[2] Leimer and Lesnoy (1982) found a computer programming error in Feldstein (1974) and called his results into question, also otherwise pointing out that in his model social security has two opposing effects: benefits reduce the need for private saving and reduced retirement age increases it (as pensions are not sufficient for consumption needs during retirement). Based on their own estimates, Leimer and Lesnoy (1982) conclude from US data that no support is found for either positive or negative effects on saving. Feldstein (1982) presented corrected estimates with larger datasets and argued that a significant negative effect on saving was confirmed.

Barro (1974) undermined the earlier results with a model where bequests to be left behind enter the utility function of agents and therefore form an infinite chain for the dynasty. Under his assumptions public policy, be it government debt or social security, has no real effects on the economy (in other words, it makes no difference whether public expenditure is financed by taxes or debt). This then became known as Ricardian equivalence, thanks to Buchanan (1976), who reminded readers that David Ricardo had looked at basically the same issue, although the label is a paradox as Ricardo himself considered that neither the assumptions nor the implications of the equivalence theorem were valid (see, for example, Elmendorf and Mankiw, 1999).

If Barro was right there would not be much point in studying the effects of population ageing on saving, or, for that matter, on public policies related to these matters. However, as Elmendorf and Mankiw (1999) put it, most economists disagree with Barro and agree more with Ricardo; they consider that the reality differs from the world à la Barro and leaves room for the effects of public policy.[3] Consequently, extensive literature followed Diamond's (1965) pioneering OLG model for finitely living economic agents, including further theoretical results, empirical testing of the model and large-scale dynamic general equilibrium (DGE) models. On 3 July 2008 "Ageing+pensions+saving" received 356 000 hits from Google and nearly 100 from JSTOR. We confine ourselves to a selective survey and make use of two recent volumes: Creedy, J. and R. Guest, eds. (2007), *New developments in the economics of population ageing,* which is a collection of reprints of papers published during the past 20 years, most of them after 2000, and Kent et al., eds. (2006), the proceedings of the *Demography and Financial Markets* conference of the Reserve Bank of Australia in 2006, which brought together a number of leading scholars in the field.

Empirical testing of the OLG model

The implications of the multi-cohort OLG model have typically been addressed by using the contemporary dependency ratio as the explanatory variable for private or total saving (see survey in McMorrow and Röger, 2004: also Loayza, Schmidt-Hebbel and Servén, 2000, and Samwick, 2000). The results are somewhat mixed, although a negative effect is prevalent. But do these studies really test the OLG model? Especially as the ongoing increase in longevity is as important as fertility decline as a determinant of the age structure, we must keep in mind that according to the OLG model it is the *expected* time in retirement that should determine the saving decisions of working adults, rather than the *contemporary* dependency ratio.[4]

Similarly, the prevailing public pension system parameters should not be assumed to represent the expectations held by working adults for these parameters when they make their lifecycle plans. Expectations, not contemporary values, should be inserted into the OLG model application. Again, there is on average a 30-year-long lead time.

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[3] In a broader context, Barro (1974) draws attention, like Modigliani and Brumberg (1954), Ando and Modigliani (1963), and Friedman (1957), to the short-term dynamics of the saving rate: if consumers optimise over a somewhat longer time horizon, it follows that the saving rates are not constant, as was assumed in simple post-Keynesian models. This undermined post-Keynesian convictions of the effectiveness of public policies to fine-tune the economy in the short to medium term. These issues are not covered here as we deal with the long term.

[4] Bloom, Canning and Graham (2003) find empirical support for a positive effect of longevity on saving from a large international dataset. According to them, this effect is in principle temporary but may last 50 years.

Additionally, if the hypothesis to be empirically tested is derived from a steady state OLG model, there is a logical problem. The demographic adjustment typically takes the lifetime of the representative individual. Therefore, assuming that observations on movements over time in yearly (or other high frequency) data represent steady states is a contradiction in terms: how can we possibly assume that an observation from a certain period reflects behaviour determined by an equilibrium which requires that the observed state of the world had prevailed for the last 80 years, and assume at the same time that an observation from another data point, say, five years earlier, was determined by another state of the world? Here, we should also recognise that state of the world also contains people's expectations.[5] Indeed, when dealing with demographic change we should not assume that we observe unbiased steady state values. Rather, the shocks we have observed in the last 40-60 years, which dominate factors now and a few decades ahead, have not been random, but lean towards aggravating the ageing process, unexpectedly for both experts and ordinary people (who are supposed to optimise under the OLG model).

Furthermore, there might be a serious question about multicollinearity: as the GDP rate of growth in the industrial world declined from its level in the golden age of Bretton Woods after WWII to the early 1970s, and as a lower saving rate is sufficient to support a given capital/output ratio, and as the dependency ratio has increased over the same period, we may easily observe a spurious correlation between ageing and saving. This has certainly not gone unnoticed in the studies, but it is not easy to cope with this problem, especially as the key underlying factors (expectations) are not observable. The ongoing population ageing process may be quite unique and also relatively similar across countries, and thus we do not have a sufficient number of observations for reliable statistical testing of the OLG hypothesis.

For all these reasons it seems that the empirical tests using aggregate data do not give much ground for assessing the validity of the OLG model. Fortunately, another line of study has been looking into data over lifetime consumption profiles to see if they fit the implications of the OLG model. Attanasio et al. (1999) find a hump shape consumption profile (from US data) and conclude that it is indeed compatible with the OLG hypothesis, but with the important qualification that the model has to incorporate various time-varying demographic factors (family size, retirement, time of death etc.) and precautionary saving due to their uncertainty for generating the time profiles observed. This is a lesson we make use of below to set up the structural factors for lifetime consumption and saving.

Dynamic effects

Following the seminal study by Auerbach and Kotlikoff (1987) entitled "Dynamic fiscal policy", a large number of simulation exercises have been undertaken with dynamic general equilibrium models (DGE) based on the OLG hypothesis.[6] An advanced example is reported by Börsch-Supan et al. (2006; see also Börsch-Supan et al., 2005, and Börsch-Supan, 2006). Their baseline projection for the total saving rate for European Union countries gives a fairly monotonic decline from seven percent of GDP in 2000 to two percent around 2040 (followed by a slight increase). Given a "fundamental pension reform", the decline in the saving rate is 1½ percentage points smaller, which is nonetheless a significant decline. The same pattern is valid under various assumptions on the international capital market and in other regions of the

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[5] Poterba (2001) rightly notes that getting the timing of changes to expectations wrong might confuse the estimation results.

[6] For an extensive list of dynamic simulation models, see Fehr et al. (2006, p. 176).

world. Thus, their results, using a dynamic model, replicate the common understanding probably partly stemming from the simple steady state analysis that population ageing is associated with a lower rate of saving.[7]

The various contributors to the Australian Central Bank conference in 2006, including Fehr et al. (2006), which is an offshoot of Auerbach and Kotlikoff (1987), left Burtless (2006, p. 372) badly confused and in doubt as to whether economists have the right model for linking population ageing to saving, investment and financial markets. Stevens (2006) was also struck by the model's uncertainty and its implications for policy design.

Our primary suspect for the cause of the confusion is that the transition path has not been modelled carefully enough: effects in the first phase, and this can last for 2-3 decades, can be very different from the ultimate long-run effects. In particular, we suspect that we should look carefully into the assumptions regarding expectations: they determine much of the dynamics, as under the OLG model people are supposed to plan for the rest of their lives, i.e. for decades.

However, before going into the analysis of dynamic effects we first need to set up the basic structure of the steady state model.

3. Setting parameter values for steady state analysis

We first specify the basic lifecycle model with regard to work and retirement, children, public pensions, bequests, and, what also turns out to be potentially relevant, a possible constraint faced by young workers of borrowing against their future labour income and inheritance.

We assume for all cases uniform labour and uniform wage and everybody entering the work force at age 20. All adults are unisex and give birth to children (their number can be a number with decimals) at age 30. A pensioner is assumed to have the same utility as a worker by spending 10% less (because direct work-related costs have vanished and more time is used for shopping, see Bullard and Feigenbaum, 2007). The unit period of time is a decade. The model using Microsoft Excel is able to cope with fractions of a decade for working at ages 60-69, i.e. any exogenously determined retirement age within this range is possible; also, longevity can be assumed to increase gradually by any fraction of a decade over ages 70-89.

Labour is assumed to be uniform over the lifetime cohorts (i.e. there is no seniority factor), except that labour productivity, and hence wage rate, increases at an exogenously determined rate, which we assume to be 1.75% p.a. The model works with real economic variables, without interference from inflation.

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[7] By way of exception, Bohn (2006, p. 57) produces from his OLG application an increase in the saving rate over the next 100 years and beyond, while he also produces results from the "Dynastic Model" for a declining saving rate. The latter follows from the Barro-type (1974) assumption that the representative individual cares about her/his heirs and effectively lives infinitely. For Bohn the reality is probably in between the two models, depending on the region in the world and the stage of development of financial markets. The latter make it possible through annuity- and insurance-type products to economise on bequests, which, according to him, mainly determine the capital stock in the world. Thus, he also concludes that the increase in capital intensity projected from the OLG model will not materialise.

Our analysis is partial in the sense that the interest rate is assumed to be at an exogenously determined constant margin (in our base case 1.5% p.a.) above the rate of growth of the wage bill. Thus, saving does not affect the interest rate. Nor do we specify the production sector, i.e. we do not have (or need) a production function that would determine the rate of interest via the marginal product of capital. All this makes the partial analysis simple and makes it possible to look in detail at the transition path, while building a dynamic general equilibrium model with the same details is excluded from the present exercise.

People optimise consumption over their lifetime under a simple utility function, meaning that the time path is determined by the specific structural factors in the model, and it is adjusted in each period for the general increase in unit wage (i.e. people are assumed to care strongly about their relative consumption vis-à-vis their peers). This means that changes in the interest rate do not affect the time profile, i.e. the rate of intertemporal substitution is zero.[8]

The starting point for calculating the effects of changes in population parameters and the related pension system parameters is an initial steady state, calibrated as a stationary population (fertility at reproduction and constant longevity). Then, fertility and longevity change, and we specify various options for the public pension system and its reform. The consistency of the model guarantees that ultimately a new, permanent population age structure and a stable public pension system emerge.

The steady state model is calibrated stepwise:

1. The simplest lifecycle

In the initial steady state retired people consume what they saved when working. Fertility is one child for every 30 year-old adult. Thus, children exist but their consumption is ignored (as so often in OLG models). People work for 40 years, die at 76 and leave no bequests. Income includes wage and interest on the assets acquired while working and depleted when retired. These parameter values imply a saving rate of 19% when working, and a peak in assets of nearly 10 years' wages in the last decade at work. The implied ratio of aggregate capital to aggregate annual income (corresponding to net national income, NNI below) is 5.4. As the economy grows at the assumed rate of 1.75% p.a., the aggregate saving rate is 9.3% of income. These details and further scenarios are reported in Table 1 and Graph 1. The capital to NNI ratio of over 5 implied by this simple model (although often referred to in the literature) is out of line with reality; thus, something essential must be lacking.

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[8] Admittedly, we emphasise the structural factors to the extreme when suppressing intertemporal substitution. It is debateable to what extent the observed strong co-movement of consumption within reference groups and along with aggregate consumption can be explained by peer group effects or whether it follows from common factors neglected in the analysis (see Maurer and Meier, 2005). However, the main results of the present exercise should not be affected whatever the correct explanation is. – For a steady state model where people care about relative consumption, see Knell (2008).

Table 1. Steady state scenarios with alternative assumptions

		Scenario number							
		1	2	3	4a	4b	5	6	7
	Child	no	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	Bequest	no	no	no	Yes	Yes	Yes	Yes	Yes
	Pensions	no	no	Yes	Yes	Yes	Yes	Yes	Yes
Initial population	**K/Y**	5.41	4.85	0.97	1.52	1.79	1.79	1.79	1.79
Ret. age 60	**S/Y (%)**	9.31	8.34	1.67	2.62	3.08	3.08	3.08	3.08
Pens. ben. 0 or 60	**IPD/Y**	0.00	0.00	3.99	3.91	3.88	3.88	3.88	3.88
	ncr	0.00	0.000	0.243	0.243	0.243	0.243	0.243	0.243
	CrC	no	no	no	no	Yes	Yes	Yes	Yes
New population	**Ret. age**	63	63	63	60	60	63	63	65
	Pens. ben.	0	0	60	60	60	60	50	50
	K/Y	7.43	6.69	1.18	2.24	2.24	1.98	2.69	2.36
	S/Y (%)	7.33	6.60	1.17	2.21	2.21	1.95	2.65	2.33
	IPD/Y	0.00	0.00	6.19	6.77	6.77	6.07	5.14	4.82
	ncr	0.000	0.000	0.354	0.440	0.440	0.354	0.292	0.254
	CrC	no	no	no	no	no	Yes	no	no
Change	**K/Y**	2.02	1.85	0.21	0.72	0.45	0.19	0.90	0.57
from initial	**S/Y (%)**	-1.98	-1.73	-0.51	-0.41	-0.87	-1.13	-0.43	-0.75
steady state	**IPD/Y**	0.00	0.00	2.21	2.85	2.89	2.19	1.26	0.94
to the new	**ncr**	0.000	0.000	0.111	0.197	0.197	0.111	0.049	0.011

Legend: for the scenarios, see the rows for Child, Bequests and Pension and the main text
Bequest assumed at one yearly wage.
K = capital stock = total assets
S = saving (private)
Y = total income = total wage income plus interest on assets
IPD = Implicit pension debt = present value to future pensions accrued to date
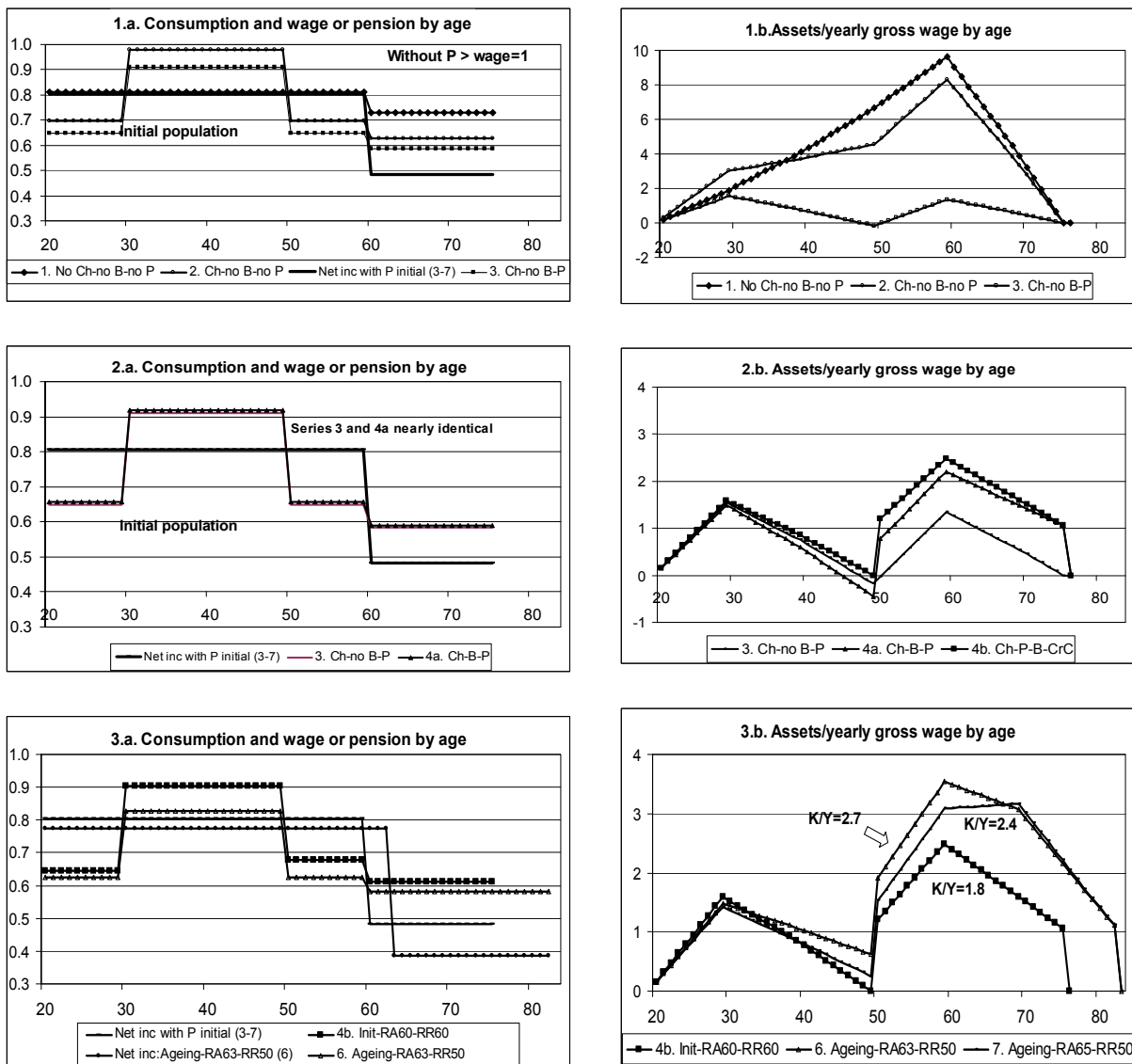ncr = net contribution rate = ratio between replacement rate and wage after pension contributions
CrC = credit constraint operative and binding or not.

2. Consumption of children

Let us now assume that children must also consume and that parents support them until age 20. Lacking reliable estimates we simply start with a rough one that the consumption of a child is 50% of that of an adult.[9] We complement this with a rough assumption that one fifth of this is covered by a child allowance from the government (in a European welfare state). Thus, the consumption of the child covered out of the parent's pocket is 40% of that of the parent (the taxes collected to finance child allowances, like other taxes, are left outside the model). This factor changes the pattern of lifecycle saving as now this particular component of expenditure during the early part of life is a counterweight to saving for retirement. The implied capital to NNI ratio falls to 4.8 (Table 1, scenario 2 and Graph 1.1.a-b).

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[9] Estimates of child consumption of market goods might be somewhat lower (see Apps and Rees, 2001), but taking a broader view and including the opportunity cost of raising children (see Aldersa and Broer, 2005) would lead to roughly 50%; this is also used by Geanakoplos et al. (2004).

Graph 1. Consumption and asset holdings over lifetime in alternative steady states

1.a. Consumption and wage or pension by age

Without P > wage=1

Initial population

Legend: 1. No Ch-no B-no P | 2. Ch-no B-no P | Net inc with P initial (3-7) | 3. Ch-no B-P

1.b. Assets/yearly gross wage by age

Legend: 1. No Ch-no B-no P | 2. Ch-no B-no P | 3. Ch-no B-P

2.a. Consumption and wage or pension by age

Series 3 and 4a nearly identical

Initial population

Legend: Net inc with P initial (3-7) | 3. Ch-no B-P | 4a. Ch-B-P

2.b. Assets/yearly gross wage by age

Legend: 3. Ch-no B-P | 4a. Ch-B-P | 4b. Ch-P-B-CrC

3.a. Consumption and wage or pension by age

Legend: Net inc with P initial (3-7) | 4b. Init-RA60-RR60 | Net inc:Ageing-RA63-RR50 (6) | 6. Ageing-RA63-RR50

3.b. Assets/yearly gross wage by age

K/Y=2.7   K/Y=2.4

K/Y=1.8

Legend: 4b. Init-RA60-RR60 | 6. Ageing-RA63-RR50 | 7. Ageing-RA65-RR50

Legend:
The scenario numbers refer to those in Table 1.
Ch = Child; B = Bequests; P = Pensions.
Initial refers to the initial population age structure.
Ageing refers to the new age structure determined
by new permanent fertility and longevity (see main
text).

RA = Retirement age
RR = Replacement rate
Note: transforming the numbers from decade model
to yearly data is approximation only.

3. Public pensions

Obviously, in European welfare states the public pension system to a great extent evens out the income profile to correspond to consumption needs over a lifetime. Samuelson (1958) and Aaron (1965) already showed the steady state implications of public pensions (or social security as the term often goes) in the OLG model. Diamond (1965) did not have (or need) them for his topic, although adding implicit pension debt along with explicit debt would have been a straightforward extension of his analysis. Feldstein (1974) made this extension (although Diamond, 1965, is missing from his references), and Auerbach and Kotlikoff (1987, p. 160) provided simulation results showing, for example, that "introducing unfunded social security can substantially crowd out long-run capital formation".

We assume for the initial steady state a pure PAYG system with pension accrual at 1.5% for each year at work, leading after 40 years at work to a pension of 60% of net wage (wage after pension contribution; we are thinking primarily of an earnings-related Defined Benefit system, although in our model with homogeneous workers other interpretations are also possible). Adding this to the calculation implies negative assets at age 40-49. The reason is that it is optimal to borrow during the time when the child is at home and pay back the loan later. The aggregate capital to NNI ratio is then only 1 (Table 1, scenario 3). Thus, it is far too low if we bear in mind that in industrial economies the capital/output ratio is in the range of 3-4. Note that in this instance we are modelling private sector behaviour and that assets owned collectively through the government (perhaps roughly one year's GDP) come on top of that. In order not to restrict our guesswork too much, we shall accept a range of 1½ - 3 for the ratio of private capital stock to total annual income.

4. Bequests

Bequests are missing by assumption from Auerbach and Kotlikoff (1987) and, for example, from Miles (1999) and Börsch-Supan et al. (2006), but are included in Auerbach et al. (1989) in simulations for Germany, Japan and Sweden. The latter uses estimates that most private wealth is bequeathed while the component determined by lifecycle smoothing of consumption is a minor proportion, as found also in Kotlikoff and Summers (1986). Using various sources, Modigliani (1988) presents detailed estimates and argues for much lower values for bequests, while Kotlikoff (1988) broadly maintains his methods and estimates.

Thus, the literature on bequests is mixed and it has to be admitted that we know very little about their determinants. But we know from everyday life that they are significant and potentially important for our topic. Just to put a rough figure on them, we can simply observe that median workers/households have normally amortised their housing loan before they retire and maintain their positive housing wealth without reverting extensively to 'negative housing loans', 'reverse mortgages' or other such financial instruments that would allow them to consume their wealth, even if these are increasingly available on financial markets. For more wealthy people it might be plausible to assume that wealth (and showing it to others) directly enters their utility function, and this pattern prevails until death. Positive wealth holding is also justified by the uncertainty of the time of death, i.e. people must hold assets for precautionary reasons and therefore leave them behind. For example, Cagetti (2003) finds from US data that the wealth level at retirement with precautionary motives is twice as high as that implied by a model without uncertainty.

Here, we keep the model simple and do not incorporate uncertainty explicitly, but it is in the background when we assume that the bequest left behind is a given proportion of average

annual wage. This simple assumption makes it fully transparent that the bequest is determined exogenously, apart from the total wage cost being the scale factor. This is in any case more realistic than assuming that it is zero, as is often assumed in OLG models.[10]

For our simple case we set the bequest left behind at death at the gross wage over one year of the representative individual, i.e. 2.5% of the total annual wage cost. This is of the same order of magnitude as in Modigliani (1986) and Altonji and Villanueva (2007). We are mindful that net housing wealth of middle income people ultimately to be left behind can typically be somewhat bigger, that wealthy individuals leave behind larger estates, that the distribution of wealth ownership and bequests are very skewed and therefore working with an assumption of uniform agents is never fully justified.[11] Note also that bequest behaviour may change in future, for example, as the annuities markets develop. That might look like a change in behaviour, but it could also be interpreted as doing away with an existing constraint. This factor should always be kept in mind for forecasting, but it is not part of the effect of population ageing on saving.

Inserting an assumption that bequests are equal to one year's gross wage implies an aggregate capital to NNI ratio of 1.5 (Table 1, scenario 4a). Graph 1.2b illustrates how heirs inherit at the age of 50, right after the child leaves home at age 20. The effect of bequests is readily seen as a higher level of assets over the rest of the person's lifetime. One implication of bequests is that, under optimal behaviour, borrowing in the third decade (age 40-49) increases. With the assumed parameter values this is not very big, just 4% of annual gross wage. However, if bequests were alternatively assumed to be equal to two years' gross wage, borrowing at age 40-49 would be as high as 70% of annual gross wage.

Conceptually, we do not include *inter vivos* transfers in bequests. In our model, if these transfers are consumed by the beneficiary (as for tuition fees, for example), it does not matter who paid for this consumption. However, if the transfer affects asset holdings over a lifetime, it is like a bequest being transferred at an earlier phase in life. This would smooth the effect of the bequest factor in the model.

5. Credit constraint

Borrowing by younger workers for consumption, as implied in the previous step, should not be accepted without question as an available option because the future wage or inheritance is not accepted as collateral (due to asymmetric information and moral hazard – note that here we do not mean borrowing for housing where the property acts as collateral). In many models, though not in all (Bohn, 2006, is an example of an exception), the question of the possible credit constraint does not arise because they lack the factors that would induce young adults to borrow for consumption (as children's consumption and bequests and sometimes even public pensions are missing).

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[10] We could extend the model by specifying an endogenously determined bequest component, but the essential features of the present analysis would still be valid as long as there is also an exogenously determined one.

[11] Our simple figure is in line with an estimate for Finland given by Vaittinen and Vanne (2008). Fehr et al. (2008) refer to a considerably higher figure for Germany, 5.2% of GDP. Their source is Braun and Metzger (2007, Tabelle 2), which, however, is not fully convincing as, for example, their procedure for estimating aggregate bequests on the basis of figures for individuals is not fully transparent. In any case, the figure of 5.2% of GDP for Germany seems high in international comparison. It can be that bequests are comparatively high in Germany or else the estimate is biased upwards.

We work here on the assumption that people are not allowed to hold negative assets at any phase of life. This is implemented in the model by not allowing the assets of a worker at age 40-49 to be negative. The optimal path is first calculated without the credit constraint, and if, and only if, the outcome is a negative asset position at age 40-49, the credit constraint is imposed; then, at age 50-59, when the child has left home and they have received their inheritance, people optimise their consumption for the rest of their lives. With the assumed initial values this gives an aggregate capital to NNI ratio of 1.8 (Table 1, scenario 4b, and Graph1.2.b). We have now reached an acceptable order of magnitude for the capital to NNI ratio for our stylised economy.

6. A demographic change

Having specified the initial steady state with the basic structural factors, we now move to the scenarios with ageing (scenarios 5-7). A demographic change occurs, and we look at the new steady state, say, at least 80 years later. Fertility declines from 1 to 0.8 (i.e. from 2.1 to about 1.7 for the ordinary fertility indicator per woman, which corresponds roughly to the decline in Western Europe from 1970 to 2000), and longevity increases gradually from 76 in the 1980s to 83 in the 2050s (a steady one year increase in a decade). The scenarios without child, bequests and public pensions are also shown in Table 1 for the ageing scenario. Obviously, following the longevity increase, the retirement age should be allowed to increase. We show options maintaining 60 as the retirement age, increasing it to 63 and further to 65. We also show the options for reducing the replacement rate from 60% to 50%. In all scenarios the capital to NNI ratio is higher than in the corresponding initial steady state (see the row for the change of K/Y). Yet the saving rate is always lower: this shows that for the saving rate the decline in the steady state rate of growth due to the decline in fertility dominates the effect of the increasing capital to NNI ratio. Table 1 also shows the implicit pension debt (IPD) of the government per NNI and the pension contribution rate under a pure PAYG pension system. These are indicators of the relative size of the public pension system, determined by the pension system parameters and population ageing. Note that we do not have explicit government debt in this model, but it could be added and its effects would be the same as those of the IPD induced by the pure PAYG pension system.

Summary of the steady state analysis

Starting with the scenarios without public pension system (regardless of how realistic they are) we see the dramatic negative effect on the capital stock of inserting the public pension system (scenario 3 compared to 2 for the initial steady state and also for the new). Public policy providing a pension replacing (a significant part of) the foregone wage reduces the capital stock considerably, and this effect is reinforced by ageing (compare the move from scenario 2 to 3 for each steady state). This indicates that introducing generous public pensions of the European type is probably the most important factor affecting saving, capital and production, at least if we speak about policy instruments. We admit that comparing two cases, one with pensions and one without but assuming the same retirement age, is a very hypothetical setting, as in the scenario without public pensions people would probably not choose to retire at the age of 60 but rather to work longer and thereby save less. However, for the purposes of this paper, a calculation with the given retirement age calibrated for current European data serves as a reference.

The result for the new steady state with retirement age maintained at 60 (implying a capital to NNI ratio of 2.2) must be judged to be only hypothetical because the pension contribution rate

would have to be 44% of the net wage. This would probably not be acceptable as people would not agree to allocate such a high a proportion of their wage to the public pension system (remembering that the proportion of their disposable income would be even higher if other taxes were taken into account); they would rather accept an increased retirement age and/or reduced replacement rate, as these would be natural remedies for the consequences of the ageing process.

Scenario 5 assumes that retirement age is increased to 63. The assets to NNI ratio is 2. It is lower than in the previous scenario primarily because the shorter time in retirement requires less saving. In this scenario the asset holdings are slightly higher than in the initial steady state, and the saving rate is significantly lower, due to the lower growth factor. This scenario might be a reference for further considerations as a relatively generous pension option, with replacement rate maintained at 60%, although we should note that the contribution rate of net wage is quite high, at 35%. In this scenario the credit constraint is binding.

Scenario 6 and Graphs 1.3a-b show reform where the retirement age is increased to 63 and the replacement rate is reduced to 50% (note that, as working life is now longer, the yearly accrual is reduced by more than one fifth). Both the capital to NNI ratio and the saving rate are higher than in the previous scenario: people save more as public pensions are now lower. This already materialises at a young age, with the effect that the credit constraint is not binding: workers at age 40-49 choose positive assets voluntarily.

The last one, scenario 7, illustrates an increase in retirement age to 65. This corresponds to maintaining the initial ratio between years in work and in retirement. This compensates for the increase in longevity, and as the replacement rate is decreased from 60% to 50%, also, more or less, for the effect of the decline in fertility, meaning that the pure PAYG contribution rate is only marginally higher than in the initial steady state (scenarios 3-7). This significant pension reform scenario implies that the capital to NNI ratio is 2.4, i.e. 0.6 points higher than in the initial steady state, and that the aggregate saving rate reduces by a quarter to 2.3%. The credit constraint is not binding in this scenario.

Assuming that bequests are equal to two years' wages rather than one (calculations not reported in detail here) gives a capital to NNI ratio 0.6-0.7 higher than in the corresponding cases in Table 1. However, the changes from the initial steady state to the new are very similar to those in Table 1. This indicates that in the model here the assumption regarding bequests is important for the *level* of capital to NNI ratio and therefore for the calibration of the model. However, the *changes* in the capital to NNI ratio caused by population ageing and pension reforms depend very little on this particular assumption (as long as the bequest parameter itself remains unchanged). These implications can be intuitively explained with the help of Graphs 2b and 3b above: higher bequests increase assets held from the age of 50 onwards, but the assets of younger workers would not change much as their behaviour is affected by the credit constraint.

4. The transition paths triggered by population ageing and pension reforms

Recognising the puzzle concerning the effects of population ageing expressed, for example, by Burtless (2006) and Stevens (2006), we now look explicitly into the transition from one population structure to another. We pay special attention to people's expectations. They might be practically redundant for the steady state analysis where the assumption that they are

identical to the values in the model can be a natural simple reference (for caveats, see, however, Molnar and Simonovits, 1998). But when changes trigger a transition, the question of expectations becomes more serious and assumptions have a crucial effect on dynamics.

Starting from the steady state model above, we set up a simple Excel spreadsheet model to illustrate alternative transition paths triggered by a gradual decline in fertility from one to 0.8 over the three decades from 1970 to 1999 and an assumed increase in longevity of one year per decade from 76 at the end of the 1980s to 83 in the 2050s.

We start from the initial steady state scenario 4a above: a pure PAYG public pension system, 60% net replacement rate after full career, bequests equal to one year's wage, no credit constraint.

We first show the results for the baseline where it is conventionally assumed that people correctly foresee both population ageing decades in advance and expect the pension rules in force to be maintained (PF for perfect foresight).

Next, we assume a fully anticipated pension reform announced in 2010. It includes an increase in the retirement age to 63 by 2030 (labelled RA 63), assumed to be frontloaded so that most of the adjustment takes place by year 2014, and by 2020 the initial ratio between years at work and in retirement is restored. Also, pension accrual is reduced from 2010 onwards so that the replacement rate (of net wage) declines from 60% to 50% (labelled RR 50). The accrual rate is reduced immediately in 2010, but replacement rates for pensions paid out decrease gradually, the effect going fully though only after 63 years.

The results for the aggregate saving rate and capital to NNI ratio are reported in Graphs 2.a-b. As the changes are perfectly foreseen, saving and capital stock already start increasing in 1940 as people react to future changes. With the unreformed pension system the saving rate (Graph 2a) already peaks in the decade 1990 when the number of workers is first affected. In the pension reform scenario the same happens in a more subdued manner, and when the retirement increase takes place in 2010, affecting the wage sum, the saving rate temporarily falls but increases again afterwards. Graph 2b shows the corresponding paths for the capital to NNI ratio. In the no-reform scenario this ratio reaches its maximum in 1920 and then levels off. In the reform scenario it increases gradually and in both it is higher than initially, as already seen from the steady state analysis above.

Neither of these two scenarios is presented as realistic as the perfect foresight assumption is so obviously unrealistic here. How could workers in 1940 already have foreseen the decline in fertility that started in 1970 and the increase in longevity from 1990 onwards? Also, with regard to the no-reform scenario, it is hardly realistic to assume that people would understand that the existing rules would mean that pension contribution rates would nearly double and at the same time accept it as a realistic basis for their private saving decisions. The correctly anticipated reform scenario is equally dubious: in addition to having correctly foreseen the demographic changes in 1940, workers would have already then correctly anticipated the pension reform in 2010. Thus, these two scenarios are obviously not realistic and therefore not useful as a basis for formulating policies. They only demonstrate that something in the model must be changed.

We now show results for a set of alternative assumptions, labelled as Surprise (S): during the four decades from the start of demographic transition in 1970 (for the labour force from 1990)

until 2010 people are assumed not to realise that the ageing process had started, but simply maintain the same age-cohort-specific consumption propensities (of their disposable income) as in the initial steady state. Obviously, this behaviour could not continue forever as the economy would be on an unstable path with ever declining savings and capital. The moment of truth is assumed to come in 2010, when a pension reform is implemented (with alternative options, including no reform). After this surprise, people are again assumed to correctly foresee the demographic and economic future from then onwards for the rest of their lives. Also, the cohorts at adult age in 2010 re-optimise their consumption paths, based on their initial assets and revised expectations. A one-off surprise and revision of expectation is only assumed here for the purposes of producing a simple example. Nothing would prevent us from producing scenarios where surprises and revisions arrive sequentially.

Graph 2. Transition paths for saving and capital accumulation, pure PAYG
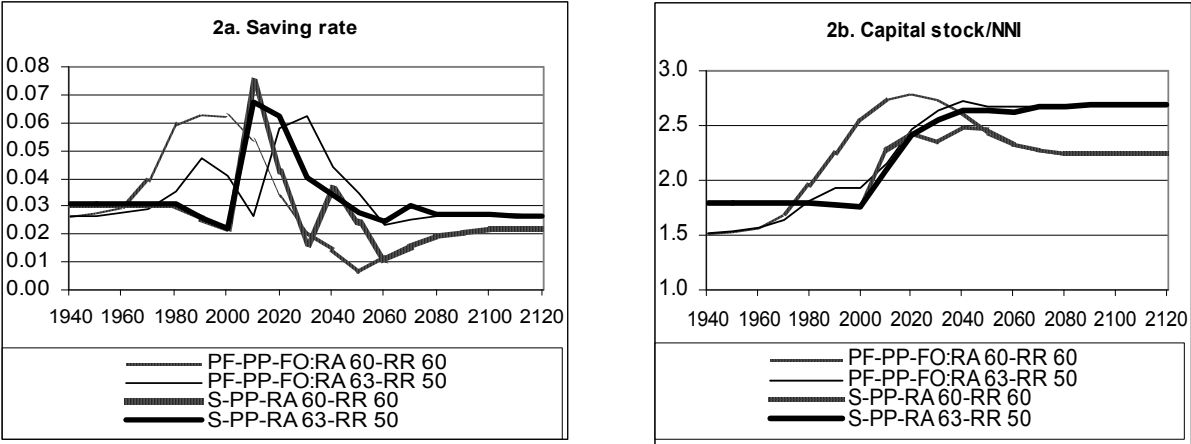
Legend:
NNI = net national income
PF = perfect foresight
PP = pure PAYG
FO = free optimisation = credit constraint (CrC) not binding; for S cases CrC binding until 2000.

S = surprise in 2010
RA = retirement age
RR = replacement rate
F = partial funding

The other two pairs of lines in Graphs 2.a-b show the results for the S(urprise) scenarios. Here, we consider the initial steady state with credit constraint (scenario 4b above) to be more realistic. Assuming that cohort-specific saving rates do not change until recognising the surprise in 2010, and first assuming no pension reform, the capital to NNI ratio increases from 1.7 to 2.2 in the new steady state (the same as with PF), with a peak at 2.5 during the transition. The saving rate first increases from 3% to 7%, oscillates and then settles at 2% (so that, for the new steady state, the negative impact from the fall in the rate of growth dominates the one from the increase in capital to NNI ratio). As already said, this scenario is not a realistic one: the pension contribution rate would increase from 24% to 44% of net wage, a level that is not likely to be acceptable for both political economy and tax evasion reasons.
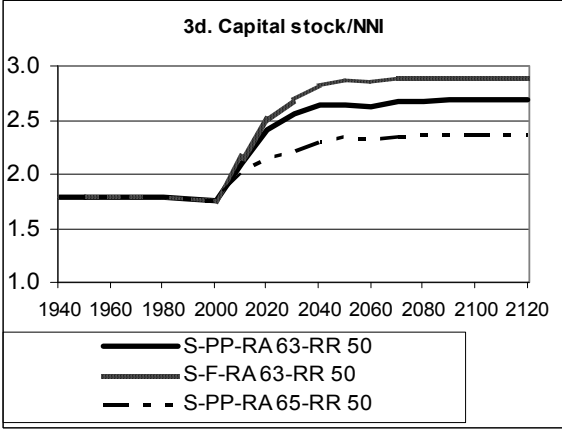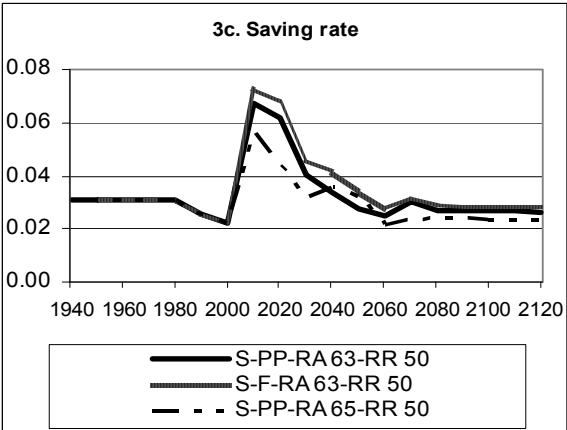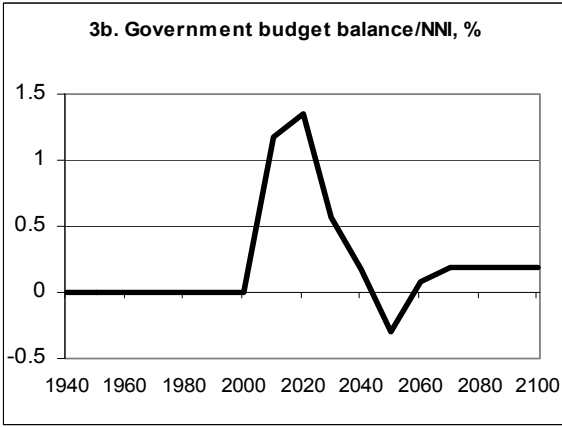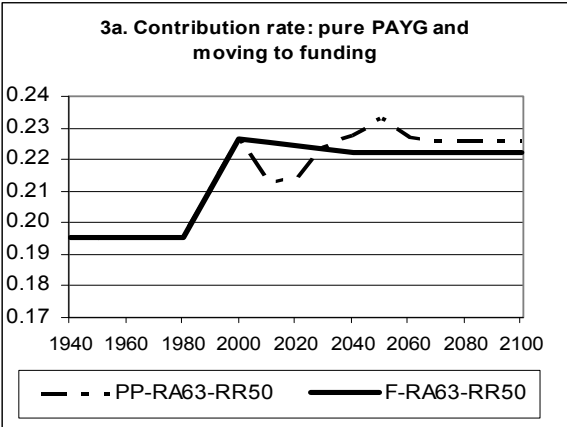
Therefore, the pension reform scenario is more interesting. As already implied by the steady state analysis, mainly because the replacement rate is reduced to 50%, people save more and the capital to NNI ratio increases to 2.7. The steady state level of saving is close to 3%, only slightly below the initial level. More importantly, the transition path shows a single peak in the saving rate, at 6-7% over two decades after the surprise.
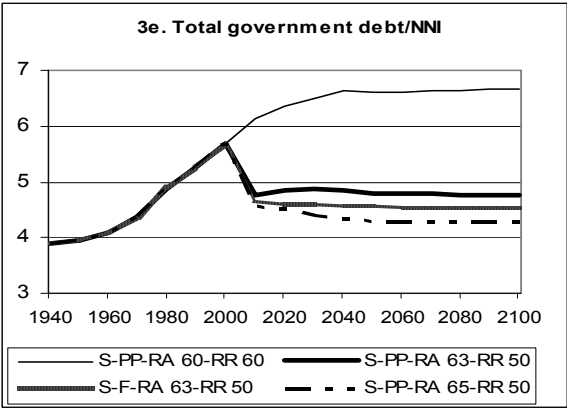
Moving to partial funding

Many pension simulation models assume that a pure PAYG system is maintained, even if with new parameters for benefits (Fehr et al., 2006, stands out as an example of a wider range of options, including debt financing and privatisation). However, as argued by Beetsma and Oksanen (2007), and earlier by Sinn (2000) and Oksanen (2004), under population ageing and pension reforms there is a case for abandoning the pure PAYG pension system (where, by definition, the contribution rate is determined by expenditure). This argument can be illustrated by an example: if fertility declines and/or longevity increases to permanently new levels, the first generation with the new demographic characteristics will gain at the expense of all future generations, as it will have paid the contributions determined by the longevity of the previous generation but enjoy the benefits over the longer retirement (this is a generalisation of the inference concerning the windfall to the first generation(s) when a pure PAYG system is established). To correct this unequal treatment, some of the policy parameter values should be changed: if the benefit rates are maintained, the contribution rate should be increased immediately it becomes evident that fertility has declined and longevity is increasing; or the future benefits should be changed by decreasing the current accrual rates and/or by increasing the retirement age. The principle, labelled actuarial neutrality, that similar generations should pay the same price for similar pension benefits opens up a wide range of reform options, with the salient feature that the system may no longer be a pure PAYG, but may move to partial funding; to be more precise, it may accumulate either assets or debt, even permanently (debt should accumulate if the accrued rights in the public pension system are significantly reduced, for example, in the context of privatisation, while recognising the rights accrued before the reform; this means that implicit debt becomes explicit).[12]

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[12] The argument above is different from the arguments put forward by, for example, Kotlikoff (1997) and Feldstein and Samwick (2002) that privatisation and funding provide efficiency gains to be shared between the current and future generations. If valid, their arguments also apply without population ageing, while the argument above should prove that there might be a case for moving to partial funding under population ageing solely for the purposes of neutral burden sharing, even without any possible efficiency gains. - Orszag and Stiglitz (2001) rightly point out that moving to partial funding of public pensions means pre-funding in the broader sense so that the system accumulates assets, while establishing individual accounts is a separate matter. - The neutrality principle referred to here is more generally applicable than Musgrave's (1986) 'fixed relative position' rule for a pure PAYG system promoted, for example, by Esping-Andersen et al. (2002). It is worth emphasising that a mandatory public pension system (paternalistically eliminating people's short-sightedness, and avoiding free-riding under social insurance based on Rawlsian principles) need not be a pure PAYG but can be funded to any degree, and this degree can also change over time. – Barr and Diamond (2008) discuss extensively all issues relating to pension reforms, including the choice of degree of funding. The framework referred to here offers a useful benchmark for moving to partial funding based on actuarial fairness across generations. It provides a benchmark where moving to partial funding under population ageing is not an excessive burden for the working generation but rather a justified measure designed to avoid an increase in the burden on future generations. In principle, this is the same as a pension system being established where everybody contributes the actuarial value of his or her own future pension, and a fully funded system emerges. Deviations from actuarial neutrality have arisen, notably where PAYG pension systems have been established, but, importantly for today's problems, also under population ageing if the benefit parameters of a (broadly mature) DB PAYG system are maintained. This actuarial neutrality framework can be used to analyse the justification of those policy decisions and to quantify their consequences.

Graph 3. Moving to partial funding of public pensions under ageing

3a. Contribution rate: pure PAYG and moving to funding

Legend: ▪▪▪ PP-RA63-RR50 ▬▬ F-RA63-RR50

3b. Government budget balance/NNI, %

3c. Saving rate

Legend: ▬▬ S-PP-RA 63-RR 50, ▬▬ S-F-RA 63-RR 50, ▪ ▪ ▪ S-PP-RA 65-RR 50

3d. Capital stock/NNI

Legend: ▬▬ S-PP-RA 63-RR 50, ▬▬ S-F-RA 63-RR 50, ▪ ▪ ▪ S-PP-RA 65-RR 50

Legend: see Graph 2 and the main text.

3e. Total government debt/NNI

Legend: ── S-PP-RA 60-RR 60, ▬▬ S-PP-RA 63-RR 50, ▬▬ S-F-RA 63-RR 50, ▬ ▬ S-PP-RA 65-RR 50

The principle of treating successive generations equally under the public pension system can be defined in strict terms only with reference to an abstract model where the generations form a chain but their working lives do not overlap. But as they do overlap when we look at the age cohorts by year or decade, we can only arrive at an approximation, albeit a useful one.

Graphs 3a-e illustrate the policy rule that leads to partial funding. The reference is the last scenario in Graph 2: retirement age is increased to 63 and replacement rate reduced to 50%.

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Graph 3a shows that the contribution rate under the pure PAYG declines in the decade starting in 2010, stemming from the increase in the total wage bill due to the retirement age increase, and increases in 2030 as adverse factors take effect and labour force starts to shrink due to decreased fertility. It peaks in 2050 and then declines as the reduced accrual rate has its full effect on total pension expenditure.

Taking the contribution rate in 2000 as the starting point, an iterative method gives a sustainable contribution rate for the new steady state, applying a rule that it is reached gradually by 2040 (one generation after the reform date of 2010). The surplus in government budget is on average over 1% of NNI over three decades (Graph 3b), and the government will have assets permanently of 20% of NNI. Total saving rate and capital to NNI ratio are shown in Graphs 3c-d. The additional component in the capital stock due to the rule leading to partial funding, comparing this scenario to the corresponding pure PAYG (PP) case, is roughly the same as the accumulated assets in government accounts. Thus, under the assumptions of our model, additional public saving does not reduce private saving.

In addition, a scenario with retirement age increased to 65, combined with the replacement rate reduced to 50%, is shown in Graphs 3c-d. Only the pure PAYG (PP) scenario is shown as the contribution rate falls smoothly back to close to its initial level. Hence the rule for treating generations equally is approximated without moving to partial funding.

In all reform scenarios capital stock to NNI ratio increases and the saving rate is significantly higher than initially for four decades. Capital accumulation is largest in the scenario of partial funding: capital is accumulated in the first decades and the proceeds benefit all future generations. In all these scenarios the permanent level of saving rate is a little lower than in the initial steady state, being lowest in the last scenario where the retirement age increases to 65 – as the time at retirement is lowest, so is the need for saving.

Graph 3e gives the ratio of total government debt to NNI in all four scenarios for pension benefits and retirement age. The total government debt is explicit government debt plus Implicit Pension Debt (IPD) in retrospect, so that IPD already starts increasing in 1940 as pension costs in the future start to increase due to ageing. The uppermost line gives the scenario of no pension reform, where it would increase from 3.8 times the annual NNI to 6.4. This is one more reason for the conclusion that the no-reform scenario is unsustainable: serving this high an amount of implicit debt and maintaining generous pensions requires an increase in pension contributions to 44% of net wage.

The IPD for the various reform scenarios implemented in 2010 show how the expansion of implicit (and total) debt is cut and contained under the alternative policies. The uppermost of them gives the IPD for the pure PAYG with retirement age increasing to 63 and replacement rate reduced to 50%. The next one is the partial funding scenario, where the public pension system accumulates funds up to 20% of NNI – hence the total government debt is lower. The lowest line is for the retirement age increase to 65 under pure PAYG (where a funding rule would not deviate much from it).

There is no simple rule for judging precisely which target for government debt would be acceptable for neutrality across generations. However, all three reform scenarios should be approximations of this rule: debt in the new steady state is higher than in the initial steady state, but lower than in 2000 where we see the peak as population ageing had already started but the reform had not yet been designed and implemented. Thus, the collectively made error

of recognising the ageing problem only slowly would require acceptance of a somewhat higher permanent total debt burden than in the initial steady state. Otherwise, the working cohorts in the early decades of the 2000s would have to bear a much larger burden than others.

5. Assessing the literature and results from the present model

Taking the above results into account, we now discuss the vast literature on ageing and saving, focusing first on the various structural factors and then on the dynamics of transition.[13]

The structural factors

Public pensions, since Feldstein (1974), now have a relatively established role in the analysis of population ageing, although, exceptionally, they were ignored, for example, by Bohn (2006) and Kulish et al. (2006). Depending on the topic, this omission can be justified, but implicitly assuming that saving in public pension systems and private retirement saving are perfect substitutes should not go without notice.

The credit constraint is operative in some studies, but it is not a standard feature, as in most models negative assets do not appear in the first place because the underlying factors (children's consumption and bequests) are missing.

Children's consumption is missing from Börsch-Supan et al. (2006), while included in Fehr et al. (2006, p. 177), although both the precise assumptions and the implied magnitude are missing from their reports. As seen from the above, this factor may affect the steady state implications and therefore affect the various parameter values when calibrating the model.

Apart from public pensions the bequest is probably the most important factor to affect the results, as already forcefully demonstrated by Barro (1974). Taking the rather high saving rates observed in 1960 in various countries, Auerbach et al. (1989) calibrated their model on fairly high estimates for bequests (much higher than accepted by Modigliani, 1988) and assumed a negative time preference rate for consumers. This assumption goes against common conceptions, although the results are claimed not to be sensitive to this assumption. A similar issue is encountered by Fehr et al. (2006) in the calibration for China: they set a large negative number, -13%, for the annual time preference. Thus, a problem in specifying bequests can easily spill over into the choice of other parameter values.

An important structural feature of the models is whether the retirement age is treated as a policy parameter (as we did above) or whether it is assumed to be determined endogenously by unconstrained choice of agents. In their seminal paper, Auerbach and Kotlikoff (1987) did not have one either, as they did not consider longevity increase. In more recent models retirement age is endogenous, in which case, depending on the choice of parameters, the length of working life increases roughly in line with longevity (e.g. Börsch-Supan et al., 2006, Kulish et al., 2006, and Fehr et al., 2006). Such simulations can be useful as the implied

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[13] For many economic issues the results from a steady state analysis are instructive and sufficient, as it is often justified to assume that full adjustment takes place within a few years. However, for population ageing-related models such an assumption would be a contradiction in terms.

retirement age increase might be regarded as a reasonable benchmark or target. However, the retirement age is directly affected by policy under most European public pension systems, either through statutory, even mandatory withdrawal from the labour force, or through pecuniary incentives that trigger retirement at a certain age. If so, both initial retirement age and any changes due to longevity increase are distorted by policy. If this is the case, a model with a fully endogenous retirement age cannot be used for policy exercises where it is a policy parameter.

The dynamics

We noted earlier that assumptions regarding expectations are crucial for the dynamics of the results. Starting from Auerbach and Kotlikoff (1987), assuming perfect foresight dominates multi-period simulation models. They defended this as "a useful benchmark because deviations from full rationality are not likely to be systematic". In their study this was reasonable as they primarily studied the effects of fiscal policy, most of their work did not cover questions related to population ageing, and the specific chapter on demographic transition excluded longevity increase and only considered changes in fertility that were already known to have happened (baby boom and the ensuing bust after WWII).

However, as their study became a model for further simulation models geared to drastic and permanent changes in fertility and longevity, we have to have a closer look at what perfect foresight really means in these models: it is the fact that private agents already correctly foresee, decades ahead, the demographic changes that are plugged into the model (the most recent expert projections).[14] This has serious consequences for OLG simulation applications: the exercises typically start with a baseline scenario under perfect foresight, thus incorporating a significant part of the *effect* of population ageing on people's saving behaviour, as – under the assumptions made – saving has already changed decades before the demographic changes actually take place. For example, using our simulation results above as an illustration, under perfect foresight the fall in fertility starting in 1970 and the increase in longevity starting in 1990 already affect saving in 1940, and quite significantly so over the 1980-2000 period. Consequently, in 2010, the "current" year of our exercise, the saving rate is already declining in our no-reform scenario.

In recent simulation studies the quantitative results for the baseline are typically reported from the "current" date without exploring, explaining or reporting what the anticipatory effects might have been before the reported period.

Having specified the baseline, these simulation exercises specify a pension reform that comes as a surprise: people observe it and re-optimise under perfect foresight from then onwards (as also in the simulations in this paper). The deviation from the baseline is then supposed to give the effect of the reform (e.g. Miles, 1999, Börsch-Supan et al., 2006, and Fehr et al., 2006; see also Ludwig, 2007, for the algorithms). This might lead to plausible results, but with the caveat that part of the effect of population ageing might already be incorporated in the baseline. For example, if the retirement age is endogenous then some of the future policy decisions might also be endogenous. This might be plausible, given that people would expect this to happen, but one has to be aware that the underlying policy reaction is then part of the

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[14] Note that the uncertainty with regard to individual lifespan causing unintended bequests that is plugged into the recent vintages of OLG applications is a separate matter. It does not undo the assumption that the individual correctly foresaw the average expected lifespan decades ahead.

baseline scenario, and yet the baseline might be misleadingly presented or wrongly understood as 'no policy change'.

Calibration and purpose: projecting or analysing the effects of ageing and policy options

It is always important to clarify whether the purpose is to produce the *most reliable projection* for economic variables for decades ahead or to analyse the *effects of both identified exogenous factors* (population ageing) and *related policy instruments (pension reforms) on economic variables*. The former does not necessarily give an estimate of the effects of population ageing while the outcome of the latter is not a reliable projection of what will happen, as other relevant factors may also change.

The recent models are typically calibrated to fit the observed data around the year 2000 (or a preceding period), plugging in the observed demographic data and finding (obviously by some trial and error) a combination of plausible structural parameters. This calibration can either mean that the initial state is in equilibrium (i.e. a steady state as we are dealing with dynamic modes), as per the model, or out of equilibrium. If it is supposed to be in equilibrium, several questions are triggered:

- Due to the decades-long adjustment (which does not stem from sluggish adjustment due to ad hoc factors but from the basic characteristics of the OLG model) the plugged-in real demographic data in practice never represent a steady state (stable) age structure.
- Equilibrium also requires the expectations of the lifecycle consumers to settle into unbiased estimates of future variables (equal to the projections for the relevant variables produced by the model). How plausible this may be, given that during the few decades used for calibration (e.g. 1960-2001 in Börsch-Supan et al., 2006) quite remarkable changes took place in both demographic variables and other drivers of economic growth (independently of demographic factors and pension policies), is a moot point. Note that the early part of the 1960-2000 period fell into the golden age of growth in industrial countries, probably full of a series of positive surprises; long-term growth and expectations of growth should then have fallen, even without population ageing, bringing about a lower saving rate.
- As the public pension systems developed fast from the 1950s to 2000, it is reasonable to assume that people only learned of the changes gradually and changed their behaviour slowly, with many surprises along the way. Not only were the changes to public pension systems remarkable over those decades, there is also evidence that people know and understand very little about them (Boeri et al., 2001 and 2002).

These remarks imply the following: if the model is supposed to be in a steady state when the simulation starts, it may not be the best option to choose the parameter values so that the outcome in 2000 fits the observed data; it might well be more advisable to set for the initial state a hypothetical equilibrium that deviates from the observed data.

If, instead, calibration is made so that the model starts in the year 2000 from out of equilibrium, then it is obvious that the resulting projection, compared to the 2000 data, should not be interpreted as an *effect* of population ageing, as the result will obviously also contain convergence towards the new equilibrium, in addition to those effects and the subsequent changes in policy parameters.

Remarks on two recent large-scale simulation models

Börsch-Supan et al. (2006) cover the main regions of the world economy and obtain plausible results for the (positive) effect on total saving from benefit-cutting pension reforms. Their study targets the international financial flows resulting from saving and investment, which are both affected by population ageing. As the interest rate is determined in the world market, it

reflects the average developments in population ageing and related factors, allowing the adjustment of current accounts to smooth the outcomes for individual regions. The story is consistent, although it does not of course prove the theory, as it is more a portrayal of the implied assumptions plugged in.

The key result is that the saving rate consistently declines in all regions, e.g. in the EU from around 7% to 2% around 2040 and then increases to 3%. These numbers are not far off simple steady state figures: if the capital to GDP ratio is around 3, these saving rates would comply with an initial steady state growth rate of slightly above 2% p.a., and ultimately about 1%. For example, the latter would be consistent with a 1½% growth rate in labour productivity and labour force shrinkage of ½% p.a. (due to fertility below reproduction level). The temporary drop around 2040 could be the result of the post-WWII baby boom generation fading away.

Children's consumption is missing from their model, as, too, are bequests (as they unrealistically assume a perfect annuity market). In the calibration, the missing bequest in particular may have affected their choice of values for the parameters behind the lifecycle profile of consumption: rate of return on savings, pure time preference, assumption on intertemporal rate of substitution, etc. The authors do not, as is also the case in most other papers, report the (implied) lifecycle profile for consumption and its possible change due to demographic and other changes. This makes it difficult to assess to what extent the various missing elements might affect the results, including for the transition path.

The relatively steady decline in the saving rate contrasts with the simulations made in this paper, which puts the emphasis on the likely (or at least possible) positive effect in the first phase of the transition. One possible reason why the first positive effect is missing from Börsch-Supan et al. (2006) is that when constructing their model they assumed that retirement age would increase and thus neutralise at least most of the effect of increased longevity. Their result for the "exogenous labour supply" variant seems to confirm this. Another possible reason is the assumption of perfect foresight concerning future demographic development in the baseline scenario: in their simulation the positive effect on saving might already take place decades in advance, i.e. well before 2000, but this is not reported.

As said, the main results in Börsch-Supan et al. (2006) in terms of the importance of international capital movements are plausible, but it has to be said that the simulated effects on current account balances might be rather uncertain, as they cumulate the uncertainty in a number of structural parameters behind both saving and investment. One such crucial factor is, again, the retirement age.[15]

The main contribution in Fehr et al. (2006) is to add China to the model of the world economy, and doing so quite forcefully, starting with the title of their paper. This is a clear step forward, as China has in the past few decades become a major player in the supply of

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[15] Börsch-Supan et al. (2006, p. 643) say that the projected decline in the saving rate is caused by population ageing. However, their exercise does not substantiate such a simple causality as, in their model, other factors also move and obviously cause part of the projected decline (it seems that in their study the effects of the exogenous factors that reduced the rate of increase in productivity from, say, 1960 to 2010 and onwards are not controlled for). - In the simulation where pension benefit rate is reduced, the saving rate immediately falls and increases only later (their Figure 2(b) on p. 644). This initial negative effect, which does not readily follow from standard assumptions, is not discussed in their text.

products to the world market and as a source of global current account imbalances. Its population age structure is projected to change in the coming few decades even more rapidly than in the old industrial world.

Fehr et al. (2006) is a rich model, including, for example, children's consumption and bequests. In their model bequests are only accidental, caused by the unknown moment of death and the missing annuity market. This specification does not therefore cover other possible explanations of bequests, including the factors behind the bequests of rich people going beyond the simple OLG model. Similarly, (net) assets, including the physical capital of the government, are missing, but calibration still targets the aggregate capital to output ratio.[16]

In the case of China, possibly due to a number of other factors behind high saving rates being missing, Fehr et al. (2006) calibrate a large negative number (-13%) for the time preference rate. In their baseline they then assume that it gradually reaches the American habits for 21 year-olds by 2040. The paper also presents a simulation where the initial (negative) time preference rate is maintained. The result is that the capital to NNI ratio increases 5-fold, compared to 1.3-fold in the baseline case, and that a similar increase not only happens in China but also spills over to the rest of the world. This result tells us that the results from the model are quite sensitive to the particular assumption regarding time preference and may therefore significantly impact on the time path over the transition. Is this a case where targeting the observed aggregate capital to output ratio and an upwards biased estimate for bequests is offset by too low a time preference rate, repeating the possible flaws in Auerbach et al. (1989)? As the results for the whole world are so sensitive to this particular assumption regarding the Chinese time preference rate, the reader is left somewhat perplexed as to what is a plausible view.

It is without doubt most useful to study the effects of population ageing and public pensions (or lack of a credible public pension system) on the Chinese economy. It is indeed plausible that those factors partly explain saving behaviour and the current account. However, trying to explain the observed saving rate in China with a standard OLG model is probably doomed to failure, as something essential might be missing or wrongly specified. Perhaps the most consistent theme about China in the last few decades and for the 2-3 decades ahead is the pool of excess supply of (unskilled) labour that has made rapid economic growth possible once the government opened the country up for exporting to the rest of the world. The infinitely elastic supply of labour should be a temporary phenomenon in a very long-run general equilibrium framework, but because the potential labour force in the 1970s was huge and remains so today, the adjustment may well take many decades and have significant repercussions on other countries.

In addition to the issues related to China, the general problems noted with regard to Börsch-Supan et al. (2006) are pertinent: even though the individual's lifetime is uncertain in Fehr et al. (2006), its expected value increases and the fertility decline is also known by agents when they optimise and determine the baseline simulation.

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[16] As noted above (footnote 11), Fehr et al. (2008) may be using an upwards biased bequest estimate for Germany.

Therefore, a significant part of the effects of ageing is probably already taking place in their study over the few decades preceding 2004, the year they started their reporting.[17]

The present model

These partly critical comments on the large-scale simulation models are not meant to discourage their use – a lot can be learned with their help – but one conclusion here is that those models deserve a critical look, which is not easy, however, as the models are complex and the reporting is not always transparent and accurate. Therefore, they should not crowd out simpler, more practical analyses and models, which should also have their place in policy advice and public discussion.

The simple model in this particular paper is meant to serve the latter purpose. Its main limitation is that it is a *partial equilibrium* analysis, especially as the interest rate is not determined fully endogenously but by an assumption of the exogenous margin over the rate of growth (of the wage bill). This assumption is partly made in order to attain a recursive structure so that saving at each point in time does not affect the interest rate. This allows the model to be constructed and used with simple computations.

In defence of the simple assumption on the interest rate, we can note that it implies a path that is not very different from those derived from the large-scale models for otherwise similar cases. As we use the model only for cases which we consider realistic or relevant for policy design, the path for the interest rate is in line with a more refined (neoclassical) theory of the interest rate. Thus, in the relevant range of scenarios the simplifying assumption on the interest rate may not create problems. Also, the interest rate in our model represents people's expectations of it when they plan their lifecycle saving. For this purpose, our simple assumption might well be as good as any other. In addition, our model could be extended to cases where the interest rate takes any other exogenously determined path.

Even though our model above is simple, agents are assumed to solve a relatively demanding optimising problem to obtain the time path of their consumption and saving under their own projections for income and other factors. Breaks in expectations can be modelled, making simple and fully transparent assumptions. The model incorporates various specific factors for improved realism: children's consumption and vanishing work-related market consumption at retirement (partly explaining the observed hump shape of the lifetime consumption path), bequests and the credit constraint preventing borrowing against future wage and/or inheritance.

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[17] We should also note that the simulations in Fehr et al. (2006) on the effect of complete privatisation of public pensions are quite extreme: they assume that 1-2 working generations over the next few decades will shoulder the full burden of the implicit pension debt that has accumulated over the past 50 years or so. The outcome would be a doubling of the capital to NNI ratio while the transition generation's welfare would be cut by 5-10% over their whole lifetime. There are probably no economic or other convincing arguments for such a policy and it is difficult to see how it could be approved under any realistically specified democratic process. The result illustrates the importance of implicit pension debt in public pension systems, and, since it exists (as a result of decisions in the past), the burden has to be borne in one way or another by the current and future generations. There is no way of getting rid of it without harming at least one generation quite significantly. This implication of their fully fledged multi-period dynamic simulation model is in line with simple back-of-an-envelope steady-state calculations.

By a simple assumption in our model, people always adjust their current consumption to the general income level determined by labour productivity. The seniority factor is missing from our model. Intertemporal substitution of consumption is assumed to be zero, i.e. the interest rate does not affect the time profile of consumption, only total consumption through the income effect. Plugging these factors into our model would probably not change the implications very much, as they would tend to shift consumption towards a younger age, but the credit constraint would prevent this from happening.

The implications of our simple assumptions are illustrated in Graph 1 above. They could potentially be compared to those of the calibrated OLG models that are based on more refined utility functions but are often missing one or another (obvious) special factor. However, as the reports on large-scale models do not usually display the longitudinal implications of their assumptions, this exercise remains for further investigation.

It can be considered to be an advantage of a partial model geared to a particular issue that it focuses on the *effects* of certain exogenous variables on endogenous variables, while other factors outside the model may always have effects as well. This way, a partial model can give guidance for policies that are supposed to cope with a particular exogenous change such as population ageing. It is perhaps fair to note that, in practice, reliable *projections* that would incorporate all possible factors are also beyond the recent large simulation models geared to population ageing and pensions. Even if they are richer, they ignore the effects of other persistent changes (supply of labour in China and elsewhere, factors related to climate change and energy), which will also play a role.

Summing up, the various simplifying assumptions allowed us to simulate the *effects* of population ageing and related policy options, taking into account the factors that seem to be the most important determinants of lifecycle saving and, by implication, of aggregate saving in the model economy. In particular, the recursive structure of the model made it possible to demonstrate the critical role of the assumptions on expectations by modelling the "Surprise" reactions of lifecycle consumers when they realise that the ageing process has taken off and the pension system has been reformed. This deviates from the standard large-scale models where it is postulated that agents foresee the ageing process (including higher longevity) and react decades ahead, i.e. already over the period 1960-2000, and increase their retirement ages. However, this latter proposition does not fit easily with the observations on the European welfare states on average: from the late 1960s on, the average effective retirement age decreased by two years per decade until around 2000.[18] Therefore, the simple assumption made in the Surprise model in this paper that people were taken by surprise and started to adjust their behaviour only some time after 2000 when the consequences of population ageing properly reached public consciousness might be more justified than the 'perfect foresight' baseline under the large OLG models.

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[18] For men, it has revived on average since around 2000; for women, the turn occurred in the early 1990s, but this can probably be explained by other socio-economic factors than ageing and pensions.

6. Conclusions

The model in this paper gives, under plausible assumptions for European welfare states, the result that population ageing (decrease in fertility and increase in longevity less than fully compensated by increase in retirement age) increases saving and leads to an increase in capital to income (or more broadly interpreted, capital to output) ratio. The saving rate first significantly increases and then decreases and settles at a level that is in most scenarios slightly lower than the initial saving rate. The ultimately lower saving rate should be understood as reflecting the lower GDP growth rate brought about by the permanently lower fertility rate (via the lower growth rate of the labour force): a lower than initial saving rate is sufficient to maintain the increased capital to income ratio under the new steady growth.

Broadly speaking, the results in this paper are in line with the OLG models developed in 1960s-1970s for steady state analysis. Here, we have looked in particular at the transition path from one steady state to another. Our selective survey of the literature, including a few large-scale simulation models, provides the generally accepted result that, under population ageing and plausible assumptions regarding people's behaviour and government policy, the capital to income ratio increases. Also, quite robustly, the ultimate saving rate is somewhat lower than initially. It is less clear from this literature what happens to the saving rate during the transition: in many studies it decreases monotonically; in others it is not clearly spelled out. We have shown that a significant increase in the saving rate over two-three decades is quite plausible. Critical reading of the literature revealed that the reasons why the positive transitory effect does not come out from most studies seem to be related to issues concerning the use of the conventional perfect foresight assumption and possibly to assumptions regarding the change in retirement age. We hope to have shown convincingly that improvements in dealing with these factors in the large-scale simulation models are both required and possible, and that the positive transitory effect would then be confirmed.

With our simple model we were able to reiterate the standard result in an OLG model that, for a given demographic structure, public pension benefits decrease aggregate saving and capital stock. Under population ageing, cutting pension benefits increases saving, while an increase in retirement age has the opposite effect, as less saving is needed for retirement. We were also able to go beyond the standard assumption of a pure PAYG system and show that moving to partial funding in the public pension system (a positive item in government balance) increases total saving by roughly one-to-one. This result admittedly stems from the structure and calibration of our model, but it should give insight into the factors that probably determine the effectiveness of public policy with regard to long-term determinants of saving. In particular, our framework is capable of dealing with questions about intergenerational equity. It links population ageing, i.e. producing fewer children and enjoying a relatively longer share of life in retirement, to saving and capital formation. Any social norm making this link can only be the result of a political process, but an analytical framework for posing the right questions should be of some help.

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