Can Media Pluralism Be Harmful to News Quality?*

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Abstract

Two stylized facts characterize the Internet: a great diversity of news sources and the proliferation of disinformation. I study a model of information design that connects these observations. I show that competition between news sources with opposite biases reduces information quality when news consumers have limited attention. The reason is the endogenous formation of echo chambers. The standard narrative is that echo chambers arise because news consumers suffer confirmation bias. I show that even unbiased and rational news consumers devote their limited attention to like-minded news sources. Confirmation bias thus arises endogenously because news sources have no incentive to provide valuable information. I show that the presence of many news sources and the widespread existence of misleading news are concurrent.

Keywords: Bayesian Persuasion, Echo Chambers, Heterogeneous Beliefs, Limited Attention, Media Bias, Media Pluralism.

JEL Classification: D82, D83, L82

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1. Introduction

A critical problem for democracies is that those who control the information flow can influence political and economic outcomes. For a long time, the Internet has been considered a very effective way to guarantee pluralism in information (Keen, 2015). Ideally, the presence of competing sources of information is beneficial. The more information an individual can receive, the more she knows about the issue and the smaller the influence of a particular source. But is competition among news sources on the Internet really beneficial? Empirical evidence suggests a deterioration of the quality of the information at one's disposal. A notable example is the proliferation of conspiracy theories and "fake news" online.[1] The explanation I suggest in this paper is the endogenous formation of echo chambers.

The Cambridge dictionary defines echo chambers as "a situation in which people only hear opinions that are similar to their own". Echo chambers are a prominent feature of the Internet.[2] Online networks show high homophily: an individual learns from those who share her worldview (Del Vicario et al., 2016; Halberstam and Knight, 2016). Within echo chambers, each individual never questions her beliefs. As a consequence, society divides into opposing factions.[3] Moreover, the presence of echo chambers affect the quality of news. As I show, the media provide low-quality news within echo chambers.

The standard explanation for echo chambers is preference-based, namely that individuals are subject to confirmation bias.[4] I provide an alternative explanation: even if individuals seek the most informative news, echo chambers arise because of the interplay between limited attention of news consumers with heterogeneous beliefs and media bias of news sources.[5]

[1] Fake news are of public concern since the 2016 US presidential election (Allcott and Gentzkow, 2017). For instance, it is hard to find reliable information about health conditions online (Swire-Thompson and Lazer, 2019). Using the taxonomy proposed by Molina et al. (2021), my model captures partisan news, misreporting and persuasive advertising, which lie in the "grey area" between objectively real and false news.

[2] There is evidence of echo chambers even in non-partisan contexts such as climate change (Williams et al., 2015), vaccinations (Cossard et al., 2020) and the financial markets (Cookson et al., 2021).

[3] Being part of an echo chamber affects individual behaviour. During the COVID-19 pandemic, Democrats and Republicans in the US show different attitudes towards social distancing (Allcott et al., 2020; Gollwitzer et al., 2020) and vaccinations (Fridman et al., 2021).

[4] Nickerson (1998) defines confirmation bias as "seeking or interpreting of evidence in ways that are partial to existing beliefs, expectations, or a hypothesis in hand".

[5] Perceived information overload is positively associated with selective exposure in online news consumption (Lee et al., 2017). Internet users fail to discriminate news based on quality (Qiu et al., 2017). My results are in line with recent advances in psychology

I study a Bayesian persuasion model with two states of the world and two actions. There are two types of agents: experts and decision-makers. Each expert is biased: his preferred action is independent of the state of the world. Each expert designs information about the state of the world to persuade decision-makers to take his preferred action. Each decision-maker is unbiased: she wants to match her action with the state. Decision-makers have heterogeneous beliefs about the state of the world and limited attention: each decision-maker can only devote attention to one expert. After observing the information such an expert provides, the decision-maker takes an action. I show that competition between experts is harmful to decision-makers when the latter strategically allocate their limited attention.

As a benchmark, I consider a single expert and two subgroups of decision-makers with different beliefs: sceptics and believers. Without information, believers choose the expert's preferred action, whereas sceptics do not. Hence, the expert designs information to change sceptics' behaviour. All decision-makers receive the same information. Thus, any attempt to change a sceptic's belief affects a believer's belief as well. Being exposed to information could change believers' behaviour (i.e., induce them to take the expert's adverse action). The expert trades off between persuading sceptics and retaining believers, and there are two candidates for the optimal strategy.

The *hard-news strategy* focuses on persuading sceptics. For this purpose, a message must be credible, that is, it can be misleading only to a limited extent. Therefore, this strategy entails the cost of revealing the unfavourable state with positive probability to all decision-makers. In such a case, believers take the expert's adverse action.

The *soft-news strategy* focuses on retaining believers. The expert sends two messages of different credibility. One is credible enough to persuade sceptics. The other one is not, but at the same time, it does not induce believers to take the expert's adverse action. With this second message, the expert leverages believers' credulity. This strategy ensures that believers will continue to choose the expert's preferred action.

I show that the hard-news strategy is more informative than the soft-news strategy according to the ordering defined by Blackwell (1953). Nevertheless, the expert prefers the soft-news strategy if decision-makers have sufficiently polarized beliefs. If the belief of sceptics is extreme, then it is very costly to persuade sceptics. To be credible, the expert has to reveal the unfavourable state with high probability. If the belief of believers is extreme, then it is very tempting to retain believers because it is easy to leverage their credulity. In

showing that politically motivated reasoning does not drive selective exposure to news online (Pennycook and Rand, 2021).

both scenarios, the soft-news strategy is more favourable for the expert. A second key parameter is given by the expert's belief. The soft-news strategy is more appealing the higher is the expert's belief of his unfavourable state. Intuitively, the expert values more his ability to mislead (at least) believers.

Next, I show how media pluralism (i.e., competition between experts) makes decision-makers worse off. Two experts with different preferred actions compete to persuade decision-makers. Because of limited attention, each decision-maker can only devote attention to one expert. Therefore, each expert behaves like a monopolist given his audience. In other words, for any expert, the allocation of attention determines the distribution of beliefs such an expert has to confront. The strategy of each expert must be optimal for a given distribution. Here, the novelty is the interaction between optimal persuasion and the endogenous allocation of attention.

The allocation of attention depends on the strategies of the experts. Each decision-maker allocates her attention to maximize her subjective probability of taking the correct action. This probability is at its minimum without information. An expert designs information to change decision-makers' behaviour. To be successful, the expert must provide sufficiently accurate information, and this makes decision-makers (weakly) better off. I define a decision-maker's information gain as the increase in her subjective probability of taking the correct action following information provision. Therefore, each decision-maker allocates her attention to maximize her information gain.

It makes a difference whether a decision-maker is a *target* of an expert. An expert targets a subgroup of decision-makers if he tailors his strategy to persuade them (for example, sceptics are the targets when the expert uses the hard-news strategy). Any target of a given expert receives zero information gain when devoting attention to such expert. Indeed, the expert does not reveal more information than what is strictly necessary to change the behaviour of targets.

The incentive of each decision-maker is to avoid being a target. At the same time, the optimal strategy of each expert features (at least) one target, unless the expert faces only his believers. This tension determines what allocations of attention can support an equilibrium.

In any symmetric equilibrium, there is at most one informative expert (i.e., an expert who uses either a hard-news strategy or a soft-news strategy). Indeed, with two informative experts, there is always (at least) one target who can get a positive information gain by changing her allocation of attention. Therefore, in any symmetric equilibrium at least one expert is babbling (i.e., provides no information). I define the audience of a babbling expert as an echo chamber. Decision-makers who cluster into an echo chamber are those with the most extreme beliefs among believers. Given babbling, there is no decision-maker who wants to join the echo chamber. At the same time, each decision-maker in the echo chamber is too sceptical to benefit from devoting attention to the informative expert.

The omnipresence of information can make all information useless. Limited attention makes media pluralism harmful to those decision-makers who cluster into an echo chamber and receive babbling. In a monopoly, the expert uses either the hard-news strategy or the soft-news strategy. Hence, decision-makers are better informed (according to Blackwell (1953)'s ordering) as both these strategies produce some dispersion in posterior beliefs.

This negative result follows from the endogenous allocation of attention by decision-makers. If both experts use the hard-news strategy, each decision-maker has an incentive to devote attention to the like-minded expert to get a positive information gain. However, the strategic response of the experts traps decision-makers into echo chambers. A platform with the ability to determine the allocation of attention can induce both experts to use the hard-news strategy. In this way, media pluralism can become beneficial.

1.1. Example

I use the COVID-19 vaccination as an example to illustrate my results. There are two possible states of the world: either a vaccine is safe or not (e.g., it has side effects). Each citizen wants to get vaccinated if and only if the vaccine is safe. Some citizens are sceptical about vaccinations being safe and are not willing to get vaccinated a priori (Paul et al., 2021). A pro-government media wants to persuade citizens to get vaccinated to reach herd immunity.

In a monopoly, the supply of news by the pro-government media depends on its confidence about vaccinations' safety. If the pro-government media is very confident, it provides "hard evidence" (e.g., the evaluations by the European Medicines Agency based on clinical trials). The pro-government media attempts to persuade sceptics to get vaccinated because it expects persuasion to be very likely. If the pro-government media is not confident enough, high polarization makes it optimal to supply also "soft evidence" (e.g., weaker statements such as "benefits are higher than risks"). In this way, the pro-government media is sure to retain those citizens who were already willing to get vaccinated.

In a competitive setting, a no-vax media opposes vaccinations to make profits with alternative treatments (Ghoneim et al., 2020). An equilibrium could be as follows: the pro-government media produces "hard evidence", whereas the no-vax media is babbling within its echo chamber.[6] Citizens

[6] Di Marco et al. (2021) find evidence of echo chambers about the COVID-19 pandemic.

who are sceptical about vaccinations understand that the pro-government media designs information to change their attitudes. Therefore, these citizens do not benefit from the information provided by the pro-government media, and thus rationally allocate their limited attention to confirmatory news. The pro-government media cannot persuade these citizens to get vaccinated. The existence of a large no-vax echo chamber can help explaining why herd immunity is difficult to reach (Diamond et al., 2021).

1.2. Plan of the Paper

In Section 2, I review the literature. In Section 3, I present the theoretical model. In Section 4, I study optimal persuasion in a monopoly. In Section 5, I describe the effects of competition. In Section 6, I examine some extensions. In Section 7, I discuss the applicability of my model. In Section 8, I conclude.

2. Related Literature

I contribute to the literature by exploring how the endogenous supply of (potentially misleading) information to decision-makers with heterogeneous beliefs interacts with limited attention. Therefore, my paper connects with the following streams in the literature.

Limited attention

> "In an information-rich world, the wealth of information [...] creates a poverty of attention and a need to allocate that attention efficiently among the overabundance of information sources that might consume it." Simon (1971)

The Internet has led to an information-rich economy as it allows news sources to reach more consumers at a lower per-consumer cost. The growth in consumers wealth and firms market power helped this process (Falkinger, 2008). Limited attention can explain many puzzling empirical patterns, for instance, asset-price dynamics (Peng and Xiong, 2006), the attraction effect (Masatlioglu et al., 2012), nominal rigidities (Matějka, 2016), persistently low inflation (Pfäuti, 2021) and the superstar effect (Hefti and Lareida, 2021).[7] In this paper, I offer new insights into the effects of limited attention. I show that

Jiang et al. (2021) show that segregation is stronger among far-right users.

[7] Gabaix (2019) and Mackowiak et al. (2020) survey the literature on behavioural and rational inattention, respectively.

limited attention can explain why rational decision-makers cluster into echo chambers and thus rationalizes the proliferation of low-quality information.

Limited attention influences price competition and advertising within and across industries (Anderson and de Palma, 2012; De Clippel et al., 2014; Hefti and Liu, 2020). In particular, Anderson and Peitz (2020) show that increasing media diversity has the undesired effect of increasing advertising clutter and thus can make consumers worse off. My findings are complementary to Anderson and Peitz (2020). Indeed, I show that media diversity can also harm news consumers by causing a reduction in information quality.

Bayesian persuasion. A standard assumption in this literature - pioneered by Aumann and Maschler (1995) and Kamenica and Gentzkow (2011) - is the existence of a common prior belief. By contrast, I examine the problem of a sender (expert) who faces many receivers (decision-makers) endowed with heterogeneous beliefs.[8] In Guo and Shmaya (2019), a separating (soft-news) strategy yields a higher payoff to the sender than a pooling (hard-news) strategy if the receiver has sufficiently accurate private information. The distribution of private information is (strategically) equivalent to receivers holding heterogeneous beliefs. From this perspective, I show that more accurate private information can lead to less accurate public information. Indeed, if polarization is above a threshold, the sender provides information of lower quality. A similar effect arises in Gitmez and Molavi (2020). However, these authors focus on the ability of a sender to gather attention from receivers with heterogeneous beliefs.

Gentzkow and Kamenica (2017a,b) argue that competition among senders weakly increases information provision and benefits receivers. I show that this conclusion fails if receivers have heterogeneous beliefs and limited attention. My model incorporates endogenous allocation of attention between competing senders and endogenous persuasion.[9] In Knoepfle (2020), senders compete to gather the attention of a receiver. By contrast, senders are concerned about receivers' actions in my model. This difference leads to opposite results: endogenous echo chambers in my model, whereas full revelation is

[8]Alonso and Camara (2016) consider a sender who persuades a receiver, and the two have heterogeneous beliefs. Beliefs are exogenous to the model, and it is beyond the purpose of this paper to study the origin of beliefs (Flynn et al., 2017). Bergemann and Morris (2019) and Kamenica (2019) survey the literature on information design.

[9]Che and Mierendorff (2019) and Leung (2020) study the problem of a receiver who has to allocate her limited attention between biased senders. In these papers, the information design is exogenous. Bloedel and Segal (2020), Gitmez and Molavi (2020), Lipnowski et al. (2020) and Wei (2020) study how limited attention by the receiver(s) affects optimal persuasion by a single sender.

the final outcome in Knoepfle (2020).

Echo chambers. Jann and Schottmuller (2019) rationalize echo chambers in a many-to-many cheap talk model with biased decision-makers.[10] By contrast, even unbiased decision-makers may cluster into echo chambers in my model. Martinez and Tenev (2020) study a model where experts are unbiased. The experts are heterogeneous in terms of information precision. A decision-maker rationally infers that an expert has higher quality if he supplies information more in line with the decision-maker's belief. By contrast, experts are biased, and precision is endogenous in my model. The strategic interaction between decision-makers and experts plays a crucial role in the formation of echo chambers.[11] Jann and Schottmuller (2019) and Martinez and Tenev (2020) argue that echo chambers can be helpful, either to enhance communication in a network or to separate high-quality and low-quality news. Instead, echo chambers have a negative effect here. The reason is the endogenous supply of information by biased experts.

Detrimental competition. A broad literature shows that competition can backfire in many different settings. Chen and Riordan (2008) show that price-increasing competition occurs when products are sufficiently differentiated. Easier entry in a setting with procrastinating consumers and switching costs may lead to higher prices (Heidhues et al., 2021). In the insurance market, competition can increase distortions when agents have heterogeneous perceptions about risk (Spinnewijn, 2013). The "unravelling" effect of competition has been disputed: with vertically differentiated firms, only high-quality firms have incentives to disclose (Board, 2009), or there is no disclosure at all (Janssen and Roy, 2014). Information overload does not allow decision-makers to identify high-quality experts (Persson, 2018) and implies higher prices because consumers get lost in diversity (Hefti, 2018). Costly information acquisition or communication reduces each expert's effort in the presence of other experts: free-riding harms decision-makers (Kartik et al., 2017; Emons and Fluet, 2019). I uncover a novel channel for detrimental competition: media bias when decision-makers have limited attention and heterogeneous beliefs.

[10] See also Giovanniello (2021) where echo chambers arise because biased voters have incentives to communicate useful information only to like-minded peers.

[11] Alternatively, echo chambers may arise because the cost of processing information is increasing in its precision (Nimark and Sundaresan, 2019) or when decision-makers look for disapproving evidence eventually supplied by like-minded experts (Hu et al., 2021). Levy and Razin (2019) survey the economics literature on echo chambers.

3. Model

There are two states of the world and two actions. That is, I denote with $\Omega := \{\omega_1, \omega_2\}$ the set of states and with $A := \{a_1, a_2\}$ the set of actions.[12] Each agent l has a prior belief $\mu_l^0(\omega_1) \in (0,1)$ that the state is ω_1. Clearly, $\mu_l^0(\omega_2) = 1 - \mu_l^0(\omega_1)$ is the agent l's prior belief that the state is ω_2. There are two types of agents: experts and decision-makers. I denote with D the set of decision-makers and with J the set of experts. Decision-makers partition in homogenous subgroups: $D := \bigcup_{i \in I} D_i$ where I is the set of subgroups of decision-makers. Two decision-makers of the same subgroup share the same belief: $\mu_d^0(\omega_1) = \mu_{d'}^0(\omega_1) = \mu_i^0(\omega_1)$ for any $d, d' \in D_i$ and any $i \in I$.

Each decision-maker (she) takes an action $a \in A$, and her goal is to match the action with the state:

$$u(a, \omega_k) := \mathbb{1}\{a = a_k\} \tag{1}$$

Before taking an action, each decision-maker $d \in D$ pays attention to one expert $j_d \in J$: she uses the information provided by the expert to update her belief. The allocation problem is analysed in greater detail in Section 5.

An expert $j \in J$ (he) cannot implement an action on its own. Therefore, he design information $\pi_j : \Omega \to \Delta(S_j)$ to manipulate decision-makers' behaviour. In words, each expert commits to the probability $\pi_j(s|\omega)$ to send message s given state ω, for any message $s \in S_j$ and any state $\omega \in \Omega$.[13] Each expert j has a unique preferred action $a_j \in A$. His payoff from a decision-maker who takes action $a \in A$, and for any state $\omega \in \Omega$ is:

$$u_j(a, \omega) = u_j(a) := \mathbb{1}\{a = a_j\}$$

In other words, each expert has state-independent preferences, and his payoff is 1 if and only if the action chosen by a decision-maker is the expert's preferred action.

The game has the following timing:

1. Each expert j designs information π_j. At the same time when experts choose their strategies, each decision-maker d pays attention to one expert j_d.[14]

2. Each decision-maker d observes the strategy π_{j_d} of the expert she pays attention to, *and* the strategy's realization $s \in S_{j_d}$ (that is, a message) chosen by Nature.

[12] In Section B.5, I discuss an extension with more than two states.

[13] I assume that the message space S_j contains at least two elements for any expert $j \in J$.

[14] In Section B.2, I consider a sequential version of the game. The effect of competition is different when experts implicitly become attention-seekers, as in Knoepfle (2020).

3. Given any posterior belief μ_d, each decision-maker d takes an optimal action. In case of indifference, I assume that decision-maker d chooses the preferred action of expert j_d.

I solve the game by backward induction, and the equilibrium notion is Perfect Bayesian Equilibrium. I assume without loss of generality that the preferred action of expert j_d is a_1. By (1), the optimal action of decision-maker d with posterior belief μ_d is given by the following function:

$$\sigma(\mu_d) := \begin{cases} a_1 & \text{if } \mu_d(\omega_1) \geq \frac{1}{2} \\ a_2 & \text{otherwise} \end{cases}$$

Each decision-maker d forms the posterior belief μ_d using Bayesian updating:

$$\mu_d(\omega_1 \,|\, s) := \frac{\pi_{j_d}(s \,|\, \omega_1)\mu_i^0(\omega_1)}{\pi_{j_d}(s \,|\, \omega_1)\mu_i^0(\omega_1) + \pi_{j_d}(s \,|\, \omega_2)\mu_i^0(\omega_2)}$$

Thus, for decision-maker $d \in D_i$ to take action a_1, upon observing message s, the following condition must hold:

$$\mu_d(\omega_1 \,|\, s) \geq \frac{1}{2} \iff \pi_{j_d}(s \,|\, \omega_1)\mu_i^0(\omega_1) \geq \pi_{j_d}(s \,|\, \omega_2)\mu_i^0(\omega_2)$$

In words, the expert must ensure that state ω_1 is more likely than state ω_2 for a decision-maker of subgroup i after receiving the message s. I label this condition *persuasion constraint*.

Definition 1 (Persuasion constraints)**.** *The persuasion constraint for a decision-maker of subgroup $i \in I$, who devotes attention to expert $j \in J$ and observes message $s \in S_j$, in order for her to take action a_1 is:*

$$\pi_j(s \,|\, \omega_2) \leq \frac{\mu_i^0(\omega_1)}{\mu_i^0(\omega_2)}\pi_j(s \,|\, \omega_1) := \phi_i \pi_j(s \,|\, \omega_1) \tag{2}$$

I denote with $H_j := \{d \in D \,|\, j_d = j\}$ the set of decision-makers who pay attention to expert j. For any $i \in I$, I define g_{ij} as the fraction of decision-makers in H_j who are of subgroup i. Mathematically,

$$g_{ij} := \begin{cases} 0 & \text{if } H_j = \varnothing \\ \frac{|\{d \in H_j \,|\, d \in D_i\}|}{|H_j|} & \text{otherwise} \end{cases}$$

These decision-makers have the same posterior belief. Therefore, the payoff of expert j from these decision-makers, upon observing message s, is:

$$v_{ij}(\pi_j, s) := g_{ij} u_j\Big(\sigma\big(\mu_d(\omega_1 \,|\, s)\big)\Big)$$

The expert j maximizes the sum of expected utilities he derives from his audience H_j:

$$\max_{\pi_j} \sum_{i \in I} \sum_{s \in S_j} \sum_{\omega \in \Omega} \pi_j(s\,|\,\omega)\mu_j^0(\omega)v_{ij}(\pi_j, s) \tag{3}$$

The expert takes his audience H_j as given. Therefore, (3) is a best-response problem in a simultaneous-move game, where each decision-maker d chooses her j_d, and each expert j chooses his π_j.

This problem entails a trade-off for the expert. On the one hand, a message must be "credible" to induce a decision-maker to take the expert's preferred action. Formally, this message must satisfy the corresponding persuasion constraint. The former imposes an upper bound to the probability of observing such a message in the state associated with a different action. On the other hand, provided that a message is persuading, the expert would like to send this message as often as possible.

Lemma 1 (Persuasion constraint)**.** *Consider any expert j and assume without loss of generality that $a_j = a_1$. In any best response π_j, either 1.) there exist a subgroup $i \in I$ of decision-makers and a message $s \in S_j$ such that $\pi_j(s\,|\,\omega_2) = \phi_i \pi_j(s\,|\,\omega_1)$ or 2.) $\pi_j(s\,|\,\omega_1) = \pi_j(s\,|\,\omega_2)$ for any $s \in S_j$.*

By Lemma 1, I can restrict the set of strategies that can be best responses: if the expert's audience includes sceptics, then at least one persuasion constraint must hold with equality. In the following section, I use this insight to find candidates for the optimal strategy.

4. Media Monism

As a benchmark, I study the problem of one expert (3), abstracting from the attention allocation problem of decision-makers (Section 5). I assume without loss of generality that the expert's preferred action is a_1, and I omit the index j for simplicity. By (2), a message s persuades a decision-maker of subgroup i to take action a_1 if and only if $\pi(s\,|\,\omega_2) \le \phi_i \pi(s\,|\,\omega_1)$. The ratio of prior beliefs ϕ_i for each subgroup $i \in I$ will play a crucial role in the following analysis. From the perspective of the expert, there are two categories of decision-makers: believers and sceptics.

Definition 2 (Believers and sceptics)**.** *Decision-makers of subgroup i are believers of state ω_1 relative to ω_2 if $\phi_i > 1$. Decision-makers of subgroup i are sceptics of state ω_1 relative to ω_2 if $\phi_i < 1$. I denote with $I_2 \subset I$ the set of subgroups of sceptics.*

Without information provision by the expert, believers choose the expert's preferred action, whereas sceptics do not. Therefore, sceptics require persuasion: the expert manipulates their beliefs through his strategy π, to induce sceptics to take action a_1. However, the expert must account for the indirect effect that persuasion of sceptics has on the behaviour of believers, as all decision-makers receive the same information. Information provision could induce believers to take the expert's adverse action a_2. Therefore, the expert trades off between persuading sceptics and retaining believers.

In this section, I assume that there are two subgroups of decision-makers, that is, $I = \{1, 2\}$. I assume that subgroup 1 of decision-makers are believers i.e. $\phi_1 > 1$, whereas subgroup 2 are sceptics i.e. $\phi_2 < 1$.[15] Thus, the expert can use a message to persuade all decision-makers or only believers or nobody to take action a_1. In the optimal strategy at least one persuasion constraint must hold with equality (Lemma 1). In particular, either only the persuasion constraint for sceptics holds with equality, or both persuasion constraints do so. Hence, I identify two candidates for the optimal strategy:

Definition 3 (Hard-news strategy)**.** *The hard-news strategy π_h consists of a persuading message s and a residual message s' such that*

$$\pi_h(s\,|\,\omega_1) = 1, \quad \pi_h(s'\,|\,\omega_1) = 0, \quad \pi_h(s\,|\,\omega_2) = \phi_2, \quad \pi_h(s'\,|\,\omega_2) = 1 - \phi_2$$

Definition 4 (Soft-news strategy)**.** *The soft-news strategy π_s consists of two messages s, s' such that*

$$\pi_s(s\,|\,\omega_1) = k, \quad \pi_s(s'\,|\,\omega_1) = 1 - k, \quad \pi_s(s\,|\,\omega_2) = \phi_2 k, \quad \pi_s(s'\,|\,\omega_2) = \phi_1(1 - k)$$

where $k := \frac{\phi_1 - 1}{\phi_1 - \phi_2}$ is strictly increasing in ϕ_1 and ϕ_2.

The hard-news strategy implies the following posterior beliefs:

$$\mu_1(\omega_1\,|\,s) = \frac{\phi_1}{\phi_1 + \phi_2} > \mu_2(\omega_1\,|\,s) = \frac{1}{2}, \quad \mu_1(\omega_1\,|\,s') = \mu_2(\omega_1\,|\,s') = 0 \tag{4}$$

whereas the soft-news strategy implies the following posterior beliefs:

$$\mu_1(\omega_1\,|\,s) = \frac{\phi_1}{\phi_1 + \phi_2} > \mu_2(\omega_1\,|\,s) = \frac{1}{2}, \quad \mu_1(\omega_1\,|\,s') = \frac{1}{2} > \mu_2(\omega_1\,|\,s') = \frac{\phi_2}{\phi_1 + \phi_2} \tag{5}$$

The hard-news strategy persuades all decision-makers after seeing s and nobody after seeing s'. Thus, decision-makers choose the expert's preferred action in the state ω_1, and sometimes in the state ω_2. The expert provides

[15] In Section 6.2 I consider the case of arbitrarily many subgroups of decision-makers.

sufficiently accurate information able to influence sceptics. However, this comes at a high cost to make the persuading message s credible. The credibility of s requires to send the residual message s' often enough when the state is ω_2. The message s' reveals the unfavourable state ω_2, inducing *all* decision-makers to choose the expert's adverse action.

The soft-news strategy persuades all decision-makers after seeing s and believers after seeing s'. Thus, believers choose the expert's preferred action *with probability one*, whereas sceptics choose it with a positive probability (but smaller than one) in either state. The expert alternates information of different accuracy. The message s' is not credible enough to persuade sceptics but ensures that believers keep choosing the expert's preferred action. The expert leverages the believers' credulity without completely giving up on the persuasion of sceptics. The value of k is the maximal extent of persuasion of sceptics, which is possible without affecting believers' behaviour.

Proposition 1 (Optimal persuasion)**.** *Let $I = \{1, 2\}$, $\phi_1 > 1$ and $\phi_2 < 1$. The unique optimal strategy is either the hard-news strategy or the soft-news strategy. The hard-news strategy is optimal if 1.) decision-makers have sufficiently similar beliefs or 2.) the fraction of believers is sufficiently small or 3.) the expert's favourable state is sufficiently likely from his perspective.*

By Proposition 1, three parameters influence optimal persuasion:

1. *Decision-makers' polarization, that is, $\phi_1 - \phi_2$*: The larger ϕ_1 is, the higher is the incentive to use the soft-news strategy. Indeed, it is easier to leverage believers' credulity using the message s'. In other words, it is easier to prevent believers from taking the expert's adverse action. The smaller ϕ_2 is, the smaller is the incentive to use the hard-news strategy. Indeed, it is more costly to persuade sceptics using the message s: the credibility of s requires revealing the unfavourable state with a higher probability. The difference $\phi_1 - \phi_2$ is a proxy for polarization, as the underlying beliefs become more extreme as such difference grows. Therefore, the higher polarization is, the higher the incentive to use the soft-news strategy;

2. *Fraction of believers, that is, g_1*: The larger the subgroup of believers (the higher g_1), the higher is the incentive to retain believers (and the lower the incentive to persuade sceptics). This implies a higher incentive to use the soft-news strategy;

3. *Expert's prior belief, that is, $\mu_j^0(\cdot)$*: The higher the expert's belief of his unfavourable state $\mu_j^0(\omega_2)$, the higher the cost of revealing the unfavourable state to all decision-makers with the hard-news strategy. In

other words, the expert values his ability to mislead (at least) believers, especially when he is very uncertain about the state of the world. It follows a higher incentive to use the soft-news strategy.

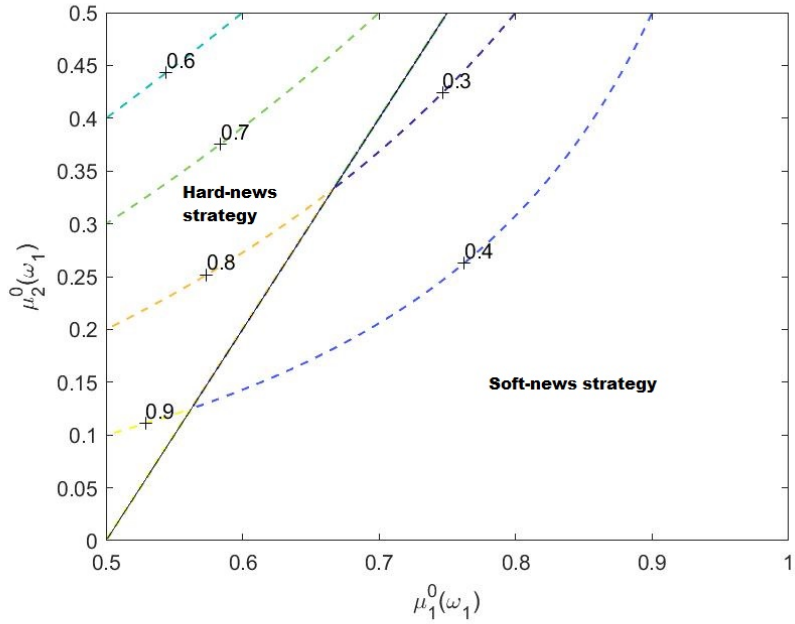
Proposition 1 relates to Kamenica and Gentzkow (2011) in the following way. Kamenica and Gentzkow (2011) assume a common prior belief and, if the decision-maker is a sceptic, the hard-news strategy is optimal. Heterogeneous beliefs give rise to a new type of optimal strategy - the soft-news strategy - pointing out the importance of decision-makers' polarization for optimal persuasion. Moreover, Kamenica and Gentzkow (2011) argue that if a decision-maker chooses the expert's adverse action, then it must be the case that the state is one where such action is optimal. However, this holds only if the expert uses the hard-news strategy. With the soft-news strategy, sceptics may choose the expert's adverse action even if it is not optimal for them. Finally, persuasion is always optimal when decision-makers have heterogeneous beliefs. The expert uses either the hard-news strategy or the soft-news strategy. Babbling is never optimal.

Lemma 2 (Blackwell's criterion)**.** *The hard-news strategy is more informative than the soft-news strategy, according to the ordering over distributions of posterior beliefs defined by Blackwell (1953).*

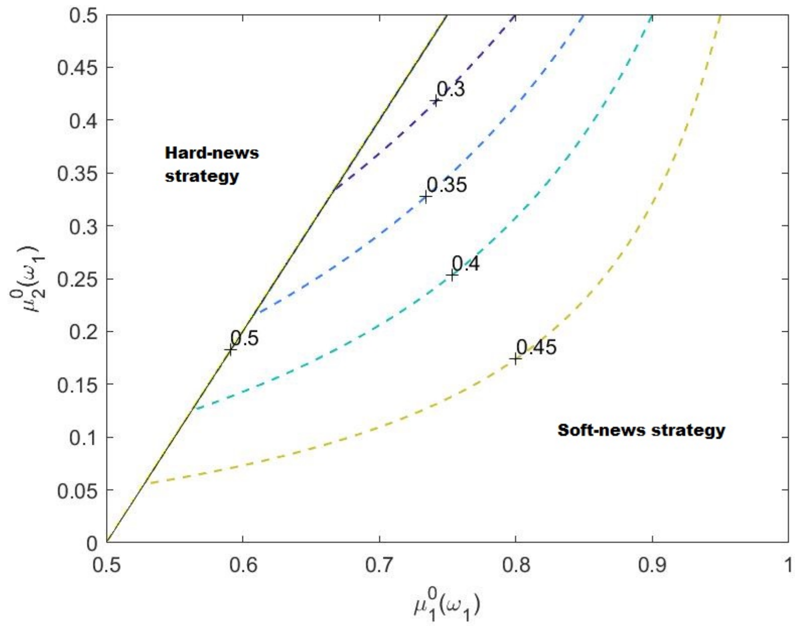
A strategy π is more informative than π' according to Blackwell (1953) if the distribution of posterior beliefs induced by π constitutes a mean preserving spread of the distribution of posterior beliefs induced by π'. Following this definition, truth-telling is the most informative strategy, as the posterior belief is either 0 or 1. Instead, babbling leaves beliefs unchanged, and thus it is the least informative strategy. The hard-news strategy is more informative than the soft-news strategy, for all decision-makers. Indeed, it induces more dispersion in the posterior beliefs through the residual message, which reveals the unfavourable state for the expert.

As Figures 1a and 1b show, the effect of polarization on the informativeness of the monopolist's strategy is non-monotonous. Polarization increases informativeness (i.e., the range of posterior beliefs). However, there is a discontinuity point, that is, when the expert shifts from the hard-news strategy to the soft-news strategy. Therefore, having some degree of heterogeneity in beliefs is beneficial, as it increases the quality of the information provided by the expert. However, if polarization becomes too high, the expert changes strategy. Lemma 2 shows that the soft-news strategy is less informative than the hard-news strategy.

Example. I consider the example from the introduction. There are two states of the world: either a vaccine is safe or it has side effects. The pro-



(a) Believers



(b) Sceptics

Figure 1: Range of posterior beliefs

government media wants to persuade citizens that the vaccine is safe. There are two groups of citizens, 1 and 2, and $g_1 = g_2$. Group 1 are believers whereas group 2 are sceptics, with prior beliefs $\mu_1^0(\text{Safe}) = 0.7$ and $\mu_2^0(\text{Safe}) = 0.2$ respectively. Each citizen decides whether to get vaccinated. The hard-news strategy is then defined as follows:

ω	Safe	Side Effects	
s	*safe*		*side effects*
$\pi(s\,\vert\,\omega)$	1	0.25	0.75

The message *safe* persuades sceptics. To be credible, the pro-government media needs to commit to sending the message *side effects* often enough when the true state is "Side Effects".

The soft-news strategy consists of two messages. The message *safe* (e.g., clinical trials) persuades sceptics but has a low chance to be misleading (that is, to induce decision-makers to choose the wrong action). The message *anecdotal safe* (e.g., vague comparisons of benefits and risks) has a higher chance to be misleading but persuades only believers.

ω	Safe		Side Effects	
s	*safe*			*anecdotal safe*
$\pi(s\,\vert\,\omega)$	0.64	0.36	0.16	0.84

The advantage of the soft-news strategy is that believers get vaccinated with probability one. With *anecdotal safe* the pro-government media leverages believers' credulity. Meanwhile, it does not give up entirely from the persuasion of sceptics (message *safe*). Given citizens' beliefs, whether the soft-news strategy is better than the hard-news strategy only depends on the pro-government media's belief. In the following table, I compare the expected payoff for the pro-government media from the two strategies for different beliefs.

Pro-government media's belief (safe)		Hard-news	Soft-news
	0.5	0.625	0.7
	0.8	0.85	0.772

When uncertain about the existence of side effects and if citizens have sufficiently polarized beliefs, the pro-government media uses the soft-news strategy.[16]

[16] In Section 7, I discuss some possible caveats of this example.

The natural question to ask is then: What happens if we allow competition by a no-vax media? Next section provides an answer.

5. Media Pluralism

In this section, I study how competition affects persuasion. I restrict attention to competition between two experts with different preferred actions.[17] The following lemma establishes the effect of competition with unlimited attention.

Lemma 3 (Competition)**.** *Let* $J = \{j_\alpha, j_\beta\}$ *with* $a_{j_\alpha} = a_1$ *and* $a_{j_\beta} = a_2$*. For any* $s_{j_\alpha} \in S_{j_\alpha}$ *and any* $s_{j_\beta} \in S_{j_\beta}$ *such that* $\pi_{j_\alpha}(s_{j_\alpha}|\omega_1), \pi_{j_\beta}(s_{j_\beta}|\omega_2) > 0$*, then* $\pi_{j_\alpha}(s_{j_\alpha}|\omega_2) = \pi_{j_\beta}(s_{j_\beta}|\omega_1) = 0$*.*

By Lemma 1, at least one persuasion constraint must hold with equality in any best response of any expert. The corresponding decision-makers are thus indifferent between either preferred action. Then, the rival has incentives to undercut the expert: it is sufficient to provide very little information to change such decision-makers' behaviour. Therefore, there cannot be an equilibrium unless any expert refrains from persuading under each other's favourable state. Full revelation (i.e., truth-telling by both experts) is the equilibrium when decision-makers have unlimited attention: given truth-telling by the rival, any attempt to persuade is futile.[18]

In the following, I introduce limited attention and show that full revelation is not an equilibrium. Competition is actually harmful to decision-makers as it deteriorates the quality of information.

Limited attention implies that each decision-maker can only devote attention to one expert.[19] In other words, either $j_d = j_\alpha$ or $j_d = j_\beta$ for any decision-maker $d \in D$. The problem for each expert j is identical to the one solved previously. However, the composition of his audience H_j is now endogenous. The distribution of prior beliefs each expert faces is the result of the optimal attention choices of decision-makers. The allocation of attention

[17]Information provision is not affected by the entry of experts with the same preferences as the incumbent. Indeed, the entrant cannot refine the optimal persuasion of the incumbent. See Section B.3 in the Appendix.

[18]This result is coherent with the literature (Gentzkow and Kamenica, 2017a,b; Ravindran and Cui, 2020).

[19]In Section B.1, decision-makers can pay attention to the second expert at a cost. I show that full revelation is achievable if and only if this cost is zero. Such a case is equivalent to attention being unlimited. Instead, my results still hold under the weaker assumption that attention is costly rather than limited.

and the optimal strategy are chosen simultaneously by each decision-maker and each expert, respectively.

The objective function of each decision-maker is her subjective probability of choosing the correct action (that is, her expected payoff). Suppose that a decision-maker $d \in D_i$ devotes attention to the expert $j \in J$. Mathematically, this probability has the following expression:

$$\lambda_i(\pi_j) := \sum_{s \in S_j} \sum_{\omega_k \in \Omega} \pi_j(s \,|\, \omega_k) \mu_i^0(\omega_k) \mathbb{1}\left\{\sigma\big(\mu_d(\omega_1 \,|\, s)\big) = a_k\right\}$$

Lemma 4 (Decision-maker's payoff)**.** $\lambda_i(\pi_j) = 1$ *if and only if* π_j *is truth-telling. If* π_j *is babbling, then* $\lambda_i(\pi_j) = \mu_i^0(\omega_m)$*, where* $m = \arg\max_{m \in \{1,2\}} \mu_i^0(\omega_m)$*. It holds that* $\lambda_i(\pi_j) \in [\mu_i^0(\omega_m), 1]$*.*

Intuitively, the subjective probability of taking the correct action is maximal when an expert reveals the state of the world. Persuasion cannot decrease such a probability compared to the no information case. In particular, an expert can change a decision-maker's behaviour. However, this requires the expert to reveal some information and makes the decision-maker (weakly) better off. Without information, a decision-maker of subgroup i chooses the action associated with her most plausible state given prior beliefs: $\mu_i^0(\omega_m)$ is the corresponding subjective probability of taking the correct action. Therefore, $\Delta_{ij} := \lambda_i(\pi_j) - \mu_i^0(\omega_m) \geq 0$ is the subjective information gain from persuasion.

Definition 5 (Target)**.** *For any expert* $j \in J$*, a target is a subgroup* $i \in I$ *of decision-makers whose persuasion constraint holds with equality, given the expert* j*'s strategy. Let* T_j *be the set of targets for expert* j*.*

By Lemma 1, the set of targets is non-empty. The hard-news strategy targets sceptics, whereas the soft-news strategy targets sceptics and believers. A subgroup being a target means that the expert tailors his strategy to persuade marginally decision-makers belonging to such subgroup and thus renders them exactly indifferent between the two actions.

Proposition 2 (Zero information gain for a target)**.** *For each expert* $j \in J$ *and each* $i \in T_j$*, it holds that* $\Delta_{ij} = 0$*.*

Proposition 2 states that when a subgroup is a target of an expert, such decision-makers receive zero information gain when devoting attention to this expert. Intuitively, an expert reveals only the information that is strictly necessary to persuade decision-makers of a targeted subgroup. Being a target is a sufficient condition for zero information gain from persuasion.[20]

[20]However, it is not a necessary condition: decision-makers whose behaviour is not affected by beliefs updating have zero information gain as well.

Proposition 2 shapes decision-makers' incentives regarding the allocation of attention. The optimal allocation of attention for a decision-maker $d \in D_i$ is given by $j_d(\pi_{j_\alpha}, \pi_{j_\beta})$, and $j_d(\cdot) = j$ requires that $j \in \arg\max_{j \in J} \Delta_{ij}$. In other words, each decision-maker devotes attention to the expert that grants her the highest information gain. Crucially, each decision-maker wants to avoid being a target, as in that case $\Delta_{ij} = 0$.

Any equilibrium is thus characterized by a vector $(\pi_{j_\alpha}, \pi_{j_\beta}, j_1, \dots, j_{|D|})$. The set of decision-makers who pay attention to the expert j (his audience) is $H_j = \{d \in D \,|\, j_d(\cdot) = j\}$. Each strategy must be a best response for the corresponding expert: for a given audience H_j, each expert j uses his optimal strategy $\pi_j(H_j)$. At the same time, the allocation of attention must be consistent with decision-makers' incentives. In particular, for any expert $j \in J$ and any decision-maker $d \in H_j$, it must hold that $j_d(\pi_{j_\alpha}(H_{j_\alpha}), \pi_{j_\beta}(H_{j_\beta})) = j$. I define two categories of equilibria:

Definition 6. *An equilibrium is "symmetric" if any two decision-makers of the same subgroup $i \in I$ pay attention to the same expert $j \in J$. Otherwise, the equilibrium is "asymmetric".*

Here, I assume $I = \{1, 2\}$ with $\phi_1 > 1$ and $\phi_2 < 1$.[21] Importantly, decision-makers of subgroup $i = 1$ ($i = 2$) are believers (sceptics) of ω_1 and sceptics (believers) of ω_2. There are three symmetric equilibrium candidates, namely:

1. *Monopoly.* All decision-makers devote attention to the same expert: $H_{j_\alpha} = D$ or $H_{j_\beta} = D$. The optimal strategy follows Proposition 1. The non-active expert is indifferent between any strategy;

2. *Echo chambers.* Each expert collects attention only by his believers: $H_{j_\alpha} = D_1$ and $H_{j_\beta} = D_2$. Therefore, the optimal strategy is babbling;

3. *Opposite-bias learning.* Each expert collects attention only by his sceptics: $H_{j_\alpha} = D_2$ and $H_{j_\beta} = D_1$. Therefore, the optimal strategy is the hard-news strategy.[22]

Proposition 3 (Equilibrium). *Let $J = \{j_\alpha, j_\beta\}$ and $I = \{1, 2\}$, where decision-makers of subgroup 1 (2) are believers from the perspective of expert j_α (j_β). Echo chambers with babbling is the unique symmetric equilibrium such that both experts are active.*

[21] In Section 6.2 I consider the case of more than two subgroups of decision-makers.

[22] The soft-news strategy allows to retain believers. It is not optimal when only sceptics devote attention.

In echo chambers, given babbling by both experts, decision-makers have no incentive to deviate, because each expert provides zero information gain. Therefore, echo chambers is an equilibrium. An equilibrium with a monopolist requires that the non-active expert provides zero information gain. Otherwise, the targets of the monopolist would find it beneficial to deviate.

By Lemma 2, opposite-bias learning would be desirable as both experts would use the hard-news strategy. However, opposite-bias learning cannot be an equilibrium because it is not coherent with each decision-maker's incentives. Each sceptic can get a strictly positive information gain by becoming a believer of the like-minded expert. Indeed, when a sceptic deviates and devotes attention to the like-minded expert, she is not a target given the like-minded expert's strategy. In other words, the like-minded expert does not tailor information to manipulate his believers' behaviour. That is why sceptics benefits from the deviation.

The game has also asymmetric equilibria. A necessary condition is that decision-makers of the same subgroup are indifferent about the allocation of attention. There exist asymmetric equilibria where one expert uses the hard-news strategy (i.e., informative expert), whereas the other is babbling (i.e., babbling expert). From the perspective of the informative expert, his sceptics are indifferent and can devote attention to either expert. Instead, his believers are strictly better off by devoting attention to him. Thus, the informative expert collects attention from all his believers and some of his sceptics. Sceptics could also join the echo chamber of the babbling expert. Any allocation of attention that makes the hard-news strategy optimal for the informative expert constitutes an equilibrium.[23]

Proposition 4 (Harmful competition)**.** *For any competitive outcome, there exists a monopoly outcome such that 1.) it is a Pareto improvement and 2.) information quality is (weakly) higher for any decision-maker.*

Proposition 4 implies that decision-makers are less informed with competition. Media pluralism harms decision-makers when the latter have limited attention and can freely allocate it between experts. Each decision-maker attempts to get positive information gain from persuasion by avoiding to devote attention to an expert that targets her. However, this leads decision-makers to cluster into echo chambers. Echo chambers are harmful because each expert faces only his believers, and the best response is babbling. Thus, decision-makers would be better informed in a monopoly. Indeed, a monopolist uses either the hard-news strategy or the soft-news strategy. Both

[23]There also exist asymmetric equilibria where both experts use the soft-news strategy. Both subgroups are targets of each expert. Thus, each decision-maker gets zero information gain independently of the allocation of attention.

these strategies produce some dispersion in posterior beliefs, whereas babbling leaves beliefs unchanged. Hence, babbling is less informative according to Blackwell (1953)'s ordering.

Example. An asymmetric equilibrium could fit the COVID-19 vaccination example. The pro-government media collects attention from believers and sceptics and, thus, uses the hard-news strategy. The no-vax media exploits his echo chamber and provides information that amounts to babbling. Therefore, decision-makers in the no-vax echo chamber are less informed than in a monopoly.

Citizens who are sceptical about vaccinations understand that the pro-government media tailors information to change their behaviour. Therefore, a sceptic has no advantage from devoting attention to the pro-government media and could decide to join the no-vax echo chamber.

The number of citizens that the pro-government media can persuade to get vaccinated depends on the equilibrium allocation of attention. Sceptics may cluster into the no-vax echo chamber and get confirmatory news. Their worldview cannot change and, thus, they are not willing to get vaccinated. An implication of this result is that herd immunity is unachievable if the no-vax echo chamber is too large.

6. Extensions

6.1. Platform

The negative effect of competition is related to the endogenous allocation of attention by decision-makers. In this section, I show that media pluralism can enhance information quality when the allocation of attention is exogenous for decision-makers. I assume that there exists a third agent (a platform) which chooses the allocation of attention to maximize news quality. In other words, the platform chooses g_{ij} for any subgroup $i \in I$ and any expert $j \in J$. Then, each expert j solves (3). Let $J = \{j_\alpha, j_\beta\}$, $a_{j_\alpha} = a_1$, $a_{j_\beta} = a_2$ and $I = \{1, 2\}$. I assume that decision-makers of subgroup 1 (2) are believers of state ω_1 (ω_2), that is, $\phi_1 > 1$ and $\phi_2 < 1$. By Lemma 2, the most informative strategy (among those that are compatible with each expert's incentives) is the hard-news strategy. By Proposition 1 (in particular equation (7) in the Appendix), each expert uses the hard-news strategy if there are not too many believers in his audience:

$$g_{1j_\alpha} \leq \frac{\mu^0_{j_\alpha}(\omega_1) + \phi_2 \mu^0_{j_\alpha}(\omega_2)}{\mu^0_{j_\alpha}(\omega_1) + \phi_1 \mu^0_{j_\alpha}(\omega_2)} \qquad g_{2j_\beta} \leq \frac{\mu^0_{j_\beta}(\omega_2) + \frac{1}{\phi_1}\mu^0_{j_\beta}(\omega_1)}{\mu^0_{j_\beta}(\omega_2) + \frac{1}{\phi_2}\mu^0_{j_\beta}(\omega_1)}$$

These conditions represent a constraint for the platform that chooses the allocation of attention to induce both experts to use the hard-news strategy. There is no equivalent constraint when the allocation of attention is chosen by decision-makers, and this explains echo chambers. Indeed, given that both experts are using the hard-news strategy, decision-makers have incentives to become believers. However, this makes the hard-news strategy suboptimal for each expert and traps decision-makers into echo chambers.

Proposition 5 (Platform). *For each decision-maker, a monopoly with the hard-news strategy is more informative than opposite-bias learning, which in turn is more informative than a monopoly with the soft-news strategy.*

The hard-news strategy is *more informative* for a believer than for a sceptic. Therefore, if there exists a monopolist willing to use the hard-news strategy, such outcome is better than opposite-bias learning for believers of the monopolist (whereas sceptics are indifferent).

The platform can do better than opposite-bias learning. A platform would like to allocate believers to like-minded experts ($g_{1j_\alpha}, g_{2j_\beta} \uparrow$). However, this is effective only if each expert is using the hard-news strategy, and this requires the presence of enough sceptics ($g_{1j_\alpha}, g_{2j_\beta} \downarrow$). Some believers can be allocated to each expert without affecting his incentives to use the hard-news strategy. There are allocations of attention which outperform a monopoly (with the hard-news strategy) in terms of aggregate informativeness. However, some decision-makers receive lower quality information than in monopoly.

6.2. Many Decision-makers

In this section, I show that my results continue to hold with any arbitrary set I of subgroups of decision-makers. First of all, I consider finitely many subgroups, each one endowed with a different prior belief.

Proposition 6 (Optimal Persuasion). *Let $I = \{1, \ldots, R\}$ with $R > 2$, $\phi_1 < 1$ and $\phi_R > 1$. The unique optimal strategy is either a hard-news strategy or a soft-news strategy. A hard-news (soft-news) strategy is optimal if a subgroup of sceptics (believers) has the highest value of being persuaded marginally.*

Proposition 6 shows that optimal persuasion is robust to heterogeneity within believers and sceptics. The expert uses a hard-news strategy if the subgroup with the highest value as a target is a subgroup of sceptics. Next, I use such insight to extend the analysis to a continuous distribution of decision-makers' beliefs.

Proposition 7 (Optimal persuasion). *Let $F(x)$ be a distribution with support $[0, \infty)$ and density $f(x) > 0$ $\forall x$. Let $\phi_i := \frac{\mu_i^0(\omega_1)}{\mu_i^0(\omega_2)} \sim F$. Then, the expert j with ratio of prior beliefs ϕ_j uses a hard-news strategy if a unique solution $\phi \in [0, 1]$ to the following equation exists*

$$h(\phi) = \frac{1}{\phi_j + \phi} \tag{6}$$

and condition (12) *holds. Note that $h(x) := \frac{f(x)}{1-F(x)}$ is the hazard rate function.*

It is possible to evaluate the quality of the information in real-world settings using condition (6). A researcher needs to know the distribution of decision-makers' beliefs and the expert's belief.[24] Then, condition (6) predicts whether the expert uses a hard-news strategy or a soft-news strategy.

Gitmez and Molavi (2020) find a similar characterization of the optimal strategy in a setting where the expert is trading-off between an extensive margin (how many decision-makers devote attention) and an intensive margin (how many decision-makers are persuaded). By contrast, in my setting devoting attention to one expert is costless, which means that all decision-makers devote attention.

As an example, I assume that F is the exponential distribution. In other words, $F(x; \lambda) = 1 - e^{-\lambda x}$ where λ is a parameter. A special property of this distribution is a constant hazard rate, that is, $h(x) = \lambda$. Therefore, equation 6 implies $\phi = \frac{1}{\lambda} - \phi_j$ and, by Proposition 7, the expert uses a hard-news strategy if $\lambda \geq \frac{1}{1+\phi_j}$. Fixing $\phi_j = \frac{1}{2}$, Figure 2 depicts two examples of density functions that imply different optimal strategies.

Lemma 5 (Blackwell's criterion). *A hard-news (soft-news) strategy is more informative the more extreme are the prior beliefs of its target(s). The ranking of the strategies in terms of informativeness is subgroup specific.*

More extreme targets (i.e., targets with beliefs closer to either 0 or 1) induce a more disperse distribution of posterior beliefs: the strategy moves closer to truth-telling. Lemma 5 extends Lemma 2: some decision-makers may find a soft-news strategy more informative than a hard-news strategy if the former targets more extreme sceptics. See condition (13) in the Appendix.

Proposition 8 (Competition with limited attention). *In any symmetric equilibrium, at least one expert is babbling.*

[24] Similar knowledge could derive, for instance, from surveys.

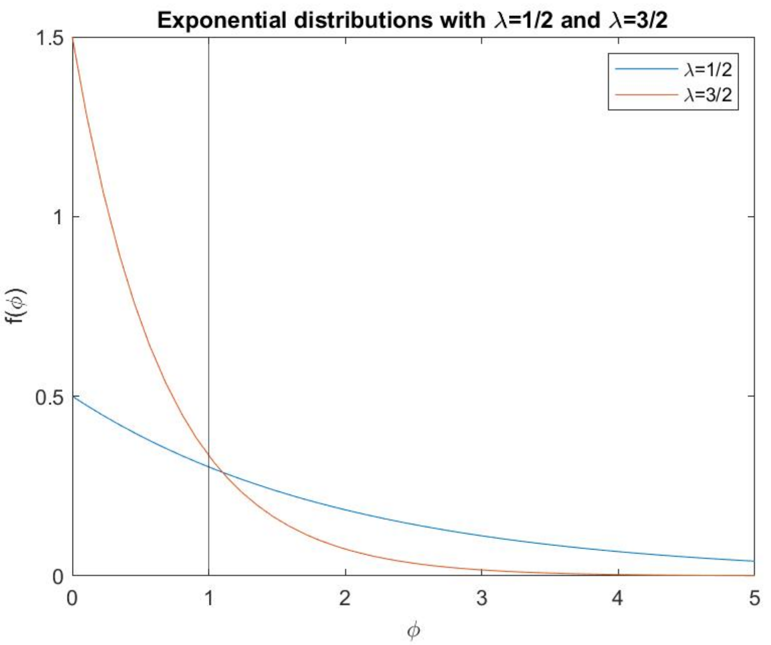


Figure 2: The black line at $\phi = 1$ separates sceptics (at the left) from believers. When $\lambda = \frac{3}{2}$, the majority of decision-makers are sceptics and, thus, a hard-news strategy is optimal. By contrast, a soft-news strategy is optimal when $\lambda = \frac{1}{2}$, because many decision-makers are believers.

The key mechanism behind this result is the following: for any allocation of attention and corresponding optimal strategies, there exists at least one target who can deviate and get a positive information gain, unless at least one expert is babbling.

The existence of more than two subgroups of decision-makers generates additional symmetric equilibria, which I label *partial echo chambers*. In these equilibria, an ordered subset of believers (those with the most extreme prior beliefs) join the echo chamber of the babbling expert. The other expert gets attention from the remaining decision-makers, including some of his sceptics. Thus, he uses either a hard-news strategy or a soft-news strategy or, in other words, he is an informative expert. Given babbling, nobody outside the echo chamber wants to join it. At the same time, any believer within the echo chamber would become the most sceptical decision-maker of the informative expert in case of a deviation: given the informative expert's strategy, her behaviour would not change. Therefore, this deviation would yield zero information gain, and this supports the equilibrium.

Proposition 9 (Harmful competition)**.** *Under a technical condition, for any* *partial echo chambers outcome, there exists a monopoly outcome such that 1.) it is a Pareto improvement and 2.) information quality is (weakly) higher for any decision-maker.*

The negative effect of competition (Proposition 4) extends in a setting with any arbitrary distribution of decision-makers' beliefs. When comparing monopoly with partial echo chambers, a case distinction is necessary. If a monopolist uses a hard-news strategy, competition is harmful because information gains are (weakly) lower, and those decision-makers who cluster into the echo chamber receive babbling. When an expert uses different soft-news strategies in monopoly and partial echo chambers, some decision-makers might be better off in partial echo chambers. In this case, competition is harmful to all decision-makers if both targets are less extreme in partial echo chambers than in monopoly. Intuitively, this sufficient condition should hold because there are fewer sceptics in partial echo chambers, and thus the expert might be tempted to retain less extreme believers.

7. Applications

Throughout the paper, I have considered the COVID-19 vaccination as an example to illustrate my results. Such an example could have some caveats. Perhaps it is controversial to assume that the pro-government media has state-independent preferences. There is a trade-off between economic outcomes and the time needed to eradicate COVID-19, which means that herd immunity is a goal. However, the pro-government media is also concerned about safety. My model applies to a vaccine that has been approved for administration. Thus, it is safe overall. However, the pro-government media could avoid disclosing possible side effects. Moreover, many citizens are irrational and cannot be persuaded. Hence, my model applies to the subset of the population that is rational. I show that endogenous echo chambers can explain why many rational citizens are still sceptical about vaccinations and can be a threat to reaching herd immunity.

In this section, I argue that the applicability of my results goes beyond the previous example. My findings require four assumptions: on the one hand, experts are biased and have commitment power; on the other hand, decision-makers have heterogeneous beliefs and limited attention. Here, I briefly discuss what is the outcome if I relax any of these assumptions:

1. Under unlimited attention, by Lemma 3 experts are in direct competition to persuade decision-makers. As a consequence, full revelation is the unique equilibrium.

2. When decision-makers share the same prior belief, experts do not face a trade-off between persuading sceptics and retaining believers. As a consequence, each decision-maker has zero information gain independently of the allocation of attention.

3. Trivially, an unbiased expert is truth-telling and collects all attention.

4. When experts have no commitment power, decision-makers anticipate that babbling is optimal for each expert. Thus, decision-makers are indifferent about the allocation of attention.

Therefore, each assumption is necessary for my results to hold. These assumptions allow me to build a model able to offer insights into the real world. By contrast, the outcome when relaxing any assumption is either full revelation or anything.

My assumptions are realistic in many contexts. The media may have commitment power, for instance, because of law or reputation concerns.[25] Limited attention is a well-established fact. Heterogeneous beliefs are also very likely to exist in all situations where the objective probability for a claim to be true is ambiguous. Whenever the true state of the world is disputed, there are likely competing interpretations of the current state of events. If this is true, the last requirement to apply my insights, namely competition between biased experts, is fulfilled. In the following, I provide a non-exhaustive list of examples where my insights may be useful.

My model applies to the design of information about political issues. A politician wants to persuade voters to support a particular point of view. The optimal design of information trades off the desire of persuading sceptical voters and the goal of keeping loyalists. As a result, some information is provided. With competition and limited attention, some voters cluster into to echo chamber(s) and get no useful information.

A recent example is Trump's claim that the US Presidential election was fraudulent. The United States show increasing political polarization (Finkel et al., 2020). My model can explain why Republicans believe Biden won because of a "rigged" election, even though Trump has failed to provide any evidence about that (Rutenberg et al., 2020).

Climate change is another relevant example. A vast majority of scientists claim that climate change is real. Many NGOs warn that immediate intervention is necessary to avoid a sharp increase in mass disasters, whereas corporations (especially coal and oil producers) try to dispute such warnings.

[25] Nguyen (2017) and Fréchette et al. (2019) provide evidence in support of the Bayesian persuasion model.

Endogenous echo chambers can explain the existence of climate change deniers. Similarly, believers of a long list of debunked conspiracy theories can survive within echo chambers. The common root is widespread scepticism about Science (Achenbach, 2015).

My model also applies to the advertising of differentiated products. A firm wants to persuade consumers to buy a product with uncertain value. Some consumers believe the product has a high value, whereas others believe it has low value. Each consumer buys if and only if she believes the product has high value. The firm designs the advertisement to maximize sales and then optimally provides some information about the product's value. With competition and limited attention, each consumer believes one product has a higher value than the other and may devote her attention only to the producer of this particular product. Echo chambers make it optimal for the firms to provide no information. My model can also rationalize asymmetric equilibria where one firm invest in informative advertising, whereas the other enjoys its market niche. If both firms design informative advertising, consumers rationally want to learn about their favoured products. But then providing informative advertising is not optimal for the firms. Cookson et al. (2021) provide evidence that investors behaviour in the financial markets is in line with this application.

8. Conclusion

The quality of information depends on agents' beliefs. When worldviews are sufficiently polarized, a monopolist provides lower quality information. Competition backfires when attention is limited: increasing the diversity of information sources reduces information quality even further. Indeed, echo chambers arise endogenously. As a consequence, the incentives to provide valuable information vanish.

My findings, therefore, provide a sobering insight into the effects of media pluralism: under media users' limited attention and heterogeneous beliefs, media pluralism leads to worse-informed media users. Information overload introduces an additional choice for decision-makers: the subset of information to process. Policymakers should account for decision-makers' incentives. Supporting media pluralism is a good idea only if decision-makers are sufficiently attentive to process information from diverse sources.

Whereas the literature has justified echo chambers with confirmation bias, I show that the opposite can be true. Even unbiased decision-makers end up devoting attention to like-minded experts. The latter, then, find it optimal to confirm decision-makers' beliefs. Therefore, I provide a rational foundation for confirmation bias.[26]

My paper leaves an open question that requires further research. How can we mitigate confirmation bias? One approach is to enhance attention, but it is unclear how to do this. An alternative is to manipulate the allocation of attention to increase information provision. In Section 6.1, I have shown how a platform that wants to maximize the informativeness of news should allocate attention. Such a platform would design each expert's audience to give him incentives to use the hard-news strategy. For instance, opposite-bias learning is an appropriate allocation of attention (but perhaps a platform can do better). Platforms such as news aggregators may have the ability to shape how users allocate attention. However, there is no guarantee that such platforms behave as a social planner would do.

[26]Goette et al. (2020) provide experimental evidence that limited attention reinforces confirmation bias.

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A. Appendix A

Proof of Lemma 1

Proof. I assume there exists $i \in I$ such that $g_{ij} > 0$ and $\phi_i < 1$. Otherwise, persuasion is not necessary and babbling is the only optimal strategy. I assume by contradiction that $\nexists\, s \in S_j$ such that $\pi_j(s \mid \omega_2) = \phi_i \pi_j(s \mid \omega_1)$ for some $i \in I$. Let $\{\phi_i\}$ be the ordered (in ascending order) set of constraints for each subgroup $i \in I$ such that $g_{ij} > 0$. If the n-th constraint holds for a message $s \in S_j$, then the m-th constraint holds too, for any $m > n$. Therefore, if n-th constraint holds there is more persuasion than if only the m-th constraint were holding, ceteris paribus. Thus, if the n-th constraint is slack, it is beneficial for the expert to increase the probability of the corresponding message, at the expense of the probability of a message which satisfy only the m-th constraint. There always exists a deviation for the expert unless at least one constraint holds with equality. □

Proof of Proposition 1

Proof. The payoff for *Babbling* is $V_u := g_1$, whereas the payoff for the *Truth-telling strategy* is $V_t := \mu_j^0(\omega_1)$. The *Hard-news strategy* is as follows:

ω	ω_1	ω_2	
s	s	s	s'
$\pi_h(s \mid \omega)$	1	ϕ_2	

$$\implies V_h := \mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_2$$

The *Soft-news strategy* is as follows:

ω	ω_1		ω_2	
s	s	s'	s	s'
$\pi_s(s \mid \omega)$	k	$1-k$	$\phi_2 k$	$\phi_1(1-k)$

$$\implies V_s := kV_h + (1-k)\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_1\right] g_1$$

where

$$1 - \phi_2 k = \phi_1(1-k) \iff k = \frac{\phi_1 - 1}{\phi_1 - \phi_2}$$

Any alternative strategy with $\pi(s \mid \omega_1) < k$ is suboptimal, because the soft-news strategy increases the probability of persuading sceptics without affecting the behaviour of believers.

Note that $V_h \geq V_t$. Hence, the expert does not use the truth-telling strategy. Moreover, $V_s > V_u$ for any $g_1 \in (0,1)$. The hard-news strategy is optimal if:

$$\begin{aligned} V_h \geq V_s &\iff \mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_2 \geq \left(\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_1\right) g_1 \\ &\iff \mu_j^0(\omega_1)(1 - g_1) \geq \mu_j^0(\omega_2)\left(\phi_1 g_1 - \phi_2\right) \end{aligned} \tag{7}$$

Note that the RHS of (7) is increasing in ϕ_1 and decreasing in ϕ_2. The difference of these two values is a proxy for decision-makers' polarization in terms of prior beliefs. The RHS (LHS) of (7) is increasing (decreasing) in g_1, the share of believers among decision-makers. Finally, the RHS (LHS) of (7) is decreasing (increasing) in $\mu_j^0(\omega_1)$, the expert's belief of his favourable state. □

Proof of Lemma 2

Proof. First of all, the distributions of posterior beliefs induced by these two strategies have the same mean, which coincides with $\mu_i^0(\omega_1)$ for any $i \in I$, following Bayesian plausibility. It follows by (4)-(5) that π_h is characterized by more dispersion then π_s. Indeed, with the hard-news strategy:

$$\mu_1(\omega_1 \mid s) - \mu_1(\omega_1 \mid s') = \frac{\phi_1}{\phi_1 + \phi_2}$$

$$\mu_2(\omega_1 \mid s) - \mu_2(\omega_1 \mid s') = \frac{1}{2}$$

whereas with the soft-news strategy:

$$\mu_1(\omega_1 \mid s) - \mu_1(\omega_1 \mid s') = \frac{\phi_1}{\phi_1 + \phi_2} - \frac{1}{2}$$

$$\mu_2(\omega_1 \mid s) - \mu_2(\omega_1 \mid s') = \frac{1}{2} - \frac{\phi_2}{\phi_1 + \phi_2}$$

Therefore, π_h is more informative than π_s following Blackwell (1953). □

Proof of Lemma 3

Proof. With unlimited attention, each decision-maker observes the strategies of all experts and the corresponding messages. In particular, she observes

$\pi(s|\omega) = \Pi_{j\in J}\pi_j(s_j|\omega)$ for any $s \in S_J := \times_{j\in J} S_j$ and any $\omega \in \Omega$, *and* a realization $s \in S_J$ chosen by Nature. Thus, for expert $j_k \in J$ with preferred action a_{j_k} with $k \in \{\alpha, \beta\}$, the *persuasion constraint* becomes:

$$\pi_{j_k}(s_{j_k}|\omega_{-k}) \leq \frac{\mu_i^0(\omega_k)}{\mu_i^0(\omega_{-k})}\frac{\pi_{-j_k}(s_{-j_k}|\omega_k)}{\pi_{-j_k}(s_{-j_k}|\omega_{-k})}\pi_{j_k}(s_{j_k}|\omega_k) := \phi_{ij_k}\pi_{j_k}(s_{j_k}|\omega_k) \tag{8}$$

Note that if $\pi_{j_\alpha}(s_{j_\alpha}|\omega_1), \pi_{j_\beta}(s_{j_\beta}|\omega_2) = 0$ then $s_{j_\alpha}, s_{j_\beta}$ cannot persuade. I assume by contradiction that $\pi^0_{j_\beta}(s_{j_\beta}|\omega_2) > 0$ and $\pi^0_{j_\beta}(s_{j_\beta}|\omega_1) > 0$ for some s_{j_β}. Thus, it holds $\phi_{ij_\alpha} > 0$ for any $i \in I$, and by Lemma 1 $\pi_{j_\alpha}(s_{j_\alpha}|\omega_2) = \phi_{i'j_\alpha}\pi_{j_\alpha}(s_{j_\alpha}|\omega_1)$ for some $i' \in I$. It follows that $\phi_{ij_\beta} > 0$ for any $i \in I$, and $\phi_{i'j_\beta} = \frac{\pi^0_{j_\beta}(s_{j_\beta}|\omega_1)}{\pi^0_{j_\beta}(s_{j_\beta}|\omega_2)}$. I assume without loss of generality that decision-makers break the ties in favour of expert j_α. In order to persuade i', s_{j_β} has to satisfy the following persuasion constraint:

$$\pi_{j_\beta}(s_{j_\beta}|\omega_1) < \phi_{i'j_\beta}\pi_{j_\beta}(s_{j_\beta}|\omega_2)$$

which requires simply to set $\pi_{j_\beta}(s_{j_\beta}|\omega_1) = \pi^0_{j_\beta}(s_{j_\beta}|\omega_1) - \epsilon$ with $\epsilon > 0$ and small. This is a beneficial deviation because j_β persuades an additional subgroup of decision-makers (i') with a negligible reduction in the probability of persuasion. By (8), it follows that the persuasion constraint for expert j_α becomes:

$$\pi_{j_\alpha}(s_{j_\alpha}|\omega_2) \leq \frac{\mu^0_{i'}(\omega_1)}{\mu^0_{i'}(\omega_2)}\frac{\pi_{j_\beta}(s_{j_\beta}|\omega_1)}{\pi_{j_\beta}(s_{j_\beta}|\omega_2)}\pi_{j_\alpha}(s_{j_\alpha}|\omega_1) < \phi_{i'j_\alpha}\pi_{j_\alpha}(s_{j_\alpha}|\omega_1)$$

that is a contradiction, which follows from the fact that this is a zero-sum game for the experts. □

Proof of Lemma 4

Proof. Assume that π_j is truth-telling. Hence, $\pi_j(s|\omega_1) = \pi_j(s'|\omega_2) = 1$ and $\pi_j(s|\omega_2) = \pi_j(s'|\omega_1) = 0$. This implies that $\lambda_i(\pi_j) = 1$. Assume that π_j is not truth-telling, and without loss of generality $\pi_j(s|\omega_2) > 0$. Note that either $\sigma\big(\mu_i(\omega_1|s)\big) = a_1$ or $\sigma\big(\mu_i(\omega_1|s)\big) = a_2$. It follows that $\lambda_i(\pi_j) < 1$.

If π_j is babbling then, for any $s \in S_j$, $\sigma\big(\mu_i(\omega_1|s)\big) = a_m$. It follows that $\lambda_i(\pi_j) = \mu_i^0(\omega_m)$. Assume that there exists $s \in S_j$ and $\omega_k \neq \omega_m$ such that $\pi_j(s|\omega_k) \neq \pi_j(s|\omega_m)$. By (2), $\sigma\big(\mu_i(\omega_1|s)\big) = a_k$ if $\pi_j(s|\omega_k) \geq \frac{\mu_i^0(\omega_m)}{\mu_i^0(\omega_k)}\pi_j(s|\omega_m)$, and this implies that $\lambda_i(\pi_j) \geq \mu_i^0(\omega_m)$. □

Proof of Proposition 2

Proof. Assume without loss of generality $a_j = a_1$. If π_j is a hard-news strategy then $T_j = \{i\}$ and $\phi_i < 1$. This implies $\lambda_i(\pi_j) = \mu_i^0(\omega_1) + \mu_i^0(\omega_2)[1 - \phi_i] = \mu_i^0(\omega_2)$. If π_j is the soft-news strategy then $T_j = \{i, i'\}$ and without loss of generality $\phi_{i'} > 1 > \phi_i$. Therefore, $\lambda_i(\pi_j) = \mu_i^0(\omega_1)k + \mu_i^0(\omega_2)[1 - \phi_i k] = \mu_i^0(\omega_2)$ and $\lambda_{i'}(\pi_j) = \mu_{i'}^0(\omega_1)$. □

Proof of Proposition 3

Proof. Echo chambers: Given $H_{j_\alpha} = D_1$ and $H_{j_\beta} = D_2$, babbling is optimal for each expert. Therefore, by Lemma 4, $\Delta_{ij} = 0$ for any $i \in I$ and $j \in J$. Therefore, $j_1 = j_\alpha$ and $j_2 = j_\beta$ is optimal for decision-makers.

Monopoly: I assume without loss of generality $H_{j_\alpha} = D$ and $H_{j_\beta} = \varnothing$. The subgroup $i = 2$ must be a target. By Proposition 2, sceptics get zero information gain, that is $\Delta_{2j_\alpha} = 0$. Therefore, $j_2 = j_\alpha$ is optimal only if $\Delta_{2j_\beta} = 0$. Note that j_β is indifferent between any strategy. This equilibrium breaks down if π_{j_β} is such that $\Delta_{2j_\beta} > 0$.

Opposite-bias learning: Given $H_{j_\alpha} = D_2$ and $H_{j_\beta} = D_1$, the hard-news strategy is optimal for each expert. By Proposition 2, $\Delta_{1j_\beta} = \Delta_{2j_\alpha} = 0$. However, $\Delta_{1j_\alpha}, \Delta_{2j_\beta} > 0$. Therefore, $j_1 = j_\beta$ and $j_2 = j_\alpha$ cannot be optimal for decision-makers. □

Proof of Proposition 4

Proof. An asymmetric equilibrium where for each subgroup $i \in I$ two decision-makers of the same subgroup devote attention to different experts requires each expert to use the soft-news strategy. These equilibria are equivalent to echo chambers: the information gain is zero for any decision-maker. A monopoly outcome is a Pareto improvement: if the expert uses the hard-news strategy, believers are better off; whereas if he uses the soft-news strategy each decision-maker is indifferent. There cannot exist an asymmetric equilibrium such that one expert uses the hard-news strategy whereas the other uses the soft-news strategy: with the hard-news strategy, believers get a positive information gain. The alternative asymmetric equilibria is such that one expert is using the hard-news strategy whereas the other is babbling. This requires the second expert to collect attention only from believers. Such asymmetric equilibria are equivalent to a monopoly using a hard-news strategy in terms of information gains. For these equilibria to exist there must be at least one expert such that as a monopolist he would use the hard-news strategy. In this case, a sufficiently small mass of sceptics can devote attention to the other expert without changing the monopolist's optimal strategy.

If both experts as monopolists would use the soft-news strategy, the mass of believers must be reduced to switch in favour of the hard-news strategy. However, this is not compatible with the second expert babbling.

In any asymmetric equilibrium with a babbling expert, those who devote attention to the latter receive information of the lowest quality. Indeed, babbling is the least informative outcome following Blackwell (1953): posterior beliefs are equal to prior beliefs. Instead, the hard-news strategy and the soft-news strategy produce both some dispersion in posterior beliefs. In any asymmetric equilibrium where both experts use the soft-news strategy, each decision-maker is equally informed. By (4)-(5),

$$\mu_1(\omega_1 \,|\, s) - \mu_1(\omega_1 \,|\, s') = \mu_2(\omega_1 \,|\, s) - \mu_2(\omega_1 \,|\, s') = \frac{\phi_1 - \phi_2}{2\left[\phi_1 + \phi_2\right]} < \frac{1}{2}$$

Therefore, in a monopoly each decision-maker is better (equally) informed if the expert uses a hard-news (soft-news) strategy. □

Proof of Proposition 5

Proof. The following table summarizes the posterior beliefs of decision-makers (divided in subgroups 1 and 2) that the state is ω_1, following the different (incentive-compatible) strategies that the experts can use:

i	Hard-news j_α		Hard-news j_β		Soft-news j_α		Soft-news j_β	
	s	s'	s	s'	s	s'	s	s'
1	$\in (0.5, 1)$	0	**0.5**	**1**	$\in (0.5, 1)$	0.5	0.5	$\in (0.5, 1)$
2	**0.5**	**0**	$\in (0, 0.5)$	1	0.5	$\in (0, 0.5)$	$\in (0, 0.5)$	0.5

With Opposite-bias learning, each decision-maker is a sceptic, and each expert uses the hard-news strategy (the beliefs are in bold in the table). Comparing it with a monopolist using the hard-news strategy, it is immediate to see that a sceptic is indifferent whereas a believer is better informed with a monopoly, according to the ordering from Blackwell (1953). Instead, the comparison with a monopolist using the soft-news strategy shows that each decision-maker is worse off with a monopoly. □

Proof of Proposition 6

Proof. Let $|I_2| = R_2 < R$. I order the subgroups of decision-makers from the most sceptical to the least:

$$\phi_1 < \cdots < \phi_{R_2} < 1 < \cdots < \phi_R$$

For any subgroup $r \in I$, I define the value for the expert of persuading marginally subgroup r as

$$E_r := \left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_r\right] \sum_{i=r}^{R} g_i \tag{9}$$

For any $r, r' \in I$, it is possible to define the following strategies:

Definition 7 (Hard-news strategy)**.** *A hard-news strategy π_r, with target $T = \{r\}$ such that $r \leq R_2$, consists of a persuading message s and a residual message s' such that*

$$\pi_r(s \,|\, \omega_1) = 1 \quad \pi_r(s' \,|\, \omega_1) = 0$$

$$\pi_r(s \,|\, \omega_2) = \phi_r \quad \pi_r(s' \,|\, \omega_2) = 1 - \phi_r$$

The hard-news strategy implies the following posterior beliefs:

$$\mu_i(\omega_1 \,|\, s) = \frac{\phi_i}{\phi_i + \phi_r}, \quad \mu_i(\omega_1 \,|\, s') = 0 \quad \forall i \in I \tag{10}$$

Definition 8 (Soft-news strategy)**.** *A soft-news strategy $\pi_{\{r,r'\}}$, with targets $T = \{r, r'\}$ such that $r \leq R_2$ and $r' > R_2$, consists of two messages s, s' such that*

$$\pi_{\{r,r'\}}(s \,|\, \omega_1) = k \quad \pi_{\{r,r'\}}(s' \,|\, \omega_1) = 1 - k$$

$$\pi_{\{r,r'\}}(s \,|\, \omega_2) = \phi_r k \quad \pi_{\{r,r'\}}(s' \,|\, \omega_2) = \phi_{r'}(1 - k)$$

where

$$k := \frac{\phi_{r'} - 1}{\phi_{r'} - \phi_r}$$

is strictly increasing in $\phi_r \in [0,1]$ and $\phi_{r'} \in [1, \infty]$.

The soft-news strategy implies the following posterior beliefs:

$$\mu_i(\omega_1 \,|\, s) = \frac{\phi_i}{\phi_i + \phi_r}, \quad \mu_i(\omega_1 \,|\, s') = \frac{\phi_i}{\phi_i + \phi_{r'}} \quad \forall i \in I \tag{11}$$

The payoff of a hard-news strategy is

$$V_r := E_r$$

whereas the payoff of a soft-news strategy is

$$V_{\{r,r'\}} := kE_r + (1 - k)E_{r'}$$

The payoff from the truth-telling strategy is $V_t = \mu_j^0(\omega_1)$ and $V_1 > V_t$. The payoff from babbling is $V_u = G_1 := \sum_{i=R_2+1}^{R} g_i$. Note that $V_{\{r,R_2+1\}} > V_u$. Therefore, babbling is not optimal. I assume that there exist a unique $r^* = \arg\max_r E_r$. It follows that a monopolistic expert is optimally using either a hard-news strategy or a soft-news strategy. This assumption rules out, for instance, any linear combination of hard-news strategies targeting different subgroups of sceptics. If $r^* \leq R_2$, a hard-news strategy with $T = \{r^*\}$ is optimal. Clearly $V_{r^*} > V_r$ for any $r \leq R_2$ and $r \neq r^*$. Moreover $V_{r^*} > V_{\{r,r'\}}$ as $E_{r^*} \geq E_r$ and $E_{r^*} > E_{r'}$ for any $r \leq R_2$ and any $r' > R_2$. If $r^* > R_2$, clearly $V_{\{r,r^*\}} > V_r$ for any $r \leq R_2$. Therefore, a soft-news strategy is optimal. However, r^* is not necessarily the target: for any $r \leq R_2$, $V_{\{r,r^*\}} < V_{\{r,r'\}}$ if there exists a subgroup of believers $r' < r^*$ such that the difference $E_{r^*} - E_{r'}$ is sufficiently small. □

Proof of Proposition 7

Proof. The value of being persuaded marginally - a generalization of expression (9) - is:

$$E_\phi := \left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi\right][1 - F(\phi)]$$

As suggested by Proposition 6, the expert uses a hard-news strategy or a soft-news strategy depending on whether the solution to $\max_\phi E_\phi$ belongs to $[0,1]$ or to $[1,\infty)$, respectively. The F.O.C. is:

$$\mu_j^0(\omega_2)[1 - F(\phi)] - f(\phi)\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi\right] = 0$$

and implies condition (6), whereas the S.O.C. is:

$$-2\mu_j^0(\omega_2)f(\phi) - f'(\phi)\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi\right] < 0$$

which implies

$$\frac{f'(\phi)}{f(\phi)} > -\frac{2}{\phi_j + \phi} \tag{12}$$

Clearly, if the F.O.C. is always negative/positive (or the S.O.C. is violated) there exist a corner solution, namely the most valuable subgroup is $x = 0$ or $x = 1$. Following Proposition 6, $x = 0$ implies the truth-telling strategy, which is a special case of a hard-news strategy in this setting. Instead, $x = 1$ does not imply necessarily that such subgroup is a target. The actual targets of the soft-news strategy depends on the shape of $F(\cdot)$. A sufficient condition for uniqueness is $f'(\phi) \geq 0$ for any $\phi \in [0,\infty)$. □

Proof of Lemma 5

Proof. Let us consider two hard-news strategies π_r and $\pi_{r'}$, with targets $T = \{r\}$ and $T = \{r'\}$ respectively, such that $r < r'$. Then, π_r is more informative than $\pi_{r'}$ for any $i \in I$, according to the ordering from Blackwell (1953). This follows by (10) and $\phi_r < \phi_{r'}$.

Now, let us consider two soft-news strategies $\pi_{\{r,r'\}}$ and $\pi_{\{r,r''\}}$, with targets $T = \{r, r'\}$ and $T = \{r, r''\}$ respectively, such that $r' > r''$. Then, $\pi_{\{r,r'\}}$ is more informative than $\pi_{\{r,r''\}}$ for any $i \in I$, according to the ordering from Blackwell (1953). This follows by (11) and $\phi_{r'} > \phi_{r''}$.

Finally, let us consider a hard-news strategy with target $T = \{r\}$ and a soft-news strategy with targets $T = \{r', r''\}$. If $r < r'$, Lemma 2 extends. If $r > r'$, there are two opposite effects: on the one hand, moving from a hard-news strategy targeting r to another targeting r' increases informativeness; on the other hand, moving from a hard-news strategy to a soft-news strategy reduces informativeness. For each subgroup $i \in I$, with the hard-news strategy, by (10):

$$\mu_i(\omega_1 \mid s) - \mu_i(\omega_1 \mid s') = \frac{\phi_i}{\phi_i + \phi_r}$$

whereas with the soft-news strategy, by (11):

$$\mu_i(\omega_1 \mid s) - \mu_i(\omega_1 \mid s') = \frac{\phi_i}{\phi_i + \phi_{r'}} - \frac{\phi_i}{\phi_i + \phi_{r''}}$$

The hard-news strategy is more informative if the following holds:

$$\frac{\phi_i + \phi_{r'}}{\phi_i + \phi_r} > \frac{\phi_{r''} - \phi_{r'}}{\phi_{r''} + \phi_i} \tag{13}$$

This condition may fail, especially if subgroup i are sceptics. □

Proof of Proposition 8

Proof. If at least one expert gathers attention exclusively from believers, then his best response is babbling. This supports the existence of an equilibrium in some cases. More details in the main text. Here, I focus on showing that this is a necessary condition. I assume that both experts gathers attention from some sceptics and some believers. By Proposition 6 each expert j uses either a hard-news strategy with target r_j or a soft-news strategy with targets $\{r_j, r'_j\}$. Consider a hard-news strategy. It follows:

$$\lambda_i(\pi_j) = \begin{cases} \mu_i^0(\omega_2) & \text{if } i \le r_j \\ \mu_i^0(\omega_1) + \frac{\mu_i^0(\omega_2)}{\mu_{r_j}^0(\omega_2)}\left[\mu_{r_j}^0(\omega_2) - \mu_{r_j}^0(\omega_1)\right] > \mu_i^0(\omega_2) & \text{if } i \in (r_j, R_2] \\ \mu_i^0(\omega_1) + \frac{\mu_i^0(\omega_2)}{\mu_{r_j}^0(\omega_2)}\left[\mu_{r_j}^0(\omega_2) - \mu_{r_j}^0(\omega_1)\right] > \mu_i^0(\omega_1) & \text{if } i > R_2 \end{cases}$$

Therefore, $\Delta_{ij} > 0 \iff i > r_j$.

Consider a soft-news strategy. It follows:

$$\lambda_i(\pi_j) = \begin{cases} \mu_i^0(\omega_2) & \text{if } i \le r_j \\ \mu_i^0(\omega_1)k + \frac{\mu_i^0(\omega_2)}{\mu_{r_j}^0(\omega_2)}\left[\mu_{r_j}^0(\omega_2) - \mu_{r_j}^0(\omega_1)k\right] > \mu_i^0(\omega_2) & \text{if } i \in (r_j, R_2] \\ \mu_i^0(\omega_1)k + \frac{\mu_i^0(\omega_2)}{\mu_{r_j'}^0(\omega_2)}\mu_{r_j'}^0(\omega_1)(1-k) > \mu_i^0(\omega_1) & \text{if } i \in (R_2, r_j') \\ \mu_i^0(\omega_1) & \text{if } i \ge r_j' \end{cases}$$

Therefore, $\Delta_{ij} > 0 \iff i \in (r_j, r_j')$.

There are three cases to analyse:

1. Both experts use hard-news strategies. It follows that each expert targets a subgroup of sceptics, and they gets zero information gain. Such sceptics can deviate, become believers of the other expert, and get a positive information gain.

2. One expert uses a soft-news strategy whereas the other uses a hard-news strategy. The sceptics targeted by the soft-news strategy can deviate, become believers of the other expert, and get a positive information gain.

3. Both experts use soft-news strategies. Let $T_{j_\alpha} = \{r_{j_\alpha}, r_{j_\alpha}'\}$ and $T_{j_\beta} = \{r_{j_\beta}, r_{j_\beta}'\}$ be the set of targets for the experts j_α and j_β respectively. I assume without loss of generality that $r_{j_\alpha} < r_{j_\beta}' \le R_2 < r_{j_\beta} < r_{j_\alpha}'$. By Proposition 2, each target experiences zero information gain. Those targets who have intermediate prior beliefs (in this case, r_{j_β}' and r_{j_β}) have incentives to deviate, in order to get a positive information gain.

□

Proof of Proposition 9

Proof. In the following, I compare the optimal strategies of an informative expert in two scenario: monopoly and partial echo chambers. The difference is that in partial echo chambers some sceptics devote attention to the other expert, who is babbling. I denote with $\hat{r}$ the least sceptical subgroup of decision-makers who in partial chambers devotes attention to the babbling expert. There are two cases to consider:

1. The expert uses a hard-news strategy both in monopoly. Let r be the target under monopoly. If $\hat{r} < r$, by Proposition 6, the subgroup with the highest value of being marginal persuaded is still r. Therefore, the

expert uses the corresponding hard-news strategy. Decision-makers of any subgroup $i \leq \hat{r}$ are indifferent about the allocation of attention, that is, get zero information gain in any case. However, because they devote attention to the babbling expert, they get lower quality information. If $\hat{r} \geq r$, then the subgroup of sceptics that is targeted must change, and the new target is $r' > r$. The new strategy could be either a hard-news strategy or a soft-news strategy. In both cases, all decision-makers have a (weakly) lower information gain and, by Lemma 5, receive information of lower quality.

2. The expert uses a soft-news strategy both in monopoly with targets $T = \{r, r'\}$. For any $\hat{r} \leq R_2$, a subgroup of believers has the highest value of being marginal persuaded. Therefore, by Proposition 6, the expert uses a soft-news strategy in partial echo chambers. If $\hat{r} < r$, the expert's payoffs do not change, thus the expert uses the same soft-news strategy. Decision-makers of any subgroup $i \leq \hat{r}$ are indifferent about the allocation of attention, but they get lower quality information. If $\hat{r} \geq r$, the new targets are $\hat{T} = \{i, i'\}$, where $i > r$. Now, if $i' \leq r'$ all decision-makers have a (weakly) lower information gain and, by Lemma 5, receive information of lower quality.

In the following, I find a sufficient condition for $i' \leq r'$. The optimal strategy in monopoly is the soft-news strategy with the highest payoff. Therefore, it is the solution of the following maximization problem:

$$\max_{\phi_r, \phi_{r'}} k\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_r\right][1 - F(\phi_r)] + (1-k)\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_{r'}\right][1 - F(\phi_{r'})]$$

subject to $k = \frac{\phi_{r'}-1}{\phi_{r'}-\phi_r}$, $\phi_r \in [0,1]$ and $\phi_{r'} \in [1,\infty)$. The F.O.C. are:

$$\Psi_{\phi_r} := \frac{\partial k}{\partial \phi_r}\Big\{\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_r\right][1 - F(\phi_r)] - \left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_{r'}\right][1 - F(\phi_{r'})]\Big\} +$$
$$+ k\mu_j^0(\omega_2)[1 - F(\phi_r)] - kf(\phi_r)\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_r\right] = 0$$
$$\Psi_{\phi_{r'}} := \frac{\partial k}{\partial \phi_{r'}}\Big\{\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_r\right][1 - F(\phi_r)] - \left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_{r'}\right][1 - F(\phi_{r'})]\Big\} +$$
$$+ (1-k)\mu_j^0(\omega_2)[1 - F(\phi_{r'})] - (1-k)f(\phi_{r'})\left[\mu_j^0(\omega_1) + \mu_j^0(\omega_2)\phi_{r'}\right] = 0$$

Because in partial echo chambers it holds $i > r$, then $i' \leq r'$ if $\frac{\partial \Psi_{\phi_{r'}}}{\partial \phi_r} \leq 0$, which is equivalent to:

$$[F(\phi_{r'}) - F(\phi_r)][\phi_j(2 - \phi_{r'} - \phi_r) + \phi_{r'} + \phi_r - \phi_{r'}\phi_r] +$$
$$+ (\phi_{r'} - \phi_r)[f(\phi_{r'})(\phi_{r'} - 1)(\phi_j + \phi_{r'}) - f(\phi_r)(1 - \phi_r)(\phi_j + \phi_r)] \leq 0$$

□

B. Appendix B

B.1. Costly Attention

The results in my paper are derived under the assumption that each decision-maker can devote attention to just one expert. Now, I endogenize this decision by allowing each decision-maker to devote attention to a second expert at a cost $c \geq 0$.

Proposition 10. *Truth-telling is an equilibrium if and only if $c = 0$.*

Assume that π_{j_α} and π_{j_β} are truth-telling strategies. It follows that $\lambda_i(\pi_{j_\alpha}) = \lambda_i(\pi_{j_\beta}) = \lambda_i(\pi_J) = 1$ for any $i \in I$. Therefore, it is sufficient to devote attention to one expert in order to maximize the subjective probability of taking the correct action. If $c = 0$, decision-makers can pay attention to both experts without any cost. This is equivalent to unlimited attention. By Lemma 3, truth-telling is indeed the equilibrium in such a setting. If $c > 0$, each decision-maker strictly prefers to devote attention to just one expert, as she gains no additional information from the second one. However, it is not optimal for the experts to reveal the true state when decision-makers pay attention to only one expert.

The equilibria of the game are robust for any $c \geq 0$. Given any equilibrium, it follows by Proposition 8 that there is no incentive to devote attention to a second expert. Multi-homing is not optimal because at least one expert is babbling. For instance, consider partial echo chambers with j_β babbling. For any $i \in H_{j_\alpha}$, it holds $\lambda_i(\pi_{j_\alpha}) = \lambda_i(\pi_J)$ because π_{j_β} does not affects posterior beliefs, hence optimal actions. For any $i \in H_{j_\beta}$ it must be the case that both experts are providing zero information gains, and $\lambda_i(\pi_{j_\alpha}) = \lambda_i(\pi_{j_\beta}) = \lambda_i(\pi_J) = \mu_i^0(\omega_m)$. Therefore, decision-makers are not willing to pay $c \geq 0$ to devote attention to a second expert.

B.2. Alternative Timing

In the main text, I assume that optimal persuasion and the allocation of attention are simultaneous. Now, I examine the possibility that the two are sequential.

If the allocation of attention is chosen *before* persuasion takes place, my results extend. Remarkably, a monopoly is a much more credible equilibrium in this case. The allocation of attention cannot react to optimal persuasion by a monopolist. Therefore, it does not matter what is the strategy of the non-active expert in the second stage of the game.

If the allocation of attention is chosen *after* persuasion takes place, babbling by both experts (with any allocation of attention) is not an equilibrium. Suppose, by contradiction, the opposite. Believers take each expert's preferred action, but any expert can deviate and persuade also his sceptics with positive probability (for instance, with the soft-news strategy). In order to do so, it is sufficient to provide a strictly positive information gain, which requires to avoid targeting sceptics.

At the same time, truth-telling is an equilibrium. If any expert deviates, he does not collect attention. Therefore, he is not able to persuade, and indifference follows. This result is in line with Knoepfle (2020). Experts are implicitly attention-seekers: persuasion is effective only if an expert gets attention in the second stage. Optimal persuasion involves targeting of some decision-makers. However, by Proposition 2 a target gets zero information gain from persuasion. Therefore, she is unlikely to devote attention in the second stage of the game.

The latter setting is in line with the literature on media bias, where consumers buy news knowing the media's reputation or slant (Gentzkow et al., 2015). In turn, the latter is influenced by the incentive to steal consumers from the rival, and this is likely to generate beneficial competition. My approach is different because I assume that persuasion is rather flexible compared to the attention habits. Experts behave strategically taking as given the allocation of attention, and this is a source of persuasion power.[27]

B.3. Competition with Homogenous Experts

With unlimited attention, having two experts with the same preferences does not affect information provision compared to a monopoly.

Proposition 11 (Homogeneous experts)**.** *Consider $J = \{j_\alpha, j_\beta\}$ and assume $a_{j_\alpha} = a_{j_\beta}$ and $\mu^0_{j_\alpha}(\omega_1) = \mu^0_{j_\beta}(\omega_1)$ for any $\omega \in \Omega$. In the equilibrium one expert (say j_α) behaves as a monopolist whereas the other one (say j_β) is babbling.*

Given babbling by j_β, j_α uses the optimal strategy as monopolist (Proposition 1). The two experts have the same preferences and the same belief. Therefore, the strategy of j_α is optimal also for j_β. There is no incentive to change the posterior beliefs by providing further information. Hence, babbling is optimal for j_β.

The entry of (potentially many) experts with the same preferences and belief as the incumbent is not affecting information provision. The intuition

[27] There exist empirical evidence that biased experts, for example politicians, respond strategically to attention habits. See for instance Eisensee and Strömberg (2007).

is that the entrant cannot refine the optimal strategy of the incumbent.[28]

With limited attention, two experts using the same strategy can be active. Indeed, each decision-maker is indifferent about her allocation of attention, as each expert provides her the same information gain.[29] This allows to extend the prediction of my model beyond a duopoly. The existence of additional experts has the effect of splitting attention, but it does not affect the equilibria of the game qualitatively.

With costly attention, a decision-maker could rationally pay attention to multiple experts providing her a positive information gain. However, multi-homing triggers a strategic response by the experts (Proposition 11). In this setting, the unique equilibrium is a monopoly.

B.4. Micro-targeting

In the paper, persuasion is public. By contrast here, I assume that decision-makers are micro-targeted: each expert uses a specific strategy for each sub-group of decision-makers. Let π_j^i be the strategy of expert $j \in J$ which targets subgroup $i \in I$. In a monopoly, π_j^i is babbling if subgroup i are believers, whereas it is the hard-news strategy if subgroup i are sceptics. This follows from Kamenica and Gentzkow (2011). With competition and single-homing, $\lambda_i(\pi_j^i) = \mu_i^0(\omega_m)$ for any $i \in I$ and any $j \in J$. In words, there cannot be a positive information gain from persuasion, for any decision-maker. This follows from Lemma 4 and Proposition 2. Therefore, decision-makers are indifferent about the allocation of attention.

An expert benefits from the possibility to target many different decision-makers. By contrast, the effect of micro-targeting on decision-makers is ambiguous: believers are always worse off, but the sceptics might benefit. For instance, assume that public persuasion is given by a soft-news strategy. With micro-targeting, each subgroup of sceptics is tailored with a specific hard-news strategy, and she could be better informed by Lemma 5.

Here, the equivalence between public and private persuasion (Kolotilin et al., 2017) fails because the expert knows the prior beliefs of each decision-maker.

[28]Experts with heterogeneous beliefs can have different optimal strategies (in monopoly). However, differently from Lemma 3, there is no incentive to undercut the rival because the preferred actions coincide.

[29]If the experts use different strategies, then decision-makers have incentive to devote attention to the most informative one.

B.5. Many States

In this section, I examine how my model can be extended allowing for more than two states of the world.

A first approach is to consider a continuous state space i.e. $\Omega := [0,1]$ while keeping the action binary i.e. $A := \{a_0, a_1\}$. Here, I adopt a setting similar to Guo and Shmaya (2019). Each agent $l \in I \cup J$ has distinct prior beliefs with full support: $\mu_l^0(\cdot) \in \Delta_+(\Omega)$, where $\mu_l^0(\omega)$ is agent l's belief that the state is ω. Following Bayesian updating, posterior beliefs are:

$$\mu_i(\omega \,|\, s) := \frac{\pi_{j_i}(s \,|\, \omega)\mu_i^0(\omega)}{\int_0^1 \pi_j(s \,|\, \omega')\mu_i^0(\omega')d\omega'}$$

I assume that each decision-maker follows a threshold rule: she wants to take action a_1 if and only if the state ω is above a threshold $\bar{\omega}$. It follows that the optimal action for each decision-maker of subgroup i becomes:

$$\sigma(\mu_i) = \begin{cases} a_1 & \text{if } \int_{\bar{\omega}}^1 \mu_i(\omega)d\omega \geq \frac{1}{2} \\ a_2 & \quad\text{otherwise} \end{cases}$$

Upon receiving message s, the implied persuasion constraint is

$$\int_{\bar{\omega}}^1 \pi_j(s \,|\, \omega)\mu_i^0(\omega)d\omega \geq \int_0^{\bar{\omega}} \pi_j(s \,|\, \omega)\mu_i^0(\omega)d\omega$$

In such a setting, I keep the restriction of two subgroups of decision-makers, believers ($i = 1$) and sceptics ($i = 2$). A believer is such that $\int_{\bar{\omega}}^1 \mu_1^0(\omega)d\omega > \frac{1}{2}$, whereas a sceptic is such that $\int_{\bar{\omega}}^1 \mu_2^0(\omega)d\omega < \frac{1}{2}$. As in the baseline model, the optimal strategy focuses either on persuading sceptics or on retaining believers. However, the structure of the optimal strategy changes.

If the focus is to persuade sceptics (hard-news strategy), then a candidate optimal strategy must satisfy the following constraint:

$$\int_{\bar{\omega}}^1 \mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \pi(s \,|\, \omega)\mu_2^0(\omega)d\omega \tag{14}$$

I denote with Π_H the subset of strategies such that (14) holds. Note that in the baseline model Π_H is singleton, whereas here the expert has degrees of freedom on the distribution of probability for each state $\omega \in [0, \bar{\omega}]$. By (3), the incentive of the expert is to pool states with high $\mu_j^0(\omega)$, while fully revealing others.

If the focus is to retain believers (soft-news strategy), then a candidate optimal strategy must satisfy the following constraints:

$$\int_{\bar{\omega}}^1 \pi(s \,|\, \omega)\mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \pi(s \,|\, \omega)\mu_2^0(\omega)d\omega \tag{15}$$

$$\int_{\bar{\omega}}^1 \pi(s' \,|\, \omega)\mu_1^0(\omega)d\omega = \int_0^{\bar{\omega}} \pi(s' \,|\, \omega)\mu_1^0(\omega)d\omega \tag{16}$$

I denote with Π_S the subset of strategies such that (15)-(16) hold, and note that in the baseline model Π_S is singleton. In this case, the goal of the expert is to maximize the probability of persuading sceptics subject to the constraint that believers chooses the preferred action with probability one. The incentives of the expert are difficult to disentangle, as these depend on $\mu_j^0(\omega)$, $\mu_1^0(\omega)$ and $\mu_2^0(\omega)$.

However, even if the structure of the optimal strategy changes, my results are not affected. In particular, Proposition 2 generalizes to this setting. Note that

$$\int_0^{\bar{\omega}} \mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \pi(s|\omega)\mu_2^0(\omega)d\omega + \int_0^{\bar{\omega}} \pi(s'|\omega)\mu_2^0(\omega)d\omega$$

which implies

$$\int_0^{\bar{\omega}} \pi(s'|\omega)\mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \mu_2^0(\omega)d\omega - \int_0^{\bar{\omega}} \pi(s|\omega)\mu_2^0(\omega)d\omega$$

It follows that sceptics get zero information gain. By (15),

$$\lambda_2(\pi) = \int_{\bar{\omega}}^1 \pi(s|\omega)\mu_2^0(\omega)d\omega + \int_0^{\bar{\omega}} \pi(s'|\omega)\mu_2^0(\omega)d\omega = \int_0^{\bar{\omega}} \mu_2^0(\omega)d\omega$$

Hence, $\Delta_2 = 0$. Proposition 2 characterizes the incentives of decision-makers about the allocation of attention. Therefore, the effect of competition with limited attention is unchanged.

The analysis of optimal persuasion becomes generally intractable when the cardinality of Ω is equal to the cardinality of A.[30] I define $\phi_i(\omega,\omega') := \frac{\mu_i^0(\omega)}{\mu_i^0(\omega')}$ for any $\omega,\omega' \in \Omega$. A message s persuades decision-makers of subgroup i that the state is ω if $\pi(s|\omega') \leq \phi_i(\omega,\omega')\pi(s|\omega)$ for any $\omega' \in \Omega$. Decision-makers of subgroup i are true believers (sceptics) of state ω if $\phi_i(\omega,\omega') \geq 1$ (< 1) for any $\omega' \in \Omega$. A hard-news strategy can target true sceptics. A soft-news strategy can solve the trade-off between persuading true sceptics and retaining true believers. Therefore, if an expert faces only true sceptics and true believers, the result of Proposition 6 extends. However, different strategies could be optimal if there exist decision-makers who believe that some states are a priori more plausible than ω, whereas others are not.

Example - I consider the COVID-19 vaccination example, and I assume that there exists a third state of the world: safe but with caution (simply caution now on). Therefore $\Omega = \{\omega_1,\omega_2,\omega_3\} = \{$caution, safe, not safe$\}$. I

[30] A full characterization of prior beliefs requires $|\Omega|!$ decision-makers. Unlike Section 6.2, there is no intuitive ordering of decision-makers. Optimal persuasion cannot be studied generically without restrictive assumptions on the distribution of beliefs.

assume that the monopolistic expert (say a politician) is biased towards caution. For instance, the politician might want to vaccinate only the elderly.

There are two subgroups of decision-makers as before: believers and sceptics, respectively, about the vaccine being safe. I assume $\phi_1(\omega_1, \omega_3) > 1 > \phi_1(\omega_1, \omega_2)$ and $\phi_2(\omega_1, \omega_2) > 1 > \phi_2(\omega_1, \omega_3)$. A soft-news strategy is not useful because there are not true believers. Let π_h be a hard-news strategy:

$$\pi_h(s|\omega_1) = 1 \quad \pi_h(s'|\omega_1) = 0$$

$$\pi_h(s|\omega_2) = \phi_1(\omega_1, \omega_2) \quad \pi_h(s'|\omega_2) = 1 - \phi_1(\omega_1, \omega_2)$$
$$\pi_h(s|\omega_3) = \phi_2(\omega_1, \omega_3) \quad \pi_h(s'|\omega_3) = 1 - \phi_2(\omega_1, \omega_3)$$

Let us consider as alternative π_s:

$$\pi_s(s|\omega_1) = k \quad \pi_s(s'|\omega_1) = 1 - k$$

$$\pi_s(s|\omega_2) = \phi_1(\omega_1, \omega_2)k \quad \pi_s(s'|\omega_2) \leq \phi_2(\omega_1, \omega_2)(1 - k)$$
$$\pi_s(s|\omega_3) = \phi_2(\omega_1, \omega_3)(1 - k) \quad \pi_s(s'|\omega_3) \leq \phi_1(\omega_1, \omega_3)k$$

The favourable state of the politician is caution, that is a compromise between opposite decision-makers' beliefs. If decision-makers have sufficiently polarized beliefs (and the politician is sufficiently uncertain about the true state), then it is optimal to use π_s. The intuition is similar to Proposition 1. With π_s, the politician randomizes between messages that either support one extreme state or the other. In other words, in order to persuade citizens that the best option is to take caution, a politician alternates positive and negative news about vaccinations. These news are not designed to move one group from one extreme to the other, but just from one extreme to a compromise. The alternative is to provide "hard evidence" that vaccinations are safe given precautions. This is extremely costly with high polarization, as both extreme views have to be contrasted at the same time. Note that π_s is not a soft-news strategy, but it works similarly: the goal is to leverage believers' credulity.

The intractability of optimal persuasion does not allow to study the whole game. However, intuitively my results should not be affected by the existence of many states of the world and corresponding actions. For instance, let us consider Proposition 3. True believers clustering into echo chambers is an equilibrium. Indeed, no information is provided and hence the decision-makers do not have incentives to deviate. Decision-makers are better informed with a monopoly, because the existence of heterogeneous beliefs makes optimal for the expert to use some informative strategy, where informativeness is defined following Blackwell (1953).

B.6. Biased Decision-makers

In the paper, decision-makers are unbiased in their utilities. All the results are driven exclusively by heterogeneous prior beliefs. Now, I show that the same results can be obtained in a setting where decision-makers share a common prior belief $\mu^0(\omega_1)$, but each subgroup of decision-makers i is endowed with a vector of biases $b_i := \{b_i^\omega\}_{\omega\in\Omega}$. The utility of a decision-maker of subgroup i is $u_i(a,\omega_k) := \mathbb{1}\{a = a_k\}b_i^\omega$. See (1) for a comparison. The corresponding optimal action is as follows:

$$\sigma(\mu, b_i) = \begin{cases} a_1 & \text{if } \mu(\omega_1) \geq \frac{b_i^{\omega_2}}{b_i^{\omega_1}+b_i^{\omega_2}} \\ a_2 & \text{otherwise} \end{cases}$$

Upon observing message s, action a_1 is chosen if and only if:

$$\mu(\omega_1\,|\,s) \geq \frac{b_i^{\omega_2}}{b_i^{\omega_1} + b_i^{\omega_2}} \iff \pi_j(s\,|\,\omega_2) \leq \frac{\mu^0(\omega_1)}{\mu^0(\omega_2)}\frac{b_i^{\omega_1}}{b_i^{\omega_2}}\pi_j(s\,|\,\omega_1) \tag{17}$$

A model with unbiased decision-makers and heterogeneous beliefs is equivalent to a model with biased decision-makers and a common belief only if, for any $i \in I$ and any $\omega \in \Omega$, $b_i^\omega = \frac{\mu_i^0(\omega)}{\mu^0(\omega)}$. This follows immediately from the comparison of conditions (2) and (17). Note that $b_i^\omega > 1$ if and only if $\mu_i^0(\omega) > \mu^0(\omega)$. Hence, a larger bias is equivalent to a decision-maker having a higher prior belief that the state ω is the true state. Remarkably, this multiplicative bias is different from the common definition of bias. In the literature, the utility of biased decision-makers depends on the action, but not on the state. By contrast here, each decision-maker has a strict preference to take the correct action given the state. The bias is limited to each decision-maker valuing some states more than others ex ante.

Hu et al. (2021) consider a model where decision-makers have different default actions. Given a common belief, each decision-maker would take her default action. Decision-makers of subgroup i are characterized by a specific threshold $c_i \in [0,1]$ for the posterior belief which makes them indifferent:

$$\sigma(\mu, c_i) = \begin{cases} a_1 & \text{if } \mu(\omega_1) \geq c_i \\ a_2 & \text{otherwise} \end{cases}$$

Thus, the models are equivalent if $c_i = \frac{b_i^{\omega_2}}{b_i^{\omega_1}+b_i^{\omega_2}}$.