Assessment of GDP forecast uncertainty

by

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Acknowledgements
I have benefited from discussions with and comments from Filip Keereman, Lars Jonung, Harry Huizinga, and Douglas Koszerek. I would also like to thank Jeanette Bell and Martine Maebe for secretarial assistance and participants at the DG ECFIN internal seminar. Shortcomings, errors, and any views expressed in this paper are solely the responsibility of the author, and should not reflect upon the European Commission, or be interpreted as the views of the European Commission.

ISBN 92-894-5643-4
KC-AI-03-005-EN-C
ECFIN/215/03-EN
This paper only exists in English.

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Abstract

This paper develops an approach to measure the uncertainty surrounding expected GDP growth that prevails in the economy. This is accomplished by making use of consensus forecasts of GDP growth and by studying the properties of distributions of forecasted euro area GDP growth. A euro area distribution is constructed from the mean distributions of individual country specific consensus forecasts. Information contained in the distributions can be used to make uncertainty assessments of future economic development. The paper shows that uncertainty varies over time, and how the levels can be compared with a historical mean and between different time periods. Furthermore, the paper shows that the constructed distributions can be asymmetric as measured by their skewness. This information can be used to assess whether risks are on the upside, or the downside. Two graphs are proposed to be used as a regular monitoring tool, illustrating the measured uncertainty and balance of risks.
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1. Introduction

The most important single measure of aggregate production in an economy is the gross domestic product (GDP), a statistic that aims to measure the total value of goods and services produced within the national territory during a given period of time. Forecasts of the future development of this measure are an essential input in both public and private decision making. Governments use forecasts to predict, for example, the sustainability and evolution of publicly financed social welfare systems. GDP forecasts are used to make forecasts of future tax returns, so that policy makers can take active measures and decide on the design of government budgets. In the private sector, GDP forecasts are inputs in the strategic decision making, e.g. when choosing which markets to penetrate, or when forming an anticipation of cash flows for decisions concerning investment expenditures etc. GDP forecasts are, thus, of vital interest to both government and private institutions.

Trying to predict the future is always a risky business. The question is not so much of whether the prediction is right or wrong, but of how much it will deviate from the actual outcome. In making predictions of GDP, forecasters by necessity take on noise or uncertainty in their forecast. First, they meet constraints on how much information can be gathered. These constraints are both physical and economical, e.g. all data are simply not available, and it costs money to gather information. Therefore, forecasters have both different types and different amounts of information to form their beliefs about future GDP developments. Second, recent data on numerical variables that are used when forecasting are usually estimations and subject to future revisions. As such they introduce noise into the forecasts. Third, subjective considerations have to be taken, concerning for instance, which theory to rely on, or which econometric methodology to adopt. Finally, there are future events that cannot be predicted, thus putting restriction on the accuracy of forecasts. All these factors affect the accuracy of the predictions, and the uncertainty that surrounds them. In this environment, assessing the amount and the form\(^1\) of forecast uncertainty is important.

Besides improving the methodology of forecasting, to acquire a higher degree of accuracy, forecasts can be complemented with assessments of the uncertainty that surrounds the predictions. One way to assess uncertainty is to use different scenarios. This can be done by sensitivity analysis, which amounts to changing the assumptions of one or more variables underlying the forecast. Another possibility is to use econometric modelling, producing a forecast with statistical confidence bands, based on the variation in the underlying data. These measures of uncertainty and estimations indicate where and to what extent the uncertainty concerning the forecast lies.

The objective of this paper is to develop an alternative methodology for assessing the amount and form of forecast uncertainty surrounding the euro area GDP growth forecast. The focus is on measuring uncertainty in practice, not on theoretical issues related to how individual forecasts are optimally combined, or how to improve forecasting methodology. The methodology for measuring uncertainty can potentially be used as an input in a forecasting exercise to set confidence bands around the forecast, for determining forecasters’ views of which direction is the more plausible one for a deviation of the forecast from the actual outcome, or to indicate in which

\(^1\) For example, whether uncertainty is on the upside or the downside.
direction a forecast will be revised. The paper is organised as follows: Section 2 develops a theoretical framework to decide what measures of uncertainty are the most appropriate, and what kind of data can be useful in reaching the objective. Section 3 describes the consensus data employed in this paper, and gives some descriptive statistics. Section 4 develops hypotheses and tests of the information contents of the data. Furthermore, suggestions are made for presentable information, such as graphs, tables, and statistics. Section 5 concludes the paper.

2. Measuring forecast uncertainty and its asymmetry

2.1. Uncertainty

A first issue to resolve is what measure of uncertainty is reasonable to use when assessing the ambiguity surrounding output growth forecasts. In the academic literature there are three main candidates for measuring uncertainty, all have been used in previous research and macroeconomic analysis:

- Disagreement among forecasters – The variation of mean predictions among forecasters.
- Average individual forecast uncertainty – The average of forecasters’ variation around their mean prediction.
- Variance of aggregate forecast distribution – The variance of an aggregated distribution made up of individual forecaster’s distributions.

The meaning of the three different uncertainty measures can be made clearer through the construction of an example. Assuming there are three different forecasters making predictions of euro area GDP growth, they all make different predictions of the growth rate, and they also differ in how uncertain they are about their predictions. Table 1 presents assumed figures of forecasters’ growth predictions and their corresponding uncertainty measures. The second column in Table 1 contains the publicly announced growth rates, and the third column contains the uncertainty that the forecasters have regarding their own growth forecasts. In the example forecasters’ uncertainty is given in terms of their variance.

Table 1: Example with three forecasters and their respective growth forecasts with corresponding uncertainties

<table>
<thead>
<tr>
<th>Forecaster</th>
<th>Predicted GDP Growth (%)</th>
<th>Uncertainty (var)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0.04</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Figure 1a shows forecasters’ distributions corresponding to the information given in Table 1. The midpoint (the mean) in each of the distributions equals the predicted growth rate.

---

GDP growth rate. The width of the distributions corresponds to the uncertainties (the variances) of forecasters’ predictions. From Table 1 and Figure 1a it is clear that forecaster A has the lowest predicted growth rate, but at the same time A is the forecaster most certain about his growth forecast. Forecaster C has the highest predicted growth rate, but is the least certain about the prediction.

Figure 1: Forecasters’ distributions corresponding to the example in Table 1

The first measure of uncertainty recognises that different forecasters disagree on what the growth rate will be (in this case 1%, 1.5%, and 2%). An observer of these forecasts would be less certain about how to interpret these growth rates if the predicted growth rates were wider apart. One way to measure this kind of uncertainty (how much forecasters disagree) is to measure the dispersion of the forecasted growth rates. One such measure is the variance of the growth rates in column 2, and can be calculated to be 0.25 in the example.

The second measure of uncertainty is a more direct measure, recognising that each individual forecaster is uncertain about its own predictions. Since some forecasters are more certain than others, an observer of forecasts can get an idea of how much uncertainty there is on average to predicted growth. In the example, A is the least uncertain forecaster with a variance of 0.04, the other two forecasters, B and C, follow in an increasing order of uncertainty. The average uncertainty can be calculated as the mean of the numbers in column 3 of Table 1, and results in an uncertainty number of 0.10 in the example.

The third measure incorporates both that forecasters disagree about the forecasted growth rate and their differences in individual uncertainty. This is done by first merging (“adding” them together) the individual distributions in Figure 1a into an aggregated distribution depicted in Figure 1b, and then calculate the variance of the aggregated distribution. By merging the distributions both the different mean predictions and the individual uncertainty are used in determining uncertainty. The variance of the aggregated distribution is in the example 0.35, the sum of the two other measures.

In order to more formally discuss the relationship between the first two and the last of these different measures, a model is constructed along the lines of Giordani and Söderlind (2001). The model is slightly extended to allow for a situation resembling
the euro area, where there are many forecasters in several countries producing individual forecasts for one specific country. This model allows for aggregating country forecasts into a euro area forecast.

In this model there are many individual forecasters that face different, but correlated information sets. Each forecaster uses a model and gathered information to produce the best possible forecast for one specific country within the euro area. The information set and the model of forecaster \( i \) is summarised by the scalar signal \( s_{c,i} \), where \( c \) denotes the country for which forecaster \( i \) is predicting the yearly GDP growth rate. The scalar signal is a single number that can be seen as a composite index, summarising all information available to a forecaster including the used forecasting methodology.

One way to formalise the discussion is to think of both future output growth and forecasters signals as random variables. First, let \( pdf(G_c|s_{c,i}) \) be the probability density function of forecasted GDP growth in country \( c \) conditional on receiving the signal of forecaster \( i \). The mean and variance for this distribution are denoted by \( \mu_{c,i} \) and \( \sigma^2_{c,i} \), which can be different for different forecasters. Second, let \( pdf(s_{c,i}) \) be the density function of receiving the signal for country \( c \) of forecaster \( i \) in the same time period. Then the aggregate country specific distribution in that period is \( pdf_A(G_c) \), which is the average distribution across forecasters, and amounts to calculating the marginal distribution of \( G_c \),

\[
pdf_A(G_c) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} pdf(G_c|s_{c,i})pdf(s_{c,i})ds_{c,i}.
\] (1)

The variance of the aggregated distribution is calculated to see how the distribution is related to individual uncertainty and disagreement among forecasters. If the moments exist, the variance of the aggregated distribution in equation (1) is

\[
Var_A(G_c) = Var(\mu_{c,i}) + E(\sigma^2_{c,i}).
\] (2)

Equation (2) shows that the variance of the aggregate country specific distribution can be decomposed into the variance of the forecasters’ means, i.e. their “collective” disagreement, and the average of the forecasters’ variance, i.e. average “individual” uncertainty.

---

3 This indicates that a first reasonable measure of output growth uncertainty is the average across forecasters individual uncertainty. At this preliminary stage that amounts to calculating the expected value of the individual variances of the probability density function for one specific country forecast (i.e. keeping \( c \) fixed), denoted by \( E(\sigma^2_{c,i}) \). This calculation only measures the uncertainty for forecasts concerned with a specific country.

4 For any random variables \( y \) and \( x \) the

\[
Var(y) = E(y^2) - [E(y)]^2 = E[E(y^2|x)] - [E[E(y|x)]]^2 = E[E(y^2|x)] - E[E(y|x)]^2 + E[E(y|x)]^2 - [E[E(y|x)]]^2 = E[Var(y|x)] + Var(E(y|x))
\]
The country specific distributions can be aggregated to a euro area distribution by adding the separate country distributions. Treating the country specific GDP growth variables as independent\(^5\), adding the country distributions, and then calculating the variance, results in a variance for the forecasted euro area GDP growth rate equal to the weighted sum of the individual country specific variances:

\[
\text{Var}_{\text{EUR}}(G_{\text{EUR}}) = \sum_c w_c^2 \text{Var}_c(G_c) = \sum_c w_c^2 \text{Var}(\mu_{c,j}) + \sum_c w_c^2 E(\sigma_{c,j}^2). \quad (3)
\]

The country weight, denoted \(w_c\), is the country’s GDP ratio to the total euro area GDP. In equation (3), the variance for the euro area retains the decomposition of forecaster disagreement (collective uncertainty) and individual uncertainty as their weighted averages.

Equation (3) contains the three possible measures of uncertainty presented in the beginning of this section, and how these measures are related to each other:

i. Disagreement on the most likely outcome, \(\sum_c w_c^2 \text{Var}(\mu_{c,j})\).

ii. Average standard deviation of individual probability density functions, \(\sum_c w_c E(\sigma_{c,j}^2)\).

iii. Variance of the aggregate probability density function, \(\text{Var}_{\text{EUR}}(G_{\text{EUR}})\), which is the sum of the two other measures.

Each of these measures has its pros and cons. The choice of uncertainty measure depends on what the measure is going to be used for and its availability. The advantages and drawbacks of these measures are reviewed as follows.

i. The first measure, disagreement on the most likely outcome, has the advantage of being readily available and easy to compute. Survey data exists, and the computation amounts to calculating the variance of forecasters’ supplied predictions. The drawbacks are that the measure becomes meaningless if the number of forecasters reduces to one, or when forecasters have the same information and employ the same model when forecasting. In this case, the measure of uncertainty simply collapses to zero.

ii. Average individual uncertainty, equal to the average variance of individual probability density functions, does not exhibit the drawbacks of the first measure. It can be viewed as the uncertainty of a representative forecaster, but it neglects the fact that forecasters may disagree on the outcome of forecasted GDP growth, and disagreement should reflect some type of uncertainty.

---

\(^5\) The assumption of independence simplifies the merging of the separate country distributions, which amounts to calculating a multidimensional integral. The assumption is necessary to handle the computation needed to form the aggregated distribution in the empirical section. The methodology applied is described in section 3.

\(^6\) The independence assumption is used directly when forming the variance of the aggregated distribution by adding the variance of the country specific distributions.
iii. The last measure of uncertainty is based on the aggregated probability density function. The variance of this distribution is higher than the average variance of individual distributions, which can be interpreted as if individual forecasters underestimate uncertainty. This distribution causes some puzzlement regarding the interpretation. It is well established, both theoretically and empirically, that combining forecasts from different forecasters reduces the forecast error variance (e.g. see Granger and Ramanathan (1984), Zarnowitz (1967), and Figlewski (1983)). On the other hand, the aggregated distribution is less informed than the individual forecaster’s distribution since it has a higher variance. The lower forecast error variance suggest that combined forecasts are better than individual forecasts, but only if disagreement is large compared to individual uncertainty (Giordani and Söderlind (2001)). The higher variance of the aggregated distribution suggests that individual forecasters consistently underestimate uncertainty.

Whether the last statement is true or not is an empirical question. It is studied by Giordani and Söderlind (2001) using survey inflation data. They show that forecasters do seem to underestimate inflation uncertainty on average, but they also manage to show that the disagreement measure is highly correlated with the aggregate measure. Their analysis suggests that the more obtainable variance of the mean-distribution is a good approximation for inflation uncertainty.

Surveys are continuously made of forecasters’ views of the mean economic outlook, but unfortunately very few of these surveys include information on individual forecasters’ uncertainty. Due to the lack of data on individual distributions and uncertainty, it is difficult to produce an overall aggregated distribution for the euro area. Limited by data availability, this paper will employ the measure \( \sum w_c \text{Var}(\mu_{c,d}) \) as the measure of uncertainty, measuring disagreement on the most likely outcome among forecasters. The problems with this measure are mitigated, as this study is not only interested in measuring the actual degree of uncertainty, but also how uncertainty evolves over time. In this respect, the high correlation with the aggregate measure is an advantage.

### 2.2 Asymmetries in uncertainty

The terms upside and downside risk to a forecast are often used when discussing predictions of economic variables. How the terms are used probably differs among forecasters. In any case the terms express some kind of asymmetry in the uncertainty that surrounds a forecast. It is beyond the scope of this paper to discuss, or define what different forecasters mean by downside or upside risks. Nonetheless it is interesting to study the asymmetry of the forecasted GDP distribution. Potentially it is possible to assess if more or less probability is assigned to a wider range of values to the left or the right in the distribution, i.e. if there is a higher variance in the left or right tail of the distribution. Furthermore, it might be possible to make a statement on which is the most plausible direction of a deviation from the initial mean forecast, or if the forecast bias (here defined as \( G_{EUR} - \tilde{\mu}_{EUR} \), where \( \tilde{\mu}_{EUR} \) is the median of the forecasted euro area growth distribution) has a higher probability to be larger to the left or to the right of the median forecast.
In a plot of a density function, the probability of an outcome within a value range is the area bounded by that range and the graph. In case data are distributed symmetrically about their central value as in Figure 2a, large values are no more likely than small ones. By contrast, the distribution in Figure 2b has a long tail to the right, with more abrupt cut-off to the left. Such distributions, which are said to be skewed to the right, have the characteristic that their mean exceeds their median. The distribution in Figure 2c depicts the opposite situation. Here, the distribution is skewed to the left, so that the lowest observations extend over a wide range, but the highest do not.

**Figure 2:** Examples of symmetric and non-symmetric distributions

Skewness is a measure of the asymmetry of a distribution like the ones in Figure 2b and Figure 2c. For symmetric distributions, e.g. like the normal distribution in Figure 2a, skewness is zero. For asymmetric distributions, the skewness will be positive if the “long tail” is in the positive direction, i.e. skewed to the right.

Developing an aggregated euro area distribution of GDP forecasts would make it possible to study the skewness, thus assessing whether probability is assigned asymmetrically to lower and higher outcomes. Unfortunately there are no easily obtainable surveys of forecasters’ distributions of their projection for the euro area or the Member States. In an attempt to draw some conclusions about asymmetric risks, a euro area distribution of the mean forecasts is derived. This distribution has its drawbacks discussed in the empirical section. The variance of this distribution is the disagreement measure discussed in the previous section.

### 3. Survey data and the construction of a euro area consensus forecast

The data employed comes from the economic survey organisation Consensus Economics. On a monthly basis Consensus Economics survey financial and economic forecasters for their estimates of a range of macroeconomic variables including future growth in gross domestic product, inflation, interest rates, and exchange rates. The survey results are published in the monthly publication Consensus Forecasts, covering mean figures of GDP growth consensus forecasts (and other variables) for more than twenty countries. More detailed information about the consensus forecasts is published for twelve countries, containing the mean, standard deviation, the maximum, and the minimum. In addition, for these twelve countries, the survey
presents the individual forecasters’ figures for each variable, which makes it possible to form sample distributions for the mean forecast, and to do analysis that go beyond the standard descriptive statistics.

The used subset of data contains forecasted GDP growth figures from individual forecasters, dating back to January 1990. The survey is conducted on a monthly basis asking for predicted annual average growth rates for the present year and the year after. At most there is thus 24 consecutive months of updated forecasts for each year predicted. Detailed information of each individual forecaster’ predicted value exists for three countries within the euro area prior to 1995: France, Germany, and Italy. From January 1995 detailed information also exists for the Netherlands and Spain and is accordingly also added to the used subset.

As a preview of the data, Table 2 shows some descriptive statistics of the survey data for each of the five countries over the entire time spanned. The number of institutes surveyed for each country is fairly constant over time. This makes comparisons of yearly mean distributions of different years more independent of the number of observations. Still, there is a sharp increase in the total number of respondents when the Netherlands and Spain are included from January 1995, which can have an effect on e.g. aggregated distributions, as will be discussed later in this paper.

The purpose of this paper is to make a risk assessment of forecasted euro area GDP growth. Since the survey does not consider the euro area as a separate economic entity, a euro area consensus forecast has to be deduced by aggregating national forecasts from euro area member countries. The data consists of the three largest economies in the euro area (Germany, France, and Italy) during the years 1990 to 1994. The three countries represent more than 69% of the total euro area GDP during this time period. After 1995 the next two largest euro area economies (Spain and the Netherlands) are also included in the detailed data set. The five countries together represent more than 85% of the total euro area economy.

The individual forecasted GDP figures for each country make up a country specific consensus distribution of the mean annual growth rate. Adding together these country specific distributions forms the forecasted euro area consensus distribution. Treating the country specific distributions as discrete enables a straightforward approach to form the euro area distribution. Forecasted GDP for each country is seen as a random variable, letting the forecasted GDP figures make up the sample distribution. The stochastic variable takes the value of the supplied growth rates with the probability equal to their relative frequency of occurrence in the data set. The country specific probability function is represented by \( P_{cG}(g_c) \), where \( G_c \) is the stochastic variable forecasted GDP for country \( c \), and \( g_c \) are the values the random variable can take, which are any real value to one decimal point.

The forecasted euro area GDP growth is the weighted sum of forecasted GDP growth in the constituent countries,

\[
G_{EUR} = \sum_c w_c G_c, \text{ where } c = \{D, F, I, E, NL\}. \tag{4}
\]
Table 2: Descriptive statistics of the consensus forecasts data set of GDP growth

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean std. of GDP forecast</th>
<th>Mean skew. of GDP forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>-0.15</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>-0.09</td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>-0.05</td>
</tr>
<tr>
<td>Netherlands</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>-0.59</td>
</tr>
<tr>
<td>Spain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.48</td>
</tr>
</tbody>
</table>

The country weight, denoted $w_c$, is the country’s GDP ratio to the total GDP of the euro area countries included, measured in euro at current prices. The country weights are taken from AMECO, a European Commission database. Assuming the forecasts for the country specific growth rates are independent of each other (i.e. independent $G_c$’s), the probability function for the euro area is

$$P_{EUR}(g_{EUR}) = \sum_{c} \prod_{c} P_{G_c}(g_{c}), \text{ where } c = \{D, F, I, E, NL\}. \quad (5)$$

The calculation becomes a programming exercise in finding all possible combinations of adding the supplied forecasts. The result is a discrete distribution with as many as 100,000 possible outcomes for the euro area distribution in any specific month. Since GDP growth is a continuous variable, rather than discrete, and many of the discrete outcomes are very close to each other, it makes sense to group them together into bins, such that a histogram is formed. In this way the euro area distribution can be made to look more like a continuous sample distribution. In order to do so, a bin size of 0.05% is chosen. Figure 3 presents one example of what the discrete and continuous distributions look like in one specific month.
4. Uncertainty and its properties

4.1. Are the mean distributions normal?

Measuring uncertainty by measuring the degree of disagreement, $Var_{A\text{EUR}}(G_{EUR})$, involves calculating the variance of a mean distribution. It is well known that the distribution of the mean of a sample tends to approximate normality as the sample size grows regardless of the distribution of the individual observations. This is a large-sample distribution theory result, which thus raises the question whether the sample size is large enough to induce the constructed euro area distributions to become normal. Thus, the first hypothesis tested is:

**Hypothesis 1**: The euro area distributions are normally distributed.

The null hypothesis of a normal distribution is tested using the Jarque-Bera test statistic. At the 5% significance level, the null hypothesis can be rejected for 95% of the distributions in the sample.

This result leads to the conclusion that the large sample properties of the data are not valid for this sample. To see what other information can be contained in the data, it is justified to further study the properties of these distributions without an assumption of normality.

4.2. Declining, but varying variance

All standard deviations and variances are presented in Table 4 and Table 5. The first column contains the predicted years, and the first row contains the months when the forecast was made. The first 12 months are associated with the year before the predicted year, and the last 12 months are associated with forecasts made during the same year as the one being predicted, e.g. for the predicted year 1992, the first 12 months comes from 1991, and the following 12 months come from 1992.

---

7 This is true for distributions with finite variance.
Two statements can be made regarding the variance of the distributions, which is the measure of uncertainty employed. First, the distribution is affected by the changing information set that occurs over time. Second, the distribution depends on the number of observations contained in the sample. How these statements affect the variance are now dealt with below.

First, since the forecasting for a specific year is done on a monthly basis over a period of two years, the information set that the forecasting institutes draw upon expand over time. As an example, to predict the euro area growth in 2002, there is much less information available in January 2001 (the first prediction month for 2002) than there will be in December 2002 (the last prediction month for 2002). Besides having about half to two thirds of the actual outcome already realised, almost everything that will determine GDP in 2002 will have occurred by December 2002. It is therefore reasonable to believe that predictions made at the end of the forecasting period for a specific year are less uncertain than predictions made in the beginning of the forecasting period. Thus, the second hypothesis is:

**Hypothesis 2:** For a specific year that is forecasted, the variance of the distributions is constant or increasing over the forecasting period, i.e. $Slope \geq 0$.

In Figure 4 the variance of the distributions in a forecasting period are plotted for each year predicted. At first glance at the graphs in Figure 4, it seems like the variance in general declines over time in the different forecasting periods (one for each year predicted). This indicates that the hypothesis should be rejected.

To test the hypothesis more formally, the time period for each year predicted is regressed upon their respective variances. The slope coefficient from this regression is the time trend of the variances in Figure 4. It turns out that for 11 out of the 14 years predicted, the distributions exhibit a decreasing variance over the prediction period, and thus decreasing uncertainty. The three years where the null hypothesis cannot be rejected are 1990, 1993, and 2003. As the graph for 1993 shows, there was considerable uncertainty regarding the future GDP growth at the end of 1992 and the beginning of 1993, associated with the currency crises that many countries experienced during this particular time period. Depicted in the graph for the forecasted year 2003, the variance increased from August to October 2002, reaching similar levels as for the forecasted year 2002 that spread right after the terrorist attack on New York, the 11 September 2001. This reflects increased uncertainty regarding growth in 2003, maybe stemming from the threat of a military conflict with Iraq and the collapse of stock markets with a negative impact on confidence.

Besides showing that the hypothesis can be rejected in most cases, the closer look at the variance of the distributions also indicate that uncertainty varies over time, and measures increased forecasting risk as different events occur and new information disseminates.
Figure 4: Variance time plots, equally scaled, 1990-2003
Turning to the second statement, it seems as if the variance in the forecasting periods prior to 1995 is higher than in the following forecasting periods. This observation is true, and is explained by the theoretical fact that the variance of the mean of a random sample has a variance equal to the variance of the underlying distribution divided by the number of observations. So, the variance of the mean declines as the sample size increases. The apparent shift in the level of uncertainty in 1995 is explained by the expanding sample that occurs with the inclusion of Spain and the Netherlands into the sample. The inclusion of these two countries in the data set increases the average number of forecasting institutes by 21, which on average gives rise to 108 times as many ways to combine predicted growth rates when deriving the euro area distributions. The result is a lower variance, but there is still considerable variation in uncertainty over time, and normality is still rejected. The advantage is that there are less obscure distributions, as is the case in the first couple of years of the sample. The variation in variance can be observed in Figure 4 and Figure 7. The only difference between the graphs in the two figures is that in Figure 7 the graphs are not identically scaled along the y-axis.

The two points made are further illustrated in Figure 5 and Figure 6. Figure 5 is a time plot of the variances for all predicted years, but since the prediction periods overlap, this plot only contains the predictions made during the same year that is forecasted. As the figure illustrates, the variance in the years prior to 1995 have considerably higher variance than from 1995 and onwards. The graph also shows how the variance declines as the forecasting period comes closer to its final forecasting month.

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\[ \bar{x} = \frac{1}{n} \sum x_i, \quad E[\bar{x}] = \frac{1}{n} \sum \mu = \mu. \]  The observations are assumed to be independent, so \[ \text{Var}[\bar{x}] = \frac{1}{n} \text{Var}[\sum x_i] = \frac{1}{n} \sum \sigma^2 = \sigma^2 / n \]
Another way to make the same illustration is by plotting the distribution from above in a contour plot. This is done in Figure 6. This plot is similar to a map with landscape contours showing how the terrain shifts from sea level to plains, from plains to mountains, and ultimately to peaks in a mountain range. Each contour line represents a level of the variance (the height above the sea). Inside the contour line variance is higher than outside. The closer the contour lines are the steeper is the rise in the distribution. As can be seen from the plot, the distributions are wider in the beginning of each year, and become narrower with more contour lines as the year-end approaches. This illustrates the declining variance over the prediction period, the distributions become narrower, more concentrated around one value for the forecasted growth rate. The reductions in variance after 1994 is also noticed, as the distributions after 1994 are narrower, and as such have more contour lines than the distributions before 1995.

Two more observations can be made using a graph like Figure 6. First, the means of the distributions can be found in Table 3, but it is also possible to follow its time path as it starts at a high of 3% in 1990, to decrease to −1% at the end of 1994, before rising again averaging around 2% for the rest of the nineties and the beginning of the 2000’s. Second, it is possible to get a sense of how the distribution is skewed (although it requires a little imagination), by looking at the contour lines to see if they are closer together on one side than on the other. If the contour lines are further a part on one side this means that the distribution is skewed towards that side, there is more probability mass assigned to values further away from the centre of the distribution, this is further explored in the next section.
4.3. Skewness – Are the distributions asymmetric?

Skewness coefficients of all derived euro area distributions are presented in Table 6. As in the case of Table 3 to Table 5, the first column contains the years predicted, and the first row contains the months in each of the two years when predictions were made. Almost 90% of these coefficients are statistically different from zero at a conventional significance level.

Skewness time plots for each year predicted are in Figure 7 graphed alongside with the time plots for the mean and variance for the same prediction period and distributions. The time plots for skewness exhibit a high degree of variation between different years predicted and their respective prediction periods. For some years there seem to be trends in how the coefficients develop over time, e.g. in 1991 the trend seems to be downward sloping, while in 1993 it looks like it is upward sloping. These trends occur over a two-year period. Other graphs look more like white noise, where the skewness oscillates around zero without a clear systematic pattern. Furthermore, many time plots seem to show that the distributions can remain skewed in one direction or the other for many months in a row, e.g. for the predicted year 1994 the distributions are negatively skewed almost over the whole two-year period when predictions were made. Similar observations can be made regarding the distributions for the years 1996, 1999, and 2000, where the distributions were skewed in one direction for long periods of time.
Figure 7: Mean, variance, and skewness time plots, 1990-2003
The last observation is further illustrated in Figure 8, where the skewness coefficient is plotted for the overall sample period. Only the last 12 prediction months are used for each year predicted, since the other 12 months overlap with the previous year predicted. As the plot shows, long time periods of positive skewness are found during 1996 and between 1999 and 2001. The periods 1994 to 1995 and 1997 to 1999 are, on the other hand, better characterised by their more frequently occurring negative skewness.

These observations suggest that there is some degree of systematic change in the distributions as they evolve over time. One possibility is that the growth forecasts and skewness moves in the opposite direction. This can be the case, e.g. if forecasted GDP growth rates are high and approaching what can be considered a turning point in the cycle. In this case it is possible that higher probability is assigned over a wider range of declining growth rates, i.e. it is more probable to have a large decline in growth than of having a large increase. This can be interpreted as if there existed a downside risk to the forecasts, and can show up as a negative correlation between forecasted growth rates and skewness.

Another possibility is that changes in forecasted growth rates and skewness are positively related to each other. If the sentiment among forecasters is negative, such that they believe in a more negative economic outlook, this gives rise to a period of continuously negative revisions of their predictions. In this case, forecaster views can show up in the distributions as negative skewness during the adjustment period.

These two possibilities do not oppose each other, rather they are complements. The first dependence relates to turning points in the cycle and the second to the transition from the perception of high to low growth, or vice versa. The two conceivable ideas give rise to two hypotheses regarding the relationship between skewness and the GDP growth predictions. The two hypotheses are:

**Hypothesis 3** There is no or a positive relation between skewness and the GDP growth forecasts, i.e. $\text{Corr}(\text{skewness}, \text{GDP}) \geq 0$. 
Figure 8: Skewness time plot from 1990 to October 2002

Hypothesis 4 There is no or a negative relation between skewness and the change in GDP growth forecasts, i.e. \( \text{Corr}(\text{skewness}, \Delta GDP) \leq 0 \).

Calculating the correlation coefficient between the forecasted growth rates and the skewness coefficients tests the first hypothesis. The correlation coefficient is -0.16 with a p-value around 0.02. The hypothesis can thus be rejected at a standard significance level. Hence, there seems to be a negative relation between skewness and the level of growth.

Calculating the correlation coefficient between two constructed time series tests the second hypothesis. The first series is constructed by assigning a 1 if skewness is positive and -1 if skewness is negative. The other series is constructed in a similar way. It takes the value 1 if the forecasted growth rate increase and the value -1 if the forecast decreases. The correlation coefficient is 0.084 with a p-value of 0.075, and can thus be rejected at a significance level of 10%, but not at 5%. Only using significant skewness as the basis for constructing the series, leads to a p-value below 5%.

The rejection of the last hypothesis suggests that there is a positive relation between the skewness and the changes in GDP forecasts, and that it is possible to make qualitative statements about the direction of the risk that GDP growth forecasts are associated with. For example, based on Figure 7 and the skewness graph for 2002, it seems reasonable to say that the consensus forecast favoured a positive economic development, with possible bigger upward revisions than downward revisions of the forecasts in the first five months of 2002. This positive position on the economic
outlook is then eased in July, turning negative in the following months, maybe because many expectations have not yet materialised.

In addition to the above tests, it is worth pointing out that significant skewness at any specific point in time contains information about where uncertainty lies momentarily. Skewness says something about how much probability is assigned to specific tail events. In general, monitoring the change in the distribution over time can say something about how uncertainty increases/decreases and if there is more risk in one tail of the distribution, or the other.

4.4. Monitoring forecast uncertainty and balance of risks

The mean distributions of forecasted euro area GDP can be used for monitoring forecasters apprehension of uncertainty and balance of risks. For this purpose uncertainty can be measured by the distribution’s standard deviation, and the balance of risks can be measured by the skewness. The monitoring can be conducted by regularly follow the evolution of the distribution as described by its two moments. The development of the distribution over time can be graphically presented in different ways, some of which have been used in the preceding sections. Presented in Figure 9 are two other illustrative examples of graphs of the standard deviation and the skewness of monthly distributions.

Figure 9: Time plots of forecast uncertainty with long-term trends and balance of risks

Figure 9a shows how the standard deviation (uncertainty) of the mean distributions for the predicted year 2002 and 2003 has varied over time, together with the overall historical trends (dotted line). In Figure 9a it is possible to compare the level of uncertainty for different years predicted, but the same prediction month has to be used. For example, the level of uncertainty for 2002 in the prediction made in October 2001 can be compared with the level for 2003 in prediction month October 2002. Furthermore, it is possible to compare the level in one month with the trend standard deviation for that month. The trend is a fitted line to the average monthly standard deviation. The average is calculated using the same prediction month for different years predicted.

Figure 9a shows that uncertainty for 2002 started off below the average in January 2001, but shot up in a spike following the terrorist attack in September 2001, reflecting the extreme uncertainty the attack caused. While uncertainty was still high
for the year 2002 in January 2002, uncertainty for growth in 2003 was contained and around the long-term trend. When the expected recovery in 2002 did not take hold, and new information arrived that did not support earlier perceptions of growth, uncertainty for 2003 increased to comparable levels to that after the terrorist attack. The increase in the standard deviation could reflect, e.g. the increased uncertainty that came about with the risk of a war with Iraq and decreasing stock markets.

Figure 9b shows how the skewness (balance of risks) of the mean distribution for the predicted year 2002 and 2003 evolved over time. Skewness varies around zero and illustrates how risks are balanced. A positive skewness means that more probability is assigned to positive “extreme” events, in some sense reflecting an upward risk. The opposite is true for negative skewness. Figure 9b shows that skewness was rather low, or balanced, in 2001 for the predicted year 2002, but in the beginning of 2002 skewness became positive for both years, slightly more so for 2003 than for 2002. This reflects the positive expectations that were perceived in the first half of the year. Forecasters believed that the economy would accelerate in 2003, giving higher probabilities to large positive deviations. As new information came, more risk was assigned to negative tail events, making the distribution skewed to the left.

5. Conclusions

The objective of this paper is to develop a risk assessment methodology for the forecast of the euro area GDP growth rate. This is accomplished by making use of consensus forecasts for GDP growth and studying the properties of euro area distributions. For this purpose a euro area distribution is necessary, and is constructed out of mean distributions of individual country specific consensus forecasts. The standard central moments like mean, variance, and skewness are analysed to discover the properties of these distributions. Information contained in the distributions can be used to make risk assessments of the future economic development. It can be used as input in a forecast exercise to set confidence bands around the forecast, for determining forecasters’ views of which direction is the more plausible one for a deviation of the forecast from the actual outcome, or to indicate in which direction a forecast will be revised.

The results of the analysis show that the variances of the consensus distributions are time varying. Furthermore, it decreases during the forecasting period as more information is revealed about the year that is predicted. Still, there are sharp shifts in the measure of uncertainty as certain events happen, or periods of time are particularly difficult to forecast. The variance measure employed in the paper can be used when comparing the change in levels between two dates, but it probably underestimates the level of risk.

Skewness also varies a lot over time. There is regularity in the time series of distributions. Time periods when a relatively positive sentiment of the economic outlook prevail, are associated with a positive skewness, i.e. there is a higher probability of a large upward revision in the coming months than of a downward revision of the same magnitude. There is also evidence that more probability is assigned to outcomes that go the opposite way when growth can be considered high, or low.
Two graphs are proposed to be used to measure and assess present sentiment about uncertainty and balance of risks regarding future growth. One graph is a time plot of the standard deviation (uncertainty), which can be compared with the historical mean trend of the standard deviation. In such a graph it is possible to compare the levels of uncertainty in the same forecasting month, but for different years forecasted. Furthermore, the level of uncertainty in any month can be compared to the historical average to assess if uncertainty is below or above the trend for that specific month. The other graph is a time plot of the skewness (balance of risks), which will fluctuate around zero. If skewness is significant in a particular month, it can give an indication to whether risks are on the downside or the upside.
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Table 4: Standard deviation of euro area mean-GDP distribution

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