# Analyzing Fiscal Sustainability\*

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#### Abstract

We develop a framework to study the implications of fiscal policy behavior for sovereign risk that exploits a country's fiscal limit, the point at which for economic or political reasons taxes and spending can no longer adjust to stabilize debt. A real business cycle model maps the economic environment—expected fiscal policy, the distribution of exogenous disturbances, and private agents' behavior—into a distribution for the maximum sustainable debt-GDP ratio. Default is possible at any point on this fiscal limit distribution. The model is consistent with a wide range of relationships between risk premia and government debt. Calibrations of the model to Greek and Swedish data illustrate how the framework can be used to study actual fiscal reforms undertaken by developed economies facing sovereign risk pressures.

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#### **1** INTRODUCTION

Sovereign debt default, once the exclusive domain of developing countries, is now a pressing concern for several advanced economies. Because it has been largely geared toward understanding the experiences of emerging economies—particularly in Latin America—the academic literature has been caught somewhat unprepared to address the sovereign debt problems that countries face today.

Strategic default models building on Eaton and Gersovitz (1981) initially employ sovereign risk as a device to help standard real business cycle models to reproduce key business cycle facts in emerging economies [Aguiar and Gopinath (2006) and Arellano (2008)]. Combining non-contingent one-period government debt with a strategic default decision by the government permits the RBC model to generate several features of data in emerging economies. Those setups generate default risk and default events as outgrowths of bad technology fundamentals. They are silent, however, about the *policy* behavior that converts the bad fundamentals into a sovereign debt crisis.

Bad luck in the form of a sequence of realizations of low productivity levels certainly plays a role in triggering sovereign default episodes. But this cannot be the whole story, at least in developed countries. The recent recession and financial crisis was widespread, yet only a handful of advanced economies have run into serious sovereign risk problems. What distinguishes countries with and without sovereign risk troubles today? Surely, at the top of the list are the past and prospective fiscal policies they pursue—not simply the realizations of their transitory productivity disturbances. This is why troubled countries, and even some less-troubled nations, are adopting drastic fiscal austerity measures intended to be long-lasting.

Understanding how fiscal policies determine a country's sovereign risk requires explicit modeling of fiscal behavior. Imprudent past fiscal policies can push an economy closer to its fiscal limit, while profligate prospective policies can reduce the fiscal limit, particularly when such policies prevail in the face of bad exogenous disturbances. Every economy faces a fiscal limit, the point at which, for economic or political reasons, taxes and spending can no longer adjust to stabilize debt: tax distortions and public intolerance of taxation typically imply Laffer curves that limit revenues; as this paper is being written, Greek citizens are rioting to protest their parliament's adoption of the latest round of austerity measures designed to push Greece farther from its fiscal limit. In the absence of a shift in monetary policy to a regime that stabilizes debt—an absence that this paper shares with virtually all recent work on sovereign debt—at the fiscal limit a government has no choice but to default on its outstanding debt obligations. A country's fiscal limit determines its maximum sustainable debt-GDP ratio. That limit depends on the entire economic and political environment: expected fiscal policy behavior, the distribution of exogenous disturbances, and private agents' behavior. In most cases a country's fiscal limit will not be revealed by historical policy choices. Rather, the fiscal limit answers the counterfactual question, "After accounting for economic and political constraints, what is the maximum expected present value of primary surpluses?" This paper answers that question with a simple real business cycle model that maps economic environments—especially fiscal policy regimes—into conditional and unconditional distributions of the fiscal limit. A conditional distribution reflects the notion that bondholders' expectations of repayment depend on the current state of the economy, including shock realizations and policy regime. For some analyses, particularly of long-run fiscal reforms, the unconditional fiscal limit distribution is more appropriate. We illustrate uses of both types of distributions.

By mapping policy behavior into fiscal limit distributions, the paper provides a tool to examine the efficacy of fiscal reforms that countries under sovereign risk pressures pursue. Both the nature and the credibility of proposed reforms matter for their likely success in reducing sovereign risk. Changes in average levels of government expenditures or revenues, for example, shift the unconditional fiscal limit, while modifications in the degree of countercyclicality of fiscal policies change the shape of the distribution. A country whose transfers programs are expected to grow as a share of GDP—as in many aging advanced economies can have a fiscal limit distribution with a fat lower tail, implying relatively high risk of default even at moderate levels of debt. As the prospects rise that credible reforms will stabilize expenditures on transfers, the fiscal limit transforms to permit higher debt levels to be essentially risk-free.

We do not model default as a strategic decision taken by an optimizing sovereign. Instead, we appeal to political frictions that make default decisions intrinsically uncertain. The distribution of the fiscal limit reports the probability that a particular debt level can be supported by taxing income at the peak of the Laffer curve, given the current state and the exogenous processes. Default is possible at any point on the distribution: in the upper tail, default requires a run of bad shocks to the economy that raise debt, while in the lower tail, default can occur even after a sequence of good shocks when debt is low. The model is consistent with the empirical fact that defaults occur at both high and low levels of debt. Randomness inherent in the politically-determined default decision is modeled as a random draw of the "effective fiscal limit" from the distribution of the fiscal limit. Agents base their expectations of default on this entirely model-determined distribution. Default occurs according to some specified rule when the current level of debt exceeds the effective limit; otherwise, debt obligations are fully honored.

Our framework models the causes of sustained increases in government debt and assesses the efficacy of alternative short-run austerity measures and long-run fiscal reforms. The framework also avoids the logical corner into which strategic default paints us: if defaults are unavoidable consequences of external disturbances, then a welfare-maximizing government cannot improve on its policy making. How can we square this theoretical logic with the observation that highly-indebted governments consistently seek—or claim to seek—fiscal reforms designed to reduce sovereign risk?

Sovereign bond prices in the model embed expected default risk. That risk depends on the current state of government debt relative to the fiscal limit distribution. Risk premia reflect both the probability of default and the expected default rate—the haircut bondholders anticipate should default occur. The framework can generate a wide range of risk premia for a given level of debt, including the very large interest-rate spreads that Greece now faces at a debt-GDP ratio of 150 percent. Sensitivity of risk premia to debt levels can be quite inelastic or extremely elastic, depending on the state of the economy and other features of the economic environment, imparting natural time variation to the premia.

The paper begins with a selective review of the recent literature on sovereign risk, including setups in which default is an optimal policy choice and models in which it is the only policy consistent with equilibrium. Missing from this existing work is detailed modeling of current and future fiscal policy regimes—how tax and spending regimes might change to reduce default risk and avoid default. By eliminating the possibility of a shift to sustainable fiscal policies, existing work cannot capture the fiscal expectations that are critical to the forward-looking nature of the pricing of sovereign bonds. It also cannot model the impacts of the fiscal reforms that highly-indebted countries are actually undertaking.

Our framework builds on Bi (2011), a closed-economy RBC model with fixed capital and a proportional tax levied against labor income. For given government purchase and transfers processes, an economic natural fiscal limit arises from the peak of the dynamic Laffer curve. Because that peak depends on the state of the economy, the joint distribution of the fundamental disturbances and private agents' optimal decision rules induce a distribution for the maximum sustainable government debt. Maximum debt equals the expected discounted present value of maximum primary surpluses, where the stochastic discount factors and tax revenues come from private behavior when tax rates maximize revenues. Of course, in a stochastic environment, "maximum debt" is a *distribution*, not a point.

Government transfers in the model may be stationary or grow as a share of GDP, depending on the prevailing regime. Growing transfers stand-in for either the old-age benefits that governments in advanced economies have promised their citizens or for the rapid growth in the government's share of the economy, as has occurred in some countries. Stationary transfers represent an entitlements reform regime. Transition probabilities between the two regimes reflect the likelihood and credibility of transfers reforms. Non-stationary transfers have several effects. First, they provide a rationale for high and rising debt and tax rates, which push the economy closer to its fiscal limit. Second, the distribution of the fiscal limit, conditional on residing in the non-stationary transfers regime, has a fat tail, so the probability of default rises with debt at debt levels observed in, say, Greece. Finally, even if the prevailing transfers regime is stationary, if the reforms are imperfectly credible, the probability of returning to the non-stationary regime shifts the fiscal limit to make debt risky at lower debt-output ratios.

Calibrating the model to Greek data, we examine a variety of policy scenarios motivated by fiscal developments in Greece. Those developments include an increase in transfers as a share of GDP since 1970 of about 12 percentage points, along with a seven-fold increase in the government debt-GDP ratio. On the heels of steady growth in transfers, bad technology realizations can generate large risk premia. Credible shifts to a regime that stabilizes transfers, however, can insure against risk premia even when fundamentals are poor. An identical shift that is less-than credible does little to bring down debt-service costs. Each of these experiments exploits the conditional fiscal limit distribution to focus on short-run matters.

Sweden in the 1990s is a case study of largely credible long-run fiscal reforms that dramatically shifted its unconditional fiscal limit distribution and reduced the riskiness of its sovereign debt. In response to the protracted recession and banking crisis that placed its solvency in question, Sweden reformed by reducing the average levels of spending and transfers as shares of GDP, reducing the degree of countercyclicality of spending and transfers, and adopting a ceiling on the nominal level of central government expenditures. When calibrated to Swedish data, the model predicts pre-crisis risk premia arising at debt levels like those that Sweden experienced in the 1990s. After the fiscal reforms, Sweden's fiscal limit shifted out substantially, allowing debt to be risk free even at debt-GDP ratios higher than observed during Sweden's crisis.

### 2 Contacts with the Literature

To place our proposed framework in context, it is useful to selectively survey existing work.<sup>1</sup> Built on Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008), an important branch of the literature models sovereign defaults as strategic decisions made by a welfare-maximizing government in response to bad productivity shocks. Sovereign default

<sup>&</sup>lt;sup>1</sup>More complete surveys appear in Eaton and Fernandez (1995), Hatchondo, Martinez, and Sapriza (2007), Hatchondo and Martinez (2010), Stähler (2011).

risk helps standard real business cycle models reproduce key business cycle facts in emerging economies, namely volatile risk premia and consumption and coutercylical interest rates and net exports. By modeling default as an optimal response to exogenous shocks, however, the strategic default literature is largely silent about the policy behavior that got the country into a sovereign debt crisis in the first place and it is equally silent about the policy reforms that might resolve the crisis. Of course, these are the issues at the heart of current policy debates in Europe and elsewhere.<sup>2</sup> These are also the issues that our approach aims to address.

Following the approach by Bohn (1998), Ghosh, Kim, Mendoza, Ostroy, and Qureshi (2011) estimate the responses of primary fiscal surpluses to debt levels for 23 advanced economies and argue that the responses are weaker at higher levels of debt, a phenomenon the authors dub "fiscal fatigue."<sup>3</sup> Imposing that the government always follows its historical surplus rule, there is a level of debt beyond which the government can no longer service its debt because at those high debt level, fatigue sets in. For each country the authors compute a debt limit that is fully determined by the risk-free interest rate, the recovery rate, and the support of the shock to primary balance. For instance, the debt limit for Ireland is 157.6 percent of GDP, if the recovery rate is 90 percent, the risk-free rate is the average of advanced economies during the period of 2003-07, and the shock support is the average of the worst five negative residuals to the Irish primary fiscal balance during 1985-2007. "Fiscal space" is defined as the difference between the long-run average debt ratio and the debt limit. The debt limits the authors estimate are single points, rather than distributions and are determined entirely by historical policy behavior.

Coming from the International Monetary Fund, the paper's methodology poses a logical puzzle. Presumably, fiscal space measures are intended to provide a basis for policy recommendations: when a country's fiscal space is limited, it is time for the government to consider policy reforms. But the fiscal space calculation *presumes* that policies cannot change. Any change in policy rules—surely what any "reform" entails—would alter the country's debt limit and make irrelevant the initial fiscal space calculations. Any approach that treats the fiscal limit as an immutable primitive of the economic environment can provide only limited policy guidance.

A paper that is in the spirit of Ghosh, Kim, Mendoza, Ostroy, and Qureshi (2011) is Juessen, Linnemann, and Schabert (2009). Those authors compute the government's debt repayment capacity using a Laffer curve argument to generate the maximum debt limit.

 $<sup>^{2}</sup>$ Cuadra and Sapriza (2008) is an exception to the silence about fiscal policy. That paper allows two types of governments, each embracing different preferences over public goods, that alternate in power to show that more polarized economies may face higher default rates.

<sup>&</sup>lt;sup>3</sup>Ghosh, Kim, Mendoza, Ostroy, and Qureshi (2011) extends Mendoza and Ostry (2008).

The authors then posit a constant tax rate, different from the revenue-maximizing one, and constant level of government expenditures—a special case of the fixed historically estimated functions that Ghosh, Kim, Mendoza, Ostroy, and Qureshi (2011) assume—to arrive at the current level of debt. Whenever current debt exceeds the debt limit, default occurs in the amount necessary for equilibrium. In this setting, the risk premium is solely determined by the stochastic level of productivity, which determines actual revenues collected at the fixed tax rate. A negative serially correlated productivity shock reduces future tax revenues and the government's capacity to service its debt. At the same time, though, lower productivity and tax revenues require the government to borrow more today.

We seek to model how a country's fiscal limit *distribution* varies systematically with the economic environment, including the specification of policy behavior. To do that, we turn to describe the framework that we propose.

### 3 Our Approach

Following Bi (2011), we lay out a closed economy model in which the fiscal limit, a measurement of the government's ability to service its debt, arises endogenously from dynamic Laffer curves.

3.1 MODEL With linear production technology, output is determined by productivity  $(A_t)$ and labor supply  $(1-L_t)$ . Household consumption  $(c_t)$  and government purchases  $(g_t)$  satisfy the aggregate resource constraint

$$c_t + g_t = A_t (1 - L_t) \tag{1}$$

where the level of productivity follows an AR(1) process with A the steady-state level of technology

$$\ln \frac{A_t}{A} = \rho^A \ln \frac{A_{t-1}}{A} + \varepsilon_t^A \qquad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2).$$
(2)

The government finances lump-sum transfers to households  $(z_t)$  and exogenous unproductive purchases by collecting tax revenue and issuing one-period bonds  $(b_t)$ . Government purchases obey

$$\ln \frac{g_t}{g} = \alpha^g \ln \frac{A_t}{A} + \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g \qquad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2) \tag{3}$$

where g represents steady-state purchases and  $\alpha^{g}$  determines the cyclicality of purchases. Government transfers to households have risen as a share of output in many developed economies since 1970. We allow transfers to follow one of two regimes: in one regime transfers are stationary and in the other they grow exponentially. The transfers regime is indexed by  $rs_t^z$ :

$$z_t \equiv z(rs_t^z, A_t) = \begin{cases} z\left(\frac{A_t}{A}\right)^{\alpha^z} & \text{if } rs_t^z = 1\\ \mu^z z_{t-1} + z\left(\left(\frac{A_t}{A}\right)^{\alpha^z} - 1\right) & \text{if } rs_t^z = 2 \end{cases}$$

with  $\mu^z > 1$  and  $\alpha^z$  measures the cyclicality.  $rs_t^z$  evolves according to the transition matrix

$$P^{z} \equiv \begin{pmatrix} p_{1}^{z} & 1 - p_{1}^{z} \\ 1 - p_{2}^{z} & p_{2}^{z} \end{pmatrix}$$
(4)

The government follows a simple tax rule with tax adjustment parameter  $\gamma$ 

$$\tau_t - \tau = \gamma \left( b_t^d - b \right), \qquad \gamma > 0 \tag{5}$$

This rule captures the observation that fiscal authorities tend to increase tax rates when government debt rises. With lump-sum taxes, any  $\gamma > 0$  guarantees an equilibrium exists. Distorting labor taxes, though, are subject to a Laffer curve that, in this economy with no long-run growth, imposes an upper bound on tax revenues. When transfers can grow explosively, the feedback rule in (5) is not sufficient to ensure that government debt is default-free, as section 3.3 explains.

The default scheme at each period depends on the effective fiscal limit  $(b_t^*)$ , which follows a time-varying distribution  $(\mathcal{B}_t^*)$ . The distribution arises endogenously from the distorting taxes, as section 3.3 details. If the government's obligations at the beginning of period t are less than the effective fiscal limit, then it repays its debt in full and no default occurs; otherwise, the government partially defaults and the stochastic default rate follows the distribution  $\Omega$ . This rule determines the default rate  $\Delta_t$ 

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta_t & \text{if } b_{t-1} \ge b_t^* \end{cases}$$

where  $b_t^* \sim \mathcal{B}_t^*$  and  $\delta_t \sim \Omega$ . The distribution  $\Omega$  is derived from empirical evidence on sovereign debt defaults and restructures observed in emerging market economies from 1983 to 2005. Moody's (2008) reports the total amounts of defaulted debt during rated sovereign bond defaults since 1983, Panizza (2008) provides a thorough dataset on public debt in developing countries, and Sturzenegger and Zettelmeyer (2008) estimate the haircuts in sovereign debt restructures. Based on these three sources, we compute the default rate, defined as the share of actual defaulted debt over total public debt. The stochastic default rate falls between 0 and 0.1 with 70 percent probability, between 0 and 0.3 with 90 percent probability, and between 0 and 0.5 with 100 percent probability. [see Bi (2011) for more details].

Let  $q_t$  be the price of a sovereign bond in units of consumption goods at time t. For each unit of bonds, the government promises to pay the household one unit of consumption in the next period. This bond contract is not enforceable: at time t, the government may partially default on its outstanding liabilities  $(b_{t-1})$  by the fraction  $\Delta_t$ , with post-default government liabilities denoted as  $b_t^d$ . These considerations yield the government's flow budget constraint

$$\tau_t A_t (1 - L_t) + b_t q_t = \underbrace{(1 - \Delta_t) b_{t-1}}_{b_t^d} + g_t + z_t \tag{6}$$

3.2 HOUSEHOLDS With access to the sovereign bond market, a representative household chooses consumption, leisure, and bond purchases to solve

$$\max \qquad E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right) \tag{7}$$

s.t. 
$$A_t(1-\tau_t)(1-L_t) + z_t - c_t = b_t q_t - (1-\Delta_t)b_{t-1}$$
 (8)

taking prices  $(q_t)$  and policies  $(\tau_t, z_t, \Delta_t)$  as given.  $E_t$  is the mathematical expectation conditional on the information available at time t, including sovereign default information. The utility function is strictly increasing and strictly concave in consumption and leisure.  $\beta \in (0, 1)$  is the discount factor.

The household's first-order conditions are

$$\frac{u_L(t)}{u_c(t)} = A_t(1-\tau_t) \tag{9}$$

$$q_t = \beta E_t \Big[ (1 - \Delta_{t+1}) \frac{u_c(t+1)}{u_c(t)} \Big]$$
(10)

requiring that the marginal rate of substitution between consumption and leisure equals the after-tax wage. Bond prices in equation (10) reflect the household's expectation about the probability and magnitude of sovereign default in the next period. The optimal solution to the household's maximization problem also implies the transversality condition

$$\lim_{j \to \infty} E_t \beta^{j+1} \frac{u_c(t+j+1)}{u_c(t)} (1 - \Delta_{t+j+1}) b_{t+j} = 0$$
(11)

As the results reveal, even this very simple model generates non-linearities that play a critical role in pricing sovereign debt. The full non-linear model is solved by discretizing the

state space and iterating on decision rules. Details appear in appendix A.

3.3 DISTRIBUTION OF FISCAL LIMIT Distorting taxes have important implications for how much revenue the government can collect. Consider an increase in the tax on labor income. If the household's work effort remained unchanged, the tax base would also remain fixed and tax revenues would rise unambiguously. A higher income tax, however, reduces the after-tax return to working and induces households to work less. The resulting impact on revenue collections is ambiguous, but generally at low tax rates, higher tax rates raise revenues, while at high tax rates, tax hikes can actually reduce revenues. This phenomenon is the basis of the "Laffer curve." In this model, the Laffer curve is dynamic, as the shape of the Laffer curve depends on the state of the economy. For given levels of productivity and government purchases  $(A_t, g_t)$ , a tax rate exists such that higher rates do not raise more revenue. At this peak of the Laffer curve, denoted by  $\tau^{max}(A_t, g_t)$ , government collects the maximum level of tax revenue for the given state, denoted by  $T^{max}(A_t, g_t)$ .

The government's ability to service its debt also depends on the size of government purchases and lump-sum transfers, which are political decisions that grow out of conflicts and compromises among parties with different ideologies [Persson and Svensson (1989) and Alesina and Tabellini (1990)]. To avoid developing a structural political economy model, we specify the processes for government purchases and transfers to capture the trends and fluctuations of government expenditures observed in the data.

We define the maximum level of debt that the government is able to pay back, or the *fiscal limit*, as the sum of discounted expected maximum primary surpluses in all future periods. The dynamic and stochastic nature of the Laffer curve and shock processes imply that the fiscal limit is stochastic with a probability distribution that depends on all the features of the economy, including private sector elasticities, the nature of policy behavior, and the properties of the random disturbances in the economy.

3.3.1 CONDITIONAL DISTRIBUTION We first consider the conditional, or state-dependent, distribution of fiscal limits, defined as

$$\mathcal{B}^{*}(A_{t}, g_{t}, rs_{t}^{z}) \sim \sum_{j=0}^{\infty} \beta^{j} \frac{u_{c}^{max}(A_{t+j}, g_{t+j})}{u_{c}^{max}(A_{t}, g_{t})} \left( T^{max}(A_{t+j}, g_{t+j}) - g_{t+j} - z(rs_{t+j}^{z}, A_{t+j}) \right)$$
(12)

 $u_c^{max}$  is the marginal utility of consumption when the tax rate is at the peak of the Laffer curve  $(\tau^{max})$ . Given the parameters of the model and specifications of the shock processes, the unique mapping between the peak of dynamic Laffer curve and the exogenous state of the economy determines the state-dependent distribution of the fiscal limit. The *conditional* 

distribution implies that households' expectations about the government's ability to pay back its debt hinge on the current state of the economy—for instance, whether current transfers are stationary or explosive—and whether the current productivity level is high or low.

3.3.2 UNCONDITIONAL DISTRIBUTION In the long run, the state of the economy today plays a less significant role in determining the government's ability to service its debt. The unconditional distribution  $\mathcal{B}^*$  is no longer time-varying, being given by

$$\mathcal{B}^* \sim E\left(\sum_{h=1}^{\infty} \beta^h \frac{u_c^{\max}(t+h)}{u_c^{\max}(t)} (T_{t+h}^{\max} - g_{t+h} - z_{t+h})\right)$$
(13)

3.3.3 DISCUSSION In linearized models, where transfers are stationary, positive feedback from government debt to taxes, like the tax rule specified in equation (5), can keep sovereign debt from exploding unless the tax adjustment parameter is too small [Leeper (1991), Bohn (1998)]. This is not guaranteed, however, if the tax rate is approaching the peak of Laffer curve or if transfers follow the Markov regime-switching process specified in section 3.1. Trabandt and Uhlig (2011) use a neoclassical growth model to show that Denmark and Sweden are already on the "slippery side" of their curves, where lower tax rates will raise revenues. Even if the average tax rate is still far from the peak of Laffer curve, which is arguably more relevant for most countries, rising transfers raise the debt level and, under the specified tax rule, also the tax rate. Under regime-switching transfers, there can be prolonged periods during which rising transfers steadily raise government debt. Forwardlooking agents may still be willing to purchase sovereign debt if they expect the explosive regime to end. If transfers stay in that regime for too long, however, debt may rise to such a level that the government will be unable to repay its debt in full, even if it consistently follows a tax rule designed to stabilize debt. A positive probability of eventually hitting the peak of Laffer curve in the future can spur sovereign default fears today even if the current tax rate is well below the peak of Laffer curve.

We assume the effective fiscal limit  $(b_t^*)$  is a draw from the fiscal limit distribution. As shown in the numerical analysis below, the distribution can be quite dispersed, especially when transfers currently reside in the explosive regime. The distribution reports the probability that a particular debt level can be supported by taxing income at the peak of the Laffer curves, given the stochastic processes for transfers, government purchases and productivity. At any point on the distribution, default is possible, but in the upper tail it would require a run of bad shocks to the economy, while in the lower tail default becomes possible even with a run of good shocks. If a debt level of  $b^{**}$  is associated with a probability of  $p^{**}$  in the distribution, then it implies that with the probability  $p^{**}$  a run of bad shocks may occur that makes the debt level  $b^{**}$  unsustainable.

Full details of how the fiscal limit distributions are computed appear in appendix B.

## 4 DEBT CRISIS IN GREECE

4.1 TIMELINE From 2001–2008, benign neglect describes financial market attitudes toward Greece's fiscal situation. The view seemed to be that the discipline instilled by membership in the euro area would force the Greek government to conform to the euro zone's fiscal standards. Throughout this period of robust economic growth, Greek fiscal policy was strongly procyclical, generating persistent deficits that maintained debt at about 100 percent of GDP. The series of U.S.-based financial events—the subprime mortgage crisis and the failures of Bear Stearns and Lehman Brothers—induced investors to more carefully assess the riskiness of Greek sovereign debt and drove Greek-German interest rate spreads to a couple of hundred basis points.

The impact on Greek rates of these global shocks may have been contained had it not been for the "data revisions" the Greek Ministry of Finance began to announce in 2009. The 2009 budget deficit, initially forecasted to be 2 percent of GDP, was revised upward to 3.7 percent in January, to 5.1 percent in a mid-year review, and 12.7 percent by late November. Eurostat eventually announced a final value of 15.8 percent of GDP. This fiscal news alerted markets to the true state of fiscal policy in Greece and triggered a steady rise in risk premia throughout 2010. Figure 1 reports daily spreads between 10-year yields on Greek sovereign bonds and German Bunds.

In May 2010, Greece's socialist PASOK-led government narrowly approved sweeping fiscal changes that cut public sector wages, reformed pensions, and raised taxes. These austerity measures were part of a bailout agreement between Greece and the troika (the IMF, the EC, and the ECB). Because the changes did not grow out of a clear political consensus on the need for fiscal consolidation, they triggered violent public protests and wide-spread criticism that raised doubts about the ruling government's ability to complete its term, which ended in 2013, much less see the reforms through. Greek risk premia confirm these doubts, as they continued their relentless rise through 2010, reaching nearly 10 percentage points by year-end.

Since that first troika agreement, each quarter Greece missed its fiscal targets and the Greek government has announced additional austerity actions. None of these actions tempered the rise in risk premia. An October 2011 EU summit sought to calm financial markets by reasserting member countries' commitments to sustainable fiscal policies. Summit leaders called on Private Sector Involvement (PSI) to help Greece reach a 120 percent debt-GDP

ratio. Under PSI, private investors would agree to a 50 percent haircut on Greek bond holdings, while euro zone countries would provide 30 billion euro to the PSI package and contribute to recapitalizing Greek banks [European Union (2011)]. Within a month of the summit, premia had risen more than 500 basis points.

Figure 1 makes clear that risk premia behaved quite different beginning in the second half of 2011. Over the 18 months from September 2009 to June 2011, the premium rose 1164 basis points—65 basis points per month on average. But in just the six months from July through December 2011, the increase was 1655 basis points, averaging 276 basis points per month. Very little news about the *current* state of Greek finances arrived in the latter period, but plenty of news arrived about the stability of the Greek government and prospective *future* fiscal states.

Following Prime Minister Papandreou's resignation in early November 2011, Lucas Papademos, former governor of the Bank of Greece and Vice President of the ECB, was named prime minister of a caretaking coalition government. On February 13, 2012 the new government approved the terms of the second troika bailout. Conditions included a 22 percent cut in the minimum wage, large reductions in public employment, and substantial cuts in pension, health, and defense spending. Among the concessions the agreement achieved were: an increase to 53.5 percent for the haircut taken by private bond holders, a schedule for coupon payments on Greek bonds through 2042, a reduction of interest rates of the Greek Loan Facility, and a promise by national central bank holders of Greek bonds to pass earnings from those bonds back to Greece [European Union (2012)]. So far in 2012, the risk premium has stabilized at about 30 percentage points.

4.2 MODEL CALIBRATION We calibrate the model to annual Greek data from 1971– 2007 to illustrate uses of the conditional (or state-dependent) fiscal limit.<sup>4</sup> Steady state government purchases are 16.7 percent of GDP, lump-sum transfers are 13.34 percent of GDP, and the tax rate—based on average revenues—is 0.32. These produce a steady state government debt-GDP ratio of 0.40. The estimated elasticity of detrended real transfers with respect to detrended real GDP per worker ( $\alpha^z$ ) is -0.45.<sup>5</sup> After applying a Hodrick and Prescott (1997) filter, the productivity shock has persistence of 0.45 and a standard deviation of 0.0328, while the government purchase shock has persistence of 0.426 and a standard deviation of 0.0294. The parameter  $\gamma$  measures the tax response to debt increase; we calibrate it to 0.32, which is the lowest bound that ensures existence of a unique equilibrium if the transfers were always stationary and default was not an option. Transfers follow a

 $<sup>^4\</sup>mathrm{Appendix}$  C describes the data sources.

<sup>&</sup>lt;sup>5</sup>We set the elasticity of government spending to real GDP per worker ( $\alpha^g$ ) to be zero, as there is no clear pattern over time.

Markov regime-switching process. In the explosive regime, transfers growth  $(\mu^z)$  is 1.015 to match the observation that transfers from the Greek government to the private sector rose by 10 percentage points of GDP over the 37 years. In the baseline case, regime persistence, parameterized by  $p_1^z$  and  $p_2^z$ , is set at 0.975 so that the regimes are symmetric with expected duration of 40 years. Table 1 summarizes the parameter settings.

Parameter	Value
Discount rate $(\beta)$	0.95
Steady state leisure $(L)$	0.75
Persistence of productivity $(\rho^A)$	0.45
Standard deviation of productivity $(\sigma_A)$	0.0328
Persistence of government purchases $(\rho^g)$	0.426
Standard deviation of government purchases $(\sigma_q)$	0.0294
Response of taxes to debt $(\gamma)$	0.32
Spending-GDP ratio $(g/y)$	0.167
Transfers-GDP ratio $(z/y)$	0.134
Tax rate $(\tau)$	0.32
Elasticity of transfers to productivity $(\alpha^z)$	-0.45
Transfers growth $(\mu^z)$	1.015
Regime-switching parameters $(p_1^z/p_2^z)$	0.975/0.975

Table 1: Baseline Calibration for the Greek Economy

4.3 BASELINE CASE The top panels of figure 2 show the cumulative conditional distributions (CDFs) for fiscal limits,  $\mathcal{B}^*(A_t = A^i, g_t = g, rs_t^z = 1)$ , and the bottom panels are the corresponding risk premia for the baseline calibration. The top left panel reports the CDFs when current productivity is at different levels (i = low, ss, high), while current government purchases are at steady state and current transfers reside in the stationary regime  $(rs_t^z = 1)$ . All future states  $(A_{t+i}, g_{t+i}, rs_{t+i}^z)$  evolve according to their stochastic specifications. Since the effective fiscal limit at each period is a random draw from the conditional distribution, the CDF illustrates the default probability at each debt level (scaled by steady-state output): if the amount of debt the government issues at period t is  $b_t$ , then the CDF illustrates the probability that the government will default in the following period,  $\Phi(b_t \ge b_{t+1}^*)$ . The solid blue line is the probability when current productivity is at steady state, while the dashed red line and the dash-dotted black line are CDFs when the current productivity is 6 percent below and above steady-state. Current productivity changes contemporaneous tax revenues directly and future tax revenues indirectly depending on the shock persistence, so productivity has a significant impact on the distributions. At the debt ratio of 200 percent, for example, the default probability is 0.15 when productivity is at steady state and less than 0.1 when productivity is 6 percent above steady state; the default probability rises to 0.4 when productivity is 6 percent below steady state. Symmetric changes in productivity produce asymmetric changes in default probabilities because the possibility of explosive transfers in the future generates fat tails in the fiscal limit distribution.

As described in Appendix A, the risky interest rate on government bond  $R_t$  can be computed in terms of the current state  $\psi_t = (b_t^d, A_t, g_t, z_{t-1}, rs_t^z)$ . The risk-free rate  $R_t^f$  is computed from an identical specification, but conditional on the assumption that the the government never defaults. The risk premium,  $r_t$ , is defined

$$r_t \equiv R_t - R_t^f \tag{14}$$

$$=\frac{1}{q_t} - \frac{1}{q_t^{\Delta_t \equiv 0}} \tag{15}$$

where  $q_t^{\Delta_t \equiv 0}$  is the equilibrium bond-pricing function when government debt is perfectly safe.

Under the assumption that the stochastic default rate follows the empirical distribution  $\Omega$  that section 3.1 describes, the bottom left panel shows the risk premia the government has to pay to issue the quantity of debt  $b_t$ . Sovereign risk premia follow the fiscal limit distribution closely when the default probability is low, but can deviate quite a bit when default becomes very likely to occur. For instance, at a debt level of 225 percent of output when productivity is below steady state, the risk premium hasn't peaked yet even though the default probability is already 1. This happens because in a closed economy the stochastic discount factor,  $\beta \frac{u_c(t+1)}{u_c(t)}$ , and the post-default rate,  $1 - \Delta_{t+1}$ , are positively correlated—higher  $\Delta_{t+1}$  reduces the debt burden and tax rate next period, reducing the marginal utility of consumption next period and the discount factor. The positive correlation gets stronger when default becomes more likely, as the bond pricing expression makes clear

$$q_t = \beta E_t \frac{u_c(t+1)}{u_c(t)} E_t(1 - \Delta_{t+1}) + cov_t \left(\beta \frac{u_c(t+1)}{u_c(t)}, (1 - \Delta_{t+1})\right)$$
(16)

The top middle panel of figure 2 reports the conditional distributions when current transfers are either in the stationary or the explosive regime, while current productivity and government spending are at steady-state. All else equal, the default probability can be significantly higher in the explosive regime: when the debt level is 150 percent of output, default occurs with 20 percent probability if the current transfers grow exponentially, but close to zero if transfers are stationary. Importantly, the distribution has a fat tail even when current transfers are stationary—the possibility that future transfers may switch to the explosive regime implies that future fiscal surpluses could be significantly lower, constraining the government's ability to service its debt today even if transfers are stationary today.

The right panels of the figure compare the conditional distributions and risk premia at different levels of government purchases while holding current productivity at steady state and current transfers in the stationary regime. Different levels of government purchases have limited impact on the fiscal limit distributions and even less effect on risk premia, as the dashed, solid and dash-dotted lines are very close. This contrasts sharply to the cases with different transfers regimes or with different productivity levels. Contrasts arise for several reasons. First, regime switches in transfers change households' expectations about fiscal policy in the long run, while shocks to government purchases are short-lived. Second, government purchases have two opposing effects on government debt: higher purchases directly increase government liabilities; higher purchases also reduce wealth, crowd out private consumption and induce households to work more, so higher output raises tax revenues to help finance government purchases.

The top panels in figure 3, which are the same as the bottom left and middle panels in figure 2, illustrate how much the government has to compensate investors in order to sell  $b_t$  units of bonds, for the given state of economy. The bottom panels in figure 3, on the other hand, address a slightly different question: given that the government has already accumulated an amount of debt at  $b_{t-1}$ , what is the risk premium the government must offer to continue selling bonds in the market? This distinction matters little in good states of the economy: when productivity is above steady state, the dash-dotted black line in the bottom left panel is very similar to its analog in the top left panel. But when productivity is low, tax revenues fall, and the government has to borrow substantially more to rollover its debt,  $b_t \gg b_{t-1}$ , raising the default probability. The risk premium curve, therefore, is much steeper in the bottom panel than in the top. As discussed below, this compounding mechanism plays a more important role when the default rate is higher.

4.4 ALTERNATIVE CALIBRATIONS In the baseline case, the stochastic default rate follows the empirical distribution  $\Omega$ , which has a mean of 0.1. Historically, much higher default rates have occurred and negotiations underway now imply a far higher default rate on Greek bonds, at least for those privately held. For comparison, we also consider alternative scenarios with a fixed default rate  $\delta_t = \delta$ : whether or not the government defaults depends on the existing debt level and the effective fiscal limit, but once default occurs, a fixed fraction of debt is written off.

The top panels of figure 4 are for the baseline case, while the second, third and bottom panels illustrate alternative calibrations in which the default rate is fixed at 10 percent, 20 percent, and 30 percent. The dashed green lines in all panels are the same, plotting the state-dependent fiscal limit distribution when current transfers are explosive, productivity is 6 percent below steady state, and government spending is at steady state. The solid blue lines in the left panels report the risk premia the government must pay to borrow  $b_t$  at end of period t. Risk premia curves flatten after the default probabilities reach 1 because of the positive correlation between the stochastic discount rate and the post-default rate.

The right panels of the figure depict the risk premia the government must when existing debt is  $b_{t-1}$ . Risk premium curves steepen with respect to the existing debt level, as the default rate rises from 10 percent to 30 percent. For instance, if existing debt is 150 percent of output, the government only needs to pay a premium of 5 percentage points when the default rate is 10 percent, but 7 percentage points when the default rate is 20 percent, and 20 percentage points when the default rate is 30 percent. This nonlinear relationship between the default rate and risk premia arises from the compounding effect discussed in section 4.3.

Figure 5 plots the corresponding decision rules for end-of-period debt  $b_t$  as functions of the existing debt level  $b_{t-1}$  under different specifications of default rate. This figure illustrates that the compounding effect can be quantitatively significant. A higher default rate can significantly raise borrowing costs, forcing the government to issue still more debt When the default rate is 0.1, end-of-period debt increases at a constant rate with respect to existing debt. If the default rate is 0.2, the slope of the debt rule becomes steeper when existing debt is between 150 and 200 percent of output, implying the government has to issue more debt to rollover the same existing debt. Once the default rate becomes 0.3, the slope becomes so steep that the end-of period debt has to increase by 35 percent of GDP to finance a 10 percentage point increase in existing debt, when debt is in 150 to 170 percent range.

These results come from the baseline calibrations that treats time averages of Greek fiscal variables between 1971 and 2007 as steady states of the model. In light of the variables' upward trends over the sample, it is interesting to recalibrate the fiscal variables to the sub-period 1987 to 2007 to reflect more recent developments in Greek fiscal policy. The key change is an increase in the share of transfers from 13.34 percent to 15.6 percent of GDP. We also revise the transfer regime probabilities,  $p_1^z$  and  $p_2^z$ , to 0.95 so that the expected duration for each regime is 20 years. Figure 6 compares the state-dependent distributions for fiscal limits under the baseline and the alternative calibrations, assuming current spending is at steady state and current productivity is 6 percent below steady state. If the current transfers reside in the stationary regime (top panel), the higher steady state transfers shift the fiscal limit distribution by 50 percent of GDP. Under the new calibration the default probability rises from 0.05 to 0.25 when debt is 150 percent of output. This dramatic increase in the probability of default translates immediately into uniformly higher risk premia. The fiscal limit is also reduced if the current transfers reside in the explosive regime, but to a less

extent because the explosive regime is less persistent under the alternative calibration.

4.5 POLICY EXPERIMENTS Greece has experienced persistent transfers growth during the past three decades which, in combination with rampant tax evasion, has led to soaring government debt. Mounting pressure from financial markets has forced the Greek government to adopt a variety of fiscal austerity measures. On February 10, 2012, the Greek cabinet approved a new austerity plan, which is estimated to improve the 2012 budget deficit by 3.3 billion euro. It remains an open question whether the fiscal austerity measures are credible.

We consider two extreme scenarios against the baseline case—a less-credible versus a more-credible reform. All else equal, the smaller is the regime-switching probability  $p_1^z$ , the more likely transfers will switch from the stationary regime to the explosive regime, and the less credible is the fiscal reform. We consider an incredible reform with  $p_1^z = 0.75$ ,  $p_2^z =$ 0.975—even if transfers are stationary today, with 25 percent probability the government will renege on the fiscal reform and revert to the explosive regime next period. This yields an expected duration for the stationary regime of only four years. We contrast this to a credible reform with  $p_1^z = 0.995$ ,  $p_2^z = 0.975$ —once the transfers are stationary, the probability of leaving that regime is only 0.5 percent, giving the reform an expected duration of 200 years.

Figure 7 compares the fiscal limit distributions for these two reform scenarios to the baseline calibration with an expected duration of 40 years. The top panel illustrates the comparison when current transfers are stationary while both current productivity and government purchases are at steady state. For a stark contrast, the dotted black line is the fiscal limit when transfers are always zero. The solid blue line is the baseline case, the dashed red line shows the incredible reform, and the dash-dotted green line illustrates the credible reform. Everything is identical across the four scenarios except the expectation of future transfers. The area under the dashed red line and above the solid blue line is the loss of fiscal limit space due to the incredible reform, which equals the expected present value of future transfers increases due to a higher probability of switching to the explosive regime. Similarly, the area between solid blue line and the dash-dotted green line is the gain in fiscal limit space due to the credible reform. The bottom panel repeats the same comparisons except the current transfers begin in the explosive regime.

Figure 7 makes clear that if fiscal reform is credible, the current transfers regime matters a great deal in determining the default probability, as the dash-dotted green line is much less dispersed in the top panel than in the bottom. On the other hand, if fiscal reform isn't credible, being able to contain the transfer growth temporarily does little to constrain the default probability and risk premia, as shown by the dashed red lines in both panels. Speculation that the general election in Greece in 2013 may overturn many fiscal austerity measures suggests that markets may not be confident in the credibility of Greek fiscal reforms.

An alternative way to model the fiscal reforms is through changes in the persistence of explosive regime. The higher the parameter  $p_2^z$ , the more likely transfers will stay in the explosive regime, and the less credible is the fiscal reform. We consider an incredible reform with  $p_1^z = 0.975$ ,  $p_2^z = 0.995$ , which yields an expected duration for the explosive regime of 200 years, and also a credible reform with  $p_1^z = 0.975$ ,  $p_2^z = 0.75$ , giving the explosive-transfer regime an expected duration of only 4 years. Figure 8 compares the fiscal limit distributions for these two reform scenarios to the baseline calibration. An incredible fiscal reform reduces fiscal space, captured by the area between the dashed red line and the solid blue line. More interesting, if the government can commit to reducing the duration of the explosive regime, such a fiscal reform can raise fiscal space regardless of whether current transfers are stationary or explosive, because the explosive regime is expected to be short-lived. In contrast, the credible reform in figure 7, taken to mean a more persistent stationary regime, but has limited impact if the current transfers still reside in the explosive regime.

# 5 Swedish Fiscal Reform in 1990s

Large and seemingly permanent changes in fiscal behavior in Sweden following the recession and banking crisis in the early 1990s illustrate uses of the unconditional distribution of fiscal limits.

5.1 TIMELINE In the early 1990s, Sweden experienced a boom-bust cycle that severely tested the prevailing policy regime.<sup>6</sup> After deregulating the financial system, the economy boomed in the late 1980s, with rapid growth in GDP and employment. By 1989–1990 the boom had ended and the bust began. The resulting recession was comparable to Sweden's experience in the Great Depression, with GDP falling for three consecutive years and unemployment rising from 1.5 percent in 1989 to over 8 percent in 1993. Large automatic stabilizers built into Swedish fiscal rules swung the general government balance from a 5 percent surplus in 1989 to nearly a 12 percent deficit in 1993. The Swedish government responded with a thorough policy reform.

The fiscal framework that was introduced in 1993 consists of three components covering both central and local governments. First, a ceiling on total expenditures, excluding interest payments, was introduced at the central government level. Sweden's Ministry of Finance prepares the budget and presents it to Riksdag, which votes on the expenditure ceiling and

<sup>&</sup>lt;sup>6</sup>This section draws liberally from Swedish Ministry of Finance (2001), Jonung and Hagberg (2005), Jonung (2009), Reinhart and Rogoff (2008), and Wetterberg (2009).

how to divide the budget into 27 expenditure areas. Second, a budget surplus target of 1 percent of GDP over the business cycle was adopted at the general government level to ensure that Sweden's aging population will not cause public finances to deteriorate. Third, a balanced budget at the local government level was introduced in 2000.

Under this fiscal framework, the Swedish government was able to reduce public expenditures from 60 percent of GDP in 1993 to 45 percent of GDP in 2007 by cutting social benefits, public subsidies, capital expenditures and public consumption. The successful fiscal reform has earned applause from sovereign debt rating agencies. After the 1993 downgrade of Swedish debt, Standard & Poor's (1997) revised its long-term foreign currency rating outlook for Sweden from negative to stable, largely due to "expected fiscal strengthening" arising from the reforms. In the context of the 2007-2009 economic downturn, Standard & Poor's (2009) wrote, "the established fiscal rules have served Sweden well" and, "the Kingdom's substantial fiscal buffers to support its creditworthiness in the current adverse economic environment." Despite the decline in fiscal performance as a result of rising government spending and declining tax revenue, rating agencies believe that the deterioration in public finances will be temporary as the Swedish government has a solid history of fiscal discipline and credible rules in place.

Figure 9 suggests that a shift in the level of transfers and government spending occurred between 1992 and 1997. Sweden's financial crisis started in 1992, while the expenditure ceiling on central government spending was introduced in 1997. Claeys (2008) identifies the breakpoint for government spending as the third quarter of 1995 and for transfers as the second quarter of 1996. We set the breakpoint to be 1997 in order to highlight the comparison before and after the fiscal reform, but different breakpoints do not affect our results qualitatively.

The degree of countercyclical behavior of government spending and transfers, as summarized by the parameters  $\alpha^g$  and  $\alpha^z$ , is estimated using Swedish data during the period of 1980 to 2007. Table 2 shows the estimated  $\alpha^g$  and  $\alpha^z$ , the average tax rate, and the ratios of government spending and transfers to GDP in different episodes. First, there was a sharp decline in the level of transfer payments, from 22 to about 19 percent of GDP. Second, government spending shifted from being countercyclical in the early period ( $\alpha^g < 0$ ) to being procyclical in the latter period ( $\alpha^g > 0$ ), which may be a consequence of the 1997 expenditure ceiling policy.

5.2 PARAMETER CALIBRATION Table 3 summarizes the calibration of the parameters. The calibrations for discount rate and the labor supply elasticity are the same as those in the Greek experiment. Using a Hodrick and Prescott (1997) filter, the productivity shock

	1980 - 2007	1980 - 1997	1997 - 2007
Response of spending to productivity $(\alpha^g)$	-0.142	-0.183	0.196
Response of transfers to productivity $(\alpha^z)$	-1.65	-1.70	-1.066
Average tax rate $(\tau)$	49.7	49.6	49.9
Spending-GDP ratio $(g/y)$	27.3	27.6	26.7
Transfers-GDP ratio $(z/y)$	21	22	19.1

BI & LEEPER: ANALYZING FISCAL SUSTAINABILITY

Table 2: Swedish Fiscal Data (1980–2007).

has a persistence of 0.661 and a standard deviation of 0.015. The degree of countercyclical government spending and lump-sum transfers,  $\alpha^g$  and  $\alpha^z$ , the transfers-GDP ratio, z/y, and the government spending-GDP ratio are calibrated to pre-crisis (1980–1997) and post-crisis (1997–2007) data.<sup>7</sup> The steady-state tax rate,  $\tau$ , is calibrated to the average level of tax ratio in the data.

Parameter	Value	
Discount rate $(\beta)$	0.95	
Steady state leisure $(L)$	0.75	
Persistence of productivity $(\rho^A)$	0.661	
Standard deviation of productivity $(\sigma_A)$	0.015	
Average tax rate $(\tau)$	0.5	
	Pre-Crisis	Post-Crisis
Response of spending to productivity $(\alpha^g)$	-0.183	0.196
Response of transfers to productivity $(\alpha^z)$	-1.70	-1.066
Spending-GDP ratio $(g/y)$	0.276	0.267
Transfers-GDP ratio $(z/y)$	0.22	0.19

Table 3: Calibration for the Swedish Economy

5.3 POLICY EXPERIMENTS We treat as the baseline the calibration from the pre-crisis period when Swedish sovereign bonds were downgraded by the rating agencies, government spending and transfers are countercyclical, and the average tax rate and the share of transfers are relatively high. We simulate the distribution of the fiscal limit for this baseline scenario and then contrast it to the distributions obtained under three alternative calibrations that are designed to capture the post-crisis fiscal reforms.

<sup>&</sup>lt;sup>7</sup>As illustrated above, the AR(1) process for government spending has very limited impact on the fiscal limit distribution and is, therefore, dropped in the Sweden calibration where we discuss the long-term unconditional distribution.

Table 4 summarizes the policy settings in the baseline model and in the three alternatives. The first alternative scenario, labeled "post-crisis", is a counter-factual exercise that asks what the fiscal limit would be if the government were to reduce the tax rate and the share of transfers in GDP to their post-crisis levels, but continued to follow the pre-crisis countercyclical expenditure rules.

The second and third alternative scenarios, labeled "post-crisis-procyclical" and "postcrisis-ceiling" respectively, offer two explanations for government expenditure data from 1997 to 2007. In the "post-crisis-procyclical" case, government spending policy is assumed to have shifted from countercyclical to procyclical, while the other fiscal policy parameters are calibrated to the data in the post-crisis period. In the "post-crisis-ceiling" case, on the other hand, expenditure ceilings on government spending and transfers are imposed, while all the fiscal policy parameters, including countercyclical spending and transfers policy, are calibrated to the pre-crisis levels. The ceiling rules are given by

$$\log \frac{g_t}{g} = \min\left(\alpha^g \log \frac{A_t}{A}, -\alpha^g \sigma_A\right) \tag{17}$$

$$\log \frac{z_t}{z} = \min\left(\alpha^z \log \frac{A_t}{A}, -\alpha^z \sigma_A\right)$$
(18)

where  $\sigma_A$  is one standard deviation for the technology shock. Equations (17) and (18) operate asymmetrically. When productivity is high, expenditures tend to be low and the constraints do not bind. When productivity is low, however, expenditures automatically tend to be higher than normal. If the productivity shock is sufficiently bad, the automatic expansion in expenditures may be bounded above as the ceiling binds, implying that the government can conduct countercyclical expenditure policies only within some range.

Parameter	Pre-Crisis	Post-Crisis	Post-Crisis (procyclical)	Post-Crisis (ceiling)
	Baseline	Case 1	Case 2	Case 3
Response of spending				
to productivity $(\alpha^g)$	-0.183	-0.183	0.196	-0.183
Response of transfers				
to productivity $(\alpha^z)$	-1.70	-1.70	-1.066	-1.70
Spending-GDP ratio $(g/y)$	0.276	0.267	0.267	0.267
Transfers-GDP ratio $(z/y)$	0.215	0.19	0.19	0.19
Expenditure ceiling	n.a.	n.a.	n.a.	$g_t \le g^{ceil}$ $z_t \le z^{ceil}$
				$z_t \leq z^{ceil}$

Table 4: Alternative Swedish Fiscal Policies

5.3.1 FISCAL LIMITS Figure 10 compares the distributions of the fiscal limit under the baseline and the three alternative scenarios. The top panel compares the pre-crisis and postcrisis cases. In the pre-crisis baseline calibration, the distribution, centered at a debt-output ratio of 70 percent, is quite dispersed, implying that Swedish sovereign debt holders may have had good reason to place probability on default in the early 1990s, even when the debt was at relatively modest levels. This, of course, was the time when Swedish sovereign debt was downgraded. On the other hand, fiscal reform that led to a smaller government in terms of the share of transfers in GDP and the average level of taxation shifted the fiscal limit markedly to the right, with the mean moving to 140 percent, as shown by the solid blue line labeled "post-crisis."

The dotted-dashed red line, labeled "post-crisis-pro," uses identical policy settings as "post-crisis" except that government spending switches from countercyclical to procyclical, with  $\alpha^g$  changing from -0.183 to 0.196, and transfers become somewhat less countercyclical, with  $\alpha^z$  changing from -1.70 to -1.066. Altering the cyclical nature of government expenditures has little effect on the mean of the distribution, but reduces its dispersion. Expenditure ceilings have a more subtle influence on the distribution of the fiscal limit, as the dashed green line shows. Asymmetry in expenditure rules induces asymmetry in the fiscal limit: the upper tail is substantially fatter than the lower tail, shifting risk away from moderate debt-output ratios. Taken together, the results for procyclical spending and expenditures ceiling policies provide some support for the argument that such policies can cushion the Swedish economy from risk premia on government debt.

Figure 10 has important implications for empirical work seeking to find a relationship between debt and interest rates. Nonlinearity means that over a wide range of "low" levels of debt, interest rates are quite insensitive to changes in debt. As debt levels rise, though, interest rates may move substantially with changes in debt. An empirical finding of a small correlation between interest rates and debt when debt is low cannot be extrapolated to higher levels of debt. Moreover, since the fiscal limit, and therefore the relationship between interest rates and debt, is time varying, it can be quite tricky to make accurate predictions of how rates will change with debt.

#### 6 CONCLUSION

[To be written]

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# A SOLVING THE NONLINEAR MODEL

Other than the end-of-period government debt, all other variables are either exogenous or can be computed in terms of the current state  $\psi_t = (b_{t-1}^d, A_t, g_t, z_{t-1}, rs_t^z)$ .

$$\tau_t = \tau + \gamma \left( b_t^d - b \right) \tag{19}$$

$$\ln \frac{g_t}{g} = \alpha^g \ln \frac{A_t}{A} + \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g$$
(20)

$$\ln \frac{A_t}{A} = \rho^A \ln \frac{A_{t-1}}{A} + \varepsilon_t^A \tag{21}$$

$$c_t = \frac{(A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}$$
(22)

$$\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta & \text{if } b_{t-1} \ge b_t^* \end{cases}$$

$$z_t \equiv z(rs_t^z, A_t) = \begin{cases} z\left(\frac{A_t}{A}\right)^{\alpha^z} & \text{if } rs_t^z = 1\\ \mu^z z_{t-1} + z\left(\left(\frac{A_t}{A}\right)^{\alpha^z} - 1\right) & \text{if } rs_t^z = 2 \end{cases}$$

The decision rule for government debt,  $b_t = f^b(\psi_t)$ , is solved in the following steps:

- 1. Define the grid points by discretizing the state space  $\psi_t$ . Make an initial guess of the decision rule  $f_0^b$  over the state space.
- 2. At each grid point, solve the following core equation and obtain the updated rule  $f_i^b$  using the given rule  $f_{t-1}^b$ .

$$\frac{b_t^d + z_t + g_t - \tau_t A_t n(\psi_t)}{f_i^b(\psi_t)} = \beta (1 - \Delta_{t+1}) E_t \frac{c(\psi_t)}{c(\psi_{t+1})}$$
(23)

where  $\psi_{t+1} = ([f_{i-1}^b(\psi_t), b_{t+1}^*, \delta_{t+1}], A_{t+1}, g_{t+1}, z_t, rs_{t+1}^z)$ . The integral on the right-hand side is evaluated using numerical quadrature.

$$E_{t} \frac{1 - \Delta_{t+1}}{c_{t+1}} = \int_{\varepsilon_{t+1}^{A}} \int_{\varepsilon_{t+1}^{g}} \int_{rs_{t+1}^{z}} \int_{b_{t+1}^{*}} \int_{\delta_{t+1}} \frac{1 - \Delta_{t+1}}{c_{t+1}}$$
$$= \int_{\varepsilon_{t+1}^{A}} \int_{\varepsilon_{t+1}^{g}} \int_{rs_{t+1}^{z}} \left(1 - \Phi(b_{t} \ge b_{t+1}^{*})\right) \frac{1}{c_{t+1}} |_{\text{no default}} + \int_{\varepsilon_{t+1}^{A}} \int_{\varepsilon_{t+1}^{g}} \int_{rs_{t+1}^{z}} \Phi(b_{t} \ge b_{t+1}^{*}) \int_{\delta_{t+1}} \frac{1 - \delta_{t+1}}{c_{t+1}} |_{\text{default}}$$

3. Check the convergence of the decision rule. If  $|f_i^b - f_{i-1}^b|$  is above the desired tolerance

(set to 1e-6), go back to step 2; other wise,  $f_i^b$  is the decision rule and used to evaluate the particle filter as described below.

## **B** SIMULATION PROCEDURE FOR FISCAL LIMITS

In this model, the choices of household consumption and labor supply only depend on the income tax rate and the exogenous state variables  $(A_t, g_t)$ . Assume the utility function is  $u(c, L) = \log c + \phi \log L$ . The household first-order conditions can be written as,

$$1 - L_t = \frac{A_t(1 - \tau_t) + \phi g_t}{A_t(1 + \phi - \tau_t)}$$
(24)

$$c_t = \frac{(A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}$$
(25)

The tax revenue  $(T_t)$  is,

$$T_{t} = \tau_{t} \frac{A_{t}(1-\tau_{t}) + \phi g_{t}}{1+\phi-\tau_{t}}$$
  
=  $(1+2\phi)A_{t} - \phi g_{t} - \left(A_{t}(1+\phi-\tau_{t}) + \frac{(1+\phi)\phi(A_{t}-g_{t})}{1+\phi-\tau_{t}}\right)$  (26)

The tax revenue reaches to the maximum level  $(T_t^{max})$  when the tax rate reaches the peak point of the Laffer curve  $(\tau_t^{max})$ .

$$\tau_t^{max} = 1 + \phi - \sqrt{\frac{(1+\phi)\phi(A_t - g_t)}{A_t}}$$
(27)

$$T_t^{max} = (1+2\phi)A_t - \phi g_t - 2\sqrt{(1+\phi)\phi A_t(A_t - g_t)}$$
(28)

B.1 CONDITIONAL FISCAL LIMIT Since there exists a unique mapping between the exogenous state space  $(A_t, g_t)$  to  $\tau_t^{max}$  and  $T_t^{max}$ , the conditional distribution of fiscal limit  $(\mathcal{B}^*(A_t, g_t, rs_t^z))$  can be obtained using Markov Chain Monte Carlo simulation:

1. For each simulation *i*, we randomly draw the shocks for productivity  $(A_{t+j})$ , government purchases  $(g_{t+j})$ , and the transfer regime  $(rs_{t+j}^z)$  for 200 periods conditional on the starting state  $(A_t, g_t, rs_t^z)$ . Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables using the house-hold's first-order conditions and the budget constraints, and the discounted sum of

maximum fiscal surplus, defined as

$$B_i^*(t) = \sum_{j=0}^{\infty} \beta^j \frac{u_c^{max}(A_{t+j}, g_{t+j})}{u_c^{max}(A_t, g_t)} \left( T^{max}(A_{t+j}, g_{t+j}) - g_{t+j} - z(rs_{t+j}^z, A_{t+j}) \right)$$
(29)

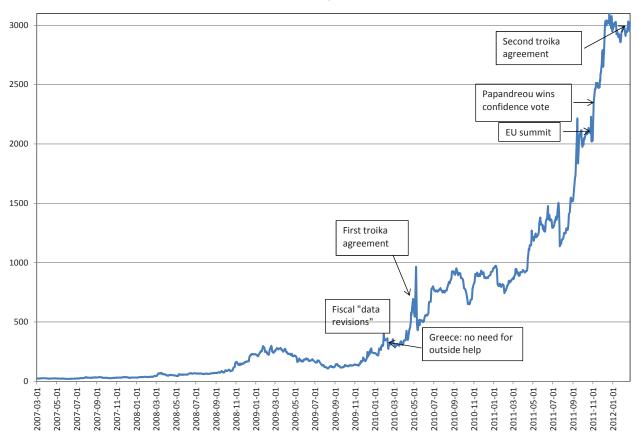
- 2. Repeat the simulation for 100,000 times and obtain the conditional distribution of  $\mathcal{B}^*(A_t, g_t, rs_t^z)$  using the simulated  $B_i^*(t)$  (i = 1, ..., 100000).
- 3. Repeat the first and second steps for all possible exogenous states  $(A_t, g_t, rs_t^z)$  within the discretized state space.

B.2 UNCONDITIONAL FISCAL LIMIT The unconditional distribution  $(\mathcal{B}^*)$  can be obtained in a similar way:

- 1. For each simulation *i*, we randomly draw the shocks for productivity  $(A_j)$ , government purchases  $(g_j)$ , and the transfer regime  $(rs_j^z)$  for 400 periods and drop the first 200 as burn-in period. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the discounted sum of maximum fiscal surplus  $B_i^*$ .
- 2. Repeat the simulation for 100,000 times and obtain the unconditional distribution of  $\mathcal{B}^*$  using the simulated  $B_i^*$  (i = 1, ..., 100000).

# C DATA

The data of government debt is from European Commission (2009), while the rest fiscal data is from the OECD Economic Outlook No. 84 (2009) for the period between 1971 and 2010. The average tax rate is defined as the ratio of the total tax revenue over the GDP, including social security, indirect and direct taxes. The government purchases are government final consumption of expenditures. Lump-sum transfers are defined as the sum of social security payments, net capital transfers and subsidies. Using a Hodrick and Prescott (1997) filter, we detrend the data of the real GDP per worker from Penn World Table Version 6.2 (see Heston, Summers, and Aten (2009)) and estimate the shock process of productivity. The elasticity of lump-sum transfers with respect to productivity ( $\alpha^z$ ) is estimated using the detrended data of real lump-sum transfers and real GDP per worker. The elasticity of government purchases with respect to productivity ( $\alpha^g$ ) is estimated using the detrended data of real government expenditures and real GDP per worker.



**Greek Interest Rate Spreads Over Bund** 

Figure 1: Greek Risk Premia: Daily Greek Sovereign Bond Yield Spreads Over German Bund (10-year yields)

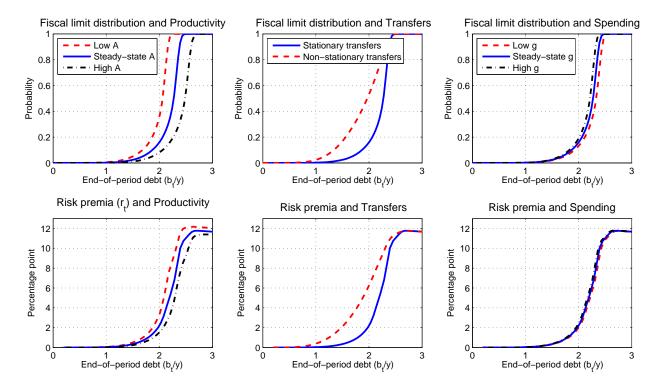


Figure 2: Risk premia and conditional (state-dependent) distributions of fiscal limits for model calibrated to Greek data: baseline case. Estimated empirical distributed from Monte Carlo Markov Chain simulation of model.

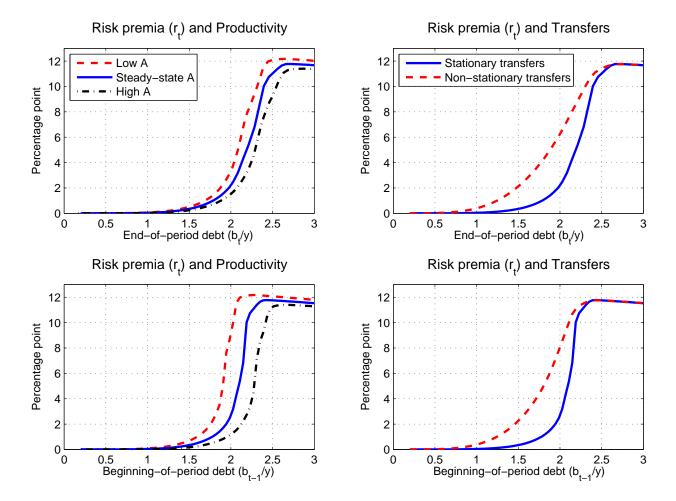


Figure 3: Risk premia  $(r_t)$  with respect to the end-of-period debt  $(b_t)$  vs. the existing debt  $(b_{t-1})$  for model calibrated to Greek data: baseline case.

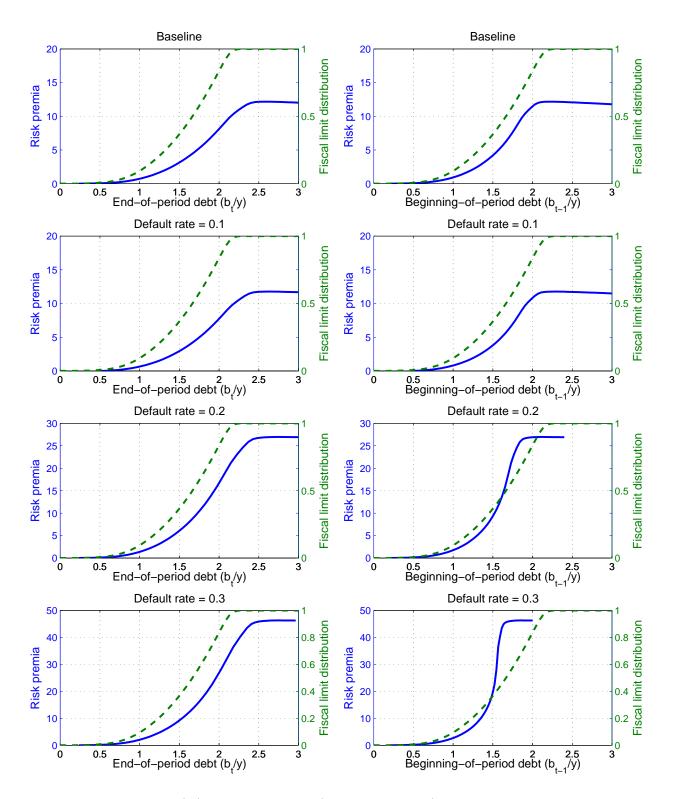


Figure 4: Risk premia  $(r_t)$  and conditional (state-dependent) distributions of fiscal limits for model calibrated to Greek data: baseline case vs. alternative specifications of default rate (assuming the current productivity is 6 percent lower than the steady state, the government spending is at steady state, and transfers reside in the explosive regime). Left panels: risk premia with respect to the end-of-period debt  $(b_t)$ ; right panels: risk premia with respect to the existing debt  $(b_{t-1})$ .

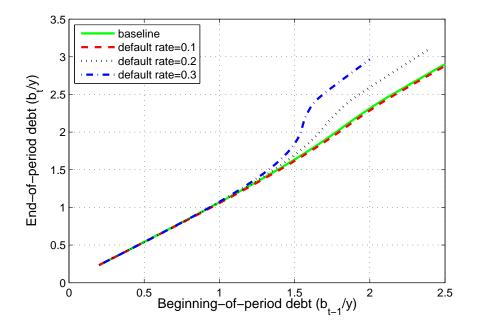
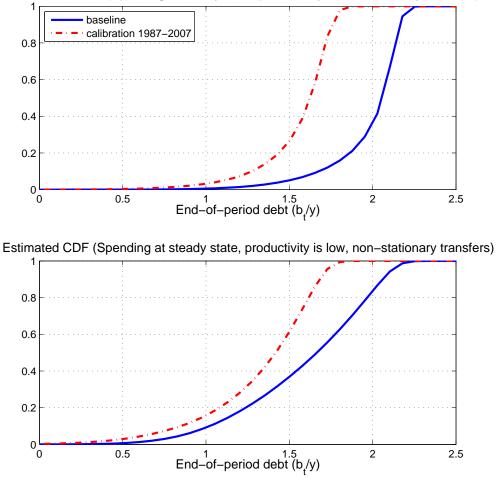


Figure 5: Decision rule of the end-of-period debt level  $(b_t)$  with respect to the existing debt level  $b_{t-1}$  for model calibrated to Greek data: baseline vs. alternative specifications of default rates (assuming the current productivity is 6 percent lower than the steady state, the government spending is at steady state, and transfers reside in the explosive regime).



Estimated CDF (Spending at steady state, productivity is low, stationary transfers)

Figure 6: State-dependent distributions of fiscal limits for two Greece calibrations: baseline vs. calibration to the period of 1987 to 2007.

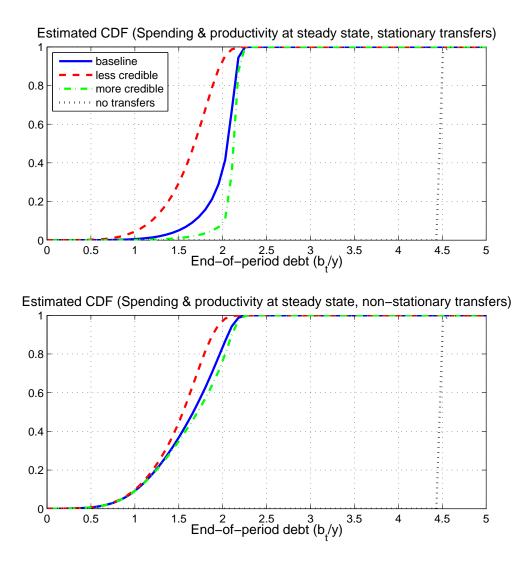


Figure 7: State-dependent distributions of fiscal limits for Greece calibration: fiscal reforms with different  $p_1^z$ . Baseline is specified as  $p_1^z = p_2^z = 0.975$ , a less credible reform features  $p_1^z = 0.75, p_2^z = 0.975$ , and a more credible reform is specified as  $p_1^z = 0.995, p_2^z = 0.975$ . Estimated empirical distributed from Monte Carlo Markov Chain simulation of model.

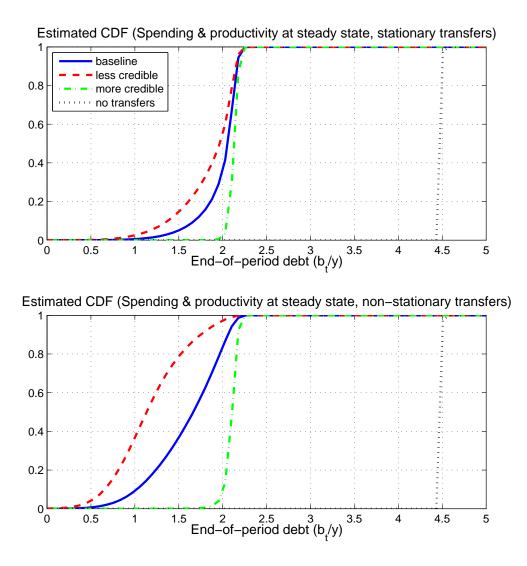


Figure 8: State-dependent distributions of fiscal limits for Greece calibration: fiscal reforms with different  $p_2^z$ . Baseline is specified as  $p_1^z = p_2^z = 0.975$ , a less credible reform features  $p_1^z = 0.975, p_2^z = 0.995$ , and a more credible reform is specified as  $p_1^z = 0.975, p_2^z = 0.975$ ,  $p_2^z = 0.75$ . Estimated empirical distributed from Monte Carlo Markov Chain simulation of model.

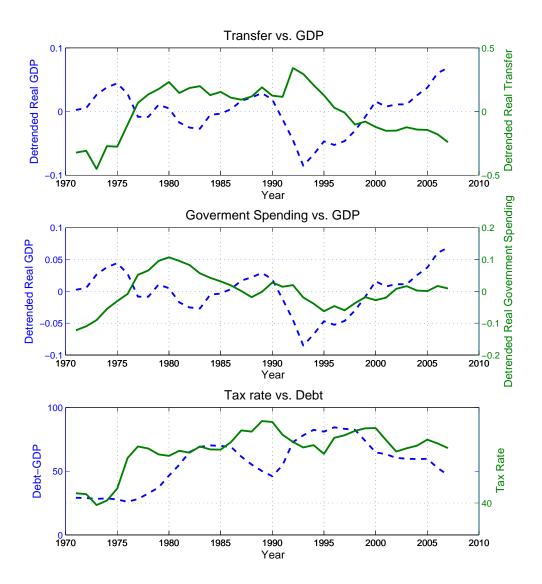


Figure 9: Swedish fiscal data: dashed lines are measured on the left axis, and solid lines are measured on the right axis. GDP, transfers, and government spending are detrended.

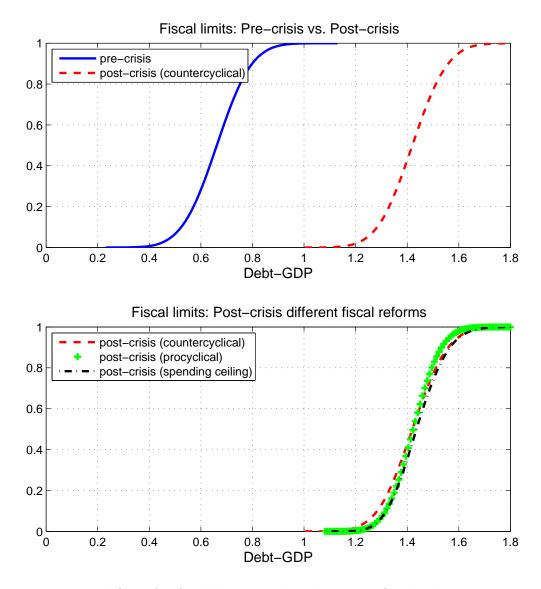


Figure 10: Estimated CDF for fiscal limits under alternative fiscal policies. Top panel compares the pre-crisis case to the post-crisis case with countercyclical government spending. Bottom panel compares three post-crisis cases: countercyclical government spending, expenditure ceiling, and procyclical government spending.