Non-performing loans and inefficient capital reallocation

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Abstract

A high share of non-performing bank loans in total bank loans (sNPL) has been shown to negatively affect aggregate investment and economic growth. While these empirical facts have been well established, neither the mechanisms causing the great cross-country heterogeneity in the sNPL, nor the channels through which they affect the real economy are well understood. A commonly invoked channel is that the sNPL leads to reduced credit supply. This paper first shows that focusing solely on this channel would provide an incomplete picture. Reduced credit supply implies higher rates of return to capital in economies with a higher non-performing loan burden as profitable projects are not met with sufficient credit supply. This can neither be observed in country cross-sections nor in time series data. The more important channel through which the sNPL affects the economy seems to stem from the credit demand side with the sNPL providing a mirror image of real capital misallocation. The paper then proposes a structural model with search frictions in used capital markets that links the sNPL with frictions in capital reallocation to explain the observed sNPL and investment dynamics. The structural model shows that long and persistent sNPL increases in response to a negative shock, are either a symptom of a low option value of foreclosure for banks due to inefficient used capital markets or a symptom of forbearance incentives for banks due to balance sheet weaknesses and regulatory requirements. Both frictions are captured parsimoniously in the model. The fact that these frictions imply different impulse responses for capital prices is used to identify and estimate the extent to which they drive variation in the sNPL over the business cycle. Both inefficiencies lead to more misallocated capital and reduce the marginal product of fresh capital, thereby impacting credit creation. A higher sNPL following a negative shock, such as the Covid-19 pandemic, will lead to more prolonged output and investment below equilibrium in countries with less efficient used capital markets and larger increases in forbearance incentives. The tractable model can provide an explanation for observed sNPL and macroeconomic outcomes and presents and identification and estimation of the two identified drivers of the sNPL over the business cycle, used capital market inefficiencies and forbearance incentives.

Keywords: *Business cycles, Search frictions, Investment, Capital allocation, Non-performing loans*

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"... At the end of 2016, the stock of gross NPLs in the EU banking sector was around € 1 trillion. This number, however, does not take into account the fact that that collateralised lending plays an important role in Europe. For example, including collateral and provisioning, the coverage of NPLs is, on average, 82% in the euro area ...
... The outstanding stock of NPLs is a consequence of cyclical and structural factors. First, the severe recession resulting from the global financial crisis led to a deterioration of the quality of banks' loan books... At the same time, structural weaknesses still persist. These include inadequate internal governance structures in banks, ineffective and costly debt recovery procedures in some Member States and misaligned incentives that prevent a quick resolution of NPLs."[1]

1 Introduction

The share of non-performing bank loans in total loans (sNPL) in a country has received a lot of attention by policymakers ever since the global financial crisis in 2007 led to a persistent rise of the share in some countries. The recession induced by the Covid-19 pandemic is expected to lead to a similar rise (Kasinger et al., 2021). A high share of badly performing loans in total loans and subdued aggregate output go hand in hand. For good reasons, both have been suspected to be the cause of the other. The sNPL may increase when negative supply or demand shocks cause the economy to contract, causing previously profitable firms and solvent households to default on payments. On the other hand, a high sNPL may lead to more restrictive lending and investment keeping the economy subdued. This paper presents the behaviour of the sNPL over the business cycle and shows then that an inefficient capital allocation and a high sNPL go hand in hand. The paper then builds on existing and predominantly empirical literature on the connection between the sNPL and macroeconomic performance to develop a structural macroeconomic model capturing sNPL statics and dynamics. Based on the model, it presents new insights explaining the highly different reaction of the sNPL to macroeconomic shocks for different countries. Specifically, the model allows for an evaluation of the importance of forbearance incentives and used capital market efficiencies. Judging which of these fictions become more prevalent in an economy during a recession, provides a basis for formulating and prioritising the most effective policies to resolve NPLs across Europe.

The paper first shows that increases in the sNPL go hand in hand with reductions in real capital returns, and provides evidence of the sNPL being linked to real capital misallocation. In a next step, a structural model with search frictions in used capital markets is developed to show that sNPL can be understood as a symptom of these capital market frictions which will encourage banks to maintain loans in low quality matches. Concretely, two kinds of frictions are studied. The first is the relative ease with which used capital can be reused compared to fresh capital relating to the efficiency of used capital markets. The second friction are forbearance incentives which mean banks incurring real cost when call-

[1] Speech by Mario Draghi, at the time President of the ECB and Chair of the European Systemic Risk Board, at the second annual conference of the ESRB, Frankfurt am Main, 21 September 2017

ing a loan. Both forbearance frictions and inefficiencies in used capital markets are shown to be drivers of the sNPL in the model. The paper then uses the model to estimate how these frictions vary over the business cycle in different economies. The paper finds inefficient used capital markets to be a more important driver of sNPL in countries with high sNPL, meaning that NPLs may be understood in these countries as a symptom of an economy's inability to efficiently reallocate capital from unproductive to productive use. Finally, the paper finds that changes in frictions in used capital markets explain more of the variation in sNPL over the business cycle.

Even though non-performing loans have recently taken a center stage in the policy discussion in many countries, we know little about what the sNPL tell us about the state or dynamic response of a country's economy to macroeconomic shocks. Figure 1 shows that much like unemployment the sNPL is commonly cyclical, and remains persistently elevated following a recession. The right graph in Figure 1 shows that the speed of recovery a recession by means of investment correlated with the ability of a country to keep NPL stocks low following the negative shock as that this relationship did not change with the great increase in NPLs during the great recession. Recently, a growing empirical and policy-oriented literature has emphasized this negative correlation of NPL stocks on consumption, investment, and more generally on the macroeconomic performance of a country. A non-exhaustive list of empirical and policy papers in this area include (Louzis et al., 2012), (Klein, 2013), (Beck et al., 2013), (Jassaud and Kang, 2015), and (Ari et al., 2020). (Balgova et al., 2016)s argue using an event study approach that reducing the sNPL leads to an increase in real growth and investment. Even though it is difficult to determine the direction of causation between macroeconomic variables and sNPL, Institutions have become concerned with levels of NPLs and started a lively discussion about approaches to reduce and prevent these loans from arising. Policy proposals focus on macro-prudential policies, asset management strategies, as well as faster default processes and capital reallocation in the form of liquidating collateral.

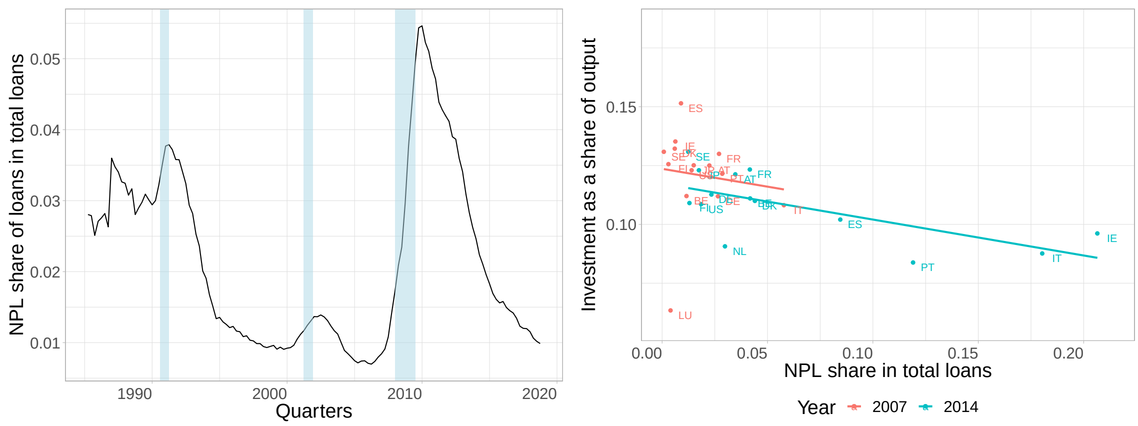


Figure 1: Left: NPLs as a share of total gross loans in the US from the FRED database. Right: Change in investment share of GDP and NPLs for OECD countries between 2007 and 2014.Investment calculated from the KLEMS database, NPL ratios from World bank and IMF data.

This policy discussion has so far largely focused on data and empirical models, while structural and business cycle models studying non-performing loan dynamics remain scarce. The general narrative for interpreting the empirical observations on sNPL has been that NPLs on a bank's balance sheet lead to lower bank profits and a regulatory need for higher cash reserves to compensate loan losses. Banks with a higher stock of NPLs on their balance sheet will then have less balance sheet space to lend out capital and thereby reduce credit supply.[2]. While the general intuition is compelling, it is difficult to find clear evidence that the lending behaviour of individual banks is differing due to their non-performing loan stocks ((Bredl, 2018) and (Accornero et al., 2017)). The cross-country comparison in this paper shows that this theorised reduction in credit supply cannot be the only channel through which the sNPL affects macroeconomic outcomes as it would imply higher real rates of return for capital in the long-run. The paradox of a policy discussion paired with a lack of structural models has been pointed out by a senior policymaker of the bank of Italy, which is due to the high stock of NPLs in Italy a major stakeholders in this discussion: *"To my knowledge, there is no clear theory suggesting that high volumes of NPLs impair the credit allocation mechanism."*[3].

The empirical part of the paper finds the sNPL to be strongly counter-cyclical. The sNPL correlates negatively with investment activity, aggregate returns on capital, capital prices, and capital reallocation. A VAR model with long-run restrictions (Blanchard and Quah, 1988) shows that positive supply shocks will lead to a reduction in the sNPL, while a rise in the sNPL has an ambiguous short-run effect on output. Meanwhile, short-run restricted VARs suggest that, controlling for output, an increase in the sNPL will lead to reduced investment, reduced capital returns, and delayed capital reallocation. On the other hand, positive investment shocks have an ambiguous effect on the sNPL, while a capital return shock reduces them. A country cross-section using aggregated firm micro and sectoral data shows that a higher sNPL is linked with lower aggregate returns on capital, lower investment, and a higher prevalence of non-profitable firms. Capital reallocation slows down as the sNPL rises.

The paper then presents a structural business cycle model where banks act as intermediaries for household lending to firms to match observed dynamic data and identify cross-country differences in sNPL outcomes as either the result of skewed bank forbearance incentives or inefficiently working markets for used capital. In the model, loans are provided with underlying collateral to firms for them to produce. The search and matching framework applied to bank-firm credit relationships combined with frameworks developed for capital reallocation ((Cao and Shi, 2017), (Ottonello, 2017), (Gopinath et al., 2017), (Lanteri, 2018), (Eisfeldt and Shi, 2018), and (Cui and Radde, 2020)) is shown to be particularly useful for modelling the decision-making of banks on whether to foreclose or forbear a loan. It can parsimoniously capture heterogeneity in the quality of a loan and heterogeneity in

[2]See, for instance, https://www.bankingsupervision.europa.eu/banking/priorities/npl/html/index.en.html

[3]Paolo Angelini, at the time Deputy Director General for Financial Supervision and Regulation, Bank of Italy; VOX EU CEPR, 12April 2018

capital allocation while allowing for a tractable model that can explain non-performing loans, investment, and capital allocation dynamics. Loans may become non-performing in the model, upon which a bank must decide whether to foreclose the loan and reallocate the foreclosed collateral or to forbear the loan incurring real cost and hoping that the loan will become profitable in the future. This dynamic decision-making problem is modelled by assuming search frictions in used capital markets. This is a way to capture the fact that used capital has high asset specificity ((Bertola and Caballero, 1994) and (Kermani and Ma, 2020)) meaning it may hold a heterogeneous value for heterogeneous firms involving information and market search problems.

The bank's foreclosure decision will depend on the value of forbearance incentives and the efficiency of markets for the collateral. Forbearance incentives are regulatory or other frictions that result in banks incurring real economic losses when foreclosing a loan. Higher forbearance incentives and lower efficiency of used capital markets will both cause higher non-performing loan levels in equilibrium. However, they imply different dynamics regarding the price of capital and new investment activity. This allows using the response of countries to output shocks to judge the extent to which strong forbearance incentives or inefficient used capital markets drive sNPL persistence. The paper finds that in most European countries struggling with a high sNPL following the global financial crisis the inefficiency of used capital markets is at fault. This result correlates well with the resolving insolvency scores from the World bank's doing business indicators which should be a combination of asset specificity and used capital market efficiency.

Models with search frictions in credit markets have recently become more popular. The model presented in this paper build on many of the insights of models from search frictions in labour markets. It is kept simple in a random search fashion with intra-period heterogeneity similar to (Mortensen and Pissarides, 1994). However, the model setup is chosen in such a way as to allow for persistent heterogeneity dynamic directed search block recursive equilibrium extensions of the type developed in (Menzio and Shi, 2010a) and (Menzio and Shi, 2010b). Papers introducing search in credit markets in a similar manner to this paper like (Beaubrun-Diant and Tripier, 2015), (Boualam, 2018) or (Cui and Radde, 2020) have mostly set up the models with firm search and bank free-entry, leading to a challenge in determining firm stocks or assuming less-intuitive fixed stocks of entrepreneurs which may gather financing. The paper also speaks to the recently re-emerging zombie firm literature (Caballero et al., 2008) and (Acharya et al., 2020), which is unsurprising as corporate NPLs are likely to stem from zombie firms. While not focusing explicitly on either the financial crisis or the financial accelerator literature (Bernanke, Gertler, and Gilchrist, 1999), this model can also be straightforwardly integrated in such models, and may be used for studying the effect of unconventional central bank policies in models of the (Gertler and Karadi, 2011) type.

The next section of this paper presents the business cycle properties of non-performing loans and shows that the sNPL has an effect on the macro-economy beyond the pure credit supply channel. The third section presents the model focusing on bank foreclosure decisions and the consequences on the sNPL, capital reallocation, and capital productivity is presented. The model section first describes a simple partial equilibrium model, which serves to provide intuition for the main mechanism for the dynamic stochastic general equilibrium of the main model. Following this, the main model is presented featuring endogenous foreclosure decisions, search frictions in used capital markets and heterogeneity in the profitability of loans, and forbearance incentives. It is shown that the efficiency of used capital markets and forbearance incentives play a crucial role in determining the outside value of foreclosing capital versus forbearing capital. The model is then calibrated to show that it can explain the correlations of the business cycle with sNPL dynamics. Furthermore, the variation of forbearance incentives and matching frictions in used capital markets over the business cycle is estimated using the proposed identification scheme. Finally, the fourth section concludes.

2 Business cycle properties of NPL shares in bank balance sheets

This section first presents the business cycle properties of the sNPL using aggregated US data for the period 1985 - 2018. US data on the sNPL and capital reallocation is available for a longer time period than for most other economies. The sNPL is shown to be counter-cyclical. Long-run restrictions suggest that output growth leads to a decline in the sNPL, while the opposite impact is ambiguous. Controlling for Real GDP short-run restrictions suggest a rise in the sNPL will drive down investment, increase delinquency rates, reduce property prices, and capital returns.

The correlations in Table 1 present aggregate business cycle properties of the sNPL. Series are downloaded from the federal reserve of St. Louis' database unless otherwise specified. The sNPL series describes the share of non-performing loans in total loans. Return on capital is calculated as the value-added accruing to capital over the capital stock calculated via a perpetual inventory method from capital formation and consumption. Investment is the GDP share of gross fixed capital formation, while property prices are captured by the house price index. Reallocation is calculated similarly to (Eisfeldt and Rampini, 2006) as the sum of firm acquisitions of existing property plant or equipment over total firm assets. Firm data is downloaded from the WRDS Compustat database for the relevant period. The reallocation series which is only available at annual frequencies is linearly interpolated. All series are Hodrick-Prescott filtered at quarterly frequencies to highlight cyclical properties and remove trends.

Table 1 shows that the sNPL is counter-cyclical. The correlations further suggest that low returns on capital lead the sNPL, while the sNPL lead reductions in investment, reduced reallocation, and reduced property prices. Delinquency rates increase as the sNPL increase. The underlying series for Table 1 and the impulse response functions identified via short-run restrictions in the next subsection can be found in Appendix B.

Variable	sNPL (-2)	sNPL (-1)	sNPL	sNPL (+1)	sNPL (+2)
Real GDP	-0.64	-0.67	-0.64	-0.56	-0.43
Return on capital	-0.34	-0.49	-0.60	-0.68	-0.73
Investment	-0.67	-0.75	-0.77	-0.74	-0.66
Reallocation	-0.51	-0.47	-0.41	-0.32	-0.20
Delinquency rates	0.75	0.81	0.81	0.77	0.68
Property prices	-0.79	-0.75	-0.69	-0.60	-0.49

Table 1: Business cycle properties of sNPL. sNPL, Real GDP, Delinquency rates, and property prices are downloaded from Fred. Aggregate capital returns are calculated based on BEA data. Capital reallocation is calculated following Eisfeldt and Rampini (JME, 2006). All series are calculated as deviations from a quarterly Hodrick-Prescott trend. Property prices stand in as capital prices. Sources: FRED, BEA, WRDS

2.1 Shock identification and impulse responses

First, we are interested in the impact real output has on non-performing loans and vice versa. Given the long-run property of the sNPL in the US presented in Figure 1 to return to an equilibrium, and their similarity to unemployment it is reasonable to identify the effects of the sNPL on output and vice versa by assuming variation in them has only a temporary effect on output. Thus one can impose long-run restrictions of the type suggested by (Blanchard and Quah, 1988) to separate demand and supply shocks with the sNPL capturing demand. While the available thirty years of data are not enough to provide clearer confidence intervals for either of the series responding to shocks from the other Figure 2 suggests that it is very likely a rise in real GDP will reduce the sNPL.

It is therefore clear that controlling for real GDP is necessary to identify correct impulse responses for a change in the NPL share with relation to other variables that provide indications about capital reallocation, such as return on capital, reallocation flows, delinquency rates, or property prices. Using the suggested ordering from the correlations and real GDP ordered before non-performing loan rates short-run restrictions are imposed. The number of lags (L) is chosen using the Hannan–Quinn information criterion. ϵ is a vector of identified shocks. Importantly, the main reason for the imposition of short-run restrictions here is not to identify causal relations, but to explore the dynamic behaviour of NPL shares in relation to other variables. Causality in this paper is implied by the mechanisms in the structural model. The VAR models for the relevant variables are found in equations (1) and (2). Equation (1) is used for variables for which the correlation table implies that they lead NPL shares, while equation (2) is used for variables that lag them.

$$\begin{bmatrix} Var\ leading\ NPL \\ Real\ GDP \\ NPL \end{bmatrix} = A(L) \begin{bmatrix} Var\ leading\ NPL(L) \\ Real\ GDP(L) \\ NPL(L) \end{bmatrix} + B\epsilon \tag{1}$$

Figure 3 suggests the sNPL declining as capital returns increase, while the effect in the opposite direction is also negative meaning that a higher NPL share causes lower returns

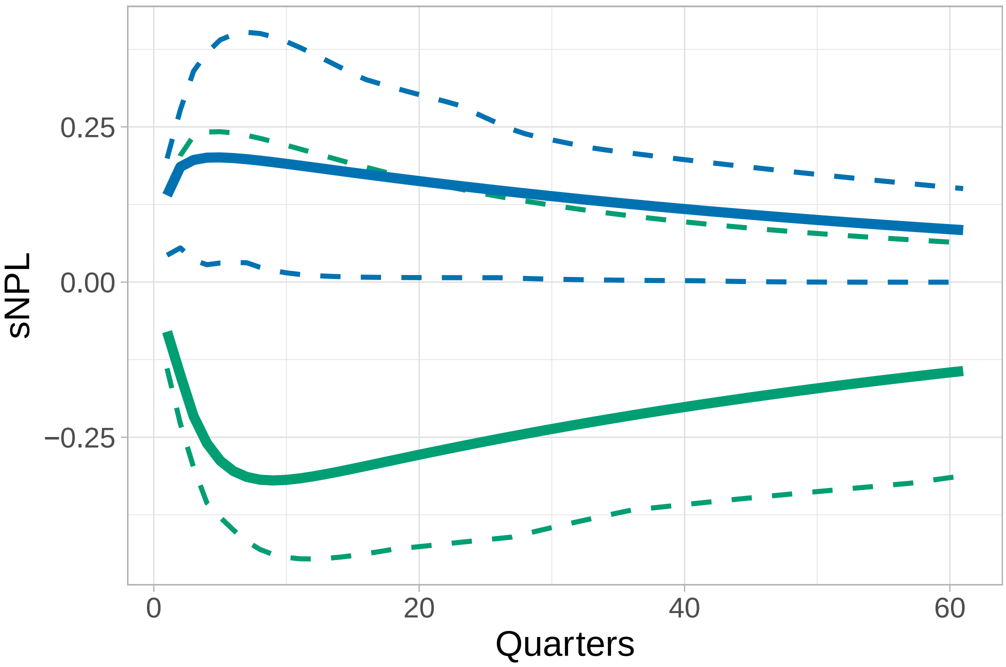


Figure 2: Impulse response of the sNPL via long-run identification. Green captures supply shocks (from real GDP) and blue demand shocks (from sNPL). Dashed lines show bootstrapped confidence intervals.

to capital while controlling for the impact of output. Meanwhile higher levels of the sNPL lead to lower investment as the graphs in Figure 4 show, and higher investment is accompanied by a reduction in the sNPL. This suggests that increased investment will result in a fewer share of weak loans driving down the sNPL. Capital reallocation flows are initially affected negatively as the sNPL rise as shown in Figure 5. Meanwhile, delinquency rates and NPLs both affect each other positively as shown in Figure 6. This means that capital reallocation flows decline with higher NPL stocks even though foreclosures by banks rise. This suggests a classic congestion mechanism inherent in markets with search frictions. Loans and their underlying collateral may become delinquent, but no immediate new productive use is found. Finally, Figure 7 shows that a higher sNPL will negatively affect the outside value of the underlying collateral capital as proxied for by property prices. Given the negative correlation of NPLs with capital reallocation, this is an expected fact highlighted by the capital reallocation literature. Low capital prices are in this literature found t go hand in hand with low reallocation. Search frictions in used capital markets are useful in explaining this strong correlation of the capital price with capital reallocation.

$$\begin{bmatrix} Real\ GDP \\ NPL \\ Var\ lagging\ NPL \end{bmatrix} = A(L) \begin{bmatrix} Real\ GDP(L) \\ NPL(L) \\ Var\ lagging\ NPL(L) \end{bmatrix} + B\epsilon \tag{2}$$

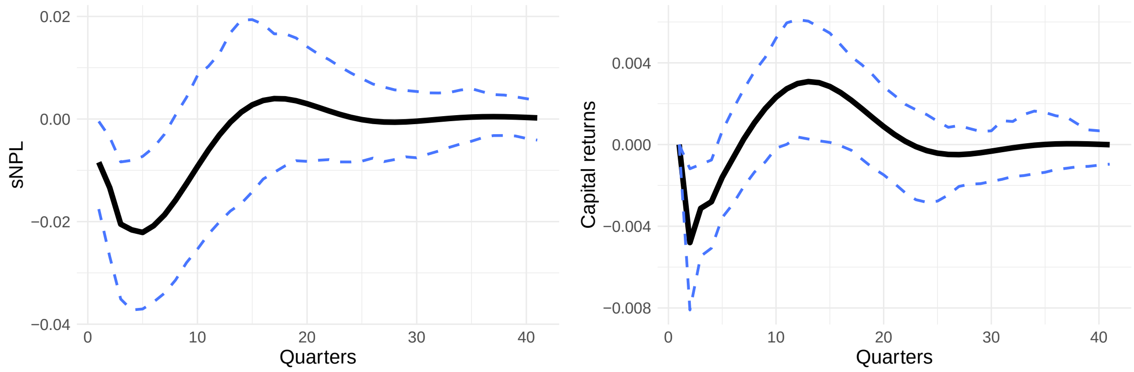


Figure 3: Impact of capital returns on NPLs (left) and vice versa (right)

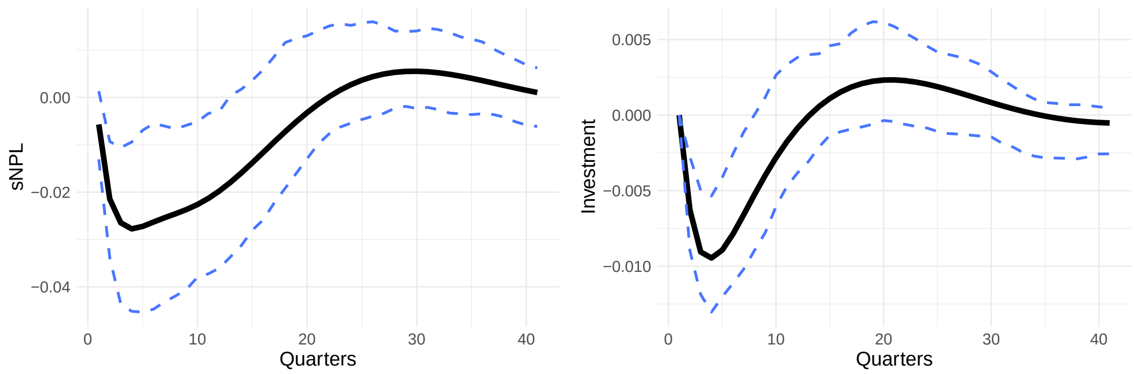


Figure 4: Impact of Investment on NPLs (left) and vice versa (right)

2.2 Cross-country comparison showing that capital misallocation and the sNPL are connected

A lemma from the assumption that high sNPL only reduces credit supply via the bank profit channel is that real returns of capital should increase. The reason for this is that in a frictionless credit market the price of capital should equal marginal returns to capital $r_k = MPK = r_l$. If the lending rate increases due to a reduction in credit supply then $r_k = MPK = r_l + u^{NPL}$. Thus reduced credit supply should lead to increased marginal real returns of capital due to unrealised opportunities.

However, Figure 8 shows that real capital returns seem at best negatively correlated with the sNPL in OECD countries for the data available. In the figure, the mean NPL share between 1995 - 2017 is plotted versus the mean capital return over the same period. This result is robust to using other measures for the correlations such as NPL shares at the start of the dataset (start_npl), NPL share growth (d_npl), capital returns corrected by value-added growth (r_g_y), and their respective growth rates. The results of these simple regressions are in Table 2 and all correlation plots can be found in Appendix B. The main takeaway from this section is that the credit supply channel cannot be the only way through which NPL shares in total loans correlate with other macroeconomic outcomes in general and capital productivity in specific. The structural model in the next section will argue that the results can be explained by NPL also indicating a reduction in marginal returns to capital

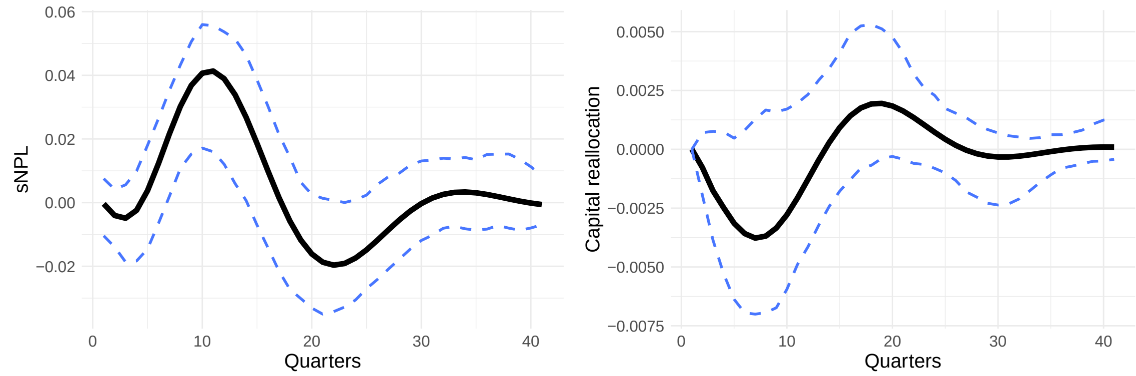


Figure 5: Impact of capital reallocation on NPLs (left) and vice versa (right)

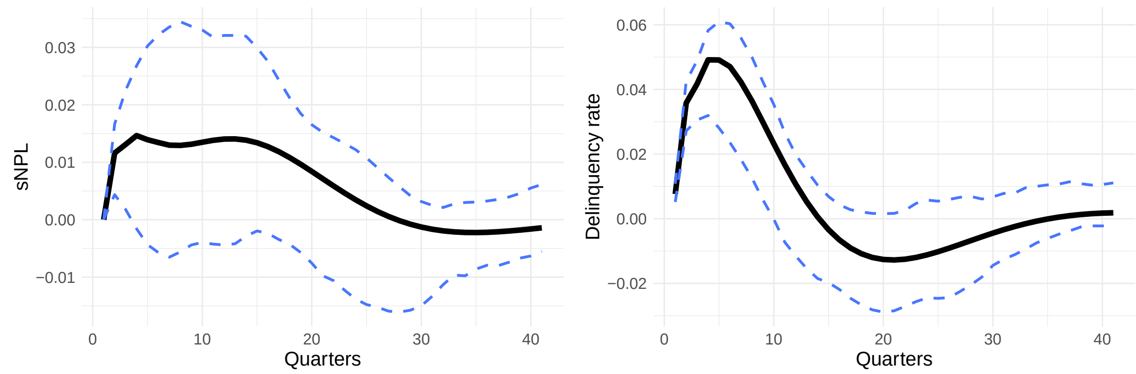


Figure 6: Impact of delinquency rates on NPLs (left) and vice versa (right)

in the economy via a misallocation channel $MP_{Misallocation}$. This can explain the equation before in very reduced form as $r_k + MP_{Misallocation} = r_l + u^{NPL}$. Thus a higher sNPL doesn't lead to higher capital returns, because higher NPL shares indicate that a larger part of the capital stock gets stuck in unproductive relationships and cannot escape these relationships due to market frictions.

Table 2: Regressions of capital returns on NPL variables

	Dependent variable:											
	r_k	r_g_y	d_r_k	dr_g_y	r_k	r_g_y	d_r_k	dr_g_y	r_k	r_g_y	d_r_k	dr_g_y
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
m_npl	-0.062	-0.053	-0.038	-0.027								
	(0.194)	(0.183)	(0.057)	(0.056)								
start_npl					-0.150	-0.128	-0.093*	-0.084				
					(0.189)	(0.178)	(0.051)	(0.051)				
d_npl									0.336	0.348	0.133	0.322
									(1.843)	(1.732)	(0.543)	(0.527)
Constant	0.119***	0.116***	-0.0001	0.0003	0.121***	0.118***	0.001	0.002	0.116***	0.113***	-0.002	-0.001
	(0.011)	(0.011)	(0.003)	(0.003)	(0.010)	(0.009)	(0.003)	(0.003)	(0.008)	(0.008)	(0.002)	(0.002)
Observations	16	16	16	16	16	16	16	16	16	16	16	16
R^2	0.007	0.006	0.032	0.016	0.043	0.035	0.189	0.160	0.002	0.003	0.004	0.026
Note:												*p<0.1; **p<0.05; ***p<0.01

The result that misallocation rises when NPL increase beyond their country mean, which

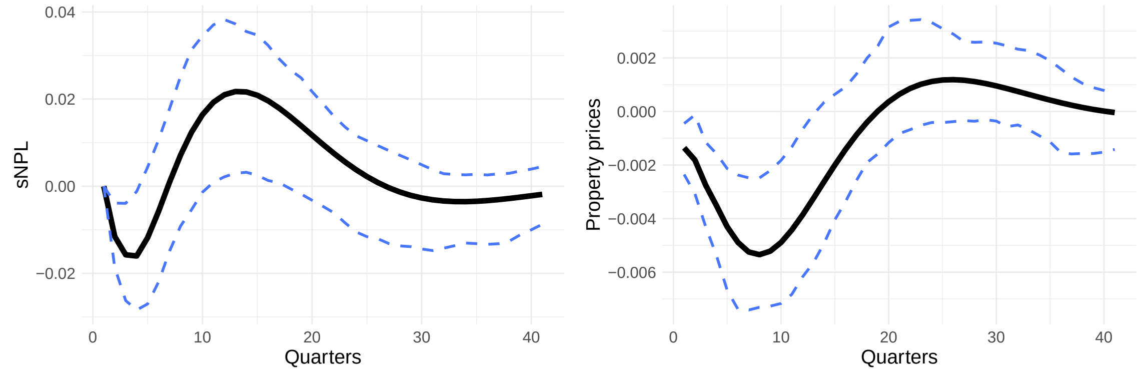


Figure 7: Impact of property prices on NPLs (left) and vice versa (right)

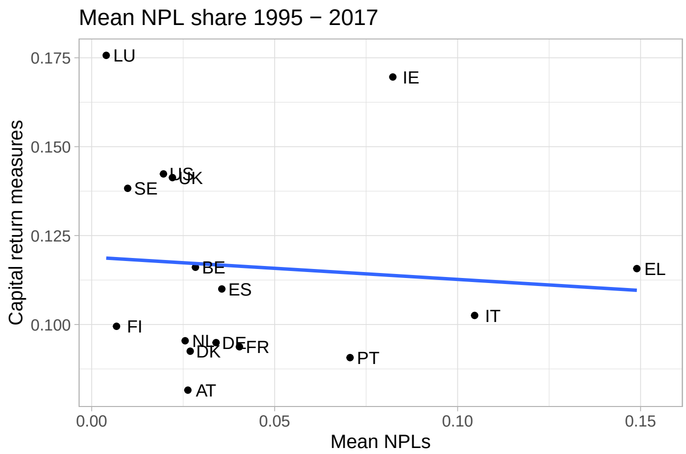


Figure 8: Capital return measures are from KLEMS data, while NPL measures are from merged world bank and IMF data.

was drawn from aggregate data is confirmed by aggregated micro-data from the 7th Vintage CompNet dataset. Figure 9 presents the changes to the coefficient of variation for total factor productivity estimated by imposing a Cobb-Douglas production function on sectors and employing ordinary least squares estimates on the log of production inputs and value-added. The figure shows that the sNPL being above its mean value in a country is going hand in hand with a larger dispersion of firm productivity. The estimates employing instead the method for total factor productivity estimation proposed in (Wooldridge, 2009) are similar. They are presented together with further dispersion and misallocation estimates in Appendix B. The structural model will capture this increase in capital misallocation via lower values of idiosyncratic capital productivity being accepted when the sNPL rises. Table 3 further confirms that it is not only output variation driving this variation in dispersion of firm productivity, but that variation in the sNPL significantly contributes to explaining it.

Finally, the variable for the suggested identification in this paper of the dynamic importance of forbearance incentives versus frictions in used capital markets is the effect of the sNPL on the price of capital. Figure 10 and Table 3 show that this variable generally reacts ambiguously to increases in sNPL, while in the US time series shows it to decline in sNPL increases in Figure 7. Overall the price of capital seems to be more likely to decrease

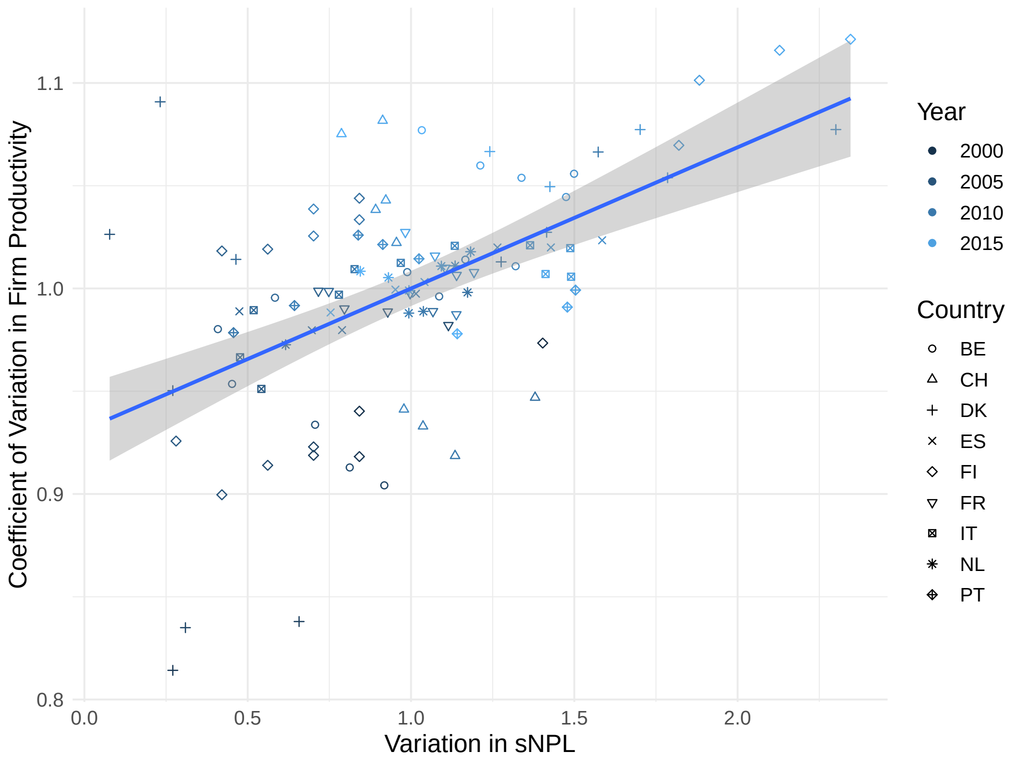


Figure 9: As the sNPL changes estimated MPK dispersion from C-D estimation by sector increases proportionally. Source: CompNet 7th vintage dataset for the TFP estimate and sNPL measures are from merged world bank and IMF data.

than increase as non-performing loans increase beyond their mean. As will be shown in Section 3, capital becoming cheaper would be an unlikely result of forbearance incentives rising strongly and driving the sNPL. The structural model shows rising forbearance incentives will lead to more tight used capital markets, which would lead to an expected increase in the real price of capital, while higher frictions in used capital markets will lead to the opposite.

3 Model

The model assumes that capital markets experience matching frictions summarising similar heterogeneities as matching frictions in labour markets and that the aggregate product of capital experiences marginally diminishing returns, whether the capital is put to use or not. Modelling used capital with higher matching frictions than fresh capital is meant to capture the heterogeneities entrepreneurs who want to productively employ used capital face when reusing this specified capital, for example, a specific type of factory. If entrepreneurs were to receive fresh capital they could build the factory to any specification. An entrepreneur seeking to employ a reused factory may find an excellent and inexpensive fit, but may also spend a long time on search not finding a suitable production location. Capital experiencing marginally decreasing returns on the aggregate is a common assumption and can be micro-founded by unused real estate or factories occupying the most productive locations, or occupying other non-modelled resources such as labour performing

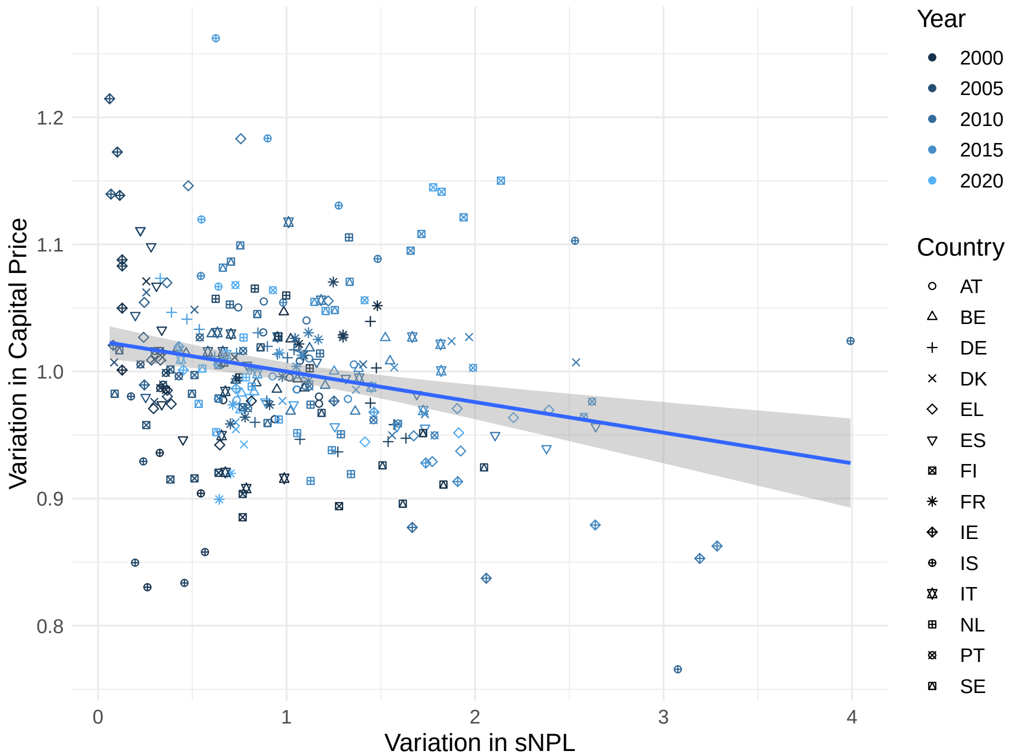


Figure 10: As sNPL increase capital prices fall. sNPL measures are from merged world bank and IMF data. The real price of capital is from Eurostat.

	Dependent variable:		
	Coefficient of Variation - OLS	Coefficient of Variation - Wooldridge	Capital Price
	(1)	(2)	(3)
NPL variation	0.083***	0.084***	-0.005
	(0.010)	(0.011)	(0.007)
Output variation	0.598**	0.634**	0.198
	(0.256)	(0.276)	(0.145)
Constant	0.917***	0.915***	1.003***
	(0.011)	(0.012)	(0.008)
Observations	85	85	203
R^2	0.453	0.425	0.014
Adjusted R^2	0.440	0.411	0.005
Residual Std. Error	0.039 (df = 82)	0.042 (df = 82)	0.048 (df = 200)
F Statistic	33.990*** (df = 2; 82)	30.277*** (df = 2; 82)	1.466 (df = 2; 200)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3: Explanatory power of variation in the sNPL versus aggregate output variation on capital productivity dispersion and capital prices.

maintenance tasks or engaging in production with foreclosed capital. As a consequence, each unit of capital exhibits a negative externality on all other units of capital.

The model is placed into a general equilibrium framework to assess the impact of frictions in used capital markets on the dynamics and long-run outcomes of reallocation, non-performing loans, and capital returns. Figure 11 sketches an overview of the workings of this general equilibrium model. Households provide fresh resources to create capital in the form of deposits to banks. Households own firms, which search for profitable opportunities to borrow this capital from banks, which then becomes a new loan with underlying collateral. Firms, which can be also viewed as entrepreneurs, make proposals for using the capital to banks, which are at the heart of the capital market. Banks provide capital in form of loans to firms. Once firms become unproductive and can no longer pay a share of profits in the form of interest to banks in return for the financing the banks have to decide whether to foreclose the loan and seize the collateral capital. If the bank decides on doing this it will seek to re-lend the foreclosed capital to entrepreneurs. Banks pass any profits or losses generated with household deposits on to households. The capital markets with search frictions in this model are closed on the loan demand side via the free entry by firms, while they are closed on the loan supply side via the deposit provision from households, which is derived from the intertemporal Euler equation.

The first subsection in this section aims to illustrate the key mechanism, namely how the efficiency of used capital markets and forbearance incentives affect the bank forbearance or foreclosure decision, in a partial equilibrium model. The second subsection then presents the full dynamic stochastic general equilibrium model with endogenous foreclosure.

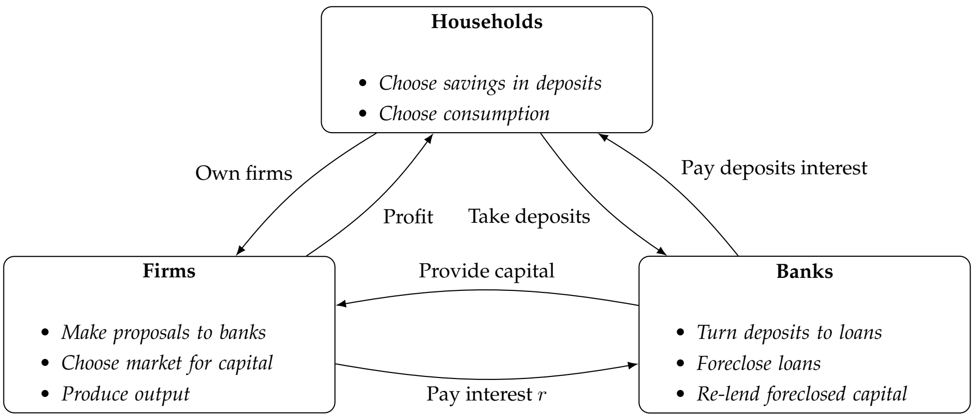


Figure 11: Model relations

3.1 Partial equilibrium foreclosure decision

The purpose of this simple model is to show that the loan foreclosure decision will be determined by the ability of the bank to reuse foreclosed capital productively and the cost it incurs in foreclosure. For illustration, very simple assumptions are taken leading to a closed-form solution. These are subsequently relaxed in the full model following this section.

Assume a model that only runs for one period. Banks come into the period with all capital lent to firms. For simplicity, banks learn about the productivity of lent capital units, decide whether to foreclose the loan, and have the option in case of foreclosure to re-lend the capital within one period.. Banks will aim to maximise profits by making an optimal foreclosure decision.

A firm matched with a bank will pay the bank a stochastic interest rate realisation $r(\epsilon)$ depending on the firms marginal productivity. $r(\epsilon)$ is assumed to be the realisation of $\bar{r} - \epsilon$, where $\log(\epsilon)$ is assumed to be normally distributed. Thus the set of possible interest rate realisations for legacy loans is $(-\infty, \bar{r})$.

Let p be the probability for the bank of finding a new firm if it decides to foreclose a current loan. p is assumed to be a function of match efficiency μ in used capital market and the tightness of the market θ. Tightness is the number of entrepreneurs offering a business plan for the capital unit g over the number of capital units searching to be matched s. $p(\mu, \theta)$ is a function with the properties $\frac{\partial p}{\partial \mu} \geq 0$ and $\frac{\partial p}{\partial \theta} \geq 0$. Let $V_u = [p(\mu, \theta)\bar{r} + (1 - p(\mu, \theta))b]$. Thus the reward of finding a new loan is to be matched at the productivity frontier $\bar{r}$. The bank will decide to foreclose a legacy loan when $V_m = r(\epsilon) < V_u$. The cutoff value $\tilde{\epsilon}$ at which a bank decides to foreclose a unit of capital is then given by (3). p is specified as the result of a Cobb-Douglas matching function with $p = \mu\theta^{0.5}$. Finally, there may be forbearance incentives for banks τ which would increase the benefit from keeping a beginning of period loan.

$$r(\tilde{\epsilon}) = \bar{r} - \tilde{\epsilon} = p(\mu, \theta)\bar{r} + (1 - p(\mu, \theta))b - \tau = p(\mu, \theta)[\bar{r} - b] + b - \tau \tag{3}$$

With an appropriate calibration of $b < 0$ to capture possible losses of capital due to depreciation it is then clear that values of $V_u < 0$ are possible depending on the productivity of used capital markets, the value of $\bar{r}$, and match probabilities $p(\mu, \theta)$. A reasonable assumption is to define an NPL cutoff where $r(\epsilon_{NPL}) = 0$, meaning those legacy loans that are maintained by banks even though $r(\epsilon) < 0$ are NPLs. The distribution presented in Figure 12 with the calibration with $\bar{r} = 1$, $b = -1$, $\mu = 0.3$, $\theta = 1$ and the distributional parameters for $\log(\epsilon)$ set to mean 0 and standard deviation 1 shows that there can be a significant mass on NPLs in this model with matching frictions.

The reason NPLs arise in this model is that foreclosing a loan doesn't necessarily mean for the bank that it will be able to re-lend the underlying capital to a new firm and gain $\bar{r}$. The bank may make a loss b on the capital instead as it stays idle, depreciates, or possibly requires maintenance costs or management by the bank. The probability of this negative event happening depends on match probabilities p. Low probabilities due to low used capital efficiencies will mean that the bank is willing to accept more and more negative interest rates increasing the NPL share in total loans given in equation (4), where $\Phi()$ is the

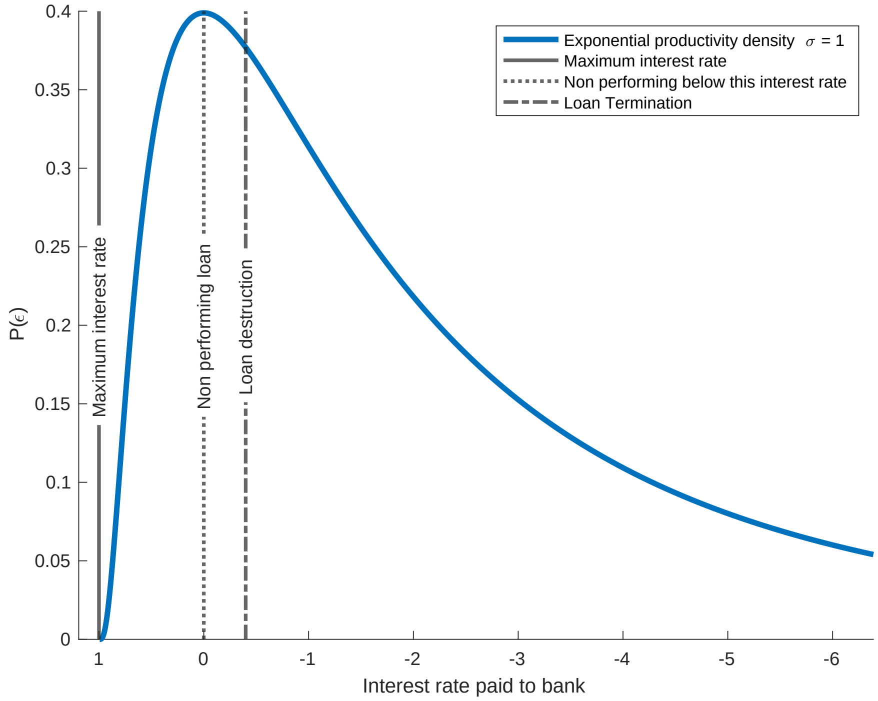


Figure 12: Simple illustration of a mass of NPLs in the distribution of existing loans

cumulative distribution function of a normal distribution.

$$NPLshare = \frac{\Phi(\log(\tilde{\epsilon})) - \Phi(\log(\epsilon_{NPL}))}{\Phi(\log(\tilde{\epsilon})) + p[1 - \Phi(\log(\tilde{\epsilon}))]} \tag{4}$$

$\Phi(\log(\tilde{\epsilon}))$ captures is the share of surviving legacy loans while $p[1-\Phi(\log(\tilde{\epsilon}))]$ is the value of newly created loans. Mean expected interest rates r_m received by banks can be computed from the mills ratio as $r_m = \bar{r} - \exp(-\frac{\phi(\log(\tilde{\epsilon}))}{\Phi(\log(\tilde{\epsilon}))})$ and 50 % variation in parameters μ, θ, and $\bar{r}$ can be compared to provide an idea of the forces in the model. Here μ is an increase in used capital market efficiency, while θ is an increase in entrepreneurial activity, thus an increase in investment and $\bar{r}$ can be interpreted as a rise in total factor productivity lifting interest rates. The underlying outcomes of these variations on the partial equilibrium model are shown in Figure 13. These variations show that a higher value of foreclosure only leads to more loans being called, hence more reallocation if the increase is due to a rise in match probabilities, which happens when θ or μ increase. While increases in θ, however, experience marginally diminishing returns increases in match productivity exponentially increase mean interest rates and decrease NPL shares due to increased reallocation. When $\bar{r}$ rises reallocation falls as shown by the rise in surviving loans. The only reason NPLs fall, in this case, is due to the rising denominator and the shift of the distribution, but not due to reallocation. This shows that match efficiency and reallocation go hand in hand. However, to study the business cycle properties of this mechanism it has to be included in the dynamic general equilibrium setting, which is done in the next section.

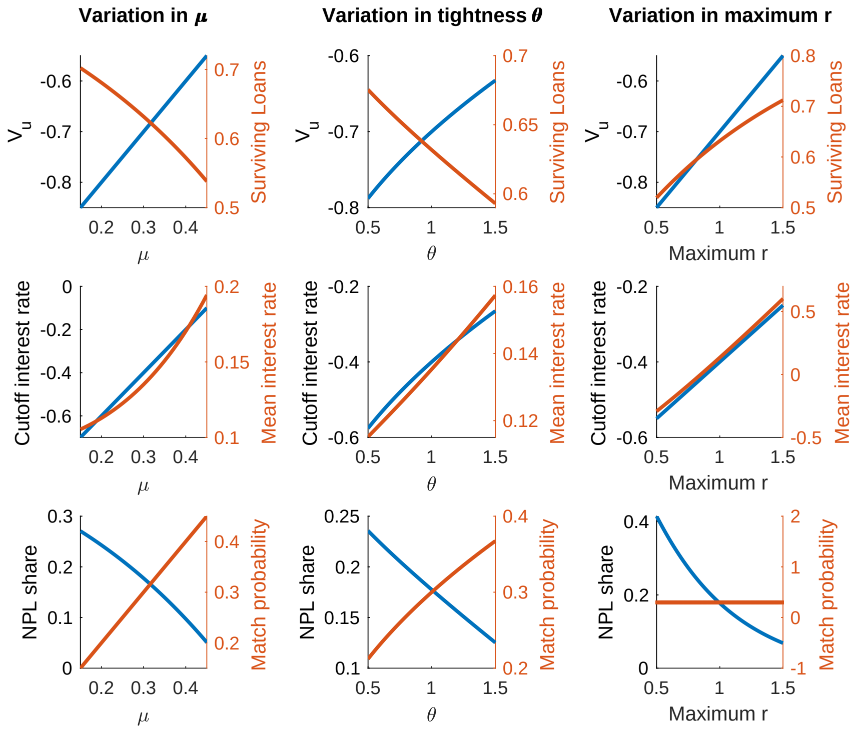


Figure 13: Comparison of the value of foreclosing a loan V_u, surviving legacy loans, the cutoff interest rate, the mean interest rate paid to banks, the share of NPLs in total loans and the match probability to variations in μ, θ, and $\bar{r}$

Figure 14 finally shows the comparison between an increase in the efficiency of used capital markets and a decrease in forbearance incentives. From the statics, it is clear that both would lead to a similarly shaped decrease in the sNPL as both affect the cutoff condition for $r(\tilde{\epsilon})$. However, while a higher forbearance incentive leads to loans being kept due to the cost of dissolving the loan, a lower efficiency of used capital markets drives down the probability of rematching and thereby V_u. This ultimately will lead to less tight capital markets as more capital is on the market searching to be matched with a lesser likelihood of success. The dynamic general equilibrium setting can use this distinction to identify the importance of forbearance incentives in comparison to the efficiency of used capital markets using the behaviour of the observed price of capital following a shock.

3.2 General equilibrium model with endogenous loan foreclosure

The decision on whether to foreclose a non-performing loan, on which the partial equilibrium model of the previous section focused, can be integrated into a dynamic stochastic general equilibrium. The expected value of a capital unit underlying as collateral a loan for banks to entrepreneurs then creates an infinite value function, which depends on the ability of the entrepreneur to pay for the capital unit of the loan hence the entrepreneur's capital productivity. Other parts of the model are kept as simple as possible. Some kind of heterogeneity in the productivity of loans will be necessary to model non-performing loans. Non-performing loans are those where capital is below a certain productivity $z < \tilde{z}$,

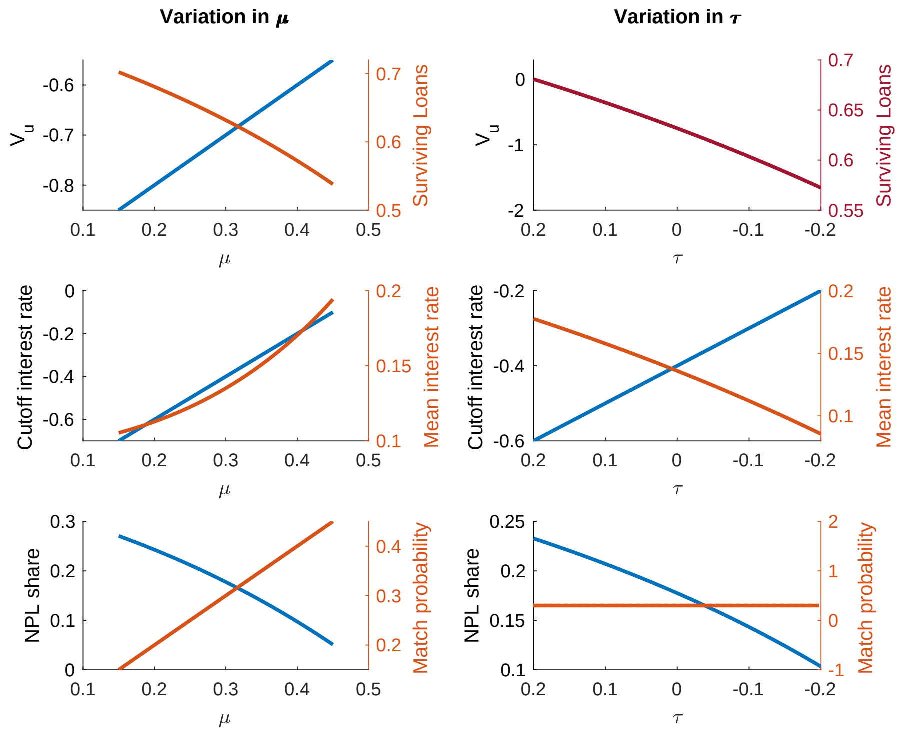


Figure 14: Comparison of the value of foreclosing a loan V_u, surviving legacy loans, the cutoff interest rate, the mean interest rate paid to banks, the share of NPLs in total loans and the match probability to variations in μ, τ.

but above the bank's cutoff value z_s. Further, there is a certain level below which a bank chooses to call the non-performing loans $\tilde{z}$. From this time on the loan will not produce and the only purpose will be to rematch the capital with another entrepreneur. Taking reasonable definitions for when to consider a loan as non-performing it is straightforward to show that with matching frictions in used capital markets $\tilde{z} < z < \hat{z}$, and thus there exists a share of loans that are non-performing but not foreclosed at all times.

3.3 Aggregate production function

Similar to (Ottonello, 2017), who distinguishes between several types of capital based on their "employment status", there are three types of capital stocks in the economy. These are matched capital in safe loans and in weak loans denoted by K_t^N and K_t^E and unmatched capital denoted by K_t^U. Matched capital is employed in firms in production. Matched capital is split into matched capital in safe loans K_t^N and capital in weaker loans where payment of interest rate is uncertain and possibly lower than the market rate and which may become non-performing K_t^E. Unmatched capital is owned by banks and only produces output, but still forms part of the capital stock. The aggregate capital stock in the economy is $K_t = K_t^N + K_t^L + K_t^U$. Firms produce output with capital units. Each capital unit can either be in a loan or be foreclosed and held by the bank. When a capital unit is in a fresh loan, which will not be foreclosed it produces output according to equation (5). When a capital unit is in a loan that may be foreclosed it produces output according to equation (6). When a capital unit is foreclosed it produces output according to equation (7).

$$\bar{y} = AK^{\alpha-1}\bar{z} \tag{5}$$

$$y_i = AK^{\alpha-1}z \tag{6}$$

$$y_i = g \tag{7}$$

The total aggregate capital stock negatively affects returns to the capital unit, but the distribution of capital over states does not. Employed capital K_t^E is the sum of all employed capital units K_i^L over all states $K_t^E = \sum_{i=0}^{Z} z_i K_i^L$. Total output is given by equation (8).

$$Y = \bar{y}K_t^N + \int_{z_s}^{\bar{z}} y(z)\, d(z) K^L + gK^U = AK^{\alpha-1}(\bar{z}K^N + \int_{z_s}^{\bar{z}} z\, d(z)K^L) + gK^U \tag{8}$$

Output may be used for consumption by the households, as well as investment into bank deposits to create more productive capital, or as a resource to set up a business plan proposal by the entrepreneur.

K_u is the capital banks have foreclosed. This capital remains idle and only produces with g, which may also be a negative output consuming resources. Banks seek to rematch foreclosed capital in secondary capital markets with productive entrepreneurs. The life-cycle of a physical capital unit underlying a bank loan is sketched in Figure 15. Fresh capital can be added to the existing capital stock via investment but matched and foreclosed unmatched capital remain in the economy. Matched capital changes exogenously with probabilities π^N and π^L from being matched in a safe or weak loan and vice versa. Capital switches states from weak loans to unmatched capital owned by banks according to the agent decision-making. The only way for physical capital to exit the economy is via depreciation.

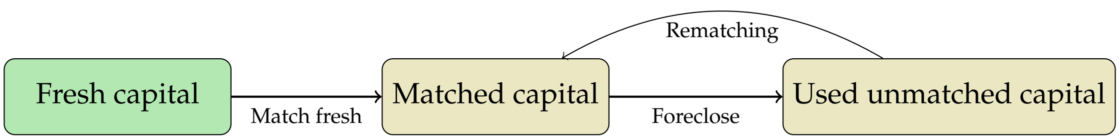


Figure 15: Life cycle of collateral underlying lent capital

3.4 Households

The economy is populated again by a unit mass of identical households. Each household has an initial deposit wealth D_0. However, not all deposits are automatically turned to capital. Thus deposits may stay idle and remain bank cash reserves X. This means real deposit values won't always depreciate in the same way in the same way as the underlying collateral values of capital.

$$\max_{C_t, I_t} E_t(\sum_{s=0}^{\infty} \frac{C_{t+s}^{1-\sigma}}{1-\sigma}) \tag{9}$$

The maximisation problem is subject to a conventional budget constraint where consumption C and investment I equal firm profits Π_t and interest rates paid on bank deposits $\rho_t D_t$.

$$C_t + I_t = \Pi_t + \rho_t D_t \tag{10}$$

Deposits evolve according to equation (11) with the current value of $\delta_{d,t-1}$ taken as given by the individual household and defined at a later time.

$$D_t = (1 - \delta_{d,t-1}) D_{t-1} + I_{t-1} \tag{11}$$

This means the inter-temporal Euler equation is a conventional function of depreciation, interest rates ρ, as well as present and future consumption.

$$C_t^{-\sigma} = \beta E_t [C_{t+1}^{-\sigma} (1 - \delta_{d,t+1} + \rho_{t+1})] \tag{12}$$

3.5 Credit market

3.5.1 Financial intermediaries (Banks)

Banks turn household deposits into physical capital units when they match with entrepreneurs making a convincing business proposal that receives financing. The rate at which proposals arrive is $p(\theta_{x,t-1})$, where $\theta_{x,t} = \frac{g_{x,t}}{s_{x,t}}$ denotes market tightness. $s_{x,t}$ is the share of deposits that may be lent out, which may be affected by regulatory policy $s_{x,t} = \psi_x x_t$. Denote deposits that have not yet been turned into capital with X_t. Fresh deposits evolve according to equation (13).

$$X_t = (1 - \delta_x)[X_{t-1} - p(\theta_{x,t-1}) S_{t-1}] + I_{t-1} \tag{13}$$

A bank's present discounted value from a safely matched unit of capital that was just created by lending out deposits is denoted by $V_{\bar{z}}^{B,L}$. The present discounted value of a capital unit in a weak lending relationship is $V_z^{B,L}$. In both cases, z describes the idiosyncratic productivity of the capital being used. The present discounted value of a capital unit that is unused because the bank has foreclosed it is $V^{B,U}$. The present discounted value of a new and a weak lent unit are defined by equations (14) and (15). For simplicity of notation define the mean expected surplus-value of a weak loan match over a foreclosed loan next period as $\hat{V}^{B,L}_{\hat{z},t+1} = \int_{z_{z,t+1}}^{\bar{z}} (\hat{V}^{B,L}_{z,t+1} - V^{B,U}_{t+1} + \tau)\, d(z)$. τ captures a loan forbearance incentive for banks, who depending on the calibration may incur real costs when foreclosing a loan and reducing the size of their balance sheet.

$$V^{B,N}_{\bar{z},t} = r_{\bar{z},t} + (1-\delta_k)E_t[\mu_{t+1}[\pi^N(V^{B,L}_{\bar{z},t+1} - V^{B,U}_{t+1}) + (1-\pi^N)(\hat{V}^{B,L}_{\hat{z},t+1} - \tau) + V^{B,U}_{t+1}]] \quad (14)$$

$r_{\bar{z}}$ denotes the interest rate paid by firms in safe loans, which will depend on the productivity of the capital underlying the loan, while $\mu_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\sigma}$ is the stochastic discount factor. δ_k is the depreciation rate of capital. π^N is the probability that a safe loan will remain safe, while it will turn into a weaker loan with probability $(1-\pi^N)$. In the case where $\pi^N = 0$ and $\pi^L = 1$ the model is then similar to the interpretation of new loans being created by businesses at the technology frontier as is assumed in (Mortensen and Pissarides, 1994) for new jobs.

$$V^{B,L}_{z,t} = r_{z,t} + (1-\delta_k)E_t[\mu_{t+1}[(1-\pi^L)(V^{B,L}_{\bar{z},t+1} - V^{B,U}_{t+1}) + \pi^L(\hat{V}^{B,L}_{\hat{z},t+1} - \tau) + V^{B,U}_{t+1}]] \quad (15)$$

r_z denotes the interest rate, which will depend on the productivity of the capital underlying the loan, while $\mu_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\sigma}$ is the stochastic discount factor. δ_k is the depreciation rate of capital. The present discounted value of a foreclosed unit of capital is in equation (16).

$$V^{B,U}_t = g + (1-\delta_k)E_t[\mu_{t+1}(p(\theta_{u,t})[V^{B,L}_{\bar{z},t+1} - V^{B,U}_{t+1}] + V^{B,U}_{t+1})] \quad (16)$$

g is the benefit or cost banks receive on a foreclosed capital unit. This may also be a cost. $p(\theta_u)$ is the probability with which a bank will find a new entrepreneur willing to take on the foreclosed capital unit. This probability will depend on market tightness in secondary capital markets θ_u.

3.5.2 Firms

Firms submit proposals for funding to banks. Opening a proposal and presenting it to financial intermediaries comes at a cost κ_j. j denotes here the possibility of the firm searching either in markets of fresh or markets of used capital. It is assumed that the expected benefit of receiving an old or a new capital unit is brought to the same level by market forces. This leads to the following equivalencies for allocating search between fresh and used capital markets in equation (17).

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = \frac{\kappa_u}{q_u(\theta_{u,t})} = (1-\delta_k)E_t[\mu_{t+1}V^{E,N}_{\bar{z},t+1}]) \quad (17)$$

The value a successful match to the firm is found in equation (19). The firm will produce the period output produced $A_t z K_t^{\alpha-1}$ with the collateral capital provided and pay interest rate $r_{z,t}$ to the bank for it. The future present discounted value of the collateral capital provided is $(1-\delta_k)E_t(\mu_{t+1}[V_{z,t+1}^{E,L}])$, which accounts for temporal discounting and capital depreciation. Again, for simplicity of notation define the mean expected surplus value of a weak loan match to the entrepreneur next period as $\hat{V}_{\hat{z},t+1}^{E,L} = \int_{z_{z,t+1}}^{\bar{z}} V_{z,t+1}^{E,L}\, d(z)$

$$V_{\bar{z},t}^{E,N} = A_t K_t^{\alpha-1}\bar{z} - r_{\bar{z},t} + (1-\delta_k)E_t\left[\mu_{t+1}[\pi^N V_{\bar{z},t+1}^{E,N} + (1-\pi^N)\hat{V}_{\hat{z},t+1}^{E,L}]\right] \tag{18}$$

$$V_{z,t}^{E,L} = A_t K_t^{\alpha-1} z - r_{z,t} + (1-\delta_k)E_t\left[\mu_{t+1}[(1-\pi^L)V_{\bar{z},t+1}^{E,N} + \pi^L \hat{V}_{\hat{z},t+1}^{E,L}]\right] \tag{19}$$

3.5.3 Equilibrium interest rate

The equilibrium interest rate is determined via Nash bargaining between the bank and the firm. This delivers a simple solution, though more complicated bargaining solutions may be implemented as well. Let η be the bargaining power of the bank.

$$\eta V_{\bar{z},t}^{E,L} = (1-\eta)(V_{\bar{z},t}^{B,L} - V_t^{B,U}) \tag{20}$$

$$\eta V_{z,t}^{E,L} = (1-\eta)(V_{z,t}^{B,L} - V_t^{B,U} + \tau) \tag{21}$$

The Nash bargaining solution for safe loans is in equation (20), while the solution for weak loans is in equation 921). For weak loans there is a possibility of separation, so not separating forms part of the surplus for banks.

$$r_{\bar{z},t} = \eta[A_t K_t^{\alpha-1}\bar{z} + \kappa_u \theta_{u,t}] + (1-\eta)g \tag{22}$$

$$r_{z,t} = \eta[A_t K_t^{\alpha-1} z + \kappa \theta_{u,t}] + (1-\eta)[g - \tau(1-\beta\pi^L(1-\delta_k))] \tag{23}$$

3.5.4 Loan creation decision

Substituting equation (22) into equation (18) and combining it with the free entry condition for entrepreneurs into capital proposals in equation (17) yields the loan creation condition in equation (24).

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = (1-\eta)(A_t K_t^{\alpha-1}\bar{z} - g) - \eta\kappa_u\theta_{u,t} + E_t\left[\pi^N \frac{\kappa_x}{q_x(\theta_{x,t+1})} + (1-\delta_k)(1-\pi^N)E_t(\mu_{t+1}\hat{V}_{\hat{z},t+1}^{E,L})\right] \tag{24}$$

3.5.5 Foreclosure decision

A bank will only choose to foreclose capital when the benefit from the foreclosed capital exceeds the benefit from keeping the loan relation. The benefit from foreclosing a loan is $V_t^{B,U}$, which is the real value of the secondary specified capital in the match. This foreclosed capital can be sold in frictionless financial markets and purchased by other banks or kept by the bank itself. In either case, the bank will choose not to foreclose a loan as long as (25) holds.

$$V_{z,t}^{B,L} - V_t^{B,U} + \tau > 0 \tag{25}$$

The foreclosure condition in equation (26) is the result of combining equation (15), (16), and (23).

$$0 = [(1-\eta)(A_t K_t^{\alpha-1} z_{s,t} - g + \tau(1-\beta\pi^L(1-\delta_k))) - \eta\kappa\theta_{u,t} + E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}_{\hat{z},t+1}^{E,L}\right]] \tag{26}$$

$$z_{s,t} = K_t^{1-\alpha} A_t^{-1}[g - \tau(1-\beta\pi^L(1-\delta_k))) + \frac{\eta}{1-\eta}\kappa_u\theta_{u,t} - \frac{1}{1-\eta}E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}_{\hat{z},t+1}^{E,L}\right]] \tag{27}$$

This means that the productivity cutoff for weak loans will increase with the total capital stock, as loans become less productive due to declining marginal returns. The cutoff decreases with higher productivity levels and decreases with lower levels of demand for newly built capital by entrepreneurs. Further forbearance incentives will lead to lower productivities being accepted by banks before they foreclose the entrepreneur.

3.5.6 Transition laws

The total real value of existing capital units should equal the total value of real deposits at all time.

$$D_t = X_t + K_t^N + K_t^L + K_t^U \tag{28}$$

The transition law for bank cash is found in equation (29). Cash is reduced by a small overhead that may be taken by the bank δ_x and the capital $p(\theta_{x,t-1})S_{t-1}$ lent out to firms. It is increased by deposits made by households I_{t-1}.

$$X_t = (1-\delta_x)[X_{t-1} - p(\theta_{x,t-1})S_{t-1}] + I_{t-1} \tag{29}$$

The transition law for collateral capital in safe loans K^N is given by equation (30) and states that present capital is the not depreciated $(1-\delta_k)$ part of past safely lent capital

K_{t-1}^N, which has also not been foreclosed turned weak with probability π^N. Further loans that have turned from weak to strong loans again $(1-\pi^L)K_{t-1}^L$. Finally, the capital which has been successfully re-lent $p(\theta_{u,t-1})K_{t-1}^U$ plus the successfully added fresh capital built from cash $p(\theta_{x,t-1})S_{t-1}$ form part of K_t^N.

$$K_t^N = (1-\delta_k)[\pi^N K_{t-1}^N + p(\theta_{u,t-1})K_{t-1}^U + p(\theta_{x,t-1})S_{t-1} + (1-\pi^L)K_{t-1}^L] \tag{30}$$

The transition law for collateral capital in weak loans K^L is given by equation (31) and states that present capital is the not depreciated $(1-\delta_k)$ part of past lent capital K_{t-1}^L, which has also not been foreclosed $(1-H(z_{s,t}))$ and not turned safe again with $(1-\pi^L)$. To this capital, the loans which have turned from safe to weak in the last period are added $(1-\pi^N)K_{t-1}^N$.

$$K_t^L = (1-\delta_k)(1-H(z_{s,t}))[\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N] \tag{31}$$

Here $H(z)$ is the cumulative distribution function of the stochastic variable z.Equally the transition law for K^U in equation (32) is the not depreciated part $(1-\delta_k)$ of past unmatched capital plus foreclosed matched capital $H(z_{s,t})[K_{t-1}^L + p(\theta_{u,t-1})K_{t-1}^U]$ minus successfully re-matched foreclosed capital $p(\theta_{u,t-1})K_{t-1}^U$.

$$K_t^U = (1-\delta_k)[H(z_{s,t})(\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N) + (1-p(\theta_{u,t-1}))K_{t-1}^U] \tag{32}$$

Overall the transition law for deposits is then the addition of cash X, lent capital K^L, and foreclosed capital K^U.

$$X_t + K_t^N + K_t^L + K_t^U = (1-\delta_x)(X_{t-1} - p(\theta_{x,t-1})S_{t-1}) + (1-\delta_k)[K_{t-1}^N + K_{t-1}^L + K_{t-1}^U + p(\theta_{x,t-1})S_{t-1}] + I_{t-1} \tag{33}$$

This transition allows for specifying the deposit depreciation $\delta_{d,t}$ in every period in equation (34).

$$\delta_{d,t} = \delta_k \frac{K_{t-1}^N + K_{t-1}^L + K_{t-1}^U + p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} + \delta_x \frac{X_{t-1} - p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} \tag{34}$$

This then leads to the aggregate deposit law of motion originally specified for the household.

$$D_t = (1-\delta_{d,t})D_{t-1} + I_{t-1} \tag{35}$$

3.5.7 Interest rates

As expected, the interest rate paid to the household ρ depends on the state of the lent capital in safe $r_{\bar{z},t}$ and weak loans $r_{\hat{z},t}$ and the proportion of lent $\frac{K_t^L}{D_t}$ and foreclosed capital

$\frac{K_t^U}{D_t}$ on the active side of the bank balance sheet versus deposits, the passive side.

$$\rho_t = \frac{r_{\bar{z},t} K_t^N + r_{\hat{z},t} K_t^L + g K_t^U}{D_t} \tag{36}$$

3.6 NPL cutoff

The NPL share is defined endogenously. This share will be loans that are no longer profitable for the bank. In this case, NPLs are defined as the share of total weak loans K^L where the interest rate paid to banks doesn't cover the long-run risk-less equilibrium rate paid to consumers. The idea is that a bank at this point would be better off if it could take back the capital unit provided with the specific loan without facing any frictions and transfer the capital into a fixed income asset providing the long-run risk-less rate.

$$r_{z,t} \leq \rho \tag{37}$$

We can thus find the real interest rate paid by firms to banks below which the loan turns non-performing by finding $\hat{z}$ as a function of equilibrium ρ.

$$r_{\hat{z},t} = \rho \tag{38}$$

3.7 Intra-period heterogeneity via the productivity distribution of loans

Each period the loan productivity of the capital in weak loans is drawn from a negative exponential distribution. The choice of an exponential distribution versus the log-normal distribution in the partial equilibrium model does not impact the model properties. The advantage of the exponential distribution over the log-normal is that it facilitates finding closed expressions for the equations above due to the closed-form expressions available for conditional expectations for this type of distribution. The idea of the negative exponential distribution is to assume that most loans are productive, but there is a subset in the tails that become costly to the banks due to being very unproductive. Thus there is a maximal value of z, which is $\bar{z}$. All safe loans are at productivity level $\bar{z}$. Weak loans may be at a productivity $z \in [\bar{z}, -\infty]$. This is the result of a realisation ζ of the exponential distribution that is subtracted from $\bar{z}$. The exponential distribution for ζ with the calibrated values is pictured in Figure 16.

$$z = \bar{z} - \zeta \tag{39}$$

In general, ζ is exponentially distributed with probability density function $\gamma \exp(-\gamma\zeta)$.

The properties of the exponential distribution allow for a closed form computation of the mean value of not foreclosed weak loans $V_{z,t}^{\hat{E},L}$. Given a cutoff z_s, the cutoff of ζ will be $\zeta_s + \bar{z}$. This leads to a mean value of the exponential distribution, with γ as the distributional

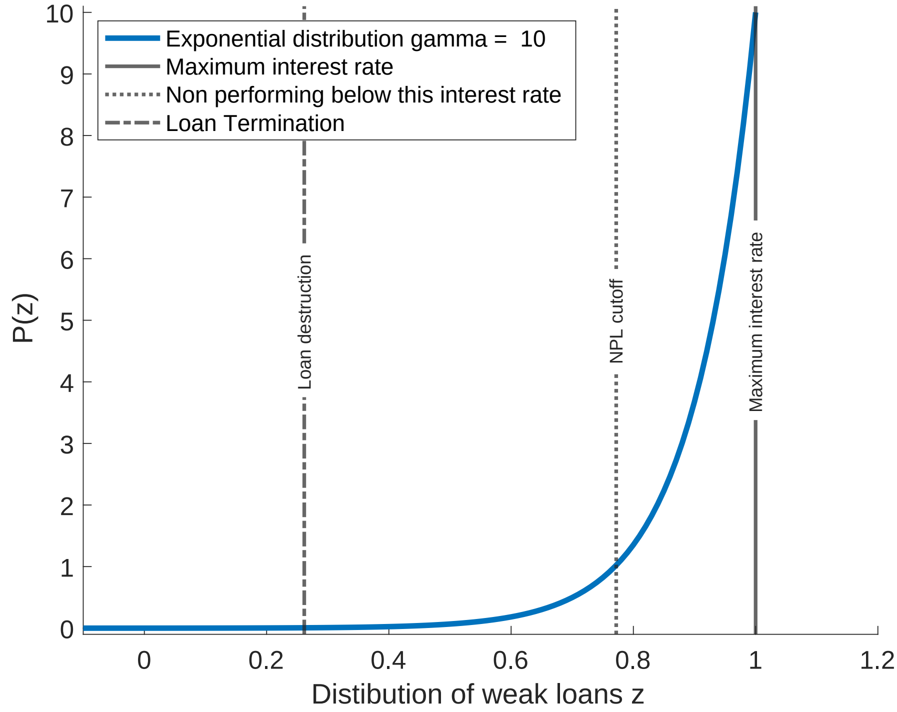


Figure 16: Pareto shape $\gamma = 2.4$

parameter of $-\zeta_s$, or transformed to z to a conditional expected value $z_s \in (-\infty, \bar{z}]$. We can transform a value of ζ to idiosyncratic productivity with equation (39). The probability of weak loans being foreclosed is then $H(z_s) = exp(-\gamma(\bar{z} - z_s))$. The mean productivity of a weak loan is then given by equation (40).

$$\hat{z} = \frac{\bar{z} - \frac{1-\gamma \exp(-\gamma[\bar{z}-z_{s,t}])(\bar{z}-z_{s,t})}{\gamma}}{1 - \exp(-\gamma[\bar{z} - z_{s,t}])} \tag{40}$$

3.8 Stochastic exogenous processes

There are three stochastic processes in this economy. Equation (41) shows the auto-regressive process capturing typical deviations in aggregate productivity. Equation (42) consists of an auto-regressive process capturing shocks to the current forbearance incentive with $t\hat{a}u = \tau_t - \pi^L(1-\delta_k))E_t(\mu_{t+1}\tau_{t+1})$. $\sigma a, \tau$, which is suspected to be less or equal to 0 captures forbearance incentives possibly rising in line with an aggregate shock. Similarly, equation (43) captures possible changes to the efficiency of used capital markets. It is probable that $\sigma a, \mu \geq 0$, especially when shocks to aggregate productivity affect the functioning of capital transactions due to other turmoil in the economy. Further, the Cobb-Douglas matching function may not capture all changes in frictions in capital markets similar as in employment markets, where deep recessions have been shown to be accompanied by a significant decline in matching efficiency (Sedláček, 2014).

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \tag{41}$$

$$\hat{\tau}_t = (1-\rho_\tau)\tau + \rho_\tau \hat{\tau}_{t-1} + \sigma_{a,\tau}\epsilon_t + \epsilon_\tau] \tag{42}$$

$$\mu_{u,t} = (1-\rho_\mu)\mu_u + \rho_\mu \mu_{u,t-1} + \sigma_{a,\mu}\epsilon_t + \epsilon_\mu] \tag{43}$$

3.9 Equilibrium

The competitive equilibrium of the economy can be summarised by the following equations in the highlighted boxes 1, 2, 3, and 4. The matching functions are specified in a Cobb Douglas form with the number of matches of new and foreclosed capital given as $M_x = \mu_x \psi X \theta_x^{1-\xi}$ and $M_u = \mu_u \psi K^U \theta_u^{1-\xi}$. $t\hat{a}u$ is defined as $t\hat{a}u = \tau(1-\beta\pi^L(1-\delta_k))$ capturing possibly dynamically changing forbearance incentives in the banking sector. The idea is that these incentives may increase when the economy is experiencing a negative shock. Similarly, the efficiency of used capital markets may decrease in economic crisis as market makers exit.

Dynamic competitive equilibrium: Exogenous processes

Stochastic aggregate process:

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t]$$

Stochastic forbearance cost process:

$$\hat{\tau}_t = (1-\rho_\tau)\tau + \rho_\tau \hat{\tau}_{t-1} + \sigma a, \tau A_t + \epsilon_\tau]$$

Stochastic used capital market efficiency process:

$$\mu_{u,t} = (1-\rho_\mu)\mu_u + \rho_\mu \mu_{u,t-1} + \sigma a, \mu A_t + \epsilon_\mu]$$

Highlighted Box 1: Competitive equilibrium: Exogenous processes

3.10 Comparative statics

The baseline calibration of the model parameters is set out in Table 4. Variations of this baseline calibration are shown in this comparative statics section. The choice of the matching functions for fresh unspecified and used foreclosed capital is made such that fresh capital markets are in the baseline three times as efficient as specified capital markets with $\mu_u = 0.2$, while $m_x = 0.6$. This corresponds to the finding in the literature that (Huljak, Martin, Moccero, and Pancaro, 2020) asset specificity on average leads to a mere recovery rate of 35 % for non-financial firms. The share of interest payments by non-financial firms compared to their profits between 1985 and 2018 in the US is around 0.297. This suggests banks being able to extract a share of around 0.3 from the joint surplus of the Nash bargain

Dynamic competitive equilibrium: Household and aggregate

Euler equation:

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta_{d,t+1} + \rho_{t+1})]$$

Resource constraint:

$$C_t + I_t = A_t K_t^{\alpha-1}(\bar{z}K_t^N + \hat{z}K_t^L) + gK_t^U - \kappa_x \theta_{x,t} \psi X_t - \kappa_u \theta_u K_{u,t}$$

Deposit depreciation:

$$\delta_{d,t} = \delta_k \frac{K_{t-1}^N + K_{t-1}^L + K_{t-1}^U + p(\theta_{x,t-1})S_{t-1}}{D_{t-1}} + \delta_x \frac{X_{t-1} - p(\theta_{x,t-1})S_{t-1}}{D_{t-1}}$$

Interest rate paid to households:

$$\rho_t = \frac{r_{\bar{z},t}K_t^N + r_{\hat{z},t}K_t^L + gK_t^U}{D_t}$$

Aggregate capital and deposits:

$$K_t = K_t^N + K_t^L + K_t^U, \quad D_t = X_t + K_t^N + K_t^L + K_t^U$$

Highlighted Box 2: Competitive equilibrium: Household and aggregate constraints

with firms. The bargaining power of banks η is set therefore set to 0.3, meaning that the matching parameter ξ is set to 0.7 to fulfill the (Hosios, 1990) condition. κ_x and κ_u are both set to 0.01 keeping the cost for new proposals for capital from firms to banks at 1% of the loan value. g is chosen at $1 - \beta(1 - \delta_k)$ allowing for foreclosed capital to be perfectly mothballed so that the real cost of foreclosed capital are close to 0. The discount factor β is set to 0.99, while physical capital depreciation δ_k is set to 0.05, following (Cooley, 1995). The share of capital in the economy α is set to 0.35 as is conventional. δ_x is set to 0 which can be considered a real cost of keeping the unmatched capital charged by the bank, but could also be set to $\delta_x > 0$ to obtain similar results. The maximum idiosyncratic productivity of a capital unit in a loan $\bar{z}$ is normalised to 1. The parameter of the exponential distribution is then chosen to calibrate endogenous loan foreclosure. A higher value will mean a steeper distribution with more foreclosed loans in every period. In this case, a value of 10 means 3% of loans are foreclosed in every period. π^N is set to 0.8 meaning safe loans have a 20% chance of turning weak and, weak loans have a 20% chance of turning safe again due to $\pi^L = 0.8$. These choices are made to target a sNPL of 0.0448 corresponding to the mean sNPL of the 15 European economies over the period 1999-2017. Finally, for foreclosure incentives $\tau = 0$ in the baseline calibration.

The persistence of the aggregate process ρ_a is set to 0.9. The other exogenous processes are assumed to be of similar persistence. In the baseline calibration $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ are set to 0 meaning that a shock to the aggregate process is not accompanied by increases in foreclosure costs or decreases in the efficiency of used capital markets. These parameters are then varied in the dynamic simulation to judge which one is more likely to capture the data.

The steady-state of the model is found with a non-linear solver solving the equations in

Dynamic competitive equilibrium: Credit markets

Loan creation condition:

$$\frac{\kappa_x}{q_x(\theta_{x,t})} = E_t((1-\delta_k)\mu_{t+1}\left[(1-\eta)(A_{t+1}K_{t+1}^{\alpha-1}\bar{z}-g) - \eta\kappa_u\theta_{u,t+1} + \pi^N\frac{\kappa_x}{q_x(\theta_{x,t+1})} + (1-\pi^N)(1-\delta_k)\mu_{t+2}\hat{V}_{\hat{z},t+2}^{E,L}\right])$$

Loan foreclosure decision:

$$z_{s,t} = K_t^{1-\alpha}A_t^{-1}[g-\tau(1-\beta\pi^L(1-\delta_k)) + \frac{\eta}{1-\eta}\kappa_u\theta_{u,t} - \frac{1}{1-\eta}E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}_{\hat{z},t+1}^{E,L}\right]]$$

Mean productivity of a weak loan and foreclosure share of weak loans:

$$\hat{z} = \frac{\bar{z}\gamma - 1 + \exp(-\gamma[\bar{z}-z_{s,t}])(\gamma[\bar{z}-z_{s,t}]+1)}{\gamma(1-\exp(-\gamma[\bar{z}-z_{s,t}]))}, \quad H(z_{s,t}) = \exp(-\gamma[\bar{z}-z_{s,t}]))$$

Mean value of a weak loan:

$$\frac{\hat{V}_{\hat{z},t}^{E,L}}{(1-\exp(-\gamma(\bar{z}-z_{s,t})))} = [(1-\eta)(A_tK_t^{\alpha-1}\hat{z} - g + \tau(1-\beta\pi^L(1-\delta_k))) - \eta\kappa\theta_{u,t} + E_t\left[(1-\pi^L)\frac{\kappa_x}{q_x(\theta_{x,t})} + \pi^L(1-\delta_k)\mu_{t+1}\hat{V}_{\hat{z},t+1}^{E,L}\right]]$$

Match probabilities:

$$p(\theta_{u,t}) = \mu_u\theta_u^{1-\xi}, \quad p(\theta_{x,t}) = \mu_x\theta_x^{1-\xi}, \quad q(\theta_{u,t}) = \mu_u\theta_u^{-\xi}, \quad q(\theta_{x,t}) = \mu_x\theta_x^{-\xi}$$

Interest rates to banks:

$$r_{\bar{z},t} = \eta[A_tK_t^{\alpha-1}\bar{z} + \kappa_u\theta_{u,t}] + (1-\eta)g, \quad r_{z,t} = \eta[A_tK_t^{\alpha-1}z + \kappa\theta_{u,t}] + (1-\eta)[g-\tau(1-\beta\pi^L(1-\delta_k))]$$

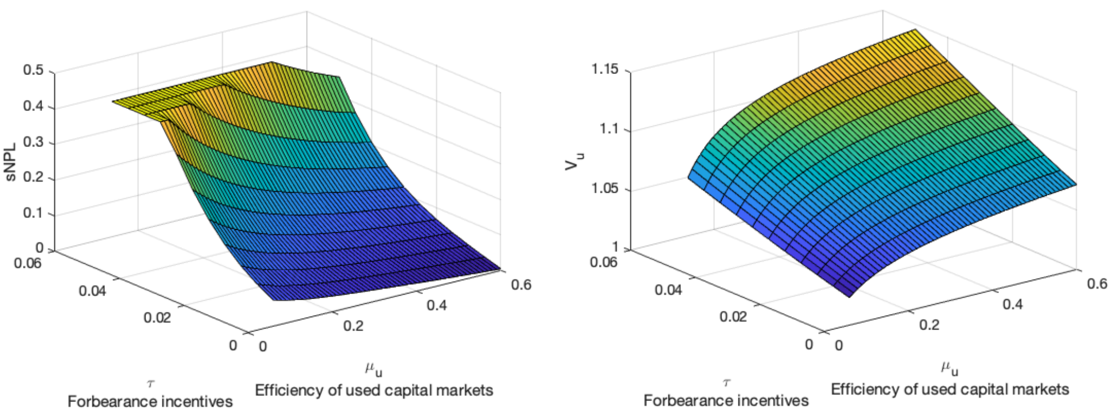
Highlighted Box 3: Competitive equilibrium: Credit market decision-making

Appendix A. Figure 17a - Figure 18b show the comparative statics when varying the efficiency of a match in used capital markets and forbearance incentives similar to the partial equilibrium section. Clearly, as the match efficiency in used capital markets increases or forbearance incentives decrease the sNPL decreases as Figure 17a shows.

On the other hand, the steady-state value of foreclosed capital in Figure 17b rises when used capital markets are more efficient, but also when forbearance incentives are larger as these will decrease the overall steady-state capital stock and thereby negative externalities for each foreclosed capital unit. The amount of foreclosed capital decreases with an increase in forbearance incentives as Figure 18a shows. An increase in efficiency of used capital markets will lead more banks to foreclose loans, because of the improved prospect of re-lending foreclosed collateral productively. Capital prices are not defined in this model. Nevertheless, as will become apparent in the section matching impulse response response functions defining a variable that can capture the dynamics of capital prices is useful. It is argued that used capital market tightness θ_u should be seen as capturing recorded capital price dynamics. The reason is that the prices already capture existing capital, which is represented by the denominator of θ_u, and the numerator is representing capital demand. Assuming conventional demand and supply behaviour the price of capital should therefore be positively correlated to θ_u, which will be used as a proxy for the price of capital in the model. Forbearance incentives mean there is less capital on offer, which given similar demand will drive up the price of capital assumed to be proportional

Parameter	Value	Description
β	0.99	Discount rate
ψ_x	0.95	Share of cash that can be lent
δ_k	0.05	Capital depreciation
δ_x	0.02	Real cash depreciation
κ_x	0.01	Proposal cost for new capital
κ_u	0.01	Proposal cost for used capital
m_x	0.6	Match productivity new capital
m_u	0.2	Match productivity for used capital
$\bar{z}$	1	Maximum loan productivity
γ	6	Exponential distribution parameter
α	0.35	Aggregate capital exponent
ξ	0.7	Parameter on matching function
η	0.3	Bargaining power of banks
τ	0	Possible forbearance incentives
g	0.07	Production value of foreclosed capital
π^N	0.8	Probability of a safe loan staying safe in the next period
π^L	0.8	Probability of a weak loan staying weak in the next period
ρ_a	0.9	Persistence of the aggregate process
ρ_τ	0.9	Persistence of the forbearance process
ρ_μ	0.9	Persistence of the process for used capital market efficiency
$\sigma_{a,\tau}$	0	Correlation of forbearance incentives with the aggregate process
$\sigma_{a,\mu}$	0	Correlation of used capital market efficiency with the aggregate process

Table 4: Baseline calibration



(a) Comparative effects of τ and μ_u on sNPL.

(b) Comparative effects of τ and μ_u on the value of foreclosed capital V^U

Dynamic competitive equilibrium: State transitions

Law of Motion for cash:

$$X_t = (1-\delta_x)[X_{t-1} - p(\theta_{x,t-1})\psi X_{t-1}] + I_{t-1}$$

Law of Motion for capital in safe loans:

$$K_t^N = (1-\delta_k)[\pi^N K_{t-1}^N + p(\theta_{u,t-1})K_{t-1}^U + p(\theta_{x,t-1})S_{t-1} + (1-\pi^L)K_{t-1}^L]$$

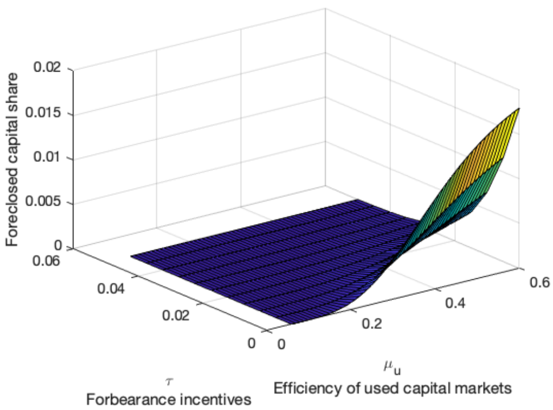
Law of Motion for capital in weak loans:

$$K_t^L = (1-\delta_k)(1-H(z_{s,t}))[\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N]$$

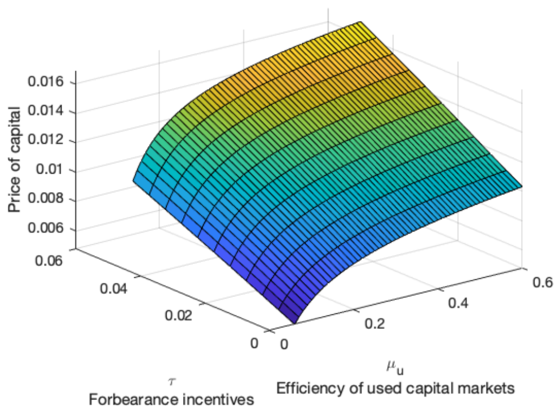
Law of Motion for foreclosed capital:

$$K_t^U = (1-\delta_k)[H(z_{s,t})(\pi^L K_{t-1}^L + (1-\pi^N)K_{t-1}^N) + (1-p(\theta_{u,t-1}))K_{t-1}^U]$$

Highlighted Box 4: Competitive equilibrium: Laws of Motion



(a) Comparative effects of τ and μ_u on the share of foreclosed capital.



(b) Comparative effects of τ and μ_u on capital prices.

to capital market tightness. In steady-state, however, agents will account for the additional cost of foreclosure making the effect of forbearance incentives for loans on capital prices neutral. Meanwhile, higher demand in more efficient markets and equal supply will mean that capital prices increase as the efficiency of used capital markets increases as shown in Figure 18b. This shows that the dynamics of capital prices provide information to separate forbearance incentives from the efficiency of used capital in their impact on the sNPL.

3.11 Dynamics

The model is simulated from the steady-state via a second order perturbation. The simulated impulse responses here show first that the model can replicate the dynamics of an aggregate output shock on sNPL. Further, the graphs show that the effect of a rise in the sNPL due to an increase in market frictions on investment, capital returns, and capital prices matches the impulse responses computed from the data. Finally, variations in the impulse response are presented when the steady-state value of frictions and the correlation of market friction shocks to output shocks is varied.

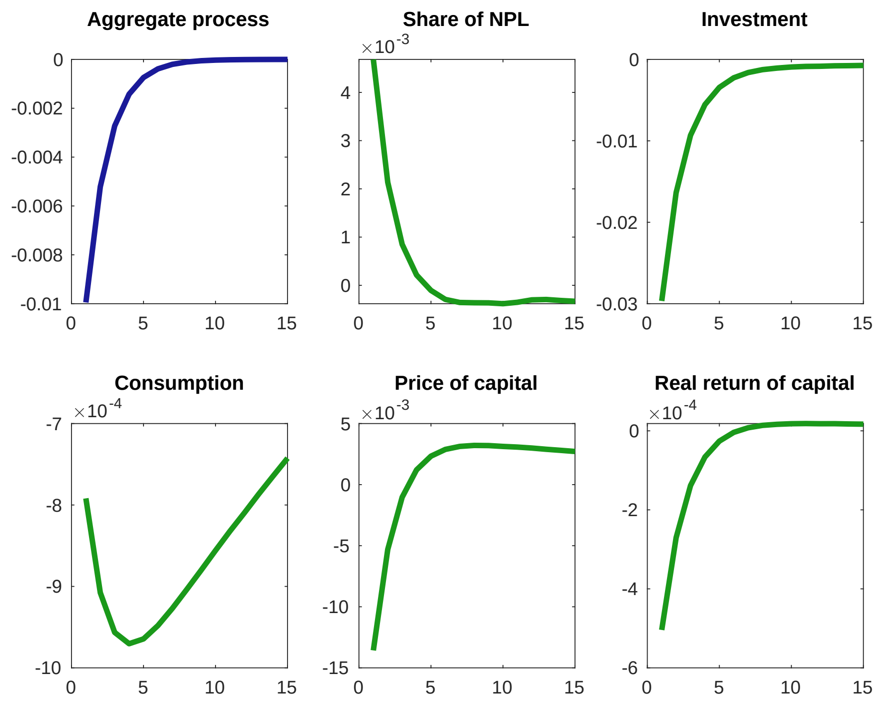


Figure 19: Impulse response to a 1% aggregate shock

The simulated impulse response functions in Figure 19 show that the model can replicate the general correlations of non-performing loans and the business cycle. The model, which is calibrated for annual rates exhibits a large rise in the sNPL following an adverse shock. As expected investment and the return to capital also decline. In the current model the sNPL forms an endogenous outcome. Hence to observe the effect of a rise in sNPL keeping aggregate output stable frictions are shocked, which will drive sNPL. Figure 20 shows the response of sNPL rising as a result of the efficiency of used capital markets declining. The mean value of a weak non-foreclosed loan will decline, meaning that banks accept not to foreclose more loans with a lower value. This leads to misallocation rising, and the real return to capital declining as well as investment falling. Note that in this case the price of capital declines. Meanwhile, a 10^-3 shock to the forbearance incentives will also increase the sNPL. Investment will fall due to the reduced value of a new loan, as will the real return to capital due to an increase in misallocation. However, tightness in used capital markets will rise, meaning there is less supply relative to demand for capital, which should lead to a proportional increase in the price of capital.

Figure 22 shows that the sNPL responds similarly if the aggregate shock has no effect on $\mu_{u,t}$ and $\hat{\tau}_t$ and only the steady-state value is varied. In both cases, higher forbearance incentives or lower used capital market efficiency will lead to a higher increase of the sNPL in response to a negative aggregate shock. Nevertheless, banks get more restrictive as shown in Figure 23 in the type of weak loan they accept, but this doesn't make up for the number of loans that fall below the performing level as the productivity of all loans

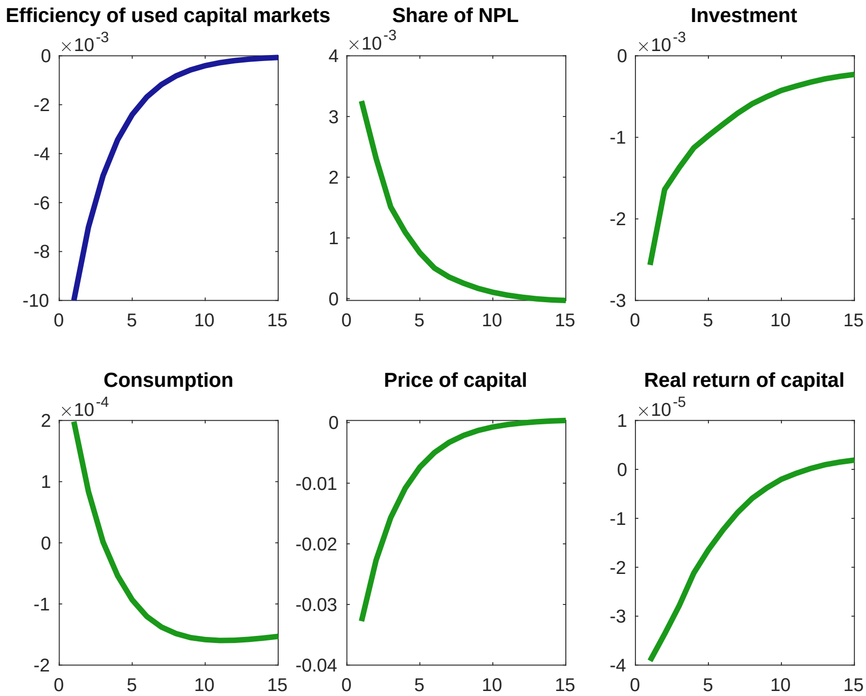


Figure 20: Impulse response to a 1% shock to the match productivity μ_u of used capital markets.

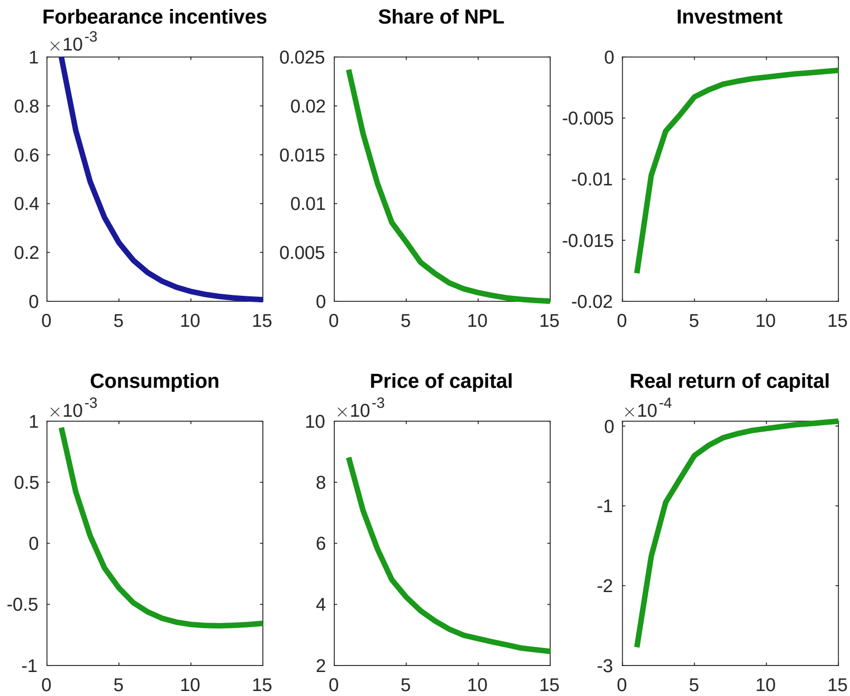


Figure 21: Impulse response to a 10ˆ-3 shock to forbearance incentives.

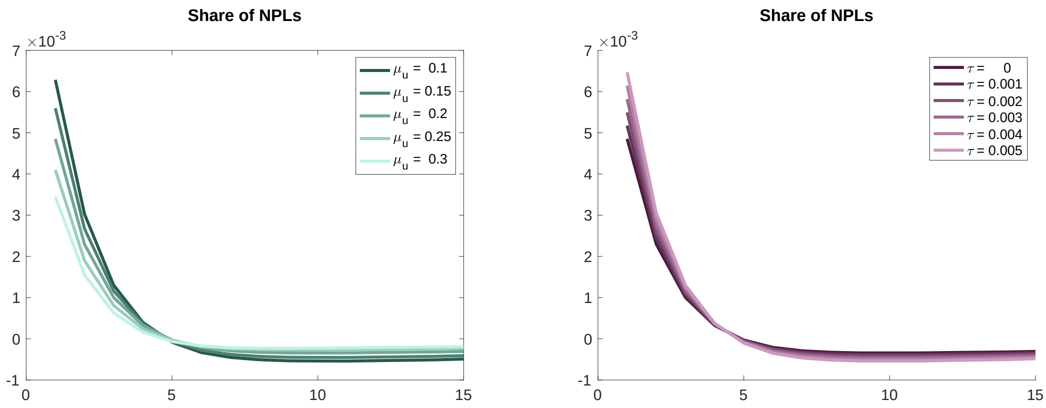


Figure 22: Dynamic responses to a negative 1% TFP shock (top row) and varying measures of μ_u (left) and $\hat{\tau}$ right.

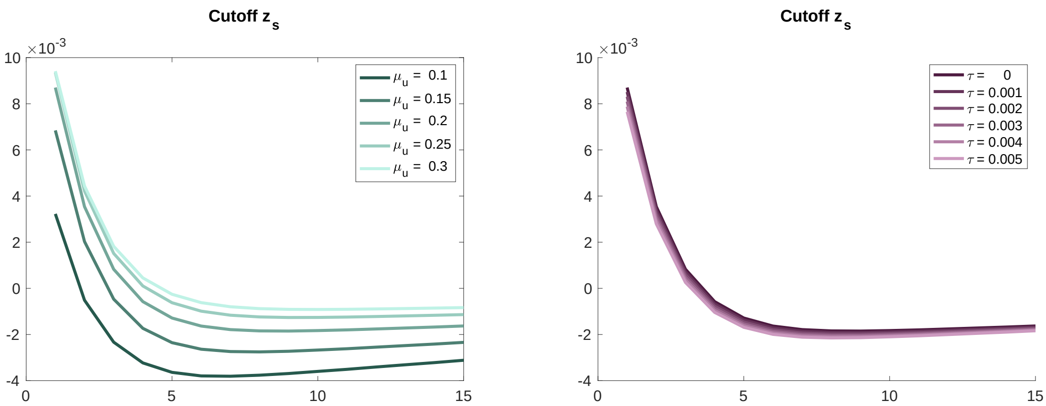


Figure 23: Dynamic responses to a negative 1% TFP shock (top row) and varying measures of μ_u (left) and $\hat{\tau}$ right.

shifts down.

The purpose of Figure 24 to Figure 26 is to show that when varying towards $\sigma_{a,\tau} < 0$ or $\sigma_{a,\mu_u} > 0$ it is clear that a negative shock is likely to be paired with a shock to the efficiency of used capital markets if the price of capital is supposed to fall as observed in the data. The model shows in Figure 24 that even small increases in forbearance incentives or changes to the efficiency of used capital markets can have very large effects on the response of the sNPL. However, Figure 25 shows that even a relatively small increase in forbearance incentives may drive the price of capital up, as less used capital is supplied to the market. Figure 26 shows this decrease in the supply of used capital to the market is not there when μ_u is decreasing, but only when $\hat{\tau}$ is increasing as foreclosure is stopped via incentives not to foreclose and not via congestion in capital markets driving down the value of foreclosed capital. This shows that a low efficiency in used capital markets and a decline in recessions is a driver the sNPL more congruent with the observed response of the sNPL and capital prices.

To summarise, the model predicts that increasing sNPL goes hand in hand with reduced capital market reallocation. This is a testable prediction. Looking at cross-country changes in countries' capital reallocation activity in Figure 9 it seems that the prediction holds. The

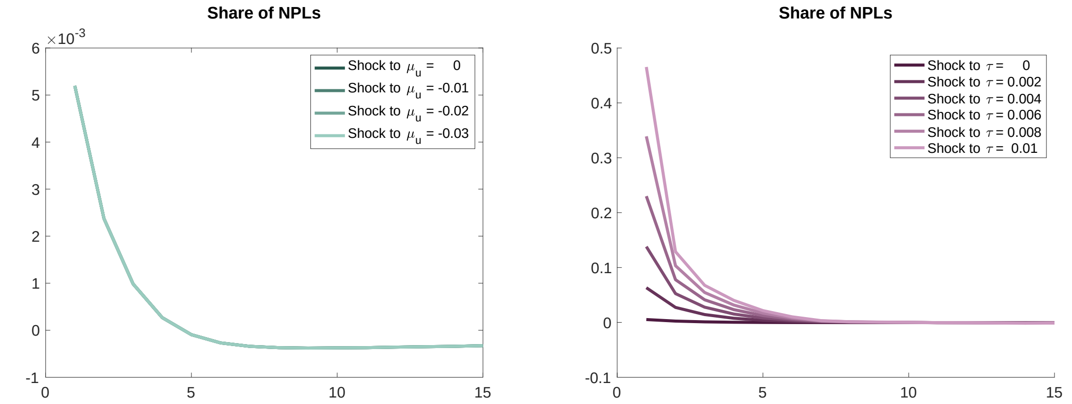


Figure 24: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the sNPL.

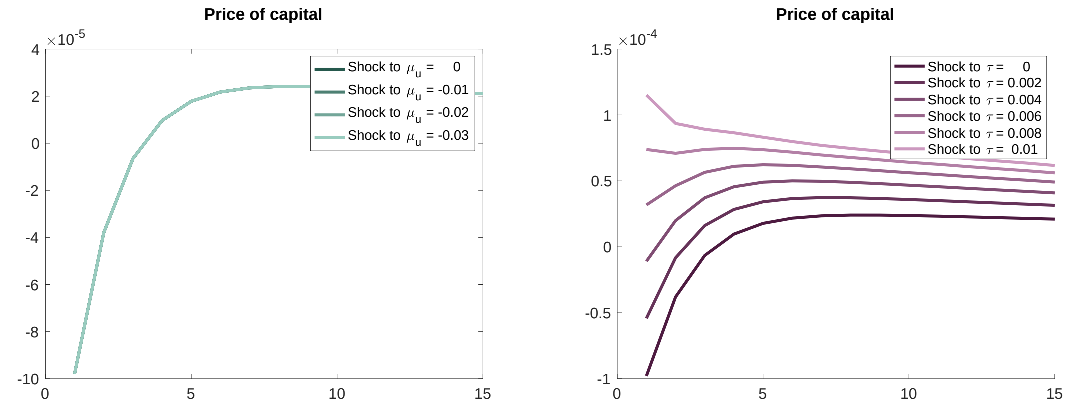


Figure 25: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the price of capital.

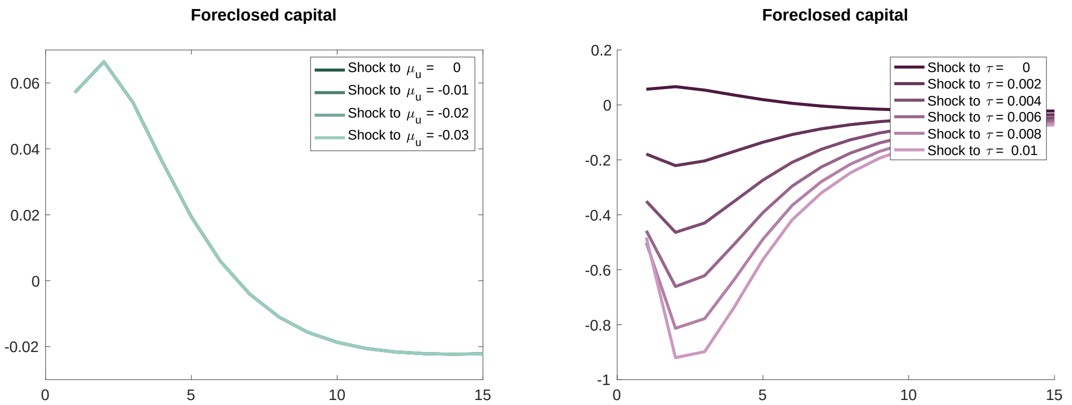


Figure 26: Dynamic responses to a negative 1% TFP shock paired with a shock to used capital market efficiency μ_u (left) or dynamic forbearance incentives $\hat{\tau}$ (right) as stated in the legend to the state of foreclosed capital seeking re-matching.

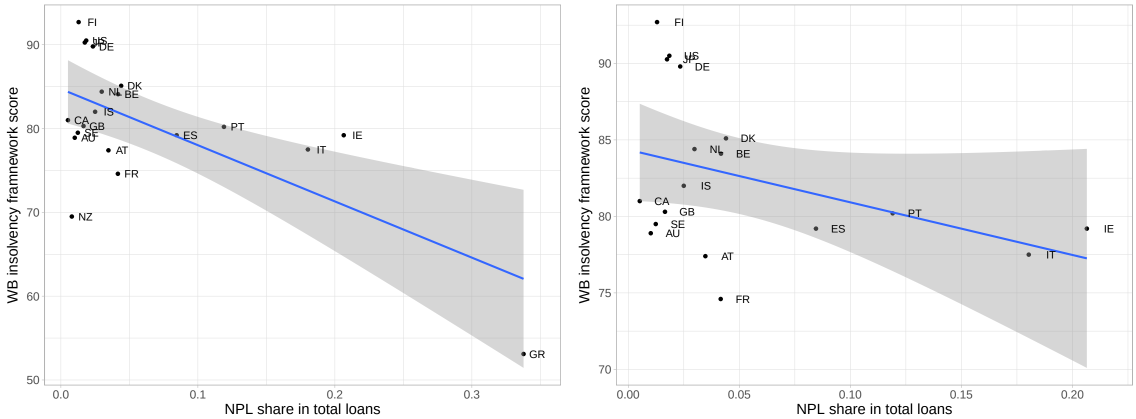


Figure 27: NPL shares and latest insolvency framework scores

model shows that both forbearance incentives and used capital market frictions drive the sNPL. Figure 27 provides credit to this theoretical result by showing the world bank insolvency scores from the ease of doing business indicators plotted against non-performing loans share for the sample of countries. We can see the difficulty in administering an insolvency process as real costs in foreclosing a loan or losses due to the low prices realised for foreclosed capital and the high matching frictions. The correlation in the two figures is negative meaning that a higher ease of winding up a company, which delivers a higher outside value to banks will lead to lower sNPL. Both figures are based on the same data, but the second figure removes outliers by only plotting countries with an insolvency score larger than 70.

3.12 Estimating capital market matching efficiency and forbearance changes

Simplifying the exogenous processes as shown in equation (44) to (46) allows for estimating the correlations $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ of fictions with the aggregate process by exploiting the different dynamic response of changes to μ_u and μ_τ on capital prices and employing impulse response function matching. The impulse response function matching method employed follows the conventional approach described in (Fernández-Villaverde, Rubio-Ramírez, and Schorfheide, 2016). The estimated correlations can reveal to which extent the efficiency of used capital markets and forbearance incentives change over the cycle.

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \tag{44}$$

$$\hat{\tau}_t = \tau + \sigma_{a,\tau} A_T \tag{45}$$

$$\mu_{u,t} = \mu_u + \sigma_{a,\mu} A_T] \tag{46}$$

The data used are the deviations from a Hodrick Prescott on annual data of real value

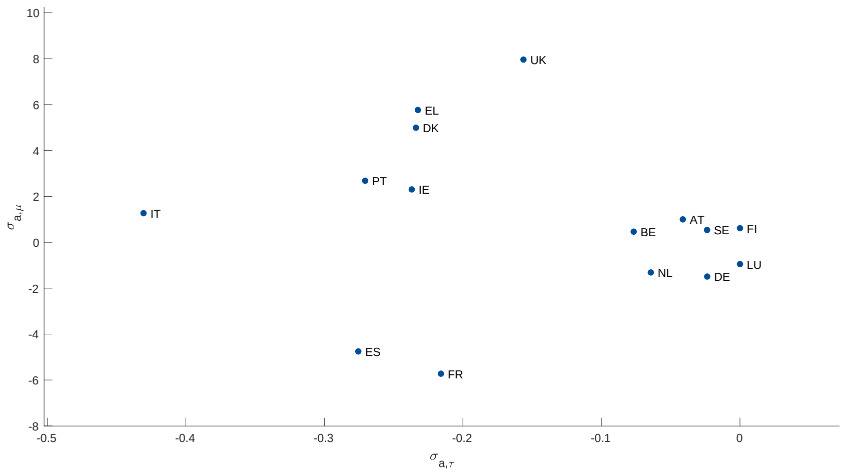


Figure 28: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes. A stronger reaction in capital market efficiency is correlated with a stronger reaction in forbearance incentives.

added to capture the actual output produced by an economy and capital prices provided by Eurostat. Furthermore, the percentage of NPLs provided by the World Bank and IMF after removing a trend. The empirical impulse to be matched is generated via the estimation is a vector auto-regression of order 1 and a constant. The unrestricted VAR is identified with short-run restrictions to identify an output shock on the three variables of interest. The impulse response of the variables and the model fit for each country can be found in Appendix C.

The estimation of $\sigma_{a,\tau}$ is restricted to the space $\sigma_{a,\tau} \in (-\infty, 0]$ meaning that a positive output shock will always reduce forbearance incentives. $\sigma_{a,\mu}$ is not restricted as there are good arguments for used capital markets getting both more and less productive in a recession $\sigma_{a,\mu} \in [0, \infty)$. The restriction on $\sigma_{a,\tau}$ is to discipline the estimation with the limited number of observations available. However, note that it would be possible for the estimation to find no effects. Figure 28 shows the estimates for several countries that were well matched by the model. It shows that in general, forbearance incentives and used capital market frictions both tend to increase with adverse shocks. A group of countries to the right experiences few changes of these frictions over the business cycle. Note that these are countries with low sNPL. Forbearance incentives increase most in Italy (IT), but other countries with high sNPL such as Greece (EL), Portugal (PT) and Ireland (IE) see both an increase in forbearance incentives paired with used capital markets becoming less efficient. The low levels of sNPL in France (FR) and Spain (ES) in the data are explained by used capital markets in these countries becoming more efficient in a recession, even though incentives for forbearance increase. For the United Kingdom (UK) and Denmark (DK) high estimates for the pro-cyclicality of capital market frictions are proposed by the model as an explanation for the highly cyclical variation of capital prices and sNPL, even though the sNPL is low in these countries.

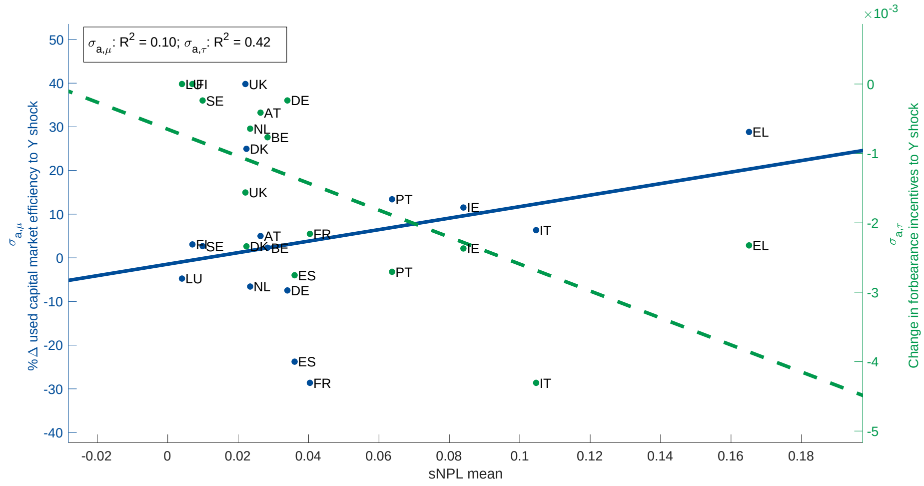


Figure 29: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes plotted against variation in NPLs.

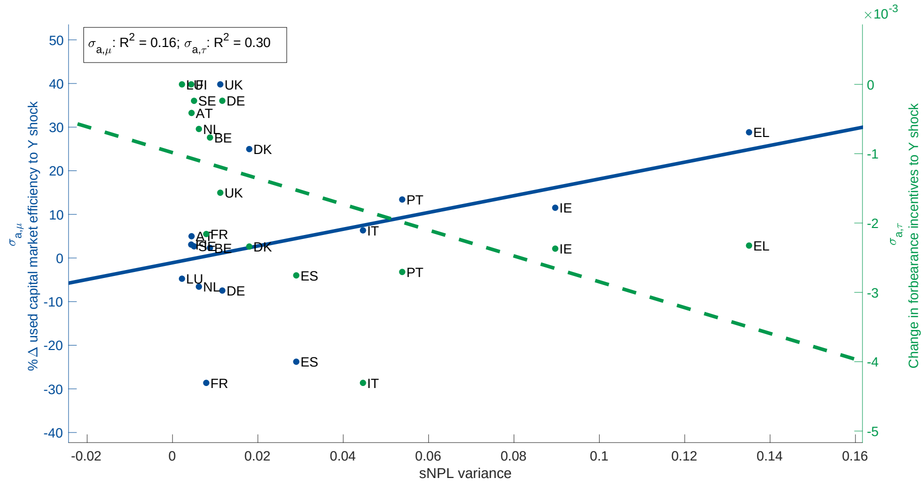


Figure 30: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes plotted against variation in NPLs.

Figure 29 shows the estimates of $\sigma_{a,\mu}$ plotted on the left axis and $\sigma_{a,\tau}$ plotted on the right axis against the mean sNPL of countries over the sample period. It is shown that both more pro-cyclical match efficiencies and counter-cyclical forbearance incentives are correlated with a higher mean sNPL. While the R^2 of $\sigma_{a,\mu}$ is low estimated changes to capital market efficiencies in response to an aggregate shock are large. Meanwhile, forbearance incentive changes cannot be computed as percentage changes due to the assumption that forbearance incentives are 0 in equilibrium. However, as shown Section 3.11 even the small forbearance incentive increases estimated in the order of 10^-3 can lead to large sNPL responses. Figure 30 shows that the estimated weights can also contribute to explaining the variance of sNPL. A pro-cyclical capital market efficiency can explain comparatively more of cross-country differences than cross-country mean differences, while the explanatory power of forbearance incentives is reduced.

Overall, the estimates show that as expected both more pro-cyclical used capital market efficiencies and counter-cyclical forbearance frictions contribute to explaining variation in the mean and variance of the sNPL between countries. Several countries with high mean sNPL seem to face declines in used capital market efficiency when adverse shocks hit the economy, making misallocation worse in these countries and increasing the sNPL.

4 Conclusion

This paper shows that the hare of non-performing loans are counter-cyclical, and correlate with reduced investment, capital reallocation, returns to capital, and investment. Further, periods of higher sNPL in a country are shown to be tightly linked to higher misallocation. A structural model with search frictions in capital reallocation can produce the sNPL endogenously and capture the empirical relations presented in Section 2. The model presents two drivers for the sNPL forbearance incentives and frictions in used capital markets, which drive down the outside value of foreclosure. It can thereby combine recent advancements in the capital misallocation literature with empirical studies regarding the sNPLs. The theoretical relations in the model imply that resolving non-performing loans is a matter of the efficiency of capital reallocation. This is defined by an economy's ability to bring used specified capital to new more productive uses.

The model further proposes a novel avenue to identify to what extent the two frictions that drive the sNPL in a country become worse over the cycle. It uses the differing impulse response of capital prices to a worsening of the matching function, in comparison to an increase in forbearance incentives. The estimates show changes in both frictions to be important to explain the data in most countries, but different countries show a different prevalence for some frictions. Countries with high sNPL over the sample period, seem to struggle more with a worsening of capital market efficiency, except Italy where bank forbearance seems to be prevalent.

The model is kept simple and tractable, but is set up in such a way that it allows for being extended to capture persistent heterogeneity extensions via a dynamic directed search block recursive equilibrium. In this way, the simple model can be expanded to allow for dealing with heterogeneity via a block recursive solution of the credit market as developed in (Menzio and Shi, 2010a). A solution would be possible in this general equilibrium setup because the aggregate return of a unit of capital remains uninfluenced by the distribution of states of capital in employed or unemployed capital linked to non-performing loans. With the block recursion persistent heterogeneous loan relationships, different types of firms and banks, refinancing, endogenous bankruptcies, and different policies to encourage solutions to non-performing loans may be handled. An example would be endogenising the empirical observation that banks with a weaker balance sheet also exhibit increased forbearance incentives (Bergant and Kockerols, 2018), or allowing for different firm and bank types which may enjoy differing levels of government loan support and insolvency protection. Finally, the tractability of the model allows for studying optimal policy towards creating or discouraging bank forbearance as well as encouraging the build-up of used capital markets or the conditions under which the founding of public asset management companies may be beneficial.

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Appendices

Appendix A Steady-States

Exogenous foreclose

The steady-state can be solved around the steady-state of the exogenous processes where $a = 1$ and $\lambda_t = \lambda$.

$$\delta K = I \tag{47}$$

$$K_E = \lambda^{-1}[\frac{\delta}{1-\delta} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda)]K_U \tag{48}$$

Define $\psi = \lambda^{-1}[\frac{\delta}{1-\delta} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1-\lambda)]$ so $K_E = \psi K_U$

$$\Pi = K_E[(1-\eta)[K_E + K_U]^{\alpha-1} - \eta p(1-\delta)(1-\lambda)\beta V_E)] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}K_U \tag{49}$$

$$\Pi = K_E[(1-\eta)K_E^{\alpha-1}[1+\psi^{-1}]^{\alpha-1} - \eta p(1-\delta)(1-\lambda)\beta V_E)] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}\psi^{-1}K_E \tag{50}$$

$$C = \Pi + (\rho - \delta)D = \Pi + (\beta^{-1} - 1)D \tag{51}$$

$$\beta^{-1} + \delta - 1 = \rho \tag{52}$$

$$r = (1+\psi^{-1})[\beta^{-1} + \delta - 1] \tag{53}$$

$$V_E = \frac{(1-\eta)[1+\psi^{-1}]^{\alpha-1}}{1-\beta(1-\delta)(1-\lambda)(1-\eta p)}K_E^{\alpha-1} \tag{54}$$

$$C = K_E^{\alpha}[1+\psi^{-1}]^{\alpha-1} - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}}\psi^{-1}K_E - \delta K_E(1+\psi^{-1}) \tag{55}$$

$$r = \eta[[1+\psi^{-1}]^{\alpha-1}K_E^{\alpha-1} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}\beta\delta(1-\lambda)V_E] \tag{56}$$

$$\beta^{-1} + \delta - 1 = r\frac{1}{1+\psi^{-1}} \tag{57}$$

Use r to solve for K_E. With K_E solve for the other variables.

$$K_E^{1-\alpha} = \frac{\eta[1+\psi^{-1}]^{\alpha-1}}{r}[1+m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}\beta(1-\delta)(1-\lambda)\frac{1-\eta}{1-\beta(1-\delta)(1-\lambda)(1-\eta p)}] \tag{58}$$

Endogenous foreclose

Steady states are solved around the aggregate process $A = 1$.

$$1 = \beta[1-\delta_d+\rho] \tag{59}$$

Creation condition

$$\frac{\kappa_x}{q_x(\theta_x)}(\frac{1}{\beta(1-\delta_k)} - \pi_N) = (1-\eta)[\bar{z}K^{\alpha-1} - g] - \eta\kappa\theta_u + \beta(1-\delta_k)(1-\pi^N)\hat{V}^E \tag{60}$$

$$\hat{V}^E = \frac{1-H(z_s)}{1-\beta(1-\delta_k)(1-H(z_s))\pi^L(1-\frac{\hat{z}}{z_s})}([(1-pa.eta)\hat{z}K^{\alpha-1}-g+\hat{\tau}]-\eta\kappa\theta_u+(1-\pi^L)\frac{\kappa_x}{q_x(\theta_x)}) \tag{61}$$

$$[\delta_x + p(\theta_x)\psi_x(1-\delta_x)]X = I \tag{62}$$

Interest to banks

$$r_{\bar{z}} = \eta[\bar{z}K_t^{\alpha-1} + \kappa\theta_u] + (1-\eta)g \tag{63}$$

$$r_{\hat{z}} = \eta[\hat{z}K_t^{\alpha-1} + \kappa\theta_u] + (1-\eta)(g-\hat{\tau}) \tag{64}$$

State steady states

K^L:

$$K^L = \frac{(1-\pi^N)(1-\delta_k).(1-H(z_s))}{1-(1-\delta_k)\pi^L(1-H(z_s))}X = \nu_1 X \tag{65}$$

K^U:

$$K^U = \frac{H(z_s)(1-\delta_k)(\nu_1\pi^L + (1-\pi^N))}{1-(1-\delta)_k)(1-p_u(\theta_u))}X = \nu_2 X \tag{66}$$

K^N:

$$K^N = \frac{(1-\delta_k)p_x(\theta_x)\psi_x}{1-(1-\delta_k)(\pi^N + (1-\pi^L)\nu_1 + \nu_2 p_u(\theta_u))} X = \nu_3 X \tag{67}$$

Foreclosure decision

$$z_s = K^{1-\alpha}[g + \frac{\eta}{1-\eta}\kappa_u\theta_u - \frac{1}{1-\eta}((1-\pi^L)\frac{\kappa_u}{q_u(\theta_u)} + \pi^L\hat{V}^E) - \hat{\tau}] \tag{68}$$

$$K^{\alpha-1} = [g + \frac{\eta}{1-\eta}\kappa_u\theta_u - \frac{1}{1-\eta}((1-\pi^L)\frac{\kappa_u}{q_u(\theta_u)} + \pi^L\hat{V}^E) - \hat{\tau}]\frac{1}{z_s} \tag{69}$$

δ_d"

$$\delta_d = \delta_k \frac{\nu_3 + \nu_3\nu_2 + \nu_3\nu_1 + p(\theta_x)\psi}{1+\nu_3+\nu_3\nu_2+\nu_3\nu_1} + \delta_x \frac{1-p(\theta_x)\psi}{1+\nu_3+\nu_3\nu_2+\nu_3\nu_1} \tag{70}$$

$$\rho = \frac{\nu_3 r_{\bar{z}} + \nu_3\nu_1 r_{\hat{z}} + \nu_3\nu_2 g}{1+\nu_3+\nu_3\nu_2+\nu_3\nu_1} \tag{71}$$

Appendix B Additional data

The series underlying the correlations table in Section 2 are presented in Figure 31.

Developed countries with higher NPLs have lower returns on capital. This relation is strong since the NPL divergence with Great Recession.

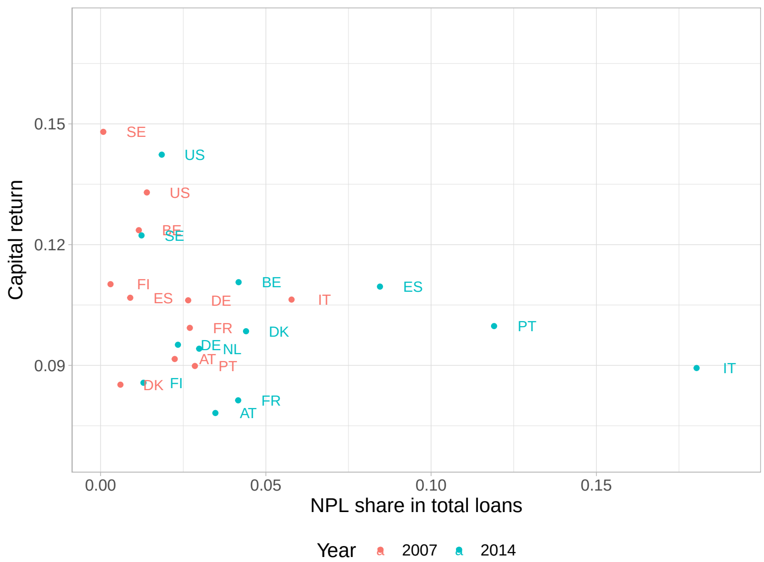

Figure 32: Returns on capital calculated from the KLEMS database, NPL ratios from World bank and IMF data

The impulse response function of demand (sNPL) and supply (real GDP) shocks on Real GDP are shown in Figure 33.

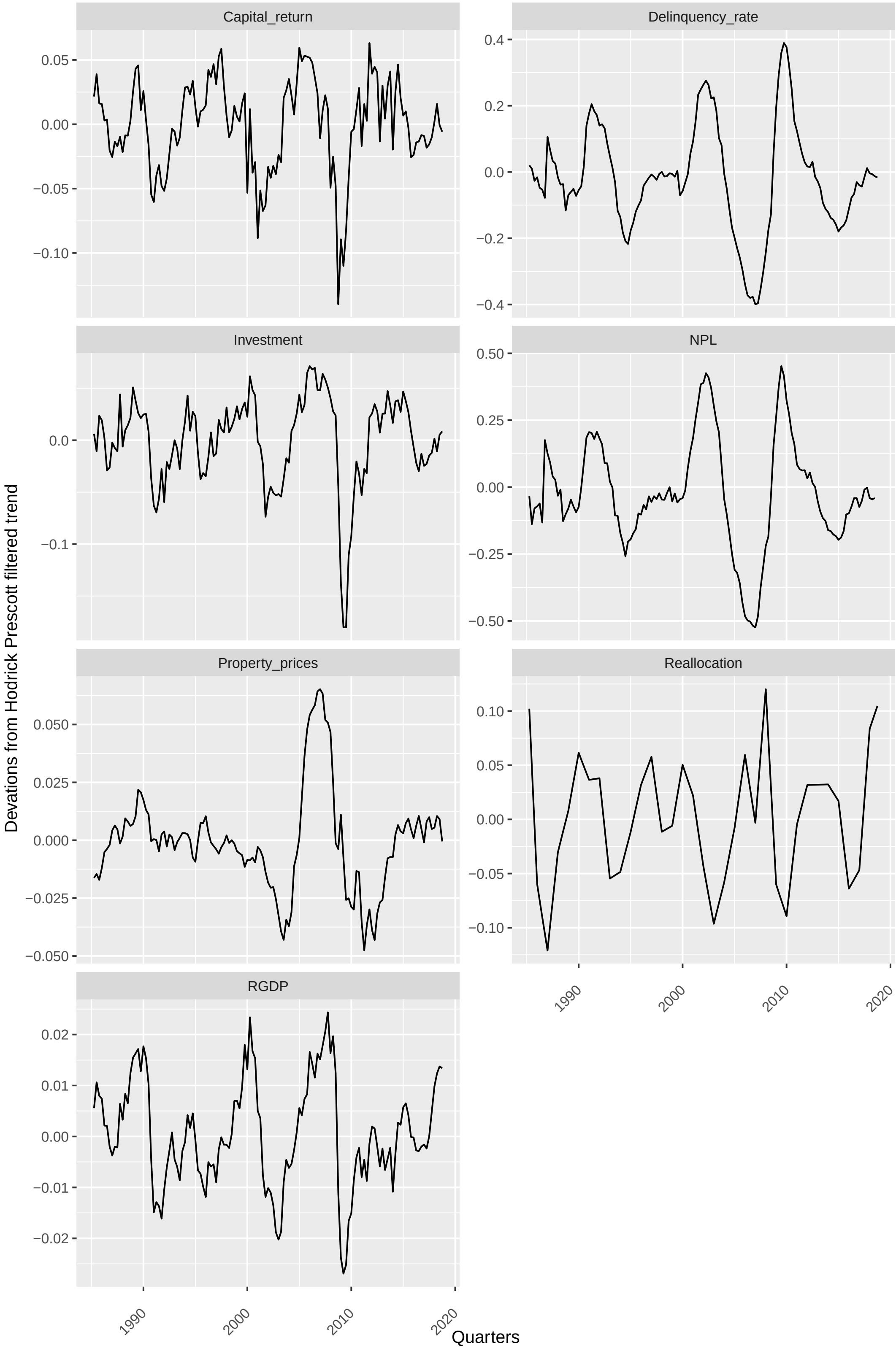

Figure 31: HP filtered US series

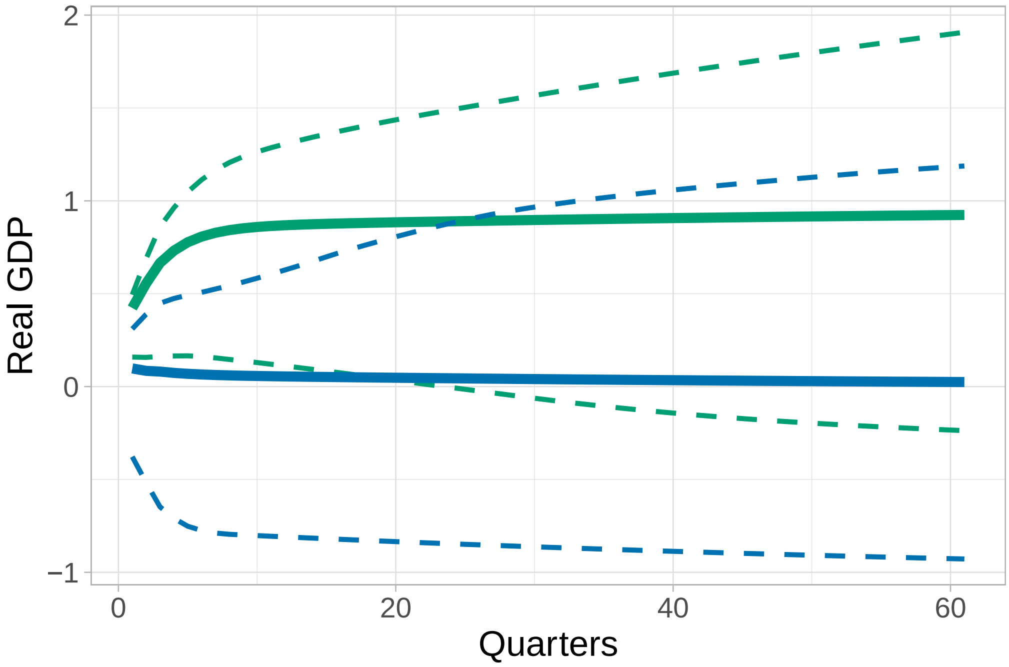

Figure 33: Green captures supply shocks (from real GDP) and blue demand shocks (from sNPL). Dashed lines show bootstrapped confidence intervals.

Other dispersion and misallocation measures

As the sNPL varies in a country estimated Coefficient of Variation in firm TFP estimated from a Cobb-Douglas production function imposed for each sector and following Wooldridge estimation (Wooldridge, 2009) increases proportionally. Figure 34 shows that dispersion increases as the sNPL increases.

Figure 35 shows how the co-variance of firm productivity and size following the estimated method in (Olley and Pakes, 1992) changes as the sNPL in a country varies.

Figure 36 shows how the dispersion marginal product of capital following (Kehrig et al., 2011) changes as the sNPL in a country varies. More information on the exact implementation of the method can be found in the user guide to the CompNet 7th vintage dataset (Deist, Amlung, Blyzniuk, Lang, Lalinskỳ, Inferrera, and Papagalli, ????).

Finding in (Balgova et al., 2016)that NPL reduction increases investment

(Balgova et al., 2016) show that countries that reduce NPLs with asset management companies experience real investment and output growth following such periods compared to countries that don’t.

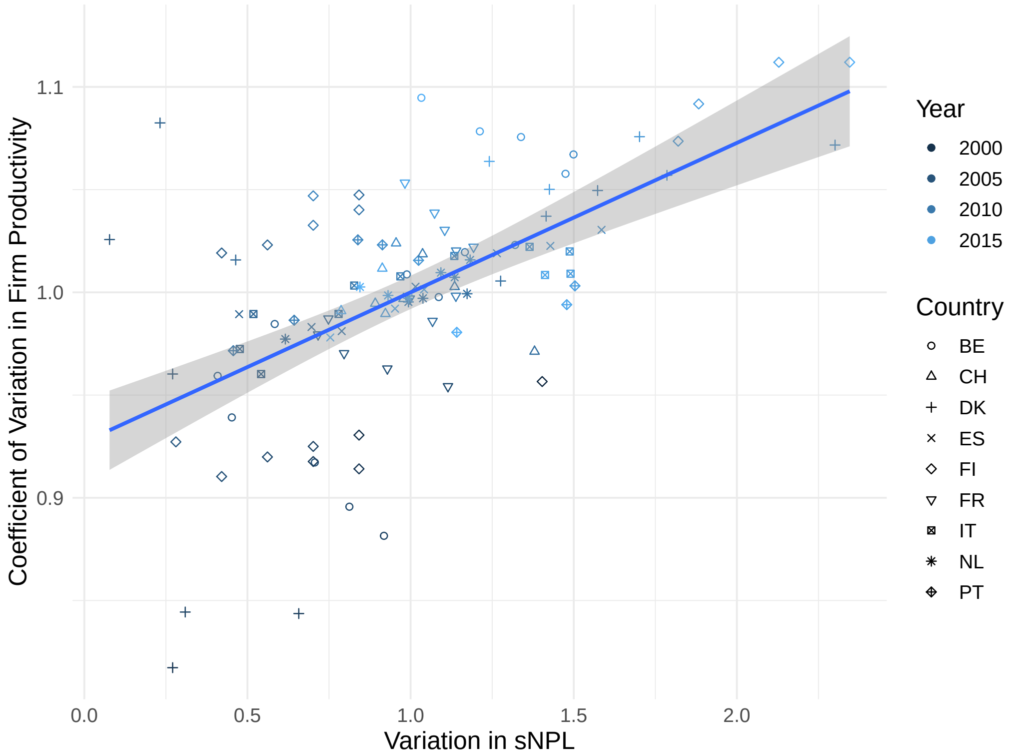

Figure 34: As the sNPL varies in a country estimated Coefficient of Variation in firm TFP estimated from a C-D production function imposed for each sector and following Wooldridge estimation increases proportionally. Source: CompNet 7th vintage dataset for TFP estimate and sNPL measures are from merged world bank and IMF data.

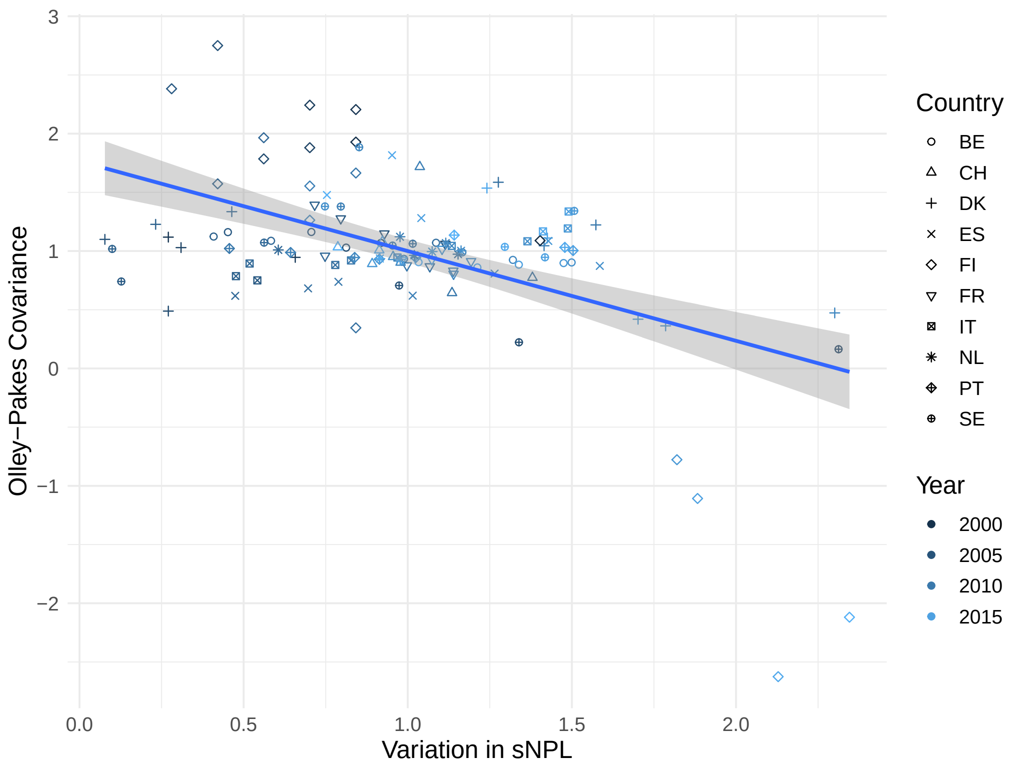

Figure 35: As the sNPL varies in a country estimated Olley-Pakes covariance declines. Source: CompNet 7th vintage dataset for Olley-Pakes estimate and sNPL measures are from merged world bank and IMF data.

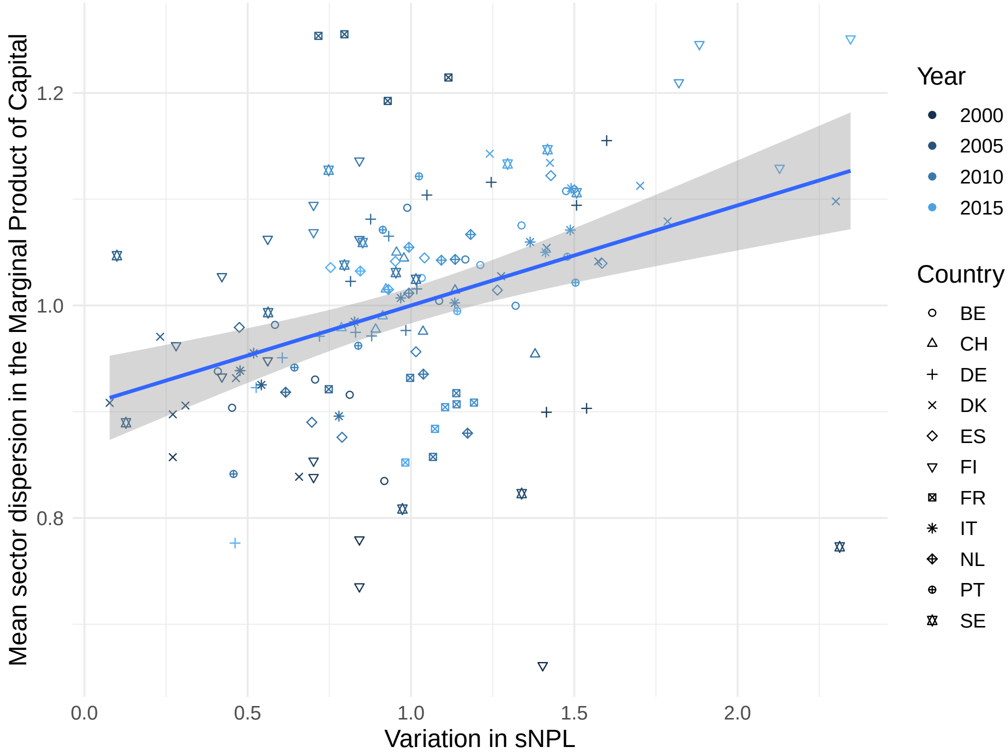

Figure 36: As the sNPL varies in a country estimated dispersion of marginal products of capital following Kehrig (2011) increases. Source: CompNet 7th vintage dataset for dispersion estimate and sNPL measures are from merged world bank and IMF data.

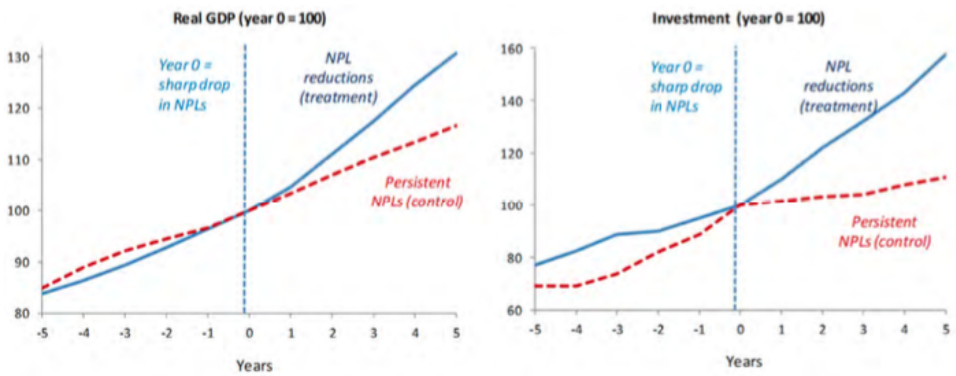

Figure 37: Increase in real output and investment following a reduction in NPLs via state intervention.

A cross-country comparison between OECD economies shows that non-performing loans tend correlate negatively with average real capital returns.

Appendix C Alternative model with exogenous loan foreclosure

The model with exogenous loan foreclosure is presented here to provide some straightforward intuition about the impact of sudden rises in unemployed capital on other aggregate variables such as investment consumption or capital productivity. Unemployed capital can be interpreted in this model both as rises in unmatched capital and exogenous rises

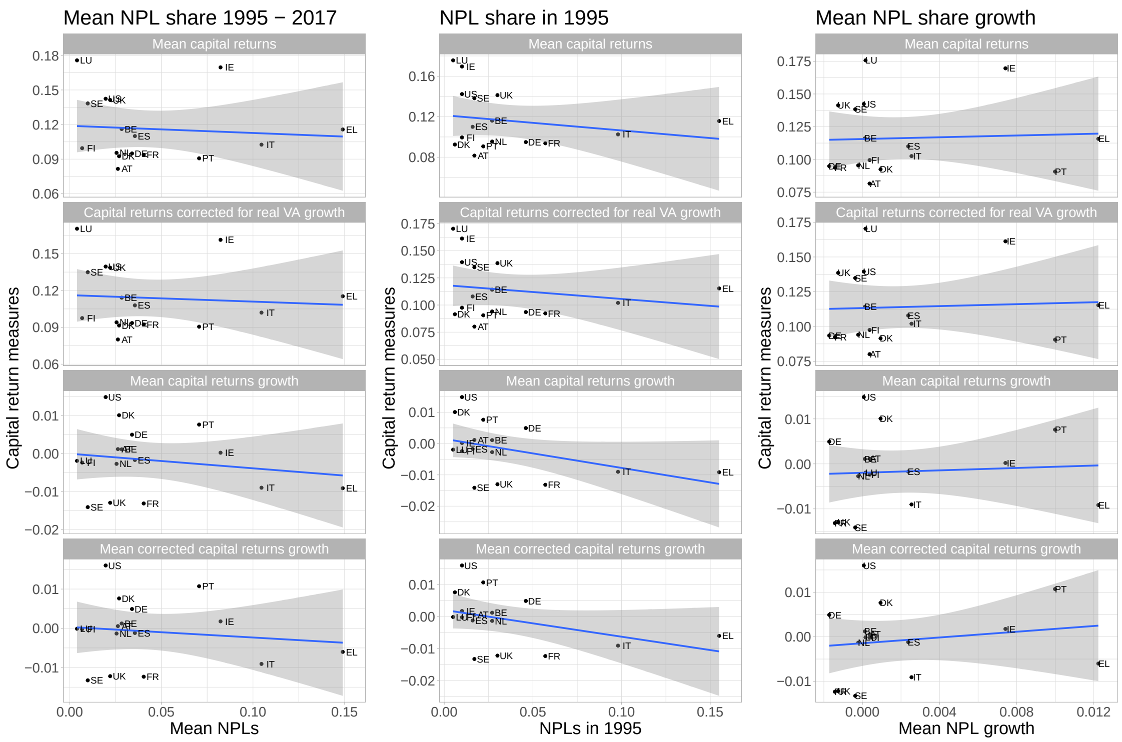

Figure 38: Mean, start and NPL share growth correlated with capital returns, capital return growth, and capital reuturns corrected for VA growth to proxy for TFP changes

in non-performing loans. The model provides a structural explanation for the empirical observations in (Balgova et al., 2016) and other policy papers investigating the impact of non performing loan increases or reductions on the real economy. It shows that an increase in misallocated capital, which can here be interpreted as both non-performing loans and foreclosed loans, can have large and persistent real effects on the economy.

Aggregate output

There are two types of capital stocks in the economy:

- Employed or productive capital K_t^E linked to performing loans or non-performing loans.
- Unemployed capital meaning non-performing or foreclosed capital K_t^U.

Both together form the aggregate capital stock in the economy $K_t = K_t^E + K_t^U$. Unemployed capital provides a negative externality for employed capital as aggregate output is given by:

$$Y_t = A_t K_{E,t}(K_{E,t} + K_{U,t})^{\alpha-1} \tag{72}$$

A is a stochastic TFP process. We commonly assume that $\alpha < 1$. Note that this function simplifies to a standard RBC function when $K_{U,t} = 0$. [4] [5]

Aggregate output in this economy can be consumed, used for investment in further capital, or used for setting up firms with provided capital.

A is an exogenous auto-regressive process describing productivity.

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t \tag{74}$$

Savings problem of the individual household

The economy is populated by a unit mass of identical households. Each household has some initial deposit wealth D_0. Each household takes credit and capital markets as given when making the individual saving decision. Thus the saving decision is a result of expected interest rates paid on deposits leading to a typical savings problem leading and a conventional Euler equation defining the savings decision.

$$\max_{C_t, I_t} E_t\left(\sum_{s=0}^{\infty} \frac{C_{t+s}^{1-\sigma}}{1-\sigma}\right) \tag{75}$$

subject to a budget constraint:

$$C_t + I_t = \Pi_t + \rho_t D_t \tag{76}$$

Here Π are profits by firms, ρ is the interest rate paid by banks on deposits, D is the stock of deposits and I is investment.

$$D_t = (1-\delta)D_{t-1} + I_{t-1} \tag{77}$$

Deposits depreciate in value with δ in this simple model as they are identical to the value of the underlying collateral of loans. In the expanded model this will not be the case.

Solving the maximisation problem it is straightforward to find the inter-temporal Euler equation.

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta + \rho_{t+1})] \tag{78}$$

One can also summarize the stochastic discount factor for firms and banks in the following

[4] Then $Y_t = A_t K_{E,t}^{\alpha}$

[5] It is also straightforward to generalise the output function to foreclosed loans also producing

$$Y_t = [K_{E,t} + \zeta K_{U,t}] A_t (K_{E,t} + K_{U,t})^{\alpha-1} \tag{73}$$

where $\zeta < 1$ and small enough that the value of a unit of performing loan capital exceeds the value of a non-performing loan to the bank.

sections as μ.

$$\mu_{t+1} = \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \tag{79}$$

Financial market

For simplicity deposits are assumed to equal lent out capital in this model $D_t = K_t$, but this is relaxed in the next section. Introducing matching frictions for new capital as well leads to a state of matched loans, a state of foreclosed loans, and a state of deposits.

All capital is assumed to be held by banks and equals the value of deposits by the household. Employed capital is used by firms, which pay interest r_t on the capital borrowed. New investment builds the capital stock. Investment of fresh capital works without frictions. However, once the firm that first received the capital defaults the capital will become unemployed, something that can be interpreted as a non-performing loan.

Foreclosed capital has to be re-matched to an entrepreneur willing to transform it for her purposes for a cost. This argument is similar to the argument in (Lanteri, 2018), who has shown that it helps provide a micro-foundation for RBC models with capital adjustment costs, which are needed to match the pro-cyclical capital reallocation observed in the data. This heterogeneous process of matching new entrepreneurs with old capital underlying non-performing loans is modelled in reduced form via a matching function.

In order to set up a firm from used capital agents need to spend κ to present a business plan to a bank and agree on an interest rate. The business proposals B_t will be matched with cash deposits and available unemployed capital. This is a frictional process, which will be summed up by the matching function in equation (80).

$$M_t = mB_t^{1-\xi} K_{U,t}^{\xi} \tag{80}$$

The cost of creating a new unit of employed capital from output in the simple model without matching frictions for fresh capital can be normalised to 1. Thus as long as there is positive investment in the economy the cost of creating a unit of employed capital from unemployed capital must equal the cost of new.

$$\kappa\theta^{\xi} = m \tag{81}$$

$$B_t = (\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} \tag{82}$$

The cost of reemploying capital must be equal to the cost of creating fresh capital to the representative household. Substituting (82) in the matching function yields.

$$M_t = (\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t} \tag{83}$$

This means $\theta_t = \theta$ in the simple model due to the fixed cost of creating fresh capital.

Benefit of an average foreclosed unemployed or an employed capital unit

The expected benefit of a single capital unit invested in the banking sector is defined by the current level of employed and unemployed capital, and aggregate investment. In the following equations, δ is the depreciation rate of capital, while λ is the exogenous probability of the firm exiting production and the capital becoming a non-performing loan.

$$V^U(K_t) = p(1-\delta)(1-\lambda_t)E_t(\beta_{t+1}[V^L(IK_{t+1}) - V^U(K_{t+1})]) + (1-\delta)E_t[\beta_{t+1}V^U(K_{t+1})] \tag{84}$$

$$V^L(K_t) = r_t + (1-\delta)(1-\lambda_t)E_t(\beta_{t+1}[V^L(K_{t+1}) - V^U(K_{t+1})]) + (1-\delta)E_t[\beta_{t+1}V^U(I_{t+1}, K_{t+1})] \tag{85}$$

The value of lent capital in the surplus notation familiar from (Mortensen and Pissarides, 1994) is in equation (86).

$$V^L(K_t) - V^U(K_t) = r_t + (1-\delta)(1-\lambda_t)(1-p)E_t(\beta_{t+1}[V^L(K_{t+1}) - V^U(K_{t+1})]) \tag{86}$$

Entrepreneurs incur a cost κ for creating a business plan and will enter the market for used capital until the benefits of entering equal the cost of entering.

The benefit of a matched unit of capital to the firm is:

$$V^E(K_t) = A_t K_t^{\alpha-1} - r_t + (1-\delta)(1-\lambda_t)E_t(\beta_{t+1}V_{E,t+1}]) \tag{87}$$

Solving for the interest rate paid by firms to banks

The interest rate for lent capital can be found by assuming Nash bargaining between firms and banks, with the bank's bargaining power being η. The equilibrium interest rate is then the result of the bargain.

$$r_t = \eta[A_t K_t^{\alpha-1} + p\beta_t(1-\delta)(1-\lambda_t)E_t(V_{E,t+1})] \tag{88}$$

This means the pooled interest rate ρ_t paid by the bank to the household for depositing a capital unit is defined by the share of productive capital in the overall capital stock, and the level of marginal returns per capital induced by the total capital stock.

$$\rho_t = r_t \frac{K_t^E}{K_t} \tag{89}$$

Transition equations for capital states

The transition equations follow from the household investment decision, and the exogenous firm destruction shock λ as well as the re-matching frictions for the market of used capital measured by non-performing loans.

$$K_{E,t} = (1-\delta)(1-\lambda_t)[K_{E,t-1} + (\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t-1}] + I_{t-1} \tag{90}$$

$$K_{U,t} = (1-\delta)[\lambda_t K_{E,t-1} + (1-p(\theta_{t-1})(1-\lambda_t))K_{U,t-1}] \tag{91}$$

Solving the model

The dynamics of this simple model can be simulated by perturbing the model around the steady-state. This can be done with ease and allows for a quick enough simulation to estimate parameters of the model such as match efficiency, which may be useful when comparing the impact of aggregate shocks and winding up of non-performing loans of different countries.

Exogenous processes Assume for now an exogenous aggregate productivity process and an exogenous default rate. The default rate can be straightforwardly endogenised. Both processes are assumed to be auto-regressive.

The aggregate productivity process is in equation (92).

$$\log(a)_t = \log(a)_{t-1}\rho_a + \epsilon_{a,t} \tag{92}$$

The exogenous process of the default rate is in equation (93).

$$\lambda_t = \lambda(1-\rho_\lambda) + \lambda_{t-1}\rho_\lambda + \epsilon_{\lambda,t} \tag{93}$$

Equilibrium equations The following are equilibrium equations and constraints describing the dynamics of the economy.

- Budget constraint:

$$C_t + D_{t+1} = \Pi_t + [1-\delta+\rho_t]D_t \tag{94}$$

- Euler Equation

$$C_t^{-\sigma} = \beta E_t[C_{t+1}^{-\sigma}(1 - \delta + \rho_{t+1})] \tag{95}$$

- Marginal input cost of investment equality

$$B_t = (\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} = \theta K_{U,t} \tag{96}$$

- The value of a fresh capital unit [6]

$$V^E(K_t) = (1 - \eta) A_t K_t^{\alpha - 1} + (1 - \delta)(1 - \lambda_t)(1 - \eta p) E_t(\beta_{t+1} V_{E,t+1}]) \tag{97}$$

- Profits from firms passed on to the households

$$\Pi_t = K_{E,t}[(1 - \eta) A_t K_t^{\alpha - 1} - \eta p(1 - \delta)(1 - \lambda_t)\beta_{t+1} V_{E,t+1})] - \kappa(\frac{m}{\kappa})^{\frac{1}{\xi}} K_{U,t} \tag{98}$$

- Transition equation of employed capital

$$K_{E,t} = (1 - \delta)(1 - \lambda_t)[K_{E,t-1} + m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}} K_{U,t-1}] + I_{t-1} - \tag{99}$$

- Transition equation of unemployed capital

$$K_{U,t} = (1 - \delta)[\lambda_t K_{E,t-1} + (1 - m(\frac{m}{\kappa})^{\frac{1-\xi}{\xi}}(1 - \lambda_t))K_{U,t-1}] \tag{100}$$

- The transition function for the capital stock, which is in the simple version assumed to equal deposits.

$$K_t = (1 - \delta) K_{t-1} + I_{t-1} \tag{101}$$

- The interest rate paid by banks to households

$$\rho_t = r_t \frac{K_t^E}{K_t} \tag{102}$$

- The interest rate charged by banks to firms per lent capital unit

$$r_t = \eta[A_t K_t^{\alpha - 1} + p\beta_t(1 - \delta)(1 - \lambda_t) E_t(V_{E,t+1})] \tag{103}$$

The steady-state equations can be found in Steady-States.

[6] It is assumed that $V^E(K_t) > 1$, which is achieved with appropriate calibration. This ensures that investment is always positive, i.e. $I > 0$. For moderate shocks the evolving non-differentiability can be ignored or alternatively a penalty function for low investment can be introduced to ensure more accurate estimation.

Simulated model

The model is simulated with $\beta = 0.99$, $\alpha = 0.66$, $m = 0.4$, $\xi = 0.8$, $\eta = 1 - \xi$, $\delta = 0.05$, $\lambda = 0.05$, and $\kappa = 0.1$. While the parameters may certainly need more calibration the dynamics of the model are nevertheless interesting. The simulated model shows that a brief rise in the default rate leads to a persistent decline of consumption and productive loans. Interest rates barely rise leading to a long recovery as investment does not rise to levels to recover destroyed productive capital due to the negative impact of non-performing loans remaining in the economy. This seems a compelling story for countries struggling with high foreclosed loan levels following financial crisis.

Meanwhile, the effect of aggregate productivity shocks is mooted due to the assumptions taking for the secondary capital market and non-endogenised defaults. These effects would change once endogenous loan calling is introduced, and search frictions are also introduced for fresh capital as the share of non-performing loans would not remain "essentially" flat and hidden. Once an endogenous decision is introduced the evidently costly foreclosure result is endogenised. Banks will decide on foreclosure and thereby also decide to forbear foreclosing decisions leading to non performing loans.

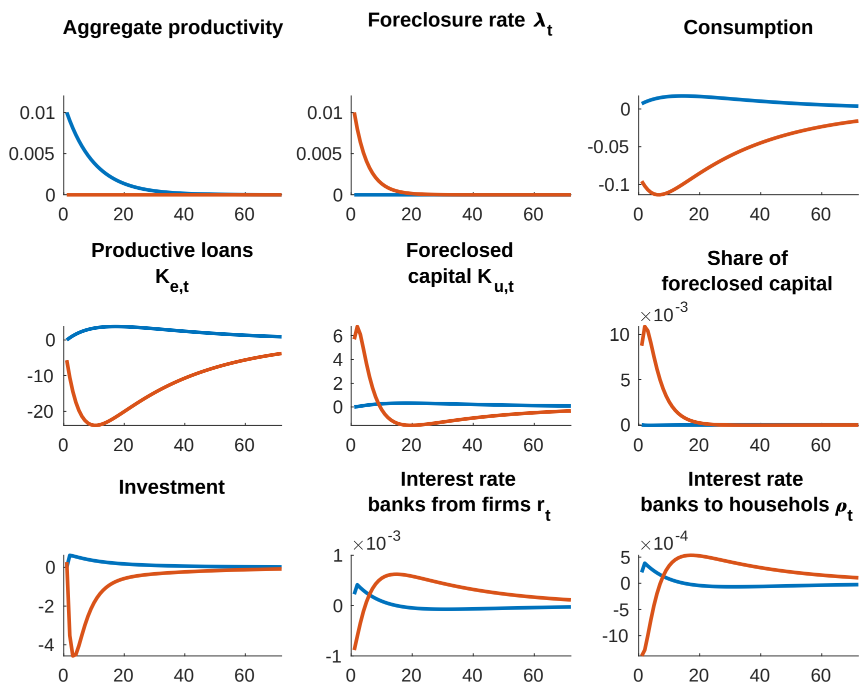

Figure 39: Impulse response functions for a 1% shock to aggregate productivity a_t (blue), and a 1% shock to the default rate λ_t (red)

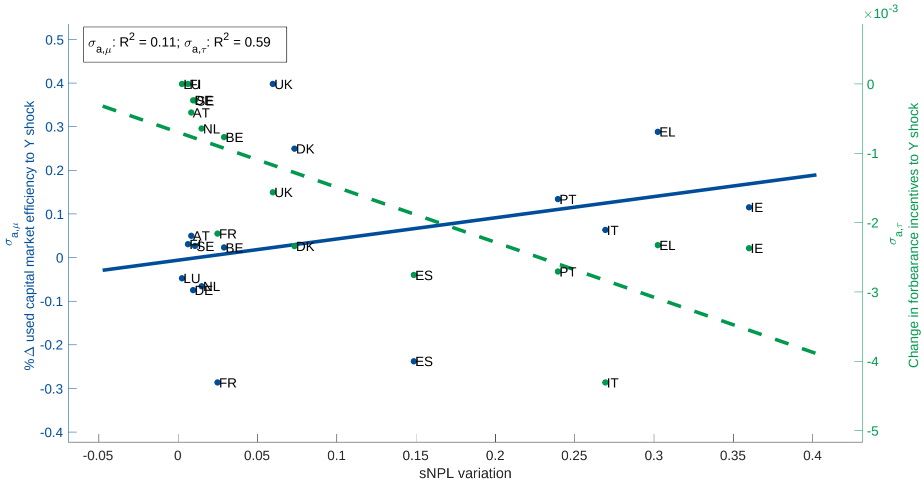

Figure 40: Estimates of $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$ for different countries in 2 digit codes plotted against variation in NPLs.

Appendix D Matched Impulse Response Functions

Figure 40 shows the result of the estimation with regard to the sNPL data variation provided for computing the empirical impulse responses. The difference to the raw variance of sNPL is that the variation is scaled to the theoretical model's steady state on which the estimation is based on. However, it can be seen that the results are similar to the explanatory power of sNPL variance, with changing forbearance explaining most of the variation, and capital market efficiency some. Appendix D NPL. A country's variation of sNPL is esymated to increase with pro-cyclical used capital market efficiencies and counter-cyclical forbearance incentives.

The impulse response functions of output, non-performing loans and value added to an output shock are plotted below. The model estimates the size of parameters $\sigma_{a,\tau}$ and $\sigma_{a,\mu}$, which will determine the reaction of capital prices and NPL to the output shock and plots the model response in red in Figure 41 and Figure 42.The two digit code of the countries forms the title of each plot.

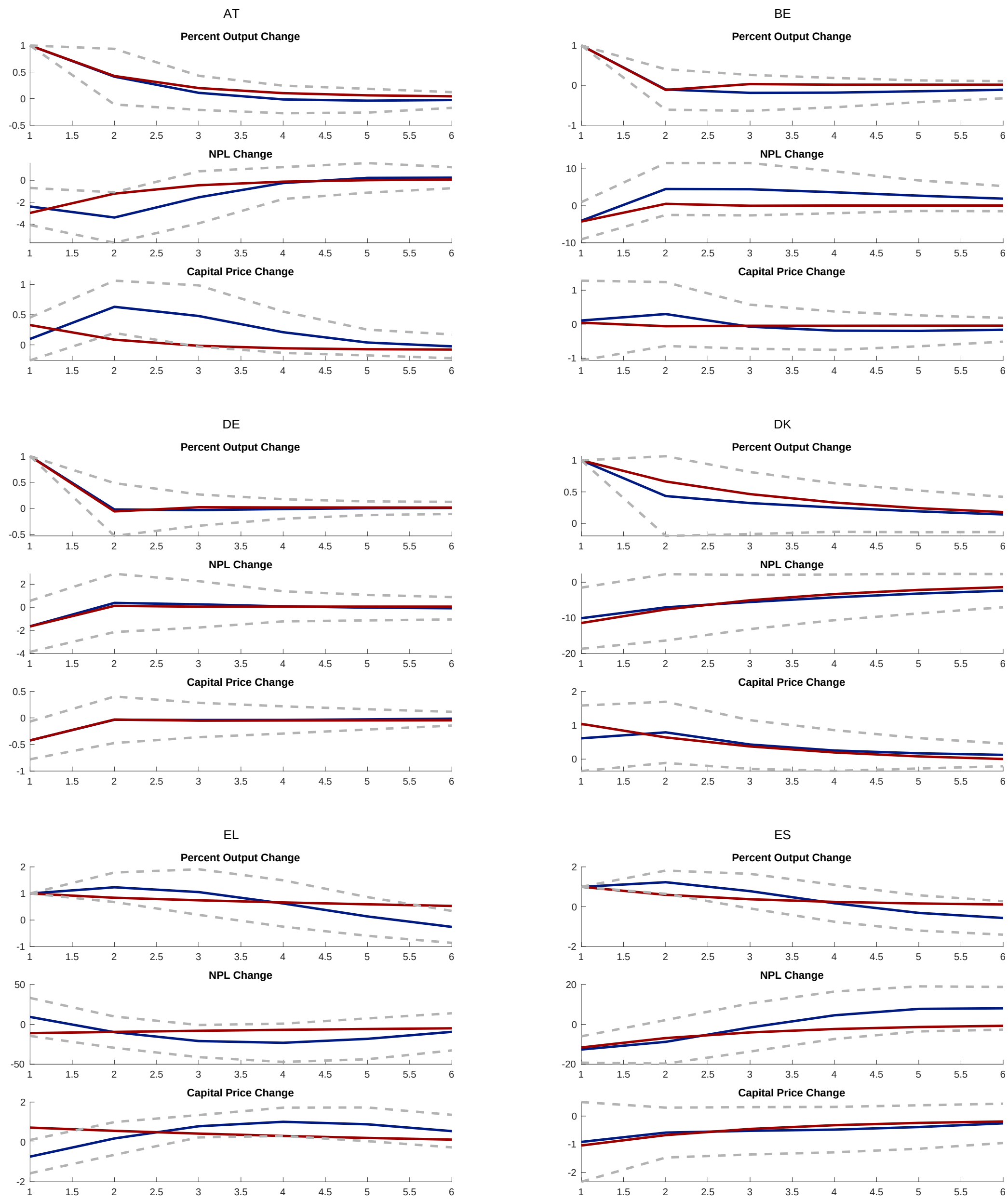

Figure 41: Short-run identified impulse responses of the variables to an output shock. The estimated model is plotted in red. Dashed lines give confidence intervals. The two-digit country code is above the plot.

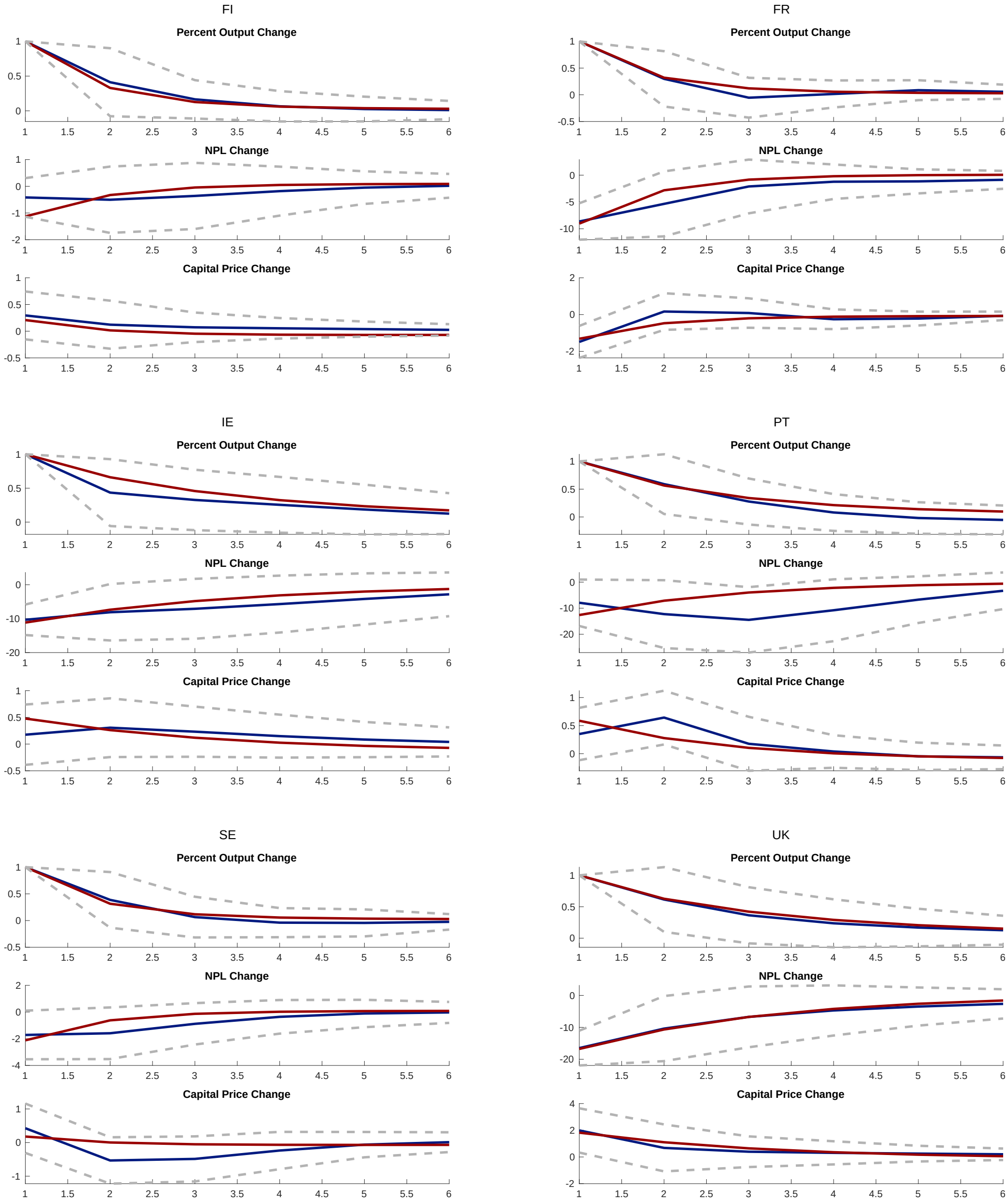

Figure 42: Short-run identified impulse responses of the variables to an output shock. The estimated model is plotted in red. Dashed lines give confidence intervals. The two-digit country code is above the plot.