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Endogenous housing risk in an estimated DSGE model of the Euro Area

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Abstract

We estimate a DSGE model with a housing sector where housing capital is used as collateral against which impatient consumers borrow from more patient lenders. While in existing estimated models with a construction sector the Loan-to-Value (LTV) ratio is imposed exogenously and constant (e.g., Iacoviello and Neri, 2010, In’t Veld et al., 2011), we introduce an endogenous LTV ratio by explicitly modeling the riskiness of loans in order to capture changing credit conditions. Using data of the Euro Area, we show that, compared to similar models with an exogenous LTV ratio, the business cycle properties of our model improve. The endogenous default mechanism allows to estimate an important amplification mechanism driven by the riskiness of collateral values and propagating, in turn, into the real economy. Housing market-related shocks appear to be the main driver of the pre-crisis growth of mortgage-backed loans and a subsequent reversal of the sentiment on the housing market may have been a trigger that led to a credit crunch, house price bubble burst and a collapse in the construction sector. Shocks on the housing market had also a substantial impact on several demand aggregates, in particular, consumption.
1 Introduction

The Great Recession that started in 2008 showed the importance of the links between the housing sector and the rest of the economy. While the causes of the recession were more complex (see, e.g., Gorton and Metrick (2012)), the burst of the house price bubble in the US and the subsequent increase in mortgage delinquencies could be seen as the direct trigger of the crisis (Blanchard (2009)). Indeed, home prices in the US increased from the trough in 1995 to the peak in 2006 by 56% (André (2009)) to fall by almost 27% between 2006 and 2011. This rendered the value of mortgages of many borrowers drop below the value of their loans. In effect, the number of foreclosures in the US increased fivefold (IMF, 2012) during this period.

The countries in the Euro Area (EA) had experienced a house price boom of a similar magnitude: between the trough and the peak (1996-2006) house prices in the EA rose by about 48%.\footnote{This and the following numbers concerning the price evolution in Europe have been taken from OECD (2012).} However, the following bust was on average much smaller than in the US: between 2006 and 2011 prices fell by only about 7%. These average numbers hide a large amount of heterogeneity (see Figure 1). During the boom period in certain EA countries (e.g. France, Spain, Netherlands) prices increased by 100% and more; in Ireland, they rose by 200%. Many of these countries experienced a bust in house prices after the break-out of the crisis. In Ireland prices fell by almost 40% in period 2007-2011. In Spain and Greece they fell by over 20%. The process of falling prices was accompanied by an increased delinquency rate by households, which greatly contributed to the first phase of the crisis.
Figure 1: House prices

Source: OECD (2012)
These developments leave no doubt that a successful macroeconomic model that aims at capturing the salient features of the business cycle should be able to account for the housing sector developments and the linkages between this sector and the rest of the economy. However, most modern Dynamic Stochastic General Equilibrium (DSGE) models until very recently did not include the construction sector.\(^2\) One exception has been the work done by Iacoviello (2005) and Iacoviello and Neri (2010) who introduced an ad hoc collateral constraint into a DSGE model with two types of households. In their model it is assumed that impatient households borrow from the patient ones against their housing wealth used as collateral. The need for collateral is loosely motivated by the absence of contract enforcement in the economy, as in Kiyotaki and Moore (1997). However, the exact form of the collateral constraint is not derived based on the optimal actions of agents. In particular, the tightness of the constraint, which directly affects the Loan-To-Value (LTV) ratio in the economy, is set exogenously and is hence a free parameter. Moreover, this model does not leave space for households’ default: Borrowers are assumed to always meet their payments and foreclosures do not emerge.

Despite its shortcomings, the Kiyotaki-Moore approach to capturing house market frictions has been an important step forward compared to the earlier generation of models. In particular, it allows for capturing amplification spirals between the house market and the rest of economy, whereby falling demand in other sectors reduces house investment and drives house prices down, which forces borrowers to deleverage and further weakens the demand. Moreover, it

\(^2\)For example the two possibly most well-known estimated DSGE models, Christiano et al. (2005) and Smets and Wouters (2007) do not have a housing sector.
allows for studying credit-crunch scenarios (modelled as exogenous tightening of the LTV ratio). One of the first policy models to incorporate the exogenous collateral constraint was QUEST III developed in the European Commission (e.g. Roeger and in ‘t Veld (2009)).

This paper makes a further step in introducing house market frictions into a large policy model. Unlike the Kiyotaki-Moore approach, we do not consider an exogenous collateral constraint. Instead, we follow Forlati and Lambertini (2011) in allowing random idiosyncratic fluctuations in the productivity of households’ house investment. Impatient households, who take credit against the houses they own, are free to decide whether the loan will be paid back. If they decide against paying back their loans, their creditors are entitled to take over whatever housing stock they owned at the moment of the default, subject to monitoring costs. Defaulting on their mortgage will be optimal if, after having faced a sufficiently negative housing shock, their housing stock falls below the value of the loan to be repaid. Since delinquent loans impose a loss on the lenders, they will require a higher interest from the borrowers. A condition that lenders’ earnings from credit is the same as earnings from other types of assets imposes then a limit on the total impatient households’ debt in terms of their housing stock. Hence, in this model, the LTV ratio is endogenous.

The strategic form of default assumed in the model implies that debt in our economy is nonrecourse: the lender’s recovery is limited to collateral and borrowers can walk away once the mortgage is seized. While this assumption accurately characterizes the legal situation in many US states, it may be less accurate for Europe, where banks, typically, are allowed to seize additional assets to compensate for the gap between the value of the seized property and the value
of the loan. There are several reasons, nonetheless, why we think this mortgage contract provides a useful approximation of contracts applied in the EA. First, for most households real property constitutes by far the most valuable asset. In many cases, hence, there may be few additional assets to be seized by the bank. Second, a seizure of additional assets often implies additional court costs, which makes it less conducive for the lender. Finally, the advantage of our approach is that it is relatively simple and already well established in the literature. In fact, to the best of our knowledge, alternative ‘ready-to-use’ approaches to modeling mortgage debt have not been proposed so far in a general equilibrium setting. We will be happy to consider an alternative approach in our future work.

Forlati and Lambertini (2011) assume state-contingent interest rate on debt. This, roughly, corresponds to adjustable mortgage rates in real economy. In reality, much of the mortgage debt is issued with fixed interest.³ For this reason, in this paper, we modify the Forlati-Lambertini setting by assuming that mortgage interest rates are set in advance, as in Benes and Kumhof (2011). The main difference between the two approaches is that, while the uncertainty due to risky house investment is fully borne by borrowers in the Forlati-Lambertini setting, it is shared between borrowers and lenders in the latter. This, in the future, should constitute a good departure point for studying amplification mechanisms working via the banking sector.⁴

³The authors of the ECB (2009) report estimated that in 2007 57% mortgage-based loans in the Euro Area were offered with fixed interest. This percentage very much varies from one country to another. In Belgium only 10% of such loans in that year had variable interest, while in Spain this share was 91%. Note, however, that the ECB defines variable rates as those that can change once a year or more frequently. From a perspective of a DSGE model with one period loans only that is estimated on quarterly basis (as it is the case for the model presented in this paper) loans with interest adjustable every year have, effectively, fixed rates as well.

⁴At the current stage financial flows between lenders and borrowers are assumed not to require any intermediation. An intermediating banking sector will be introduced in a future
The key feature of the model is the uncertainty shock, which affects the variance of the idiosyncratic risk on house investment. An increase in the variance of the house investment risk is shown to drive up the rate of default (RoD) on mortgages. In consequence, the interest rate demanded on debt (relative to the policy rate) instantaneously increases, while the mortgage debt gradually falls. The fall in demand due to households’ deleveraging then leads to contraction in output. Note that an increase in variance of the idiosyncratic housing shock leaves the mean productivity of house investment unchanged. Hence, in our model, RoD may be driven up even by a pure increase in the uncertainty.\(^5\) An increase in RoD is then followed by a credit crunch and spills over to the rest of the economy. It is in contrast to a Kiyotaki-Moore-based setting, in which a credit-crunch scenario can be studied only with the help of an exogenous decrease in the LTV ratio.

We estimate the model on the Euro Area data (sample 1995-2011) and document its fit. The share of borrowers in total households is found to be around 30\%, which we consider a plausible value. The monitoring cost for lenders in the case of a default on a loan they gave turns out to be rather high, at around 25\% of the value of the collateral. This is twice as high as the value Forlati and Lambertini (2011) use for their model’s US calibration. Our estimates are consistent with a steady state LTV ratio of about 60\%. The implied steady state RoD lies slightly below 1.9\%, comparable with 2.34\% rate of default assumed by Forlati and Lambertini for the US in their benchmark calibration.

\(^5\)RoD is fully endogenous and can be affected also by shocks not related to the productivity of house investment. In particular, contractionary shocks in monetary policy, leading to an increase in the real interest rate, temporarily increase the share of defaulting households.
We use the model to cast some light on the economic developments in the Euro Area during the last 15 years, with the special emphasis given to the housing and economic boom in the years 2004-2007 and the following financial and economic meltdown. We document that the house market demand shock and the house price shock as well as shocks to the uncertainty on the housing market were the main drivers of the pre-crisis boom on the housing market and substantially contributed to the growth of the mortgage-backed loans observed in this period. We also find strong evidence that the subsequent reversal of the sentiment on the housing market was a trigger that led to a credit crunch, house price bubble burst and a collapse in the construction sector. While other, more structural weaknesses, concealed during the boom years, played a more significant role in the economic crisis that ensued, uncertainty and other shocks on the housing market had also a substantial impact on several demand aggregates, in particular consumption. These findings, hence, provide a strong support for the strategy taken in this paper to model the housing market in much more detail than it is usually done.

The rest of the paper is organized as follows. The following sections discusses related literature. In section 3 we describe the model. Section 4 provides a discussion of the estimation strategy and presents the parameter estimates. It also discusses the fit of the model and its dynamic properties. The penultimate section of the paper provides a model-consistent interpretation of the economic developments during the last 1.5 decades, focusing on the period of Great Recession. The last section concludes.
2 Literature Review

Our paper is at the cross-roads of many research streams. After the recent economic turbulences, a growing body of literature in DSGE models has added an explicitly modeled housing sector. Davis and Heathcote (2005) and Erceg and Levin (2006) investigate a DGSE model with durable (residential investment) and non-durable sectors. Their calibrated models are used to analyze and compare the different responses of the two sectors to exogenous shocks, typically monetary shock and technology shock. One typical characteristics of durable consumption goods in the monetary business cycle is the degree of interest rate sensitivity. Barsky et al. (2007) focus on the implications of different housing prices flexibilities. They emphasize that flexible durable goods prices present problems for sticky price models of the business cycle. More precisely, when housing prices are flexible, the durable sector shrinks in response to a monetary expansion, offsetting the expansion in the non durable good sector and leaving GDP unchanged. Carlstrom and Fuerst (2006), starting from the premise that durable good prices are much more flexible than non-durable prices, propose nominal wage stickiness and credit-constraint as possible solutions to this “co-movement puzzle”. Following these contributions, in our model we allow for wage stickiness. The degree of price and wage stickiness is then estimated based on relevant price deflators.

A second group of contributions incorporates financial frictions à la Kiyotaki and Moore (1997) into models with a durable sector. Iacoviello and Neri (2010) extend the work of Iacoviello (2005) and propose an estimated DSGE model with housing and collateralized mortgage loans. As Iacoviello and Neri, we do
not commit to a particular view of the business cycle, rather we analyze the role of several shocks to explain the stylized facts and moments observed in the data. Monacelli (2009) emphasizes how the introduction of a collateral constraint in a housing model à la Iacoviello helps reconciling the model with data, especially for what concerns the observed co-movement of durable and non-durable sector in response to a monetary shock (provided the assumption of some degree of price stickiness for what concerns the durable sector).

These contributions impose an exogenously determined loan-to-value ratio which defines the ability to borrow by credit-constrained households. In order to micro-found the behavioral aspects of the financial contract, we rely on the financial accelerator mechanism from the seminal paper by Bernanke et al. (1999), but apply it to the homeowners’ borrowing problem rather than to the problem of entrepreneurs’ as it is done in the original work. In this respect, our model is similar to Forlati and Lambertini (2011), who calibrate a DSGE models with housing and endogenous LTV for the US economy. Forlati and Lambertini analyze a dynamic response of the economy to an increase in mortgage risk under alternative specifications of monetary rule and show how an increase in the volatility of the residential investment can have a substantial impact on real GDP, even though their calibration does not allow for fully accounting for the size of the decline in GDP that is often observed following disturbances on the housing market. In their model, loans are repaid at a state-contingent interest rate and lenders do not face any risk. In order to obtain a more realistic mortgage contract and to allow for lenders’ realized losses due to an unexpected increase in the riskiness of the collateral value, we follow the calibrated model of Benes and Kumhof (2011) in modeling pre-determined interest rates for the financial
contract. To the best of our knowledge, our paper constitutes a first attempt to estimate a DSGE model with an endogenous LTV ratio.

3 The Model

We consider a New-Keynesian model of an open economy, which produces goods which are imperfect substitutes to goods produced in the rest of the world (RoW). There are three production sectors, a monopolistically competitive final goods production sector, an investment goods producing sector and a monopolistically competitive housing production sector. We consider two types of households: patient households (Savers) work, consume and accumulate housing. They own the productive capital of the economy and lend to impatient households. Impatient households (Borrowers) work, consume and borrow from the Savers in order to finance their consumption and house investment needs. The loans they receive are backed by the housing stock they own that is used as collateral and will be lost in full should the household default. In modeling the housing sector, differently from the housing models by Iacoviello (2005) and Iacoviello and Neri (2010), we do not rely on an exogenous borrowing constraint, but, rather, derive it endogenously.

Our work is an extension of the QUESTIII model (Ratto et al. (2009)), to which a fully-fledged housing sector has been added. In particular, the RoW and the fiscal policy blocks of the model are identical to those described in Ratto et al. (2009) and we have omitted the description of these parts in the paper.
3.1 Households

The economy is populated by a continuum of measure 1 of households, divided in 2 groups. Savers constitute a fraction $s^s$ of all households, while the remaining $1 - s^s$ households are called Borrowers. Savers are patient and are thus characterized by a relatively lower intertemporal rate of subjective time preference than Borrowers. Hence, Borrowers’ discount factor $\beta^b$ is lower than the discount factor of the Savers, $\beta^s$, i.e., $\beta^b < \beta^s$.

Within each group, a representative household maximizes the following utility function, which is a nested constant elasticity of substitution (CES) between aggregate of consumption, $C^j_t$, and housing services, $H^j_t$, and separable in leisure:

$$\sum_{t=0}^{\infty} \beta^j tE_0 \left[ U^j(C^j_t, N_t, H^j_t) \right]$$

$$= \sum_{t=0}^{\infty} \beta^j tE_0 \left( \frac{\sigma_H}{\sigma_H - 1} \right) \log \left( \left( C^j_t - hC^j_{t-1} \right)^{\frac{\sigma_H - 1}{\sigma_H}} \right) + \left( s_{H,j} \right)^{\frac{1}{\sigma_H}} H^j_t \left( s_{H,j} \right)^{\frac{\sigma_H - 1}{\sigma_H}}$$

$$+ \frac{1}{1 - \kappa} \left( \exp(e^N_t) \omega^N (1 - \text{retir}_t - N_t) \right)^{1-\kappa},$$

with $j = b, s$. The term $s_{H,j}$ denotes the relative share of housing in the consumption index, $h$ defines habits in nondurable consumption and the term $\sigma^H$ is the elasticity of substitution. $N_t$ refers to the employment ratio, $\text{retir}_t$ defines the share of retired households and $e^N_t$ denotes an exogenous shock to the employment. The parameter $\kappa$ is related to the inverse of the elasticity of labor supply with respect to wages. In the subsections below the superscript $j$ will be skipped unless needed to avoid confusion.
3.1.1 Impatient Households (Borrowers)

Impatient households work, consume non-durable goods, buy houses and borrow an amount $L_t$ from the patient households in order to finance their consumption expenses. The loan is received in the form of a one-period collateralized financial contract against the value of the Borrowers’ house stock. Lenders receive a pre-determined nominal interest rate $i^L_t$ on the non-defaulting loans, while they seize the collateral in the case of default. At each period $t$, Borrowers are subject to the following budget constraint:

$$L_t = \left(1 + t^C\right) p^H_t \bar{I}^{H,b}_t + \left(1 + t^C\right) p^C_t C^b_t - (1 - \text{retir}_t - N_t) \text{BEN}_t + T_t$$

$$+ (1 - F(\bar{\omega}_t)) (1 + r^L_{t-1}) L_{t-1} - N_t (1 - t^w_t - ssc) w_t - TR_t,$$

(1)

with $r^L_{t-1} = i^L_{t-1} - \pi^Y_t$ the real interest rates on loans. Borrowers borrow $L_t$, supply labor $N_t$, which is remunerated at a gross wages $w_t$, receive benefits, $\text{BEN}_t$, transfers $TR_t$ and pay taxes $T_t$. They choose residential investment $\bar{I}^{H,b}_t$ and consumption $C^b_t$, priced at $p^H_t$ and $p^C_t$, respectively, where the relative prices are defined as the ratio of the price index with respect to the price index of final output, $P_t$:

$$p^H_t = \frac{P^H_t}{P_t},$$

(2)

$$p^C_t = \frac{P^C_t}{P_t}.$$

(3)

Finally, Borrowers (as all households) face several taxes and other payments; $t^C_t$, $t^w_t$ and $ssc$ denote consumption tax, labor tax and social contribution rates,
respectively.

As in Forlati and Lambertini (2011) - FL hereafter, we assume that Borrowers can default on their loans: the function $F(\bar{\omega}_t)$ denotes the probability of default and the term $[1 - F(\bar{\omega}_t)]$ represents the fraction of loans which are repaid at the pre-determined interest rate, $i^L_t$. Each household consists of many members $i$ with housing stock $H^i_{t+1}$ such that, $\int H^i_{t+1} di = H_{t+1}$. Each period, after the mortgage contract is finalized, each member’s housing stock is subject to an idiosyncratic shock, $\omega^i_t$, which is i.i.d. across members of the same household, with cumulative distribution function $F(\omega^i_t)$.

In our framework with pre-determined loan interest rate, we need to make a distinction between the expected cut-off value of the idiosyncratic housing shock, $\bar{\omega}_t^{ex-ante}$ and the ex-post realized cut-off value, $\bar{\omega}_t$. We define the ex-post cut-off value of the idiosyncratic housing shock, as the value such that, if the realization of the idiosyncratic housing shock falls below it, the borrower defaults. The ex-post cut-off value $\bar{\omega}_t$ is hence defined with the equation:

$$\bar{\omega}_t + 1 (1 - \delta^H) p^H_{t+1} H^b_t = (1 + r^L_t) L_t$$

with $\delta^H$ being the depreciation rate of the housing stock and where the left-hand-side of the equation is the threshold value of the house, below which an individual $i$ defaults and the right-hand-side is the value of the loan to be repaid including interest payments.

While FL assumes that Borrowers will absorb all of the aggregate risk and Savers hold risk-free portfolios by issuing state-contingent contracts, the pre-determined loan interest rate allows the possibility for Savers to make losses.
Indeed, the pre-determined interest rate, $i_t^L$, is set on the basis of the expectation of the realized value of the debtors’ housing stock, in order to satisfy the following equation:

$$\bar{\omega}_t^{ex-ante}E_t \left[ (1 - \delta^H) p_{t+1}^H H_t^b \right] = (1 + r_t^L) L_t,$$

(5)

where, $\bar{\omega}_t^{ex-ante}$ denotes the ex-ante threshold value of the idiosyncratic shock.

The debt contract ensures that in expected terms patient households receive risk-free interest on the loan. Therefore, Lenders set the loan interest rate so that the expected returns from lending is equal to its opportunity cost:

$$(1 + i_t - E_t \pi_{t+1}) L_t = E_t \left( \int_0^{\bar{\omega}_t^{ex-ante}} \omega_t \left[ (1 - \mu) \left( 1 - \delta^H \right) p_{t+1}^H H_t^b \right] f_t (\omega) d\omega \right) + E_t \left( \int_{\bar{\omega}_t^{ex-ante}}^{\infty} (1 + i_t^L - \pi_{t+1} L_t) f_t (\omega) d\omega \right).$$

(6)

The left-hand-side of the equation is equivalent to the value of the loan $L_t$ in real terms lent at a (nominal) risk-free rate $i_t$. The first term on the right-hand-side is equal to the total value of collateral seized by Lenders from defaulting Borrowers. Following the literature on the financial accelerator mechanism, we assume costly state verification so that Lenders pay a monitoring cost (defined by $\mu$), proportional to the collateral value to be seized. The second term on the right-hand-side of the equation is equal to the value of non-defaulting loans, gross of the borrowing interest rate.

For convenience, we define the expected value of the idiosyncratic shock conditional on the shock being less or equal to the threshold value, $\bar{\omega}_t^{ex-ante}$, as:
\[
G(\bar{\omega}_t^{ex-ante}) = \int_0^{\bar{\omega}_t^{ex-ante}} \omega_t f(\omega_t) d\omega_t, \quad (7)
\]

and the expected share of the housing value that goes to Lenders, gross of monitoring costs as follows:

\[
\Gamma(\bar{\omega}_t^{ex-ante}) = \bar{\omega}_t^{ex-ante} \int_{\bar{\omega}_t^{ex-ante}}^{\infty} f(\omega_t) d\omega_t + G(\bar{\omega}_t^{ex-ante}), \quad (8)
\]

and using eq.(5) to substitute the value of the loan interest rate, \(i_L^t\), the participation constraint of Lenders can be rewritten as:

\[
(1 + i_t - E_t \pi_{t+1}^Y) L_t = E_t \left( \chi(\bar{\omega}_t^{ex-ante}) \left( 1 - \delta^H \right) p_t^H H_t^b \right). \quad (9)
\]

where \(\chi(\bar{\omega}_t^{ex-ante}) = \Gamma(\bar{\omega}_t^{ex-ante}) - \mu G(\bar{\omega}_t^{ex-ante}).\) Note how this aggregate participation constraint resembles the more standard aggregate collateral constraint derived in models with Kiyotaki-Moore-like financial frictions. However in our model the loan-to-value ratio, represented by the term \(\chi(\bar{\omega}_t^{ex-ante}),\) is endogenous and depends on the deep parameters of the model (the monitoring cost \(\mu,\) the variance of the idiosyncratic shock to house investment and the parameters affecting the equilibrium value of \(\bar{\omega}_t^{ex-ante}\) and the Borrowers’ housing stock) as well as on the shocks hitting the economy during the business cycle.

Due to the unpredictable nature of the shocks, the number of defaulting households will typically differ from the Lenders’ ex-ante expectations, they will incur a net loss (a loss or a gain in the case of unexpected positive eco-
nomic developments) defined as the difference between the (potential) proceeds from a safe investment and the total proceeds from the mortgage-backed loan:

\[
Loss_t^s = (1 + i_{t-1} - \pi^Y_t) L_{t-1}
- \left[ \Gamma(\bar{\omega}_t) - \mu G(\bar{\omega}_t) \right] \left( 1 - \delta^H \right) p_t^H H^b_{t-1}, \tag{10}
\]

where the ex-post values of the \(\Gamma\) and \(G\) functions are defined as follows:

\[
\Gamma(\bar{\omega}_t) = \bar{\omega}_t \int_{\bar{\omega}_t}^{\infty} f(\omega) d\omega + G(\bar{\omega}_t), \tag{11}
\]

\[
G(\bar{\omega}_t) = \int_{0}^{\bar{\omega}_t} \omega f(\omega) d\omega. \tag{12}
\]

Then, the accumulation equation for borrowers’ residential investment is given by:

\[
\tilde{I}_t^{H,b} = H_t^b - (1 - \delta^H) (1 - G(\bar{\omega}_t)) H^b_{t-1},
\]

with the term \(G(\bar{\omega}_t)\) representing the fraction of defaulting borrowers’ housing stock which has been seized by the lenders.

Using the aggregate participation constraint, we can substitute the expression \((1 + i_{t-1}^L - \pi^Y_t) L_{t-1}\) in the budget constraint of Borrowers as a function of
the pre-determined rate, $i_{t-1}$, which allows us to rewrite this constraint as:

\begin{align*}
L_t &= T_t + (1 + t^c) \ p^H \ i^H, b_t \\
&\quad + (1 + t^c) \ p^C \ C_t + \left( 1 + i_{t-1} - \pi^Y_t \right) L_{t-1} \\
&\quad - N_t \ (1 - t^w - ssc) \ w_t - TR_t - (1 - retir_t - N_t) \ BEN_t \\
&\quad - p^H_t \ (1 - \delta^H) \ G \left( \bar{\omega}^{ex-ante}_{t-1} \right) \ (1 - \mu) \ H^b_{t-1}. \quad (13)
\end{align*}

In order to rewrite the budget constraint more compactly, we can introduce a new definition of residential investment:

\begin{align*}
I^H, b_t &= \tilde{i}^H, b_t - (1 - \delta^H) \ G \left( \bar{\omega}^{ex-ante}_{t-1} \right) \ (1 - \mu) \ H^b_{t-1} \quad (14)
\end{align*}

which allows us to define the Borrowers’ budget constraint as:

\begin{align*}
L_t &= T_t + (1 + t^c) \ p^H \ i^H, b_t \\
&\quad + (1 + t^c) \ p^C \ C_t + \left( 1 + i_{t-1} - \pi^Y_t \right) L_{t-1} \\
&\quad - N_t \ (1 - t^w - ssc) \ w_t - TR_t - (1 - retir_t - N_t) \ BEN_t, \quad (15)
\end{align*}

where the housing accumulation equation is defined as:

\begin{align*}
H^b_t &= \tilde{i}^H, b_t + (1 - \delta^H) \left[ (1 - \mu) G(\bar{\omega}^{ex-ante}_t) + 1 - G(\bar{\omega}_t) \right] H^b_{t-1}. \quad (16)
\end{align*}
Therefore, borrowers maximize the lifetime utility function

\[
\sum_{t=0}^{\infty} \beta^{b,t} E_0 \left[ U^b(C^b_t, N^b_t, H^b_t) \right]
\]

subject to budget constraint, eq.(15), the Lenders’ participation constraint, eq. (9) and the housing accumulation equation, eq.(16).

The first order condition with respect to \(C^b_t\) defines the Lagrange multiplier, \(\lambda^b_t\):

\[
C^b_t = \frac{U^b_{C,t}}{p^C_t (1 + r^c)} = \lambda^b_t,
\]

while the first order condition with respect to debt, \(L_t\), defines the stochastic discount factor of the Borrowers, \(m^b_t\):

\[
L_t : \beta^b E_t \frac{\lambda^b_t \psi^b_t}{\lambda^b_t} = \frac{1}{(1 + r_t) (1 - \psi^b_{t+1})} = m^b_t,
\]

with \(\psi^b_t\) being the Lagrange multiplier related to the Lenders’ participation constraint. The investment decision with respect to housing stock is given by:

\[
H^b_t : q^H_t = \frac{U^b_{H,t}}{U^b_{C,t}} \frac{p^C_t}{p^H_t} + \frac{m^b_t}{1 + e_t^p H} q^H_t (1 - \delta^H) E_t \frac{1 + \pi^H_{t+1}}{1 + \pi^H_{t+1}} \times
\]

\[
\times \left[ (1 - \mu) G (\bar{\omega}^{ex-ante}_{t+1}) + (1 - G (\bar{\omega}_{t+1})) \right]
\]

\[
+ \psi^b_t (1 - \delta^H) E_t \frac{1 + \pi^H_{t+1}}{1 + \pi^H_{t+1}} \left[ \Gamma (\bar{\omega}^{ex-ante}_t) - \mu G (\bar{\omega}^{ex-ante}_t) \right]
\]
where $q_t^{H,b} = \frac{z_t^b}{p_t^b}$, with $\xi^b_t$ being the Lagrange multiplier of the housing accumulation equation and the terms $\pi_t^H$ and $\pi_t$ denote the housing and output inflation rates, respectively. Note that we introduced a housing demand shock, $e_t^{rpH}$, in the arbitrage equation, as in Kollmann et al. (2013). This shock modifies directly the discount factor $m_t^b$ specifically used by Borrowers to discount the value of their houses and hence may be thought of as linked to the housing bubble.

We add adjustment costs of the residential investment, making a distinction between real investment expenditure, $I_t^{b,H}$ and physical expenditure, $J_t^{b,H}$:

\[ I_t^{b,H} = J_t^{b,H} + \frac{\gamma_H}{2} \left( \frac{J_t^{b,H} - \delta^H (1 - \tilde{G}(\tilde{\omega})) H_{t-1}^b}{H_{t-1}^b} \right)^2 + \frac{\gamma_{H,I}}{2} \left( \frac{J_t^{b,H} - J_{t-1}^{b,H}}{H_{t-1}^b} \right)^2, \tag{21} \]

with $\tilde{G}(\tilde{\omega})$ being the steady-state level of the term $G(\tilde{\omega}_t)$. Then, the optimal decision with respect to physical investment is given by:

\[ J_t^{b,H} : q_t^{b,H} = 1 + \gamma_H \left( \frac{J_t^{b,H}}{H_{t-1}^b} - (1 - (1 - \delta^H)(1 - \tilde{G}(\tilde{\omega}))) \right) + \gamma_{H,I} \left( J_t^{b,H} - J_{t-1}^{b,H} \right) - m_t^b \gamma_{H,I} \left( J_{t+1}^{b,H} - J_t^{b,H} \right) \frac{1 + \pi_{t+1}^H}{1 + \pi_{t+1}}. \tag{22} \]

Finally, in this economy, Borrowers decide on the optimal default decision, $\tilde{\omega}_t^{ex-ante}$:

\[ \tilde{\omega}_t^{ex-ante} : \psi_c^c \Gamma' \left( \tilde{\omega}_t^{ex-ante} \right) E_t \frac{1 + \pi_{t+1}^H}{1 + \pi_{t+1}} = G' \left( \tilde{\omega}_t^{ex-ante} \right) \left[ -m_t^b q_t^{H,d} + \mu \psi_c^c E_t \frac{1 + \pi_{t+1}^H}{1 + \pi_{t+1}} \right] \tag{23} \]
From eq. (5) and eq. (4), it is then possible to compute, respectively, the loan interest rate, \( r^L_t \) and the ex-post value of \( \bar{\omega}_t \), which determines the rate of default:

\[
RoD_t = F(\bar{\omega}_t); \quad (24)
\]

Finally, the external finance premium is defined as the difference between the loan rate and the risk-free rate:

\[
EFP_t = i^L_t - i_t. \quad (25)
\]

**Distributional assumptions**  We assume that the idiosyncratic shock \( \omega^i_t \) follows a lognormal distribution:

\[
\log(\omega^i_t) \sim N(\mu_{\omega^i,t}, \sigma_{\omega^i,t}^2) \quad (26)
\]

where \( \mu_{\omega^i,t} = -\frac{\sigma_{\omega^i,t}^2}{2} \) so that \( E_t(\omega^i_{t+1}) = 1 \) and \( \sigma_{\omega^i,t} = \sigma_{\omega} + e_{\omega,t} \), with \( e_{\omega,t} \) being an AR(1) innovation. This implies that the functions \( G(\cdot) \) and \( \Gamma(\cdot) \) can be defined as follows:

\[
G(\bar{\omega}_t) = \int_0^{\bar{\omega}_t} \omega_t f(\omega_t)d\omega_t
\]

\[
= \exp\left(\mu_{\omega^i} + \frac{\sigma_{\omega^i,t}^2}{2}\right) \left[\frac{1}{2} + \frac{1}{2}erf\left(\frac{ln(\bar{\omega}_t) - (\mu_{\omega^i,t} + \sigma_{\omega^i,t}^2)}{\sqrt{2}\sigma_{\omega^i,t}}\right)\right] (27)
\]
\[
\Gamma(\omega_t) = \bar{\omega}_t \int_{\omega_t}^{\infty} f(\omega) \, d\omega_t + G(\omega_t) \\
= \bar{\omega}_t \left( 1 - erf \left( \frac{ln(\omega_t) - \mu_{\omega_t}}{\sqrt{2}\sigma_{\omega_t}} \right) \right). \tag{28}
\]

where \( f(\cdot) \) is the probability density of the lognormal distribution.

### 3.1.2 Patient households (Savers)

Patient households own the productive sector of the economy. In each period they collect profits from the consumption goods and construction sector firms. As Borrowers, they work, earn wages \( w_t \), consume, invest in the residential sector, \( I^{H,S}_t \), and accumulate houses, receive transfers, benefits and pay taxes. They also lend \( L_t \) to Borrowers and receive back the loans lent out in the previous period gross of interest rate.\(^6\) The budget constraint for the patient households is:

\[
Pr_t + Pr^H_t = p^H_t I^{H,S}_t + \left( 1 + t^C \right) p^C_t C^*_t + L_t \\
- N_t (1 - t^w - ssc) w_t - TR_t - (1 - retir_t - N_t) \\
- L_{t-1} (1 + i_{t-1} - E_{t-1} \pi_t). \tag{29}
\]

where profits from the productive (consumption goods and housing) activities \((Pr_t \text{ and } Pr^H_t, \text{ respectively})\) are defined in detail in the following section. The

\(^6\)We define the budget constraint in terms of risk-free interest rate, similarly to the Borrowers' problem, eq.15.
residential investment is defined as follows:

\[ I^{H,s}_t = H^s_t - (1 - \delta^H) H^s_{t-1}, \quad (30) \]

and the adjustment costs of the residential investment is now defined as

\[
I^{s,H}_t = J^{s,H}_t + \frac{\gamma^H}{2} \left( \frac{J^{s,H}_t - \delta^H H^s_{t-1}}{H^s_{t-1}} \right)^2 + \frac{\gamma^{H,t}}{2} \left( \frac{J^{s,H}_t - J^{s,H}_{t-1}}{H^s_{t-1}} \right)^2.
\quad (31)

Savers optimally choose \( C^s_t, L_t, H^s_t \) and \( J^{H,s}_t \), maximizing their lifetime utility function:

\[
\sum_{t=0}^{\infty} \beta^s_t E_0 [U^s(C^s_t, N^s_t, H^s_t)]
\quad (32)
\]

subject to the budget constraint (29) and eq. (30). The first order conditions are:

\[
C^s_t : \frac{U^s_{C,t}}{p^C_t (1 + r^c)} = \lambda^s_t ;
\quad (33)
\]

\[
L_t : \beta^s_t \frac{\lambda^s_{C,t+1}}{\lambda^s_{C,t}} = \frac{1}{(1 + i_t - E_t \pi^C_{t+1})} = m^s_t ;
\quad (34)
\]

\[
H^s_t : q^{H,s}_t = \frac{p^C_t U^s_t}{p^H_t} + m^s_t \frac{(1 - \delta^H)}{1 + e^{r^p H^s_t}} q^{H,s}_{t+1} ,
\quad (35)
\]
and

\[ J_t^s : q_{t-1}^{H,s} = 1 + \gamma_H \left( \frac{J_{t-1}^{H,s}}{H_{t-1}} - \delta_H \right) + \gamma_{H,t} \left( J_{t-1}^{H,s} - J_{t-1}^{H,s} \right) - m_t^s \gamma_{H,t} \left( J_{t+1}^{H,s} - J_{t}^{H,s} \right) \] (36)

Note the presence of the housing demand shock, \( e_t^{rPH} \), defined in the previous subsection, entering the first order condition with respect to housing stock. Otherwise the first order conditions are standard.

### 3.1.3 Wage setting

A trade union is maximizing a weighted average of the utility functions of Savers and Borrowers. We assume that types of labor are distributed equally over constrained and unconstrained households. Wages are obtained by equating the weighted average of the marginal utility of leisure with the weighted average of the marginal utility of consumption times the real wages of the two types of households, adjusted for a wage mark-up factor, \( \eta^W \). In addition, we also allow for a sluggish adjustment of the real consumption wage, with the degree of sluggishness governed by the parameter \( \gamma^W \):

\[
(1 - t^w - ssc) w_t = BEN_t + \frac{1}{\eta^W} \left( \frac{V_{N_t} (1 + t^C)}{U^s_{N_t} s^e + U^h_{N_t} (1 - s^e)} \right) - \frac{1 - \eta^W}{\eta^W} \gamma^W w_t \left( \pi^W_t + \beta^s N_{t+1}^w \right). \] (37)

where \( V_{N_t} \), the marginal disutility from labor, is defined as:

\[ V_{N_t} = \exp \left( \epsilon_t^N \right) \omega^N (1 - retir_t - N_t)^{-\kappa}. \] (38)
3.2 Technology

3.2.1 Final output production

There are \( n \) monopostically competitive final goods producers. We make the assumption that individual firms are small and that they take input prices as given. Output of the firm \( j \) is produced with a Cobb-Douglas production function using capital \( u_{cap,j} K^j_t \) and labor force \( N^j_t \):

\[
Y^j_t = A \left( u_{cap,j} K^j_t \right)^{1-\alpha} \left( N^j_t \epsilon^{fp} \right)^{\alpha} K^{G,G}_{t},
\]

where the term \( u_{cap,j} \) determines the capacity utilization and the term \( K^G_t \) is government capital and \( A \) is a scaling parameter. We assume that individual firms are small enough so that they take the consumption price index as given.

The firms choose physical capital, capital utilization rate, labor and prices optimally in each period in order to maximize the present discounted value of profits \( Pr^j_t \):

\[
Pr^j_t = \frac{p^j_t}{P_t} Y^j_t - \frac{W_t}{P_t} J^j_t - ad J^{u_{cap}} (u_{cap}) - ad J^P (P^j_t) - ad J^N (N^j_t)
\]

(40)

We consider several types of adjustment costs related to the capacity utilization rate, final good price and employment, defined by the following convex functional forms:

\[
ad J^{u_{cap}} (u_{cap}) = \frac{p^j_t}{P_t} K^j_t \left( \gamma^{u_{cap},1} (u_{cap} - 1) + \gamma^{u_{cap},2} (u_{cap} - 1)^2 \right),
\]

(41)

\[
ad J^P = Y^j_t \frac{\gamma^P (P_t - P_{t-1})^2}{2 P_t},
\]

(42)
We also consider adjustment cost of investment in the following form:

\[
I_t = J_t + \frac{\gamma_K}{2} \left( J_t - \delta K_{t-1} \right)^2 + \frac{\gamma}{2} \left( J_t - J_{t-1} \right)^2,
\]

(44)

where \(\delta\) is the capital depreciation rate.

The optimal conditions for the firm \(j\) are given by

\[
J_t : q_t = 1 + \gamma_K \left( \frac{J_t}{K_{t-1}} - \delta \right) + \gamma (J_t - J_{t-1}) - \gamma m^s_t (J_{t+1} - J_t),
\]

(45)

\[
K_t : \frac{Y_t (1 - \alpha) \eta_t}{K_t} = q_t - \left( \frac{m^s_t (1 - \delta)}{1 + r p^k + e^{r p^k}} \right) q_{t+1} + \left( 1 - t^k \right) \times \\
\times \left( \gamma_{acap,1} (ucap_t - 1) + \gamma_{acap,2} (ucap_t - 1)^2 \right),
\]

(46)

\[
N_t : w_t = \frac{Y_t \alpha \eta_t}{N_t} - w_t \gamma_L (N_t - N_{t-1}) + m^s_t (N_{t+1} - N_t) \gamma_L w_{t+1},
\]

(47)

\[
ucap_t : \frac{Y_t (1 - \alpha) \eta_t}{K_t} = ucap_t p^s_t \left( \gamma_{ucap,1} + (ucap_t - 1) 2 \gamma_{ucap,2} \right),
\]

(48)
\[ Y_t : \eta_t = \eta^P - e^\eta_t - \gamma_p E_t (m_t^* ((\pi_{t+1}) - (\pi_t))). \]  

Equations (46) and (48) together determine the optimal capital stock and the capacity utilisation by equating the marginal value of product of capital to the rental price and the marginal value of capital service to the marginal costs of increasing capacity. Equation (47) shows that firms equate the marginal product of labor, net of marginal adjustment costs, to wage costs. Finally, eq.(49) determines the prices mark-up as a function of demand elasticity of substitution and changes in inflations.

### 3.3 House investment goods production

House investment \( I_t^{H,j} \) is produced from construction goods investment \( I_t^{H,j} \) with decreasing returns to scale:

\[ I_t^{H,j} = \left( I_t^{constr,j} \right)^{1-\alpha^H}, \]

where \( 0 < \alpha^H < 1 \).

Construction goods themselves are produced from final goods using a linear production function, \( I_t^{constr} = Y_t \), by perfectly competitive firms, which implies that the construction price index in terms of the final goods, \( p_t^{constr} = 1 \). House investment firm \( j \) is monopolistically competitive and maximizes its profits

\[ P r_t^{H,j} = I_t^{H,j} I_t^{H,j} - I_t^{constr,j}, \]
subject to the usual demand equation:

\[ I_{t}^{H,j} = \left( \frac{p_{t}^{H,j}}{p_{t}^{H}} \right)^{-\theta_{t}^{H}} I_{t}^{H}, \]

with \( I_{t}^{H} \) and \( p_{t}^{H} \) properly defined aggregates over all house investment firms.

The optimization problem of a monopolistic firm \( j \) becomes, therefore:

\[
\max \left[ \left( I_{t}^{\text{constr,j}} \right)^{1-\alpha} p_{t}^{H,j} - I_{t}^{\text{constr,j}} \right] \\
\text{s.t.} \quad \left( I_{t}^{\text{constr,j}} \right)^{1-\alpha} = \left( \frac{p_{t}^{H,j}}{p_{t}^{H}} \right)^{-\theta_{t}^{H}} I_{t}^{H},
\]

where \( I_{t}^{H,j} \) was substituted out using the definition of the production function and where we defined \( \eta^{H} = 1 - \frac{1}{\theta_{t}^{H}} \). The first order condition w.r.t. \( p_{t}^{H,j} \) is therefore:

\[
\left( I_{t}^{\text{constr,j}} \right)^{1-\alpha} = \lambda_{t} \theta_{t}^{H} \left( 1 - e^{ho_{t}^{PH}} - \gamma_{t}^{PH} (m_{t}^{H} E_{t} r_{t+1}^{H} - r_{t}^{H}) \right) p_{t}^{H,j-1} \left( \frac{p_{t}^{H,j}}{p_{t}^{H}} \right)^{-\theta_{t}^{H}} I_{t}^{H},
\]

where \( \lambda_{t} \) is a non-negative Lagrange multiplier on the house investment demand equation and the parameter \( \gamma_{t}^{PH} \) allows to include some frictions in the adjustment of house price, subject to a price mark-up shock \( e_{t}^{PH} \). In the equilibrium \( p_{t}^{H,j} = p_{t}^{H} \) and \( I_{t}^{H,j} = I_{t}^{H} \) so that, using again the definition of the production function, we obtain:

\[
\lambda_{t} = \left( \theta_{t}^{H} \right)^{-1} p_{t}^{H,j}. \tag{50}
\]
The optimal decision with respect to construction investment, $I_{t}^{\text{constr},j}$, is:

$$I_{t}^{\text{constr}} = \eta^H \left(1 - e_{t}^{PH} - \gamma_{P,H} \left(m_{t}^{H}E_{t}^{H} - \pi_{t}^{H} \right) \right) \left(1 - \alpha^{H} \right) p_{t}^{H} \pi_{t}^{H},$$

where we used the definition of $\lambda_{t}$ in eq.(50) and the fact that in the equilibrium $I_{t}^{\text{constr},j} = I_{t}^{\text{constr}}$.

### 3.4 Monetary policy

We assume that the central bank sets the policy rate as a response to inflation and output gap according to the following Taylor rule:

$$i_{t} = \phi^{i}i_{t-1} + (1 - \phi^{i}) \left(\bar{r} + \phi^{\pi} \left(0.25 \sum_{j=0}^{3} \pi_{t-j}^{C} \right) + \phi^{\gamma} \left(0.25 \sum_{j=0}^{3} g_{t-j}^{y} \right) \right) + e_{t}^{M},$$

where $g_{t}^{y}$ is defined as the growth rate of output. Parameter $\phi^{i}$ governs the degree of interest rate inertia in the Taylor rule. The last term, $e_{t}^{M}$, denotes an autonomous monetary policy shock.

### 3.5 Equilibrium

The aggregate house investment in period $t$ may be shown to be described by the equation:

$$J_{t}^{H} = s^{s}J_{t}^{H,s} + (1 - s^{s}) \left(H_{t}^{b} - (1 - \delta^{H}) (1 - \mu G(\bar{\omega}_{t}))H_{t-1}^{b} \right),$$

which takes into account the fact that a part of the Borrowers’ housing stock, proportional to the monitoring cost, $\mu G(\bar{\omega}_{t})$, paid by the Savers, is effectively
lost due to default.

The aggregate of any other household specific variable \( X^h_t \) in per-capita terms is given by \( X_t = \int_0^1 X^h_t dh = s^s X^s_t + (1 - s^s) X^b_t \) since households are identical within each group. Finally, in equilibrium, we have:

\[
C_t + I_t + I^H_t + G_t + TBN_t = Y_t,
\]

where the trade balance, \( TBN_t \), is defined as export minus import.

### 3.6 Innovations

A general form of the process governing a shock \( e^x_t \) is assumed to be:

\[
e^x_t = \rho_x e^x_{t-1} + \epsilon^x_t,
\]

where \( \epsilon^x_t \) is white noise process with a finite variance. It is further assumed that most shocks are persistent but stationary, implying an AR(1) innovation process with the persistence parameter \( 0 < \rho_x < 1 \). There are a few exceptions from this rule. We assume a white noise innovation to the autonomous monetary policy decision, \( e^M_t \), so that \( \rho_M = 0 \). We also assume that the total factor productivity shock, \( e^{tfp} \) and investment-specific productivity shock, \( e^{upi} \), follow random walks, implying \( \rho_x = 1 \) for these two innovations.
4 Estimation results

4.1 Estimation strategy

The model, in a linearized form, was estimated on quarterly Euro Area (EA) data over the period 1995Q1-2011Q3, using Bayesian estimation techniques. Model estimation was performed using the parallel Dynare toolbox for Matlab (Adjemian et al. (2011)) with $4 \times 100000$ Metropolis runs. A large two region version of QUEST (EA vs. the rest of the world, RoW) was estimated. Altogether 26 data series were used, allowing for a thorough description of the three main blocks of the model. The data source for most series is Eurostat, with the exception of the effective exchange rates and interest rate variables, for which the ECB database was used. The series were not detrended, although the model explicitly includes two non-stationary trends, for TFP and investment-specific productivity. 9 of the used data series describe the macroeconomic block of the model. They are: real output growth and output price inflation; aggregate consumption, corporate investment, construction investment and compensation to employees, all in the form of output ratios; consumption, investment and house price indexes, as ratios to the output price deflator; and employment rate and nominal short-term interest rate.

A number of parameters were calibrated based on the data averages over the estimation period (see Table 1, for some of the ratios). In particular, parameters were chosen to set the steady state corporate investment ratio to about 10% of

---

7These blocks are the EA macroeconomic block, the EA fiscal block and the RoW block. As was already stressed, since in this paper we focus on the EA economy and in particular on its housing sector, in this and the following sections we limit our attention to issues related directly to the EA macroeconomic block of the model, skipping the discussion of the remaining two large blocks, the EA fiscal block and the RoW block.
Table 1: Steady-state ratios

<table>
<thead>
<tr>
<th>Variable</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/GDP</td>
<td>Business investment share</td>
<td>0.1076</td>
</tr>
<tr>
<td>IH/C</td>
<td>house investment share over consumption</td>
<td>0.1216</td>
</tr>
<tr>
<td>N</td>
<td>Employment</td>
<td>0.4426</td>
</tr>
<tr>
<td>wL/GDP</td>
<td>Wage share</td>
<td>0.6008</td>
</tr>
</tbody>
</table>

GDP and housing investment ratio to about 12% of aggregate consumption. The steady state wage share was set to 60% and the employment was assumed to be about 44% of the total population.

Table 2 shows the calibrated parameters. We set the rate of intertemporal preferences, $\beta^s$, for Savers equal to 0.995, implying a steady state annual risk-free interest rate, $i_r = 2\%$. The impatient households were assumed to discount future at a higher rate, with a discount factor $\beta^b = 0.9615$. The housing shares of Savers and Borrowers, $s_{H,s}$ and $s_{H,b}$, are derived from the aggregate steady-state share of residential investment over consumption, assuming the Borrowers’ residential investment over consumption equal to the aggregate share. The depreciation rate of corporate capital, $\delta$, was set to 10% annually; for housing stock the assumed depreciation, $\delta_H$, is 4% annually. The output elasticity of employment, $\alpha$, was set to 0.67, slightly higher than the steady state of the wage share. The steady-state gross price wage markups, $1/\eta^p$, is assumed 1.11.

The risk premium, $rpk$ is set to 0.021 in order to match the steady state business investment share. The adjustment cost parameter, $\gamma_{ucap,1}$ is set to 0.0713 in order to match full capital utilization rate in the steady state. The parameter

---

8A high discount rate of borrowers is needed to keep the lagrange multiplier on their borrowing constraint positive during the whole estimation sample. Iacoviello and Neri (2010) choose a discount factor for Savers and Borrowers equal to 0.9925 and 0.97, respectively.
\( \omega^N \) is set to 0.0107 in order to match the steady-state fraction of employment. Normalizing GDP to 1 fixes the value production function scaling parameter \( A \). Finally, we also impose values for several taxes and social contribution rates; in particular, the consumption tax rate \( t^C \) is set equal to 0.2 and the capital tax rate \( t^k \) equal to 0.35. The general social contribution tax rate \( ssc \), is set equal to 0.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta^s )</td>
<td>0.995</td>
</tr>
<tr>
<td>( \beta^b )</td>
<td>0.9615</td>
</tr>
<tr>
<td>( A )</td>
<td>1.0245</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.667</td>
</tr>
<tr>
<td>( \alpha_G )</td>
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</tr>
<tr>
<td>( \delta )</td>
<td>0.025</td>
</tr>
<tr>
<td>( \delta^H )</td>
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</tr>
<tr>
<td>( \eta^P )</td>
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</tr>
<tr>
<td>( \gamma_{acap,1} )</td>
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</tr>
<tr>
<td>( \omega^N )</td>
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<tr>
<td>( rpk )</td>
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<tr>
<td>( t^C )</td>
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</tr>
<tr>
<td>( t^k )</td>
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</tr>
<tr>
<td>( ssc )</td>
<td>0.2</td>
</tr>
<tr>
<td>( s_{H,b} )</td>
<td>7.1934</td>
</tr>
<tr>
<td>( s_{H,s} )</td>
<td>6.5009</td>
</tr>
</tbody>
</table>

### 4.2 Parameter estimates

Tables 3 and 4 display estimates of parameters governing shock processes and the main structural parameters, respectively. As Table 3 shows, processes driving various shocks in the model tend to be relatively persistent. In particular, the housing price mark-up shock \( \varepsilon_i^{PH} \) is very persistent, possibly indicating non-stationarity of housing price inflation series. Another very persistent innovation
Table 3: Estimated and calibrated shocks standard deviations and autoregressive coefficients. Results from Metropolis Hastings

<table>
<thead>
<tr>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
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<tr>
<td>Distrib</td>
<td>Mean</td>
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<tr>
<td>Variances</td>
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</tr>
<tr>
<td>$\varepsilon^\omega$</td>
<td>invg</td>
</tr>
<tr>
<td>$\varepsilon^\eta$</td>
<td>invg</td>
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<tr>
<td>$\varepsilon^{PH}$</td>
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<td>invg</td>
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<tr>
<td>$\varepsilon^{rph}$</td>
<td>invg</td>
</tr>
<tr>
<td>$\varepsilon^M$</td>
<td>invg</td>
</tr>
<tr>
<td>$\varepsilon^{tfp}$</td>
<td>invg</td>
</tr>
<tr>
<td>$\varepsilon^{upi}$</td>
<td>invg</td>
</tr>
</tbody>
</table>

**Autoregressive coefficients**

<table>
<thead>
<tr>
<th>Distrib</th>
<th>Mean</th>
<th>Std</th>
<th>Mean</th>
<th>Std</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^\omega$</td>
<td>beta</td>
<td>0.850</td>
<td>0.0750</td>
<td>0.945</td>
<td>0.0269</td>
</tr>
<tr>
<td>$\rho^\eta$</td>
<td>beta</td>
<td>0.500</td>
<td>0.2000</td>
<td>0.655</td>
<td>0.0956</td>
</tr>
<tr>
<td>$\rho^{PH}$</td>
<td>beta</td>
<td>0.500</td>
<td>0.2000</td>
<td>0.965</td>
<td>0.0145</td>
</tr>
<tr>
<td>$\rho^N$</td>
<td>beta</td>
<td>0.500</td>
<td>0.2000</td>
<td>0.894</td>
<td>0.0308</td>
</tr>
<tr>
<td>$\rho^{rpk}$</td>
<td>beta</td>
<td>0.850</td>
<td>0.0750</td>
<td>0.837</td>
<td>0.0539</td>
</tr>
<tr>
<td>$\rho^{rph}$</td>
<td>beta</td>
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<td>0.0750</td>
<td>0.979</td>
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<td>(−)</td>
<td>(−)</td>
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<td>(−)</td>
</tr>
<tr>
<td>$\rho^{tfp}$</td>
<td>(−)</td>
<td>(−)</td>
<td>(−)</td>
<td>1</td>
<td>(−)</td>
</tr>
<tr>
<td>$\rho^{upi}$</td>
<td>(−)</td>
<td>(−)</td>
<td>(−)</td>
<td>1</td>
<td>(−)</td>
</tr>
</tbody>
</table>

Results from Metropolis-Hastings. “invg” and “beta” stand for the Inverse-Gamma and Beta distributions, respectively.
Table 4: Estimated Parameters. Results from Metropolis-Hastings

<table>
<thead>
<tr>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
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<tr>
<td>Distrib</td>
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<tr>
<td>Preference parameters</td>
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<tr>
<td>$\kappa$</td>
<td>gamma</td>
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<tr>
<td>$h$</td>
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<tr>
<td>$\sigma^H$</td>
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<tr>
<td>Housing sector parameters</td>
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<tr>
<td>$\mu$</td>
<td>gamma</td>
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<tr>
<td>$\sigma^\omega$</td>
<td>gamma</td>
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<tr>
<td>$\alpha_H$</td>
<td>beta</td>
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<tr>
<td>$\eta^H$</td>
<td>beta</td>
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<tr>
<td>Adjustment cost parameters</td>
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</tr>
<tr>
<td>$\gamma_{ucap,2}$</td>
<td>gamma</td>
</tr>
<tr>
<td>$\gamma_H$</td>
<td>gamma</td>
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<tr>
<td>$\gamma_{H,I}$</td>
<td>gamma</td>
</tr>
<tr>
<td>$\gamma_K$</td>
<td>gamma</td>
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<tr>
<td>$\gamma_I$</td>
<td>gamma</td>
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<td>$\gamma_L$</td>
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<td>$\gamma_P$</td>
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<td>$\gamma_{P,H}$</td>
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<tr>
<td>Other</td>
<td></td>
</tr>
<tr>
<td>$s^i$</td>
<td>beta</td>
</tr>
</tbody>
</table>

Results from Metropolis-Hastings. “gamma” and “beta” stand for the Gamma and Beta distributions, respectively.
is the housing demand shock.

Turning to the structural parameters of the model, the estimation results shown in Table 4 suggest that adjustment costs, both for quantities and prices, play an important role in the economy. In particular, output prices and wages have been found to be rigid. On the other hand, house prices turn out to adjust relatively quickly (in line with the durable goods prices literature, eg. Carlstrom and Fuerst (2006)).

Passing to the preference parameters, parameter $\kappa$, related to the inverse of the elasticity of labor supply with respect to wages was found to be around 2.4, indicating relatively weak sensitivity of households to changes in the real wage rate. The parameter governing the degree of substitution between consumption goods and housing in the households’ utility function, $\sigma_H$, has a value of about 0.37, implying a high degree of complementarity between these two types of goods. Consumption goods habits, $h$, at around 0.67, are relatively deep. Finally, the share of patient households, $s^p$, has been estimated to be around 68%. The remaining 32% of households are hence Borrowers.9

The parameters describing the Taylor rule are close to values typically found in the literature. The weight put by the central bank on current inflation ($\phi^\pi \approx 2.04$) is more than 3 times higher than the weight put on the output gap ($\phi^y \approx 0.60$). The central bank has also a preference for smoothing its policy rate: the coefficient on the lagged interest rate in the Taylor rule is relatively high, $\phi^i \approx 0.86$.

From the perspective of this paper the most important set of coefficients

---

9Note that, as is the case with most other parameter priors, we followed Iacoviello and Neri (2010), who used a relatively narrow prior distribution for parameter $s^p$. The use of a wider prior could result in a slightly different estimated share of patient households in the model.
concerns the housing sector in the model. At $\mu$ equal to 25.1% of the value of a mortgaged property, the monitoring costs are about double of what Forlati and Lambertini (2011) assumed for the US. Arguably, this result is consistent with the idea of a house foreclosure process being more expensive in Europe than in the US, due to better institutionalized and simpler default and foreclosure procedures in this country. However, the confidence bands on this parameter are relatively wide (see the table) and it may not be excluded that the true cost of foreclosure in the EA is substantially lower.

The standard deviation, $\sigma_{\omega}$ of the idiosyncratic uncertainty shock has been found to be 0.30, 50% larger than what Forlarti and Lambertini assumed in their benchmark calibration for the US. These values imply the steady state loans-to-value ratio of about 60% and the rate-of-default of about 1.9%, slightly below the rate of seriously delinquent mortgages in the 4th quarter of 2006 in the US, reported by Forlati and Lambertini to be around 2.34%. For the remaining two house sector-related parameters, the value of $\alpha_H \approx 0.25$ points towards decreasing returns to scale in this sector. This finding may stem from the fact that an important housing production input, land, is not included in the production function. Finally, the gross mark-up in the housing sector, $1/\eta_H \approx 2.7$ is relatively high.

### 4.3 Estimation fit

In this Section we compare the second order statistical moments of the growth rates main aggregate variables implied by the model with the data (see Table 5) in order to investigate the business cycle properties of the model. For this comparison, the data second order moments have been computed by taking the
model steady state as the sample mean for the observed variables, in order to exactly indicate the actual values that are implicitly used in the likelihood computations during estimation. In our estimation, in fact, we do not apply any prefiltering nor detrending and we assume balanced growth between real variables and a common trend for domestic demand prices. These model assumptions imply some obvious mismatch with respect their pure empirical counterpart for mean growth and inflation rates and when comparing the model with the data one has to be precise about their implications on the data moments that are actually ‘seen’ by the likelihood. In Table 5 we also provide the $R^2$ of the 1-step ahead predictions of the main aggregate variables, which is defined as $R^2 = 1 - \frac{\Sigma_t ((x_{t+1}-\hat{x}_t)^2)}{\Sigma_t ((x_t - \bar{x})^2)}$, where \( \bar{x} \) denotes the model implied mean (i.e. the steady state). In a standard OLS context, the $R^2$ ranges between 0 and 1. Important cases where the computational definition of $R^2$ can yield negative values arise where the least squares predictions which are being compared to the corresponding observations have not been derived from a model-fitting procedure using those data, or where a linear regression is conducted without including an intercept or in any other case where restrictions are imposed on the regression coefficients. Additionally, negative values of the $R^2$ may occur when fitting non-linear functions to data. In DSGE model estimation, negative $R^2$ may occur as well. In cases where negative values arise, the mean of the data provides a better fit to the observations than do the fitted function values. In the case of the quarterly growth and inflation rates discussed here, given the significant volatility of these series, relatively small but positive values of the $R^2$ have to be regarded as an indication of a reasonably good fit.

The results shown in Table 5 indicate that the fit of the model is quite sat-
isfactory, especially regarding the consumption series, which is typically very
difficult to fit in models with a separate group of borrowers whose implied con-
sumption is very sensitive to credit conditions and volatile, often implying nega-
tive values for the $R^2$ and very low estimated share of Borrowers in the economy.
Here, in turn, we obtained $s^*$ equal to 0.70, which seems a realistic value keeping
at the same time the fit at a reasonable level. This results is allowed by the en-
dogenous loan-to-value ratio mechanism, which makes the consumption pattern
smoother with respect to a baseline model with exogenous default and loan-
to-value ratio. The standard deviations and correlations of most variables are
within the 95 percent probability interval computed from the data. The model
replicates well the autocorrelation of housing prices observed in the data as well
as the co-movements between housing prices with consumption and residential
investment. The $R^2$ is positive within the 95 percent probability for the growth
rates of all the variables considered and it is quite large for output, construction
investment and housing prices.

4.4 Dynamic behavior of the model

This subsection presents a set of impulse responses of the main variables in
the model to five shocks: two standard ones, a (positive) TFP shock, $e_{t}^{TFP}$, and
a (negative) shock $e_{t}^{M}$ to the nominal policy rate and three shocks linked to
the housing market, a shock increasing the uncertainty of idiosyncratic house
investment, $e_{t}^{\omega}$, a house price shock, $e_{t}^{PH}$, and a house market confidence shock,
$e_{t}^{PH}$. The impulse-response functions (IRFs) are calculated imposing estimated
values on model parameters. In particular, the variances of the shocks used for
calculating IRFs are equal to the values found in the estimation. We start by
discussing the more standard TFP and monetary policy shocks.

**TFP and monetary policy shocks**  The reaction of the economy to a permanent TFP shock (Figure 2) is standard. All main aggregates (consumption, corporate investment, house investment, output) permanently increase. Employment, driven by a persistent increase in real wages, falls and reverts to its initial level only very gradually. Output price deflator falls permanently, while relative house prices initially increase, an effect of decreasing returns to scale in the housing sector. Expecting higher productivity also in the future, impatient households increase their debt level in the attempt to smooth their consumption over time. Due to a slow accumulation of the housing stock, the quick growth of debt leads to the loans-to-value ratio temporarily shooting up. A higher LTV ratio provides a stronger incentive to default when a household is faced with an adverse idiosyncratic shock to its house value.\(^{10}\) Hence, the rate-of-default temporarily rises, and with it the external finance premium required by the lenders.

\(^{10}\)When the value of the house is small compared to the loan, relatively smaller shocks to housing wealth may drive its value below that of the loan, triggering a rational default on its debt by a household. Hence, the rate-of-default is an increasing function of the LTV ratio.
Table 5: Business cycle properties of the model.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Median</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>97.5%</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>97.5%</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard Deviation (percent)</strong></td>
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<td></td>
</tr>
<tr>
<td>$Y$</td>
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</tr>
<tr>
<td>$C$</td>
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<td>0.56</td>
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<tr>
<td>$I$</td>
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<td>$I_{constr}$</td>
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<td>$p^H$</td>
<td>0.98</td>
<td>0.85</td>
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<tr>
<td>$I,Y$</td>
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</tr>
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<td>$p^H,C$</td>
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<tr>
<td>$p^H,I_{constr}$</td>
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<table>
<thead>
<tr>
<th><strong>Autocorrelations</strong></th>
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<tbody>
<tr>
<td>$Y$</td>
</tr>
<tr>
<td>$C$</td>
</tr>
<tr>
<td>$I$</td>
</tr>
<tr>
<td>$I_{constr}$</td>
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<tr>
<td>$p^H$</td>
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</table>

<table>
<thead>
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<th><strong>R-squared</strong></th>
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<tbody>
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<td>$I$</td>
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<tr>
<td>$I_{constr}$</td>
</tr>
<tr>
<td>$p^H$</td>
</tr>
</tbody>
</table>

The reported values have been calculated using growth rates of respective variables.
Figure 2: Posterior Impulse response functions to a positive shock (estimated standard deviation) to the Total Factor Productivity with Bayesian uncertainty bounds.

The interest rates and the external finance premium are annualized and expressed as deviations in percentage points from the steady-state. The inflation rates, the rate of default and the Loan-to-Value ratio are expressed as deviations in levels from the steady state (percentage numbers). All the other variables are in percentage deviations from the steady state. Uncertainty bounds refer to the 90% Highest Probability Density (HPD) interval.
Figure 3 depicts the effects of a negative shock to the policy rate. Prices, due to the presence of adjustment costs, increase only gradually, inducing a fall in the real interest rate. In effect, corporate investment shoots up while saving becomes less attractive. As a consequence, consumption, output and employment temporarily increase. The level of debt decreases as borrowers try to smooth their consumption. The decreasing borrowers’ indebtedness, coupled with increasing house investment and lower real interest rates lead to a temporary decrease in the loan-to-value ratio of impatient households. The rate of default is reduced, accompanied by a fall in the external finance premium.

\[11\] Decumulating one’s debt implies relatively lower consumption today and higher consumption at future dates. Hence, debt decumulation smoothes consumption when the economy is temporarily booming.
Figure 3: Posterior Impulse response functions to a positive shock (estimated standard deviation) to the nominal interest rate with Bayesian uncertainty bounds.

The interest rates and the external finance premium are annualized and expressed as deviations in percentage points from the steady-state. The inflation rates, the rate of default and the Loan-to-Value ratio are expressed as deviations in levels from the steady state (percentage numbers). All the other variables are in percentage deviations from the steady state. Uncertainty bounds refer to the 90% Highest Probability Density (HPD) interval.
Shock to the idiosyncratic riskiness of house investment  The effects of a positive shock to the variance of the idiosyncratic productivity of house investment, $e^{t_0}$, are shown on Figure 4. The immediate effect of the shock is to increase the rate of default, which in turn increases monitoring costs borne by the lenders. Consequently, the borrowing rate increases, as reflected in the higher external finance premium. A higher borrowing rate forces impatient households into a costly deleveraging process and further increases their financial distress. In effect their loan-to-value ratio falls. The financial distress also implies that borrowers have to limit their house investment, which drives relative housing prices down.

The negative effect on housing prices noted above is, however, mild and short-lived, a finding consistent with the results reported by Forlati and Lambertini (2011). Falling house prices make house investment more attractive for patient consumers. Moreover, with Borrowers’ credit needs curtailed, Savers now have more resources to invest in other assets. As a consequence, Lenders’ housing demand picks up, exerting an upward pressure on house prices. After around 10 quarters also the house investment demand by borrowers recovers and, in fact, increases above its steady state value, as credit conditions gradually improve and the impatient households seek to increase their credit by rebuilding their housing stock. These developments have the effect of pushing the relative housing price up.

The overall impact of the shock on the economy is negative. Financially strained Borrowers are forced to reduce their consumption. While patient households use a part of the resources freed by lower lending to increase their consumption, the total effect on consumption is strongly negative. Through the
demand effects, output, employment and real wages all fall. However, the aggregate effect on the economy is relatively short-lived: output, consumption, borrowing rates and the rate of default all return to the 'pre-crisis' level after 2 to 3 years. One exception to this rule is employment, which remains below its steady state for 10 years and longer. Also the level of credit and the loan-to-value ratio revert to their pre-crisis levels only very slowly.

The effects of the idiosyncratic shock to risk in the housing sector remind those of the 'credit-crunch' scenarios discussed in papers with an ad hoc collateral constraint (Iacoviello and Neri (2010); In ’t Veld et al. (2011)). Our approach can be viewed as providing one kind of micro-foundations to these more traditional models. We show that, as in the case of a shock applying directly to the Borrowers’ LTV ratio like in the aforementioned papers, a credit-crunch and forced deleveraging may result from an increase in the idiosyncratic risk on the house value. Additionally, however, the risk shock also leads to an increase in the default rate and the external finance premium. Hence, our model captures the linkages between households and the financial sector in a more comprehensive, and possibly realistic, manner.
Figure 4: Posterior Impulse response functions to a positive shock (estimated standard deviation) to $\sigma_\omega$ with Bayesian uncertainty bounds.

The interest rates and the external finance premium are annualized and expressed as deviations in percentage points from the steady-state. The inflation rates, the rate of default and the Loan-to-Value ratio are expressed as deviations in levels from the steady state (percentage numbers). All the other variables are in percentage deviations from the steady state. Uncertainty bounds refer to the 90% Highest Probability Density (HPD) interval.
**House market demand and house price shocks** Two other shocks of a particular interest for this paper are the (non-fundamental) house market demand shock and the house price shock. The house market demand shock, \( e^{rph}_t \), is best viewed as a driving force of the house market bubble (see Figure 5). When the house market demand exogenously increases, so does house investment and, as a consequence, the relative housing price. Expenditure switching reduces the demand for consumption and, initially, corporate investment. However, total output increases. Since the house bubble shock is inflationary, real wages fall, inducing a rise in employment and hence output.

In reaction to positive trends in the economy loans increase. This increase, however, is slower than the build-up of the housing wealth as a part of the increase in house investment is founded via lower consumption. In effect the LTV ratio temporarily drops. The rate-of-default falls and with it the EFP.
Figure 5: Posterior Impulse response functions to a positive demand shock (estimated standard deviation) to housing market with Bayesian uncertainty bounds

The interest rates and the external finance premium are annualized and expressed as deviations in percentage points from the steady-state. The inflation rates, the rate of default and the Loan-to-Value ratio are expressed as deviations in levels from the steady state (percentage numbers). All the other variables are in percentage deviations from the steady state. Uncertainty bounds refer to the 90% Highest Probability Density (HPD) interval.
The interest rates and the external finance premium are annualized and expressed as deviations in percentage points from the steady-state. The inflation rates, the rate of default and the Loan-to-Value ratio are expressed as deviations in levels from the steady state (percentage numbers). All the other variables are in percentage deviations from the steady state. Uncertainty bounds refer to the 90% Highest Probability Density (HPD) interval.
A (positive) house price shock, $e_t^{PH}$, has the effect of a typical (negative) supply shock. The shock affects directly the mark-ups in the housing sector. As it is the case for the positive changes in house market demand, in reaction to this shock, relative house prices temporarily rise (Figure 6). However, unlike for the demand shock, now demand for housing falls. Moreover, a typical substitution effect takes place: households, facing relatively more expensive housing, shift some of their expenditure to consumption, leading to its temporary increase. Since a rise in house prices is inflationary, the real interest rate initially decreases. However, tightening of monetary policy by the central bank leads to an increase in the policy rate in the medium run. This has a negative effect on corporate investment. Falling corporate and house investment, despite a hike in consumption, depress employment and, in the medium run, output, which then recovers only very slowly.

Increasing house prices have a mildly positive effect on Borrowers. Faced with an increasing value of their homes, they can afford taking up additional mortgage-backed loans. However, given the overall pessimistic prospects of the economy, debt rises less than proportionally with the value of housing wealth. In effect, the LTV ratio decreases and so does the rate-of-default. Following the temporary drop in defaults, also the EFP decreases.

5 The first years of the XXI century through the lenses of the model

This section looks at the quantitative importance of different shocks for explaining the boom-and-bust cycle from the first decade of the XXI century. When all the shocks are fed simultaneously to the estimated model, the fit of the model
and actual data coincide exactly. The shocks can therefore be used to provide an additive decomposition of the data (e.g. Christiano et al. (2008) and In ’t Veld et al. (2011)). Since the model contains a large number of shocks, many of them included to fit the fiscal and the RoW data, here we focus only on those disturbances directly linked to the main topic of the paper. Seven types of shocks have been chosen: house market demand shock, $e_{t}^{rph}$, house price shock, $e_{t}^{pH}$ and a shock to idiosyncratic riskiness of house investment, $e_{t}^{\omega}$ (three shocks specifically related to the housing sector); productivity shocks (including the TFP shock, $e_{t}^{Tfp}$, and the investment-specific productivity shock, $e_{t}^{upi}$, stack together), the wage mark-up shock, $e_{t}^{\eta}$, and the stock market risk premium shock, $e_{t}^{rpk}$ (together constituting a group of more classic supply-side shocks); and a monetary policy shock, $e_{t}^{M}$. As it turns out, this small number of shocks are capable of explaining most of the variability in the main demand aggregates like output and consumption as well as in several important financial and house sector-related variables.

**House investment growth and house price inflation** We start the discussion with the two main house sector-related variables, house investment growth and house price inflation. A shock decomposition of the house investment growth is shown on Figure 7. Not surprisingly, house market-related shocks explain much of the variation in this variable during the last 15 years. The (non-fundamental) house demand shock drove house investment demand up in the years 1998-2007 contributing to a steady growth of the construction sector in the EA. The sudden reversal of sentiments between 2007 and 2008 led then to a collapse of house investment. During the decade preceding the Great Recession,
on the other hand, elevated mark-ups in the construction sector (as captured by positive house price innovations), contributed negatively to housing demand. The situation changed during the crisis, with falling price mark-ups contributing positively to housing demand. The combined effect of both shocks was to strongly increase house investment growth in years 2003-2007 and decrease it during the Great Recession.

More classic supply shocks turned out not to be particularly important for the behavior of house investment in the decade preceding the Great Recession. However, the productivity shocks, which capture the general deterioration of the economy during the crisis, were found to significantly contribute to the collapse in house investment in this period.

Figure 7: house investment growth decomposition
The house market-related factors were also the main drivers of house demand specifically among Borrowers (Figure 8). However, in contrast to the aggregate house investment, the shock to idiosyncratic risk turns out the single most important factor explaining the historical evolution of the Borrowers’ part of house investment. The reason is that house investment plays a more pronounced role in the case of the impatient households as it determines their capacity to borrow. When the house investment uncertainty in the economy increases, as it happened during the Great Recession, credit becomes more expensive and Borrowers are forced to delever. These factors decrease the value of housing stock for borrowers and the housing demand shrinks. The opposite is, of course, true when idiosyncratic uncertainty gets smaller.

As in the case of the aggregate house investment, movements in house prices
are mainly explained by two shocks, the house demand shock and the house price shock (Figure 9). However, unlike for investment, in the case of house price inflation these shocks tend to reinforce each other. During the 1998-2007 period house prices grew in the EA, a development mainly caused by increasing mark-ups and a bullish market. These trends inverted during the great recession. Also the idiosyncratic uncertainty shock on the housing market contributed to the house price developments over the last 15 years, working through its impact on housing demand, as explained above.

Monetary policy has been found to have had only little influence on house prices, via the usual anti-inflationary channel (high real interest rates tend to have a negative effect on prices at large). The supply shocks generally did not have much impact on house price inflation.

Figure 9: House Price Inflation decomposition
Financial variables  The financial sector in the model is motivated by the presence of an agency problem between Borrowers and Lenders. The need for a collateral in the form of housing stock to ease this problem implies that most financial variables: the level of borrowers’ debt, the EFP and the LTV ratio, are found to have been historically strongly influenced by the shocks occurring in the housing sector.

Loans growth (Figure 10) is closely linked to supply and demand conditions in the housing market. Loans are positively affected by rising house demand and house price inflation. The latter increases loans by increasing the value of homes which can be used as collateral.

Historically, monetary policy and supply shocks have had only a limited impact on the level of mortgage-backed debt in the economy.

Figure 10: Loans growth decomposition
The house market-related shocks are also the main factors explaining the LTV ratio (Figure 11). An important positive driver of the LTV in the boom period was reduced idiosyncratic loan risk. The decline of the LTV since 2009 is largely driven by increased risk perceptions for mortgage loans and less by a declining demand for housing.

Interestingly, comparing figures 11 and 10, one immediately notes that a positive loan growth is not always associated with an increasing loan-to-value ratio. A rising demand for housing is partially self financed by households via a reduction of consumption. In effect, housing stock tends to grow faster than debt in boom periods, decreasing the loan-to-value ratio. An additional effect comes from goods and house consumption-smoothing in boom periods: anticipating higher output in the future, households tend to increase their consumption of
goods and house services by reducing (or increasing at a relatively slower rate) their current debt. This behavior further suppresses the level of LTV.

Figure 12: External Finance Premium decomposition

The final financial sector-related variable we will look at is the EFP. Its shock decomposition (Figure 12) indicates that for most of the analyzed period (years 1999-2009) the EFP remained below or close to its 1995 value. This general description, however, hides two salient trends: the EFP had a decreasing trend until the dot-com bubble burst in 2001; it then increased and stayed around its 1995 value until the eruption of the Great Recession. Around 2008 the EFP started steeply growing and, after having stabilized for a short period of time between 2009 and 2010, continued growing until the end of the sample. A combination of three factors (lower idiosyncratic uncertainty on the housing
market and two shocks that led to positive valuation effects, low risk premia and high housing price mark-ups) contributed to a low value of EFP in the decade preceding the outburst of the Great Recession. However, an increase in the uncertainty was by far the most important factor that led to a sudden increase of the EFP right before and during the Great Recession.

**GDP growth** Loan growth and house investment had a limited impact on GDP (Figure 13). However, the housing risk shock and associated credit squeeze had some negative impact on output during the Great Recession. Still, the model explains the recession mostly as an effect of negative productivity and investment risk premium shocks. With a more complete banking model it would be possible to generate more amplification from mortgage losses onto corporate investment, this is for example shown in Kollmann et al. (2013).

**Consumption growth** House sector-related shocks explain a larger part of variation in consumption growth (Figure 14). Due to a substitution effect, falling demand for houses contributed positively to consumption growth during the Great Recession.\(^{12}\) Quantitatively more importantly, positive shocks to riskiness of idiosyncratic house investment, \(e_\omega\), tended to depress consumption growth, during the periods of crises and in particular in the years following the dot-com bubble burst and during the Great Recession. Conversely, this shock helped increase consumption growth during the boom years, especially in the second half of the 90s. These effects are mainly driven by the behavior

\(^{12}\)Analogically, increasing stock market risk premia during this period incited households to substitute away from corporate investment and towards increasing consumption, an effect also visible on Figure 14.
of impatient households’ (see Figure 15). Higher idiosyncratic risk increases the borrowing rate and pushes borrowers into a lengthy and painful deleveraging process. The increased financial strain depresses borrowers’ consumption. While, as discussed in the previous section, a lower level of lending lets patient households consume more, the negative effect on borrower’s consumption is comparably stronger. In consequence, the effect on aggregate consumption is also negative. A converse mechanism helps increase total consumption at times of low uncertainty.

**Real interest rate** The last variable to discuss here is the real interest rate (Figure 16). The evolution of this rate is consistent with loosening of the monetary policy following the dot-com bust as well as during the Great Recession.
Monetary policy appears to have had been restrictive just before the dot-com bubble formed and immediately preceding the outburst of the current financial and economic crisis.

Apart from the monetary policy, other shocks had an influence on the real rates as well. Increased idiosyncratic uncertainty on the housing market was associated with higher rates, while the rates fell when the uncertainty was lower. Variation in stock market risk premia had a similar effect, with their impact most strongly visible during the Great Recession. Finally, shocks to wages contributed positively to the level of real interest rate during the formation of the dot-com bubble, had a largely neutral effect in the years following the bubble and contributed negatively during the Great Recession.
Figure 15: Consumption growth decomposition (Borrowers)
Figure 16: Real interest rate decomposition
6 Conclusions

Our work in this paper extends a standard DSGE model to include a micro-founded housing sector block with housing stock playing a double role of a housing services provider and a collateral against which financial contracts can be written. Unlike in other papers with the focus on the interlinkages between the financial and the housing sector, the loan-to-value ratio in our model is derived based on an agency problem between the lenders and the borrowers, as in Forlati and Lambertini (2011). In other words, the degree of leverage in our model is endogenous and reacts to, among other shocks, the idiosyncratic shock to the riskiness of the housing stock. We document how this shock as well as the housing market demand and the house price mark-up shocks were important drivers of the house investment dynamics, house prices, total consumption and of a number of financial variables including the loans, loan-to-value ratio and the external finance premium during the 1996-2011 period, while the traditional supply shocks (technology, wage mark-up and risk-premium shocks) did not have much impact on those variables.

In particular, increasing mark-ups and falling risk premia in the housing market were found to stand behind the observed increase in house prices during the 1998-2007 period. Also the house price collapse before and during the Great Recession was mainly driven by these two shocks as it was further amplified by an increase in the uncertainty on the housing market. This last shock was also the main factor behind the business cycle-frequency behavior of the debt growth and the degree of the leverage of the economy, with the periods of low uncertainty associated with the boom in credit and high uncertainty initiating painful
periods of deleveraging. Another important variable, the external finance premium, was found counter-cyclical and also closely related to the dynamics of the shock to the house market uncertainty. The counter-cyclicality of the external finance premium is a well known empirical fact and hence adds credence to our results. The next step in the process of cross-validation of our model could be to verify whether the implied dynamics of this variable matches the behavior of its empirical counterpart also quantitatively. We intend to proceed with this test in the future by estimating our model using a series of interest rates on mortgage-based loans as an additional variable.

The estimation fit of our model has been found to be good. In fact, the introduction of endogenous default dramatically improves the fit of the consumption series as compared to a model with an exogenous loans-to-value ratio, a finding that we are going to elaborate on in a separate paper. According to the results of our estimation borrowers, or credit-constrained households, constitute almost a third of all households in the EA. The relatively high share of credit-constrained households in the sample implies that financial frictions, to which they are subject, are transmitted to the rest of the economy via the consumption channel. However, since consumption is only one of the components of total demand, the pass-through of the financial and housing sector shocks to the overall aggregate economic activity is more limited. The total output performance is mainly driven by productivity shocks and stock market risk premium shocks, a finding typical for this class of models. We believe that allowing for an explicit role for banking sector and a more sophisticated description of its regulation would, however, reinforce the effects of households’ deleveraging on the rest of the economy. We leave these extensions for future research.
7 References


