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Stochastic debt simulation using VAR models and a panel fiscal reaction function – results for a selected number of countries

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Abstract

This paper uses vector auto-regression (VAR) models and a panel fiscal reaction function (FRF) to simulate debt ratios for fifteen EU Member States according to four regimes which are the Cartesian product of two types of errors (normal or bootstrapped) with two alternative assumptions on the (structural) primary balance (unchanged at the last observed value or endogenously determined using a panel FRF). Results suggest that debt ratio paths are not normally distributed being positively skewed; and that primary balances show "fiscal fatigue" and partial mean reversion to historical trends. It is suggested that debt sustainability scenarios should also be run using a FRF or some equivalent "mean reversion" hypothesis.

JEL classification: C53, E37, H68.
Key words: public debt projection, stochastic debt simulation, vector auto-regression (VAR), fiscal reaction function (FRF), primary balance, bootstrapping, panel regression, fiscal fatigue, mean reversion.

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* The views expressed in this paper are those of the author and should not be attributed to the European Commission.

1. Introduction

This paper proposes a probabilistic approach to debt dynamics using unrestricted vector auto-regression (VAR) models and a panel fiscal reaction function (FRF). The methodology for simulating the path of public debt combines disturbances to macroeconomic variables (growth, prices, interest rates, and real exchange rates), the endogenous policy response estimated through a FRF, and shocks arising from fiscal policy itself. This paper emphasises the joint role played by the structure of macroeconomic disturbances facing the economy and those associate with fiscal policy in shaping the risk profile of public debt.[1]

In contrast with standard debt sustainability analysis carried out by European Commission Services,[2] the VAR/FRF methodology is not "anchored" on any central/baseline scenario, neither assumes unchanged policies. On the one hand, the simulation of non-budgetary macroeconomic variables relies on past statistical trends as captured in estimated country-specific VAR models, while on the other, the primary balance responds to lagged values of the debt-to-GDP ratio and to contemporaneous values of the output gap. Due to the scarcity of budgetary data, panel data techniques are used to estimate a FRF, although assuming country fixed-effects. All else being equal, FRF estimates suggest that countries tend to register consecutive/prolonged periods of higher or lower primary balances (relative to average expected values).[3] In addition, and in line with important research on debt sustainability (Ghosh et al., 2011), evidence is also found of "fiscal fatigue", namely that the primary balance tends to decline, although remaining positive, at (very) high levels of the public debt-to-GDP ratio. According to common practice in the relevant literature, debt-to-GDP trajectories are simulated for five years (from 2012 to 2016).

The methodology consists of three building blocks. In the first block, a country-specific joint distribution of economic shocks is estimated that reproduces the statistical proprieties of historical data. In the second block, fiscal behaviour is modelled through estimation of a panel FRF. The third block combines the simulated macroeconomic scenarios with the estimated fiscal policy response in order to generate public debt trajectories. Debt outcomes reflect the interplay of model-consistent macroeconomic projections for growth, prices, real interest and real exchange rates, with the expected fiscal policy response. Macroeconomic shocks are repeatedly drawn[4] to generate the probabilistic distributions of debt trajectories. These distributions can then be characterised by calculating various moments of the empirical distribution (e.g. deciles) and through graphs (e.g. boxplots or 'fan-charts').

As regards the estimation of disturbances to macroeconomic variables, this paper suggests a "Protocol" for VAR model selection in order to guarantee equal treatment based on objective criteria across Member States covered by the analysis. VAR model selection and testing involves four main

[1] Previous work along these lines can be found in Celasun O., Debrun X., Ostry D. (2006), "Primary Surplus Behavior and Risks to Fiscal Sustainability in Emerging Market Countries: A "Fan-Chart" Approach", *IMF Working Paper 06/67*; and, Burger P., Stuart I., Jooste C., Cuevas A. (2011), "Fiscal sustainability and the reaction function for South Africa", *IMF Working Paper 11/69*.
[2] "Sustainability report 2009", European Commission, European Economy 9/2009.
[3] The estimated residuals derived from a FRF are 1st order autoregressive AR(1), meaning that they tend to persist over time.
[4] Estimates of VAR models can be used to simulate random shocks in two ways: i) using the variance/covariance matrix and assuming that errors are normally distributed; or ii) using the estimated residuals to bootstrap errors.

steps: i) choice of endogenous variables; ii) testing for the presence of unit roots; iii) choice of the maximum lag of the VAR process; and iv) testing for the presence of structural change in VAR models.

As regards the estimation of panel FRFs, the methodology used assumes "fiscal fatigue", implying some degree of mean reversion of primary balances to historical values. FRFs for EU countries are estimated using annual panel models with fixed effects and instruments (Ghosh et al., 2011).

Combining unrestricted VAR models with a panel FRF, the following typology for stochastic risk analysis of the debt ratio can be considered.

Table 1: Typology of debt stochastic analysis (four types)

Main assumption on the primary balance / Type of disturbances	Unchanged (structural) primary balance	Primary balance determined by a panel FRF with country fixed-effects and instruments
Normal errors	I	III
Bootstrapped residuals	II	IV

Stochastic debt simulations are calculated on two dimensions. The first dimension refers to the type of disturbances assumed for macroeconomic variables: i) drawn from a normal joint distribution using the estimated variance/covariance matrix of the VAR model or; ii) bootstrapped from estimated VAR residuals. The second dimension refers to the assumption on the (structural) primary balance: i) unchanged policy i.e. unchanged (structural) primary balance at the last observed value or; ii) assuming some mean reversion to historical values by using the estimated parameters of a panel FRF.

Applying this methodology, public debt ratios are simulated from 2012 to 2016 i.e. for five years. Two thousand simulations are calculated for each typology of shocks. Given the relevance of this methodology for debt sustainability analysis, the probability for the debt ratio to exceed given thresholds (e.g. 60%) by 2016 is also calculated. Finally, VAR models are tested for the presence of structural change, using empirical fluctuation processes (EFP) (Zeileis et al., 2002).

This paper is organised as follows. First a "Protocol" for the estimation of VAR models is proposed and the models are estimated accordingly. Second, panel fiscal reaction functions are estimated using country fixed-effects and instrumental variables. Third, a four-way typology of debt stochastic simulations is presented through various tables and graphs. Fourth, the paper concludes.

2. Unrestricted VAR models to estimate the joint probability distribution of economic shocks

Measuring risk to debt dynamics requires a stochastic framework to simulate the effects of likely macroeconomic shocks. It is necessary to derive the (empirical) frequency distribution of the debt ratio in order to be able to make explicit probabilistic assessments on debt sustainability.

The joint distribution of macroeconomic shocks is estimated from the statistical proprieties of historical data. Unrestricted VAR models are estimated to describe the co-movements among major macroeconomic variables determining debt dynamics. The proposed "Protocol" for model selection is flexible in relation to the number of endogenous variables that can be included, varying between three and six. VAR models should include a minimum of three variables which represent the strict minimum necessary to simulate debt trajectories: Inflation, Growth, and Real Interest Rate; and can have a maximum of six variables: Inflation, Growth, Real Interest Rate, Real Effective Exchange Rate (REER), Growth in Germany, and Real Interest Rate in Germany.

Based on the following considerations (and the literature), estimated VAR models include six variables.[5] First, given the relevance of macroeconomic imbalances[6] (and of the new macroeconomic imbalances procedure), it is considered preferable to include a REER variable in VAR models. Second, given the importance of the German economy in the European context, it is also decided to explore the correlation between national and German variables.

The estimation of VAR models requires quarterly data.[7] Eurostat is the main data source used. For interest rates, OECD data are used. When necessary, IMF's IFS data are used to back-cast Eurostat and OECD series. Despite the use of multiple sources, limited time coverage for some countries allows the estimation of VAR models for only 15 EU Member States: BE, DK, DE, ES, FR, IT, NL, AT, PL, PT, SI, SK, FI, SE, and the UK. Furthermore, results obtained for PL, SI, SK, and FI should be taken with caution given a data coverage well below 120 quarters (Table 2).

Variables included in the VAR system have to be tested for the presence of unit roots. The "Protocol" for model selection uses either one of the following two unit root tests: the augmented Dickey-Fuller (ADF) or the Phillips-Perron (PP). The PP test seems preferable, because it is robust with respect to unspecified autocorrelation and heteroscedasticity in the error process. Variables to be included in the VAR model are first differentiated only if the unit root test fails to reject the existence of a unit root.[8]

The "Protocol" uses the following rule for choosing the maximum lag order of a VAR system. For sample sizes lower than 120 observations, the Schwarz information criterion (SIC) is used, for sample sizes larger than 120, the Hannan-Quinn criterion (HQC) is used. This rule for lag selection has been proposed in the literature (Ivanov and Kilian, 2001).

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[5] Except for Germany, where for obvious reasons, the VAR model can include only a maximum of four variables.

[6] http://ec.europa.eu/economy_finance/economic_governance/macroeconomic_imbalance_procedure/index_en.htm

[7] Therefore, quarterly projections generated using VAR models need to be annualised in order to interplay with annual general government data.

[8] Given that some variables to be included in the VAR model are already defined as growth rates (e.g. Inflation, and Real Growth), they should only be differentiated if they are not stationary (i.e. according to a unit root test), because differentiation of a stationary variable reduces its information content, possible to the status of white noise, rendering meaningless the application of this methodology.

Formally, country-specific unrestricted VAR model are estimated with the variables and number of lags as described in Table 2.

$$Y_t = \gamma_0 + \sum_{k=1}^{p} \gamma_k Y_{t-k} + \varepsilon_t \qquad (1)$$

where $Y_t = (p_t, g_t, i_t, r_t, \hat{g}_t, \hat{\imath}_t)$; p denotes GDP inflation; g the domestic real GDP growth rate; i a non-weighted average of short and long term real interest rates (deflated using GDP inflation); r the (log of the) real effective exchange rate; $\hat{g}$ real GDP growth rate in Germany; and $\hat{\imath}$ a non-weighted average of short and long term real interest rates (deflated by GDP inflation) in Germany. γ_0 and γ_k are vectors of coefficients.

Estimation of equation 1 provides two useful outcomes for the simulation of debt dynamics: the variance-covariance matrix of residuals, and the realisation of (non-fiscal) macroeconomic variables that are consistent with the simulated shocks.

Let $\hat{\varepsilon}_t$ be the estimated residuals of equation 1. Bootstrapped simulations draw (with repetition) errors from the estimated residuals ($\hat{\varepsilon}_t$).

Instead, assuming that errors are normally distributed allows using the estimated variance-covariance matrix of residuals to simulate errors. Specifically, assume that residuals are normally distributed i.e. $\hat{\varepsilon}_t \sim N(0, \widehat{\Omega})$, where $\widehat{\Omega}$ is the estimated variance-covariance matrix of residuals. Simulations assuming normality use sequences of errors generated by: $\hat{\varepsilon}_t = W v_t$ where $v_t \sim N(0,1)$ and W is the Choleski factorisation matrix of Ω i.e. $\Omega = W'W$.

VAR residuals were tested for normality using the Jarque-Bera (portmanteau) test. All estimated VAR models rejected the null hypothesis of a joint normal distribution. Therefore, on this basis simulations based on bootstrapping seem preferable.

Due to the large amount of printout material generated, it was decided to present only a summary, regarding time coverage, variables and lag selection in Table 2. Country-specific estimates can be provided upon request.

Table 2: Main parameters used in the estimation of unrestricted VAR models.

Countries	Time coverage			Variables		Maximum lags	
	Start	End	Total quarters	Variables and degree of Integration	Test and alpha	Test	Number
BE	1981 Q1	2010 Q3	119	p(0), g(0), i(0), r(1)	PP (5%)	SC	1
DK	1978 Q1	2011 Q3	135	p(0), g(0), i(0), r(1)	PP (5%)	HQ	1
DE	1976 Q1	2011 Q3	143	p(1), g(0), i(0), r(0)	PP (5%)	HQ	2
ES	1980 Q2	2011 Q3	126	p(1), g(1), i(1), r(1)	PP (5%)	HQ	1
FR	1980 Q2	2011 Q3	126	p(1), g(0), i(1), r(1)	PP (5%)	HQ	1
IT	1981 Q2	2011 Q3	122	p(1), g(0), i(1), r(1)	PP (5%)	HQ	1
NL	1978 Q1	2011 Q3	135	p(0), g(0), i(0), r(1)	PP (5%)	HQ	1
AT	1976 Q1	2011 Q3	143	p(0), g(0), i(0), r(1)	PP (5%)	HQ	1
PL	1996 Q2	2011 Q3	62	p(1), g(1), i(0), r(1)	PP (5%)	---	2*
PT	1978 Q1	2011 Q3	135	p(0), g(0), i(0), r(1)	PP (5%)	HQ	1
SI	1998 Q4	2011 Q3	52	p(1), g(0), i(1), r(0)	PP (5%)	---	2*
SK	1995 Q3	2011 Q3	65	p(0), g(0), i(0), r(1)	PP (5%)	---	2*
FI	1987 Q2	2011 Q3	98	p(0), g(0), i(1), r(1)	PP (5%)	SC	1
SE	1981 Q1	2010 Q3	119	p(0), g(0), i(0), r(1)	PP (5%)	SC	1
UK	1976 Q1	2011 Q3	143	p(1), g(0), i(0), r(1)	PP (5%)	HC	1

Legend: **p**: GDP price inflation (source Eurostat, back-casted using IFS data); **g**: real GDP growth (source Eurostat, back-casted using IFS data); **i**: a non-weighted average of short and long-term real interest rates deflated by GDP prices (source OECD, back-casted using IFS data); **r**: real effective exchange rate (source Eurostat, back-casted using IFS data).

x(0) a non-differentiated variable; x(1) a 1st order differentiated variable.

PP (Phillips-Perron) test; ADF (augmented Dickey-Fuller) test at (y%).

SC (Schwarz information criterion); HQ (Hannan-Quinn criterion).

* Maximum number of lags limited to 2 due to the short time period considered.

3. Estimation of panel fiscal reaction functions (for types III and IV of the typology)

This section estimates annual panel fiscal reaction functions (FRF),[9] covering either 21 or all of the 27 EU Member States, depending on the list of instruments being used in a two stages panel regression. A narrower list of instruments allows the inclusion of all 27 EU Member States, while a broader one excludes 6 Member States, namely BG, CY, LV, LT, MT and RO.[10] In order to correct for possible sources of endogeneity bias, it is preferable to use the FRF with the enlarged set of instruments.

The literature suggests the existence of a FRF, meaning that the primary balance is positively correlated with lagged debt levels (after controlling for the economic cycle and other non-debt determinants of the primary balance), but also that this relation might be non-linear. The size of primary balance response may vary with the level of the debt ratio, reacting more strongly when the debt ratio exceeds a given threshold, but then the responsiveness eventually begins to weaken, and then actually decreases at very high debt levels (Ghosh et al., 2011).

[9] Use of panel data is due to the scarcity of budgetary data. Annual data are used because higher frequency data (e.g. quarterly) generally have a low "signal-to-noise ratio" and therefore are mainly used for cash management purposes rather than policy evaluation.

[10] The "narrower" list of instruments corresponds to the "broader" one, excluding a single variable which captures "foreign demand".

A strand of the literature on debt sustainability uses the FRF as the main empirical tool of analysis (e.g. Public Finance Report 2011, Part IV, Chapter 4). The main reference of this strand is the seminal paper of Bohn (1998) which argues that governments are solvent under very general conditions as long as the primary balance increases with the stock of debt.[11] However, recent research suggests that "fiscal fatigue" (at very high debt ratios) is a robust statistical finding, after controlling for other factors (Ghosh et al., 2011).

Following the literature (e.g. Celasun et al., 2006; Mendoza and Ostry, 2008; Burger et al., 2011; Ghosh et al., 2011), besides the debt ratio, a range of explanatory variables are also included in the regression, namely the output gap to control for cyclical fluctuations, and political and institutional variables.

In fact, the use of political and institutional variables is very common in this literature. In the Public Finance Report of 2011 (Part IV, Chapter 4), an annual panel FRF regression is estimated using, inter alia, the size of the government's parliamentary majority, ideology and fragmentation of government, the strength and coverage of fiscal rules.[12] The latter is summarised by a Fiscal Rules Index (FRI), which assigns larger scores to more stringent rules and/or rules with a broader coverage.[13] In this paper, the FRI is the only institutional variable considered and only in one specification of the FRF, because it is available only since 1990, thereby its inclusion significantly reduces the sample, besides being problematic given the objective of simulating (out of sample) trajectories up to 2016.

A fiscal reaction function describes the average response of actual primary balances after controlling for cyclical fluctuations, the lagged public debt-to-GDP ratio, and political and institutional variables. Given that the focus of this analysis is on debt sustainability, the dependent variable is the actual primary balance rather than the cyclically adjusted (or structural) primary balance.[14]

In line with the literature, the general specification of the FRF is:

$$pb_{i,t} = \alpha_0 + \eta_i + \rho * d_{i,t-1} \big| X_{i,t-2} + \gamma * ygap_{i,t} \big| X_{i,t-1} + \nu * fri_{i,t} \big| X_{i,t-1} + \varepsilon_{i,t} \qquad (2)$$

where $pb_{i,t}$ is the ratio of the primary balance to GDP in country *i* and year t; $d_{i,t-1}$ is the public debt-to-GDP ratio at the end of period t-1; $ygap_{i,t}$ is the output gap; $fri_{i,t}$ is the fiscal rules index; η_i is the unobserved, country fixed-effect; $X_{i,t}$ is a vector of instrumental variables; and $\varepsilon_{i,t}$.are the errors, which can follow a 1st order autoregressive process AR(1) i.e. $\varepsilon_{t+1} = \theta\varepsilon_t + u_t$. The (broader) vector of instrumental variables includes lagged endogenous variables, the primary expenditure gap, openness, inflation, terms of trade, impact of terms of trade on household income, foreign demand, and fuel prices.[15]

[11] A practical advantage of such criterion is that it is independent of information regarding interest rates and intertemporal preferences.
[12] Gosh et al. (2011) also use a number of institutional variables, such as a policy stability index, a fiscal rules index, etc.
[13] The FRI is compiled by Commission services, DG ECFIN.
[14] The latter better reflects discretionary fiscal behaviour, while the former reflects both discretionary attempts to stabilise output and the effect of automatic stabilisers.
[15] The narrower list of instruments differs from the broader one by the exclusion of the foreign demand variable.

It should be noted that the literature also uses a set of (exogenous) variables $X_{i,t}$ as regressors. In this paper, most exogenous regressors typically used in the literature were not found to be significant (e.g. the primary expenditure gap to measure the effect of temporary fluctuations in government outlays, trade openness, inflation, foreign demand, and fuel prices). The use of exogenous/policy variables as regressors would also present the disadvantage of requiring making assumptions on their out of sample trajectories in order to be able to simulate debt paths until 2016.

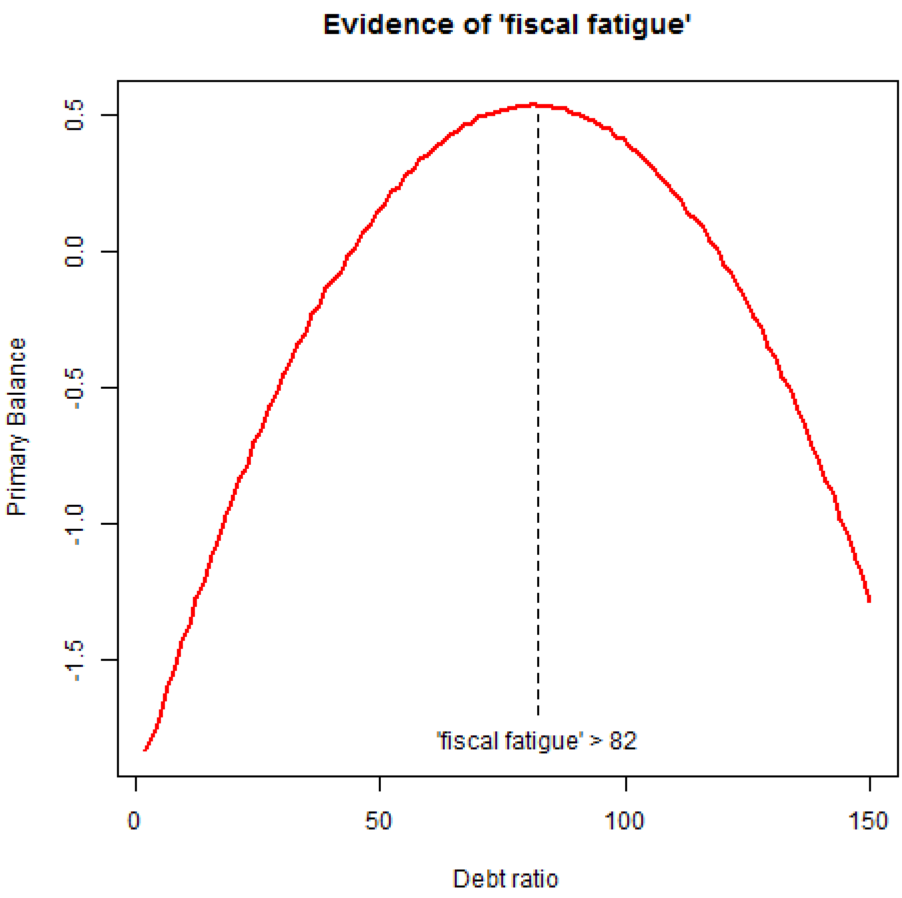
Estimation of equation 2 has to take into account three potential sources of endogeneity bias.[16] The first source is due to the contemporaneity of the output gap and the fiscal policy shock ($\varepsilon_{i,t}$). The second comes from the dependence of lagged debt on past values of the primary balance. Countries able to generate higher primary balances – reflected in higher values of the fixed-effects η_i – will tend to have lower public debt ratios. If this effect is not taken into account, the negative correlation between debt levels and country fixed-effects would exert a downward bias on the estimated debt coefficient in equation 2 (ρ). A third source of endogeneity can stem from the persistence of errors ($\varepsilon_{i,t}$), making lagged debt endogenous. All these potential endogeneity problems can be addressed by using adequate (exogenous) instruments which are still correlated with the endogenous regressors.

Table 3 presents estimation results for five specifications of panel FRFs. All estimates use instruments for lagged debt and the contemporaneous output gap.[17] The first two specifications take first differences to eliminate country fixed-effects. The first specification simply checks the existence of a panel FRF. In line with the literature, significant results are found showing that lagged debt and the output gap tend to increase (as expected) the primary balance.

The second specification in differences, thereby eliminating unobserved country heterogeneity, tests the important notion of "fiscal fatigue" in the budgetary adjustment process, which should be observed at (very) high levels of the debt ratio. In a sample covering advanced economies, Ghosh et al. (2011) find strong empirical evidence in favour of "fiscal fatigue". In practice, this means successfully fitting a "cubic" polynomial in lagged debt to explain the primary balance. In the sample used in this paper, results for the second specification suggest also the existence of "fiscal fatigue". According to the results of specification 2, "fiscal fatigue" occurs already in a debt-to-GDP range of 80% to 90% (see Figure 1).[18] This result is remarkably close to the one in Ghosh et al. (2011), which report that although the primary balance response to debt levels remains positive, it starts declining when the debt ratio reaches around 90 to 100% of GDP.

[16] Celasun et al. (2006).
[17] For the list of instrumental variables see the legend of Table 3.
[18] More precisely at around 82%. Figure 1 is drawn for the results of specification 2, assuming a constant output gap.

Figure 1: 2nd specification of a panel FRF



The third specification is in levels, thereby allowing for country fixed-effects and includes the fiscal rules index FRI (ν). Inclusion of the FRI significantly reduces the sample. Its inclusion is found to be significant only at 10%.

The fourth and fifth specifications are also in levels. They exclude the FRI, and find that errors follow a first order autoregressive process AR(1). In this paper, the fourth specification is the one being used to simulate debt paths using a panel FRF. The fifth FRF uses a reduced set of instruments and should be used only for BG, CY, LV, LT, MT and RO. Moreover, given that no VAR model is being estimated for those countries, macroeconomic variables and shocks would have to be simulated using an alternative methodology, such as the one based on the estimation of variance-covariance matrices of historical shocks (Di Giovanni and Gardner, 2008; Berti, 2012, forthcoming).

In the simulations using a panel FRF (for types III and IV of the typology), the primary balance and the debt ratio are recursively determined by:

$$\hat{\varepsilon}_{i,t} = \hat{\theta} * \hat{\varepsilon}_{i,t-1} + u_t \qquad (3i)$$

$$\widehat{pb}_{i,t} = \hat{\alpha}_0 + \hat{\eta}_i + \hat{\rho} * \hat{d}_{i,t-1} + \hat{\gamma} * \widehat{gap}_{i,t} + \hat{\varepsilon}_{i,t} \qquad (3ii)$$

$$\hat{d}_{i,t} = \frac{1+\hat{\jmath}_{i,t}}{1+\hat{n}_{i,t}} * \hat{d}_{i,t-1} - \widehat{pb}_{i,t} + \hat{\Gamma}_{i,t} \qquad (3iii)$$

where u_t is the innovation in the errors, which is normally distributed as:

$$u_t \sim N\left(0, \sqrt{\left(1-\hat{\theta}^2\right)} * \hat{\sigma}_\varepsilon\right) \tag{4}$$

$\hat{\sigma}_\varepsilon$ is the estimated standard error of the regression; j is the implicit interest rate on the debt; n is the nominal GDP growth rate; and $\hat{\Gamma}$ are exogenous factors of debt accumulation, namely those related to ageing costs.[19]

Estimates of the panel FRF are used to simulate debt paths in the following way. First, for the last year of the estimation period (time 0 of the simulation),[20] residuals are calculated as:

$$\hat{\varepsilon}_{i,0} = pb_{i,0} - \hat{\alpha}_0 - \hat{\eta}_i - \hat{\rho} * d_{i,-1} - \hat{\gamma} * gap_{i,0} \tag{5}$$

Second, for each simulation year, realisations of u_t are drawn from equation 4, and the errors are build using equation 3*i*. Third, equation 3*ii* is used to simulate the primary balance. Fourth, the primary balance is replaced in equation 3*iii* to calculate the debt ratio.

The implicit interest rate on the public debt ($\hat{j}_{i,t}$) is calculated as:

$$\hat{j}_{i,t} = \left(1 - \frac{1}{a_i}\right) * \hat{j}_{i,t-1} + \frac{1}{a_i} * \widehat{m}_{i,t} \tag{6i}$$

$$\widehat{m}_{i,t} = \hat{\imath}_{i,t} + \hat{p}_{i,t} \tag{6ii}$$

where the implicit interest rate on the public debt is a weighted average of the past implicit rate and of new loans at the nominal rate $\widehat{m}_{i,t}$. The latter is the sum of the real interest rate ($\hat{\imath}_{i,t}$) and the change in the GDP deflator ($\hat{p}_{i,t}$). a_i gives the average maturity of public debt.[21] Historical values for the implicit interest rate on public debt are taken from the Ameco database.

The nominal GDP growth rate is:

$$\hat{n}_{i,t} = \hat{r}_{i,t} + \hat{p}_{i,t} \tag{7}$$

Finally, for each of the 2000 simulations, the Hodrick-Prescott (HP) filter is run to estimate potential GDP and the output gap.[22]

[19] Sustainability Report 2012, European Commission, forthcoming.
[20] The year 2011.
[21] Elaborated by Commission services (DG ECFIN/A2), based on Bloomberg data.
[22] Using the recommended lambda value of 1600 for quarterly data. To address the end-period bias in HP filter estimates of the output gap, the sample period is extended by 4 quarters.
Instead, Burger et al. (2011) use the HP filter to estimate a single potential output based on the median outcomes of 1000 simulations for real GDP. In addition, they extend the sample period by 12 quarters to overcome the end-period bias problem.

Table 3: Estimation results of panel fiscal reaction functions (FRF)

Specification	(1)	(2)	(3)	(4)	(5)
Sample	1976-2011	1976-2011	1990-2009	1976-2011	1972-2011
Type of estimation	First differences/IV	First differences/IV	IV	IV	IVb
Constant	---	---	-2.368**	-4.058**	-3.782***
			(0.942)	(1.955)	(1.399)
Lagged debt (ρ)	0.177***	-1.926**	0.0536***	0.0783**	0.0737***
	(0.055)	(0.814)	(0.0136)	(0.0307)	(0.0208)
Lagged debt squared	---	0.0306**	---	---	---
		(0.014)			
Lagged debt cubic	---	-1.27E-4*	---	---	---
		(6.95E-5)			
Output gap (γ)	0.334**	0.499***	0.527***	0.6914***	0.7270***
	(0.139)	(0.146)	(0.113)	(0.217)	(0.151)
Fiscal rules index (ν)			0.725*	---	---
			(0.437)		
AR(1) coefficient (θ)	---	---	---	0.7010***	0.687***
				(0.07)	(0.06)
Country fixed-effects					
AT	---	---	-------------Reference country------------		
BE	---	---	0.163	-2.189	-1.948
CZ	---	---	-1.737**	-0.920	-1.040
DE	---	---	-1.028	-0.313	-0.260
DK	---	---	3.082***	4.147***	4.173***
EE	---	---	0.296	2.147	1.854
EL	---	---	-2.515*	-4.811*	-3.930*
ES	---	---	-0.291	-0.535	-0.522
FI	---	---	3.123***	5.534***	5.463***
FR	---	---	-1.470***	-0.721	-0.737
HU	---	---	-2.217**	-0.796	-0.822
IE	---	---	1.898**	-1.586	-1.177
IT	---	---	-0.614	-3.023*	-2.834**
LU	---	---	3.819***	4.833**	4.615***
NL	---	---	0.088	0.332	0.352
PL	---	---	-2.338***	-1.785	-1.836
PT	---	---	-1.046*	-1.732	-1.724
SE	---	---	1.587**	3.130**	3.052**
SI	---	---	-0.223	-0.500	-0.612
SK	---	---	-2.149***	-1.519	-1.629
UK	---	---	-1.374	-0.937	-0.952
BG	---	---	---	---	1.937
CY	---	---	---	---	-1.722
LT	---	---	---	---	-0.483
LV	---	---	---	---	-0.174
MT	---	---	---	---	-2.318
RO	---	---	---	---	0.374
Observations (unbalanced)	499	499	354	499	603
Number of countries	21	21	21	21	27
Adjusted R-squared	---	---	0.516	0.682	0.652
S.E. of regression	2.24	2.75	2.27	2.06	2.14
F-statistic	---	---	15.4	42.2	37.7
Prob(F-statistic)	---	---	0.00	0.00	0.00

Legend: Dependent variable is the general government primary balance-to-GDP ratio. Robust standard errors reported in parenthesis: * denotes significance at 10%; ** at 5%; *** at 1%.

Difference means that the equation was estimated in first differences, thereby eliminating the constant and country fixed-effects.

IV means that the equation was estimates using instrumental variables. The instruments used are: lagged values of the primary balance, the debt ratio, and the output gap; the primary expenditure gap, openness, inflation, terms of trade, impact of terms of trade in household income, foreign demand, and fuel prices.

IVb IV excluding the foreign demand variable from the list of instrumental variables.

4. Debt ratio assuming unchanged structural primary balances (for types I and II of the typology)

In the standard debt sustainability analysis carried out by Commission Services, the primary balance is the sum of a constant structural primary balance, reflecting an unchanged policy assumption, and a cyclical adjustment component. The former is usually the structural primary balanced forecast for the last year covered by the European economic forecast,[23] using the production function methodology to calculate output gaps.[24] The cyclical component is calculated by multiplying the output gap by OECD country specific estimates of the budget balance sensitivity to the economic cycle. In addition, output gaps are assumed to close three years after the last year covered by the forecast (i.e. a t+3 closure rule is assumed).

$$\hat{d}_{i,t} = \frac{1+\hat{j}_{i,t}}{1+\hat{n}_{i,t}} * \hat{d}_{i,t-1} - \overline{sb}_i - \widehat{cc}_{i,t} + \hat{\Gamma}_{i,t} \qquad (8)$$

where $\overline{sb}_i$ is a constant primary structural balance; and $\widehat{cc}_{i,t}$ is the cyclical adjusted primary balance, which is calculated as:

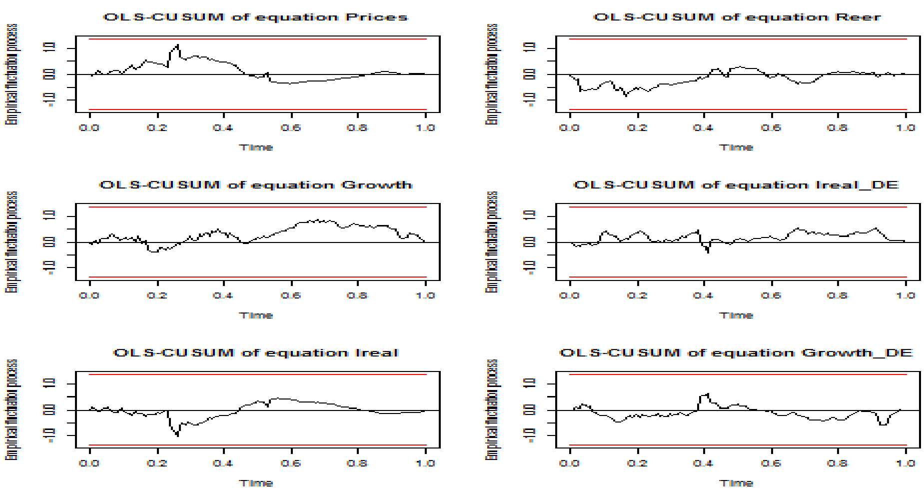
$$\widehat{cc}_{i,t} = \alpha_i * \widehat{gap}_{i,t} \qquad (9)$$

where α_i is the OECD based elasticity of the budget balance to the output gap.[25]

In the four types of simulations carried out in this paper, the output gap is derived endogenously applying the HP filter to each of the 2000 simulations in order to calculate potential output (i.e. no closure rule is assumed).

5. Identification of structural change using empirical fluctuation processes (EFP)

Figure 2: EFP for Portugal ($\alpha = 5\%$)



For each unrestricted VAR model, an EFP test was carried out to detect structural change in the linear regressions. The EFP test was calculated using the residuals of VAR estimates[26], capturing their

[23] Published twice a year by the European Commission: (http://ec.europa.eu/economy_finance/eu/forecasts/2011_autumn_forecast_en.htm)
[24] See Denis et al. (2006).
[25] See Girouard (2005).
[26] The EFP test is based on the cumulative sum of the OLS VAR residuals.

fluctuation. For these empirical processes their limits are known, so that boundaries can be computed, whose crossing probability under the null hypothesis of no structural change is α (set to 5%) (Zeileis, 2002). For all the countries for which an EFP test was run (BE, DK, DE, ES, FR, IT, NL, AT, PT, FI, SE, and the UK),[27] the hypothesis of no structural change could not be rejected. As an example, see in Figure 2, the EFP test for Portugal.

6. Simulated debt trajectories

2000 simulations are run for each of the four types of stochastic debt trajectories over five years (2012-2016): i) normal errors and unchanged policies (type I); ii) bootstrapped errors and unchanged policies (type II); normal errors and "changing" policy[28] (type III), and; iv) bootstrapped errors and "changing" policy.

In order to avoid excessive repetition, detailed results are given for only one country and for illustrative purposes (Portugal), together with some summary tables. A more complete set of results can be found in Annex (Table 6).

Table 4: Deciles of the debt-to-GDP distribution for Portugal

	2011	2012	2013	2014	2015	2016		2011	2012	2013	2014	2015	2016
Deciles	Type I						Deciles	Type III					
0%	101,6	90,8	84,0	77,5	71,3	66,2	0%	101,6	85,0	74,3	64,5	54,9	49,6
10%	101,6	99,5	96,9	94,9	92,4	89,6	10%	101,6	95,5	89,3	83,5	77,9	73,5
20%	101,6	101,3	100,4	98,9	98,0	97,1	20%	101,6	97,9	93,1	88,3	83,5	79,5
30%	101,6	102,7	102,7	102,5	102,2	102,0	30%	101,6	99,3	95,5	91,8	88,5	84,5
40%	101,6	104,0	105,0	105,7	106,4	107,2	40%	101,6	100,7	98,0	94,9	92,0	88,7
50%	101,6	105,1	107,0	108,7	110,1	111,5	50%	101,6	102,0	100,0	97,9	95,4	93,0
60%	101,6	106,1	109,0	111,5	114,2	116,7	60%	101,6	103,1	102,4	100,6	98,9	96,8
70%	101,6	107,4	111,4	114,5	118,0	122,3	70%	101,6	104,6	104,8	103,8	102,7	101,7
80%	101,6	108,8	113,9	119,1	123,9	128,5	80%	101,6	106,3	107,6	108,1	107,9	107,8
90%	101,6	110,9	118,3	125,0	131,9	140,3	90%	101,6	108,6	112,2	114,6	116,6	118,3
100%	101,6	129,6	148,1	166,7	189,0	205,1	100%	101,6	129,4	144,1	155,3	170,1	177,0
Deciles	Type II						Deciles	Type IV					
0%	101,6	89,9	81,8	71,8	59,0	52,3	0%	101,6	86,7	72,8	61,4	45,2	35,8
10%	101,6	100,3	98,2	95,4	92,6	90,0	10%	101,6	96,6	90,1	84,0	78,1	73,0
20%	101,6	102,4	101,5	100,3	98,5	97,1	20%	101,6	98,8	93,7	88,7	83,7	79,5
30%	101,6	103,6	103,7	103,4	102,8	102,4	30%	101,6	100,2	96,5	92,6	87,9	83,6
40%	101,6	104,5	105,5	106,0	106,4	107,3	40%	101,6	101,3	98,7	95,3	91,2	88,4
50%	101,6	105,3	107,2	108,6	110,1	111,4	50%	101,6	102,4	100,7	98,2	95,1	92,8
60%	101,6	106,2	109,0	111,3	113,9	116,3	60%	101,6	103,4	102,7	100,9	99,4	97,1
70%	101,6	107,2	111,2	114,7	118,4	122,0	70%	101,6	104,6	104,9	104,2	103,3	102,4
80%	101,6	108,5	113,9	118,7	123,9	128,8	80%	101,6	106,2	107,6	108,0	108,3	108,3
90%	101,6	110,5	118,2	125,2	131,8	139,3	90%	101,6	108,3	111,9	114,2	115,6	117,4
100%	101,6	130,3	150,3	176,1	188,7	210,8	100%	101,6	129,9	141,7	163,2	167,9	175,2

For Portugal, in 2016 median results for the debt-to-GDP ratio vary from 92.8% (type IV) to 111.5% (type I). Across countries there are small differences between assuming normal errors or bootstrapped residuals i.e. small differences between comparing types I and II on the one hand, or types III and IV on the other hand. Conversely, there are significant differences between assuming

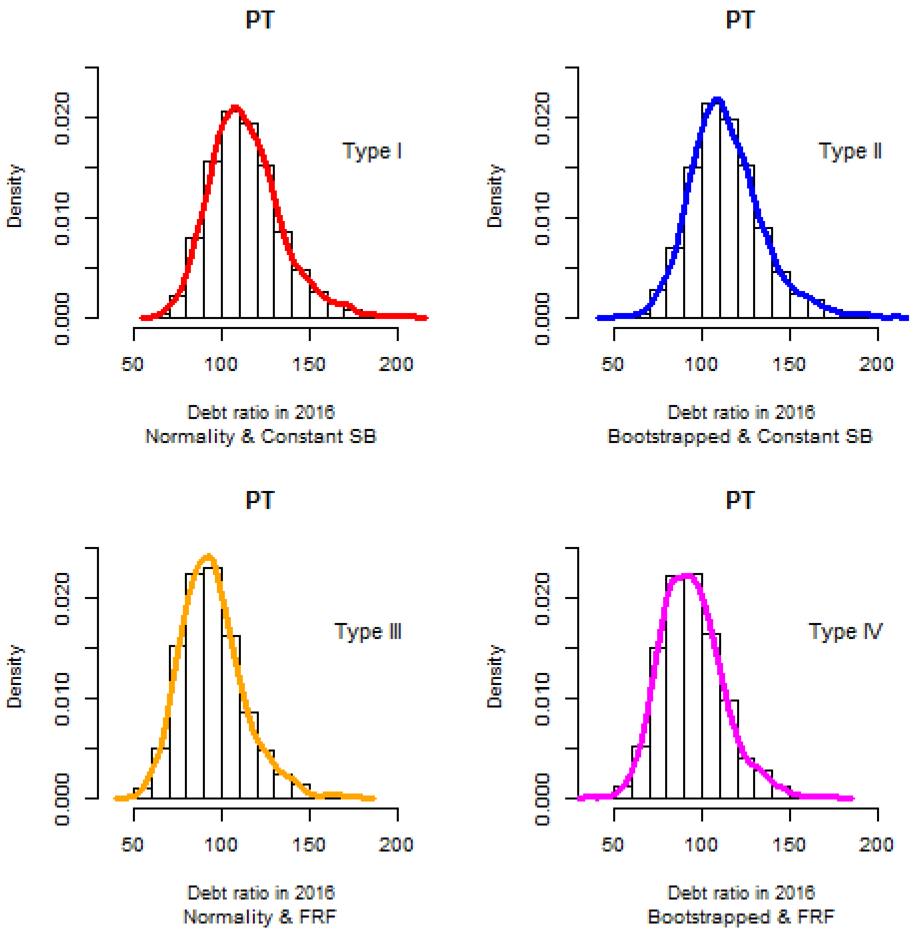
[27] For PL, SI, and SK the EFP test was not carried out due to the limited sample.
[28] Using the estimated parameters of the 4th specification of the FRF (Table 3).

unchanged policies (types I and II) or assuming changing policies determined according to the estimated parameters of a panel FRF (types III and IV). With the exception of SE, median values for the debt ratio are lower for the two FRF types III and IV (see Table 6 in Annex).

The distribution of the debt-to-GDP ratio in 2016 is plotted using histograms (Figure 3). Note that all histograms have the same scale to facilitate comparison. Across a majority of countries (not the case for Portugal), simulations that assume changed policies (types III and IV) tend to have higher variability as measured by the standard deviation (Table 5). This is an expected result, because budgetary uncertainty (introduced by a FRF) tends to add to the uncertainty linked to VAR model estimation of non-fiscal macroeconomic variables.[29]

Table 5 also suggests that (across countries and simulation types), debt-to-GDP empirical distributions are asymmetric and skewed to the left i.e. the mean is higher than the median.

Figure 3: Histograms and (kernel) density functions for Portugal



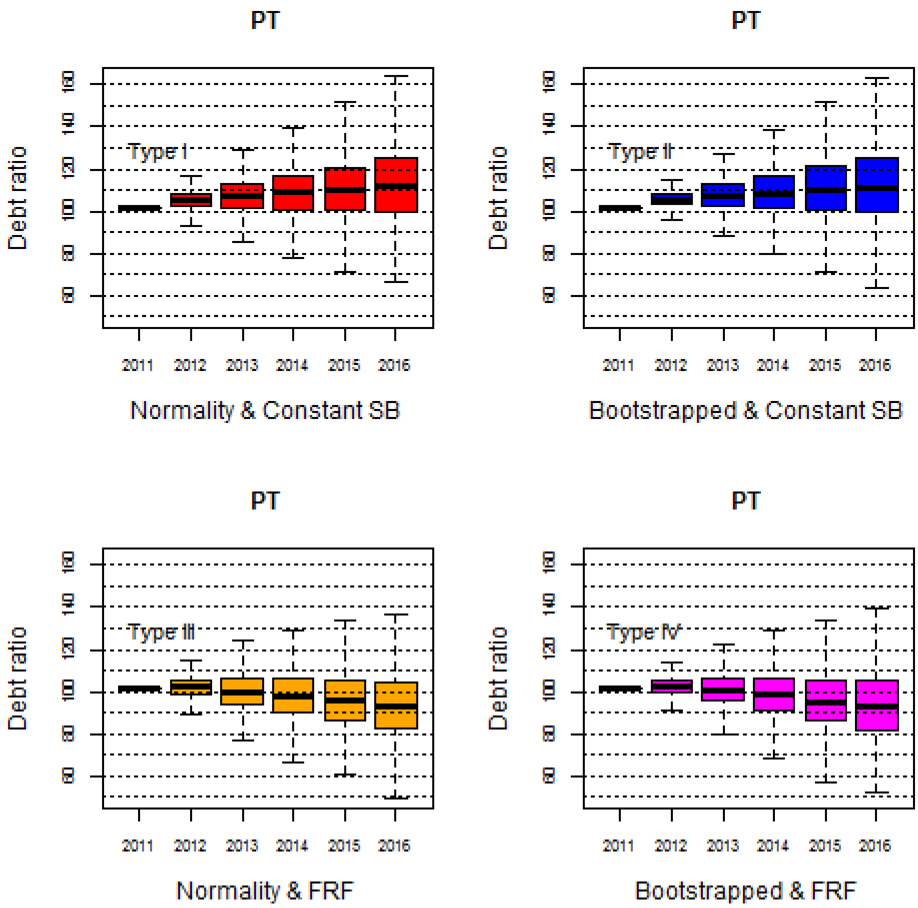
[29] However, this is not necessarily the case, partly because errors associated with budgetary policy (FRF) are autoregressive i.e. depending on initial conditions.

Table 5: Mean, median and standard deviation of debt-to-GDP distributions

		2012			2014			2016					2012			2014			2016		
		mean	median	sd a)	mean	median	sd	mean	median	sd			mean	median	sd	mean	median	sd	mean	median	sd
	I	72,3	72,3	1,4	72,8	72,7	3,8	73,5	73,2	5,6		I	65,6	65,6	1,6	66,7	66,7	3,4	67,5	67,4	3,9
AT	II	72,3	72,4	1,4	72,9	73,0	3,9	73,6	73,5	5,5	NL	II	65,7	65,7	1,5	66,6	66,7	3,4	67,3	67,2	3,8
	III	71,5	71,6	2,5	69,7	69,5	6,7	67,7	67,5	8,6		III	64,8	64,8	2,5	62,7	62,6	6,1	59,6	59,4	7,5
	IV	71,6	71,7	2,4	69,6	69,7	6,2	67,6	67,5	8,6		IV	64,9	64,9	2,3	62,5	62,2	5,8	59,2	59,3	7,4
	I	97,0	97,0	1,8	96,7	96,7	4,2	97,0	96,7	5,4		I	57,7	57,7	1,5	56,9	56,6	4,9	55,8	54,5	10,4
BE	II	97,0	96,9	1,7	96,6	96,3	3,9	96,9	96,6	5,3	PL	II	57,7	57,8	1,3	56,8	56,8	4,0	55,2	54,3	8,5
	III	96,5	96,5	2,8	94,7	94,6	6,7	93,2	93,1	8,6		III	57,0	57,0	2,7	54,9	54,9	6,9	51,9	51,1	10,5
	IV	96,5	96,4	2,7	94,6	94,5	6,6	93,0	92,8	8,7		IV	57,2	57,2	2,4	54,9	54,7	6,3	51,3	50,9	9,4
	I	81,3	81,2	2,7	81,3	80,8	8,4	82,6	81,2	13,5		I	105,2	105,1	5,1	109,5	108,7	13,3	114,1	111,5	21,9
DE	II	81,5	81,3	2,8	81,7	81,0	8,4	83,1	80,9	14,1	PT	II	105,6	105,3	5,1	109,9	108,6	13,8	113,9	111,4	22,4
	III	81,2	81,1	3,8	80,6	80,2	10,6	80,7	79,6	15,0		III	102,1	102,0	5,8	98,7	97,9	13,4	94,8	93,0	19,4
	IV	81,5	81,4	3,9	80,9	80,2	10,7	80,9	79,4	15,3		IV	102,6	102,4	5,6	99,0	98,2	13,7	94,4	92,8	19,7
	I	45,8	45,8	1,4	48,6	48,5	3,5	51,5	51,3	4,8		I	33,4	33,4	1,6	28,7	28,7	3,4	25,0	24,9	3,9
DK	II	45,9	45,8	1,4	48,7	48,6	3,4	51,5	51,2	4,7	SE	II	33,4	33,4	1,5	28,6	28,4	3,5	24,8	24,6	3,8
	III	45,6	45,6	2,5	44,9	44,8	6,4	42,3	42,2	8,3		III	33,3	33,4	2,7	29,1	29,1	6,4	26,2	26,2	8,2
	IV	45,7	45,6	2,3	44,9	44,8	6,0	42,1	42,1	8,1		IV	33,4	33,4	2,6	28,9	28,9	6,4	26,0	25,8	8,1
	I	73,8	73,7	1,9	83,9	83,3	9,1	98,0	95,6	22,4		I	47,8	47,8	2,2	49,1	48,9	6,0	51,2	50,4	9,2
ES	II	74,0	73,8	2,0	84,3	83,0	9,5	98,5	94,7	23,8	SI	II	47,9	47,9	1,8	49,0	48,6	5,1	50,8	50,1	7,6
	III	72,7	72,7	2,8	77,3	76,7	10,3	82,6	80,4	20,7		III	48,2	48,2	3,6	48,5	48,2	9,5	49,4	48,6	11,8
	IV	72,9	72,8	2,8	77,6	76,4	10,4	82,8	79,0	21,7		IV	48,3	48,2	3,0	48,2	47,9	8,2	48,8	48,2	10,2
	I	45,8	45,8	1,9	41,2	40,9	5,2	38,8	38,5	6,6		I	47,3	47,3	2,2	50,8	50,8	5,1	55,1	54,7	5,9
FI	II	46,0	46,0	1,8	41,2	41,0	5,1	38,7	38,3	6,6	SK	II	47,4	47,2	2,0	50,8	50,4	4,7	54,7	54,2	5,9
	III	44,8	44,8	3,1	36,9	36,7	8,4	31,1	30,9	10,1		III	46,5	46,5	4,3	47,2	47,2	10,1	48,6	48,5	10,6
	IV	45,0	45,0	2,9	36,8	36,7	8,1	30,8	30,4	10,1		IV	46,8	46,6	4,0	47,3	46,8	9,5	48,1	47,7	9,9
	I	87,2	87,2	1,5	90,9	90,7	6,1	95,9	95,2	12,0		I	90,7	90,6	1,8	104,0	103,7	7,0	118,9	117,5	14,8
FR	II	87,3	87,2	1,4	90,8	90,6	6,0	95,8	95,1	12,3	UK	II	90,7	90,7	1,7	104,2	103,8	7,5	119,4	117,7	17,2
	III	86,2	86,1	2,4	84,9	84,8	7,6	83,5	82,8	12,6		III	89,2	89,2	2,7	94,2	94,0	8,8	96,5	95,5	15,0
	IV	86,3	86,2	2,3	84,9	84,5	7,5	83,3	82,4	12,8		IV	89,3	89,2	2,6	94,3	93,6	9,4	96,6	94,9	16,4
	I	121,0	121,1	2,5	122,1	121,7	9,2	125,2	124,4	17,7											
IT	II	121,2	121,0	2,3	122,3	121,5	8,7	125,4	123,6	16,7											
	III	121,1	121,1	3,3	121,6	121,4	10,3	123,6	122,7	17,3											
	IV	121,2	121,1	3,2	121,7	121,1	9,9	123,4	121,8	16,9											

a) Standard deviation

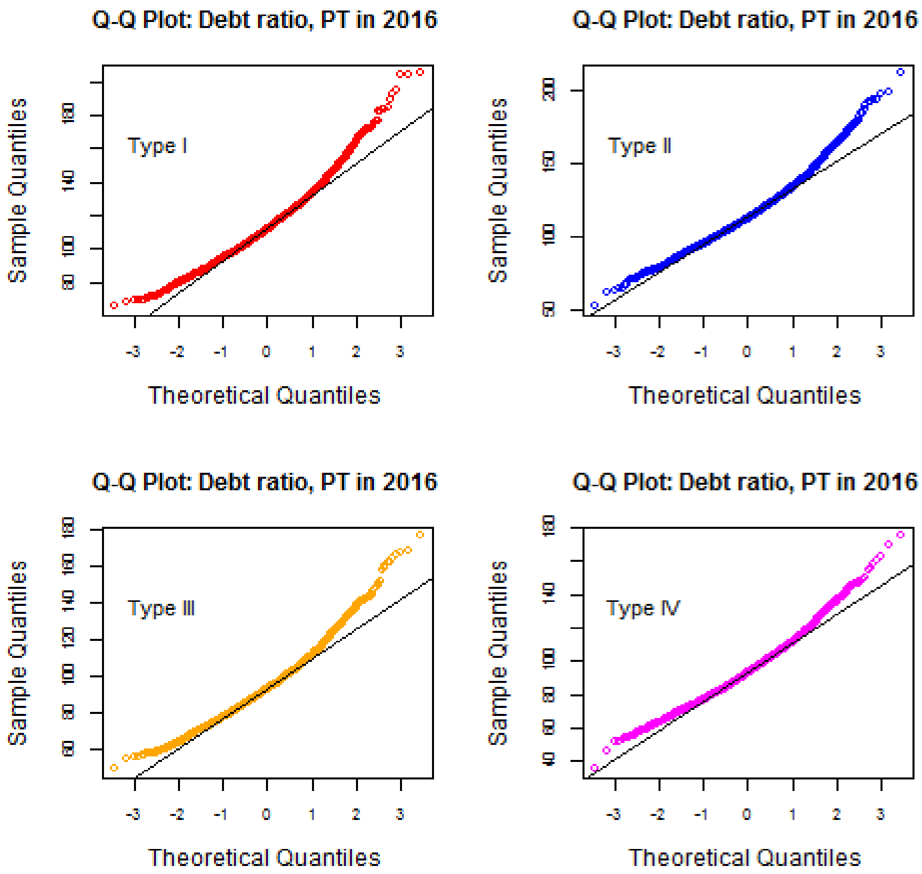
Figure 4: Boxplots of the debt-to-GDP ratio for Portugal



In the literature on stochastic debt simulation, it is common to summarise the distribution of debt-to-GDP paths using "fan-charts". Instead, this paper prefers using boxplots (Figure 4).[30] Note that all boxplots have the same scale to facilitate comparison.

The debt ratio distributions are generally asymmetry, tending to be positively skewed[31]. The Shapiro-Wilk test rejects the null hypothesis of a normal distribution. A normal Quantile-to-Quantile plot also strongly suggests that the distributions are not normal, being positively skewed (Figure 5).

Figure 5: Normal Quantile-to-Quantile plots for Portugal



It should not come as a surprise that debt ratio simulations are not normally distributed, because portmanteau tests on the residuals of VAR models had already rejected the null hypothesis of normality. Consequently, although not making dramatic differences in terms of the median (see Table 6 in Annex), it seems preferable to use results based on bootstrapping residuals i.e. type II and IV simulations, particularly when making probabilistic assessments.

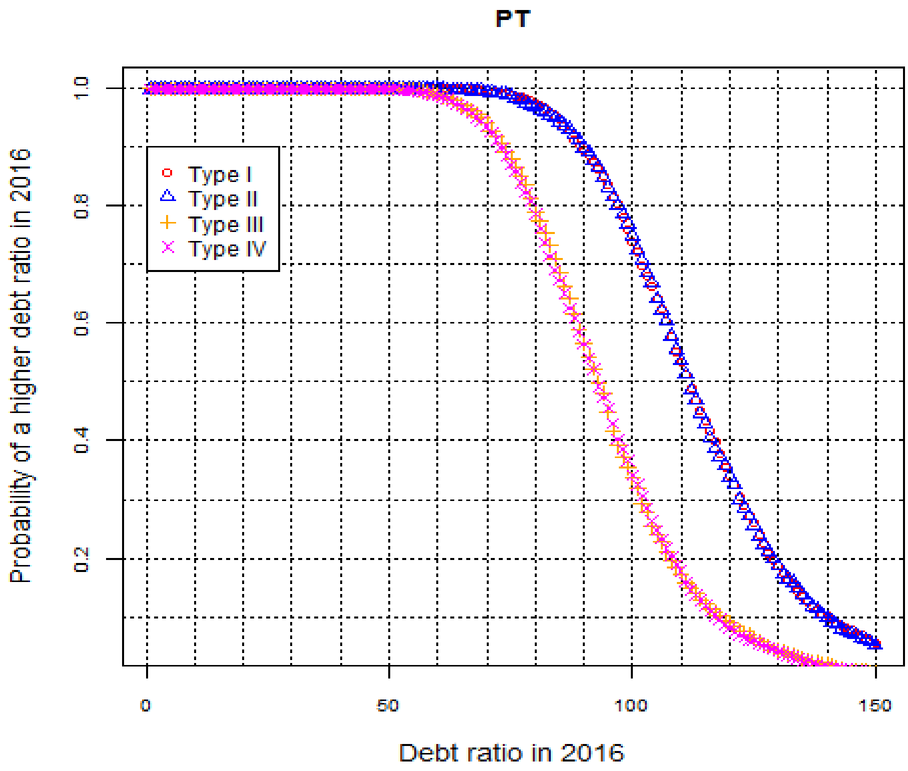
[30] Boxplots drawn in this paper summarise debt ratio distributions through five numbers: i) the lowest datum still within 1.5 times the inter-quartile range; ii) the highest datum still within 1.5 times the inter-quartile range; iii) the lower quartile; iv) the median, and; iv) the upper quartile. The inter-quartile range is the difference between the upper and lower quartiles and is considered to be a robust measure of statistical dispersion. The presence of outliers is not indicated.

[31] A positive skew indicates that the tail on the right side is longer than on the left side and that the bulk of values lies to the left of the mean i.e. the median is lower than the mean.

7. Probability of the debt ratio exceeding given values

The simulated empirical distributions allow measuring the probability of particular events occurring. An example that uses data for Portugal is the following. The unchanged policy assumption (type I and II simulations) suggest a probability of about 70% for the debt ratio to stay above 100% in 2016, which compares with only about 30% for the simulations of type III and IV (changed policies) (Figure 6).

Figure 6: Empirical probabilities for the debt ratio to exceed given values by 2016 in Portugal



8. Interpretation of results for the four stochastic types

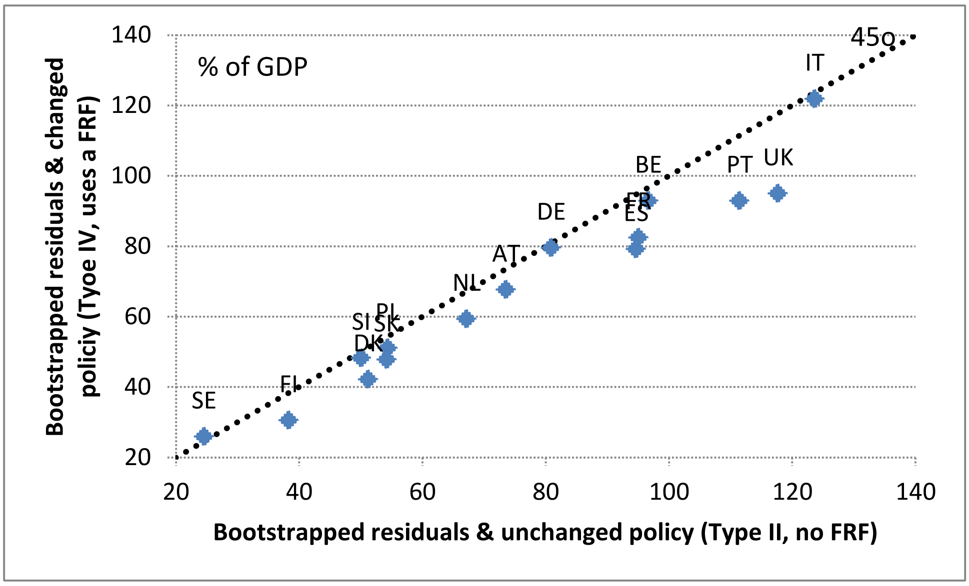
Comparing normal errors with bootstrapping

The differences between the results obtained assuming normal errors or bootstrapping residuals are relatively small (compare types I with II or III with IV). However, there is strong evidence that both the macroeconomic residuals of VAR models and the simulated debt ratios are not normally distributed. The evidence on the latter suggests that the debt ratios are asymmetric, displaying a general skew to the left. Therefore, using results based on bootstrapped residuals seem preferable, particularly when inferring probabilities from the simulated empirical distributions (e.g. the probability of the debt ratio exceeding a certain value in a given year).

Comparing unchanged policy with changed policy (i.e. use or not of a panel FRF)

Significant differences exist between the unchanged policy simulations (I and II) and the changed policy ones (III and IV). Assuming that the stance of budgetary policy tends to return to historical values, as estimated by a panel FRF with country fixed-effects, yields a considerably lower (median) debt ratio by the end of the simulation period (2016) than assuming a constant structural primary balance throughout the simulation period (Figure 7).[32]

Figure 7: Median of the simulated debt ratio in 2016 for bootstrapping types II and IV

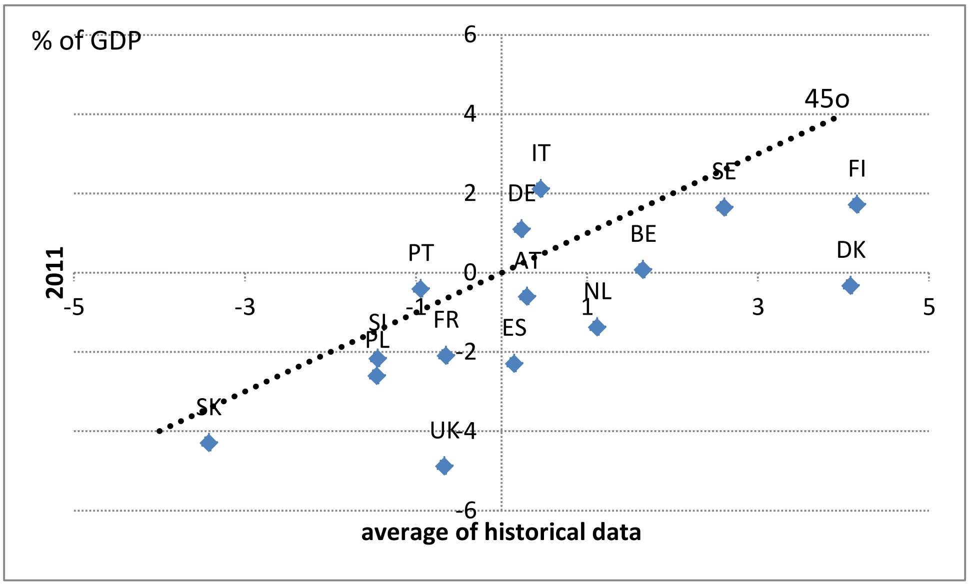


Simulations based on an estimated panel FRF usually obtain lower debt ratios, because average historical values for the (structural) primary balance tend to be higher than the values observed in the base year i.e. 2011 (Figure 8). Using a panel FRF can be interpreted as assuming "mean reversion" to historical trends.

The objection could be raised that "mean reversion", to the average across the 21 Member States included in the panel FRF[33] is not relevant on a country-by-country basis in order to assess future budgetary policy. A counterargument to this would be that idiosyncratic country effects are (largely) controlled for by the introduction of country fixed-effects and the autoregressive structure of the errors. Furthermore, the budgetary surveillance framework set in place with the onset of EMU is also likely to have strengthened cooperation and synchronicity of policy action, making average responses/outcomes across the EU more relevant at national level.

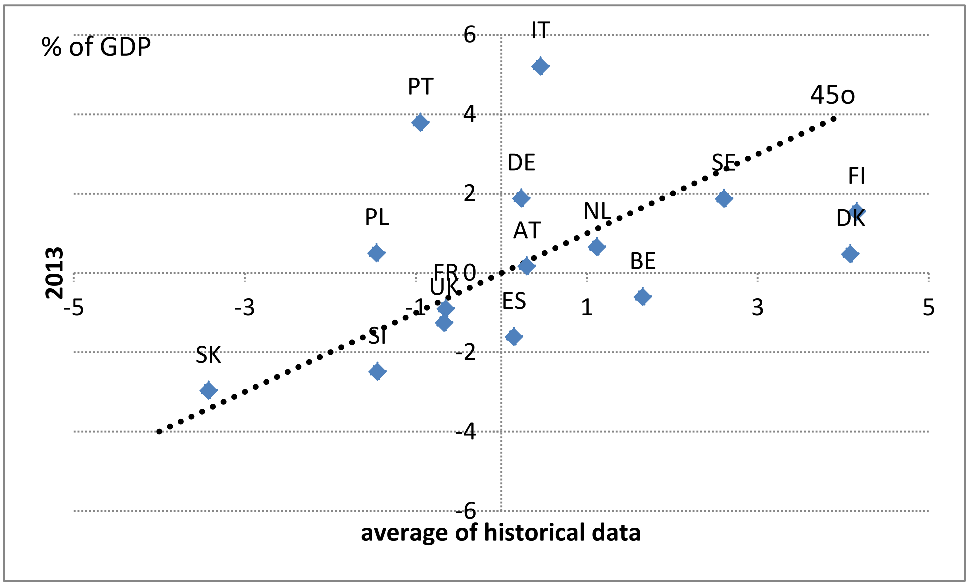
[32] The value used for the structural primary balance is the one estimated for 2011 (Economic Forecasts, DG ECFIN, Autumn 2011).
[33] Using the 4th specification.

Figure 8: Historical structural primary balance and 2011 values



It should be noted that using the year 2011 as the base for the (structural) primary balance in simulations I and II plays a role in the results obtained.[34] As an example, using the year 2013 as base year would have resulted in smaller differences relatively to historical values (Figure 9), possibly yielding a more convergence set of simulation results between those obtained using or not a panel FRF.

Figure 9: Historical structural primary balance and 2013 values



[34] The base year is 2011 and the simulations cover the period 2012-2016.

9. Conclusions

VAR models and a panel FRF are used to simulate stochastic debt ratios for fifteen EU Member States: BE, DK, DE, ES, FR, IT, NL, AT, PL, PT, SI, SK, FI, SE, and the UK.

Results are not cast in stone but should rather be seen as a snapshot of applying this methodology to the available datasets at the cut-off date of April 2012. Given the reliance on econometric methods, results based on this methodology may be subject to frequent and significant revisions. The VAR/FRF methodology should be used to make probabilistic assessments on the debt ratio rather than for providing point estimates. A major value added of this analysis is to assess the impact of major assumptions, such as unchanged fiscal policy versus "mean reversion" to historical trends.

In line with recent literature, there is evidence of "fiscal fatigue" at (very) high levels of the debt ratio, meaning that although the primary balance might remain positive, it starts declining when the debt ratio exceeds certain thresholds.

Strong evidence suggests that budgetary policy reacts to lagged debt and the contemporaneous output gap. Existence of an overall well-behaved FRF provides support for calculating also debt trajectories under the assumption of (partial) "mean reversion" to past trends. Mean reversion might be partial due to changes in institutional and/or policy variables, and the setting-up (and strengthening) of the EMU framework for budgetary surveillance.

Simulations based on bootstrapping residuals are preferred to those that assume normal errors, because both residuals in VAR models and debt ratio paths are not normally distributed; the distribution of debt paths is asymmetric, tending to be positively skewed, meaning that the bulk of values lies to the left of the mean (i.e. the median is lower than the mean).

Simulations based on a FRF tend to increase the spread of debt ratio distributions, because they involve considering an additional source of uncertainty, namely that associated with budgetary policy on top of the uncertainty due to non-fiscal macroeconomic variables in VAR models. It should be acknowledged that using 2011 as base year tends to affect simulation results, because of structural fiscal slippages due to the economic crisis. In fact, simulations based on a panel FRF yield lower median values for the debt ratio than those based on an unchanged (structural) primary surplus using 2011 as base year, because the latter values are lower than the primary balance values obtained using a panel FRF i.e. assuming "mean reversion".

Summing up, the evidence strongly suggests that debt ratio paths are not normally distributed, being instead asymmetric and positively skewed; and that primary balances show "fiscal fatigue" and (partial) "mean reversion" to past trends, calling for running also debt sustainability scenarios based on a panel FRF or some equivalent "mean reversion" hypothesis.

10. References

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11. Annex

Table 6: Deciles of the debt ratio simulations

	Deciles		0%			10%			20%			30%			40%			50%		
	Types	2011	2012	2014	2016	2012	2014	2016	2012	2014	2016	2012	2014	2016	2012	2014	2016	2012	2014	2016
AT	I	72,2	67,3	61,3	58,8	70,6	68,3	67,0	71,2	69,7	69,0	71,6	70,8	70,7	71,9	71,8	72,0	72,3	72,7	73,2
	II	72,2	66,5	59,8	55,1	71,0	68,5	67,2	71,6	70,2	69,4	71,9	71,3	71,0	72,1	72,1	72,4	72,4	73,0	73,5
	III	72,2	63,8	49,1	43,0	68,6	61,8	57,5	69,5	64,6	60,7	70,2	66,5	63,4	70,9	68,1	65,3	71,6	69,5	67,5
	IV	72,2	61,8	48,6	44,3	68,8	62,3	57,5	69,9	64,9	60,5	70,6	66,6	63,2	71,2	68,2	65,5	71,7	69,7	67,5
BE	I	97,2	90,6	80,0	78,3	95,0	92,0	90,9	95,6	93,7	92,9	96,1	94,7	94,2	96,6	95,7	95,4	97,0	96,7	96,7
	II	97,2	91,6	86,4	83,1	95,0	92,0	90,8	95,6	93,5	92,7	96,1	94,5	94,0	96,5	95,4	95,5	96,9	96,3	96,6
	III	97,2	87,2	74,6	67,6	93,1	86,5	82,9	94,2	89,5	86,4	95,1	91,4	89,0	95,9	93,1	91,0	96,5	94,6	93,1
	IV	97,2	88,7	76,0	68,1	93,2	86,7	82,2	94,4	89,5	86,3	95,2	91,4	88,9	95,8	93,0	90,8	96,4	94,5	92,8
DE	I	81,7	72,1	59,6	46,6	78,2	71,7	67,6	79,2	74,6	72,2	79,9	77,1	75,8	80,6	78,9	78,4	81,2	80,8	81,2
	II	81,7	70,3	54,3	45,5	78,8	72,9	69,3	79,7	75,9	73,1	80,4	77,9	76,1	80,9	79,4	78,5	81,3	81,0	80,9
	III	81,7	68,1	52,6	44,9	76,8	68,4	63,8	78,4	72,1	68,9	79,4	75,1	72,8	80,3	77,7	76,3	81,1	80,2	79,6
	IV	81,7	66,3	43,7	39,6	77,4	69,8	64,6	79,0	73,3	69,3	79,8	76,0	73,1	80,7	78,1	76,5	81,4	80,2	79,4
DK	I	44,1	41,1	37,2	38,2	44,1	44,6	46,0	44,6	45,8	47,8	45,0	46,9	49,1	45,4	47,7	50,1	45,8	48,5	51,3
	II	44,1	42,1	39,9	39,2	44,3	44,7	46,3	44,8	46,2	47,9	45,2	47,1	49,0	45,5	47,9	50,0	45,8	48,6	51,2
	III	44,1	37,6	25,5	17,3	42,7	37,5	32,8	43,6	39,9	35,8	44,3	41,9	38,1	45,0	43,4	40,2	45,6	44,8	42,2
	IV	44,1	39,6	25,8	18,8	42,9	37,8	32,1	43,9	40,0	35,6	44,5	41,9	37,9	45,1	43,5	40,2	45,6	44,8	42,1
ES	I	69,6	68,0	60,3	51,7	71,5	73,7	73,7	72,3	76,7	80,5	72,9	79,1	85,3	73,3	81,4	91,0	73,7	83,3	95,6
	II	69,6	69,2	65,9	53,1	71,9	74,4	74,9	72,6	77,2	80,8	73,0	79,3	85,6	73,4	81,1	90,0	73,8	83,0	94,7
	III	69,6	64,3	51,2	37,4	69,4	65,8	59,6	70,6	69,0	66,4	71,4	72,0	71,4	72,0	74,5	76,2	72,7	76,7	80,4
	IV	69,6	65,2	54,2	41,5	69,8	66,1	60,4	70,8	69,5	66,0	71,5	72,1	70,8	72,2	74,2	75,2	72,8	76,4	79,0
FI	I	49,1	39,4	25,7	17,2	43,6	34,8	31,3	44,3	37,1	33,8	44,9	38,7	35,6	45,3	39,9	37,0	45,8	40,9	38,5
	II	49,1	40,4	27,5	21,1	43,9	35,4	31,4	44,6	37,2	33,6	45,2	38,6	35,3	45,6	39,7	36,7	46,0	41,0	38,3
	III	49,1	34,9	13,6	-3,1	41,2	27,0	19,5	42,3	30,2	23,2	43,2	32,5	26,2	44,0	34,5	28,6	44,8	36,7	30,9
	IV	49,1	35,4	9,8	1,3	41,5	27,5	19,3	42,7	30,7	23,0	43,5	32,6	25,7	44,3	34,7	27,9	45,0	36,7	30,4
FR	I	85,4	82,8	75,3	67,8	85,6	83,9	82,2	86,1	86,1	86,1	86,5	87,8	89,5	86,9	89,3	92,3	87,2	90,7	95,2
	II	85,4	82,8	71,0	56,8	85,8	84,3	82,4	86,3	86,4	86,7	86,6	88,0	89,7	87,0	89,4	92,5	87,2	90,6	95,1
	III	85,4	79,1	62,9	50,3	83,4	76,1	69,2	84,4	78,8	73,6	85,0	81,1	76,9	85,6	82,9	80,0	86,1	84,8	82,8
	IV	85,4	80,0	62,8	49,4	83,5	76,2	69,0	84,5	79,2	73,4	85,1	81,1	76,8	85,7	82,8	79,7	86,2	84,5	82,4
IT	I	120,5	113,2	95,4	82,0	118,0	111,4	104,9	119,1	114,9	110,9	119,9	117,6	115,7	120,4	119,6	120,1	121,1	121,7	124,4
	II	120,5	114,5	97,9	86,6	118,5	112,5	106,3	119,4	115,6	111,5	120,0	117,7	116,3	120,6	119,7	120,3	121,0	121,5	123,6
	III	120,5	111,1	92,3	82,0	117,2	109,4	103,9	118,5	113,2	109,8	119,4	116,4	114,3	120,3	119,0	118,6	121,1	121,4	122,7
	IV	120,5	111,9	92,6	79,8	117,6	110,3	104,5	118,8	114,2	110,1	119,7	116,6	114,1	120,4	118,9	118,1	121,1	121,1	121,8
NL	I	64,2	60,3	55,1	55,9	63,7	62,6	62,8	64,4	64,1	64,3	64,9	65,0	65,5	65,2	65,9	66,5	65,6	66,7	67,4
	II	64,2	60,3	54,9	55,9	64,0	62,6	62,9	64,7	64,0	64,2	65,1	64,9	65,5	65,3	65,9	66,3	65,7	66,7	67,2
	III	64,2	57,5	44,0	37,0	61,9	55,5	50,9	62,8	57,9	53,7	63,6	59,7	55,7	64,1	61,1	57,6	64,8	62,6	59,4
	IV	64,2	57,5	45,9	37,5	62,1	55,7	50,0	63,1	57,9	53,0	63,8	59,5	55,4	64,4	60,9	57,4	64,9	62,2	59,3
PL	I	56,7	53,3	43,7	34,8	56,0	51,2	45,0	56,5	53,1	47,7	57,0	54,3	50,2	57,3	55,6	52,4	57,7	56,6	54,5
	II	56,7	53,4	44,3	34,3	56,3	52,1	45,4	56,7	53,8	48,5	57,1	54,8	50,6	57,5	55,9	52,4	57,8	56,8	54,3
	III	56,7	48,0	35,7	18,7	53,9	46,7	40,8	54,9	49,3	43,9	55,6	51,1	46,4	56,5	53,0	48,7	57,0	54,9	51,1
	IV	56,7	50,4	34,4	25,9	54,3	47,5	40,5	55,2	50,1	44,0	55,9	51,9	46,7	56,5	53,3	49,1	57,2	54,7	50,9
PT	I	101,6	90,8	77,5	66,2	99,5	94,9	89,6	101,3	98,9	97,1	102,7	102,5	102,0	104,0	105,7	107,2	105,1	108,7	111,5
	II	101,6	89,9	71,8	52,3	100,3	95,4	90,0	102,4	100,3	97,1	103,6	103,4	102,4	104,5	106,0	107,3	105,3	108,6	111,4
	III	101,6	85,0	64,5	49,6	95,5	83,5	73,5	97,9	88,3	79,5	99,3	91,8	84,5	100,7	94,9	88,7	102,0	97,9	93,0
	IV	101,6	86,7	61,4	35,8	96,6	84,0	73,0	98,8	88,7	79,5	100,2	92,6	83,6	101,3	95,3	88,4	102,4	98,2	92,8
SE	I	36,3	27,9	18,4	13,4	31,5	24,6	20,4	32,2	26,0	21,9	32,6	27,1	23,0	33,0	27,8	24,0	33,4	28,7	24,9
	II	36,3	28,9	17,8	14,5	31,7	24,5	20,5	32,2	25,9	21,9	32,6	26,8	22,9	33,0	27,7	23,8	33,4	28,4	24,6
	III	36,3	24,6	9,1	0,8	30,1	21,5	16,6	31,3	23,9	19,9	32,1	25,8	22,0	32,8	27,7	24,2	33,4	29,1	26,2
	IV	36,3	24,8	9,0	3,3	30,2	21,3	16,3	31,4	24,0	19,7	32,1	25,8	22,0	32,8	27,4	23,8	33,4	28,9	25,8
SI	I	45,5	39,6	31,7	29,3	45,3	41,9	41,2	46,1	44,2	44,1	46,8	45,9	46,3	47,3	47,4	48,4	47,8	48,9	50,4
	II	45,5	42,2	36,4	31,4	45,8	43,2	42,2	46,5	44,9	44,8	47,0	46,2	46,8	47,4	47,3	48,3	47,9	48,6	50,1
	III	45,5	35,9	23,1	17,0	44,0	36,9	36,1	45,3	40,7	40,3	46,4	43,5	43,2	47,4	45,9	46,0	48,2	48,2	48,6
	IV	45,5	40,3	26,2	24,3	44,8	38,5	36,8	45,9	41,6	40,3	46,8	43,8	43,3	47,5	45,9	45,8	48,2	47,9	48,2
SK	I	44,5	40,3	35,2	39,3	44,7	44,9	48,2	45,6	46,9	50,4	46,2	48,2	52,1	46,7	49,6	53,2	47,3	50,8	54,7
	II	44,5	42,4	38,3	39,6	45,2	45,6	48,6	45,9	47,3	50,4	46,4	48,4	51,8	46,8	49,5	53,0	47,2	50,4	54,2
	III	44,5	30,3	18,3	14,4	41,6	35,2	36,2	43,2	39,1	40,3	44,4	42,2	43,2	45,6	44,8	46,0	46,5	47,2	48,5
	IV	44,5	35,3	20,6	20,1	42,5	36,8	36,5	43,9	40,2	40,4	44,9	42,6	43,1	45,8	44,7	45,3	46,6	46,8	47,7
UK	I	84,0	84,8	86,3	80,4	88,5	95,6	102,3	89,3	98,3	107,1	89,8	100,3	110,7	90,3	102,1	114,0	90,6	103,7	117,5
	II	84,0	84,9	85,7	80,3	88,7	96,0	102,6	89,5	98,4	107,2	89,9	100,5	110,8	90,3	102,2	114,3	90,7	103,8	117,7
	III	84,0	80,6	70,4	51,2	86,0	83,9	79,8	87,1	87,3	84,6	87,9	89,7	88,8	88,6	91,9	92,2	89,2	94,0	95,5
	IV	84,0	80,7	66,9	58,0	86,2	84,2	80,0	87,2	87,3	85,2	88,0	89,6	88,7	88,7	91,6	91,5	89,2	93,6	94,9

	Deciles	60%			70%			80%			90%			100%		
	Types	2012	2014	2016	2012	2014	2016	2012	2014	2016	2012	2014	2016	2012	2014	2016
AT	I	72,6	73,7	74,5	73,0	74,7	76,0	73,4	75,8	77,7	73,9	77,5	80,1	77,3	85,5	94,9
	II	72,6	73,7	74,8	72,9	74,6	76,1	73,2	75,6	77,7	73,8	77,0	79,9	77,0	88,7	94,7
	III	72,1	71,2	69,6	72,8	72,9	71,6	73,5	75,0	74,3	74,6	77,5	77,7	78,9	95,9	98,6
	IV	72,2	71,2	69,4	72,8	72,8	71,6	73,4	74,6	74,1	74,3	76,8	77,6	79,7	90,8	101,7
BE	I	97,4	97,8	98,1	97,9	98,8	99,5	98,4	100,0	101,0	99,1	101,7	103,5	102,8	109,0	116,6
	II	97,3	97,3	97,8	97,7	98,3	99,2	98,3	99,5	100,6	99,0	101,3	102,8	103,3	111,2	117,0
	III	97,2	96,3	95,2	97,8	98,0	97,1	98,7	100,1	99,8	99,8	103,1	103,7	105,2	116,2	120,5
	IV	97,1	95,9	94,7	97,8	97,6	96,9	98,5	99,5	99,9	99,7	102,6	103,4	106,3	117,3	122,7
DE	I	81,8	82,7	84,1	82,6	84,8	88,2	83,4	87,4	92,4	84,6	91,5	98,4	89,9	114,4	136,3
	II	81,9	82,7	83,9	82,4	84,6	87,4	83,2	86,9	91,7	84,4	90,9	98,9	92,7	116,5	148,2
	III	82,1	82,5	82,9	83,0	85,1	87,1	84,1	88,5	91,4	85,7	93,3	98,5	93,1	124,3	146,6
	IV	82,1	82,5	82,9	83,0	84,9	86,4	84,0	88,0	91,2	85,7	92,8	98,9	96,1	124,2	145,6
DK	I	46,2	49,4	52,3	46,5	50,1	53,4	46,9	51,2	54,8	47,5	52,6	57,0	50,3	61,2	72,8
	II	46,1	49,3	52,3	46,5	50,1	53,5	46,9	51,2	54,8	47,5	52,6	57,1	51,2	62,2	71,6
	III	46,2	46,3	44,2	46,8	48,0	46,4	47,6	49,7	48,6	48,6	52,4	51,9	53,2	70,2	72,8
	IV	46,2	46,1	44,1	46,8	47,8	45,9	47,5	49,7	48,3	48,6	52,2	51,7	53,5	65,1	71,7
ES	I	74,2	85,6	100,5	74,7	87,7	106,5	75,2	90,4	113,6	75,9	94,4	123,2	81,7	123,9	205,5
	II	74,2	85,0	99,2	74,5	87,0	104,8	75,0	90,2	113,2	75,9	95,0	126,1	85,9	133,8	222,4
	III	73,4	79,2	84,7	74,1	81,5	90,2	74,9	84,7	97,2	76,0	89,6	106,1	83,2	120,5	182,4
	IV	73,4	78,7	83,8	74,1	81,7	89,7	74,8	84,8	96,9	76,0	89,9	108,4	86,1	123,2	188,6
FI	I	46,2	42,0	40,1	46,8	43,4	41,8	47,3	45,1	44,0	48,1	47,7	46,8	52,0	59,6	62,1
	II	46,4	42,2	39,9	46,8	43,4	41,6	47,3	45,2	43,5	48,1	47,5	46,5	52,5	59,8	65,6
	III	45,5	38,7	33,3	46,4	40,9	35,7	47,3	43,7	39,2	48,5	47,2	43,1	54,6	64,2	64,8
	IV	45,7	38,7	32,7	46,4	40,5	35,2	47,2	42,9	38,3	48,5	46,7	42,4	54,0	62,8	71,0
FR	I	87,6	92,1	98,2	87,9	93,6	101,1	88,3	95,4	104,8	88,9	97,9	110,2	93,4	117,8	145,7
	II	87,5	91,9	97,9	87,8	93,3	100,7	88,2	95,0	104,4	88,8	97,6	109,4	93,4	117,3	150,8
	III	86,7	86,5	85,7	87,4	88,4	88,7	88,0	90,6	92,6	89,0	94,1	98,5	94,4	113,8	134,9
	IV	86,7	86,4	85,3	87,3	88,2	88,5	88,0	90,5	92,5	88,9	93,6	98,3	95,4	114,1	140,3
IT	I	121,6	124,1	128,5	122,2	126,3	132,6	122,9	128,8	137,7	124,0	132,4	145,7	131,4	161,5	205,2
	II	121,5	123,6	127,5	122,1	125,9	131,7	122,8	128,9	137,8	123,9	132,7	146,7	130,2	156,0	188,5
	III	121,9	123,8	127,0	122,6	126,3	131,0	123,6	129,1	135,7	124,9	133,5	143,5	133,3	161,7	201,6
	IV	121,8	123,2	125,6	122,6	125,8	130,3	123,6	128,9	136,1	124,9	134,0	144,2	134,1	159,5	187,6
NL	I	66,0	67,5	68,3	66,4	68,3	69,3	66,8	69,3	70,5	67,5	70,8	72,2	71,9	78,3	81,1
	II	66,0	67,5	68,1	66,3	68,2	69,1	66,7	69,1	70,3	67,4	70,7	71,9	70,7	79,2	79,3
	III	65,4	64,1	61,4	66,0	65,6	63,4	66,7	67,4	65,7	67,8	70,0	68,5	73,0	85,0	84,3
	IV	65,4	63,7	61,1	65,9	65,3	63,0	66,6	67,0	65,1	67,6	69,7	67,9	72,6	81,1	84,1
PL	I	58,0	57,7	56,8	58,4	58,8	59,4	58,8	60,3	62,6	59,4	62,5	67,4	63,5	79,3	111,2
	II	58,1	57,7	56,4	58,3	58,7	58,6	58,7	60,0	61,4	59,2	61,6	65,9	61,6	68,9	91,5
	III	57,7	56,5	53,4	58,3	58,5	56,4	59,2	60,5	59,2	60,2	63,2	64,1	65,8	75,8	99,3
	IV	57,7	56,3	53,0	58,3	58,0	55,2	59,0	59,8	58,1	59,9	62,4	61,9	66,3	75,1	93,1
PT	I	106,1	111,5	116,7	107,4	114,5	122,3	108,8	119,1	128,5	110,9	125,0	140,3	129,6	166,7	205,1
	II	106,2	111,3	116,3	107,2	114,7	122,0	108,5	118,7	128,8	110,5	125,2	139,3	130,3	176,1	210,8
	III	103,1	100,6	96,8	104,6	103,8	101,7	106,3	108,1	107,8	108,6	114,6	118,3	129,4	155,3	177,0
	IV	103,4	100,9	97,1	104,6	104,2	102,4	106,2	108,0	108,3	108,3	114,2	117,4	129,9	163,2	175,2
SE	I	33,7	29,5	25,8	34,1	30,3	26,8	34,6	31,4	27,9	35,4	32,7	29,7	38,6	40,7	38,2
	II	33,8	29,3	25,5	34,1	30,1	26,5	34,6	31,2	27,5	35,2	32,8	29,4	38,5	41,6	39,8
	III	34,0	30,7	28,2	34,6	32,3	30,2	35,5	34,2	32,4	36,5	36,8	35,9	42,2	48,9	54,9
	IV	34,0	30,4	27,9	34,7	31,9	29,7	35,4	33,9	32,1	36,6	36,7	35,5	41,0	48,1	54,2
SI	I	48,3	50,3	52,6	48,9	51,9	54,7	49,5	53,8	57,6	50,4	56,5	61,4	54,7	69,2	93,5
	II	48,3	49,9	51,8	48,8	51,3	53,8	49,4	52,8	56,3	50,2	55,2	60,1	54,3	67,9	81,6
	III	49,0	50,6	51,3	50,0	53,2	54,8	51,0	55,7	58,1	52,7	60,1	63,0	58,7	81,2	98,5
	IV	49,0	49,9	50,2	49,8	52,0	53,1	50,7	54,6	56,6	52,2	58,0	61,5	57,4	76,0	86,9
SK	I	47,8	51,9	56,2	48,3	53,1	57,8	49,0	54,5	59,6	49,9	57,0	62,0	54,3	70,0	77,8
	II	47,6	51,4	55,4	48,1	52,5	56,7	48,7	53,8	58,4	49,6	56,2	61,1	57,0	71,7	87,4
	III	47,5	49,7	51,0	48,6	52,2	53,8	49,9	55,1	56,9	51,6	58,8	61,3	59,3	83,1	82,6
	IV	47,4	49,0	50,1	48,3	51,2	52,6	49,4	53,7	55,5	51,2	58,3	59,7	64,6	89,0	83,8
UK	I	91,1	105,3	121,4	91,5	107,1	125,3	92,0	109,4	129,8	92,8	112,7	136,7	96,7	129,2	176,2
	II	91,1	105,3	121,3	91,5	107,2	124,9	92,0	109,5	129,1	92,7	112,6	136,1	96,1	139,2	225,8
	III	89,9	95,8	99,2	90,5	98,2	102,4	91,3	100,9	107,4	92,5	104,9	113,9	96,9	127,2	158,0
	IV	89,8	95,8	98,6	90,5	98,1	102,6	91,3	101,0	107,0	92,5	104,8	114,0	96,5	137,3	189,4

Figure 10: Boxplots of the debt-to-GDP ratio for Belgium

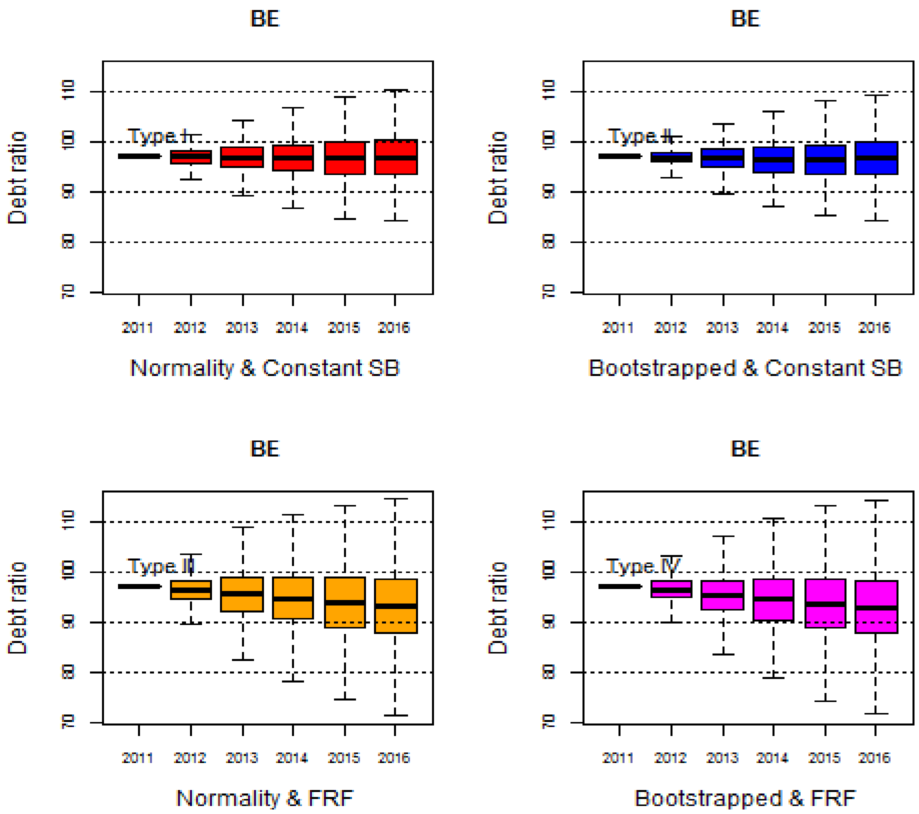


Figure 11: Boxplots of the debt-to-GDP ratio for Denmark

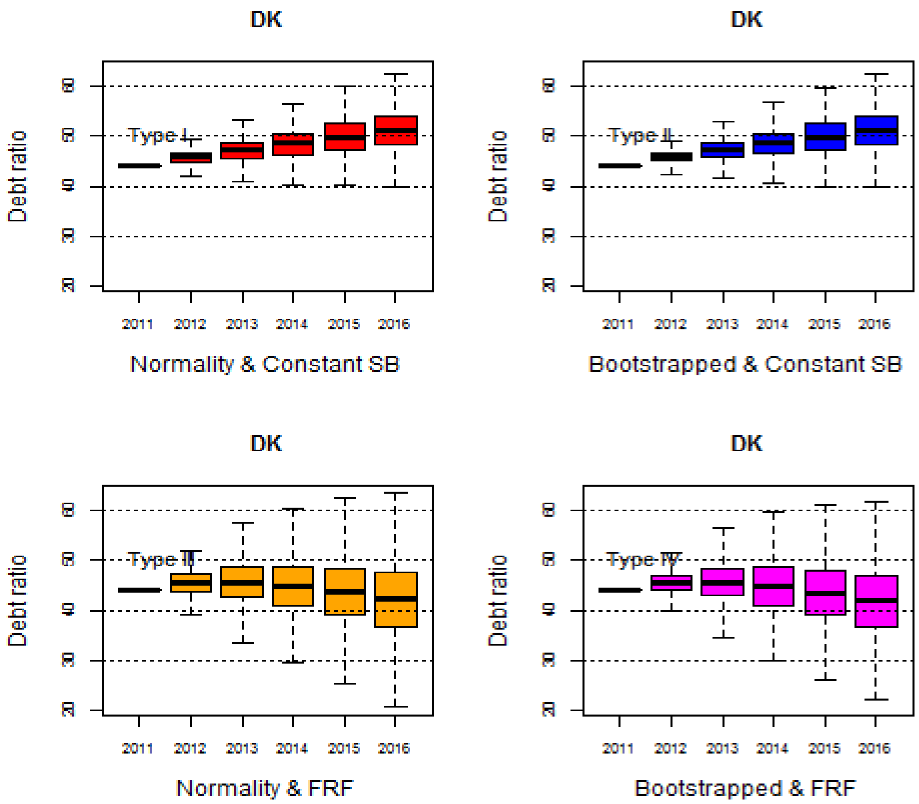


Figure 12: Boxplots of the debt-to-GDP ratio for Germany

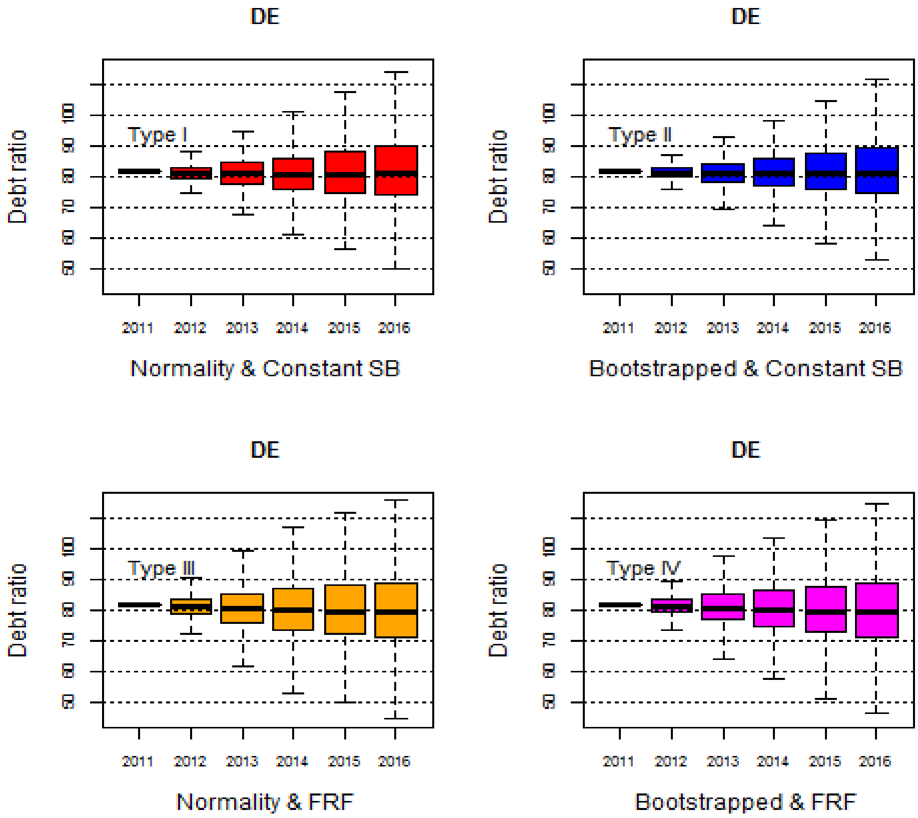


Figure 13: Boxplots of the debt-to-GDP ratio for Spain

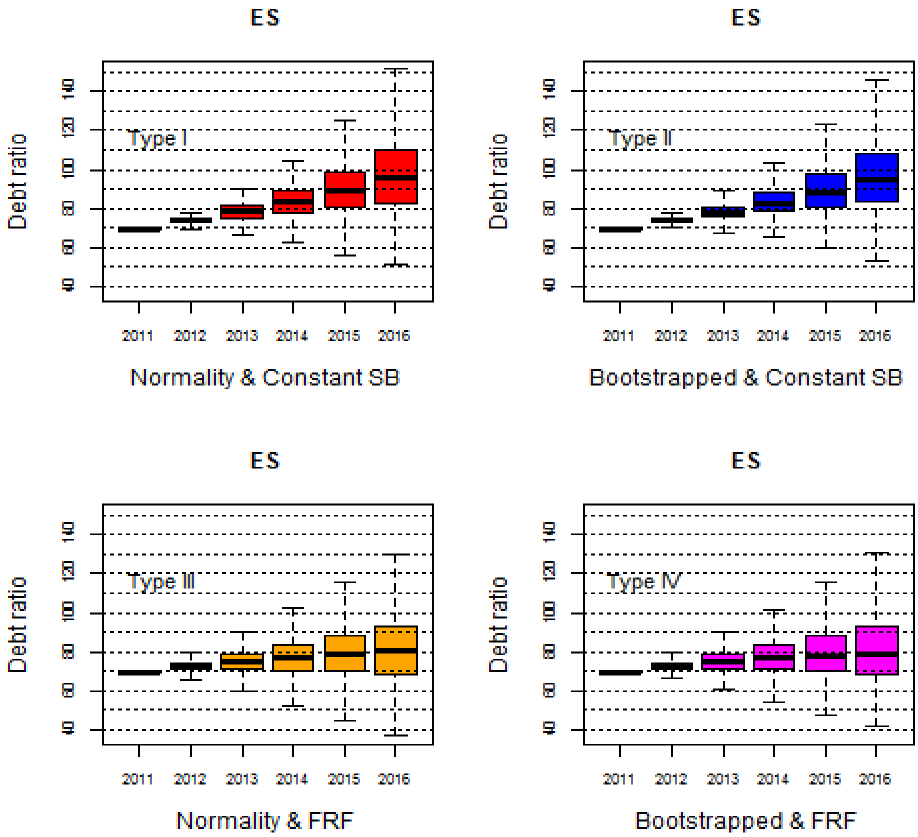


Figure 14: Boxplots of the debt-to-GDP ratio for France

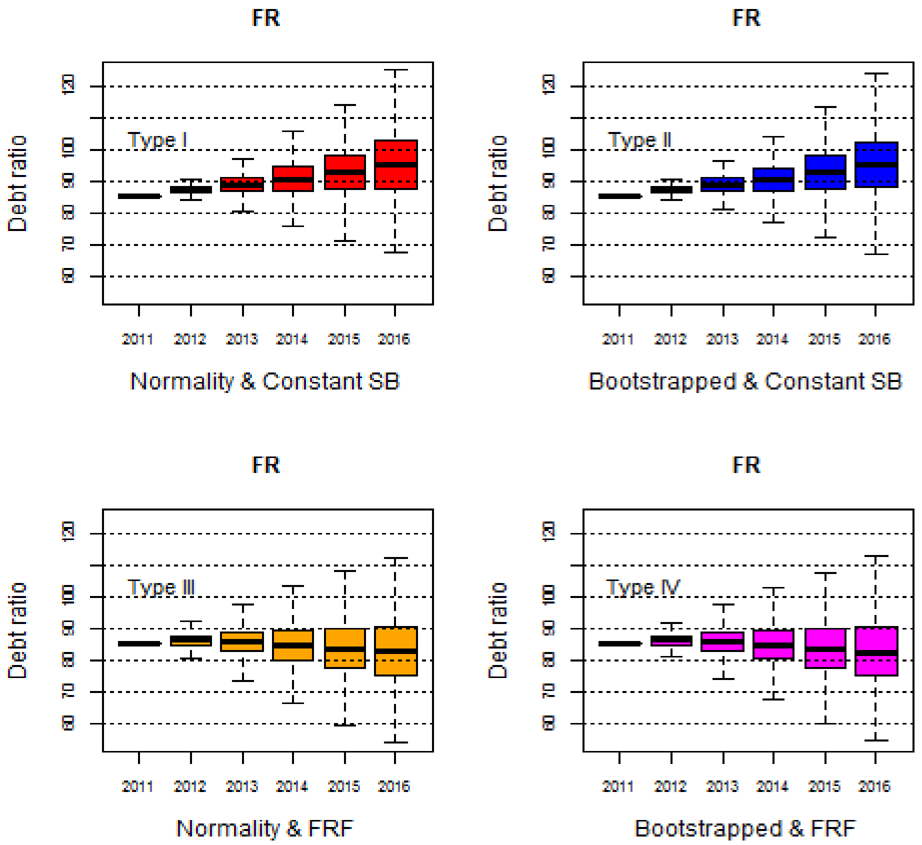


Figure 15: Boxplots of the debt-to-GDP ratio for Italy

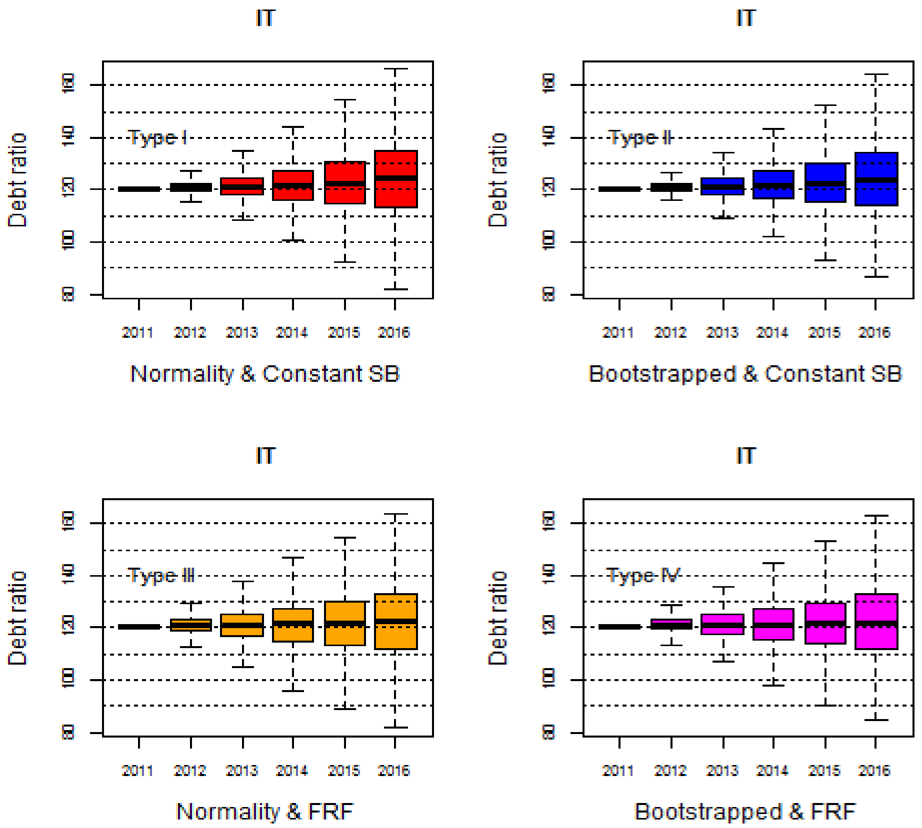


Figure 16: Boxplots of the debt-to-GDP ratio for the Netherlands

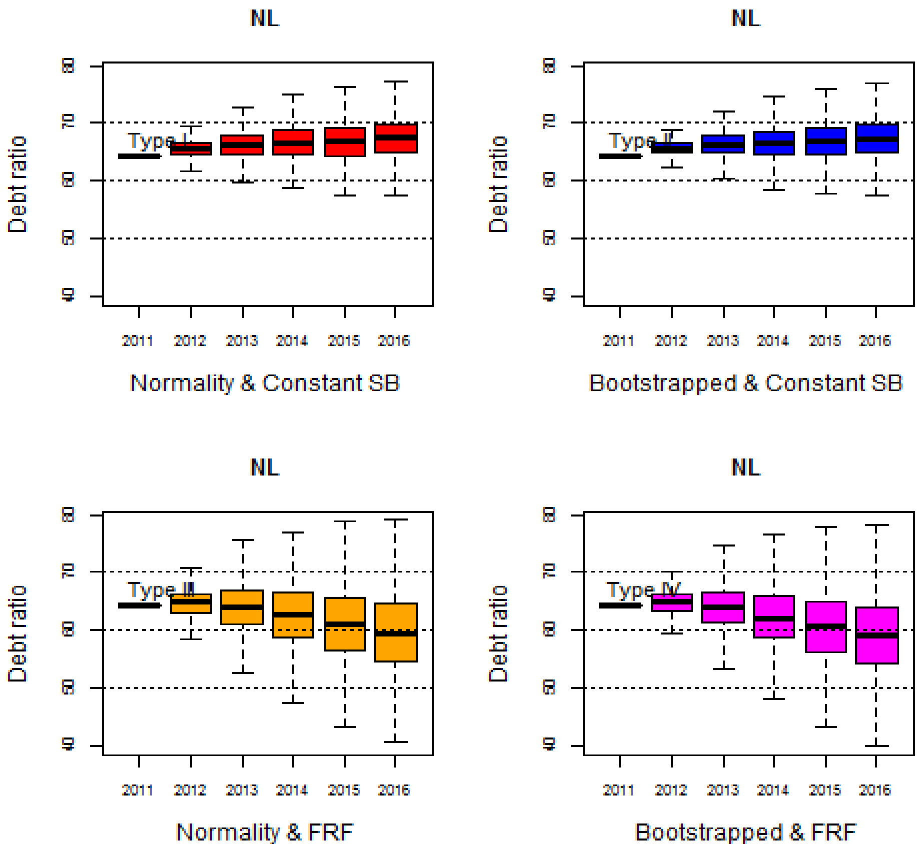


Figure 17: Boxplots of the debt-to-GDP ratio for Austria

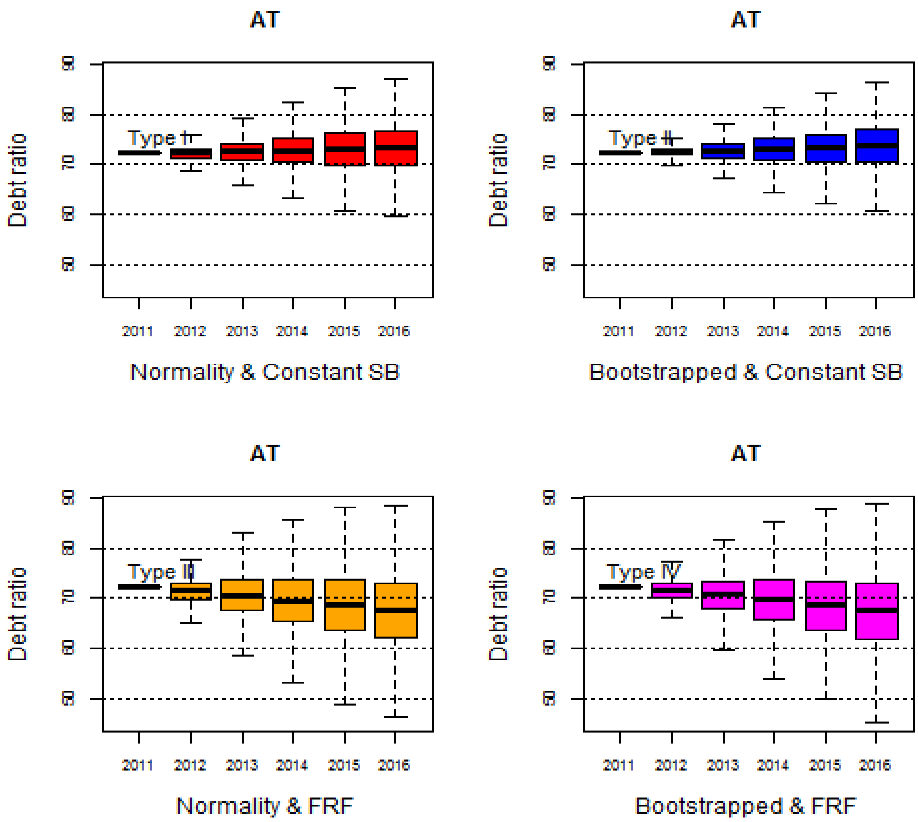


Figure 18: Boxplots of the debt-to-GDP ratio for Poland

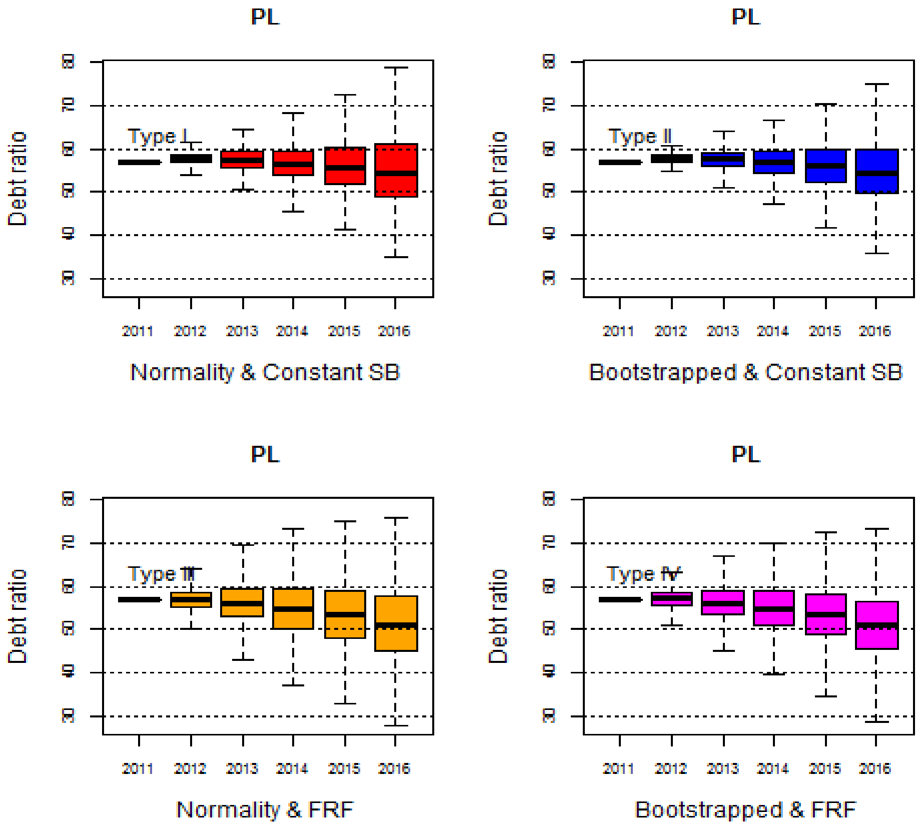


Figure 19: Boxplots of the debt-to-GDP ratio for Portugal

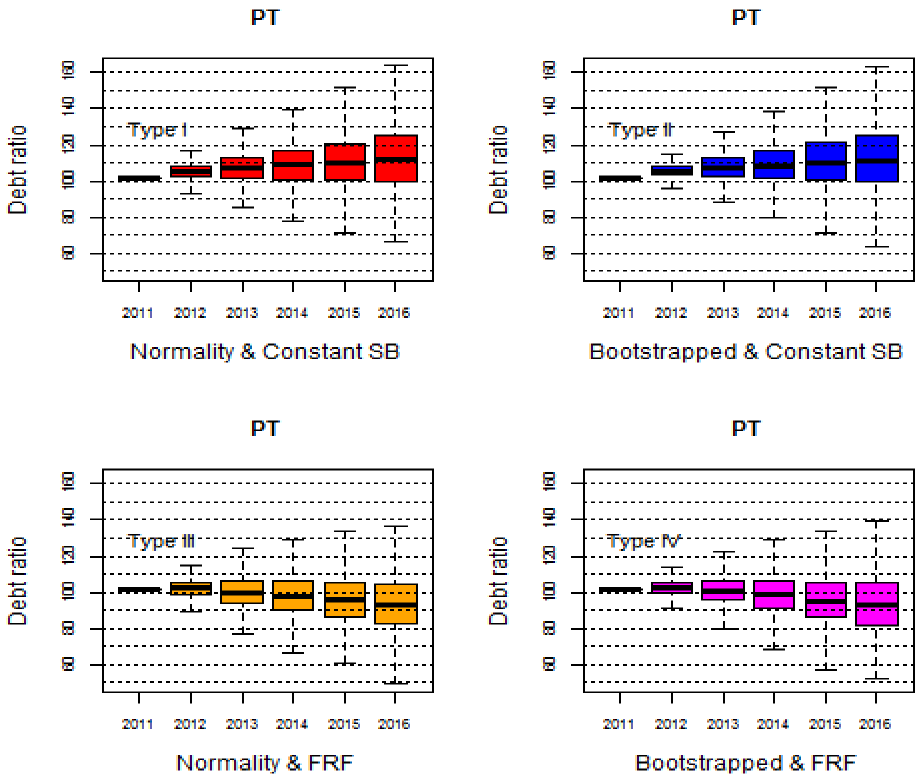


Figure 20: Boxplots of the debt-to-GDP ratio for Slovenia

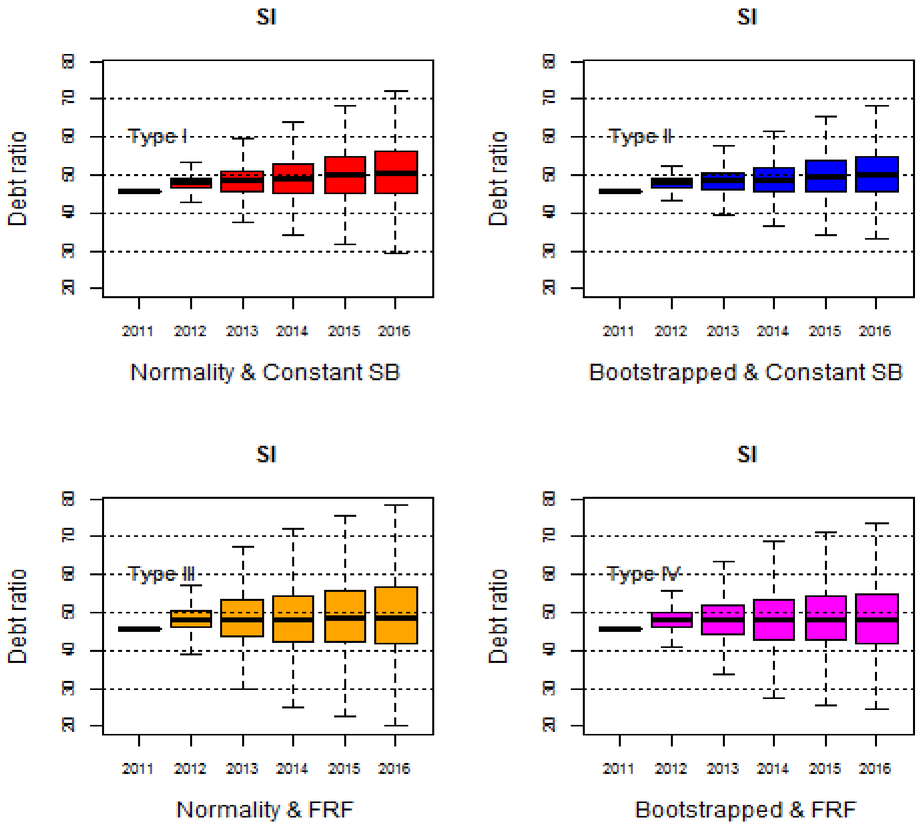


Figure 21: Boxplots of the debt-to-GDP ratio for Slovakia

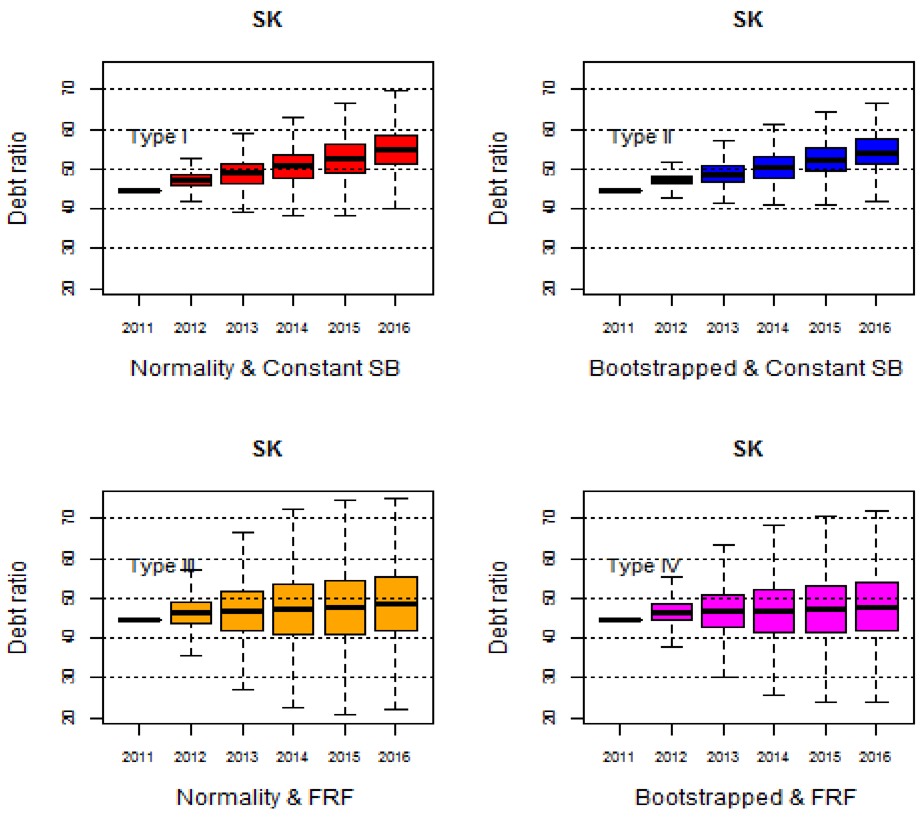


Figure 22: Boxplots of the debt-to-GDP ratio for Finland

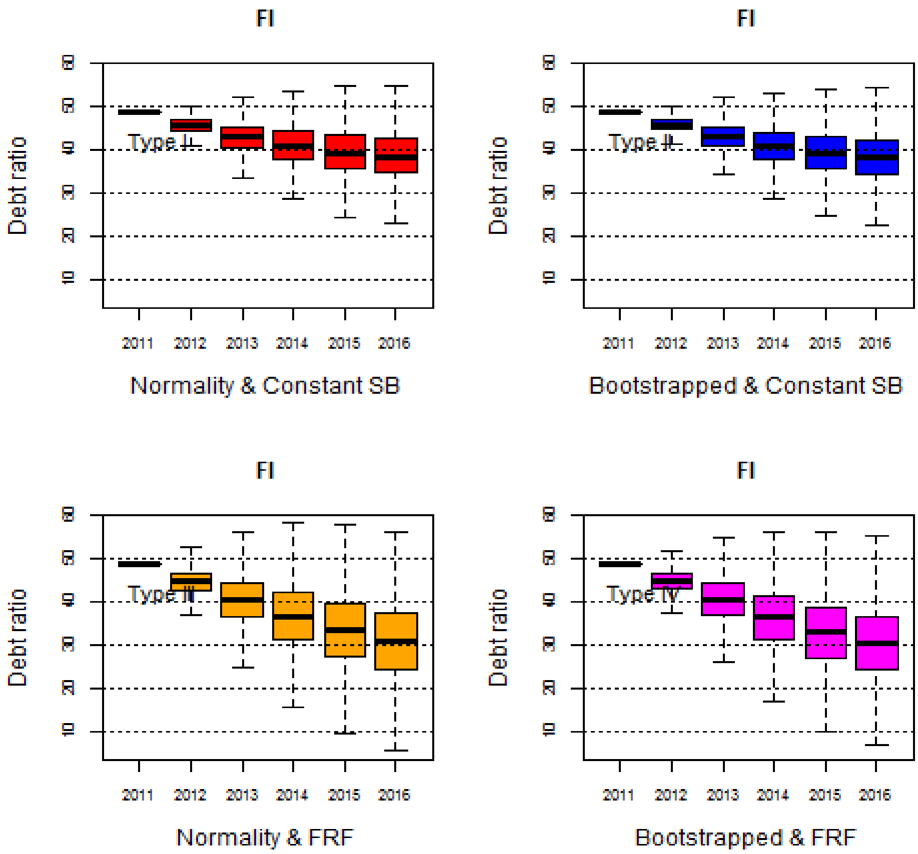


Figure 23: Boxplots of the debt-to-GDP ratio for Sweden

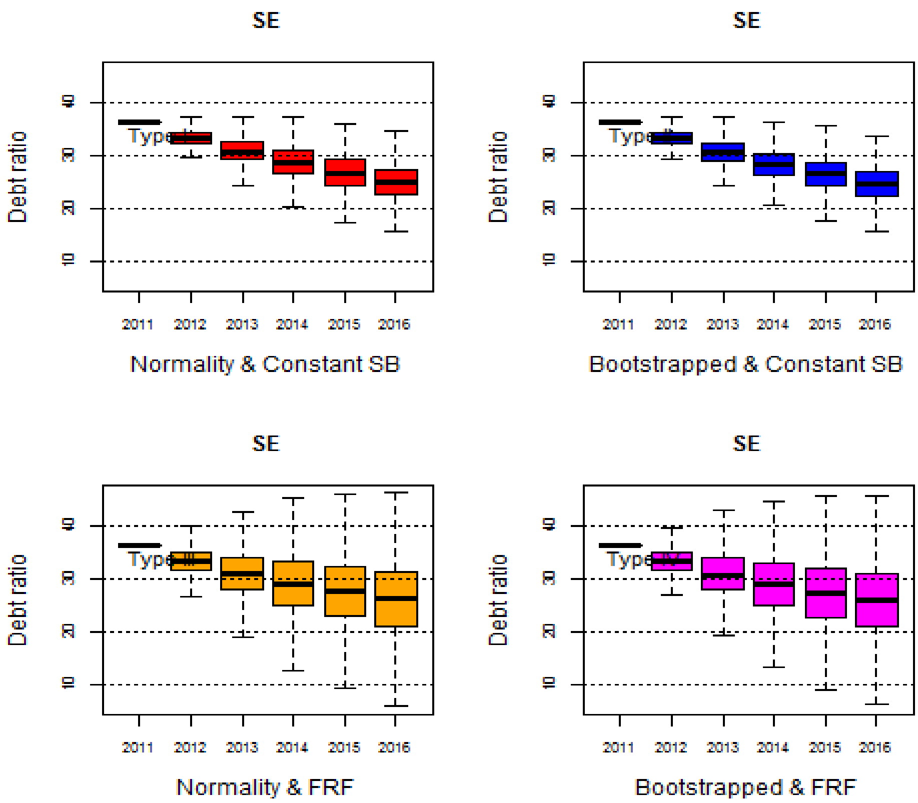
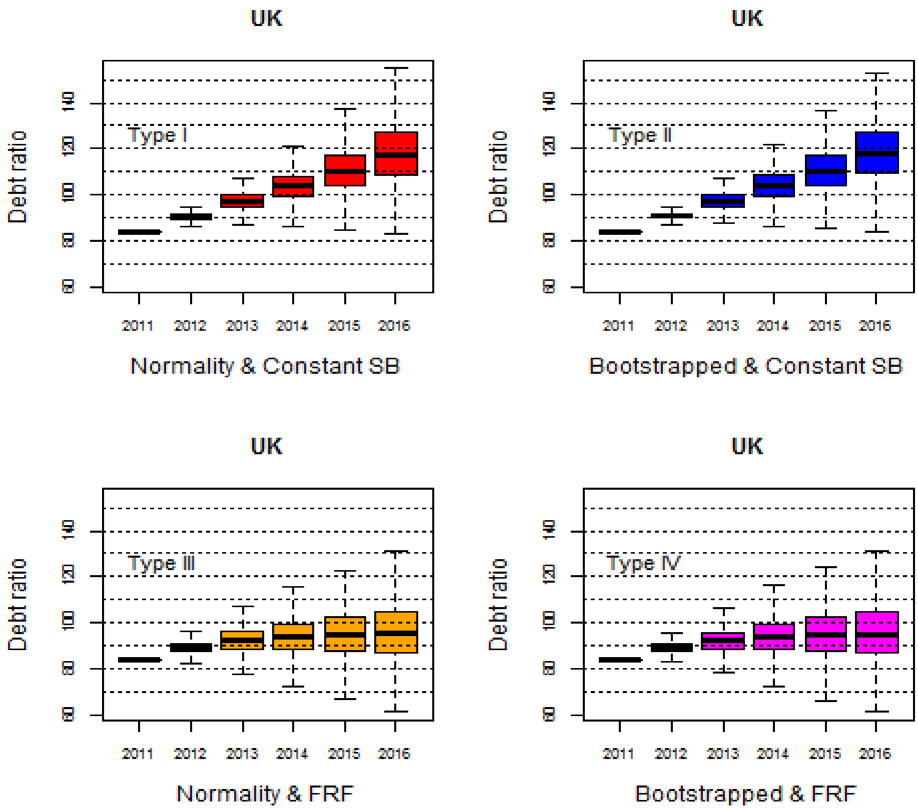


Figure 24: Boxplots of the debt-to-GDP ratio for the United Kingdom



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