Constructing a conditional GDP fan chart with an application to French business survey data

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« Prediction is very difficult, especially if it is about the future »

Niels Bohr
Once upon a time, there were three nice assumptions:

- A great moderation
- Stable and smooth forecast errors
- Nice gaussian fan charts
Fan Chart: Insee

Source: INSEE
Insee Methodology

1. Assumption: gaussian error $N(0, \sigma)$

2. $\sigma$ estimated on the sample of past forecast errors

3. Density forecast: $y_q \approx \hat{y}_q + N(0, \hat{\sigma})$
...and a big bad wolf

Insee fan chart

Source: INSEE
...a VERY big bad wolf

Insee fan chart
A need for a safer fan chart (1/2)

1. Parametric shape:

Error distribution is asymmetric with a left fat tail
A need for a safer fan chart (2/2)

2. Unconditional forecasting
   – distribution shape remains constant over time.
   – In particular, no change in the volatility of growth figures.
Let $Z_q \in \{A, D\}$ with probability $p$, $1-p$, the state of the economy.

Let $Y_q$ q-o-q GDP growth rate.

Suppose $\text{Var}(Y_t|Z_t) = \sigma^2_{Z_t}$, $E(Y_t|Z_t) = \mu_{Z_t}$ with $\mu_D < \mu_A$ and $\sigma^2_A < \sigma^2_D$.

Then, the average quadratic error is

$$E(\text{Var}(Y_t|I_t)) = p\sigma^2_D + (1-p)\sigma^2_A$$

But

$$\sigma^2_A < p\sigma^2_D + (1-p)\sigma^2_A < \sigma^2_D$$
Our goals

Derive with the Business Survey Data:

1. Conditionnal confidence intervals
2. Conditionnal Fan charts
3. A Forecasting Risk Index
Out-of-sample Results: Confidence Intervals

Out-of-sample forecasts

GDP growth rate (%)

-1.5 -1.0 -0.5 0.0 0.5 1.0 1.5

2006 2007 2008 2009 2010

- OoS Quant forecasts 5 %
- OoS Quant forecasts 95 %

Dr. CORNEC
Conditional GDP fan chart
Results: Fan chart

Conditional GDP fan chart
Forecasting Risk Index

Forecasting Risk Index

sqrt(variance) over time from 1995 to 2010.
Methodology (1/3)

› Goal: sequentially predict \( f(y_{q+1} \mid z_q) \)
With \( Z_t := (1, Y_{t-1}, I_t, \Delta I_t \mid \Delta I_t) \) and \( I_t \)
the business climate indicator

› For this, estimate the conditional quantil function

\[
\theta \rightarrow Q_{Y_{q+1}} (\theta \mid z_q) := \inf \left\{ t \in R : F_{Y_{q+1}} (t \mid z_q) \geq \theta \right\}
\]
Since \( Q_Y(\theta|z) \in \arg \min_{m(.)} E\rho_\theta(Y - m(z))|X = z \)

With the pinball loss

Thus \( \hat{Q}_Y(\theta|z) := z'\hat{\beta}_\theta \)

with \( \hat{\beta}_\theta \in \arg \min \sum_{i=1}^{t} \rho_\theta(y_i - z'i\beta) \)
Recall if $U$ is uniform on $[0,1]$, then $Q_y(U|z) = Y|z$

Algorithm to estimate $\hat{f}_y(y|z)$

- Large $N$

- Denote $y_i := \hat{Q}(u_i|z)$, $u_i := \frac{i}{N}; 1 \leq i \leq N$

- Let $\hat{f}_y(y|z) := \frac{1}{Th} \sum_i K\left(\frac{y - y_i}{h}\right)$

with $K$ a kernel, $h$ the window
Forecasting risk index

• For the $L^2$ norm, we define the intrinsic difficulty of the forecasting exercise

$$\sqrt{\text{Var} (Y|z)}$$

• The estimation is

$$FRI_t := \sqrt{\hat{\text{Var}}(Y_t|z_t)}$$
Happy end?

Will our new assumptions live happily ever after?