Climate Policies, Macroprudential Regulation, and the Welfare Cost of Business Cycles*

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Abstract

We study the performance of alternative climate policies in a dynamic stochastic general equilibrium model that includes an environmental externality and agency problems associated with financial intermediation. Heterogeneous polluting producers finance their capital acquisition by combining their resources with loans from banks, are subject to environmental regulation, are hit by idiosyncratic shocks, and can default. The welfare analysis suggests that a cap-and-trade system will entail substantially lower costs of the business cycle than a carbon tax if financial frictions are stringent, firm leverage is high, and agents are sufficiently risk-averse. Simple macroprudential policy rules can go a long way in reining in business cycle fluctuations, aligning the performance of price and quantity pollution policies, and reducing the uncertainty inherent to the chosen climate policy tool.

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1 Introduction

There is a widespread consensus among economists and policy analysts on the need for dramatic reductions in anthropogenic emissions of greenhouse gases (GHGs) to limit disastrous climate change. Yet, it is less clear which policies would best serve this purpose. Carbon pricing is considered a cost-effective policy tool that governments can use as part of their broader climate strategy. According to the World Bank (2022), carbon pricing instruments cover about 23% of global greenhouse gas emissions. As of April 2022, worldwide, there are 68 carbon pricing instruments in operation and 3 scheduled for implementation. This includes 37 carbon taxes and 34 emission trading systems. Therefore, to date, at the policy implementation level, neither carbon pricing instrument seems to prevail, at least for the number of initiatives. On the other hand, carbon taxes cover around 6% of global GHG emissions, while cap-and-trade systems cover about 17%. On the academic side, dating back to Weitzman (1974), the "prices versus quantity" literature has focused on the problem confronting the environmental regulator: whether it would be better to control pollution by pricing emissions with a tax or by fixing a quantity target through a cap-and-trade scheme.[1]

In the presence of uncertainty and in the domain of climate actions, the close connection between these two modes of environmental control becomes more problematic, and their comparative performance may favor, in turn, either prices or quantities as policy instruments. This is because policies to control emissions will also influence the macroeconomic response to exogenous disturbances. On the one hand, a cap-and-trade scheme entails more certainty about future emission levels, but it implies greater uncertainty about compliance costs, given the unpredictable trajectory of allowances price; on the other hand, a price instrument, such as a carbon tax, limits the uncertainty related to compliance costs but, allowing emissions to move procyclically with economic activity, implies more uncertainty regarding the achievement of the pollution target. See e.g. Metcalf (2009), Aldy and Stavins (2012), and Aldy and Armitage (2020). The point is that the compliance costs of environmental regulation, directly and indirectly, affect all agents of the economy, changing their incentives and, eventually, their behavior toward uncertainty and shocks. Whether regulators should adopt prices or quantities as planning instruments then depends on the characteristics of the economy under analysis (such as available technologies, preferences, and other market failures). Thus "there is

[1] Since the seminal contribution of Weitzman (1974) the issue of price versus quantity regulation has been studied thoroughly in several papers, such as Stavins (1996), Hoel and Karp (2001, 2002), Newell and Pizer (2003), Kelly (2005), and Karp and Traeger (2018), among others. See Stavins (2020, 2022) for a comprehensive discussion and review of this literature and related policy implications.

no basic or universal rationale for a general predisposition toward one control mode or the other" (Weitzman, 1974, p. 479).

This paper reconsiders this controversial subject and compares the welfare costs of business cycles in a cap-and-trade regime with auctioned allowances to those arising in the economy where environmental policy takes the form of a carbon tax. The theoretical framework we use is a dynamic stochastic general equilibrium (DSGE) model with pollution, in which business cycle fluctuations are amplified by the existence of a "financial accelerator" mechanism, as modeled in Christiano et al. (2008, 2014). The financial intermediary sector is characterized by an agency problem arising from asymmetric information and monitoring costs, as in the earlier work of Bernanke and Gertler (1989) and Bernanke et al. (1999). At the heart of the model are producers in the capital-intensive sector who borrow from banks, are subject to idiosyncratic shocks to their productivity, and whose activity generates polluting emissions that negatively affect the economy. These producers are subject to environmental regulation and must identify the least-cost combination of emissions abatement costs and total carbon tax payments or allowance purchases. Financial market imperfections determine the conditions in which credit is granted and interact with the performance of environmental policies. Business cycle fluctuations are generated by shocks to the total factor productivity in the final-good sector and by risk shocks in the capital-intensive polluting sector. The latter are disturbances to the cross-sectional dispersion of idiosyncratic shocks hitting polluting firms. Firms hit by adverse shocks may be unable to repay their loans, experience failure, and go bankrupt. In this setting, greater uncertainty leads to increasing risk premia and expands the size of the left-tail default events.[2]

Our main results show that without any macroprudential regulation, a cap-and-trade system keeps the economy significantly more stabilized and entails substantially lower welfare costs of business cycles than those observed under a carbon tax. During an economic upturn, under a cap policy, since the emission permit price moves procyclically, producers bear more costs to comply with the environmental regulation; in contrast, the opposite occurs in the face of a recession. As a result, a cap-and-trade scheme works to dampen business cycle fluctuations. On the other hand, under a tax regime, firms pay a constant fee to pollute and face slightly countercyclical (relative) compliance costs. In this case, firms can take advantage of an economic upturn and expand their production by more than under a cap. At the same time, in the face of recessions, polluting producers

[2]Notably, risk shocks and higher uncertainty have been emphasized in the literature as essential drivers of macroeconomic fluctuations. See, for example, Christiano et al. (2014), Segal et al. (2015) and Caldara et al. (2016).

will be forced to reduce their output by more than under a quantity restriction because of the increase in their compliance costs.

The differences between the two environmental regimes are magnified by the imperfectly functioning financial market and the possibility of polluting firms' default. In particular, the higher the credit leverage of polluting firms, the stronger the channel of propagation of shocks exerted by financial effects, and the more unstable the economy will be under price regulation. On the contrary, for a lower exposure of polluting firms to external financing, the channel of propagation of financial effects weakens, and the two environmental regimes will entail more similar dynamics and lower welfare costs of business cycles. Our results show that in an economy hit by shocks under a cap-and-trade system, the financial accelerator mechanism temporarily works in the opposite direction, smoothing out business cycle fluctuations. In this context, introducing a macroprudential policy envisaging countercyclical reserve requirements tends to substantially reduce the welfare cost of business cycles and align the performance of different environmental regulations.

This paper is related to a growing body of literature that studies the relationship between business cycles and environmental policy in the context of DSGE models. These models embodying environmental features into dynamic stochastic macroeconomic frameworks are also known as environmental DSGE or E-DSGE models, following the terminology introduced by Khan et al. (2019). Early contributions in this literature are those of Fischer and Springborn (2011), Heutel (2012), and Angelopoulos et al. (2013), who study pollution policies and the optimal responses to the business cycle in environmental variants of the baseline real business cycle (RBC) model.[3] The basic E-DSGE model has been later extended to include other sources of shocks (see Khan et al. 2019), multiple sectors and sources of energy (e.g., Dissou and Karnizova 2016), nominal rigidities, and monetary policy (e.g., Annicchiarico and Di Dio 2015, 2017; Annicchiarico and Diluiso 2019), and credit market imperfections (e.g., Carattini et al. 2021, Diluiso et al. 2021, Huang et al. 2022). In particular, Huang et al. (2022) use a framework with borrowing constraints and endogenous default in the spirit of the costly-state-verification model of Christiano et al. (2008, 2014), like the one we adopt in this paper, and study the macro-financial impact of tightening the environmental regulation. Therefore, the focus of their analysis is related to the transition risk rather than the study of the performance of different environmental regulations over the economic cycle, as we do in this paper.

[3]For an early discussion on this topic, see Bowen and Stern (2010), while for a review of the literature and discussion on the policy implications, see Annicchiarico et al. (2021). On emission dynamics over the business cycle, see Doda (2014) and Klarl (2020).

Among all these contributions, one of the most relevant for our analysis is that of Fischer and Springborn (2011), who compare the performance of alternative environmental policies (price, quantity, and emission intensity) during the business cycle, showing that a quantity regulation has a built-in dampening effect on the business cycle. A similar exercise is conducted in Annicchiarico and Di Dio (2015) in the context of a New Keynesian model. That paper shows that the ability of a quantity regulation to dampen business cycle fluctuations is increasing in the degree of nominal rigidities. However, both contributions find that, when comparing the welfare performances of quantity and price regulations, the two policies are not significantly different. By contrast, in their multi-sector environmental DSGE model Dissou and Karnizova (2016) show that, when energy-related shocks are the main driving force of economic fluctuations, a cap policy is significantly less costly than a tax in terms of welfare, besides the fact that a quantity restriction delivers a lower level of macroeconomic volatility. To the best of our knowledge, our paper is the first contribution that looks at the welfare cost of business cycles under price or quantity environmental regulations elucidating their potential interactions with financial frictions. Unlike previous contributions, our findings show a stark difference between the two market-based policies regarding welfare effects. This difference is only partially attenuated when the cap or the carbon tax is allowed to react to economic fluctuations optimally.

Finally, by exploring the potential role of macroprudential regulation in shaping the performance of environmental policies over the business cycle, our paper contributes to the ongoing debate about the potential role that central banks and financial regulators can have in the fight against climate change (e.g., Carney 2015, Rudebusch 2019, Bolton et al. 2020, NGFS 2020a,b). Our results suggest that simple countercyclical financial regulations, designed to stabilize the economy, can positively reduce the uncertainty inherent to the chosen environmental policy tools and align their performances, thus broadening the menu of climate policy options.

The remainder of the paper is organized as follows. Section 2 presents the utility-based theoretical framework in which we conduct our analysis. Section 3 describes the calibration of the structural parameters of the model. Section 4 looks at model dynamics and evaluates the welfare cost of business cycles under price and quantity pollution policies. Section 5 analyzes the potential role of macroprudential regulation in affecting the macroeconomic performance of environmental policies. Section 6 concludes the paper.

2 The Model Economy

We introduce environmental features in a framework close to the one developed by Christiano et al. (2008, 2014) incorporating the debt-contracting model of Bernanke and Gertler (1989) and Bernanke et al. (1999). The core of our model is at the level of the capital-intensive sector, where intermediate-good firms, differing in their net worth, experience idiosyncratic shocks and, via the production process, generate polluting emissions that negatively affect the overall output of the economy. Emissions can be reduced by sustaining extra costs through an abatement technology. Before the occurrence of shocks, each intermediate-good firm purchases capital from perfectly competitive capital-good producers by using internal financing (net worth) and loans obtained from perfectly competitive banks. The structure of the model allows for the possibility of default. In the case of bankruptcy, the banks seize the assets of firms that cannot repay their loans after sustaining a monitoring cost. The supply side of the model is closed by a final-good sector which combines the intermediate good with labor. On the other end of the economy, households enjoy consumption, supply labor, and hold bank deposits. Finally, the model features a government setting the environmental policy.

2.1 Households

There is a large number of identical households, each of which owns a large number of intermediate-good firms. The representative household derives utility from consumption C_t and disutility from hours worked H_t. Households' preferences are of the following non-additively separable type:

$$U_0 = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\sigma_L} (1 - H_t)^{1-\sigma_L} \right]^{1-\eta}}{1-\eta} \right\}, \tag{1}$$

where $\mathbb{E}_0$ is the rational expectations operator, $\beta \in (0, 1)$ is the subjective discount factor, $\sigma_L \in (0, 1)$ and $\eta \geq 0$ are preference parameters.
The period-by-period budget constraint is:

$$C_t + B_{t+1}^H \leq W_t H_t + (1 + R_{t-1}) B_t^H + T_t. \tag{2}$$

Households' sources of funds are earnings from labor $W_t H_t$, risk-free deposits B_t^H carried from $t-1$ and lump-sum payments T_t that include transfers from firms and the government. These funds are allocated to consumption and savings in the form of new deposits B_{t+1}^H. The rate of return R_{t-1} on deposits is assumed to be preset. In equilibrium, this induces a predetermined return for lenders so that borrowers end up absorbing all risk.[4] In period t the typical household chooses C_t, H_t and B^H_{t+1} with the objective of maximizing (1) subject to (2).

2.2 Final-Good Producers

The final good Y_t is produced by a representative firm through a combination of the intermediate good X_t purchased at price r^x_t and labor supplied by households according to the following Cobb-Douglas technology:

$$Y_t = A_t X_t^{\alpha} H_t^{1-\alpha}, \tag{3}$$

where $\alpha \in (0,1)$ and A_t is a measure of total factor productivity (TFP) which is negatively affected by the pollution:

$$A_t = \bar{A}_t(1 - D_t(M_t)), \tag{4}$$

where $\bar{A}_t$ is an exogenous process subject to shocks and D_t refers to a damage function that depends on cumulative emissions M_t. This function captures the negative externality of pollution that motivates environmental regulation.

2.3 Capital-Good Producers

At the end of each period t competitive capital-good producers purchase capital from intermediate-good producers for the price $Q_{K,t}$, rebuild depreciated capital, and construct new capital K_{t+1} with the following technology embodying installation costs increasing in the rate of investment growth:

$$K_{t+1} = (1 - \delta)K_t + (1 - S(I_t/I_{t-1}))I_t, \tag{5}$$

where $\delta \in (0,1)$ denotes the rate of depreciation on capital, I_t stands for investments and $S(\bullet)$ is an increasing and convex function such that in steady state $S = S' = 0$. The new capital stock is then sold for the same price $Q_{K,t}$.

[4] This assumption, together with myopia and risk neutrality of intermediate-good producers, is key in shaping the amplification of shocks in the economy. Recent works by Carlstrom et al. (2016) and Dmitriev and Hoddenbagh (2017) show that relaxing these assumptions implies an attenuation of the financial accelerator effect of the model.

2.4 Intermediate-Good Producers

The intermediate-good sector of the economy is populated by a mass of heterogeneous firms differing in their net worth and using physical capital as a production input. Firms have different levels of wealth since they experienced idiosyncratic shocks in the past.

After production in period t, the state of a typical intermediate good producer is summarized by its net worth, $N \geq 0$. Let $f_t(N)$ be the density of producers with a net worth of N, then the total net worth of the economy is:

$$N_{t+1} = \int_0^\infty N f_t(N) dN. \tag{6}$$

Henceforth, following Christiano et al. (2014), we will focus on the behavior of the typical producer with net worth N and use the superscript N to refer to variables of this N-type firm. At the end of the period t, when intermediate-good producers' net worth is known, each N-type firm obtains a loan, B_{t+1}^N, from a bank. This loan is then combined with firm's net worth to purchase capital goods, K_{t+1}^N, in an anonymous and competitive market at a price of $Q_{K,t}$. This implies that at the end of period t the balance sheet of the N-type firm is equal to $Q_{K,t}K_{t+1}^N = N + B_{t+1}^N$, from which we can define a measure for leverage, L_t^N, as follows:

$$L_t^N = \frac{Q_{K,t}K_{t+1}^N}{N} \quad \text{or} \quad L_t^N = \frac{N + B_{t+1}^N}{N}. \tag{7}$$

After purchasing capital, firms undertake the period $t+1$ production process according to the following linear technology:

$$X_{t+1}^N = \omega K_{t+1}^N, \tag{8}$$

where ω is the idiosyncratic productivity level. Following Bernanke et al. (1999) and Christiano et al. (2014), it is assumed that ω is a unit-mean lognormally distributed random variable that is independently drawn across time and across firms with a cumulative distribution function denoted by $F(\omega)$. Let σ_t denote the standard deviation of $\log \omega$. This dispersion is allowed to vary stochastically over time and is the source of risk shocks that determine the extent of the cross-sectional dispersion of the idiosyncratic productivity level. The realization of the random variable ω is observed by the producer but can only be detected by the bank if it pays a monitoring cost.[5]

[5]The random variable σ_t captures the risk to which capital-intensive producers are subject. Broadly speaking, it could also be understood as describing the exposure to risks associated with possible technological breakthroughs, ecological transitions, or paths of emissions reduction for the economy.

In the spirit of the DICE model by Nordhaus (2008) and as in Heutel (2012) and Annicchiarico and Di Dio (2015) among others, we assume that the production process is polluting and that emissions, say E_t, depend on abatement effort and production:

$$E_{t+1}^N = \chi(1-\kappa_{t+1}^N)X_{t+1}^N, \tag{9}$$

where $\chi > 0$ is a scale parameter and $\kappa_{t+1}^N \in (0,1)$ is the fraction of emissions abated. Clearly, the level of emissions also depends on the realization of the idiosyncratic shock ω. The abatement activity is assumed to be costly, and total abatement spending is described by a cost function that depends on the effort made to reduce emissions and on the level of production, that is $\theta_1 \left(\kappa_{t+1}^N\right)^{\theta_2} X_{t+1}^N$ where $\theta_1 > 0$ and $\theta_2 > 1$. Firms are subject to environmental policy and face an extra cost, P_{t+1}^E, that must be paid for each emission unit. In each period, producers can choose between incurring abatement costs or paying the regulation price (i.e., purchasing emission permits on the market in the case of quantity regulation, or paying a specific tax, in the case of price regulation). The optimal abatement choice will eventually be such that:

$$\theta_1\theta_2(\kappa_{t+1}^N)^{\theta_2-1} = \chi P_{t+1}^E. \tag{10}$$

The above condition implies that the abatement effort will be equated across intermediate-good firms, regardless of their net worth level and their specific productivity level (i.e., $\kappa_{t+1}^N = \kappa_{t+1}$). At the end of the production process of period t+1, productive capital depreciates, and firms are left with $(1-\delta)\omega K_{t+1}^N$ units of it. This capital stock is then sold in a competitive market to capital-good producers for the price $Q_{K,t+1}$.

Taking everything into account, and recalling that the intermediate good is sold for r_{t+1}^x to final-good producers, in period $t+1$ an intermediate-good producer enjoys a (gross) rate of return $\omega\left(1+R_{t+1}^k\right)$, where:

$$1+R_{t+1}^k = \frac{r_{t+1}^x + (1-\delta)Q_{K,t+1}}{Q_{K,t}} - \frac{\theta_1\kappa_{t+1}^{\theta_2} + P_{t+1}^E\chi(1-\kappa_{t+1})}{Q_{K,t}}. \tag{11}$$

where the first term on the right-hand side measures the returns on capital, while the second one measures environmental regulation compliance costs per unit of capital, that is our measure for relative compliance costs that will come in handy when discussing our results.

As anticipated, firms can self-finance only a fraction of their capital stock and rely on external finance to complement their net worth as a funding source. The loan obtained by each producer in period t takes the form of a standard debt contract that specifies Z_{t+1}, as the gross rate of interest on debt, and $\bar{\omega}^N_{t+1}$, as the value of ω that divides intermediate-good producers who cannot repay the interest and principal from those who can repay, that is:

$$\bar{\omega}^N_{t+1} K^N_{t+1}(1 + R^k_{t+1}) Q_{K,t} = B^N_{t+1} Z^N_{t+1}. \tag{12}$$

Firms experiencing an idiosyncratic shock below the cut-off level $\bar{\omega}^N_{t+1}$ go bankrupt.

Intermediate-good producers value a particular debt contract according to the expected return from operating risky technology over the return from depositing net worth in a bank, that is:

$$\begin{aligned} &\frac{\mathbb{E}_t \left\{ \int_{\bar{\omega}^N_{t+1}}^{\infty} \left[\omega(1 + R^k_{t+1}) Q_{K,t} K^N_{t+1} - B^N_{t+1} Z^N_{t+1} \right] dF(\omega) \right\}}{N\,(1 + R_t)} \\ &= \mathbb{E}_t \left[1 - \Gamma\left(\bar{\omega}^N_{t+1} \right) \right] \frac{1 + R^k_{t+1}}{1 + R_t} L^N_t, \end{aligned} \tag{13}$$

where in the second line we have used (7)and (12) to express this expected return as a function of the leverage L^N_t, while $1 - \Gamma\left(\bar{\omega}^N_{t+1}\right)$ represents the share of average earnings received by producers, with $\Gamma\left(\bar{\omega}^N_{t+1}\right) = (1 - F(\bar{\omega}^N_{t+1}))\bar{\omega}^N_{t+1} + G(\bar{\omega}^N_{t+1})$ and $G(\bar{\omega}^N_{t+1}) = \int_0^{\bar{\omega}^N_{t+1}} \omega dF(\omega)$. See Appendix A for details.

Banks specialize in lending to intermediate-good producers with specific net worth levels, and each of the identical banks holds a large portfolio of loans that is perfectly diversified across producers. Banks obtain resources by issuing B^N_{t+1} in deposits to households at the predetermined interest rate R_t. Moreover, they monitor intermediate-good producers and collect assets (net of monitoring costs) from those who default, hence the following cash constraint with the free-entry condition holds:

$$\begin{aligned} &(1 - F(\bar{\omega}^N_{t+1})) B^N_{t+1} Z^N_{t+1} + (1 - \mu) \int_0^{\bar{\omega}^N_{t+1}} \omega dF(\omega)(1 + R^k_{t+1}) Q_{K,t} K^N_{t+1} \\ &= B^N_{t+1} \left(1 + R_t\right). \end{aligned} \tag{14}$$

where the first term on the left-hand side indicates revenues received from the fraction of firms with $\omega_{t+1} \geq \bar{\omega}^N_{t+1}$, namely, those which do not go bankrupt, while the second term measures the revenues obtained from bankrupt firms, with μ denoting the proportion of assets lost for monitoring.[6] Condition (14) also highlights the main market failure of

[6]Note that (14) holds with strictly equality since we assume there is free entry, and it also implies

the model, namely the fact that the risk-free interest rate R_t is equated to the average and not to the marginal return on production. The equilibrium is inefficient because the marginal return on credit exceeds the average return.

Making use of (12) and of the previously defined functions, (14) can be rewritten in a more compact way as:

$$\Gamma\left(\bar{\omega}_{t+1}^N\right) - \mu G(\bar{\omega}_{t+1}^N) = \frac{1+R_t}{1+R_{t+1}^k} \frac{L_t^N - 1}{L_t^N}. \tag{15}$$

Intermediate-good producers choose the debt contract that maximizes their objective (13) among the $(\bar{\omega}_{t+1}^N, L_t^N)$ combinations that satisfy (15). Since the constraint is independent of net worth (which only appears as a constant of proportionality in the objective function), all firms will eventually select the same debt contract that can be represented as $(\bar{\omega}_{t+1}, L_t)$ or equivalently as (Z_{t+1}, L_t), irrespective of their net worth (see Appendix A for more details on the derivations).

Finally, it is assumed that at the end of the period $t+1$, a random fraction $1-\gamma$ of each firm's assets are eventually transferred to their household, while the rest remains with the producer. The reason for this is that firms are owned by households who, in turn, instruct producers to maximize their expected net worth. The larger the level of net worth, the greater the amount of resources transferred to households in each period. In addition, producers receive an exogenous lump-sum transfer from the household, say W_{t+1}^p.

2.5 Aggregation and the Resource Constraint

At the end of the period t the quantity of capital purchased by intermediate-good producers must equal the amount produced, K_{t+1}, by capital producers:

$$K_{t+1} = \int_0^\infty K_{t+1}^N f_t(N) dN. \tag{16}$$

Recalling (8), the aggregate supply of intermediate good to be used in the final-good sector immediately follows:

$$X_t = \int_0^\infty \int_0^\infty \omega K_t^N f_t(N) dN dF(\omega) = K_t. \tag{17}$$

By the law of large numbers, at the end of the period t, the aggregate profits of all

that the banks' return from deposits is equal to the predetermined rate. Condition (14) determines the 'menu' of state-contingent debt of contracts $(\bar{\omega}_{t+1}^N, L_t^N)$ that can be offered in equilibrium.

$N-$type intermediate-good producers are $[1-\Gamma_{t-1}(\bar{\omega}_t)](1+R_t^k)Q_{K,t-1}K_t^N$, therefore recalling that a fraction $1-\gamma$ of each producer's net worth is transferred to households as a lump-sum, aggregate net worth evolves as follows:

$$N_{t+1} = \gamma\left[1-\Gamma_{t-1}(\bar{\omega}_t)\right](1+R_t^k)Q_{K,t-1}K_t + W_t^p, \tag{18}$$

where W_t^p denotes the amount of lump-sum transfers made by households. The aggregate quantity of debt in period t, say B_{t+1}, is then:

$$B_{t+1} = \int_0^\infty B_{t+1}^N f_t(N)dN = Q_{K,t}K_{t+1} - N_{t+1}. \tag{19}$$

In equilibrium, the total funds supplied to intermediate-good producers must be equal to the deposits held by households, that is $B_{t+1} = B_{t+1}^H$. The state-contingent interest rate Z_t can be obtained by integrating (12) relative to the density $f_t(N)$.

Aggregate abatement costs and aggregate emissions immediately follow:

$$\theta_1\kappa_t^{\theta_2}K_t = \theta_1\kappa_t^{\theta_2}\int_0^\infty\int_0^\infty \omega K_t^N f_t(N)dNdF(\omega), \tag{20}$$

$$E_t = \chi(1-\kappa_t)\int_0^\infty\int_0^\infty \omega K_t^N f_t(N)dNdF(\omega) = \chi(1-\kappa_t)K_t. \tag{21}$$

Finally, the resource constraint of the economy is:

$$Y_t = I_t + C_t + \theta_1\kappa_t^{\theta_2}K_t + \mu G(\bar{\omega}_t)(1+R_{t+1}^k)Q_{K,t-1}K_t, \tag{22}$$

where the last term on the right represents the aggregate monitoring costs. Net output, say Y_t^n, is then simply equal to $Y_t - \theta_1\kappa_t^{\theta_2}K_t - \mu G(\bar{\omega}_t)(1+R_{t+1}^k)Q_{K,t-1}K_t$.

2.6 Pollution, Damage, and Environmental Policy

As seen, production at the intermediate-good level generates emissions. Polluting gases accumulate into a stock M_t according to the following law of motion:

$$M_t - \bar{M} = \sum_{s=0}^{t+T}(1-\delta_M)^s\left(E_{t-s}+E_{t-s}^*\right), \tag{23}$$

where $\bar{M}$ denotes the pre-industrial concentration of pollutants, $\delta_M \in (0,1)$ measures the natural decay rate of greenhouse gases in the atmosphere, and E^* refers to rest-of-the-world emissions and is kept constant for simplicity. Similarly to Golosov et al.

(2014) and consistently with Nordhaus (2008), the accumulation of polluting emissions negatively affects total factor productivity through a damage function D_t:

$$1 - D_t(M_t) = \exp\left(-\xi\left(M_t - \bar{M}\right)\right), \tag{24}$$

where $\xi > 0$ is a damage parameter measuring the intensity of the negative environmental externality on production or, analogously, the fraction of output lost for each extra unit of pollutants. Climate change is then a stock externality since it is a function of the accumulated stock of emissions rather than of emissions *per se* at any time.[7] With a low decay rate δ_M, marginal damage costs will be less affected by temporary changes in emissions over the business cycle.

The government can implement two alternative environmental policies to control pollution: a carbon tax and a cap-and-trade system where allowances are auctioned. We consider these two policies since they represent two benchmarks usually contrasted in the academic and policy debates. Under a carbon tax regime, a tax rate per unit of emission is imposed: in this scenario, P_t^E is set constant, say $\bar{P}^E$, and can then be interpreted as a carbon tax. Under a cap-and-trade regime instead, a cap, say $\bar{E}$, is applied to the overall emissions E_t generated by the economy.

Finally, we assume that the fiscal authority runs a balanced budget at all times and that the carbon pricing revenues, $P_t^E E_t$, are redistributed to households as lump-sum transfers, say Tr.[8]

The equilibrium conditions describing the model economy are summarized in Appendix B.

3 Calibration

This section describes our calibration strategy. Time is measured in quarters and the model is calibrated to US data. We partition model parameters into three categories: standard macroeconomic parameters, parameters related to financial frictions, and parameters associated with the environmental externality and the pollution policy. Table 1 summarizes the calibration.

[7]Note that this is a parsimonious way to introduce the effects of climate change into the model. This is standard practice in several aggregate models where it is implicitly assumed that climate change is a function of the atmospheric stock of greenhouse emissions.

[8]This is a standard assumption in the absence of any distortionary taxation. Note that we also abstract from the higher administrative costs that even the simplest cap-and-trade systems may require for implementation.

In the context of the standard parameters related to the backbone of the macroeconomic model, the discount factor β is set to a value consistent with a real interest rate of 4% per year. In agreement with Christiano et al. (2014), the depreciation rate of capital δ is set to 0.025, while the production parameter α to 0.4. We set the relative risk aversion (RRA) to 2 and the preference parameters $\sigma_L = 0.21$ and $\eta = 5.72$. The implied value for the time spent working is then 0.17. The level of total factor productivity, net of the environmental damage, $\bar{A}$ is set to 1.26, so the steady-state value of production Y is equal to 1.

To calibrate the parameters related to the financial part of the model, we mainly follow Christiano et al. (2014). We set the parameter measuring monitoring cost, μ, to 0.21, while the fraction of net worth transferred to households by intermediate-good producers, $1 - \gamma$, is fixed at 0.035. Finally, the standard deviation of the log of the idiosyncratic shock ω is set to 0.3 to deliver a risk premium of 0.52 percentage points, closed to the one observed in the US data for the period 1985Q1-2019Q4.[9]

To calibrate the environmental block of the model, we mainly rely on the DICE model by Nordhaus (2018a). We start by anchoring the stock of atmospheric concentration of carbon to 891 gigatons of carbon (GtC), the approximated value observed in the no policy scenario in the DICE model in 2020. Knowing that the pre-industrial atmospheric concentration of carbon, $\bar{M}$, is about 581 GtC, we obtain M. The quarterly decay rate of greenhouse gases δ_M is fixed at 0.003 to reflect a half-life of carbon in the atmosphere of nearly 83 years, consistently with Reilly and Richards (1993). The overall level of emissions immediately follows from (23). To pin down E we use World Bank data for the period 1985-2018 and observe that the average share of worldwide carbon dioxide emissions ascribed to the US is around 20%. The emission intensity parameter χ is then implied. Using the simulation value of the DICE model on the fraction of output lost because of environmental damages in 2020, which is 0.002438, we can compute the damage function parameter ξ. The parameter θ_2 of the abatement function is fixed at 2.6 following Nordhaus (2018b), while the scale coefficient θ_1 is normalized to one so that total abatement costs as a fraction of output are around 0.0019% of GDP. The steady-state price of carbon, P^E, is fixed so that environmental tax revenues as a share of output are 0.7% in steady state, which is a level consistent with the US environmental tax revenues in 2017, according to OECD data.[10] Note that the non-stochastic steady

[9]The quarterly Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on the 10-Year Treasury Constant is 0.58 percentage points. See Federal Reserve Bank of St. Louis https://fred.stlouisfed.org/series/BAA10YM

[10]See OECD (2022), Environmental tax (indicator). doi: 10.1787/5a287eac-en (Accessed on 18 May 2022).

state associated with each pollution policy regime is the same.

Finally, the business cycle is driven by shocks to the TFP and the standard deviation of the idiosyncratic shocks. We assume that $\bar{A}_t$ evolves as $\bar{A}_t = \bar{A}exp(a_t)$, where $a_t = \rho_a a_{t-1} + \epsilon_{a,t}$, with $\epsilon_{a,t}$ being an i.i.d. shock, while σ_t evolves as $\sigma_t = \rho_\sigma \sigma_{t-1} + \sigma\epsilon_{\sigma,t}$ with $\epsilon_{\sigma,t}$ being an i.i.d. shock. We set ρ_a to 0.9, the standard deviation of $\epsilon_{a,t}$ to 0.0034, ρ_σ to 0.97 and the standard deviation of $\epsilon_{\sigma,t}$ to 0.065. To conclude, the curvature parameter of the investment installation cost function is set to 20. This last batch of parameters has been calibrated to match some second moments for the main macroeconomic aggregates observed for the US economy in 1985Q1-2019Q4, using a simulated minimum distance routine. The model solved under a carbon tax policy can reproduce the observed standard deviation of the GDP, the relatively lower consumption volatility, and the relatively higher volatility of investments observed in the US data over the period 1985Q1-2019Q4. See Appendix C.

4 Business Cycle Fluctuations: Cap Versus Tax

The present section investigates the macroeconomic performance of a cap-and-trade scheme and a carbon tax in the presence of business cycle uncertainty. We first consider the impulse response to isolated shocks and then study the welfare costs of business cycles. In the last part of this section, we relax the assumption that policymakers are constrained to choose between a constant price and a quantity instrument and study what happens when we derive optimal pricing and cap rules according to which environmental policy is set optimally as a function of current economic conditions.

4.1 Dynamic Analysis

We start the dynamic analysis by exploring the response of the economy to a positive one-standard-deviation shock to the TFP under the alternative environmental policy regimes. See Figure 1. In response to this expansionary shock, consumption, investment, and net output immediately rise. As the marginal productivity of final-good producers increases, the demand for intermediate good rises, pushing firms at the intermediate-good level to expand their production. Consequently, the capital stock starts building up, especially after the initial surge in price $Q_{K,t}$ has settled down. Due to the initial jump in the return on production, intermediate-good producers enjoy higher net worth levels, so their balance sheet shifts more heavily toward capital. Since firms can finance their production activity in a more significant part through their own acquired resources,

Table 1: Calibrated parameters and steady state ratios

	Description	Value
Steady state ratios and values		
C/Y^n	Private consumption	0.80
I/Y^n	Total investment	0.20
Tr/Y^n	Environmental tax revenues	0.007
H	Hours	0.17
$Z-(1+R)$	Spread p.p.	0.52
$F(\bar{\omega})$	Percent of bankrupt business p/quarter	1.5
M	Stock of concentration of carbon	891
$E/(E+E^*)$	Share of US emissions	0.20
Standard Macroeconomic Parameters		
β	Discount factor	0.99
δ	Depreciation rate of capital	0.025
α	Capital share	0.4
S''	Investment installation cost curvature	20
σ_L	Preference parameter (implied)	0.21
η	Preference parameter (implied)	5.72
RRA	Coefficient of relative risk aversion	2
$\bar{A}$	Total factor productivity (implied)	1.26
Financial Parameters		
μ	Monitoring cost	0.21
$1-\gamma$	Fraction of net worth to households	0.035
σ	Standard deviation of log ω	0.30
Environmental Parameters		
$\bar{M}$	Pre-industrial concentration of carbon	581
δ_M	Decay rate of greenhouse gases	0.0021
E^*	Rest-of-the-world emissions	0.51
χ	Emission intensity parameter (implied)	0.017
ξ	Damage function parameter (implied)	7.86e-06
θ_1	Abatement cost function parameter	1
θ_2	Abatement cost function parameter	2.6
Shocks		
ρ_A	Autocorrelation TFP shock	0.90
ρ_σ	Autocorrelation risk shock	0.97
$sd\ \epsilon_A$	Standard deviation TFP shock	0.0034
$sd\ \epsilon_\sigma$	Standard deviation risk shock	0.065

leverage declines, and credit from banks decreases. The probability of default falls as the cut-off value $\bar{\omega}_t$ goes down, with monitoring costs following suit, while banks end up reducing the interest rate charged on loans, leading to a decline in the spread.

Besides the above considerations, the most evident result from Figure 1 is that a cap-and-trade scheme keeps the system substantially more stabilized than a carbon tax policy in the face of an economic upturn. Indeed, under a carbon tax regime, overall emissions can respond freely and pro-cyclically to the shock. All the positive effects described above are magnified, as the marginal cost related to abatement and the price of emissions stay constant. In contrast, the relative compliance costs slightly decrease on impact due to the upward jump in the price of capital. In this sense, the tax instrument imposes a lower burden on polluters than the quantity instrument. This mechanism induces a higher return on production for intermediate-good producers, which in turn leads to a more pronounced decrease in the cutoff value than in the cap-and-trade scenario. Putting it differently, implementing a carbon tax allows more firms to have enough resources to repay their loans, reducing the probability of bankruptcy. As a result, banks end up charging lower rates, contributing to the decline in the spread and the easing of credit conditions.

Under a cap policy, higher environmental compliance costs limit the increase in the price of capital, leading to an attenuated effect on net worth and the financial premium. The cost of borrowing declines by much less, investments are less stimulated, and the impact on the price of capital is further restrained. These effects prevent firms from fully taking advantage of the economic upturn and give them less room to repay their loans, leading to a smaller decrease in the probability of going bankrupt than under a carbon tax policy. It is interesting to see how these effects pile up during the adjustment process, pushing the risk premium temporarily above its pre-shock level. The financial accelerator mechanism is then somehow reversed under a cap.

We now turn our attention to the dynamic response of the economy to the risk shock. Specifically, Figure 2 shows the negative consequences for the economy of a one-standard-deviation shock to the volatility of the idiosyncratic shocks. Higher uncertainty increases the probability of bankruptcy, by expanding the size of the left-tail default events. An increase in the probability of a low ω, in turn, pushes banks to raise the interest rate charged on loans to producers, and credit conditions tighten. It follows that intermediate-good producers are bound to purchase less capital. This, in turn, entails lower investments, thus leading to a contraction in economic activity and consumption. The measure for leverage increases on impact as the decline in the value of the net worth of intermediate firms offsets the decrease in the valuation of capital.

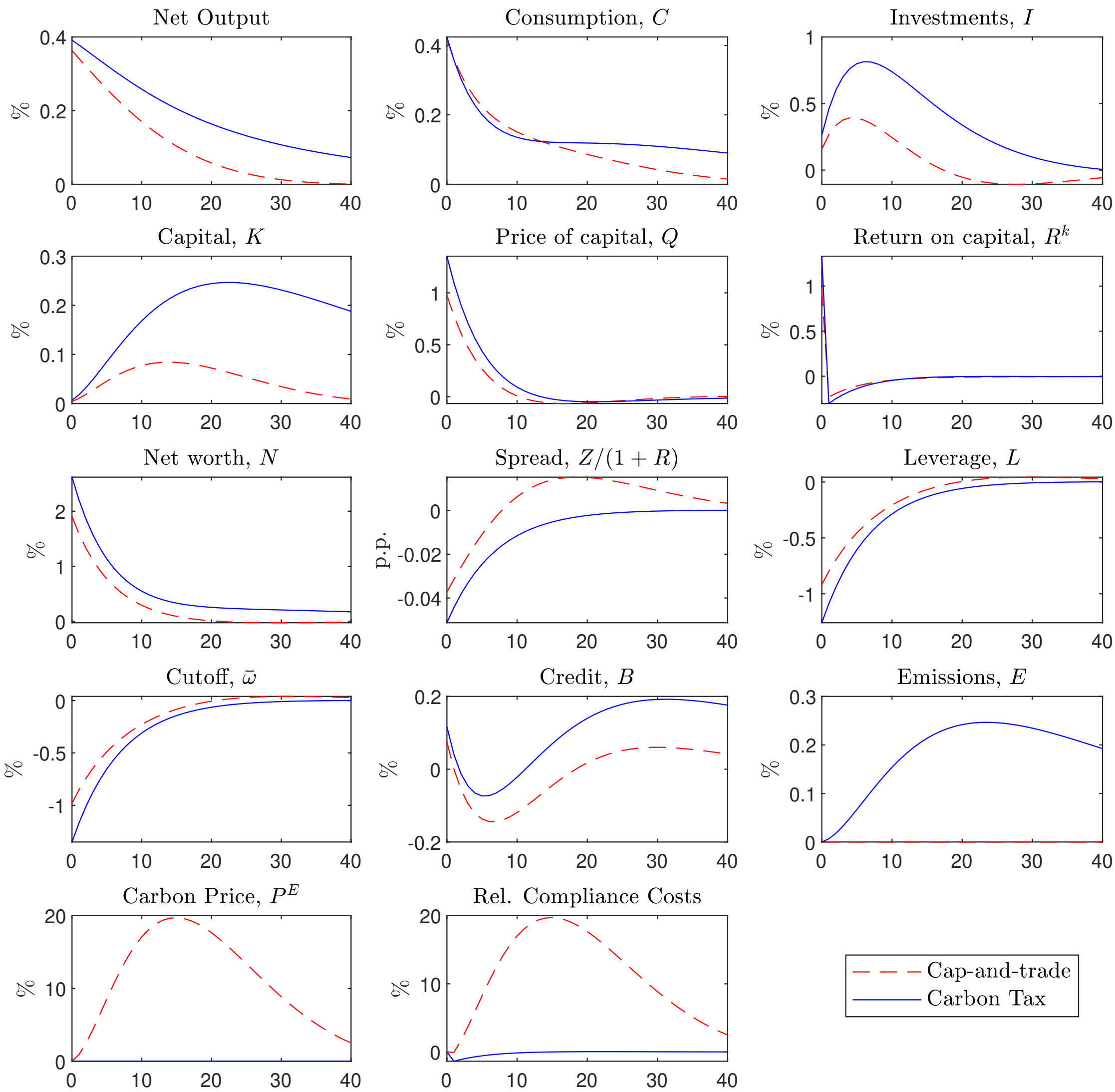
When comparing the economy's behavior under the two environmental regimes, Figure 2 confirms the above findings, with the economy being much more stable under a carbon trading scheme. In the face of a downturn, a cap-and-trade policy prevents the economy from experiencing a more profound crisis. Under a cap regime, the abatement effort and the price of emissions move pro-cyclically, allowing polluting firms to reduce their environmental compliance costs when faced with a crisis. Contrary to what happens in the case of a fixed price instrument, producers then can limit the decline in the return of their production so that the rise of the threshold level of productivity necessary to break even is partially contained. In this sense, a cap policy acts as an automatic stabilizer. At the same time, implementing a carbon tax effectively enhances the financial fragility of the system in the case of an economic slowdown. As shown in Figure 2, relative compliance costs sharply decline under a cap, while slightly increasing under a tax. Consistently with what is observed in response to a TFP shock, during the adjustment path, the acceleration mechanism is reversed under a cap, with the spread that falls below its pre-shock level.

As the discussions already made evident, the differences between the two alternative regimes are accentuated by the presence of a financial sector, whose mechanisms magnify the economy's response to shocks under the less stabilizing scenario of a carbon tax. On the other hand, a cap dampens the financial accelerator significantly and reverses the acceleration mechanism, working in the opposite direction. The following section will shed more light on the matter by shifting the scope of the analysis to the welfare costs of the business cycle.

4.2 Welfare Costs

In this section, we compare the welfare costs of the business cycle of the two environmental policy regimes. The welfare costs of business cycles associated with each environmental policy are measured as the gap between the welfare under the deterministic steady-state and mean welfare associated with the policy under consideration. More precisely, we measure how much individuals, under different environmental policy regimes, would stand ready to give up to live in a world not subject to economic uncertainty. To this end, we define the welfare associated with the time-invariant equilibrium as $U = u(C, H)/(1 - \beta)$, with $u(C, H)$ being the period-by-period utility specified in 1 and the welfare associated with a particular environmental policy, say EP, as U^{EP}. As

Figure 1: Dynamic Response to a One-Standard-Deviation TFP Shock



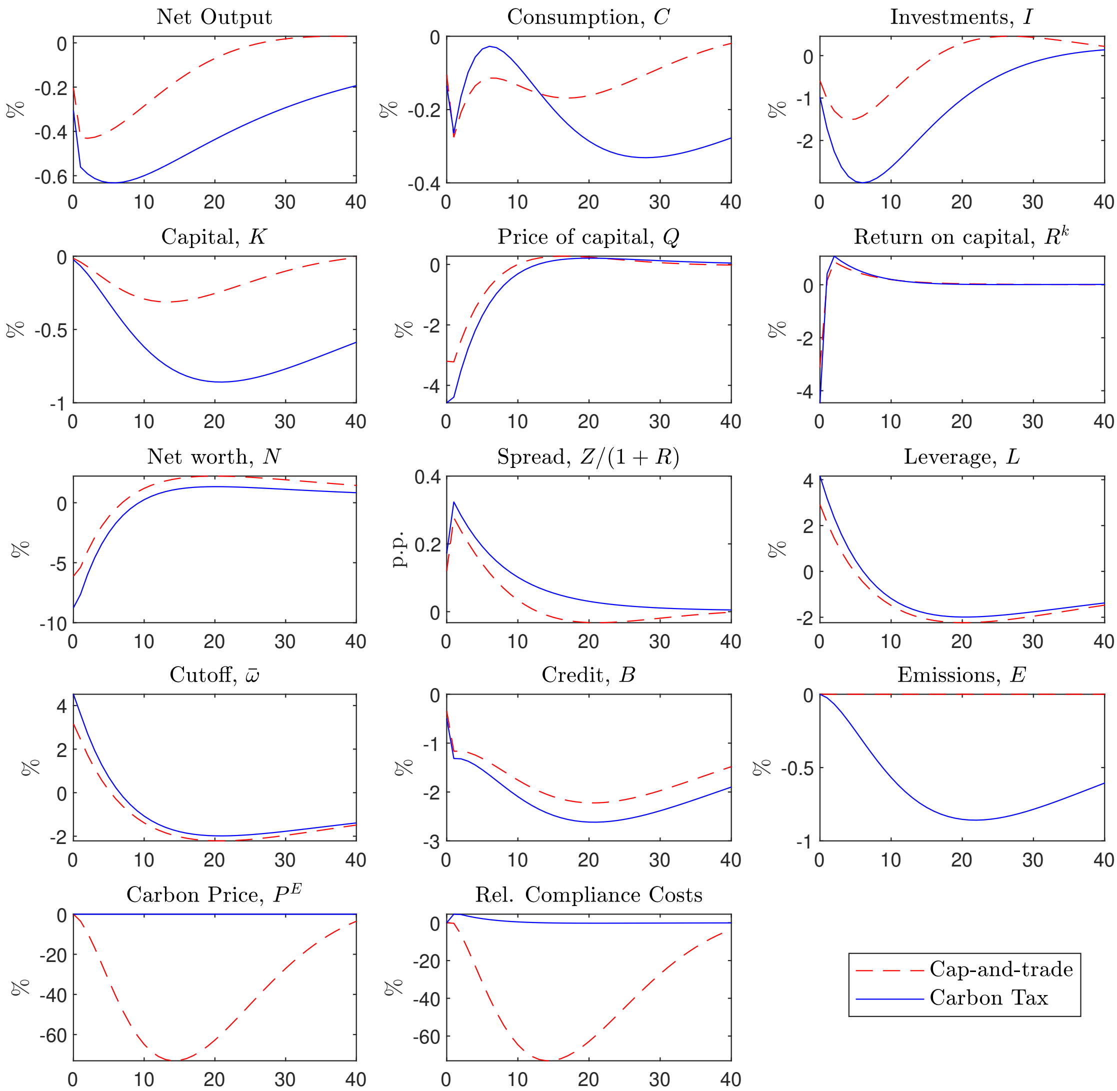
Note: Results are reported as percentage deviations from the initial steady state, except for the spread, reported as percentage point deviations from the steady state.

a welfare measure, we use the unconditional expectation of lifetime utility, that is:

$$U^{EP} = \mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t u(C_t^{EP}, H_t^{EP})\right\}, \tag{25}$$

where C_t^{EP} and H_t^{EP} are the equilibrium stochastic processes of consumption and hours under a particular environmental policy regime. Thus the cost of business cycles under

Figure 2: Dynamic Response to a One-Standard-Deviation Risk Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread, reported as percentage point deviations from the steady state.

a specific policy is given by ς, such that $u(C(1-\varsigma), H)/(1-\beta) = U^{EP}$. It follows that the higher ς, the higher the welfare costs of the business cycle under a particular environmental policy.

Our results point to a stark difference between the two regimes, with the carbon tax entailing costs about three times larger than those observed under a cap-and-trade. Table 2 provides some insight into this result, reporting mean and volatility values for a selection of macroeconomic variables under both environmental regimes, along with the implied welfare costs of the business cycle.[11]

As expected, the price instrument entails higher volatility than the quantity instrument for all the macroeconomic variables considered. The carbon tax also causes greater financial fragility than the cap, as highlighted by the higher mean and volatility of both the bankruptcy and spread indicators and the minor level of capitalization observed under the price regulation. On the environmental side, both instruments provide contrasting indications: emissions are lower in mean under the tax, but they are not stable over the business cycle. At the same time, relative compliance costs are reduced on average under the cap, but they are highly volatile since, as we have seen, they are strongly procyclical. This effect is mainly driven by the high volatility of the price of carbon. Considering all these effects, our measure of welfare costs of business cycles seems to favor a cap-and-trade regime.

Figure 3: Welfare Costs of the Business Cycle over Different Values of Risk and Leverage

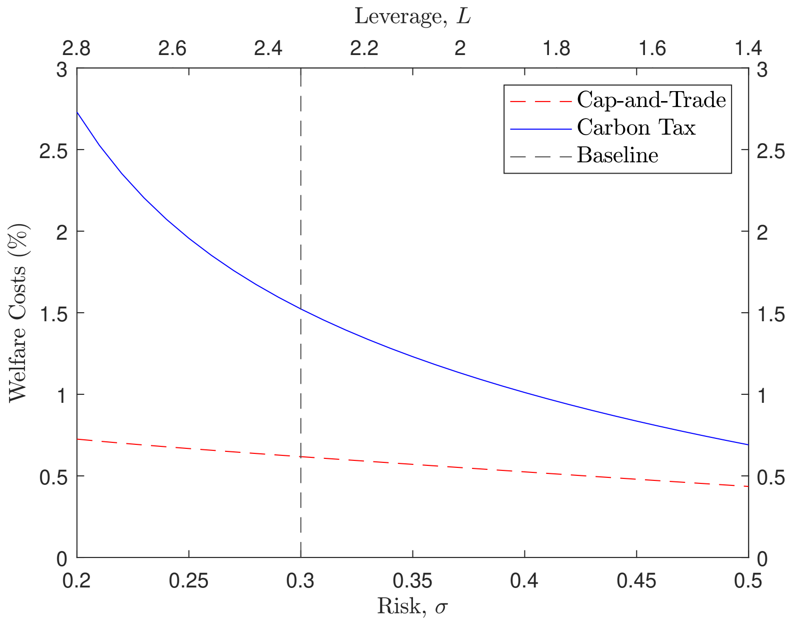


Table 3 reports the welfare costs under the two regimes for different values of a selection of parameters. An increase in the volatility σ of the random variable ω brings about a reduction in welfare costs under both scenarios, especially in the carbon tax case. In general terms, as the probability of a low ω rises, banks raise the interest rate

[11] In Appendix D we report the results using a welfare cost measure based on conditional welfare.

Table 2: Mean (and Volatility) for a Selection of Variables and Welfare Costs

	Cap-and-Trade	Carbon Tax
Net Output	-0.5691	-2.0569
	(0.0189)	(0.0360)
Consumption	-0.4837	-1.5935
	(0.0115)	(0.0196)
Investment	-0.9034	-3.8697
	(0.0113)	(0.0236)
Bankruptcy	0.5357	0.7028
	(0.0267)	(0.0345)
Net worth	3.0069	0.9986
	(0.7103)	(0.8225)
Spread	0.1573	0.2728
	(0.0082)	(0.0111)
Emissions	-	-4.1252
	-	(0.0073)
Carbon Price	-60.1306	-
	(0.1781)	-
Rel. Compliance Costs	-62.5687	0.8998
	(0.0029)	(0.0001)
Welfare costs	0.6178	1.5231

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate and the spread, reported as percentage point deviations. Standard deviations are in parentheses.

charged on loans to producers to cover the higher resulting costs, which, in turn, leads to lower borrowing by firms. This effect becomes even more evident in Figure 3: as risk rises, leverage decreases, and the welfare costs of the business cycle under the two alternative environmental policies converge. This exercise sheds light on one of the most relevant features of the analysis: when the volatility of production outcomes increases, firms opt for a reduction of their borrowing, i.e., the channel through which financial accelerator effects propagate. When the financial transmission mechanism is weakened, welfare costs unambiguously decrease and do so more intensively under the scenario in which financial effects are magnified.

Changing the abatement cost function coefficient by increasing θ_2 leaves the welfare costs under a carbon tax substantially unchanged, as the marginal abatement effort stays constant over the business cycle due to the fixed price for emissions. On the other hand,

Table 3: Welfare Costs of the Business Cycle over Different Values of Parameters

	Cap-and-trade	Carbon Tax
Baseline	0.6178	1.5231
σ = 0.2	0.7253	2.7296
σ = 0.4	0.5246	1.0111
θ_2 = 2	0.2477	1.5230
θ_2 = 3	0.7969	1.5233
μ= 0.1	0.5660	1.1905
μ = 0.3	0.6327	1.6679
μ = 0.8	0.6529	2.0039
RRA = 1.5	0.6045	1.4231
RRA = 3	0.6583	1.7854
RRA = 5	0.8618	3.1093

in a cap-and-trade scenario, uncertainty, by Jensen's inequality, implies higher average marginal costs bearing on polluting firms. For this reason, the stabilizing effect of the cap is reduced, and the policy entails higher welfare costs compared to the benchmark case, getting closer to the carbon tax regime.

Higher monitoring costs μ intensify imperfections of the financial markets as banks lose efficiency in collecting revenues and consequently lending. As the premium to be paid for external funds grows, following the higher costs for monitoring, the return on production diverges more from the risk-free interest rate, and the financial market failure gets deeper. Higher monitoring costs also translate into higher sensitivity of the premium on external funds to changes in the leverage position. In dynamic terms, increased financial market imperfections eventually induce greater economic volatility. By this, welfare costs rise as μ increases and more under the less stable scenario of a carbon tax. Under a cap-and-trade regime, the effects are milder because, as explained in the previous section, the acceleration mechanism is partially reversed.

Finally, Table 3 shows that a higher coefficient of relative risk aversion entails increased welfare costs, especially with tax regulation. As households are more risk-averse, their desire for smooth consumption increases and welfare costs rise and do so more under the regime that is more exposed to risk and variations over the business cycle.

Our simulation results point to substantial differences between the two environmental regimes. The fact that permit prices react endogenously to the business cycle favors cap-and-trade programs over taxes, that is why in the next section, we focus on optimal quantity and price environmental policy rules whose level of stringency varies with the business cycle.

4.3 Optimal Environmental Policy

So far, we have considered environmental policies that, like most real-world policies, do not explicitly respond to economic fluctuations and maintain a constant stringency over cycles.[12] In this section, we derive simple, optimal environmental policy rules according to which the cap on emissions or the carbon tax is allowed to vary in response to economic fluctuations. Since emission abatement is costly, while the damages from pollution are roughly smooth over the business cycle, it makes sense to design a flexible environmental policy rule that responds to short-run market conditions. To this end, we start by considering a hybrid environmental regulation according to which the level of cap adjusts endogenously to the deviation of the net output from its non-stochastic steady state, that is:

$$E_t = E\left(\frac{Y_t^n}{Y^n}\right)^{\nu}, \tag{26}$$

where variables without subscript are steady-state values. We search for the value of the parameter ν that minimizes welfare costs.[13] The first panel of Figure 4 displays the result and shows that the relationship between the policy parameter ν and the welfare cost is convex, with a minimum cost for ν at -2.3380. This result would suggest that a certain degree of temporal flexibility of the cap is desirable, with the cap level that moves countercyclically in the attempt to mitigate the negative impact of uncertainty on bank net worth and to reduce the bankruptcy rate. The welfare costs of the business cycle decline significantly under this optimal cap policy, as shown in Table 4.

Following the same approach, we then look for the optimal carbon tax rule, where the tax rate is designed to automatically adjust to changes in economic conditions:

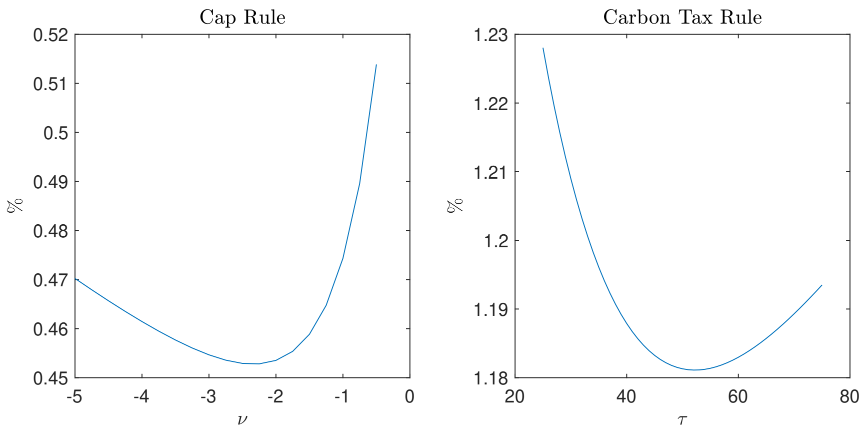
$$P_t^E = P^E\left(\frac{Y_t^n}{Y^n}\right)^{\tau}, \tag{27}$$

where τ is the policy parameter governing the response of the carbon tax. Figure 4 shows that the welfare costs of business cycles are minimized at $\tau = 52.2245$. Consistently with the previous result, the optimal carbon tax then prescribes a vigorous response

[12] Business-cycle adaptations can be considered of second-order importance, in contrast to the need to set the right degree of stringency of regulation that is instead seen as of first-order importance to limit environmental damages. See Annicchiarico et al. (2022) for a discussion.

[13] In our grid-search for the optimal cap rule the policy parameter ν is restricted to lie in the interval [-50, 50], with a step of size 0.0005.

Figure 4: Welfare Costs of the Business Cycle under Variable Carbon Pricing Rules



to current economic conditions with a significant reduction in the welfare costs of the business cycle.[14] Also, under this rule, the price of carbon is allowed to decrease when the net output goes down, that is to say that environmental regulation becomes permissive during recessions and strict during expansions. As a result of this lean-against-the-wind policy, the welfare cost of business cycles is substantially reduced, and the distortionary effects of macroeconomic volatility are milder than under the constant carbon pricing policy.

In both cases, the environmental regulation is designed to reduce the effects of the financial accelerator, stabilizing the economy in the face of shocks. It is essential to keep in mind, however, that this is an indirect way to tackle financial frictions and reduce the amplitude of macroeconomic fluctuations. The usually preferred way to deal with financial instability remains to use policy tools specifically designed for the task. [15] In the next section, we modify the baseline model by introducing macroprudential policy and explore whether there is scope to align the business cycle properties of environmental policies.[16]

[14]This strong reactivity of the carbon price to net output suggests that the optimal tax rule is designed to mimic the behavior of a cap where the reactivity of the permit price to economic fluctuations is similar in the order of magnitude. See Figures 1 and 2.

[15]Unlike what presented in this section, the standard design of climate policies should, at least traditionally, mainly concern itself with the achievement of climate targets. Reducing the financial frictions that amplify shocks and induce high volatility is not among the climate policy goals. Nevertheless, in extraordinary circumstances, countercyclical climate policies have been considered, in policy and academic circles, as a way to cushion economic shocks – a prominent example is the current rise in energy prices. For a discussion on countercyclical environmental regulations, see Dominioni and Faure (2022).

[16]In Appendix E, we show the dynamic response of the economy to shocks under optimal environmental policy rules.

Table 4: Means (and Volatility) and Welfare Costs under Optimal Environmental Policy Rules

	Optimal Cap-and-Trade	Optimal Carbon Tax
Net Output	-0.5325 (0.0108)	-1.5911 (0.0269)
Consumption	-0.4171 (0.0082)	-1.2266 (0.0149)
Investment	-0.9843 (0.0056)	-3.0171 (0.0176)
Bankruptcy	0.3113 (0.0182)	0.5975 (0.0294)
Net worth	1.0564 (0.6622)	0.9824 (0.7166)
Spread	0.1135 (0.0075)	0.2245 (0.0090)
Emissions	1.2752 (0.0034)	-3.0437 (0.0044)
Carbon Price	-3.6366 (0.3821)	-5.3199 (0.0788)
Rel. Compliance Costs	-17.1351 (0.0062)	-10.2686 (0.0012)
Welfare costs	0.4528	1.1811

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate and the spread, reported as percentage point deviations. Standard deviations are in parentheses.

5 Macroprudential Regulations

The previous section highlighted the relevance of the financial channel in defining the differences in welfare costs between a cap-and-trade and a carbon tax policy. It is now worth asking whether introducing a financial regulatory system can play a significant role in aligning the performance of different carbon pricing schemes, reducing the uncertainty surrounding their functioning over the business cycle.

Following Leduc and Natal (2018), we introduce a macroprudential policy akin to reserve requirements for lending institutions. In particular, banks are now required to keep a portion of their funds in reserves, which are assumed to be in "cash" and earn a zero rate of return.[17] Analytically, this leads to the following rewriting of the cash

[17]Notably, in most countries banks are required to hold a fraction of their funds as liquid assets on

constraint for banks:

$$(1-F(\bar{\omega}^N_{t+1}))B^N_{t+1}Z^N_{t+1}+(1-\mu)\int_0^{\bar{\omega}^N_{t+1}}\omega dF(\omega)(1+R^k_{t+1})Q_{K,t}K^N_{t+1} = \frac{B^N_{t+1}}{\Phi_t}(1+R_t)\,, \tag{28}$$

where Φ_t defines the fraction of deposits that banks can loan out. In other words, financial intermediaries must now issue B_{t+1}/Φ_t deposits to finance a quantity B_{t+1} of loans to firms.[18]

Reserve requirements are set so that conditions to extend loans tighten as financial activity accelerates, leading to the following general rule:

$$\Phi_t = \Phi^{\star}\left(FI_t\right)^{-\psi}\,, \quad \Phi_t \in (0,1], \tag{29}$$

where FI_t is an indicator of the level of financial activity. In the absence of reserve requirement, $\Phi_t = 1$, we retrieve the case analyzed until now. We set $\Phi^{\star}$ to 0.98, so that when $\psi = 0$, a static rule implying a reserve requirement of 2% is implemented.[19]

As Table 5 shows, introducing a similar regulation goes already a long way in reducing welfare costs under both environmental policies and making them converge. As banks are limited in the amount of funds they can convert into loans, credit is reduced in the economy, and firms have fewer resources to invest, which leads to lower leverage. As a result, the financial accelerator effect is weakened, and fluctuations are strongly dampened over the business cycle.

We now explore the role of dynamic macroprudential regulation where the parameter defining the sensitivity to financial activity indicators is set optimally to minimize welfare costs, while the steady-state level of reserves is kept at 2%. First, reserve requirements are allowed to vary countercyclically with respect to credit growth:

$$\Phi_t = \Phi^{\star}\left(\frac{B_{t+1}}{B_t}\right)^{-\psi^B}. \tag{30}$$

Credit growth represents an immediate choice as a financial indicator, given its role in

accounts at their national central bank. These are known as minimum reserves and function like a valve, allowing banks to face short-term needs for liquidity and unexpected changes in the interbank market where banks lend to each other.

[18] In equilibrium, now the condition $B_{t+1} = \Phi_t B^H_t$ must be satisfied.

[19] As in Leduc and Natal (2018), we opt to set the reserve requirement to 2% to reflect the average rate observed in industrialized countries in the period 1975-2011, as documented by Federico et al. (2014). Since then, this ratio has been significantly lowered by several central banks (i.e., to 1% in the euro area and 0% in the US).

Table 5: Welfare Costs of the Business Cycle under Macroprudential Regulations

	Cap-and-Trade	Optimal Cap-and-Trade
Baseline $\Phi_t = 1$	0.6178	0.4528 $\nu = -2.3380$
$\Phi_t = \Phi^\star$	0.1957	0.1883 $\nu = -0.3695$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-1.0465}$	0.1207	0.1164 $\nu = -0.3010$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-0.7220}$	0.1807	0.1776 $\nu = -0.2345$
	Carbon Tax	Optimal Carbon Tax
Baseline $\Phi_t = 1$	1.5231	1.1811 $\tau = 52.2245$
$\Phi_t = \Phi^\star$	0.3863	0.3455 $\tau = 24.2990$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-0.9935}$	0.3231	0.2697 $\tau = 44.0400$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-0.6790}$	0.2300	0.2215 $\tau = 9.4645$

amplifying economic imbalances in periods leading to financial crises (see, i.e., Schularick and Taylor 2012). In this respect, and to our purposes, anchoring the reserve requirement to credit growth is a way of introducing macroprudential regulation consistently with this model setting and in the spirit of the Basel III accord that suggests countercyclical capital requirements in periods of high credit growth.[20]

As a second option, we also link reserve requirements to the variation in asset prices:

$$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-\psi^Q} . \tag{31}$$

As the price of assets in the model reflects the conditions at which capital is traded and, consequently, net worth levels, it represents another straightforward indicator summa-

[20]The use of reserve requirements as a macroeconomic stabilization tool is well documented in Federico et al. (2014) showing that mainly emerging economies have used this tool. However, in the face of the recent coronavirus crisis, the Federal Reserve stepped in with a broad array of actions to keep credit flowing, including the elimination of banks' reserve requirements.

rizing the movements in the overall financial activity.[21]

Table 5 shows that implementing any of the two optimal dynamic rules improves the static regulation: welfare costs decrease significantly under both environmental scenarios. This is because the financial regulation is designed for engineering a procyclical response of the spread that stabilizes the economy and reverses the financial accelerator mechanism.[22] It is interesting to notice that under a cap-and-trade regime, an optimal rule reacting to credit growth leads to lower welfare costs than a rule responding to asset price growth. Under a tax policy, the reverse is true.

Finally, in the last column of Table 5, we derive the optimal environmental policy rules under different macroprudential regulatory frameworks and report the related welfare costs. As expected, when financial regulations already stabilize the economy, there is less scope for reducing welfare costs via environmental policy. In this case, in fact, both optimal environmental rules prescribe a weaker response to current economic conditions than in the absence of macroprudential regulation.

Finally, to explore whether the macroprudential policy can alleviate the uncertainty inherent to the chosen environmental policy tool under a climate policy shock in Table 6 we report the coefficient of variation for emissions, CV_E, and the permit price, CV_{PE}, under different policy combinations. Results show the non-trivial role of macroprudential regulation in fostering symmetry between quantity and price environmental regulations. Under a cap-and-trade regime, the volatility of allowances prices sharply declines when we introduce reserve requirements. Similarly, under a carbon tax, financial regulation substantially stabilizes emissions reducing the uncertainty surrounding the use of price regulation. Volatility declines more under dynamic financial rules, especially when the reserve requirement adjusts to credit growth.

6 Conclusions

Even though there is a general agreement among economists and policy analysts that ambitious climate actions are needed to reduce greenhouse gas emissions and limit climate disasters, considerable debate continues on the choice of measures to tackle this problem. Under uncertainty, the close connection between cap-and-trade and carbon tax options becomes more challenging, and their comparative performance gives ambiguous

[21] In our search for the optimal macroprudential rule the policy parameters ψ^B and ψ^Q are restricted to lie in the interval [-5, 5], with a step of size 0.0005. Since $\Phi_t \in (0, 1]$, we rule out solutions such that $\Phi_t + 2\sigma_\Phi > 1$, where σ_Φ is the standard deviation of Φ_t.

[22] In Appendix F, we show the response of the economy to TFP and risk shocks under the optimal credit rule for both environmental regimes.

Table 6: Volatility of Emissions and Permit Prices under Macroprudential Regulations

	Cap-and-Trade	Optimal Cap-and-Trade
Baseline $\Phi_t = 1$	$CV_E = 0$ $CV_{PE} = 8.3578$	$CV_E = 0.0255$ $CV_{PE} = 7.4192$
$\Phi_t = \Phi^\star$	$CV_E = 0$ $CV_{PE} = 2.5125$	$CV_E = 0.0036$ $CV_{PE} = 2.7692$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-1.0465}$	$CV_E = 0$ $CV_{PE} = 1.0935$	$CV_E = 0.0024$ $CV_{PE} = 1.3120$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-0.7220}$	$CV_E = 0$ $CV_{PE} = 2.3682$	$CV_E = 0.0024$ $CV_{PE} = 2.5130$
	Carbon Tax	Optimal Carbon Tax
Baseline $\Phi_t = 1$	$CV_E = 0.0550$ $CV_{PE} = 0$	$CV_E = 0.0343$ $CV_{PE} = 1.5576$
$\Phi_t = \Phi^\star$	$CV_E = 0.0271$ $CV_{PE} = 0$	$CV_E = 0.0220$ $CV_{PE} = 0.3962$
$\Phi_t = \Phi^\star \left(\frac{B_{t+1}}{B_t}\right)^{-0.9935}$	$CV_E = 0.0152$ $CV_{PE} = 0$	$CV_E = 0.0091$ $CV_{PE} = 0.4921$
$\Phi_t = \Phi^\star \left(\frac{Q_{K,t}}{Q_{K,t-1}}\right)^{-0.6790}$	$CV_E = 0.0282$ $CV_{PE} = 0$	$CV_E = 0.02610$ $CV_{PE} = 0.1637$

Note: The table reports the coefficient of variation as a measure of volatility.

results.

This paper contributes to these long-lasting discussions by comparing the welfare cost of business cycles under quantity and price regulations in an environmental dynamic stochastic general equilibrium model with heterogeneous polluting firms that borrow from banks and may default. Under a carbon tax scenario, the financial accelerator mechanism renders the economy more prone to business cycle fluctuations. By contrast, under a cap-and-trade system, financial frictions tend to stabilize business cycle fluctuations since, under this environmental regulation, the financial accelerator mechanism is reversed. As a result, a cap-and-trade system produces a more stabilized environment and entails substantially lower welfare costs of business cycles than a carbon tax. On the other hand, the ability of the cap to dampen business cycle fluctuations crucially depends on the degree of leverage of the economy: the lower the leverage, the smaller and the closer the welfare costs of fluctuations are under the two alternative regimes. Our results suggest that the better performance of one instrument over another also depends on other features of the model at hand, such as the degree of risk aversion and the convexity of the abatement cost function. The choice of either policy must then be assessed, taking into account other market failures and underlying policies of the economy under analysis.

A macroprudential regulation that links reserve requirements to credit growth or asset prices forces a smoothed response of real variables under both environmental regimes, significantly reducing the welfare cost of business cycles. In other words, a macroprudential regulatory framework able to minimize the distorting effects of financial frictions and stabilize the economy can *de facto* align the performance of different carbon pricing schemes reducing the uncertainty surrounding their functioning over the business cycle. In this respect, our results have important policy implications when discussing the potential role that central banks and financial regulators may have during the transition to a low-carbon economy. By stabilizing financial markets and the economy, central banks and financial regulators can help reduce the uncertainty inherent to each environmental policy tool, thus enlarging the array of policy options on the table.

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Appendix A

This appendix gives details on the derivation of the debt contract chosen by intermediate-good producers. First, let's recall some relevant functions:

$$G_t(\bar{\omega}^N_{t+1}) = \int_0^{\bar{\omega}^N_{t+1}} \omega dF(\omega)$$
$$\Gamma_t\left(\bar{\omega}^N_{t+1}\right) = (1 - F_t(\bar{\omega}^N_{t+1}))\bar{\omega}^N_{t+1} + G_t(\bar{\omega}^N_{t+1}),$$
$$G'_t(\bar{\omega}^N_{t+1}) = \frac{d}{d\omega}\int_0^{\bar{\omega}^N_{t+1}} \omega dF(\omega) = \bar{\omega}^N_{t+1}F'_t(\bar{\omega}^N_{t+1}),$$
$$\Gamma'_t\left(\bar{\omega}^N_{t+1}\right) = 1 - F_t(\bar{\omega}^N_{t+1}) - \bar{\omega}^N_{t+1}F'_t(\bar{\omega}^N_{t+1}) + G'_t(\bar{\omega}^N_{t+1}) = 1 - F_t(\bar{\omega}^N_{t+1}).$$

Intermediate good producers value a particular debt contract according to the expected return from operating risky technology over the return from depositing net worth in a bank, that is:

$$\frac{\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left[\omega(1+R^k_{t+1})Q_{K,t}K^N_{t+1} - B^N_{t+1}Z^N_{t+1}\right]dF(\omega)\right\}}{N\left(1+R_t\right)}. \quad \text{(A-1)}$$

Recall $\bar{\omega}^N_{t+1}K^N_{t+1}\left[(1+R^k_{t+1})Q_{K,t}\right] = B^N_{t+1}Z^N_{t+1}$, then:

$$\frac{\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\left(\omega - \bar{\omega}_{t+1}\right)(1+R^k_{t+1})Q_{K,t}K^N_{t+1}dF(\omega)\right\}}{N\left(1+R_t\right)}.$$

Using $Q_{K,t}K^N_{t+1} = N + B^N_{t+1}$ and $L^N_t = (N + B^N_{t+1})/N$, we get:

$$\frac{\mathbb{E}_t\left\{\int_{\bar{\omega}^N{}_{t+1}}^{\infty}\left(\omega - \bar{\omega}_{t+1}\right)(1+R^k_{t+1})L^N_t N dF(\omega)\right\}}{N\left(1+R_t\right)} =$$
$$\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\omega dF(\omega) - \int_{\bar{\omega}^N_{t+1}}^{\infty}\bar{\omega}^N_{t+1}dF(\omega)\right\}\frac{1+R^k_{t+1}}{1+R_t}L^N_t.$$

Solving the second integral by parts, we obtain:

$$\mathbb{E}_t\left\{\int_{\bar{\omega}^N_{t+1}}^{\infty}\omega dF(\omega) - \bar{\omega}^N_{t+1}F(\infty) + \bar{\omega}^N_{t+1}F(\bar{\omega}^N_{t+1}) - \int_{\bar{\omega}^N_{t+1}}^{\infty}0dF(\omega)\right\}\frac{1+R^k_{t+1}}{1+R_t}L^N_t =$$
$$\mathbb{E}_t\left\{1 - G_t(\bar{\omega}^N_{t+1}) - (1 - F_t(\bar{\omega}^N_{t+1}))\bar{\omega}^N_{t+1}\right\}\frac{1+R^k_{t+1}}{1+R_t}L^N_t =$$
$$\mathbb{E}_t\left[1 - \Gamma_t\left(\bar{\omega}^N_{t+1}\right)\right]\frac{1+R^k_{t+1}}{1+R_t}L^N_t. \quad \text{(A-2)}$$

On the side of banks, we start from the cash constraint

$$(1 - F(\bar{\omega}^N_{t+1}))B^N_{t+1}Z^N_{t+1} + (1-\mu)\int_0^{\bar{\omega}^N_{t+1}}\omega dF(\omega)(1+R^k_{t+1})Q_{K,t}K^N_{t+1} = B^N_{t+1}\left(1+R_t\right). \quad \text{(A-3)}$$

Recalling the cutoff definition $\bar{\omega}_{t+1}^{N} K_{t+1}^{N_{t+1}}\left[(1+R_{t+1}^{k})Q_{K,t}\right] = B_{t+1}^{N_{t+1}} Z_{t+1}^{N}$ again and $L_t^N = Q_{K,t}K_{t+1}^N/N$ or $L_t^N = (N + B_{t+1}^N)/N$, we can rewrite (A-3) as:

$$(1-F_t(\bar{\omega}_{t+1}^N))\bar{\omega}_{t+1}^N + (1-\mu)\int_0^{\bar{\omega}_{t+1}^N} \omega dF(\omega) = \frac{1+R_t}{1+R_{t+1}^k}\frac{B_{t+1}^N}{Q_{K,t}K_{t+1}^N} =$$
$$(1-F_t(\bar{\omega}_{t+1}^N))\bar{\omega}_{t+1}^N + (1-\mu)\int_0^{\bar{\omega}_{t+1}^N} \omega dF(\omega) = \frac{1+R_t}{1+R_{t+1}^k}\frac{L_t^N-1}{L_t^N}.$$

Making use of the previously defined functions, we can eventually recast the zero-profit condition for banks as:

$$\Gamma_t\left(\bar{\omega}_{t+1}^N\right) - \mu G_t(\bar{\omega}_{t+1}^N) = \frac{1+R_t}{1+R_{t+1}^k}\frac{L_t^N-1}{L_t^N} \tag{A-4}$$

which can also be rewritten in terms of leverage as:

$$L_t^N = \frac{1}{1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}^N\right) - \mu G_t(\bar{\omega}_{t+1}^N)\right]} \quad \text{or} \tag{A-5}$$
$$Q_{K,t}K_{t+1}^N = \frac{N}{1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}^N\right) - \mu G_t(\bar{\omega}_{t+1}^N)\right]}.$$

Intermediate-good producers finally choose the contract that maximizes their objective (A-2) among the $(\bar{\omega}_{t+1}^N, L_t^N)$ combinations satisfying (A-5). Since the constraint is independent of net worth, we get rid of the superscript N from now on, and we have:

$$\max_{\{L_t,\bar{\omega}_{t+1}\}} \mathbb{E}_t\left[1-\Gamma_t\left(\bar{\omega}_{t+1}\right)\right]\frac{1+R_{t+1}^k}{1+R_t}L_t$$
$$\text{s.t.}\quad L_t = \frac{1}{1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]}.$$

The problem can be rephrased as:

$$\max_{\bar{\omega}_{t+1}} \frac{1}{1+R_t}\mathbb{E}_t(1+R_{t+1}^k)\frac{\left[1-\Gamma_t\left(\bar{\omega}_{t+1}\right)\right]}{1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]}.$$

The first-order condition is then found to be:

$$\frac{\left[-\Gamma_t'\left(\bar{\omega}_{t+1}\right)\right]\left(1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]\right)}{\left(1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]\right)^2} +$$
$$-\frac{\left[1-\Gamma_t\left(\bar{\omega}_{t+1}\right)\right]\left(-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t'\left(\bar{\omega}_{t+1}\right) - \mu G_t'(\bar{\omega}_{t+1})\right)\right.}{\left(1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]\right)^2} = 0.$$

Rearranging, we get:

$$\frac{\Gamma_t'\left(\bar{\omega}_{t+1}\right)}{1-\Gamma_t\left(\bar{\omega}_{t+1}\right)}=\frac{\frac{1+R_{t+1}^k}{1+R_t}[\Gamma_t'\left(\bar{\omega}_{t+1}\right)-\mu G_t'(\bar{\omega}_{t+1})]}{1-\frac{(1+R_{t+1}^k)}{1+R_t}[\Gamma_t\left(\bar{\omega}_{t+1}\right)-\mu G_t(\bar{\omega}_{t+1})]}. \tag{A-6}$$

Recalling again the functions defined initially, we eventually obtain the form of the intermediate-good producers' FOC:

$$\frac{1-F_t\left(\bar{\omega}_{t+1}\right)}{1-\Gamma_t\left(\bar{\omega}_{t+1}\right)}=\frac{\frac{1+R_{t+1}^k}{1+Rt}[1-F_t\left(\bar{\omega}_{t+1}\right)-\mu\bar{\omega}_{t+1}F_t'(\bar{\omega}_{t+1})]}{1-\frac{1+R_{t+1}^k}{1+R_t}[\Gamma_t\left(\bar{\omega}_{t+1}\right)-\mu G_t(\bar{\omega}_{t+1})]}. \tag{A-7}$$

All firms eventually select the same debt contract that can be represented as $(\bar{\omega}_{t+1}, L_t)$ or equivalently as (Z_{t+1}, L_t), irrespective of their net worth.

Appendix B

This appendix describes the equilibrium conditions for the economy, which is characterized by 20 endogenous variables $\{C_t,\ H_t,\ \lambda_t,\ X_t,\ K_t,\ E_t,\ \kappa_t,\ P_t^E,\ R_t^k,\ r_t^x,\ Q_{K,t},\ N_t,\ R_t,\ \bar{\omega}_t,\ Y_t,\ A_t,\ W_t,\ M_t,\ I_t,\ Tr_t\}$ and two exogenous processes $\{\bar{A}_t,\ \sigma_t\}$.
According to environmental policy regime in place, an additional equation has to be considered. Under a cap-and-trade regime, $E_t = \bar{E}$, while under a carbon tax, $P_t^E = \bar{P}^E$.

Households

$$\sigma_L \left(C_t^{\sigma_L}(1-H_t)^{1-\sigma_L}\right)^{-\eta} C_t^{\sigma_L-1}(1-H_t)^{1-\sigma_L} = \lambda_t \tag{B-1}$$

$$(1-\sigma_L)\left(C_t^{\sigma_L}(1-H_t)^{1-\sigma_L}\right)^{-\eta}(1-H_t)^{-\sigma_L}C_t^{\sigma_L} = W_t\lambda_t \tag{B-2}$$

$$\lambda_t = (1+R_t)\,\beta\,\mathbb{E}_t\,\lambda_{t+1} \tag{B-3}$$

Intermediate-Good Producers and Banks

$$X_t = K_t \tag{B-4}$$

$$E_t = \chi(1-\kappa_t)K_t \tag{B-5}$$

$$\theta_1\theta_2\kappa_t^{\theta_2-1} = \chi P_t^E \tag{B-6}$$

$$1+R_{t+1}^k = \frac{r_{t+1}^x + (1-\delta)Q_{K,t+1} - \theta_1(\kappa_{t+1})^{\theta_2} - P_{t+1}^E\chi(1-\kappa_{t+1})}{Q_{K,t}} \tag{B-7}$$

$$Q_{K,t}K_{t+1} = \frac{N_{t+1}}{1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]} \tag{B-8}$$

$$\frac{1-F_t\left(\bar{\omega}_{t+1}\right)}{1-\Gamma_t\left(\bar{\omega}_{t+1}\right)} = \frac{\frac{1+R_{t+1}^k}{1+R_t}\left[1-F_t\left(\bar{\omega}_{t+1}\right) - \mu\bar{\omega}_{t+1}F_t'(\bar{\omega}_{t+1})\right]}{1-\frac{1+R_{t+1}^k}{1+R_t}\left[\Gamma_t\left(\bar{\omega}_{t+1}\right) - \mu G_t(\bar{\omega}_{t+1})\right]} \tag{B-9}$$

$$N_{t+1} = \gamma\left[1-\Gamma_{t-1}\left(\bar{\omega}_t\right)\right](1+R_{t+1}^k)Q_{K,t-1}K_t + W_t^p \tag{B-10}$$

Final-Good Producers

$$Y_t = A_t X_t^{\alpha} {H_t}^{1-\alpha} \tag{B-11}$$

$$W_t = (1-\alpha)\frac{Y_t}{H_t} \tag{B-12}$$

$$r_t^x = \alpha\frac{Y_t}{X_t} \tag{B-13}$$

$$A_t = \bar{A}_t\left(1-D_t(M_t)\right) \tag{B-14}$$

Capital-Good Producers

$$K_{t+1} = (1-\delta)K_t + \left(1 - \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t \tag{B-15}$$

$$\begin{aligned} Q_{K,t} =& 1 + Q_{K,t}\frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \gamma_I Q_{K,t}\left(\frac{I_t}{I_{t-1}} - 1\right)\frac{I_t}{I_{t-1}} + \\ & - \gamma_I \beta\, \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} Q_{K,t+1}\left(\frac{I_{t+1}}{I_t} - 1\right)\frac{I_{t+1}^2}{I_t^2} \end{aligned} \tag{B-16}$$

where we have assumed $S(I_t/I_{t-1}) = \frac{\gamma_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$.

Pollution and Damage

$$1 - D_t(M_t) \equiv \exp\left(-\xi\left(M_t - \bar{M}\right)\right)$$

$$M_t = (1-\delta_M)\, M_{t-1} + E_t + E_t^* \tag{B-17}$$

Market Clearing

$$Y_t = I_t + C_t + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega)(1 + R_{t+1}^k) Q_{K,t-1} K_t + \theta_1 (\kappa_t)^{\theta_2} K_t \tag{B-18}$$

$$E_t P_t^E = Tr_t \tag{B-19}$$

Exogenous Processes

$$\log \bar{A}_t = \rho_A \log \bar{A}_{t-1} + (1-\rho_A)\log A + \varepsilon_{A,t} \tag{B-20}$$

$$\log \sigma_t = \rho_\sigma \log \sigma_{t-1} + (1-\rho_\sigma)\log \sigma + \varepsilon_{\sigma,t} \tag{B-21}$$

where variables without subscript are steady-state values

Appendix C

Starting from model's variables consumption C, investments I and net output Y^n, we define $c_t = log(C_t) - log(C)$, $i_t = log(I_t) - log(I)$ and $y_t^n = log(Y_t^n) - log(Y^n)$. We then compute the simulated moments generated by the model and compare them to those observed in the US data. Table C-1 reports the results.

Table C-1: Model and Data - Moments

	Model	Data
Standard Deviation		
σ_{y^n}	0.010	0.010
σ_i/σ_{y^n}	3.72	4.67
σ_c/σ_{y^n}	0.77	0.85
Cross-Correlations		
ρ_{i,y^n}	0.80	0.89
ρ_{c,y^n}	0.67	0.92
First-Order Autocorrelation		
ρ_{y^n}	0.79	0.90
ρ_i	0.94	0.88
ρ_c	0.67	0.86

Note: the table reports the moments generated by the model (under carbon tax) and those of the US HP-filtered quarterly data over the period 1985Q1-2019Q4, retrieved from FRED.

Appendix D

Table D-1 provides results on welfare costs using a different measure based on the conditional expectation of lifetime utility. The difference in relative terms between regimes is robust to this alternative specification, with the carbon tax still entailing costs more than two times larger than those observed under a cap.
Some insight into this result is provided by reporting the risk-adjusted steady state for a selection of macroeconomic variables under both environmental regimes. Following Coeurdacier et al. (2011, p. 398), we define the risky steady state as "the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is zero at this date". In the table, we report the risky steady state in deviations from the deterministic steady state to give a more precise idea of the effects of uncertainty on the economy.

Table D-1: Risk-Adjusted Steady State for a Selection of Variables and Welfare Costs

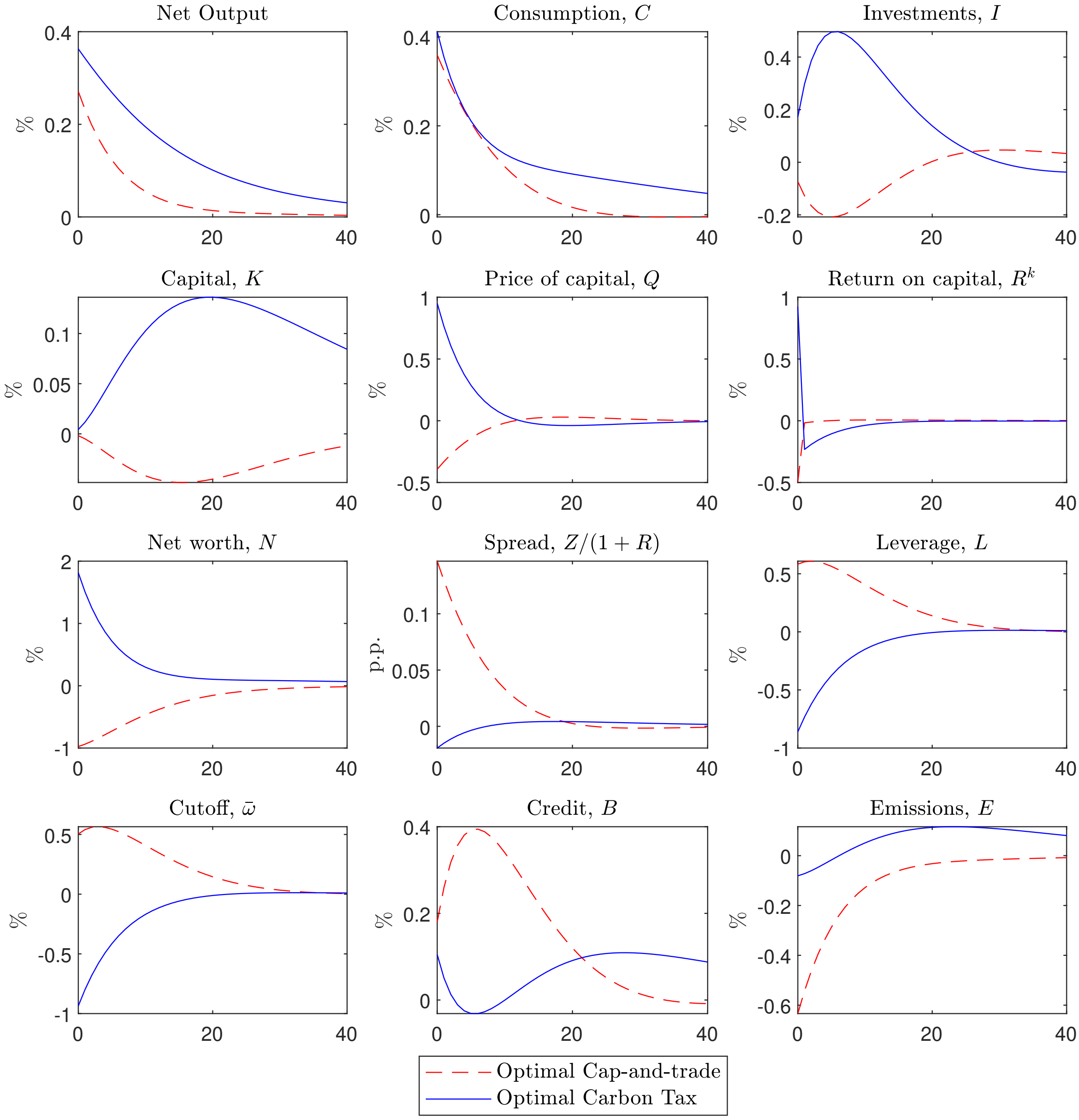
	Cap-and-Trade	Carbon Tax
Net Output	-0.1842	-0.5066
Consumption	-0.0687	-0.1871
Investment	-0.6358	-1.7565
Bankruptcy	0.3239	0.8882
Net worth	-5.0259	-13.7821
Spread	0.0985	0.2701
Welfare costs	0.0047	0.0098

Note: Results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate and the spread, reported as percentage point deviations.

Appendix E

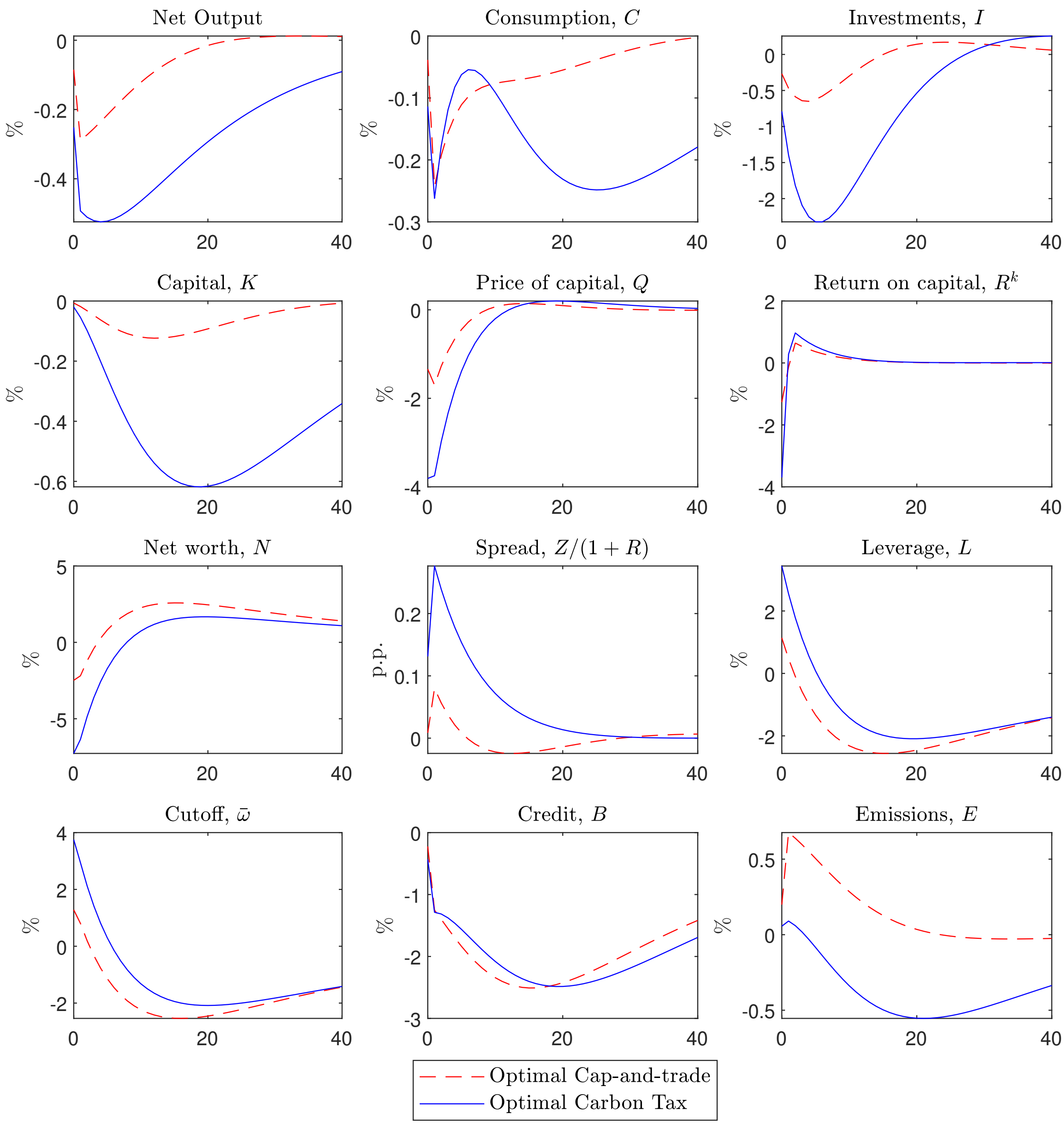
Figures E-1 and E-2 show the response of the economy to positive shocks to TFP and risk under the two optimal environmental policies (with no active macroprudential instrument).

Figure E-1: Dynamic Response to a One-Standard-Deviation TFP Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread, reported as percentage point deviations from the steady state.

Figure E-2: Dynamic Response to a One-Standard-Deviation Risk Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread, reported as percentage point deviations from the steady state.

Appendix F

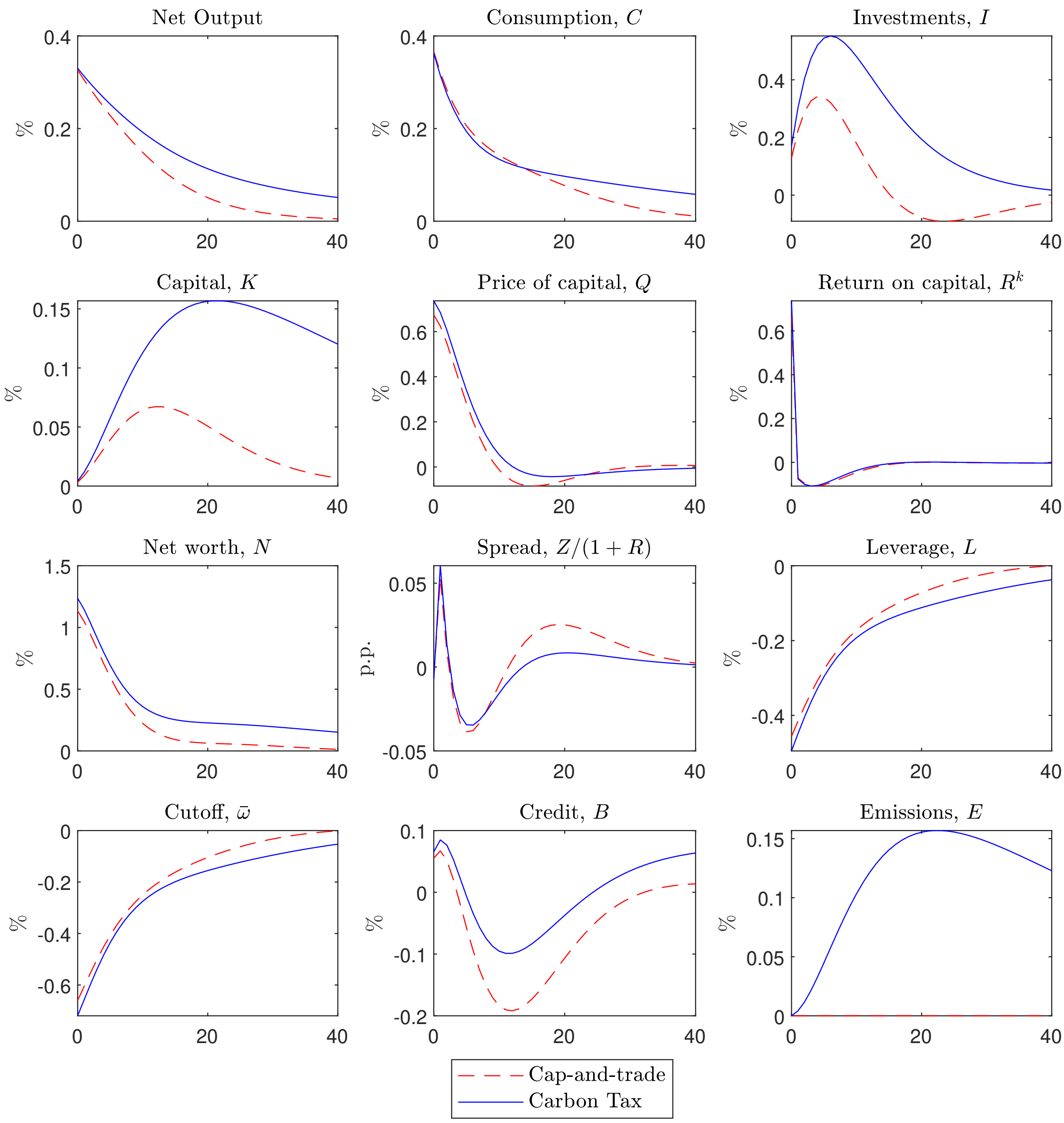
Figures F-1 and F-2 display the impulse response of the economy to positive shocks to TFP and risk under a macroprudential policy optimally responding to credit growth. Tables F-1 and F-2 report moments generated by the model under a static and an optimal macroprudential policy.

Table F-1: Mean (and Volatility) for a Selection of Variables and Welfare Costs under Static Macroprudential Policy Rule

	Cap-and-Trade	Carbon Tax
Net Output	-0.2007 (0.0089)	-0.4796 (0.0147)
Consumption	-0.1672 (0.0069)	-0.3864 (0.0098)
Investment	-0.3655 (0.0044)	-0.9397 (0.0089)
Bankruptcy	0.1694 (0.0083)	0.1856 (0.0088)
Net worth	5.4258 (0.3884)	5.0493 (0.3544)
Spread	0.0437 (0.0025)	0.0670 (0.0027)
Emissions	- -	-0.9951 (0.0025)
Carbon Price	-19.7031 (0.1078)	- -
Rel. Compliance Costs	-20.6689 (0.0018)	0.1574 (0.00003)
Welfare costs	0.1957	0.3863

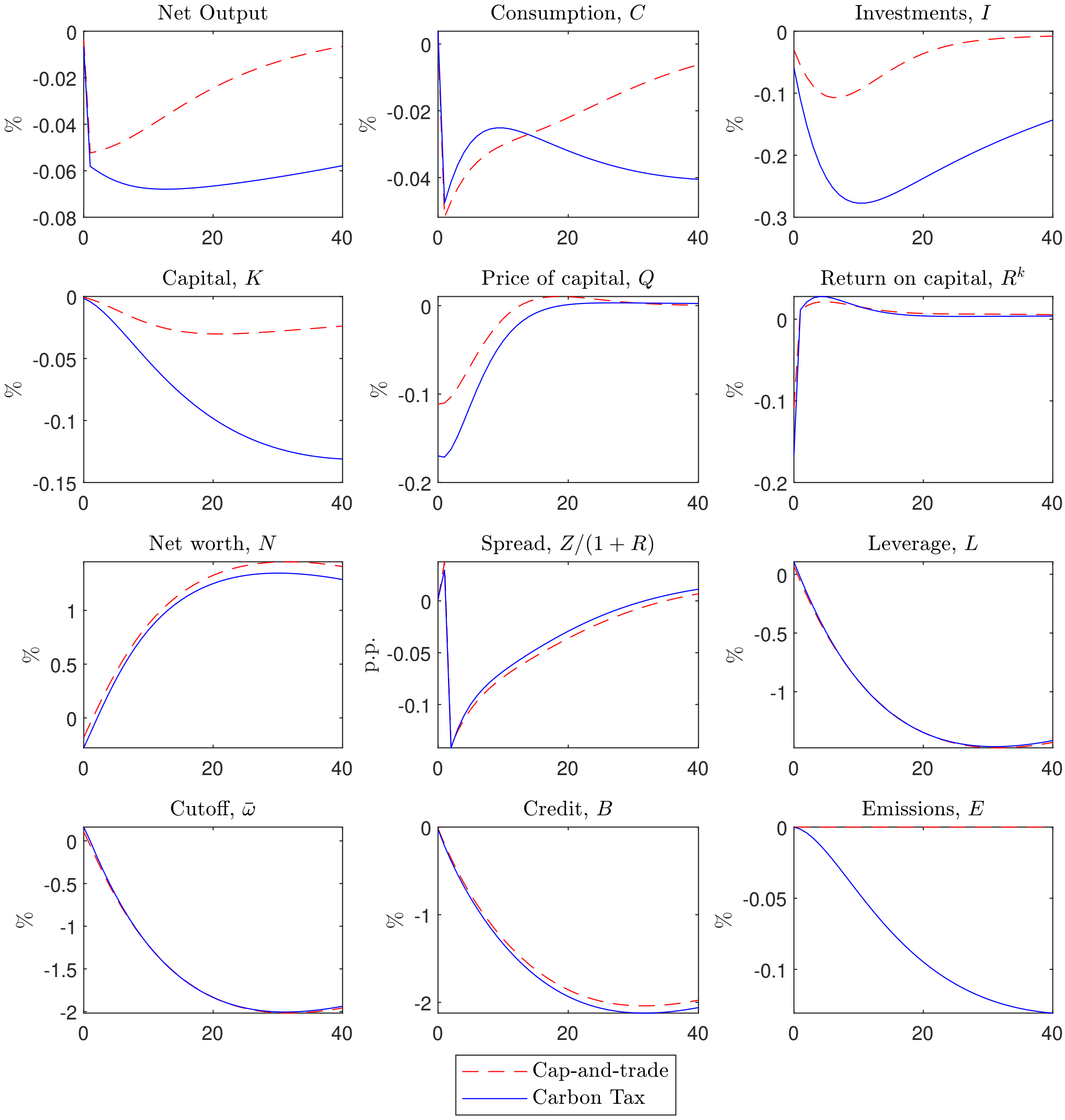
Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate and the spread, reported as percentage point deviations. Standard deviations are in parentheses.

Figure F-1: Dynamic Response to a One-Standard-Deviation TFP Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread, reported as percentage point deviations from the steady state.

Figure F-2: Dynamic Response to a One-Standard-Deviation Risk Shock



Note: Results are reported as percentage deviations from the initial steady state, except for the spread, reported as percentage point deviations from the steady state.

Table F-2: Mean (and Volatility) for a Selection of Variables and Welfare Costs under a Macroprudential Policy Rule Optimally Responding to Credit Growth

	Cap-and-Trade	Carbon Tax
Net Output	-0.0957	-0.4256
	(0.0075)	(0.0102)
Consumption	-0.0836	-0.3349
	(0.0063)	(0.0072)
Investment	-0.1550	-0.8731
	(0.0016)	(0.0036)
Bankruptcy	0.1548	0.1561
	(0.0073)	(0.0074)
Net worth	5.1327	4.3272
	(0.3847)	(0.3547)
Spread	0.0340	0.0573
	(0.0047)	(0.0045)
Emissions	-	-0.8819
	-	(0.0014)
Carbon Price	-26.3458	-
	(0.0430)	-
Rel. Compliance Costs	-26.4482	0.0103
	(0.0007)	(0.00001)
Welfare costs	0.1207	0.3231

Note: Mean results are reported in percentage deviations from the deterministic steady state, except for the bankruptcy rate and the spread, reported as percentage point deviations. Standard deviations are in parentheses.