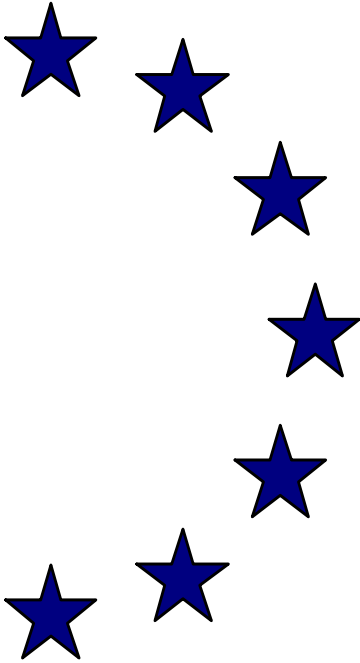


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**An estimated new keynesian dynamic stochastic
general equilibrium model of the Euro area**

by

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1 Introduction

In recent years a new consensus has emerged in macroeconomics in general and in model building in particular, the so called New Keynesian Paradigm (NKM). This paradigm is well established as can be seen from the prominent treatment in recent textbooks (see, for example Obstfeld/Rogoff) and literature surveys (see for example Clarida, Gali Gertler). In a sense the NKM paradigm combines elements from the RBC literature with more traditional Keynesian ideas. Traditional Keynesian models suffered from underdeveloped microfoundations and a lack of long term factors influencing the economy which made them subject to the Lucas critique. It also did not have a coherent theoretical explanation for the sluggish behaviour of prices it assumed. On the other hand, RBC modellers built their models up from the actions of optimising economic agents whose choices are made within specified constraints. The New Classical view of RBC modellers saw business cycles as largely the result of shocks to productivity and preferences, and downturns as merely the optimal adjustment of the economy to such disturbances. NKM models correct the RBC models by introducing frictions in goods, labour and financial markets in order to provide a better fit with actual data, but at the same time tries to model the frictions explicitly as constraints faced by households and firms. This allows combining optimal behaviour with rigidities in a way which avoids the Lucas critique.

The QUEST model has been set up in the spirit of a NKM model, with a strong emphasis on theoretical consistency of the behavioural equations. However at the time when QUEST II was introduced the estimation technology for DSGE models was not sufficiently developed to allow for rigorous estimation and testing of these models. Large parts of these models needed to be calibrated. Following recent developments in Bayesian estimation techniques (see, e.g., Geweke 1999 and Schorfheide 2000), it has become possible to estimate these type of models. Smets and Wouters (2003) have been the first to estimate such a model for the Euro area. They followed Cristiano, Eichenbaum, and Evans (2001) and designed a DSGE model for the Euro area featuring price and wage stickiness, partial indexation of prices and salaries, external capital formation, variable capital utilisation rate and stochastic shocks to each structural equation of the resulting model. Smets and Wouters (2003) show that the current generation of New-Keynesian DSGE models is sufficiently rich to capture the time-series properties of the data, as long as a sufficient number of structural shocks is considered. In particular, it is able to match the degree of empirical persistence found in the euro area data for inflation and wages quite well.

This paper applies Bayesian estimation techniques to a time series data set of the euro area and presents estimates of a DSGE model. The purpose of this paper is not to estimate the current version of the QUEST model directly with these methods but rather to estimate a prototype new generation New-Keynesian DSGE model. This model can then serve as a benchmark for an estimation of a QUEST specification. In fact in some dimensions the QUEST model may need to be adjusted to come closer to a DSGE model.

One of the common features between the QUEST II model and the estimated New-Keynesian DSGE model presented in this paper are that both, in the long run, closely resemble the standard neoclassical growth model. All behavioural relations are derived from dynamic optimisation problems of households and firms, with optimisation subject to technological constraints, budget constraints and/or institutional constraints, often captured as adjustment

costs. This leads to a description of economic behaviour that is a mixture of backward and forward looking behaviour. The main differences with the QUEST II model are in the specification of consumption. While consumption in the QUEST model is based on a permanent income model for finitely-lived households, as popularised by Blanchard (1984), in this DSGE model in contrast, consumption is derived from intertemporal optimisation for infinitely lived households, like in all other current DSGE models. However, consumption is modelled more backward looking by allowing for habit persistence (i.e. consumption decisions today depend partially on the previous pattern of consumption). On the other hand, QUEST allowed for liquidity constraints, a feature still missing in the consumption framework here. The differences in the investment specification are only of minor importance, with a stronger emphasis on adjustment costs here. The modelling of the labour market is substantially different. Here, like in other DSGE models, this is based on a neoclassical labour supply with monopoly power for workers. One of the distinguishing features of the QUEST model is the labour market specification derived from a theoretical search model based on the work by Pissarides (1990).

The estimated model as presented here is still incomplete since it treats the Euro area as a closed economy. The closed economy setting was chosen because we first wanted to concentrate on the main aggregates consumption and investment as well as on prices and wages and their interactions. However, adding a trade sector would be among our first priorities for further extensions of this model. The model will then include a more explicit modelling of trade frictions within the framework of convex adjustment costs, which would distinguish it from the QUEST model, where trade is modelled through an ad-hoc specification of adjustment lags in quantities and prices.

An important reason for estimating a DSGE model was also to be able to compare estimation results with the existing literature and to make sure that the estimation yields results which are consistent with the results obtained with similar specifications and similar datasets.

The main goals of this exercise are:

- 1) Demonstrate that models derived from economic theory can fit Euro area data, provided one allows for sufficient institutional restrictions. We compare the predictive performance of the estimated DSGE model with that of a VAR model estimated over the same euro area data set. (We intend to conduct a similar exercise for the US economy to see whether institutional constraints play the same role there).
- 2) Identify the main structural shocks hitting the Euro area economy in a theoretically consistent way. An advantage of an explicit structural model is the fact that residuals can be given a structural interpretation, *i.e.* we can identify shocks which originate from consumption, technology, labour supply, labour demand, investment and fiscal and monetary policy. This may be of added value in trying to understand the nature of the current economic situation. For example, the model identifies a declining trend in government spending, which is reversed in recent years, a decline in price mark-ups, reflecting increased competitive pressures, a trend increase in total factor productivity in the 1980s, followed by a decline in the late 1990s and a trend increase in labour supply, reflecting a declining NAIRU.
- 3) Provide the typical response of the economy to the individual shocks in the form of impulse responses, like in VAR studies, with the additional benefit that confidence intervals can be provided to show the uncertainty surrounding these responses.

The outline of the paper is as follows. Section 2 presents the model. In section 3 discusses the estimation methodology. Since estimation of these models is non standard a fairly comprehensive explanation will be provided. Section 4 presents the estimation results. In order to provide a more intuitive understanding for the quality of the fit of this model, a comparison with a simple VAR model is given. Section 5 presents and interprets the structural shocks identified by the model estimates and section 6 describes the dynamic adjustment of the euro area economy to structural shocks.

2 The DSGE Model

Households:

The household sector decides about consumption and asset accumulation (including fixed capital). Each household supplies a specific variety of labour in a monopolistically competitive fashion, i.e. the household sector sets the wage given the demand curve for labour. When making decisions the household also faces adjustment costs for changing wages. These adjustment costs are borne by the household (see budget constraint). The household maximises a utility function subject to a budget constraint. The Lagrangian of this maximisation problem is given by

(1)

$$\begin{aligned} \text{Max } U_0 = & \sum_{t=0}^{\infty} \beta^t \left(U(C_t^i) + V(1 - L_t^i) + Z(M_t^i / P_t) \right) \\ & - \sum \lambda_t \beta^t \left(\frac{PC_t}{P_t} C_t^i + \frac{M_t^i}{P_t} + \frac{B_t^i}{P_t R_t} + \frac{V_t^i}{P_t R_t} - \frac{W_t^i}{P_t} L_t^i + \frac{\gamma_w L_t^i}{2} \left(\frac{\Delta w_t^i}{w_t} \right)^2 - \frac{M_{t-1}^i}{P_t} - \frac{B_{t-1}^i}{P_t} - \frac{V_{t-1}^i}{P_t} + TAX \right) \end{aligned}$$

The household maximises a utility function over consumption, leisure and real money balances. Following the recent literature we allow for habit persistence in consumption. This is an important modification w. r. t. the current consumption specification in QUEST which was based entirely on a pure life cycle model. The current version allows for lagged adjustment of consumption and we choose a logarithmic specification

$$(2a) \quad U(C_t^i) = \varepsilon_t^C \log(C_t^i - hab C_{t-1}^i)$$

where ε_t^C denotes a stochastic preference shock for consumption in period t. This specification yields the following expression for the marginal utility of consumption

$$(2b) \quad U_{C,t}^i = \varepsilon_t^C \frac{1}{(C_t^i - hab C_{t-1}^i)}$$

The consumption index is itself an aggregate over different goods which are imperfect substitutes. The preferences of households are expressed by a CES utility function

$$(2c) \quad C_t^i = \left[\int_0^1 C_t^j{}^{\frac{1-\tau_t}{\tau_t}} dj \right]^{\frac{\tau_t}{1-\tau_t}} \text{ with } \tau_t \geq 0$$

where τ_t measures the inverse of the time varying elasticity of demand of households for consumption goods of type j . The term τ_t is given by

$$(2d) \quad \tau_t = \tau^0 + \tau^1(Y_t - Ypot_t) + \varepsilon_t^\tau$$

where ε_t^τ is a autocorrelated shock to the demand elasticity.

For labour supply we use a CES utility function

$$(2e) \quad V(1-L_t^i) = \varepsilon_t^L \frac{\omega}{1-\kappa} (1-L_t^i)^{1-\kappa} \quad \text{with } \kappa > 0,$$

where ε_t^L is a possibly autocorrelated labour supply shock. The marginal utility of leisure is given by

$$(2f) \quad V_{L,t}^i = -\varepsilon_t^L \omega (1-L_t^i)^{-\kappa}.$$

The household decides about consumption, asset accumulation and the supply of labour (or more correctly about wages) and real money holdings¹. The first order conditions of the household (FOCs) with respect to consumption and financial wealth are given by the following equations:

$$(3a) \quad \frac{\partial U_0}{\partial C_t^i} \Rightarrow U_{C,t}^i - \lambda_t \frac{PC_t}{P_t} = 0$$

$$(3b) \quad \frac{\partial U_0}{\partial B_t^i} \Rightarrow -\lambda_t \frac{1}{P_t R_t} + \lambda_{t+1} \beta \frac{1}{P_{t+1}} = 0$$

$$(3c) \quad \frac{\partial U_0}{\partial M} \Rightarrow \frac{M_t^i}{P_t} - Y_t^i R_t^\zeta = 0$$

The labour supply decision is slightly more complex, since it is assumed that workers have a certain market power in the labour market, because they offer services, which are imperfect substitutes to services offered by other workers. That means aggregate labour demand of firms is a composite of labour supplied by individual workers. Total employment in production is characterised by a CES function

¹ With an interest rate rule as specified below, an optimality condition for money would only determine the desired money holdings of the household sector without any further consequence for the rest of the economy. For that reason any further discussion on money demand is dropped here.

$$(4a) \quad L_t^i = \left[\int_0^1 L_t^i \frac{\theta-1}{\theta} di \right]^{\frac{\theta}{\theta-1}} \text{ with } \theta > 1$$

where the parameter θ determines the degree of substitutability between labour supplied by individual households. Corresponding to the CES aggregator there exists a wage index

$$(4b) \quad W_t = \left[\int_0^1 W_t^i \frac{1}{\theta} di \right]^{\frac{\theta}{\theta-1}}$$

This technology yields a labour demand equation as perceived by household i

$$(4c) \quad L_t^i = \left(\frac{W_t^i}{W_t} \right)^{-\theta} L_t$$

In a monopolistic labour market the elasticity of substitution between different types of labour is important for determining the mark-up of wages over the equilibrium wage. This elasticity is defined by

$$(4d) \quad \frac{\partial L_t^i}{\partial W_t^i} = -\theta \left(\frac{W_t^i}{W_t} \right)^{-\theta} L_t \frac{1}{W_t^i} = -\theta \frac{L_t^i}{W_t^i}.$$

Now the wage setting rule can be derived taking derivatives of the Lagrangian w.r.t. wages. Using symmetry: $W_t^i = W_t$ and neglecting second order terms allows us to write

$$(5a) \quad \frac{\partial U_0}{\partial W_t^i} \Rightarrow V_L = \lambda_t \left[\frac{(1-\theta) W_t}{-\theta P_t} - \gamma_w \pi_t^w \right] + \lambda_{t+1} \beta \gamma_w \pi_{t+1}^w,$$

where π_t^w is the growth rate of nominal wages. This can be reformulated as a wage setting rule

$$(5b) \quad \pi_t^w = \frac{1}{\gamma_w} \left[\frac{C_t}{\omega(1-L)^\kappa} - (1 - mup^w) \frac{W_t}{P_t} \right] + \frac{1}{R_t} \pi_{t+1}^w \quad \text{with } mup^w = -\frac{1}{\theta}$$

where wage inflation is determined by the gap between the reservation wage and the real wage adjusted for a wage mark up. The forward looking nature of wage setting is reflected by the forward wage inflation term. This formulation generalises the neoclassical labour supply model along two dimensions. First, by introducing convex wage adjustment costs ($\gamma_w > 0$), workers want to smooth wage adjustments, taking into account current and future expected labour market conditions. Second, because workers offer services which are imperfect substitutes to services offered by other workers, they can demand wages which are above their reservation wage². The reservation wage is the marginal value of leisure, divided by the

² Notice in the limiting case of perfect substitutability ($\lim \theta \rightarrow \infty$), the mark up approaches zero.

marginal utility of consumption. That means for a given utility of leisure the reservation wage increases with a decline in the marginal utility of consumption that an additional unit of labour can buy. In estimating the wage rule two further generalisations have been introduced. Some search theoretic generalisations of the neoclassical wage rule suggest rules where wages are indexed to both the reservation wage and the marginal value product of labour with weight bg reflecting the bargaining strength of workers (see, for example, Shi et al. (1999)). In order to allow for backward looking behaviour it is assumed that only a fraction sfw of workers form rational expectations of future wages, while the remaining workers follow a simple rule of thumb where expectations are determined by past inflation. These two modifications lead to the following wage equation

$$(5c) \quad \pi_t^w = \frac{1}{\gamma_w} \left[(1-bg) \frac{C_t}{\omega(1-L)^{\kappa}} + bg\alpha \frac{Y_t}{L_t} \eta_t - (1-mup^w) \frac{W_t}{P_t} \right] + \frac{1}{R_t} (sfw\pi_{t+1}^w + (1-sfw)\pi_{t-1}^w)$$

$$0 \leq sfw \leq 1$$

Firms:

There are N firms indexed by j . Because goods produced by individual firms are imperfect substitutes, firms are monopolistically competitive in the goods market and face a demand function for goods given by

$$(6) \quad Y_t^j = s^j \left(\frac{P_t^j}{P_t} \right)^{-\frac{1}{\epsilon_t}} (C_t + G_t + I_t)$$

Output is produced with a Cobb Douglas production function

$$(7) \quad Y_t^j = (ucap_t^j K_t^j)^{1-\alpha} (U_t L_t^j)^\alpha$$

with capital and labour as inputs. Firms can also decide about the degree of capacity utilisation. The level of technology is subject to random technology shocks (ε_t^U) and follows the autoregressive process

$$(8) \quad \log(U_t) = \rho^U \log(U_{t-1}) + (1-\rho^U) \log(U) + \varepsilon_t^U.$$

The objective of the firm is to maximise the present discounted value of its cash flows. Dynamic considerations enter the problem of the firm because firm faces quadratic costs of changing capital, employment and prices. Finally firms must also choose the optimal level of capacity utilisation.

$$(9)$$

$$\begin{aligned}
Max V_0 = & \sum_{t=0}^{\infty} d^t \left[P_t^j (.) Y_t^j - W_t L_t^j - P_t I_t^j - adc^P (P_t^j) - adj^K (K_t^j, I_t^j) - adj^L (L_t^j) - adj^{CAP} (ucap_t^j) \right] \\
& - \sum \eta_t d^t \left[Y_t^j - (ucap_t^j K_t^j)^{1-\alpha} (L_t^j U_t^j)^\alpha \right] \\
& - \sum q_t d^t \left[K_t^j - I_t^j - (1-\delta)K_{t-1}^j \right]
\end{aligned}$$

where $d^t = \prod_{i=0}^t \left(\frac{1}{1+r_i+rp_i} \right)$ is the discount factor, which consists of the short term interest rate and a risk premium (rp). The risk premium can be subject to random shocks and generated by the following autoregressive process

$$(10) \quad rp_t = \rho^{rp} rp_{t-1} + (1-\rho^{rp})rp_{t-1} + \varepsilon_t^{rp}$$

For adjustment costs we choose the following convex functional forms

(11)

$$adj^L (L_t^j) = \varepsilon_t^W \frac{W_t}{P_t} L_t^j + \frac{\gamma_L}{2} \Delta L_t^j{}^2$$

$$adj^P (P_t^j) = \frac{\gamma_P}{2} \Delta \pi_t^j{}^2, \text{ with } \pi_t^j = P_t^j / P_{t-1}^j - 1$$

$$adj^K (K_t^j, I_t^j) = \frac{(\gamma_K + \varepsilon_t^I)}{2} \frac{I_t^j{}^2}{K_{t-1}^j} + \frac{\gamma_I}{2} \Delta I_t^j{}^2$$

$$adj^{CAP} (ucap_t^j) = a_1 (ucap_t^j - ucap^*) + a_2 (ucap_t^j - ucap^*)^2$$

The firm determines labour input, the capital stock, capacity utilisation and prices optimally in each period given the technological and administrative constraints as well as demand conditions. The first order conditions are given by:

$$(12a) \quad \frac{\partial V_0}{\partial L_t^j} \Rightarrow \left(\alpha \frac{Y_t^j}{L_t^j} \eta_t^j + \frac{\gamma_L}{R_t} (L_{t+1}^j - L_t^j) - \gamma_L (L_t^j - L_{t-1}^j) \right) = \frac{W_t}{P_t^j} (1 + \varepsilon_t^W)$$

$$(12b) \quad \frac{\partial V_0}{\partial I_t^j} \Rightarrow (\gamma_K + \varepsilon_t^I) \frac{I_t^j}{K_{t-1}^j} + \gamma_I (\Delta I_t^j - \frac{1}{R} \Delta I_{t+1}^j) = q_t - 1$$

(12c)

$$\frac{\partial V_0}{\partial K_t^j} \Rightarrow (1-\alpha) \frac{Y_t^j}{K_t^j} \eta_t^j - ((a_2 - a_1) + (a_1 - 2a_2)ucap_t^j + a_2 ucap_t^j{}^2) = q_t - (1-r_t - rp_t - \delta)q_{t+1}$$

$$(12d) \quad \frac{\partial V_0}{\partial ucap_t^j} \Rightarrow ((a_1 - 2a_2) + 2a_1 (ucap_t^j)) = (1-\alpha) \frac{Y_t^j}{K_t^j ucap_t^j} \eta_t^j$$

$$(12e) \quad \frac{\partial V_0}{\partial Y_t^j} \Rightarrow \eta_t^j = 1 - (\tau^0 + \tau^1(Y_t - Ypot_t) + \varepsilon_t^\tau) + \gamma_p [\beta \pi_{t+1}^j - \pi_t^j]$$

Firms equate the marginal product of labour, net of adjustment costs, to wage costs. Wage costs include a stochastic wage cost shock. This should be seen as shocks to administrative burdens related to current employment. As can be seen from the left hand side of equation (12a), the convex part of the adjustment cost function penalises in cost terms accelerations and decelerations of changes in employment. Equations (12b-d) jointly determine the optimal capital stock and optimal capacity utilisation. The firm equates the marginal product of capital to the rental price of capital, adjusted for capital costs. The firm also equates the marginal product of capital services (K^*ucap) to the marginal cost of capacity utilisation. Equation (12e) defines the mark up factor as a function of the elasticity of substitution and changes in inflation. We follow Smets and Wouters and allow for additional backward looking elements by assuming that a fraction ($1-sfp$) of firms keep prices fixed at the t-1 level. This leads to the following specification:

$$(12e') \quad \eta_t^j = 1 - (\tau^0 + \tau^1(Y_t - Ypot_t) + \varepsilon_t^\tau) + \gamma_p [\beta(sfp * \pi_{t+1}^j + (1-sfp)\pi_{t-1}^j) - \pi_t^j] \quad 0 \leq sfp \leq 1$$

Government sector:

The government sector and fiscal policy is treated in a rather rudimentary fashion. The share of government purchases

$$(13a) \quad G_t / Y_t = gs_t + \varepsilon_t^G$$

fluctuates systematically with the business cycle according to the following rule

$$(13b) \quad gs_t = t_g^Y (Y_t - Ypot_t)$$

where t_g^Y measures the degree of automatic stabilisation of government expenditure. Discretionary fiscal action is characterised by the variable ε_t^G which is allowed to be autocorrelated process. Implicitly it is assumed that government expenditure is financed, by lump sum taxes.

Central bank policy rule (interest rate rule):

Monetary policy is modelled via the following Taylor rule, which allows for some smoothness of the interest rate response to the inflation and output gap

$$(14a) \quad inom_t = ilag * inom_{t-1} + (1-ilag) * (Ex.R + \pi_t^T + t_M^\pi (\pi_{t-1} - \pi_t^T) + t_M^Y (Y_t - Ypot_t) + t_M^{\Delta\pi} (\pi_t - \pi_{t-1}) + t_M^{\Delta Y} (Y_t - Ypot_t - (Y_{t-1} - Ypot_{t-1}))) + \varepsilon_t^M$$

The term ε_t^M captures random discretionary shocks to monetary policy and π_t^T is a time varying inflation target, specified as follows

$$(14b) \quad \pi_t^T = \rho^{\pi^T} \pi_{t-1}^T + (1 - \rho^{\pi^T}) \pi^T + \varepsilon_t^{\pi^T} .$$

$\varepsilon_t^{\pi^T}$ is an i.i.d. shock to the inflation target. It is assumed that both fiscal and monetary authorities base their policies on a concept of potential output which is a smooth function of past output

$$(15) \quad Ypot_t = \rho_{ypot} Ypot_{t-1} + (1 - \rho_{ypot}) Y_t$$

3 Estimation Methodology

We present the first attempt to apply a Bayesian estimation approach to bring the model directly to the data. This approach has been discussed by many authors in the literature in the last few years (e.g. Schorfheide 2000, Lubik and Schorfheide, 2003, Smets and Wouters, 2003). Schematically, the method consists of the following steps:

- the non-linear DSGE model is solved via a linear approximation: a linear rational expectation system is obtained that must be obtaining a ‘standard’ linear model in state space form;
- the state-space approximation of the original non-linear model allows the identification of a likelihood function (via e.g. Kalman recursions) and a subsequent inference based on it (maximum likelihood estimation, etc.);
- usually theoretical model imply few, well defined shocks; unfortunately this often implies singularities in the determination of the likelihood (in the Kalman filter the number of shocks must at least be as large as the number of observables), implying the introduction of additional structural shocks and/or measurement errors;
- likelihood-based inference presents a series of issues: specifically the lack of identification (global: multiple maxima; local: over-parameterisation, i.e. the maximum is given by a complex multidimensional combination/interaction structure rather than by a single point in the parameter space);
- the Bayesian analysis is performed: prior distributions for model parameters have to be defined, representing the prior beliefs of the analyst on their plausible values, which, in combination with the likelihood function, allows to obtain the posterior distribution;
- the Bayesian inference needs the use of stochastic simulations, specifically Markov Chain Monte Carlo (MCMC) techniques, allowing to obtain samples from the posterior joint pdf of the model parameters and subsequently to make an inference in which the parameter uncertainty and the shape of the likelihood are taken into account;
- the model is finally compared to an empirical model; in the literature this is usually a VAR model.

From the computational point of view, the linear approximation and the solution of the obtained LRE can be done automatically using the DYNARE program (Juillard, 1996, 2003), which applies the generalised Schur decomposition solution method (Klein, 2000). DYNARE is a software for the simulation of DSGE models, freely available and totally open source. Presently, an estimation module is implemented on DYNARE, to include the most recent developments in Bayesian estimation macro-economic models in an extremely efficient and easy way. DYNARE is also extremely flexible, and allows to easily incorporating problem specific methodological issues or customisations.

3.1 Solving the model with linear approximations

Let a model be defined and first order conditions identified. This can be expressed as:

$$(16) \quad \begin{aligned} E_t \{f(y_{t+1}, y_t, y_{t-1}, \varepsilon_t; \theta)\} &= 0 \\ E\{\varepsilon_t\} &= 0 \\ E\{\varepsilon_t \varepsilon_t'\} &= \Sigma \end{aligned}$$

where y is vector of endogenous variables, ε is the vector of exogenous stochastic shocks, θ is the vector of parameters and E is the expectation operator.

The non-linear model is solved via a linear approximation around the deterministic steady state \bar{y} such that $f(\bar{y}, \bar{y}, \bar{y}, 0; \theta) = 0$. A linear rational expectation (LRE) system is obtained, with forward looking components

$$(17) \quad A^+ E_t \hat{y}_{t+1} + A^0 \hat{y}_t + A^- \hat{y}_{t-1} + B \varepsilon_t = 0, \text{ where } \hat{y}_t = y_t - \bar{y}$$

The system is solved for the reduced form state equation in its predetermined variables (Blanchard and Kahn, 1980; generalised Schur form, Klein, 2000). An observation equation is also added to link the observed variables y_t^* to the predetermined ones, obtaining:

$$(18) \quad \begin{aligned} y_t^* &= M(\bar{y}(\theta) + \hat{y}_t) + \eta_t \\ \hat{y}_t &= G(\theta) \hat{y}_{t-1} + H(\theta) \varepsilon_t \\ E(\eta_t \eta_t') &= V(\theta) \\ E(\varepsilon_t \varepsilon_t') &= Q(\theta) \end{aligned}$$

where η_t is the measurement error, if any. The system matrices G , H , V and Q and the steady state vector $\bar{y}(\theta)$ are functions of the vector of structural parameters θ of the original model. Vector θ includes the noise parameters Σ . The state space representation (18) allows use of Kalman filtering for the computation of the log-likelihood, and a subsequent inference based on it (maximum likelihood estimation, etc.).

In the original model specification (16), well-defined (and relatively few) shocks are usually present. If the number of shocks is smaller than the number of observed variables, singularities in the Kalman filter will be present, i.e. the probability distribution of the observables $p(Y^T | \theta)$ (the likelihood) can be degenerate. This implies the introduction of additional shocks until the system becomes non-singular, including either measurement errors η_t (as e.g. in Ireland, 2004, who also models the measurement error as a VAR(1) process) or additional structural shocks in the state equation (as e.g. in Smets and Wouters, 2003). Rigorously, in such cases, as clearly stated by Schorfheide (2000), the evaluation approach here applied will “lead to an assessment of the modified model rather than the original one”. In such cases, the parameter vector θ will be augmented for the additional noise terms and, if any, also for the VAR coefficients in the measurement errors as in Ireland (2004). In this paper we follow the Smets and Wouters approach and introduce a sufficient number of structural shocks in the state equations.

3.2 Maximum likelihood estimation and inference

The state space form of the linear approximation obtained can be subject to a ‘classical’ likelihood analysis. The system can be fed to a Kalman filter and the likelihood function $p(Y^T | \theta)$ can be computed in a standard way (Kalman, 1960, Kalman and Bucy, 1961) or, in the case of non-stationary models, with exact initial Kalman filtering (Koopman, 1997). This allows performing maximum likelihood estimation, using a numerical optimisation routine, obtaining:

$$\hat{\theta}_{ML} = \arg \max_{\theta \in \mathbb{R}^k} p(Y^T | \theta)$$

where Y^T is the information set given by a series of observations Y_t with $t=1, \dots, T$. Given the particular nature of the state space model fed to the ML optimisation, in which all matrices of coefficients are functions of the structural parameters $\theta: A = A(\theta), B = B(\theta), C = C(\theta), D = D(\theta)$, the algorithm for optimisation also implies that, for each parameter trial, the whole procedure previously presented (log-linearisation, solution of the LRE model, implementation of the Kalman filter and computation of the likelihood value) must be repeated.

The likelihood-based inference can present further problems, specifically regarding the lack of identification.

- global: the likelihood function may have multiple maxima;
- local: the likelihood function does not have a unique maximum in the neighbourhoods of some θ^* .

To better explain the latter case, in such situations there exist many combinations of model parameters that provide the same likelihood value, i.e. the maximum is not given by a single point in the parameter space, but by a complex multidimensional structure. In some disciplines this is referred as over-parameterisation, i.e. there are many parameter values or model specifications that are compatible with the same empirical evidence. This also means that the number of parameters to estimate is too large. A trivial remedy to it can be to fix some parameters (in some cases most of them!) and maximise with respect to the remaining ones, even if this solution can be regarded as arbitrary.

This also implies that the maximisation is computationally more difficult than for standard state space models. Moreover, also the representation and summary of results is difficult. For example, ML inference is often accompanied by asymptotic theory to provide confidence intervals, sampling distribution of the ML estimates, etc. But what if the maximum is not unique? Moreover, the lack of identification often leads to ill-conditioned covariance matrices (i.e. the Hessian matrix is often nearly singular). Taking into account parameter uncertainties (or in other words the shape of the likelihood function) can be therefore a very difficult problem.

All these issues call for a Bayesian approach, in which prior information is combined with the likelihood and which is especially useful in problems with many parameters and few observations. The use of priors is very ‘natural’, since economists have strong beliefs about plausible values of structural parameters; parameters have a well-defined interpretation and have a bounded domain. From the computational point of view, the use of a prior makes the optimisation algorithm more stable, namely because curvature is introduced in the objective function. Maximisation of the posterior is hence (relatively) easier than the maximisation of the likelihood. Moreover, parameter uncertainties and the shape of the likelihood (or better of the posterior distribution) are treated ‘naturally’ by applying stochastic simulation approaches.

The price to pay is that Bayesian methods are extremely computationally intensive. We describe the Bayesian route in the next section.

3.3 Bayesian estimation and inference

Roughly speaking, Bayesian inference is based on pulling the maximum likelihood estimates toward values thought as plausible a priori. From the Bayes theorem, the posterior distribution is obtained as

$$(19) \quad \begin{aligned} p(\theta | Y^T) &= \frac{p(Y^T | \theta)p(\theta)}{\int p(Y^T | \theta)p(\theta)} \\ &\propto p(Y^T | \theta)p(\theta) , \\ &\propto L(\theta | Y^T)p(\theta) \end{aligned}$$

summarising all our information (prior and likelihood) about the parameter vector θ . The likelihood function is any function $L(\theta | Y^T) \propto p(Y^T | \theta)$.

Knowing the posterior distribution, allows implementing the Bayesian inference. In general, the objective of Bayesian inference can be expressed as

$$E[g(\theta) | Y^T]$$

where $g(\theta)$ is a function of interest (a forecast, the vector of model parameters itself, etc.) and

$$E[g(\theta) | Y^T] = \int g(\theta)p(\theta | Y^T)d\theta = \frac{\int g(\theta)p^*(\theta | Y^T)d\theta}{\int p^*(\theta | Y^T)d\theta} = \frac{\int g(\theta)L(\theta | Y^T)p(\theta)d\theta}{\int L(\theta | Y^T)p(\theta)d\theta}$$

where $p^*(\theta | Y^T) \propto p(\theta | Y^T) \propto p(\theta)L(\theta | Y^T)$ is any posterior density kernel for θ . For example, for a quadratic loss function, the point estimate of model parameters is given by the posterior mean:

$$\hat{\theta} = \int \theta p(\theta | Y^T)d\theta .$$

The problem in Bayesian inference is that the integrals involved have almost never an analytical solution and need a numerical approach, specifically through stochastic simulation.

The key strategy is to generate draws of θ from the posterior distribution $p(\theta | Y^T)$. This is discussed in the next section.

3.3.1 Implementation: MCMC (Metropolis-Hastings)

The key concept of Monte Carlo simulation is as follows. Assume a vector of random variables θ with a joint pdf $\pi(\theta)$. If we can draw an *i.i.d.* sample $\theta_1, \theta_2, \dots, \theta_n$ from $\pi(\theta)$, we can approximate the integrals by discrete sums:

$$(20) \quad \bar{g} = 1/n \sum_{i=1}^n g(\theta_i) \rightarrow E(g(\theta)) = \int g(\theta)\pi(\theta)d\theta \quad \text{“almost surely” as } n \rightarrow \infty .$$

If the variance σ^2 of $g(\theta)$ is finite, then

$$(21) \quad \sqrt{n}(\bar{g}(\theta) - E(g(\theta))) \sim N(0, \sigma^2)$$

provides an estimation error.

The generic distribution $\pi(\theta)$ can be the posterior distribution $p(\theta | Y^T)$ and hence Monte Carlo simulation can be applied to solve the Bayesian inference problem. The Monte Carlo approximations can be then used to compute predictions, impulse response functions, etc.

The “almost surely” in equation (20) above means that convergence is subject to some regularity conditions of the function $g(\theta)$; specifically absolute convergence of the integral must be satisfied, see Geweke (1999).

The required sample from the posterior distribution is a multivariate sample. This is not an easy problem and has been the subject of a huge amount of literature to find techniques for this sampling problem: from acceptance sampling, importance sampling, to Markov Chain Monte Carlo approaches (Gibbs sampler and Metropolis-Hastings algorithm). The latter approach is probably the best suitable for the problem at hand and is the one which is applied in all the recent literature on Bayesian analysis of DSGE models.

Let us first define an m -states Markov process x_t . We denote the possible states of x_t by $S = \{s_1, \dots, s_m\}$ and define the transition probabilities

$$P = \begin{pmatrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{pmatrix}$$

where p_{ij} is the probability of moving from state i to state j . Let $w(t) = [w_1(t), \dots, w_m(t)]$ an $1 \times m$ vector of probabilities of x_t being in state i in period t , then the corresponding probabilities for period $t+1$ are

$$w(t+1) = w(t)P$$

- The Markov chain has an equilibrium distribution if there exists a distribution π such that $\pi = \pi P$;
- A Markov chain is reversible if probability of $i \rightarrow j$ is the same as $j \rightarrow i$:
 $\pi p_{ij} = \pi_j p_{ji}$.

A chain that is reversible has an equilibrium distribution and to sample from the equilibrium distribution one can start the chain from any $w(0)$ until it settles down to the equilibrium distribution.

A Markov chain is not *iid*, since the sample is serially correlated.

The idea of the Metropolis algorithm is to construct the transition matrix P from an ‘easy’ transition matrix Q (e.g. corresponding to a multivariate normal distribution), such that P has the desired equilibrium distribution π (i.e. the posterior distribution). This because we are not able to draw from the posterior distribution (corresponding to π in our case), but we are able to draw sample from a normal distribution (corresponding to using Q). Of course, only the Q transition is not sufficient to assure convergence to π , so we have to add an additional rule for the transition to one state to another. Suppose at t iteration we are in state s_i and based on Q we draw a proposed state s_j . We define a probability α_{ij} that the proposed state is accepted (or a probability $1 - \alpha_{ij}$ that the new state is rejected and we stay in s_i). To define the probability α_{ij} , we use the objective distribution π as follows:

$$\alpha_{ij} = \min[1, \pi_j / \pi_i]$$

and the resulting chain is reversible and has equilibrium distribution π .

In our specific problem, the Metropolis –Hastings algorithm is implemented as follows:

- 1) Conditional on data Y^T and a set of parameter values θ , the Kalman Filter is used to evaluate the log-posterior density up to a constant: $p(\theta | Y^T) \propto L(\theta | Y^T)p(\theta)$;
- 2) With a numerical optimisation routine the mode $\tilde{\theta}$ of the posterior density can be estimated, and the inverse Hessian $\tilde{\Sigma}$ at the mode computed;
- 3) Implement the Random Walk Metropolis Algorithm:
 - a) Draw a candidate parameter vector \mathcal{G} from a jumping distribution $J_s(\mathcal{G} | \theta^{(s-1)})$, with $J_s \sim N(\theta^{(s-1)}, c^2 \tilde{\Sigma})$ [the Q transition defined above]
 - b) The jump from $\theta^{(s-1)}$ is accepted ($\theta^{(s)} = \mathcal{G}$) with probability $\alpha_{s,s-1} = \min(r, 1)$ and rejected ($\theta^{(s)} = \theta^{(s-1)}$) otherwise, with

$$r = \frac{L(\mathcal{G} | Y^T)p(\mathcal{G})}{L(\theta^{(s-1)} | Y^T)p(\theta^{(s-1)})}$$

The series of draws $\{\theta^{(s)}\}$ is serially correlated (not *iid*) but, after a burn in period, converges to the desired posterior distribution (e.g., draw 10,000 samples and reject the first 2,000). The speed of convergence is a critical issue of MCMC methods. There no general rule of criterion that can assure that the chain has converged. There are a number of informal techniques to assess convergence, such as:

- plot $1/n_s \sum_{s=1}^{n_s} g(\theta^{(s)})$ as a function of n_s ;
- start the Markov-Chin at over-dispersed (i.e. extreme) values of θ and check whether different runs of the chain settle to the same distribution;
- more general methods, which combine in a more rigorous way the two above ‘empirical’ ideas, such as the potential scale reduction factor (PSRF) and its multivariate extension (Brooks and Gelman, 1998; implemented in DYNARE). Roughly speaking, this test aims at verifying that the samples obtained with a number of parallel chains are drawn from the same distribution.

When converged, the chain satisfies a weak law of large numbers, i.e. the approximations (20) and (21) apply for the Markov chain, which can then be used for the Bayesian inference.

3.4 Model comparison

In the Bayesian framework, models are compared and ranked according to the integrated likelihood (or marginal data density). Having a set of models $i=1, \dots, M$, the posterior weight of the i -th model is

$$(22) \quad w_i(Y^T) = \int_{\Theta_i} p_i(Y^T | \theta_i) p(\theta_i) d\theta_i$$

and, if the models have equal prior probabilities, the posterior probability on model i is $w_i / \sum_j w_j$. As usual, the computation of this integral is unfeasible analytically in most cases, but can be estimated using a sample from the posterior distribution. Specifically, the marginal data density of the DSGE model is here approximated with Geweke's (1999) modified harmonic mean estimator (implemented in DYNARE).

In the present report we make a preliminary comparison with a VAR(1) model, using RMSE's. Recently, Sims (2003) provided a general discussion about pitfalls of Bayesian model comparison methods, highlighting several ways they tend to misbehave. In this view, there is no point in showing a ‘preliminary’ Bayesian comparison, comparing, e.g., the

marginal data density obtained here with the data density of a VAR(1) where the priors are defined with a training set. As discussed by Sims, such kind of comparison *could be* totally arbitrary and meaningless. A full Bayesian comparison is being implemented, trying to carefully address the issues raised by Sims.

4 Estimation

4.1 Prior distributions

Structural shocks. After in initially setting priors to inv-gamma, the traditional priors of standard errors in Bayesian analysis, we preferred to set up flat priors in a relatively large interval of values, reflecting more clearly our prior ‘ignorance’ about possible values of shocks. Above all in view of a complete Bayesian comparison with other models (VARs), this assumption might be revisited by considering, e.g., a training set. This because too large a prior range might unduly penalise the present model, by giving too low weight to the likelihood (a totally uninformative prior in the range $[-\text{inf}, \text{inf}]$ would give a uniformly zero weight to any likelihood value, implying the rejection of any model; see Sims, 2003, for a full discussion on these matters).

Concerning the shock to time-varying $\delta(\varepsilon_t^\delta)$, we set a much smaller range, since we do not want δ to absorb whatever is missed by the rest of the model, but we just allow the minimum shock necessary to reconstruct the depreciation path.

Table 1.a Priors structural shocks

	Distrib.	Min	Max
Firms:			
TFP shock ε_t^U	uniform	1.e-6	0.2
Depreciation shock ε_t^δ	uniform	1.e-6	0.0001
Risk premium shock ε_t^{rp}	uniform	1.e-6	0.2
Mark-up shock ε_t^r	uniform	1.e-6	0.2
Wage cost shock ε_t^w	uniform	1.e-6	0.2
Households:			
Consumption Preference shock ε_t^C	uniform	1.e-6	0.2
Labour supply shock ε_t^L	uniform	1.e-6	0.2
Policy:			
Government expenditure shock ε_t^G	uniform	1.e-6	0.2
Inflation target shock $\varepsilon_t^{\pi^T}$	uniform	1.e-6	0.2
Interest rate shock ε_t^M	uniform	1.e-6	0.2

Table 1.b Priors shock persistence

Parameter	Distribution	Mean	St. dev.	Support
Firms:				
TFP ρ^U	beta	0.9	0.04	[0 1]
Depreciation ρ^δ	beta	0.9	0.04	[0 1]
Risk premium ρ^{rp}	beta	0.5	0.2	[0 1]
Wage costs ρ^w	beta	0.5	0.2	[0 1]
Households:				
Labour supply ρ^L	beta	0.5	0.2	[0 1]
Policy:				
Government expenditure ρ^G	beta	0.9	0.04	[0 1]
Inflation target ρ^{π^r}	beta	0.5	0.2	[0 1]

Model parameters.

The model parameters to be estimated have a structural economic interpretation and are therefore restricted to lie in certain intervals dictated by economic theory or implied by long run constraints. The following ranges have been chosen for the individual coefficients:

Table 2 Priors model parameters

Parameter	Distribution	Mean	St. dev.	Support
Firms:				
Depreciation rate δ	beta	0.015	0.005	[0 0.2]
Capacity utilisation a_2	beta	0.05	0.028	[0 0.1]
Adjustment cost, capital γ_K	beta	15	5	[0 30]
Adjustment cost, inv. γ_I	beta	10	3	[0 20]
Adjustment cost, labour γ_L	beta	15	5	[0 30]
Adjustment cost, price γ_P	beta	15	5	[0 30]
Adjustment cost, wage γ_W	beta	15	5	[0 30]
Mark-up, cyclical τ_1	beta	-0.1	0.03	[-0.2 0]
Share of fwd looking price setters sfp	beta	0.6	0.05	[0.5 1]
Households:				
Habit persistence hab	beta	0.6	0.15	[0 0.9]
Labour supply elast. κ	gamma	0.5	0.4	[0 Inf]
Labour supply const ω	gamma	0.2	0.15	[0 Inf]
Bargaining strength bg	beta	0.375	0.18	[0 0.75]
Wage mark-up θ	gamma	2	0.8	[1 Inf]
Share of fwd looking wage setters $sfpw$	beta	0.8	0.1	[0.5 1]

Policy:				
Fiscal response to ygap t_G^Y	beta	0	0.4	[-1 1]
Interest rate smoothing $ilag$	beta	0.8	0.1	[0 1]
Interest rate response, $t_M^{\Delta\pi}$	beta	0.2	0.09	[0 0.4]
Interest rate response, $t_M^{\Delta Y}$	beta	0.1	0.045	[0 0.2]
Interest rate response, t_M^π	beta	1.25	0.3	[0.5 2]
Interest rate response, t_M^Y	beta	0.3	0.06	[0 0.5]
Smoothness, trend GDP ρ_{YPOI}	beta	0.9	0.05	[0.7 1]

Note: The following parameters were fixed: output elasticity of labour $\alpha = 0.5940$, discount factor $\beta = 0.989$, interest elasticity money demand $\zeta = -0.4$, Mark-up level $\tau_0 = 0.1$;

The remaining parameters are determined by steady state constraints.

$$a_1 = (r_{ss} + r_p + \delta)(\gamma_I \delta + 1) \quad \text{1st parameter of capacity utilisation}$$

$$A = L_{ss}^\alpha K_{ss}^{1-\alpha} \quad \text{Technology constant.}$$

$$r_p = (1 - \tau)(1 - \alpha)\delta / I_{ss} / (\gamma_I \delta + 1) - \delta - r_{ss} \quad \text{Risk premium.}$$

We identified beta or gamma prior distributions for model parameters³. The prior specification of sfp and t_M^Y required a particular attention, whereby we had to give lower weight to values that were “preferred” by the likelihood, but that implied unreasonable dynamical behaviour in the impulse responses. So, we set asymmetric distributions that privileged the lower part of range for sfp and higher part for t_M^Y . This implied only a slightly worse fit, but a much better model behaviour in terms of theoretical considerations. This kind of approach is legitimate in a Bayesian framework, and distinguishes from a plain constrained optimisation, which would be considered much more arbitrary. The fact that we give a smaller (but non-zero!) prior probability to some portion of the parameter ranges, always gives the possibility to the likelihood to override this assumption, if the data strongly supports hypotheses about such values that were unlikely a priori. Moreover, this approach does not rule out possible misspecification or the rejection of the present model with respect to competing ones. In the latter case, the integrated likelihood of the present model would be penalised with respect to a competing one that provided more “agreement” between prior assumptions and likelihood shape.

The plots of the prior distributions are given below. The model was estimated using the following eight series as observations: $Y, I, C, K, L, \pi, W/P, inom$.

³ Please note that the support of beta distributions might be larger than the ranges specified in Section 1, but means and the standard deviations are set in such a manner that prior probability is larger than zero only in the acceptable range.

Figure 1 Prior distributions

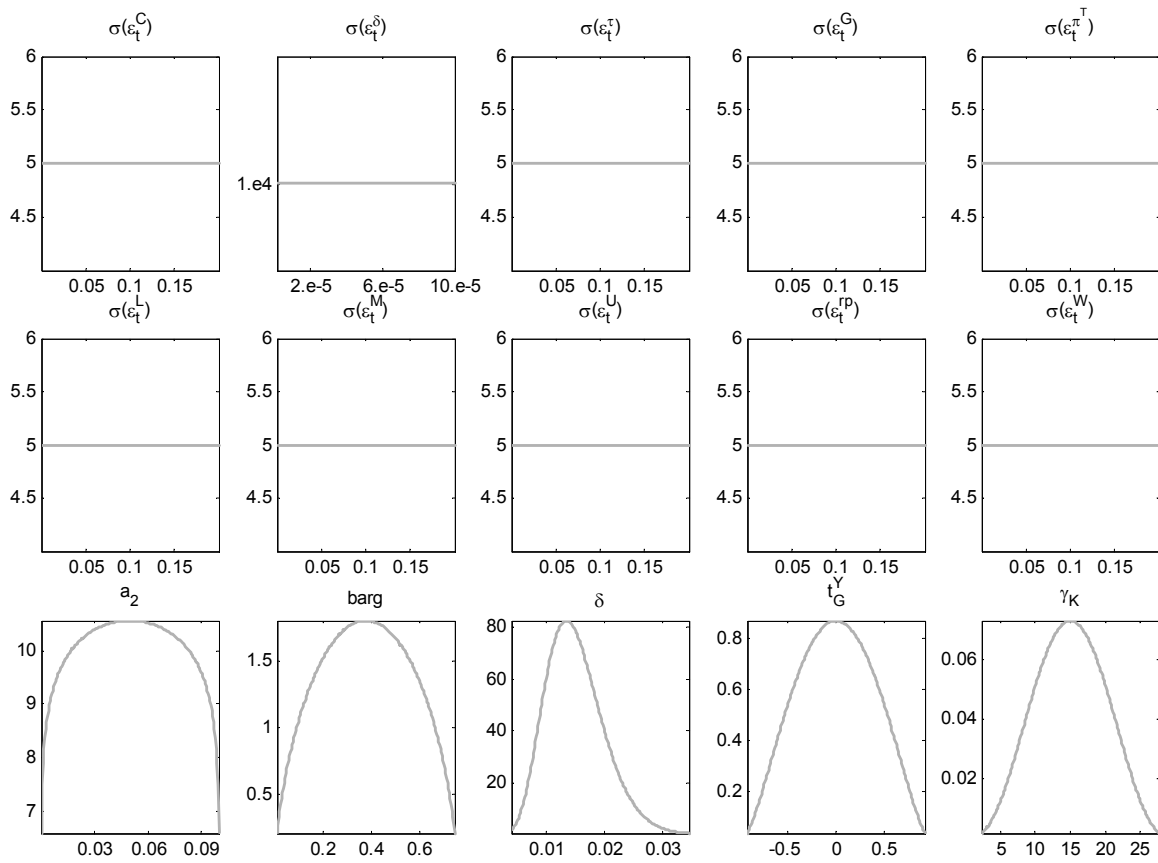
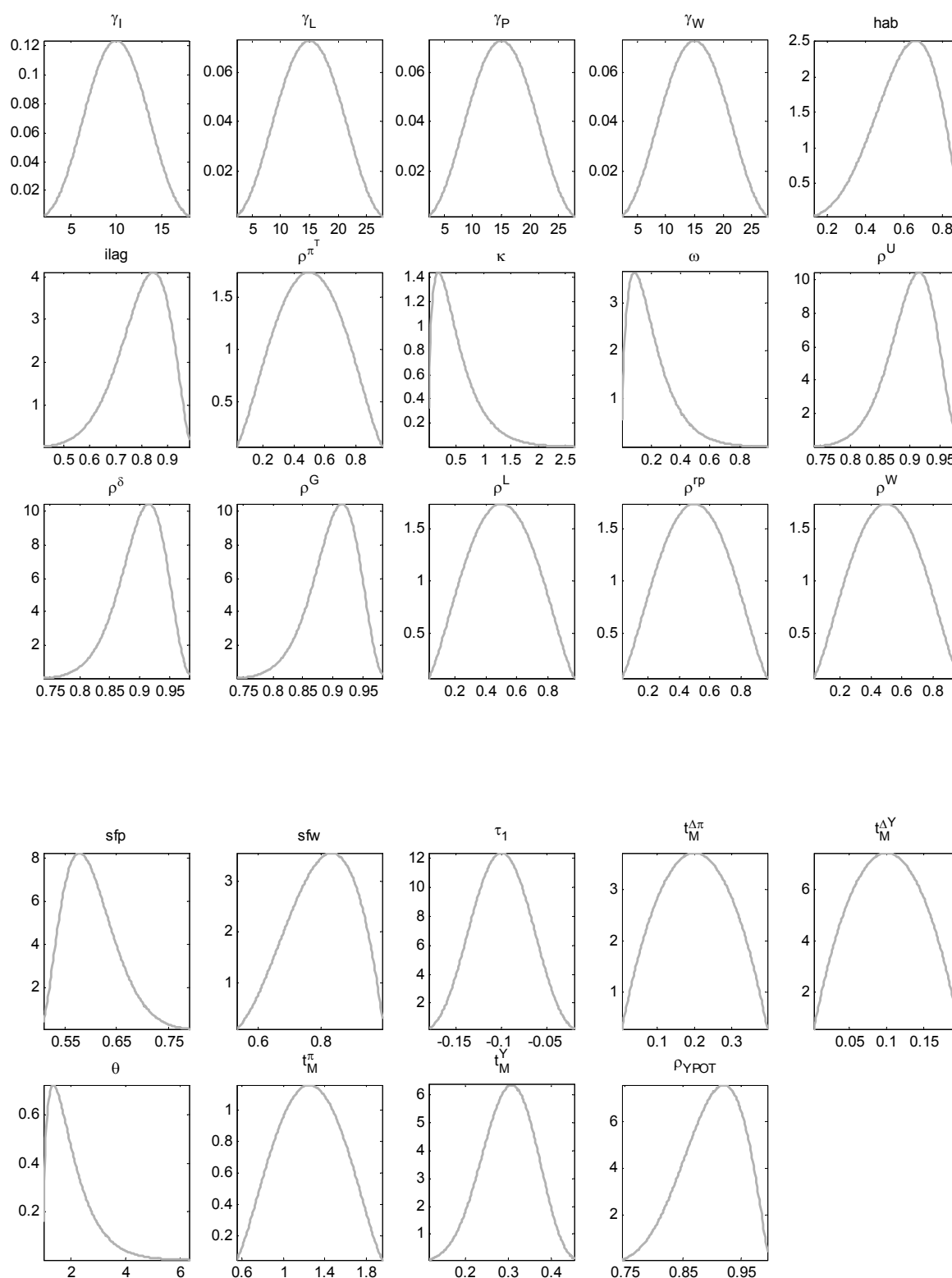


Figure 1 (cont'd) Prior distributions

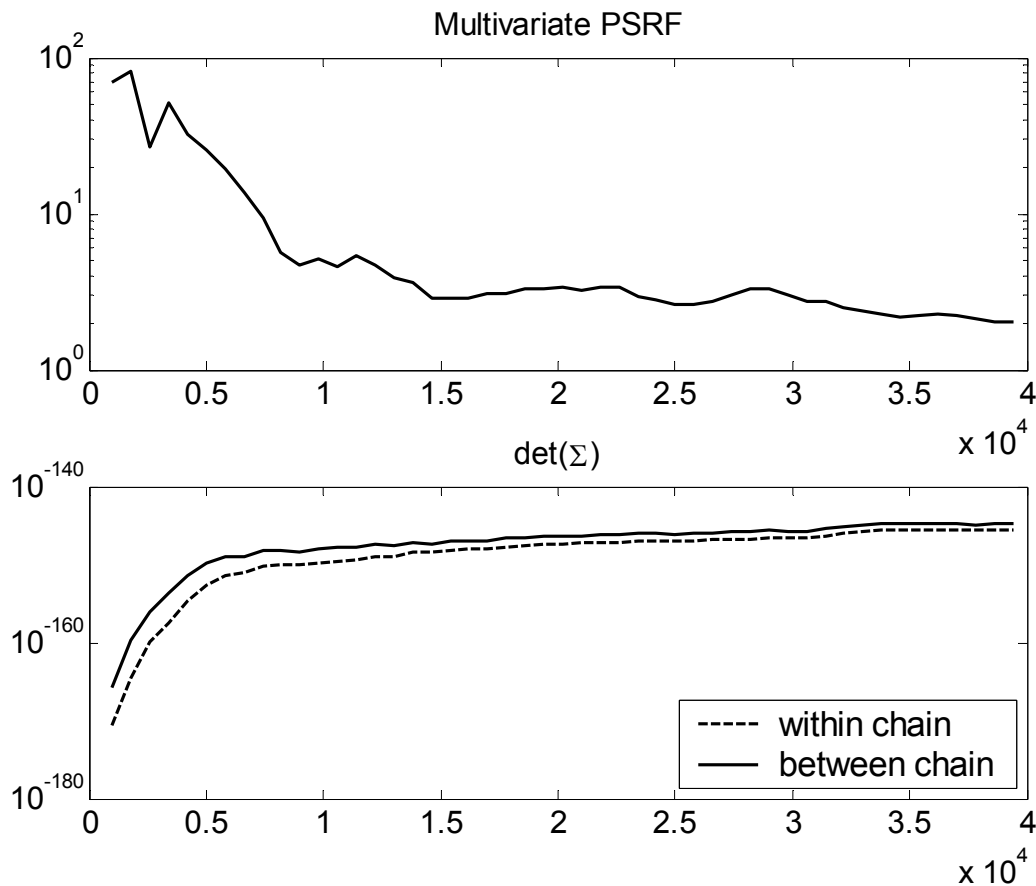


4.2 Parameter estimates and shocks identified

The posterior estimation followed the methodology of Section 2. First the mode of the posterior is estimated using a non-linear optimisation routine (values reported in the Annex).

Then, a sample from the posterior distribution is obtained with the Metropolis algorithm using the inverse Hessian at the posterior mode as the covariance matrix of the jumping distribution. The scale coefficient was set to 0.25, allowing a good acceptance rate (25%). We ran 4 parallel Markov chains. Since the refinement of the convergence tests proceeded slowly by increasing the length of the chains, we decided to update the covariance matrix of the jumping distribution according to the last portion (30%) of the chains based on the inverse Hessian. This allowed us to obtain good convergence tests of 4 new chains (of 40,000 runs each) based on the updated covariance matrix.

Figure 2 Convergence test Metropolis MCMC



This Figure shows the convergence tests for Metropolis MCMC. Upper panel shows the multivariate potential scale reduction factor, which should be near to 1 at convergence. Lower panel shows the determinant of the ‘between chains’ and ‘within chains’ covariance matrices of the Monte Carlo sample.

After discarding the initial 70% of runs, we could proceed to the Bayesian inference. The following figures show the estimated marginal posterior distributions (black lines), compared to priors (grey lines) and the point estimate of the multivariate mode (vertical dashed lines). It is interesting to note that for some parameters the maximum of the marginal distribution is shifted with respect to the mode of the multivariate distribution (in particular γ_K and γ_I). This implies that such a local maximum is in a very narrow region with almost zero mass, related to very specific parameter combinations. To give an idea of this, it is interesting to

note that the log-posterior at the mode is about 3800, while the Markov chains evolve in a range [3760 – 3792] i.e. almost 10 log-points lower with respect to the mode. In spite of this quite high difference in level, even imposing a starting point very near to the mode, the evolution of the Markov chain evolved similarly to the ones shown here, implying that such a local optimum is located in a region so small to imply an almost zero probability for a chain to fall there. In the Annex we also report the values of the posterior mean with confidence bands for the estimated parameters. The logarithm of the marginal likelihood for this model is about 3663.

Finally, Figure 4 shows the 1-period ahead predictions of the model for the main model variables, including the depreciation rate δ and government expenditure G . Dashed lines are observations; continuous lines are model predictions. On the whole, the model fits the data remarkably well. One point that is particularly noteworthy is that the model over-predicts *inom* in the last years (coupled with loose ε_t^M).

Figure 3 Prior and posterior distributions standard errors structural shocks and parameters

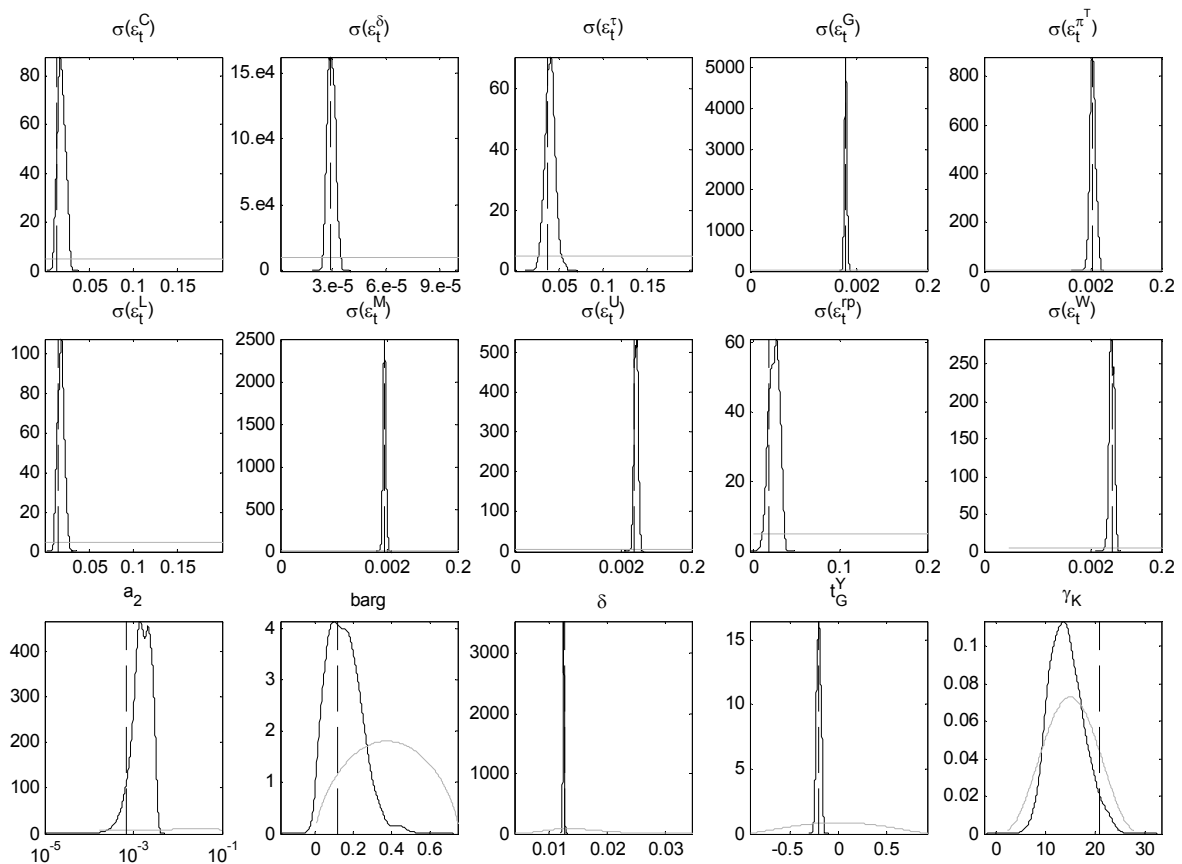


Figure 3 (cont'd) Prior and posterior distributions standard errors structural shocks and parameters

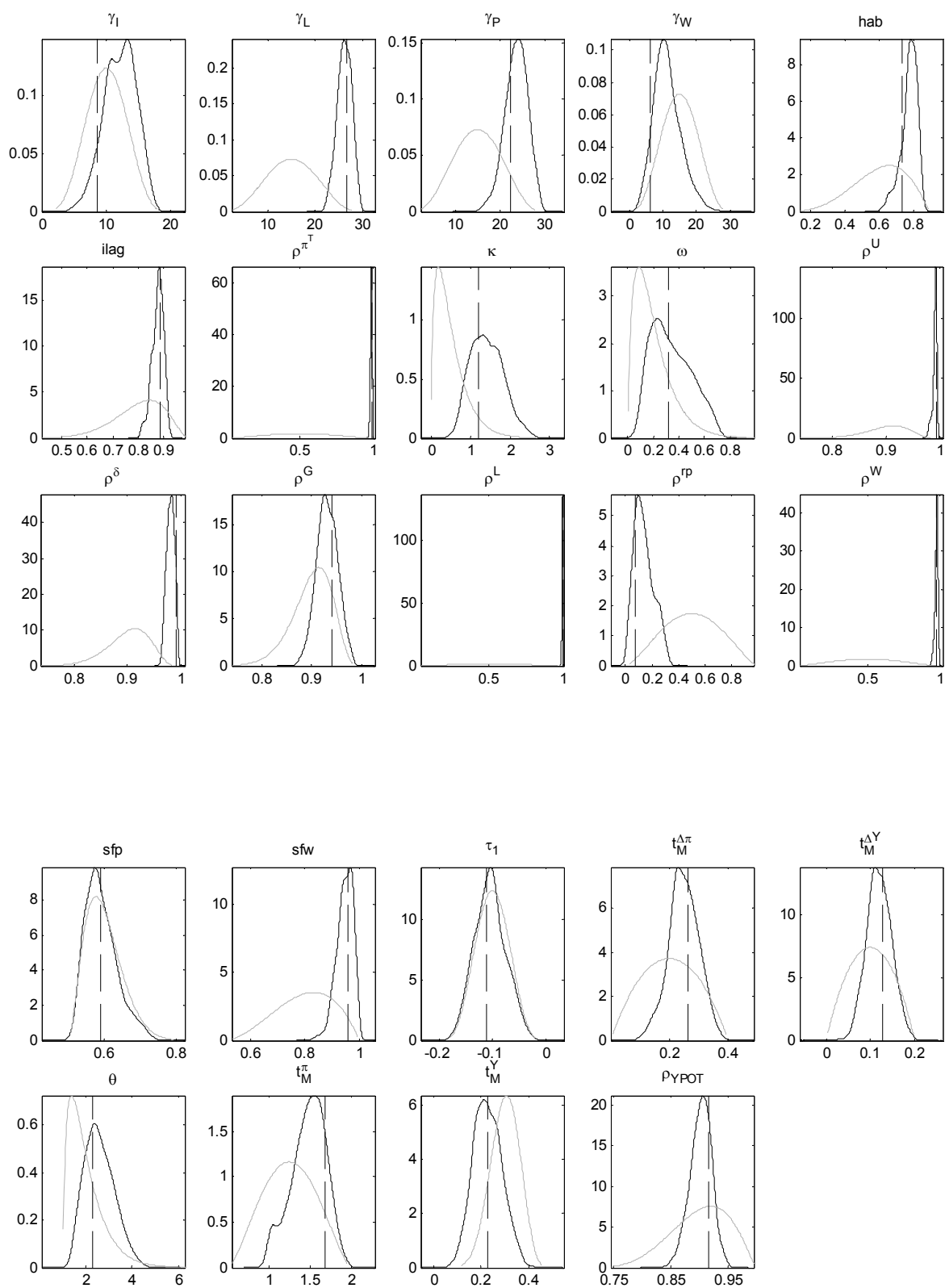
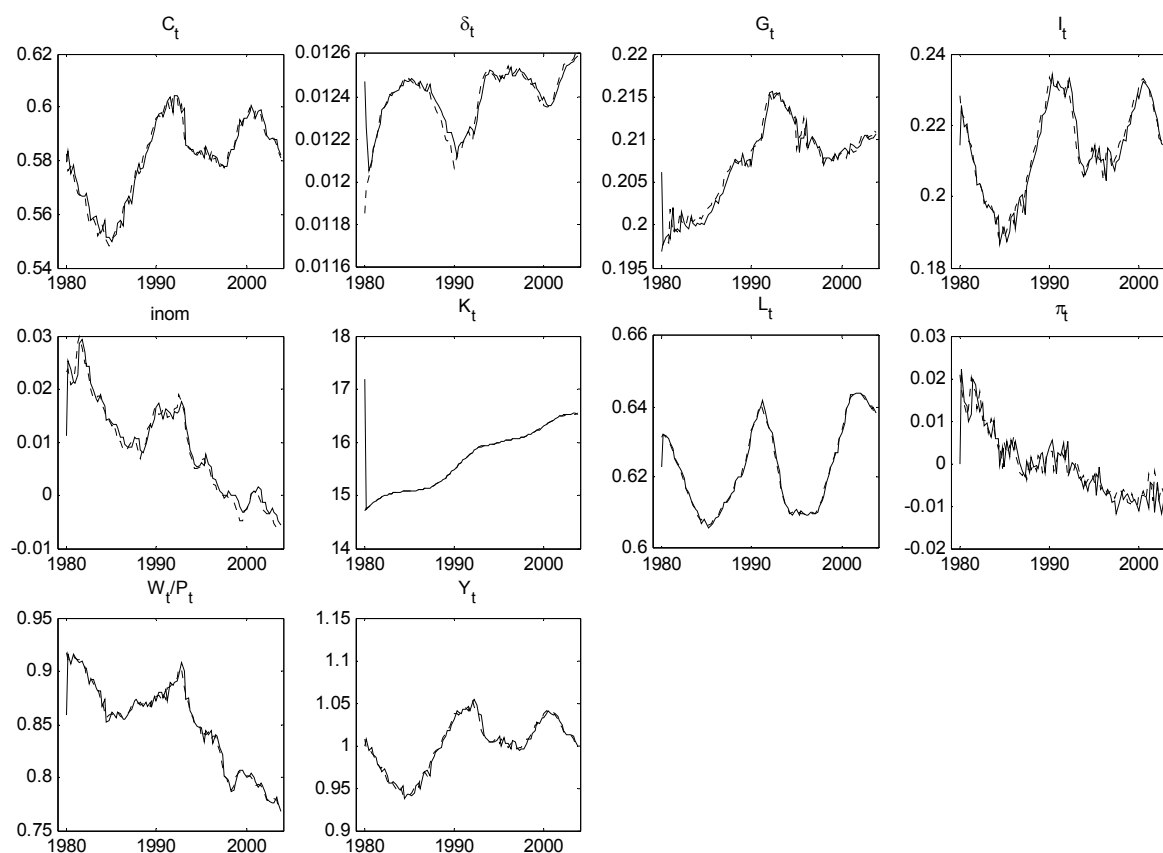


Figure 4 1-step ahead prediction



4.3 VAR comparison

If we compare the RMSE's of the DSGE model computed at the posterior mean, with the RMSE's of a VAR(1) model with the same 8 observed series : $Y, I, C, K, L, \pi, W/P, inom$.

Table 3 RMSE comparison with VAR

RMSE's	VAR	model (post. mean)
C	5.9398e-006	8.5811e-006
I	5.5508e-006	8.388e-006
inom	1.4206e-006	2.2686e-006
K	0.00037479	1.0741e-005
L	5.2953e-007	6.7359e-007
π	4.5673e-006	9.9136e-006
W/P	1.9294e-005	2.3478e-005
Y	1.9422e-005	2.6547e-005

RMSE's of the DSGE are higher but of the same order of magnitude than the VAR, except for K, where the VAR performs much worse (RMSE is forty times larger).

5 Which structural shocks drive the euro economy?

One of the major advantages of the modelling approach used here is that system estimation of the model yields, besides the posterior distribution of the model parameters, structural shocks which have an unambiguous interpretation and which can help us understand the current economic situation. The structural shocks identified in the estimation of this model are shocks to households, firms, government and fiscal policy. Households are affected by shocks to preferences for consumption and labour supply. Firms are hit by shocks to technology, mark-ups, adjustment costs for labour and capital and a risk premium shock.

Several aspects of the estimated shocks (figure 5) and implied unobserved variables (figure 6) are worth highlighting.

Demand Shocks

The consumption preference shock ε_t^C appears slightly negative at the end of the estimation sample. This suggests lower preferences for consumers spending and may be a reflection of savings uncertainty concerning future pensions and tax liabilities. Notice, however, the size of the shock is not extraordinary large, given the fluctuations of ε_t^C over the entire sample period. Investment is hit by two autonomous shocks, a risk premium shock and a shock to adjustment costs. The latter are unimportant and not further considered. The risk premium shock (ε_t^p) does not show any particular trend and appears to behave normally in recent years. This suggests that investment fluctuations are explained by fundamentals. The smoothed auto-correlated fiscal policy shock $z(\varepsilon_t^G)$ displays a turnaround in 2000-01. The fall in this auto-correlated shock shows clearly the fiscal consolidation period starting in the late 1980s, but the declining trend in government spending is reversed in the early 2000s. This refutes the view that fiscal policy has been overly restrained by the SGP and been less countercyclical over the last years.

Supply shock

Our Kalman filter estimation allows to decompose observed total factor productivity into a capacity utilisation and a ‘true TFP’ component (denoted as U). According to these estimates U shows a trend increase in the second half of the 1980s, a movement along the trend in the 90s, but a sharp decline in the late 1990s- early 2000s. Thus the fall in TFP in the recent past is largely a structural phenomenon and not the result of a lack in demand, since capacity utilisation ($UCAP$) shows a normal cyclical behaviour in recent years. Notice, TFP is also one of the major driving forces of investment and is therefore one of the fundamental factors for the slowdown of investment. Trends and fluctuations in mark-ups are important measures for the supply potential of the euro area economy. According to these estimates the mark-up has declined on average since the early 1990s ($\eta=1$ -mark-up) from around 10 to 8%. This could reflect increased competitive pressure due to goods market reforms (internal market programme) but also increased pressure from global competition.

Labour market shocks

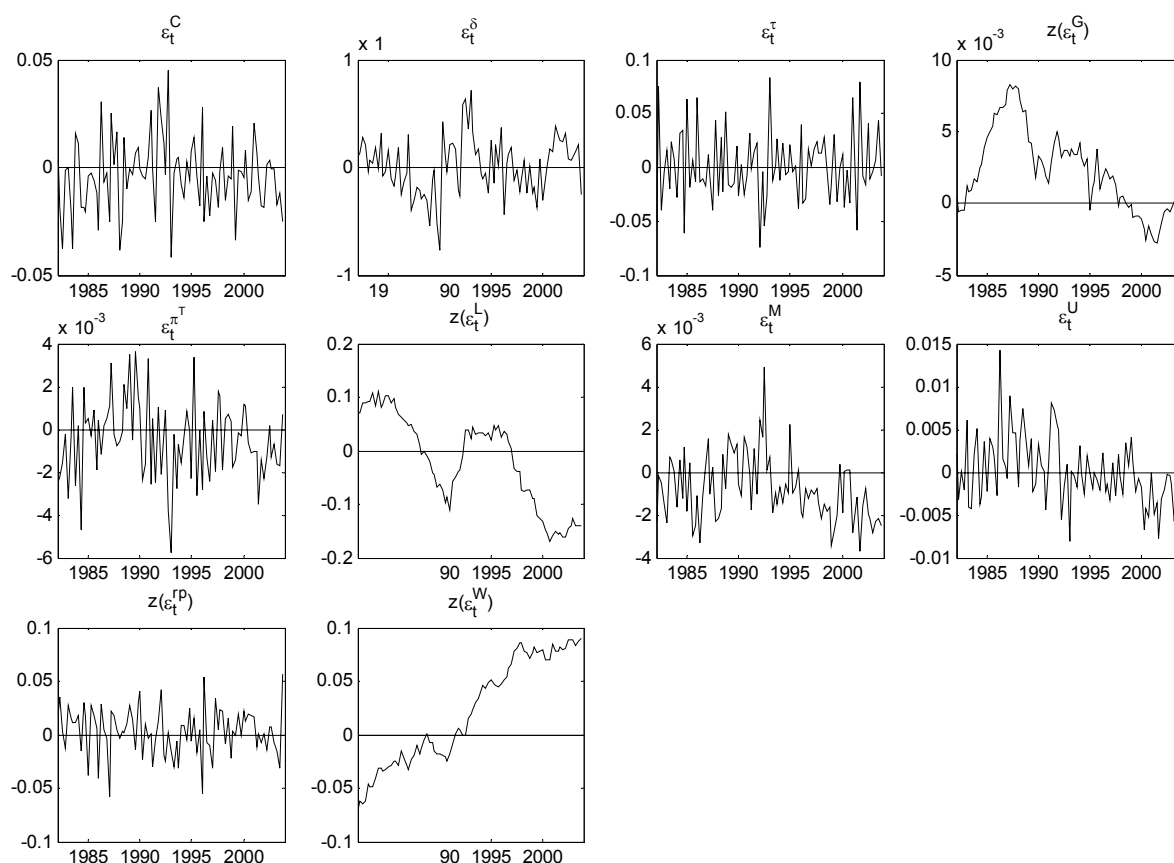
The model identifies a labour supply (ε_t^L) and a labour demand shock (ε_t^W). The trend increase in labour supply (or in model terms a trend decline in the preference for leisure ($Z(\varepsilon_t^L)$) after 1995 is consistent with the observation of increased labour force participation and a declining NAIRU in the Euro area. Notice, however, the shock to labour supply has a pronounced cyclical pattern, which suggests that the simple wage rule used here does not

properly account for the dynamic adjustment of wages over the business cycle. On the other hand, the upward trend of $Z(\varepsilon_t^W)$ reflects the increase in non-wage labour costs over the sample. Interestingly this trend has stopped in the late 1990s, possibly reflecting a success of various labour market reform measures intended to reduce regulatory burdens for firms related to employment.

Monetary policy shocks

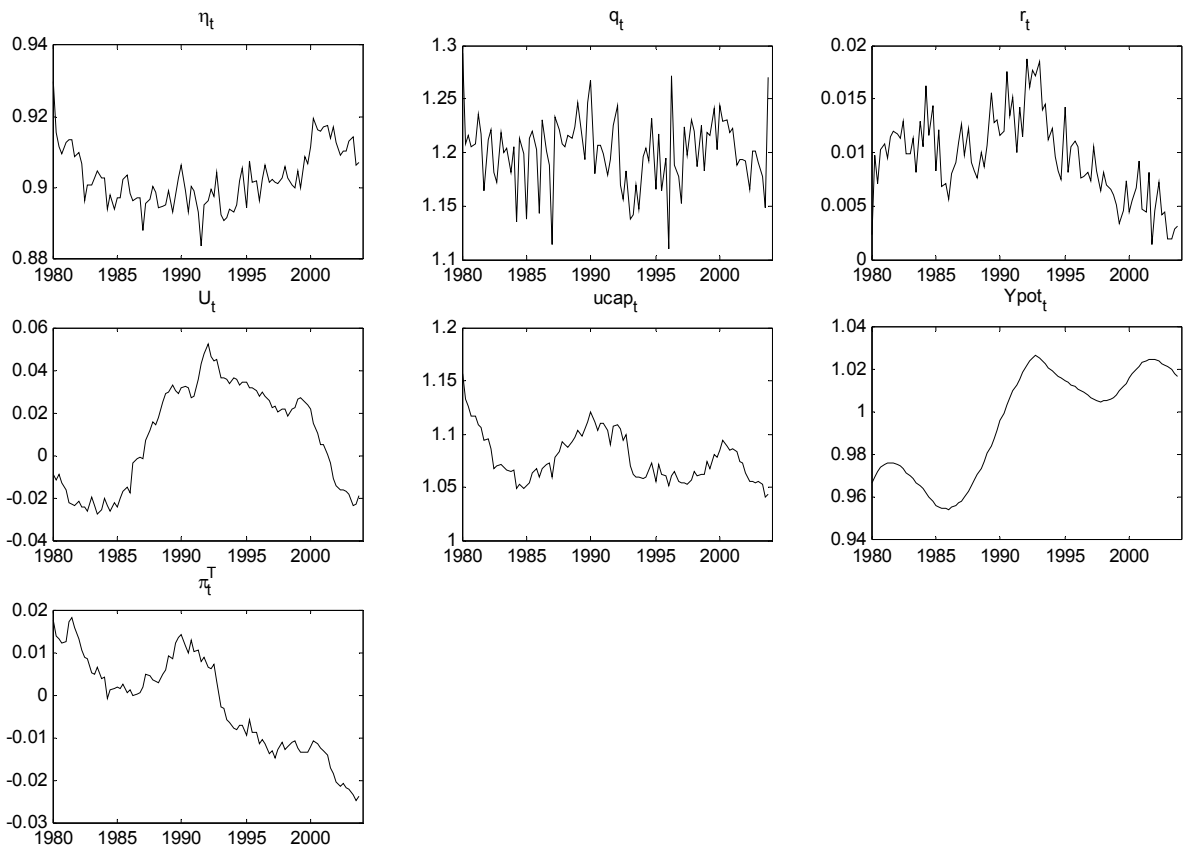
The monetary policy shock ε_t^M is negative for the years after 2000. This would suggest a looser monetary stance than suggested by the estimated Taylor rule. This could be linked to an underestimation of the decline in the inflation objective π^T , which shows a clear trend decline, but may nevertheless underestimate the actual decline in the monetary policy's inflation objective. This is one aspect that may need further attention in future extensions of this model.

Figure 5 Estimated smoothed shocks at the posterior mean



Note: if shocks are auto-correlated, the auto-correlated function is plotted ($z(\varepsilon)$)

Figure 6 Unobserved variables



6 Estimated impulse responses of structural shocks

In this section, we present the estimated impulse responses of the nine structural shocks in our model. The impulse response are generated on the basis of the reduced form representation of the model (policy and reaction functions - see annex). This is particularly simple, since the reduced form is a linear model (formally equivalent to a multivariate ARMA). They depict the responses for the endogenous variable following a one-period shock to each of the structural shocks (which are in most cases auto-correlated), each for a 5 year (20 periods) horizon. A full Bayesian IRF analysis is here presented, picking 1,000 samples out of the full Monte Carlo sample and computing IRF's for each of them. Finally, the mean path (solid lines) and the confidence band (dashed lines) can be obtained, as shown in the Figures.

Figure 7 Consumption preference shock ε_t^C

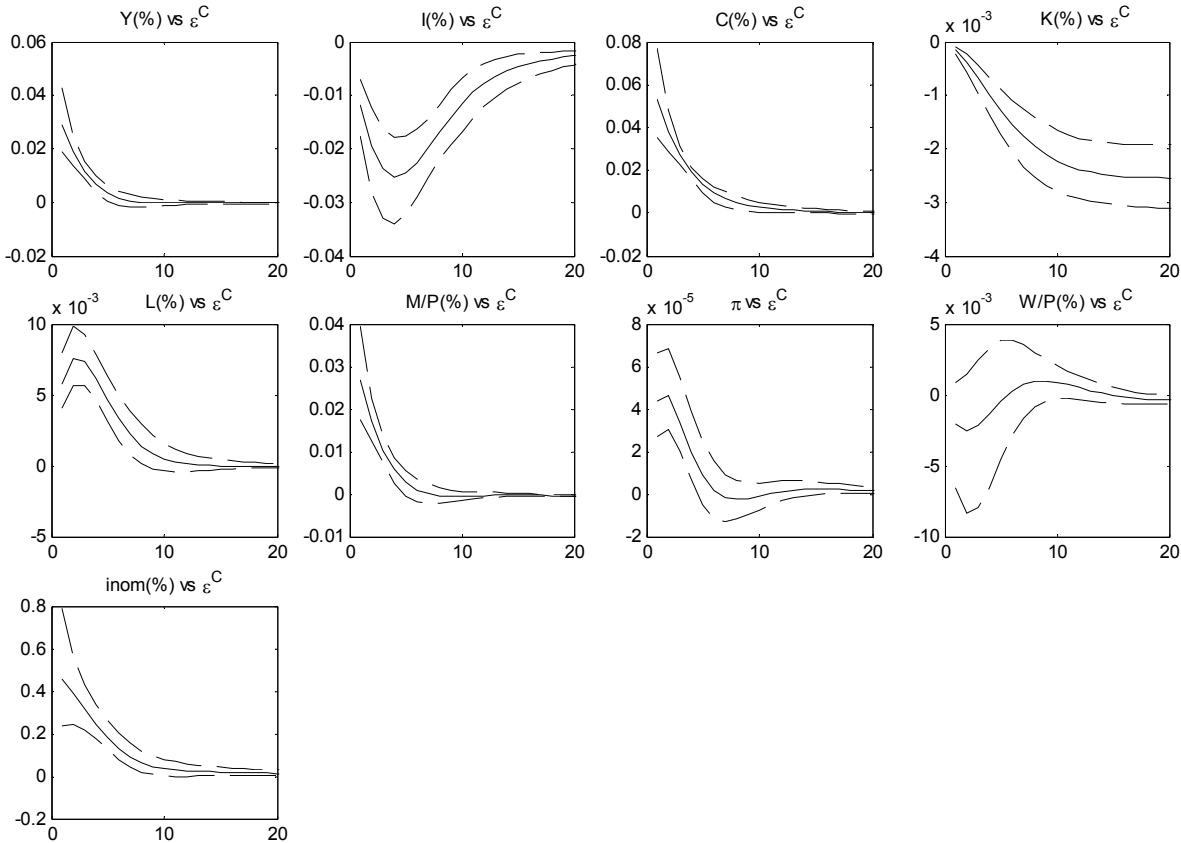


Figure 7 presents the estimated effect of a consumption preference shock (ε_t^C). This shock is a combined shock affecting consumption and leisure choice and has a direct impact on consumption and labour supply (through λ). The effect of this preference shock is to raise consumption by 0.06 percent, and employment by 0.007 per cent (in the second period after the shock). The boost to demand raises inflation and nominal interest rates rise, but the presence of adjustment costs limits the extent of the price rise. The shock leads to crowding out of investment and a decumulation of capital.

Figure 8 Shock to depreciation rate (ε_t^δ)

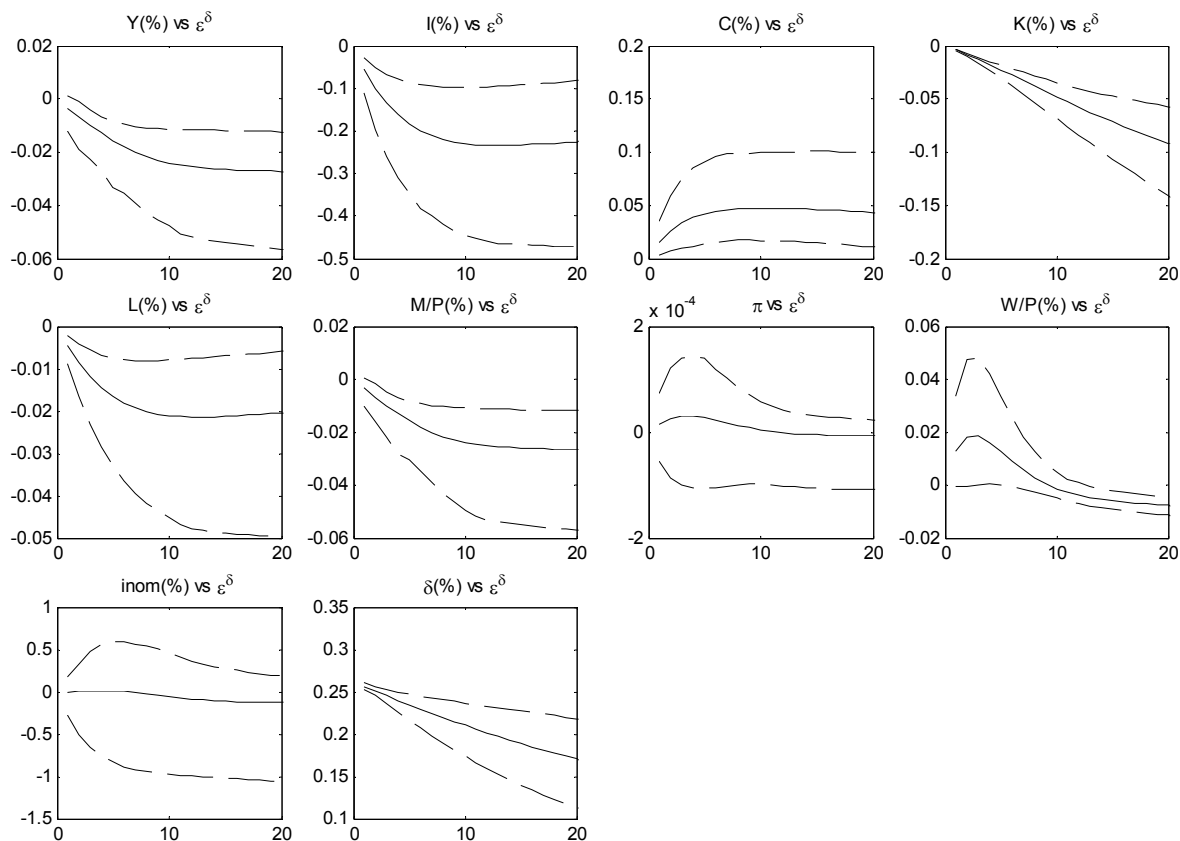


Figure 8 depicts a shock to the depreciation rate of 0.25 per cent on impact. This implies an increase in the cost of capital of 0.012 percentage points. This shock is highly persistent, in fact it has not disappeared after 100 periods, due to the large estimated autocorrelation term in the depreciation rate. The increase in the cost of capital leads to a decline in investment and the capital stock, lower real rates, higher consumption, higher real wages and lower employment. Note however, the large confidence bands, which suggest a large margin of uncertainty surrounding this type of shock.

Figure 9 Shock to price mark-up (ε_t^r)

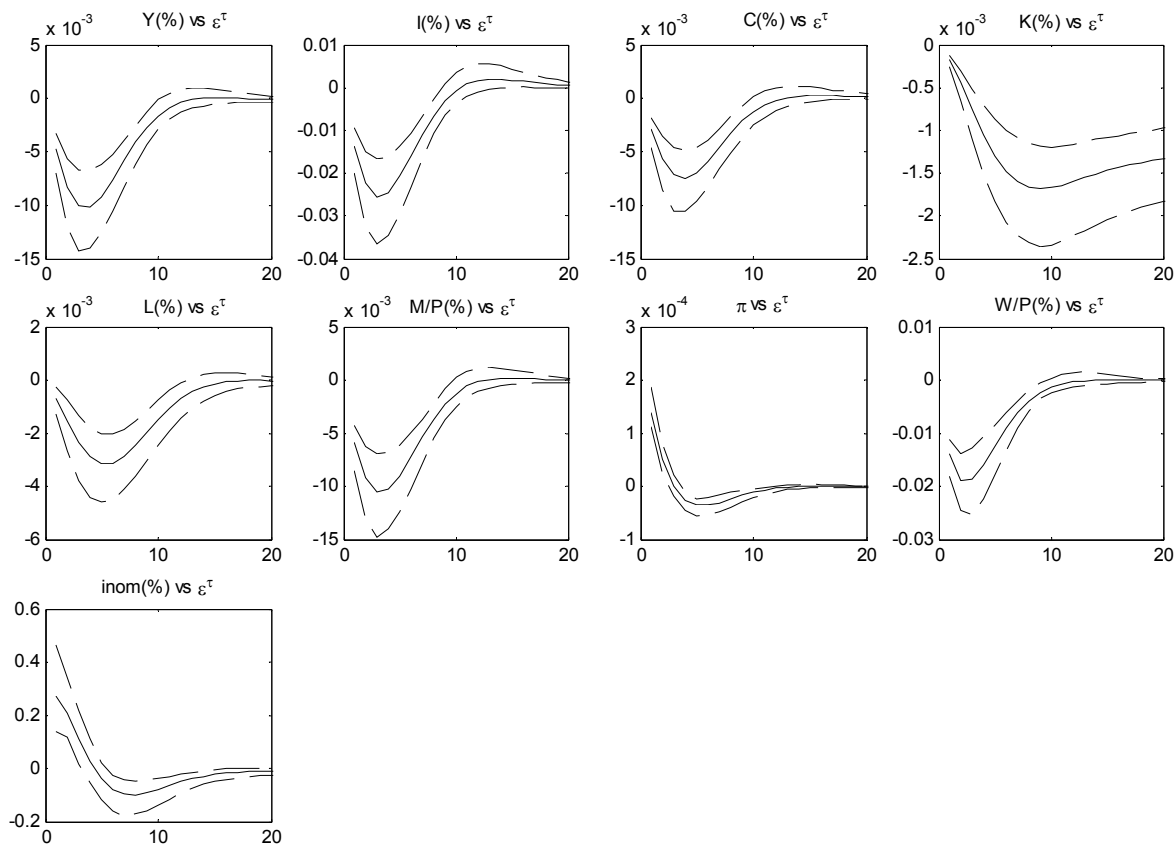


Figure 9 shows that, following a positive price mark-up shock, there is a jump increase in inflation and investment, output and consumption decline. As output and investment fall, labour demand is also lower and employment falls. As the shock is transitory, the effects fade away and have disappeared after 3 years.

Figure 10 Shock to government spending (ε_t^G)

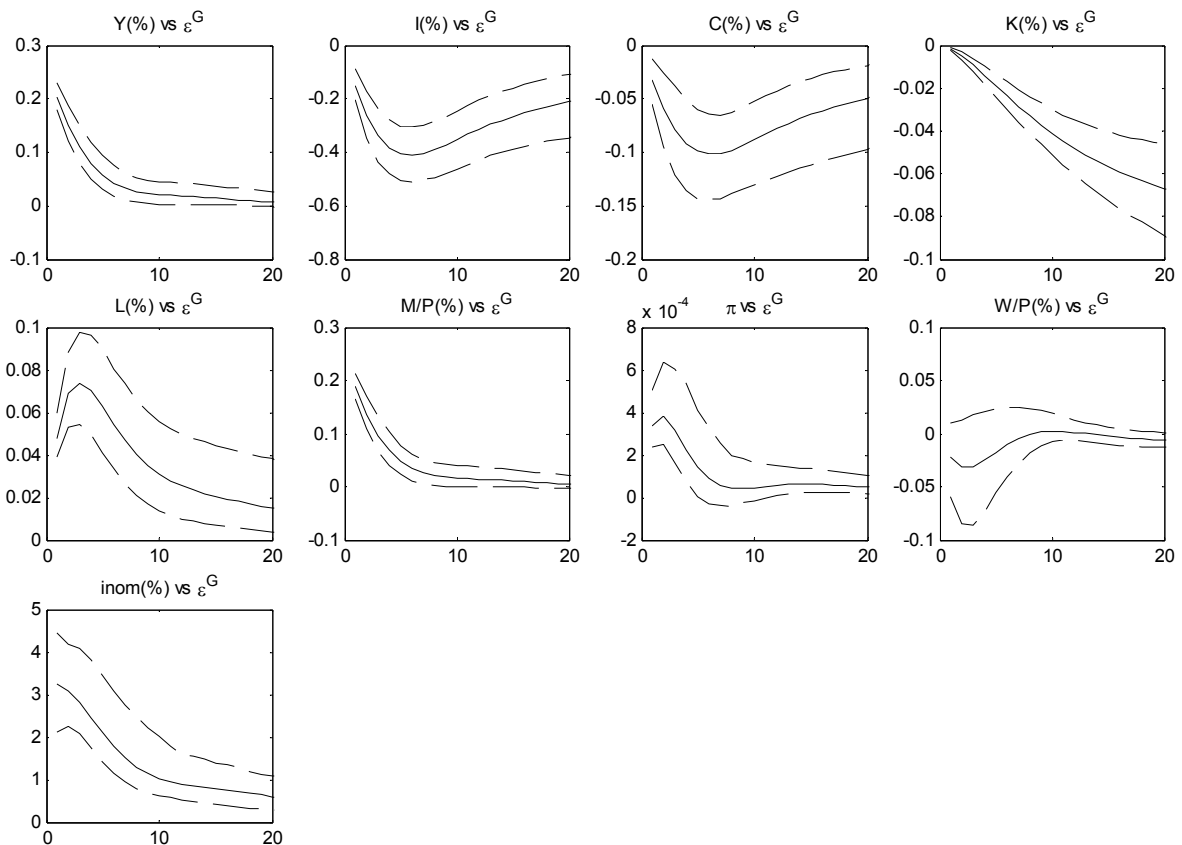


Figure 10 shows the impulse response to a government spending shock. This shock is a persistent shock with an autocorrelation of 0.98, but the overall effect on GDP is short-lived. The impact multiplier is small, not larger than 0.2, and the increase in spending leads to crowding-out of consumption, and in particular of investment, which falls by 0.4 per cent at its peak. This crowding-out of investment leads to a reduction in the capital stock. The effect on output is temporary and over a longer horizon, output becomes negative. The small multiplier for a persistent fiscal spending shock is in line with results of the QUEST model, in which permanent fiscal expansions have much smaller output effects than transitory shocks. The jump in inflation is surprising and seems at odds with empirical regularities. It appears that the peak inflation response is already reached in the second quarter. This high responsiveness of inflation is partly due to the high estimate of the forward inflation term *sfp*. Empirical studies employing VARs show generally positive output effects of an increase in government spending, but often these estimates are surrounded by large confidence intervals. Perotti (2002) finds for most countries positive output effects after an increase in spending, but in the post 1980 sample, these positive effects are small and short-lived. The effect on investment is in accordance with many empirical studies which find the strongest crowding-out of spending shocks for investment (Alessina et al., 2002).

Figure 11 Negative shock to employment ε_t^L

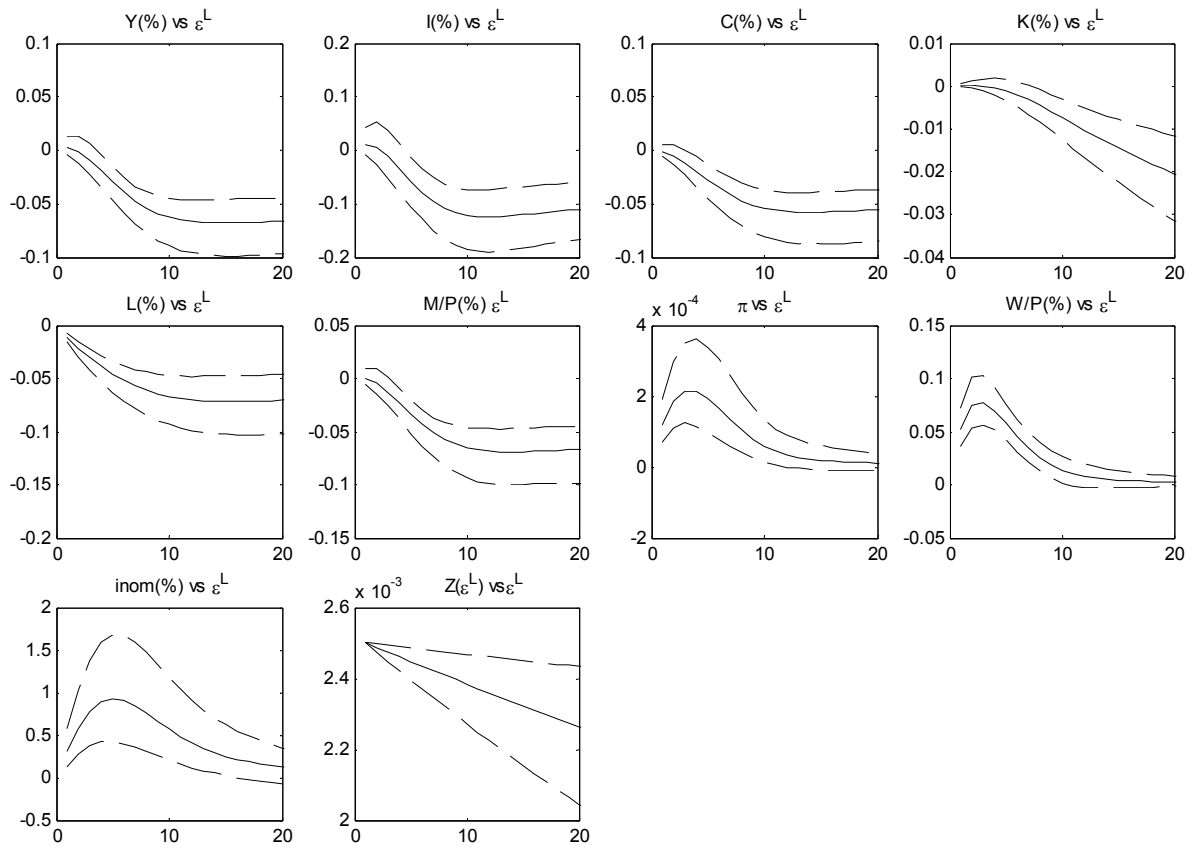


Figure 11 shows the impulse response of a positive shock to leisure (a negative employment shock). This is a highly auto-correlated shock with a high persistence of 0.99, as is clear from $Z(\varepsilon_t^L)$. Employment falls, and the reduction in labour supply has a negative impact on investment and output. As consumers anticipate lower incomes, consumption also falls.

Figure 12 Productivity shock ε_t^U

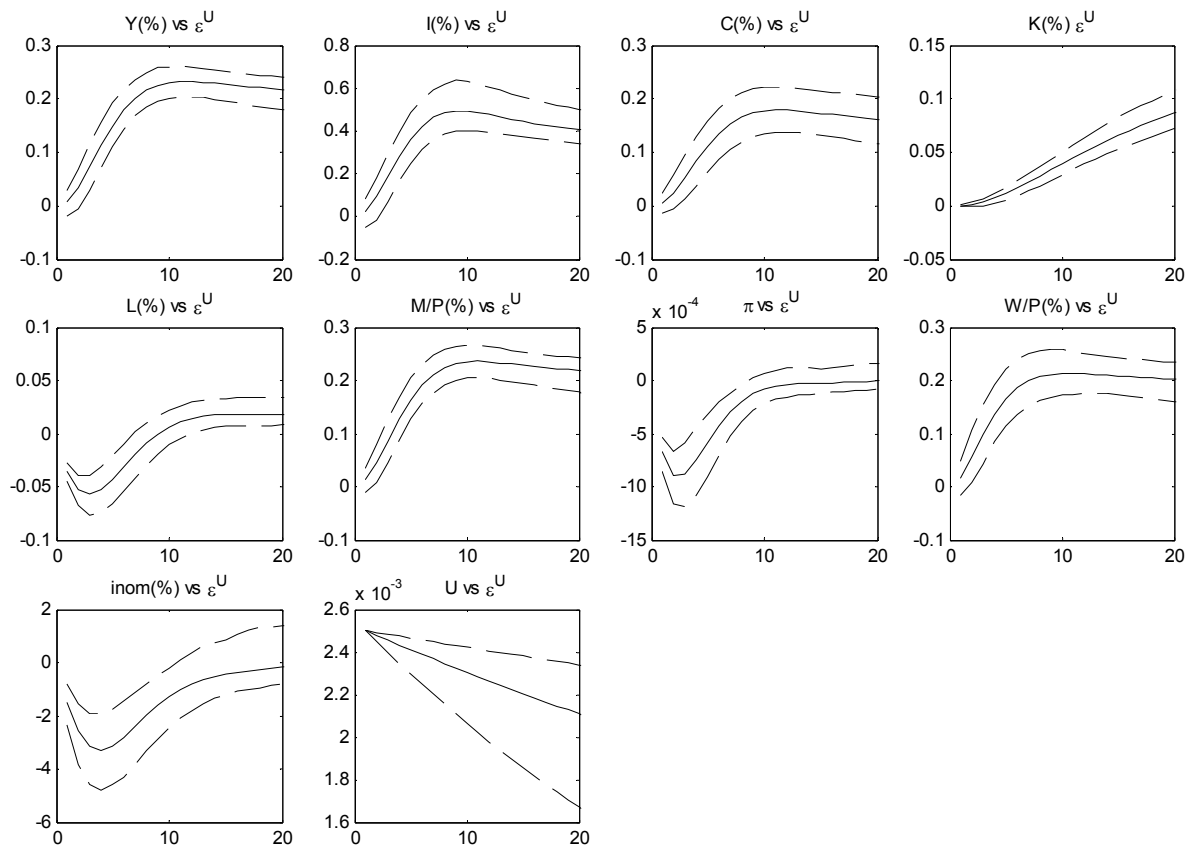
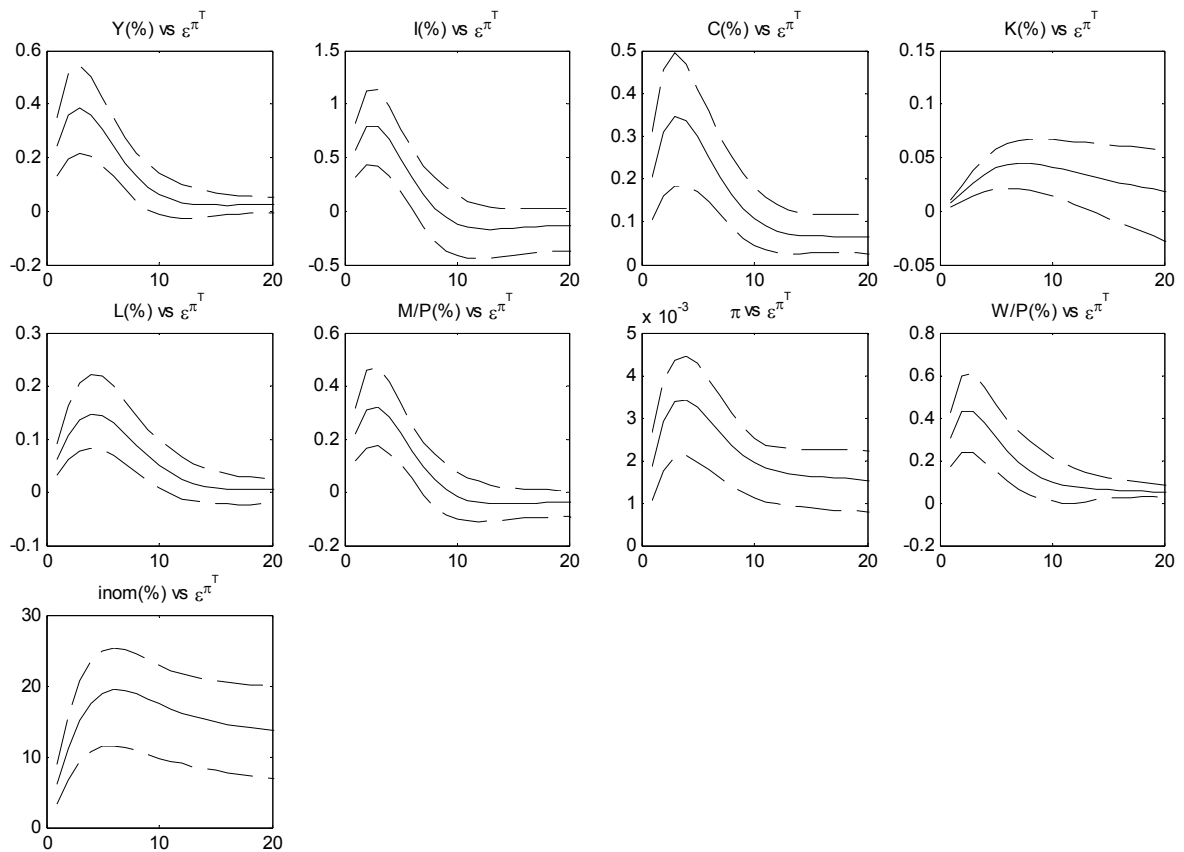


Figure 12 plots the estimated effect of a productivity shock in the model. Output, investment and consumption rise, while prices fall. However, the fall in inflation is moderated by the presence of price adjustment costs. Employment falls on impact, but over time labour supply increases as real wages are higher. Monetary policy reacts by lowering nominal interest rates, but monetary policy is not accommodating enough to prevent prices falling. It is important to note that this productivity shock is not a permanent supply shock, but fades away gradually.

Figure 13 Inflation objective shock $\varepsilon_t^{\pi^T}$



Two types of monetary policy shocks can be considered with the model. Figure 13 shows the effects of a persistent change in the inflation objective. Nominal interest rates increase immediately as inflation expectations rise. With inflation up by 0.15 percentage points, nominal interest rates are also higher by 15 percent (roughly 60 basispoints for annualised interest rates). Consumption, investment and output are all higher, with the peak response reached after three quarters.

Figure 14 Monetary policy shock ε_t^M

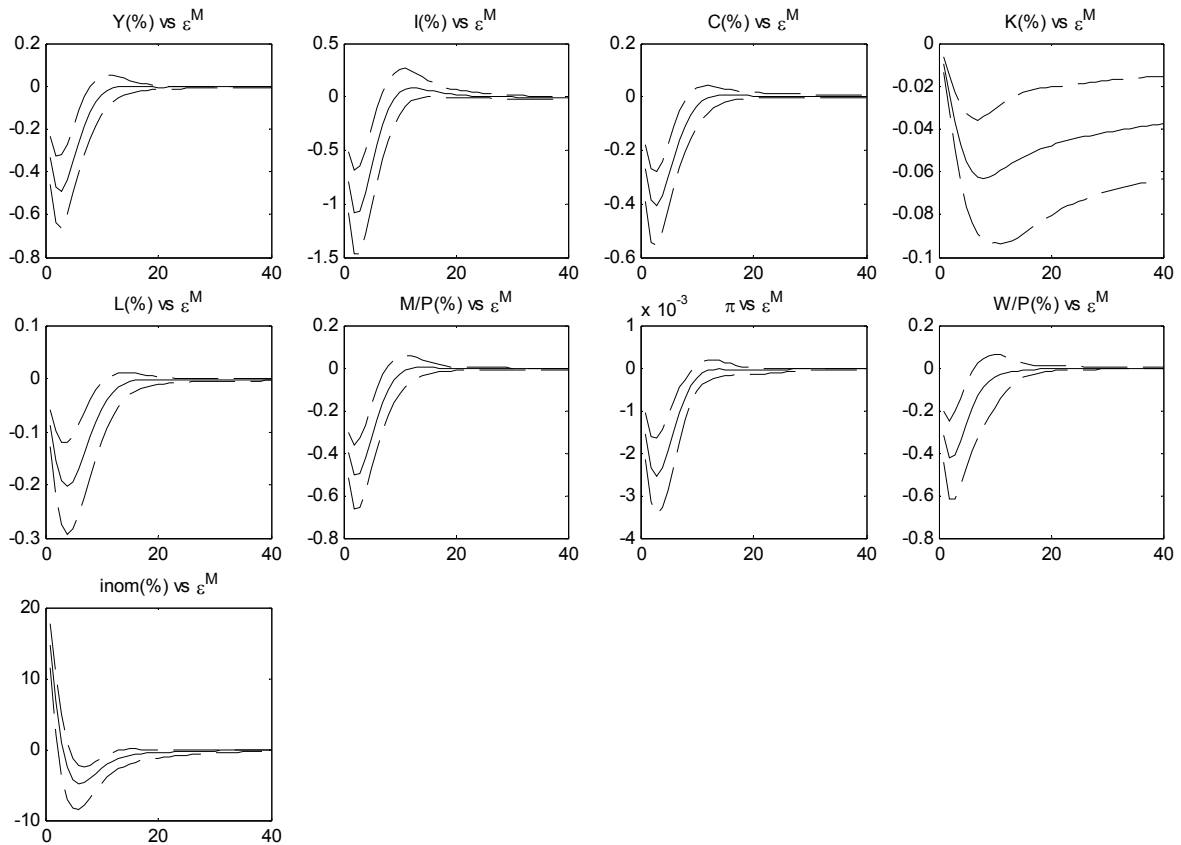


Figure 14 shows the impulse responses of a direct monetary policy shock. This shock is a transitory 1 percentage point hike in interest rates, which has disappeared after one year. The temporary increase in interest rates lowers output, investment and consumption, which all display a hump-shaped response with the peak occurring in the second and third quarter. The maximum effect on output is between 0.4-0.5%, within the range of VAR estimates and similar to simulations with the QUEST model. The output effect is mainly driven by the sharp response of investment, which maximum impact is more than twice than that of consumption.⁴ Prices fall on impact (there is thus no ‘price puzzle’ as in some VAR studies), and the speed with which prices react to the interest rate tightening is remarkably fast, with the peak inflation response reached in the third quarter. Some VAR studies have found a slower response of inflation. Again, this seems to be linked to the degree of forward lookingness in the inflation determination. The maximum effect on inflation is -0.25 percentage points on an annualised basis, but displays little persistence. Both output and price effects are within the range found in many VAR studies. Paul De Grauwe and Claudia Costa Storti (2004) find in a meta-analysis of VAR studies the output effect after one year to lie in the range between 0 and -0.7 (mean -0.33) and the price level between 0 and -0.4 (mean -0.07).

⁴ In QUEST, following a 1 percentage point interest rate shock, consumption declines on average by 0.3 per cent in the euro area and investment by 1.1 per cent.

Figure 15 Negative risk premium shock ε_t^{rp}

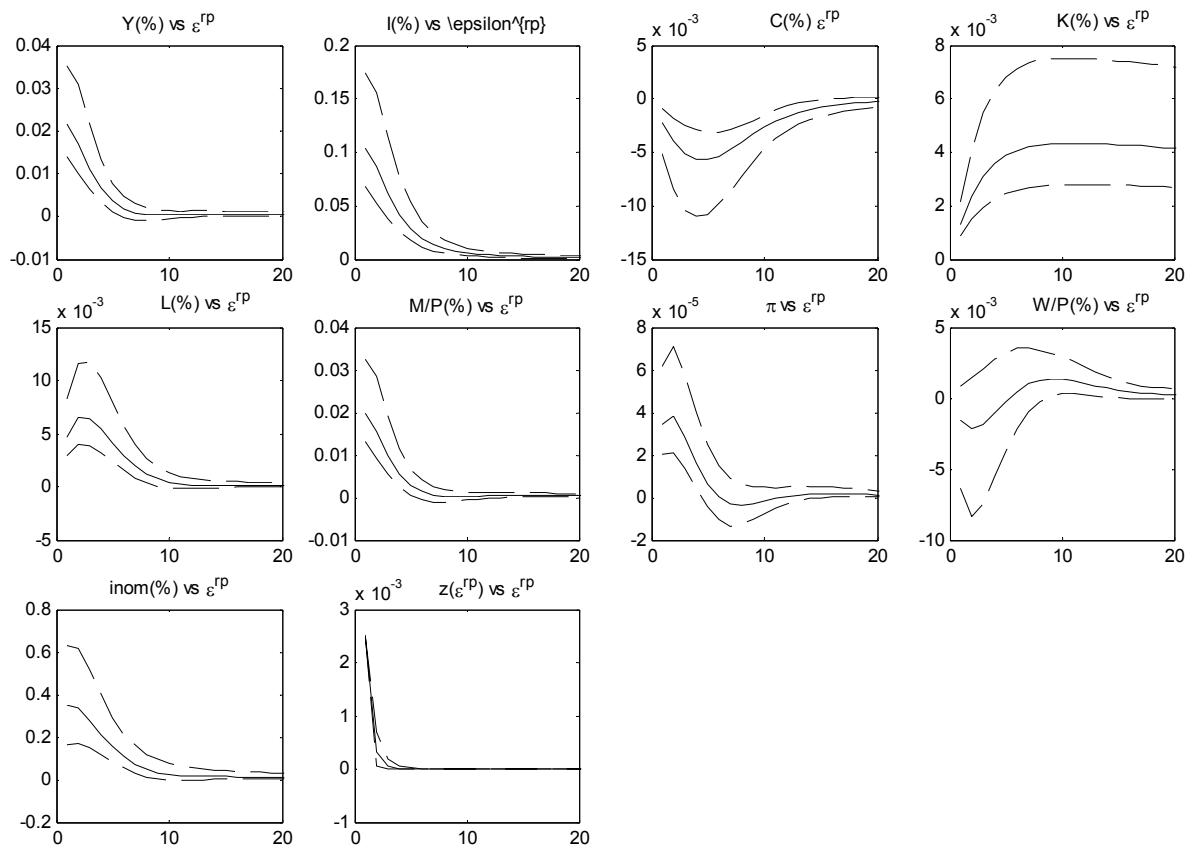
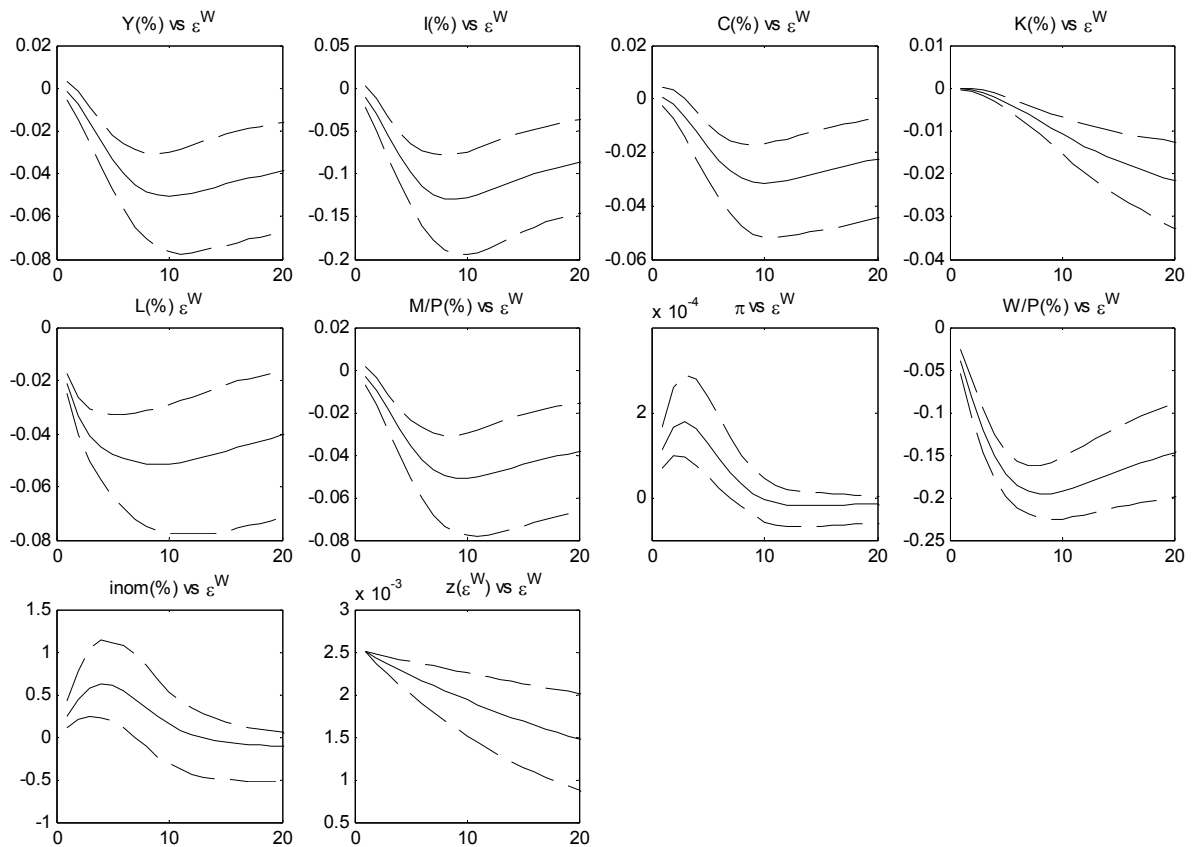


Figure 15 shows the effects of a reduction in the risk premium. This transitory reduction in the cost of capital, which displays almost no persistence, has the effect of boosting investment. Output increases, and higher interest rates raise savings and lower consumption. The increase in employment is smaller than the output effect and productivity gains lead to higher real wages.

Figure 16 Wage costs shock ε_t^W



Finally, Figure 16 shows the effects of a persistent shock to wages (non-wage labour costs) in the model. This has an immediate and persistent negative impact on employment. Investment and consumption also fall, as does output. This is a highly persistent shock, as is clear from the last chart $Z(\varepsilon_t^W)$, and the effects are long-lasting.

7 Conclusions

This paper has described our first efforts to estimate a small-scale model in the spirit of the New Keynesian models that have become the new workhorse in the macro modelling industry. Estimated for the euro economy over the period 1980-2003, the model is able to show confidence intervals of the estimated parameters which are in some cases remarkably precise.

It also is able to identify several structural shocks that have impacted on the euro economy over that period. Examples are a declining trend in government spending, reversed in recent years, a decline in price mark-ups, reflecting increased competitive pressures, a trend increase in total factor productivity in the 1980s, followed by a decline in the late 1990s and a trend increase in labour supply, reflecting a declining NAIRU. The model also produces impulse responses which are generally in line with those found in other empirical studies, with the additional benefit of providing confidence intervals for these responses which show the uncertainty surrounding them.

The model described in this paper is only our first attempt to estimate a model of this type and there are many aspects which can be improved upon in future work. As already mentioned in the introduction, the model estimated here differs in some important characteristics from the QUEST II model used in the Commission for macro economic analysis. The first objective should therefore be to adapt the model in a direction that makes it more suitable for macro economic analysis. First and foremost, the closed-economy setting of this model should be abandoned to allow for trade interactions with the rest of the world. The consumption framework should also be adjusted to allow for non-Ricardian households to make the model better suitable for fiscal policy analysis. This can be done by the introduction of liquidity constrained households in the model. The labour market specification could also be brought more in line with the QUEST model as the richness of the search bargaining labour market model would enhance the model's usefulness in analysing labour market problems.

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Annex:

A1. Results from posterior maximization

Objective function at mode: 3800.713558

parameters

	prior mean	mode	s.d.	t-stat	prior	pstdev
A2	0.050	0.0007	0.0004	1.6228	beta	0.0280
BARG	0.375	0.1176	0.1008	1.1670	beta	0.1800
DELTA	0.015	0.0126	0.0001	154.1201	beta	0.0050
G1	0.000	-0.2029	0.0213	9.5167	beta	0.4000
GAMI	15.000	20.9047	4.8567	4.3043	beta	5.0000
GAMI2	10.000	8.6001	2.7083	3.1755	beta	3.0000
GAML	15.000	26.7436	1.7125	15.6163	beta	5.0000
GAMP	15.000	22.4146	3.6221	6.1883	beta	5.0000
GAMW	15.000	6.1163	4.3002	1.4223	beta	5.0000
HAB	0.600	0.7353	0.0583	12.6078	beta	0.1500
ILAG	0.800	0.8893	0.0197	45.1071	beta	0.1000
INFLAG	0.500	0.9894	0.0047	211.4941	beta	0.2000
KAPPA	0.500	1.2019	0.3272	3.6733	gamm	0.4000
OMEG	0.200	0.3222	0.1328	2.4266	gamm	0.1500
RHO	0.900	0.9953	0.0017	572.8255	beta	0.0400
RHODELT	0.900	0.9925	0.0031	321.7083	beta	0.0400
RHOG	0.900	0.9419	0.0193	48.8082	beta	0.0400
RHOL	0.500	0.9974	0.0013	761.1845	beta	0.2000
RHOTPINF	0.500	0.0776	0.0548	1.4140	beta	0.2000
RHOW	0.500	0.9779	0.0095	102.7016	beta	0.2000
SFP	0.600	0.5890	0.0465	12.6697	beta	0.0500
SFW	0.800	0.9597	0.0376	25.5012	beta	0.1000
TAU1	-0.100	-0.1120	0.0338	3.3091	beta	0.0300
TDINF	0.200	0.2636	0.0527	4.9981	beta	0.0900
TDY	0.100	0.1282	0.0334	3.8401	beta	0.0450
THETA	2.000	2.2816	0.7984	2.8576	gamm	0.8000
TINF	1.250	1.6794	0.2004	8.3820	beta	0.3000
TY	0.300	0.2261	0.0661	3.4218	beta	0.0600
YLAG	0.900	0.9165	0.0196	46.6441	beta	0.0500

standard deviation of shocks

	prior mean	mode	s.d.	t-stat	prior	pstdev
EPS_C	0.000	0.0142	0.0040	3.5351	unif	0.2000
EPS_DELT	0.000	0.0000	0.0000	13.6287	unif	0.0001
EPS_ETA	0.000	0.0371	0.0066	5.5873	unif	0.2000
EPS_G	0.000	0.0010	0.0001	13.2884	unif	0.2000
EPS_INF	0.000	0.0022	0.0003	6.9760	unif	0.2000
EPS_L	0.000	0.0153	0.0031	4.8651	unif	0.2000
EPS_M	0.000	0.0018	0.0002	10.4079	unif	0.2000
EPS_TFP	0.000	0.0048	0.0007	6.6825	unif	0.2000
EPS_TPINF	0.000	0.0173	0.0056	3.0852	unif	0.2000
EPS_W	0.000	0.0082	0.0011	7.2337	unif	0.2000

A = 0.41207
A1 = 0.021508
BETA = 0.989
RP = -0.0067164
TAU = 0.1
ZET = 0.4

A.2 Posterior simulation:

4 parallel chains of 40000 runs; first 70% of runs discarded.
ESTIMATION RESULTS

Log data density is 3662.778532.

parameters

	prior	mean	post. mean	conf. interval	prior	pstdev
A2	0.050	0.0020	0.0007	0.0031	beta	0.0280
BARG	0.375	0.1655	0.0088	0.3003	beta	0.1800
DELTA	0.015	0.0125	0.0123	0.0127	beta	0.0050
G1	0.000	-0.2060	-0.2410	-0.1699	beta	0.4000
GAMI	15.000	14.3751	8.5182	20.2044	beta	5.0000
GAMI2	10.000	12.0733	8.0709	16.1478	beta	3.0000
GAML	15.000	26.1438	23.3680	28.7457	beta	5.0000
GAMP	15.000	22.7744	17.9836	28.1251	beta	5.0000
GAMW	15.000	12.0756	4.7057	18.8104	beta	5.0000
HAB	0.600	0.7767	0.7033	0.8526	beta	0.1500
ILAG	0.800	0.8795	0.8398	0.9162	beta	0.1000
INFLAG	0.500	0.9844	0.9740	0.9969	beta	0.2000
KAPPA	0.500	1.4513	0.7917	2.0542	gamm	0.4000
OMEG	0.200	0.3401	0.1001	0.5799	gamm	0.1500
RHO	0.900	0.9909	0.9846	0.9967	beta	0.0400
RHODELT	0.900	0.9781	0.9636	0.9932	beta	0.0400
RHOG	0.900	0.9367	0.9085	0.9691	beta	0.0400
RHOL	0.500	0.9947	0.9905	0.9982	beta	0.2000
RHOTPINF	0.500	0.1235	0.0174	0.2225	beta	0.2000
RHOW	0.500	0.9718	0.9554	0.9895	beta	0.2000
SFP	0.600	0.5936	0.5256	0.6554	beta	0.0500
SFW	0.800	0.9451	0.8970	0.9976	beta	0.1000
TAU1	-0.100	-0.1085	-0.1563	-0.0596	beta	0.0300
TDINF	0.200	0.2624	0.1814	0.3472	beta	0.0900
TDY	0.100	0.1233	0.0768	0.1766	beta	0.0450
THETA	2.000	2.6593	1.5455	3.6551	gamm	0.8000
TINF	1.250	1.5153	1.1577	1.8927	beta	0.3000
TY	0.300	0.2324	0.1286	0.3332	beta	0.0600
YLAG	0.900	0.9053	0.8745	0.9411	beta	0.0500

standard deviation of shocks

	prior	mean	post. mean	conf. interval	prior	pstdev
EPS_C	0.000	0.0193	0.0116	0.0274	unif	0.2000
EPS_DELT	0.000	0.0000	0.0000	0.0000	unif	0.0001
EPS_ETA	0.000	0.0394	0.0279	0.0499	unif	0.2000
EPS_G	0.000	0.0011	0.0009	0.0012	unif	0.2000
EPS_INF	0.000	0.0026	0.0017	0.0033	unif	0.2000
EPS_L	0.000	0.0187	0.0129	0.0251	unif	0.2000
EPS_M	0.000	0.0018	0.0015	0.0020	unif	0.2000
EPS_TFP	0.000	0.0054	0.0042	0.0065	unif	0.2000
EPS_TPINF	0.000	0.0246	0.0147	0.0346	unif	0.2000
EPS_W	0.000	0.0087	0.0066	0.0107	unif	0.2000

A = 0.41748
A1 = 0.021268
BETA = 0.989
RP = -0.0055613
TAU = 0.1
ZET = 0.4

RMSE's	VAR	model (post. mean)
C	5.9398e-006	8.5811e-006
I	5.5508e-006	8.388e-006
INOM	1.4206e-006	2.2686e-006
K	0.00037479	1.0741e-005
L	5.2953e-007	6.7359e-007
PHI	4.5673e-006	9.9136e-006
WR	1.9294e-005	2.3478e-005
Y	1.9422e-005	2.6547e-005

STEADY-STATE RESULTS at the posterior mean:

C	0.5795
DELTA	0.0124731
ETA	0.9
G	0.2062
GS	0.2062
I	0.2143
INOM	0.0111223
K	17.181
L	0.62301
LAM	1.72563
MR	0.995585
PHI	0
Q	1.1793
R	0.0111223
TFP	0
UC	1.72563
UCAP	1
VL	1.40093
WPHI	0
WR	0.858092
WRPHI	0
Y	1
YPOT	1
ZEPS_G	0
ZEPS_L	0
ZEPS_TPINF	0
ZEPS_W	0
ZPHIT	0

MODEL SUMMARY

Number of variables:	28
Number of stochastic shocks:	10
Number of state variables:	17
Number of jumpers:	6
Number of static variables:	9

POLICY AND TRANSITION FUNCTIONS

	Y	I	C	K	L	MR	PHI	WR	INOM	TFP	DELTA	ZEPS_L	ZEPS_TPINF	ZEPS_W
Constant	1.000000	0.214300	0.579500	17.181037	0.623010	0.995585	0	0.858092	0.011122	0	0.012473	0	0	0
C	(-1) 0.691604	-0.061061	0.739018	-0.061061	0.089079	0.640533	0.103558	-0.044643	0.121919	0	0	0	0	0
DELTA	(-1) -0.921541	-3.207231	2.303876	-20.012038	-0.756693	-0.917438	0.480831	2.937257	-0.000088	0	0.978102	0	0	0
INOM	(-1) -1.113647	-0.559801	-0.531870	-0.559801	-0.187520	-1.341397	-0.519187	-0.920738	0.590744	0	0	0	0	0
K	(-1) 0.000350	0.000720	-0.000377	0.988247	-0.000407	0.000504	-0.001692	-0.000218	-0.000396	0	0	0	0	0
TFP	(-1) 0.036566	0.025032	0.010812	0.025032	-0.088443	0.061370	-0.260680	0.058703	-0.063387	0.990885	0	0	0	0
WR	(-1) -0.007737	-0.006642	-0.000942	-0.006642	-0.033940	-0.008817	0.014825	0.654867	0.002830	0	0	0	0	0
Y	(-1) 0.156157	0.078496	0.074580	0.078496	0.026294	0.188092	0.072801	0.129107	-0.082835	0	0	0	0	0
YPOT	(-1) 0.261156	0.031192	0.038343	0.031192	0.040185	0.239184	0.117195	0.088222	0.052860	0	0	0	0	0
ZEPS_G	(-1) 0.753786	-0.122981	-0.074826	-0.122981	0.113085	0.697144	0.122341	-0.078216	0.135366	0	0	0	0	0
ZEPS_L	(-1) 0.003813	0.006729	-0.002991	0.006729	-0.028943	-0.001178	0.046149	0.176897	0.012631	0	0	0.994667	0	0
ZEPS_TPINF	(-1) 0.010028	0.010458	-0.000628	0.010458	0.001385	0.009281	0.001562	-0.000679	0.001784	0	0	0	0.123527	0
ZEPS_W	(-1) -0.007810	-0.009548	0.001892	-0.009548	-0.051924	-0.011851	0.043509	-0.133395	0.010346	0	0	0	0	0.971833
ZPHIT	(-1) 0.965873	0.471622	0.475191	0.471622	0.151350	0.857092	0.739999	1.055642	0.265371	0	0	0	0	0
I	(-1) 0.686645	0.711347	-0.038252	0.711347	0.087450	0.636371	0.098649	-0.046270	0.119951	0	0	0	0	0
L	(-1) 0.064856	0.047840	0.015737	0.047840	0.642295	0.076473	-0.149045	0.611962	-0.030221	0	0	0	0	0
PHI	(-1) -0.067443	-0.052890	-0.013222	-0.052890	0.001112	-0.089388	0.554715	-0.350633	0.056476	0	0	0	0	0
WPHI	(-1) -0.000398	-0.000343	-0.000047	-0.000343	-0.000170	-0.000453	0.000760	0.031513	0.000145	0	0	0	0	0
EPS_C	0.115219	-0.010173	0.123118	-0.010173	0.014840	0.106711	0.017253	-0.007437	0.020311	0	0	0	0	0
EPS_DELT	-0.942172	-3.279035	2.355455	-20.460072	-0.773634	-0.937977	0.491596	3.003017	-0.000090	0	1.000000	0	0	0
EPS_ET	-0.018206	-0.011203	-0.006644	-0.011203	-0.001719	-0.022811	0.054838	-0.046910	0.011894	0	0	0	0	0
EPS_G	0.804709	-0.131289	-0.079881	-0.131289	0.120725	0.744241	0.130606	-0.083500	0.144511	0	0	0	0	0
EPS_INF	0.981222	0.479117	0.482742	0.479117	0.153755	0.870712	0.751758	1.072418	0.269588	0	0	0	0	0
EPS_L	0.003834	0.006765	-0.003007	0.006765	-0.029098	-0.001185	0.046397	0.177845	0.012699	0	0	1.000000	0	0
EPS_M	-1.266181	-0.636475	-0.604719	-0.636475	-0.213204	-1.525125	-0.590299	-1.046849	0.671656	0	0	0	0	0
EPS_TFP	0.036902	0.025263	0.010912	0.025263	-0.089257	0.061934	-0.263078	0.059243	-0.063970	1.000000	0	0	0	0
EPS_TPINF	0.081179	0.084659	-0.005082	0.084659	0.011212	0.075134	0.012647	-0.005499	0.014440	0	0	0	1.000000	0
EPS_W	-0.008037	-0.009825	0.001947	-0.009825	-0.053429	-0.012194	0.044770	-0.137261	0.010646	0	0	0	0	1.000000

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
Y	1.0000	0.0259	0.0007
I	0.2143	0.0119	0.0001
C	0.5795	0.0113	0.0001
K	17.1810	0.7255	0.5263
L	0.6230	0.0065	0.0000
MR	0.9956	0.0254	0.0006
PHI	0.0000	0.0145	0.0002
WR	0.8581	0.0210	0.0004
INOM	0.0111	0.0120	0.0001
TFP	0.0000	0.0186	0.0003
DELTA	0.0125	0.0002	0.0000
ZEPS_L	0.0000	0.0242	0.0006
ZEPS_TPINF	0.0000	0.0025	0.0000
ZEPS_W	0.0000	0.0106	0.0001

VARIANCE DECOMPOSITION (in percent)

	EPS_C	EPS_DELT	EPS_ETA	EPS_G	EPS_INF	EPS_L	EPS_M	EPS_TFP	EPS_TPINF	EPS_W
Y	0.02	1.27	0.01	1.42	10.82	9.12	13.53	62.85	0.01	0.95
I	0.01	7.90	0.01	7.95	9.27	6.31	12.53	54.76	0.07	1.19
C	0.15	1.35	0.01	3.65	21.12	8.46	17.54	47.22	0.01	0.50
K	0.00	20.49	0.00	3.74	0.77	8.61	0.82	64.73	0.01	0.83
L	0.02	1.67	0.01	3.15	12.06	52.76	18.12	5.84	0.01	6.36
MR	0.02	1.28	0.01	1.21	7.41	9.44	14.63	65.02	0.01	0.98
PHI	0.00	0.01	0.01	0.21	86.39	0.11	11.64	1.55	0.00	0.07
WR	0.00	0.53	0.02	0.16	16.58	0.53	10.73	56.24	0.00	15.20
INOM	0.01	0.01	0.00	0.47	95.62	0.05	3.22	0.60	0.00	0.02
TFP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00
DELTA	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ZEPS_L	0.00	0.00	0.00	0.00	0.00	100.00	0.00	0.00	0.00	0.00
ZEPS_TPINF	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	0.00
ZEPS_W	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
Y	0.9779	0.9379	0.8925	0.8494	0.8125
I	0.9774	0.9328	0.8818	0.8336	0.7927
C	0.9760	0.9269	0.8688	0.8118	0.7616
K	0.9999	0.9997	0.9993	0.9988	0.9982
L	0.9846	0.9484	0.9008	0.8499	0.8012
MR	0.9758	0.9351	0.8900	0.8478	0.8121
PHI	0.9767	0.9284	0.8712	0.8156	0.7671
WR	0.9770	0.9341	0.8878	0.8460	0.8113
INOM	0.9807	0.9589	0.9365	0.9146	0.8940
TFP	0.9909	0.9819	0.9729	0.9640	0.9552
DELTA	0.9781	0.9567	0.9357	0.9152	0.8952
ZEPS_L	0.9947	0.9894	0.9841	0.9788	0.9736
ZEPS_TPINF	0.1235	0.0153	0.0019	0.0002	0.0000
ZEPS_W	0.9718	0.9445	0.9179	0.8920	0.8669