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Non-Ricardian Fiscal Policies in an Open Monetary Union*

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Abstract

The fiscal theory of the price level has challenged the conventional view that monetary factors drive prices and exchange rates and has also provided a rationale for fiscal restrictions in a monetary union. This paper extends the main results of this theory in the context of an open monetary union model. First, it analyzes solutions to the indeterminacy of the exchange rate, some of which have non-standard macroeconomic implications. Second, it shows in a calibrated model the consequences for the monetary union of fiscal misdirection in one its members.

JEL classification: E31, E63, F42, H3

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1. Introduction

Fiscal behavior has been of central interest in the European Monetary Union process. The concern with macroeconomic stability led first to the imposition of specific ceilings on the public debt and deficits of candidate States as a precondition for joining the Union. Once the Union was formed, the Growth and Stability Pact imposed an even tighter fiscal discipline on Member States. However, although price stability is perceived as desirable, its connection with fiscal restrictions is controversial. In fact, the most widely held view seems to be that price stability can be achieved with an independent central bank and a credible monetary policy, whereas fiscal policy needs to be more flexible to confront asymmetric shocks within the union.[1]

That view is questioned by the *fiscal theory of the price level* on two grounds. First, there is no reason why central bank independence per se should guarantee fiscal discipline. Second, if tax and spending decisions are taken by the public sector without regard for their effects on the present value of net surpluses, an independent central bank with a clear anti-inflationary objective may not be sufficient to guarantee price stability. Macroeconomists have long recognized the connection between fiscal and monetary decisions induced by the fact that the government has two sources of revenue, taxes and seignorage, which allow it to achieve public sector solvency through alternative coordination schemes. This connection imposes limits on the efficacy of monetary policies (Sargent and Wallace (1981)). The fiscal theory of price determination takes that interaction one step further. According to this approach, monetary policy determines the expected inflation rate but, to the extent that the price level may be determined by the government's fiscal stance, fiscal shocks may determine the observed (*ex-post*) inflation rate at each period of time.

The implications of the fiscal theory of the price level for a monetary union were first analyzed by Woodford (1996) and Sims (1997) and later on extended by Bergin (2000). In Bergin's model, the monetary union is itself a closed economy, so that important issues, like that of exchange rate determination, are left out. In this paper, we look at the case of an open monetary union in a model in which there are three fiscally independent authorities and just two currencies. Our aim is twofold: first we revisit the issue of the determination of the exchange rate of the monetary union vis-à-vis the rest of the world, and second we explore the spillover effects of fiscal shocks in one of the countries of the monetary union.

[1] See, for example, Eichengreen and Wyplosz (1998).

In section 2 we show how the indeterminacy of prices and exchange rates may be avoided in a multi-country setting with pegged interest rates, if some of the countries involved follow non-Ricardian fiscal policies. Nominal determinacy is achieved provided that there is incomplete risk sharing among the countries. This extends the analysis of exchange rate indetermination made by Dupor (2000). Although the conditions for a unique equilibrium are very demanding, we find that a wide range of combinations of risk sharing and policy rules generate such a result in a multi-country model. Furthermore, some of these policy combinations have non-standard economic implications.

In section 3 we simulate the effect of fiscal shocks in some cases of special interest for policy purposes, against the background of the standard monetary (Ricardian) model. The benchmark case is one in which monetary policy is loose within the union and tight in the rest of the world, and one of the countries belonging to the union follows a non-Ricardian fiscal policy. This is the open economy counterpart of the non-Ricardian regime that has received more attention in the literature. We find that fiscal shocks in that country may affect other countries unless the exchange rate adjusts; this adjustment does not take place in a monetary union, which allows this contagion to operate at full strength. Moreover, the instability in the union is largely independent of the fiscal behavior in the other countries of the union and of the tightness of the monetary policy exerted by the central bank. But this case is not the only departure from the Ricardian world in which a unique equilibrium exists. Other policy combinations may also yield uniqueness and are of interest since they involve different conditional correlations among macroeconomic variables.

2. Nominal determinacy in an open monetary union

2.1. The model

Let us consider a two-country model with two independent monetary authorities (countries 1 and 2), in which one of the countries is a monetary union with one currency and two independent fiscal authorities (1A and 1B). Seignorage within the union is split on equal grounds between its members and there is perfect capital mobility. We assume complete risk sharing, which means that each country may issue limitless amounts of debt as long as some other country is willing to hold it, although this assumption will be relaxed later. We leave aside the analysis of the repercussions on the distribution of wealth and consumption across countries, by assuming that the representative household consumes and produces goods in both countries, and also holds assets according to their rate of return, regardless of their currency of denomination. Goods must be purchased in each country using of the currency of that country.

The setup of the model is in the spirit of Obstfeld and Rogoff's (1995), which assumes monopolistic competition and sticky prices. The j_{th} representative consumer-producer (where $j \in [0,1]$) chooses the vector c^A_{1jt}, c^B_{1jt}, c_{2jt}, y_{1jt}, y_{2jt}, B^A_{1jt}, B^B_{1jt}, B_{2jt}, M_{1jt}, M_{2jt}, P_{1jt} and P_{2jt} to maximize (2.1) subject to (2.2) - (2.5). Individual outputs, y_{1jt}, y_{2jt} can be thought of as intermediate inputs of two composite final, goods y_{1t}, y_{2t}. As a consumer, each household purchases a share of these composite final goods in a perfectly competitive market, and as a producer of intermediate goods she faces a finite elasticity demand curve for her variety. Money and government bonds are defined in terms of the home currency in each case.

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_{jt})}{1-\sigma}^{1-\sigma} - \frac{(y_{1jt})^\gamma}{\gamma} - \frac{(y_{2jt})^\gamma}{\gamma} + \frac{\chi \left(\frac{M_{1jt}}{P_{1t}}\right)^{1-\varepsilon}}{1-\varepsilon} + \frac{\chi \left(\frac{M_{2jt}}{P_{2t}}\right)^{1-\varepsilon}}{1-\varepsilon} \right\} \tag{2.1}$$

$$P_{1t} \sum_{l=A,B} c^l_{1jt} + P_{2t} c_{2jt} e_t + \sum_{l=A,B} B^l_{1jt} + M_{1jt} + e_t B_{2jt} + e_t M_{2jt} + P_{1t} \sum_{l=A,B} \tau^l_{1jt} + e_t P_{2t} \tau_{2jt} \leq$$

$$P_{1jt} y_{1jt} + e_t P_{2jt} y_{2jt} + (1+R_{1t-1}) \sum_{l=A,B} (B^l_{1jt-1} + (1+R_{2t-1}) e_t B_{2jt-1} + M_{1jt-1} + e_t M_{2jt-1}$$

$$-P_{1t} AC_{1jt} - e_t P_{2t} AC_{2jt} \tag{2.2}$$

$$y_{ijt} = y_{it} \left(\frac{P_{ijt}}{P_{it}} \right)^{-\theta} \ ; \ i = 1,2 \tag{2.3}$$

$$AC_{ijt} = \frac{\phi_i}{2} \left(\frac{P_{ijt}}{P_{ijt-1}} - \Omega_i \right)^2 y_{it} \ ; \ i = 1,2 \tag{2.4}$$

$$\lim_{T\to\infty} E_0 \left(\prod_{s=0}^{T-1} (1+R_{1s})^{-1} \right) W_{jT} = 0 \tag{2.5}$$

where $c_{jt} = c^A_{1jt} + c^B_{1jt} + c_{2jt}$ and M_{1js} and M_{2js} are non-negative for all s; the assumption of complete risk sharing means that no such condition is imposed on B^A_{1js}, B^B_{1js}, B_{2js} (all that is needed is that $B^A_{1s}+ B^B_{1s}+ B_2 \geqslant 0$). Equation (2.2) is the flow budget constraint of the j_{th} consumer. Equation (2.3) is the demand for product j in period t.[2] Equation (2.4) is the producer's adjustment cost function of prices. Equation (2.5) is the transversality condition. Wealth is defined in country 1 currency as,

$$W_{jt+1} = (1 + R_{1t}) \sum_{l=A,B} B^l_{1jt} + M_{1jt} + (1 + R_{2t})e_{t+1}B_{2jt} + e_{t+1}M_{2jt} \tag{2.6}$$

Defining Ω_i as the steady state inflation in country i, the (symmetric aggregate equilibrium) first order conditions of the problem can be written as follows:

$$c_t^{-\sigma} = \beta\left(1 + R_{1t}\right) E_t\left(c^{-\sigma}_{t+1}\frac{P_{1t}}{P_{1t+1}}\right) \tag{2.7}$$

$$P_{1t} = P_{2t}e_t \tag{2.8}$$

$$e_t\left(1 + R_{1t}\right) E_t\left(\frac{c^{-\sigma}_{t+1}}{P_{1t+1}}\right) = \left(1 + R_{2t}\right) E_t\left(c^{-\sigma}_{t+1}\frac{e_{t+1}}{P_{1t+1}}\right) \tag{2.9}$$

$$m_{1t} = \frac{M_{1t}}{P_{1t}} = c_t^{\frac{\sigma}{\varepsilon}}\left[\chi\left(1 + \frac{1}{R_{1t}}\right)\right]^{\frac{1}{\varepsilon}} \tag{2.10}$$

$$m_{2t} = \frac{M_{2t}}{P_{2t}} = c_t^{\frac{\sigma}{\varepsilon}}\left[\chi\left(1 + \frac{1}{R_{2t}}\right)\right]^{\frac{1}{\varepsilon}} \tag{2.11}$$

[2] To derive (2.3) we may think of two aggregators, one in country 1 and the other in country 2, who buy the different varieties y_{1jt}, y_{2jt}, and produce two composite goods y_{1t}, y_{2t}. The i_{th} country aggregator ($i = 1, 2$) solves the following maximization problem:

$$\underset{y_{ijt}}{Max}\ P_{it}\left[\int_0^1 (y_{ijt})^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} - \int_0^1 P_{ijt}y_{ijt}dj$$

These demand functions, along with the zero-profit condition for the aggregator, imply the following aggregate price index for the i_{th} country:

$$P_{it} = \left[\int_0^1 (P_{ijt})^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\theta y_{1t}\left[(y_{1t})^{\gamma-1}-c_t^{-\sigma}\right]-\phi_1 y_{1t}c_t^{-\sigma}\left(\frac{P_{1t}}{P_{1t-1}}\right)\left(\frac{P_{1t}}{P_{1t-1}}-\Omega_1\right)+$$

$$+\beta\phi_1 E_t y_{1t+1}c_{t+1}^{-\sigma}\left(\frac{P_{1t+1}}{P_{1t}}\right)\left(\frac{P_{1t+1}}{P_{1t}}-\Omega_1\right)=0 \tag{2.12}$$

$$\theta y_{2t}\left[(y_{2t})^{\gamma-1}-c_t^{-\sigma}\right]-\phi_2 y_{2t}c_t^{-\sigma}\left(\frac{P_{2t}}{P_{2t-1}}\right)\left(\frac{P_{2t}}{P_{2t-1}}-\Omega_2\right)+$$

$$+\beta\phi_2 E_t y_{2t+1}c_{t+1}^{-\sigma}\left(\frac{P_{2t+1}}{P_{2t}}\right)\left(\frac{P_{2t+1}}{P_{2t}}-\Omega_2\right)=0 \tag{2.13}$$

Equation (2.8) establishes that the law of one price holds, since there are no restrictions on the movement of goods, which are perfect substitutes in consumption; (2.7) captures the intertemporal substitution in consumption, whereas (2.10) and (2.11) are the implied demands for both currencies. In a frictionless model, the price level is an asset price which is determined only by the expectation of future returns on holding money (expected inflation). By contrast, in this sticky price model, equations (2.12) and (2.13) are the (new Keynesian) Phillips curves relating inflation to the output gap in each country. Since both economies are open, demand pressure is represented both by home production (y_{it}) as well as by world consumption (c_t). The Phillips curve is positively sloped as long as there are non-negligible costs of changing prices ($\phi_i \neq 0$).

The symmetric aggregate equilibrium must satisfy three budget balance constraints (one for each independent fiscal authority) and the overall resource constraint of the economy,

$$P_{1t}g_{1t}^A - P_{1t}\tau_{1t}^A = \left(\frac{1}{2}\right)(M_{1t}-M_{1t-1}) + B_{1t}^A - (1+R_{1t-1})\,B_{1t-1}^A \tag{2.14}$$

$$P_{1t}g_{1t}^B - P_{1t}\tau_{1t}^B = \left(\frac{1}{2}\right)(M_{1t}-M_{1t-1}) + B_{1t}^B - (1+R_{1t-1})\,B_{1t-1}^B \tag{2.15}$$

$$P_{2t}g_{2t} - P_{2t}\tau_{2t} = M_{2t} - M_{2t-1} + B_{2t} - (1+R_{2t-1})\,B_{2t-1} \tag{2.16}$$

$$c_t + g_{1t}^A + g_{1t}^B + g_{2t} = y_{1t} + y_{2t} \tag{2.17}$$

The definition of equilibrium is incomplete until the behavior of the monetary and fiscal authorities is defined. In the most common case of and exogenous money supply and Ricardian fiscal policies, there is no unique equilibrium for prices. In this case, the unique equilibrium for prices and the exchange rate is found by assuming that the price level is bounded. If both central banks adopt a policy of pegging the interest rate, the conditions for a unique nominal equilibrium are more demanding.

2.2. Equilibria under complete risk sharing

Consider a situation of interest rate pegging, which we assume for the sake of simplicity:

$$R_{1t} = R_{2t} = R > 0 \tag{2.18}$$

To keep things simple, let us consider the perfect foresight equilibrium in the flexible price version of the model, in which $\phi_1 = \phi_2 = 0$. In this case, the supply curves are vertical in both countries, and $c_{t,}$ y_{1t} and y_{2t} are the solution of (2.12), (2.13) and (2.17), which for constant values of g_{it} are also constant for all t. Equations (2.9) and (2.7) imply $e_t = e_{t+1}$ and $\frac{P_{1t}}{P_{1t-1}} = \frac{P_{2t}}{P_{2t-1}} = \frac{P}{P_{-1}}$ for all t. Finally, the demands for real balances set the equilibrium value of $m_{1t} = m_{2t} = m$ and (2.14), (2.15) and (2.16) determine real debt. Thus, all real variables are uniquely determined whereas the nominal values of P_1, P_2 and e are not. This is the standard indeterminacy result associated with interest rate pegging shown by Sargent (1987).

The fiscal theory of prices has been invoked in this type of environment to obtain a unique value for the price level and (*ex-post*) inflation (Woodford, 1994). This theory relies on the transversality condition of the household's optimization problem to obtain an additional condition that the price level must satisfy. This can be easily seen in our model, under perfect foresight. Substituting the first order conditions (2.7), (2.8) and (2.9) in the government budget constraints (2.14), (2.15) and (2.16), and adding them up, we obtain:

$$W_t = P_{1t}\left[\theta_{1t}^A + \theta_{1t}^B + \theta_{2t} + \Delta_{1t}m_{1t} + \Delta_{2t}m_{2t}\right] + \frac{W_{t+1}}{(1+R_{1t})}$$

where the surplus in each country is $\theta_{1t}^l = \tau_{1t}^l - g_{1t}^l$, $l = A, B$, $\theta_{2t} = \tau_{2t} - g_{2t}$ and $\Delta_{it}m_{it}$ represents the resources from seigniorage, with $\Delta_{it} = \left(\frac{R_{it}}{1+R_{it}}\right)$. Iterating forward and applying the transversality condition (2.5) we obtain the following condition that must be satisfied at any t:

$$\frac{W_t}{P_{1t}} = \frac{\left[\theta^A_{1t+k} + \theta^B_{1t+k} + \theta_{2t+k} + \Delta_{1t+k}m_{1t+k} + \Delta_{2t+k}m_{2t+k}\right]}{\sum_{k=0}^{\infty}\left(\prod_{s=t}^{t+k-1}(1+r_{1s})\right)} \tag{2.19}$$

where $(1+R_{1s}) = (1+r_{1s})\left(\frac{P_{1s+1}}{P_{1s}}\right)$ and the real interest rate, $(1+r_s)$, is equal to the ratio of marginal utilities of consumption between s and $s+1$ (discounted). Under the assumption of interest rate pegging, this condition at any t, say $t=0$, looks like:

$$\frac{W_0}{P_{10}} = \frac{2\Delta m}{(1-\beta)} + \sum_{t=0}^{\infty}\beta^t\left[\theta^A_{1t} + \theta^B_{1t} + \theta_{2t}\right] \tag{2.20}$$

This expression means that the household transversality condition of bounded wealth imposes one joint present value constraint on the fiscal behavior of the three governments in the model. This present value constraint need not be satisfied by each fiscal authority: all that is needed is that some government is willing to hold limitless amounts of other country's debt if necessary, so that the present value of worldwide public sector surpluses suffices to pay back the outstanding debt.

W_0 is predetermined and Δm is given by the monetary policy, but still this condition may or may not pin down P_{10} depending on the way the governments set their fiscal policies. To see this, assume that all three fiscal authorities in the model design their fiscal policies as a constant surplus (θ_i for all t) whose size is set as a function of the price level, such that the following relations hold for any value of P_{10} and P_{20}.

$$\theta_1^{l*} = \frac{(1-\beta)\left(B^l_{1(-1)}(1+R) + \frac{1}{2}M_{1(-1)}\right)}{P_{10}} - \left(\frac{1}{2}\right)\Delta m, \ l = A, B$$

$$\theta_2^* = \frac{(1-\beta)\left(B_{2(-1)}(1+R) + M_{2(-1)}\right)}{P_{20}} - \Delta m$$

These policy reaction functions can be added to obtain the following expression that also holds for any P_{10}.

$$\theta_1^{A*} + \theta_1^{B*} + \theta_2^* = \frac{(1-\beta)W_0}{P_{10}} - 2\Delta m$$

These policies (Ricardian policies) render (2.20) redundant and the transversality condition cannot be used to select a unique level of prices. If some, or all, governments set θ_i disregarding the present value constraint (i.e. any non-Ricardian policy $\theta_i \neq \theta_i^*$) the second term in the right-hand side of (2.20) is exogenous and the transversality condition is satisfied only for a unique value of P_{10}. This is the basis of the fiscal theory of prices.

However, as Dupor (2000) has shown, non-Ricardian policies do not determine a unique equilibrium in a two-country model. Even if there were a unique value of P_{10} that satisfied (2.20), P_{20} and e_0 would remain indeterminate. The explanation of this indeterminacy is that the transversality condition implies a single present value condition on aggregate debt. Debt in each country need not be bounded; in other words, the possibility of one country issuing a limitless amount of debt cannot be ruled out, as long as other countries become net lenders. One country's debt may be growing or falling without limit as long as the aggregate debt is not.[3]

2.3. Equilibria under incomplete risk sharing

The assumption of complete risk sharing among countries is a strong one, since it may imply a substantial wealth transfer from countries with fiscal surpluses to those with persistent deficits. In the absence of compensatory payments (risk insurance) this equilibrium can hardly be sustained. Let us assume instead that there is incomplete risk sharing. This means that one country cannot accumulate limitless amounts of other country's debt, which imposes a lower bound for B_{it}. Let us first assume that there is risk sharing among the countries belonging to the union (1A,1B) but not among those and country 2. This is represented by the constraints,

$$(B_{1t}^A + B_{1t}^B) \geq 0,\ \ B_{2t} \geq 0,\ \ \forall t \tag{2.21}$$

which along with the transversality condition of the consumer's problem, imply the following necessary conditions for equilibrium,

$$\lim_{T\to\infty} E_0 \left(\prod_{s=0}^{T-1} (1 + R_{1s})^{-1} \right) W_{1T} = 0 \tag{2.22}$$

[3] Dupor (2000) shows that this result holds even if the substitutability of currencies is not restricted.

$$\lim_{T\to\infty} E_0 \left(\prod_{s=0}^{T-1} (1+R_{2s})^{-1} \right) W_{2T} = 0 \tag{2.23}$$

where $W_{1t+1} = (1+R_{1t})(B_{1t}^A + B_{1t}^B) + M_{1t}$, $W_{2t+1} = (1+R_t)B_{2t} + M_{2t}$.

Conditions (2.22) and (2.23) impose two present value constraints on government balances that, under interest rate pegging, may be written as,

$$\frac{W_{10}}{P_{10}} = \frac{\Delta m}{(1-\beta)} + \sum_{t=0}^{\infty} \beta^t \left[\theta_{1t}^A + \theta_{1t}^B\right] \tag{2.24}$$

$$\frac{W_{20}}{P_{20}} = \frac{\Delta m}{(1-\beta)} + \sum_{t=0}^{\infty} \beta^t \theta_{2t} \tag{2.25}$$

Now, if fiscal policies in both countries are non-Ricardian (i.e. θ_{1t}^A, θ_{1t}^B, θ_{2t} are exogenous), all nominal variables in the model are uniquely pinned down. In the particular case in which current and future surpluses are expected to be constant, we have P_{10}, P_{20}, e_0 determined by:

$$P_{10} = \frac{(1-\beta)W_{10}}{\Delta m + \theta_1^A + \theta_1^B} \tag{2.26}$$

$$P_{20} = \frac{(1-\beta)W_{20}}{\Delta m + \theta_2} \tag{2.27}$$

$$e_0 = \left[\frac{(1-\beta)W_{20}}{\Delta m + \theta_2}\right]^{-1} \left[\frac{(1-\beta)W_{10}}{\Delta m + \theta_1^A + \theta_1^B}\right] \tag{2.28}$$

It should be noticed at this point that the fiscal stance is not only important insofar as it helps to determine prices and the exchange rate. When the central bank follows a policy of pegging the money supply and the fiscal policy is Ricardian, a tax cut today is offset by an expected surplus rise at some point in the future that would leave permanent income and consumption unaltered, and with no effect on prices. By contrast, under non-Ricardian fiscal policies, the response of most macroeconomic variables to exogenous shocks differs from what would be obtained in the standard monetary equilibrium. Expressions (2.26) to (2.28) trace out the effect of fiscal shocks on prices and the exchange rate. For instance, a tax cut in country 1B (reduction in θ_1^B) generates a rise in prices in country 1 and an exchange rate depreciation. Nevertheless, to explore the conditional correlation of variables under alternative shocks we have to move to a more general model with more realistic policy rules. This is discussed at length in Section 3.

2.4. Alternative policy combinations and risk sharing assumptions

The last two subsections showed that, under interest rate pegging, Ricardian fiscal polices produce the well known result of nominal indeterminacy, both for prices and the exchange rate. Non-Ricardian policies, which serve to fix a unique price level in a closed economy, do not achieve the same in a multi-country model under perfect capital mobility and complete risk sharing. Imperfect risk sharing, though, allows to achieve nominal determinacy.[4]

In an open monetary union, however, the number of policy combinations that may generate a unique equilibrium with the price level determined through the intertemporal budget constraint is much larger than in the closed economy case. Although we shall not describe all of them here, two might be of some interest. First consider the following situation: the central bank in the monetary union fixes the money supply and both countries, 1A and 1B, follow Ricardian fiscal policies, while country 2 pegs its interest rate and follows a fiscal policy of exogenous surplus. In this case we would still have a unique equilibrium, but one with different properties to those discussed above. In particular, P_2 would react to shocks to θ_2 while P_1, which is now determined by the money demand in country 1, would not respond to shocks to θ_1; hence, the pattern of response of the exchange rate would differ from both the standard Ricardian and non-Ricardian regimes discussed above.

A more striking case, also resulting in uniqueness of equilibrium, would be one with the following policy combination: one of the countries belonging to the union follows a non-Ricardian fiscal policy while the union's central bank fixes the money supply and, simultaneously, country 2 pegs the interest rate and follows a Ricardian fiscal policy. In a closed economy framework P_{20} would be indeterminate while P_{10} would be over-determined. However, since these two countries are connected through their goods and financial markets, nominal variables would be uniquely determined. Basically, the over-determination in country 1 is eliminated since now there are two prices to be set: P_{10} and e; the purchasing power parity condition then determines P_{20}, thus removing the indeterminacy in country 2. It goes without saying that in this case the international transmission of shocks is different from all the cases discussed above, with a predominant role for monetary and fiscal shocks originating in country 1. This case will also be considered in Section 3.

Notice further that all the above equilibria have been obtained in a model

[4] Canzoneri, Cumby and Diba (1998) have noticed that result in a simpler environment.

under the assumption of incomplete risk sharing among countries. A different assumption about risk sharing would require alternative policies to generate a unique equilibrium. For example, let us take the assumption of imperfect risk sharing to the limit, imposing that countries 1A, 1B and 2 face some lower bound limit for their debt. In such case the determination of nominal variables could not be guaranteed:

$$B_{1t}^{A},\ B_{1t}^{B},\ B_{2t} \geq 0,\ \forall t$$

which implies, along with the transversality condition and (2.8), the following relationships that P_{10}, P_{20} and e_0 must satisfy:

$$P_{10} = \frac{(1-\beta)W_{10}^{A}}{\left(\frac{1}{2}\right)\Delta m_1 + \theta_1^{A}}$$

$$P_{10} = \frac{(1-\beta)W_{10}^{B}}{\left(\frac{1}{2}\right)\Delta m_1 + \theta_1^{B}}$$

$$P_{20} = \frac{(1-\beta)W_{20}}{\Delta m_2 + \theta_2}$$

$$e_0 = \frac{P_{10}}{P_{20}}$$

where P_{10}, and thus the whole system, would be overdetermined, unless for example only one of the countries belonging to the monetary union (1A or 1B) is following a non-Ricardian policy. The bottom line is that our multi-country model displays a variety of equilibria whose properties depend on the combinations of risk sharing assumptions and policy schemes. Woodford (2000) has argued that the US economy has gone through periods in which monetary policy was passive and, nonetheless, prices were determined, which he interprets as evidence of underlying non-Ricardian fiscal policies. His argument is a closed economy one, and our results show that in an open economy a unique equilibrium can be obtained with alternative non-Ricardian policy combinations.

3. The effect of fiscal shocks under endogenous policy rules

This section explores the effects of fiscal shocks in the open monetary union under a more realistic environment in which both monetary and fiscal authorities react to the state of the economy. First we discuss the stability of this model and later on we calibrate it to analyze the effects of shocks under alternative monetary and fiscal regimes.

3.1. Stability of the model

As discussed previously, there are conditions under which a unique equilibrium exists with all nominal and real variables uniquely determined. These conditions require that either the monetary policy or fiscal policy, but not both, pin down the price level. The effects of fiscal shocks in the polar cases of interest rate pegging and exogenous surplus have been analyzed in Section 2. However, purely exogenous interest rates and government surpluses are an extreme form of policies rarely seen in practice.

The recent literature has chosen to represent the behavior of monetary and fiscal authorities by means of feedback rules, whereby policy instruments are chosen to react to the level of some endogenous variable. Interest rate rules are well known after the work of Taylor (1993 and 1999). He represents the policy reaction function as an adjustment of the nominal interest rate to current inflation and the output gap,[5]

$$R_{1t} = \delta_{10} + \delta_{11}\left(\frac{P_{1t}}{P_{1t-1}}\right) + \delta_{12}\widehat{y}_{1t} + \varepsilon_{1t}^{R} \tag{3.1}$$

$$R_{2t} = \delta_{20} + \delta_{21}\left(\frac{P_{2t}}{P_{2t-1}}\right) + \delta_{22}\widehat{y}_{2t} + \varepsilon_{2t}^{R} \tag{3.2}$$

In these expressions, δ_{10} and δ_{20} are positive constants, which depend on the steady state real interest rate. δ_{11} and δ_{21} capture the response of the nominal rate to deviations of inflation from its target value. The response to deviations of output from trend is captured by δ_{12} and δ_{22}. Finally, ε_{1t}^{R}, ε_{1t}^{R} are monetary policy innovations that produce unanticipated movements in the nominal interest rate.[6]

Similarly, Bohn (1998) has found empirical support for fiscal rules of the type,

$$\tau_{1t}^{A} - g_{1t}^{A} = \alpha_{10}^{A} + \alpha_{11}^{A}\left(\frac{B_{1t-1}^{A}}{P_{1t-1}}\right) + \alpha_{12}^{A}\widehat{y}_{1t} + \varepsilon_{1t}^{\tau A} \tag{3.3}$$

$$\tau_{1t}^{B} - g_{1t}^{B} = \alpha_{10}^{B} + \alpha_{11}^{B}\left(\frac{B_{1t-1}^{B}}{P_{1t-1}}\right) + \alpha_{12}^{B}\widehat{y}_{1t} + \varepsilon_{1t}^{\tau B} \tag{3.4}$$

[5] Alternatively, it may also respond to expected inflation, as in Clarida, Galí and Gertler (2000).

[6] These rules are identical to the standard Taylor rules of the form $R_{it} = \widetilde{\delta}_{i0} + \delta_{i1}(\pi_{it} - \Omega_i) + \delta_{i2}\widehat{y}_{it}$ where π_{it} is the (net) inflation rate and Ω_i its steady-state value.

$$\tau_{2t} - g_{2t} = \alpha_{20} + \alpha_{21}\left(\frac{B_{2t-1}}{P_{2t-1}}\right) + \alpha_{22}\widehat{y}_{2t} + \varepsilon^{\tau}_{2t} \tag{3.5}$$

Besides responding to a cyclical indicator (α^A_{12}, α^B_{12}, α_{22}), and to an innovation ($\varepsilon^{\tau A}_{1t}$, $\varepsilon^{\tau B}_{1t}$, ε^{τ}_{2t}), the surplus in each country reacts to the amount of real outstanding debt as state variable, with coefficients α^A_{11}, α^B_{11} and α_{21} respectively.

In this section we return to the original setup in which producers act in a monopolistic competitive environment and set prices whose adjustment is costly ($\phi_s \neq 0$); its log-linear version appears in Appendix A. We assume that imperfect risk sharing applies also within the monetary union.[7] The path followed by the endogenous variables in the model is driven by the dynamic behavior of real debt, prices and consumption. Inspection of the log-linear model suggests that a subset of parameters are of crucial importance to shape the dynamics of these variables: β, δ_{11}, δ_{21}, α^A_{11}, α^B_{11}, α_{21}. The discount rate enters into the autorregressive process of the system, whereas the other parameters determine the feedback from past inflation into current inflation (δ_{11}, δ_{21}, through the effect of monetary policy on the excess demand) and from the stock of debt into future debt (α^A_{11}, α^B_{11}, α_{21}). Existence of a unique equilibrium is not guaranteed, since there are plausible parameter combinations for which the system (with six dynamic equations) has either too many or too few eigenvalues outside the unit circle.

Given the large size of the system we have carried out the stability analysis using a slightly simplified version; the details are described in Appendix B. The parameter combinations that define the regions of non-equilibrium, indeterminacy and unique equilibrium are a generalization of those obtained by Leeper (1991) in a closed economy model, with exogenous output. [8] The critical values for the relevant parameters are:

$$\beta\delta_{i1} \lesseqgtr 1, \; i = 1, 2$$

$$\beta^{-1} - \alpha^l_{i1} \lesseqgtr 1, \; i = 1, 2 \text{ and } l = A, B$$

Unlike the closed economy model, we now find many other combinations that produce a unique nominal equilibrium, but which cannot be labeled either Ricardian or non-Ricardian. Some of these combinations are described in the Appendix. The cases that have received most attention in the literature in a closed economy environment are the Ricardian and non-Ricardian regimes. A Ricardian regime is defined as one in which the monetary policy is active ($\beta\delta_{i1} > 1$), whereas fiscal

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[7]We are imposing: $B^A_{1t}, B^B_{1t}, B_{2t} \geq 0, \forall t$.

[8]See also Leith and Wren-Lewis (2000).

policy adjusts to prevent real debt from exploding (i.e. fiscal policy is passive, $\beta^{-1} - \alpha_{i1}^{l} < 1$). Non-Ricardian regimes have a unique equilibrium too, but monetary policies hardly respond to inflation ($\beta\delta_{i1} < 1$), and it is fiscal policy that pins down the level of nominal variables (i.e. it is active, $\beta^{-1} - \alpha_{i1}^{l} > 1$).

Notice that a pure interest rate peg renders monetary policy unable to fix the level of prices. In equations (3.1) and (3.2) an interest rate peg is represented by low values of δ_{11}, δ_{21}. Taylor (1999) has identified the post-Volcker US monetary policy as one in which δ_{i1} is well above 1, so that it has helped to stabilize prices (i.e. active monetary policy). A similar logic applies to the degree of fiscal response to the stock of debt, although in this case the benchmark value that separates active and passive fiscal policies is $\alpha_{i1}^{l} \approx 0$. A purely exogenous fiscal surplus ($\alpha_{i1}^{l} = 0$) would make the stock of debt explosive, forcing an adjustment of prices to prevent the economy getting onto such a path. Bohn (1998) obtains evidence of such a fiscal reaction to the stock of debt that, although tiny, suffices to characterize US fiscal policy as passive (in the sense of accommodating to prevent the stock of debt from rising out of control).[9]

3.2. Calibration

The calibration sets most parameters at values that are common in the business cycle literature. The utility function is assumed to be logarithmic in consumption, and the discount rate is set at 0.994 to yield an implicit steady-state annual real interest rate of 2.5% and an inflation rate of 1.5%.The income elasticity of money demand is one and the interest elasticity is taken from Chari, Kehoe and McGrattan (2000). The ratio of government consumption to output is 0.3. The ratio $\frac{m}{y}$ is set at 5%. The steady-state surplus is chosen to ensure that the debt-to-output ratio is the one in Table 1. Finally, the slope of the Phillips curves $\left(\frac{\theta}{\phi\Omega^2}\right)$ of all three countries is assumed to be the same, whith a value whith the order of magnitude obtained in the literature (Sbordone (1998)). Higher nominal inertia would be represented by a lower elasticity of this curve.

Aside from the possibility of different policy regimes and from differences in the ratio $\frac{b}{y}$, countries 1 and 2 are assumed to be symmetric in all respectss. The calibration of the model is summarized in Table 1. The benchmark values for the policy rules are displayed in Table 2.

[9] Bohn's results are consistent with those by Canzoneri, Cumbi and Diba (1999). Woodford (2000) argues that these tests are not conclusive and that the possibility of non-Ricardian fiscal policies in the US cannot be ruled out.

3.3. Fiscal Shocks

In this section we assess the relevance of some of the effects discussed so far. In particular we are interested in the response of inflation within the monetary union and the exchange rate to a fiscal shock that takes place in country 1B. As discussed above, the macroeconomic effect of such a shock will depend mainly on the combination of policies in all the countries.

Figure 1 compares the effects of an expansionary fiscal shock in country 1B under two different policy combinations. The shock is formulated as an unexpected cut of 1% in the tax rate of that country. The starred line represents the situation under what we call a Ricardian regime in both countries. The magnitude of the fiscal authorities' tax response to the level of debt is 0.05, the estimated value found by Bohn (1998) for the US, whereas the monetary authorities follow a Taylor rule. The continuous line is a non-Ricardian regime and corresponds to the policy parameters described in Table 2. Country 2 follows a tight fiscal ($\alpha_{21} = 0.05$) and monetary policy ($\delta_{21} = 1.5$), while in the monetary union country 1A also displays a strong fiscal response to the level of debt ($\alpha_{11}^{A} = 0.05$), whereas the deficit in country 1B is virtually exogenous ($\alpha_{11}^{B} = 0.001$); the monetary authority of the union hardly responds to the level of inflation ($\delta_{11} = 0.1$). When the Ricardian regime prevails worldwide, consumption is determined by permanent income, which is not affected by tax changes; thus, a fiscal shock in 1B does not have inflation or output effects. A temporary tax cut leads to a temporary rise in real debt that returns to the steady state as a result of the response of the surplus driven by the fiscal policy reaction function of that country.

Under a non-Ricardian regime the fiscal authority in 1B does not react to the rise in the level of debt and another way of preventing a debt explosion is needed. This is achieved by means of a sharp rise in the price level (and inflation) of the union, as observed in the continuous line in the figure. This brings debt in country 1B onto a path that asymptotically converges with its steady state. Also, the reduction in the level of real debt in country 1A induces a reduction in taxes. The interest rate rises as a result of the attempt by the monetary authorities in the union to prevent the inflation increase. The implied fall in the real interest rate induces a larger output effect.

In the standard sticky price open macro model, a positive temporary fiscal shock creates an excess demand for domestic goods relative to foreign goods that generates a temporary exchange rate appreciation. Under the simulated Ricardian regime this is limited by the immediate fiscal response. Surprisingly, in the non-Ricardian regime the impact effect is a currency depreciation, that is expected to proceed further until the interest rate differential disappears. The expected depreciation is consistent with the difference in the nominal interest rate across countries, but the initial depreciation is somewhat counterintuitive. It comes about because of the purchasing power parity condition that ties P_2 with (P_1/e): since P_2 reacts very little, the exchange rate simply follows the path of P_1.

This sequence of events is virtually independent of the response of the fiscal authority in country 1A. To see to what extent country 1A could offset the weak response of taxes in 1B, we simulate the effects of the same temporary tax cut than before, but assuming that the fiscal rule of 1A is much more responsive to its own debt: $\alpha_{11}^A = 1.0$ (continuous line). Both impulse-responses are depicted in Figure 2 and we see that they are almost identical in terms of the aggregated variables. The only variables affected are the paths of taxes and debt in country 1A.

Similarly, a stronger response by the union's monetary authority is of little help. Figure 3 compares two non-Ricardian regimes, one being again the benchmark case of Table 2 (represented here by the starred line) and the other assuming that the central bank of the union sets a tighter inflation response, $\delta_{11} = 0.8$ (continuous line). The effects of the tax cut are qualitatively similar. With a tighter monetary policy the price level increases by a similar amount but this increase is more persistent. The only difference is to be found in the output response that is now flatter, mostly because of the flat response of the real interest rate. In any case, these results confirm Woodford's (1996) findings in the context of a closed monetary union and have the implication that a sufficient degree of endogeneity of surpluses is needed to bring all countries into a union-wide Ricardian regime in which monetary policy can be held responsible for keeping prices stable.

To complete the picture, Figure 4 compares the effects of the same tax cut in country 1B in two non-Ricardian scenarios that differ not merely in the size of the parameters (as in Figures 2 and 3) but in the policy combinations themselves. Again, the starred line represents the responses of the variables in the benchmark non-Ricardian case discussed so far. The continuous line represents a policy combination that yields uniqueness at a worldwide level, although it would have generated either overdetermination or indeterminacy if both countries were closed economies.[10] Under this parameterization, country 2 follows a loose monetary ($\delta_{21} = 0.1$) policy and a Ricardian fiscal one ($\alpha_{21} = 0.05$), whereas monetary policy is tight in country 1 ($\delta_{11} = 1.5$) and fiscal policies differ within the union: country 1A is Ricardian ($\alpha_{11}^A = 0.05$) whereas country 1B is not ($\alpha_{11}^B = 0.001$).

[10] This corresponds to the first example in case 4 discussed in Appendix B.

There are four remarkable differences between the two cases: first, the real interest rate now increases, second, the fiscal shock in 1B is now much less inflationary for the union as a whole, third, the tax cut now generates a recession in the union and, finally, the exchange rate suffers a substantial appreciation, whereas it depreciated in the benchmark case. These two sets of impulse responses could lead the observer to mistakenly interpret them as resulting from completely different shocks.

4. Conclusions

The fiscal theory of the price level has challenged the conventional view that monetary factors drive prices and exchange rates and has also provided a rationale for fiscal restriccions in a monetary union. This paper revisits the results of this theory in the context of an open monetary union model. In contrast with the closed economy, in this extended framework the discussion of equilibrium determination becomes richer and more complex. A wider range of assumptions on risk sharing and policy coordination schemes generate a unique equilibrium, each with specific properties. Some of these combinations reproduce the results obtained in the closed economy, but others display significantly different outcomes.

Appendix A: The log-linear model

$$E_t\widehat{c}_{t+1} - \widehat{c}_t = \sigma^{-1}\left(\frac{\overline{R}_1}{1+\overline{R}_1}\right)\widehat{R}_{1t} - \sigma^{-1}\left(E_t\widehat{P}_{t+1} - \widehat{P}_t\right) \quad \text{(A.1)}$$

$$\widehat{P}_{1t} = \widehat{P}_{2t} + \widehat{e}_t \quad \text{(A.2)}$$

$$\left(\frac{\overline{R}_1}{1+\overline{R}_1}\right)\widehat{R}_{1t} - \left(\frac{\overline{R}_2}{1+\overline{R}_2}\right)\widehat{R}_{2t} = E_t\widehat{e}_{t+1} - \widehat{e}_t \quad \text{(A.3)}$$

$$\widehat{M}_{1t} - \widehat{P}_{1t} = \left(\frac{\sigma}{\varepsilon}\right)\widehat{c}_t - \left(\frac{\overline{c}^{\sigma}\chi}{\overline{R}_1\varepsilon\left(\frac{\overline{M}_1}{\overline{P}_1}\right)^{\varepsilon}}\right)\widehat{R}_{1t} \quad \text{(A.4)}$$

$$\widehat{M}_{2t} - \widehat{P}_{2t} = \left(\frac{\sigma}{\varepsilon}\right)\widehat{c}_t - \left(\frac{\overline{c}^{\sigma}\chi}{\overline{R}_2\varepsilon\left(\frac{\overline{M}_2}{\overline{P}_2}\right)^{\varepsilon}}\right)\widehat{R}_{2t} \quad \text{(A.5)}$$

$$\widehat{P}_{1t} - \widehat{P}_{1t-1} = \beta\left(E_t\widehat{P}_{1t+1} - \widehat{P}_{1t}\right) + \left(\frac{\theta(\gamma-1)}{\phi_1\Omega_1^2}\right)\widehat{y}_{1t} + \left(\frac{\theta\sigma}{\phi_1\Omega_1^2}\right)\widehat{c}_t + \varepsilon_{1t}^P \quad \text{(A.6)}$$

$$\widehat{P}_{2t} - \widehat{P}_{2t-1} = \beta\left(E_t\widehat{P}_{2t+1} - \widehat{P}_{2t}\right) + \left(\frac{\theta(\gamma-1)}{\phi_2\Omega_2^2}\right)\widehat{y}_{2t} + \left(\frac{\theta\sigma}{\phi_2\Omega_2^2}\right)\widehat{c}_t + \varepsilon_{2t}^P \quad \text{(A.7)}$$

$$\left(\widehat{B}_{1t}^A - \widehat{P}_{1t}\right) = \beta^{-1}\left(\widehat{B}_{1t-1}^A - \widehat{P}_{1t}\right) + \left(\frac{\overline{B}_1^A}{\overline{P}_1}\right)^{-1}\left(\overline{g}_1^A\widehat{g}_{1t}^A - \overline{\tau}_1^A\widehat{\tau}_{1t}^A\right) +$$

$$+\left(\frac{\overline{R}_1}{\Omega_1}\frac{\overline{B}_1^A}{\overline{P}_1}\right)\widehat{R}_{1t-1} - \left(\frac{\overline{M}_1}{2\overline{B}_1^A}\right)\left(\widehat{M}_{1t} - \widehat{P}_{1t}\right) - \left(\frac{\overline{M}_1}{2\overline{B}_1^A\Omega_1}\right)\left(\widehat{M}_{1t-1} - \widehat{P}_{1t}\right) \quad \text{(A.8)}$$

$$\left(\widehat{B}_{1t}^B - \widehat{P}_{1t}\right) = \beta^{-1}\left(\widehat{B}_{1t-1}^B - \widehat{P}_{1t}\right) + \left(\frac{\overline{B}_1^B}{\overline{P}_1}\right)^{-1}\left(\overline{g}_1^B\widehat{g}_{1t}^B - \overline{\tau}_1^B\widehat{\tau}_{1t}^B\right) +$$

$$+\left(\frac{\overline{R}_1}{\Omega_1}\frac{\overline{B}_1^B}{\overline{P}_1}\right)\widehat{R}_{1t-1} - \left(\frac{\overline{M}_1}{2\overline{B}_1^B}\right)\left(\widehat{M}_{1t} - \widehat{P}_{1t}\right) - \left(\frac{\overline{M}_1}{2\overline{B}_1^B\Omega_1}\right)\left(\widehat{M}_{1t-1} - \widehat{P}_{1t}\right) \quad \text{(A.9)}$$

$$\left(\widehat{B}_{2t} - \widehat{P}_{2t}\right) = \beta^{-1}\left(\widehat{B}_{2t-1} - \widehat{P}_{2t}\right) + \left(\frac{\overline{B}_2}{\overline{P}_2}\right)^{-1}\left(\overline{g}_2\widehat{g}_{2t} - \overline{\tau}_2\widehat{\tau}_{2t}\right) +$$

$$+\left(\frac{\overline{R}_2}{\Omega_2}\frac{\overline{B}_2}{\overline{P}_2}\right)\widehat{R}_{2t-1}-\left(\frac{\overline{M}_2}{\overline{B}_2}\right)\left(\widehat{M}_{2t}-\widehat{P}_{2t}\right)-\left(\frac{\overline{M}_2}{\overline{B}_2\Omega_2}\right)\left(\widehat{M}_{2t-1}-\widehat{P}_{2t}\right) \tag{A.10}$$

$$\overline{c}\widehat{c}_t+\overline{g}_1^A\widehat{g}_{1t}^A+\overline{g}_1^B\widehat{g}_{1t}^B+\overline{g}_2\widehat{g}_{2t}=\overline{y}_1\widehat{y}_{1t}+\overline{y}_2\widehat{y}_{2t} \tag{A.11}$$

$$\overline{R}_1\widehat{R}_{1t}=\delta_{11}\Omega_1\left(\widehat{P}_{1t}-\widehat{P}_{1t-1}\right)+\delta_{12}\widehat{y}_{1t}+\widehat{\varepsilon}_{1t}^R \tag{A.12}$$

$$\overline{R}_2\widehat{R}_{2t}=\delta_{21}\Omega_2\left(\widehat{P}_{2t}-\widehat{P}_{2t-1}\right)+\delta_{22}\widehat{y}_{2t}+\widehat{\varepsilon}_{2t}^R \tag{A.13}$$

$$\overline{\tau}_1^A\widehat{\tau}_{1t}^A-\overline{g}_1^A\widehat{g}_{1t}^A=\alpha_{11}^A\left(\frac{\overline{B}_1^A}{\overline{P}_1}\right)\left(\widehat{B}_{1t-1}^A-\widehat{P}_{1t-1}\right)+\alpha_{12}^A\widehat{y}_{1t}+\widehat{\varepsilon}_{1t}^{\tau A} \tag{A.14}$$

$$\overline{\tau}_1^B\widehat{\tau}_{1t}^B-\overline{g}_1^B\widehat{g}_{1t}^B=\alpha_{11}^B\left(\frac{\overline{B}_1^B}{\overline{P}_1}\right)\left(\widehat{B}_{1t-1}^B-\widehat{P}_{1t-1}\right)+\alpha_{12}^B\widehat{y}_{1t}+\widehat{\varepsilon}_{1t}^{\tau B} \tag{A.15}$$

$$\overline{\tau}_2\widehat{\tau}_{2t}-\overline{g}_2\widehat{g}_{2t}=\alpha_{21}\left(\frac{\overline{B}_2}{\overline{P}_2}\right)\left(\widehat{B}_{2t-1}-\widehat{P}_{2t-1}\right)+\alpha_{22}\widehat{y}_{2t}+\widehat{\varepsilon}_{2t}^{\tau} \tag{A.16}$$

Appendix B: Stability analysis

In this appendix we calculate the regions of stability of the model, in the case of incomplete risk sharing among all three countries (so that the independent dynamics of all three debts matter).[11] Further we make the following simplifying assumptions:

$$\frac{\overline{c}}{\overline{y}} = \gamma = 1,\ \alpha_{12}^{A} = \alpha_{12}^{B} = \alpha_{22} = \delta_{12} = \delta_{22} = 0$$

After some manipulations, the dynamic representation of the linear version of the model (disregarding exogenous variables) is,

$$\widehat{c}_{t+1} - \widehat{c}_t = \left(\frac{\varepsilon_y}{\sigma\beta}\frac{\overline{c}}{\overline{y}}\right)\widehat{c}_t + \left[\frac{\beta}{\sigma}\left(\delta_{11} - \frac{1}{\beta^2}\right)\left(\frac{\overline{\pi}}{1+\overline{\pi}}\right)\right]\widehat{\pi}_{1t} \tag{B.1}$$

$$\widehat{\pi}_{1t+1} - \widehat{\pi}_{1t} = -\left(\frac{1+\overline{\pi}}{\overline{\pi}}\right)\left(\frac{\varepsilon_y c}{\beta y}\right)\widehat{c}_t + \left(\frac{1}{\beta} - 1\right)\widehat{\pi}_{1t} \tag{B.2}$$

$$\Delta\widehat{e}_{t+1} - \Delta\widehat{e}_t = \beta\left(\delta_{11} - \delta_{21}\right)\left(\frac{\overline{\pi}}{1+\overline{\pi}}\right)\widehat{\pi}_{1t} + \left(\beta\delta_{21} - 1\right)\Delta\widehat{e}_t \tag{B.3}$$

$$\widehat{b}_{1t+1}^{A} - \widehat{b}_{1t}^{A} = H_{11}^{A}\widehat{c}_t + H_{12}^{A}\widehat{\pi}_{1t} + H_{14}^{A}\widehat{b}_{1t}^{A} \tag{B.4}$$

$$\widehat{b}_{1t+1}^{B} - \widehat{b}_{1t}^{B} = H_{11}^{B}\widehat{c}_t + H_{12}^{B}\widehat{\pi}_{1t} + H_{15}^{B}\widehat{b}_{1t}^{B} \tag{B.5}$$

$$\widehat{b}_{2t+1} - \widehat{b}_{2t} = H_{21}\widehat{c}_t + H_{22}\widehat{\pi}_{1t} + H_{23}\Delta\widehat{e}_t + H_{26}\widehat{b}_{2t} \tag{B.6}$$

in which there are three forward-looking variables, $(\widehat{c}_t,\ \widehat{\pi}_{1t},\ \Delta\widehat{e}_t)$ and three predetermined ones $\left(\widehat{b}_{1t}^{A},\ \widehat{b}_{1t}^{B},\ \widehat{b}_{2t}\right)$. Also, $\widehat{\pi}_t = \left(\frac{1+\overline{\pi}}{\overline{\pi}}\right)\left(\widehat{P}_{1t} - \widehat{P}_{1t-1}\right)$, the $H's$, are functions of the parameters in the model, ε_y is the output elasticity in the price equation $\left(\frac{\theta((\gamma-1)+\sigma)}{\phi_1\Omega_1^2}\right)$ and, $H_{14}^{A} = \left(\frac{1}{\beta} - \alpha_{11}^{A} - 1\right)$, $H_{15}^{B} = \left(\frac{1}{\beta} - \alpha_{11}^{B} - 1\right)$, $H_{26} = \left(\frac{1}{\beta} - \alpha_{21} - 1\right)$. In matrix form:

$$\Delta\widehat{x}_{t+1} = A\widehat{x}_t$$

where,

[11] Under perfect risk sharing there would be either two (risk sharing among countries 1A and 1B but not among 1 and 2) or one (risk sharing among countries 1A, 1B and 2) dynamic equations, representing the time pattern of debt.

$$\widehat{x}_t' = \left[\begin{array}{cccccc} \widehat{c}_t & \widehat{\pi}_{1t} & \Delta\widehat{e}_t & \widehat{b}_{1t}^A & \widehat{b}_{1t}^B & \widehat{b}_{2t} \end{array}\right]$$

$$A = \left[\begin{array}{llllll} \left(\frac{\varepsilon_y}{\sigma\beta}\right) & \frac{\beta}{\sigma}\left(\delta_{11} - \frac{1}{\beta^2}\right)\left(\frac{\pi}{1+\pi}\right) & 0 & 0 & 0 & 0 \\ \left(\frac{-\varepsilon_y}{\beta}\right)\left(\frac{1+\pi}{\pi}\right) & \left(\frac{1}{\beta} - 1\right) & 0 & 0 & 0 & 0 \\ 0 & \beta\left(\delta_{11} - \delta_{21}\right)\left(\frac{\pi}{1+\pi}\right) & \left(\beta\delta_{21} - 1\right) & 0 & 0 & 0 \\ H_{11}^A & H_{12}^A & 0 & H_{14}^A & 0 & 0 \\ H_{11}^B & H_{12}^B & 0 & 0 & H_{15}^B & 0 \\ H_{21} & H_{22} & H_{23} & 0 & 0 & H_{26} \end{array}\right]$$

Given the form of the dynamic system, the characteristic roots do not depend on the parameters H_{11}, H_{12}, H_{21}, H_{22}, H_{23}. The eigenvalues of this system are:

$$z_1 = \left(\frac{1}{\beta} - \alpha_{11}^A - 1\right)$$

$$z_2 = \left(\frac{1}{\beta} - \alpha_{11}^B - 1\right)$$

$$z_3 = \left(\frac{1}{\beta} - \alpha_{21} - 1\right)$$

$$z_4 = \left(\delta_{21}\beta - 1\right)$$

$$z_5 = \frac{1}{2}\left[\left(\frac{\varepsilon_y}{\sigma\beta} + \frac{1}{\beta} - 1\right) - \sqrt{\left(\frac{\varepsilon_y}{\sigma\beta} + \frac{1}{\beta} - 1\right)^2 - 4\left[\frac{\varepsilon_y}{\sigma\beta}\left(\delta_{11}\beta - 1\right)\right]}\right]$$

$$z_6 = \frac{1}{2}\left[\left(\frac{\varepsilon_y}{\sigma\beta} + \frac{1}{\beta} - 1\right) + \sqrt{\left(\frac{\varepsilon_y}{\sigma\beta} + \frac{1}{\beta} - 1\right)^2 - 4\left[\frac{\varepsilon_y}{\sigma\beta}\left(\delta_{11}\beta - 1\right)\right]}\right]$$

where $z_6 > 0$ (provided that δ_{11} is not too large) and

$$sign\ (z_5) = sign\ (\delta_{11}\beta - 1)$$

The stability regions are a generalization of those in Leeper (1991), although many more combinations are now possible.[12] By way of illustration, the following

[12] For the general case of $\delta_{21}, \delta_{22} > 0$, the limits of the regions are slightly different. In the closed economy model, the benchmark value for δ_1 is given by:

$$\delta_1 \gtrless \frac{1}{\beta} - \frac{1}{\varepsilon_y}\left(\frac{1}{\beta} - 1\right)\left(\frac{\delta_2}{1+R}\right)$$

Since the interest rate would respond to the output gap, a lower direct response to demand shocks is required to stabilize prices, i.e. to achieve an active monetary policy interesting cases arise:

1) Unique equilibrium under Ricardian policies: Ricardian regime. There is only one possible combination that produces this result ($z_1, z_2, z_3 < 0$, $z_4, z_5, z_6 > 0$):

$$\alpha_{11}^A > \frac{1}{\beta} - 1,\ \alpha_{11}^B > \frac{1}{\beta} - 1,\ \alpha_{21} > \frac{1}{\beta} - 1,\ \delta_{11}\beta > 1,\ \delta_{21}\beta > 1$$

2) A unique equilibrium under non-Ricardian policies in all countries cannot be obtained.

$$\alpha_{11}^A < \frac{1}{\beta} - 1,\ \alpha_{11}^B < \frac{1}{\beta} - 1,\ \alpha_{21} < \frac{1}{\beta} - 1,\ \delta_{11}\beta < 1,\ \delta_{21}\beta < 1$$

In this case there is no equilibrium since there are more unstable roots than non-predetermined variables: $z_1, z_2, z_3, z_6 > 0$, $z_4, z_5 < 0$. A unique non-Ricardian equilibrium can be obtained if we make:

$$\text{either } \alpha_{11}^A > \frac{1}{\beta} - 1 \text{ or } \alpha_{11}^B > \frac{1}{\beta} - 1$$

in such a case P_1 is determined by the budget constraint of the non-Ricardian government.

3) Price levels are indeterminate whenever there are less unstable roots than non-predetermined variables. One such case is: $z_1, z_2, z_4, z_5 < 0$, $z_3, z_6 > 0$, i.e.

$$\alpha_{11}^A > \frac{1}{\beta} - 1,\ \alpha_{11}^B > \frac{1}{\beta} - 1,\ \alpha_{21} < \frac{1}{\beta} - 1,\ \delta_{11}\beta < 1,\ \delta_{21}\beta < 1$$

4) Finally, there are other combinations which may produce a unique equilibrium without a clear Ricardian or non-Ricardian pattern. One example would be: $z_1, z_3, z_5 < 0$, $z_2, z_4, z_6 > 0$, i.e.

$$\alpha_{11}^A > \frac{1}{\beta} - 1,\ \alpha_{11}^B < \frac{1}{\beta} - 1,\ \alpha_{21} > \frac{1}{\beta} - 1,\ \delta_{11}\beta > 1,\ \delta_{21}\beta < 1$$

In this parameter combination, country 2 is in a situation in which monetary policy hadly responds to the level of inflation, whereas the fiscal authority reacts quickly and strongly to prevent the level of debt from exploding. With this policy scheme, the price level in country 2 would not be uniquely determined in a closed economy framework (i.e., this economy on its own would display price level indeterminacy). Country 1 is in a different situation: since the fiscal authority of country 1B lacks discipline, the price level has to adjust to prevent b_{1t}^B from exploding, but at the same time, an aggressive monetary reaction function is at work to determine the level of P_{1t} . Thus, there would be no equilibrium in a closed economy framework (i.e., this economy on its own would display price level over-determination). Nevertheless, in the open economy context, this combination of policies helps to determine two nominal variables whereas the third one is determined by the PPP condition.

Another interesting example would be: $z_1, z_2, z_5 < 0$, $z_3, z_4, z_6 > 0$, i.e.

$$\alpha_{11}^A > \frac{1}{\beta} - 1,\ \alpha_{11}^B > \frac{1}{\beta} - 1,\ \alpha_{21} < \frac{1}{\beta} - 1,\ \delta_{11}\beta > 1,\ \delta_{21}\beta < 1$$

In this case country 1 is in a Ricardian regime, so that P_1 is determined by the stance of monetary policy in that country. Since country 2 is non-Ricardian, P_2 is fixed by the present value of this country's surpluses.

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TABLE 1

PARAMETER AND STEADY-STATE VALUES		
Parameter values	Definition	Value
Risk aversion	σ	1.0
Discount Rate	β	0.994
Elasticity of disutility of effort	γ	2.0
output elasticity of inflation	$\frac{\theta}{\phi\Omega^2}$	0.12
Consumption elasticity of money demand	$\frac{\sigma}{\varepsilon}$	1.0
Interest rate elasticity of money demand	$\frac{\bar{c}^{\frac{\sigma}{\varepsilon}}\chi}{\bar{R}\varepsilon m^{\varepsilon}}$	0.4
Steady-state values		
Nominal interest rate	R	1.01
Real exchange rate	$\frac{P_1}{eP_2}$	1.0
Money to output ratio	$\frac{m}{y}$	0.05
government consumption to output ratio		
Country 1A	$\frac{2g_1^A}{y_1}$	0.30
Country 1B	$\frac{2g_1^B}{y_1}$	0.30
Country 2	$\frac{g_2}{y_2}$	0.30
Debt to output ratio		
Country 1A	$\frac{2b_1^A}{y_1}$	0.6
Country 1B	$\frac{2b_1^B}{y_1}$	1.0
Country 2	$\frac{b_2}{y_2}$	0.5

TABLE 2

POLICY RULES: A NON-RICARDIAN REGIME					
Fiscal Rule $\tau_{it}^L - g_{1t}^L = \alpha_{i1}^L \left(\frac{B_{1t-1}^A}{P_{1t-1}}\right) + \alpha_{i2}^L \widehat{y}_{1t}$			Interest Rate Rule $R_{it} = \delta_{i1} \left(\frac{P_{it}}{P_{it-1}}\right) + \delta_{i2} \widehat{y}_{it}$		
	Country 1	Country 2		Country 1	Country 2
α_{11}^A	0.05		δ_{11}	0.1	
α_{11}^B	0.001		δ_{12}	0.5	
α_{12}^L	0.00				
α_{21}		0.05	δ_{21}		1.5
α_{22}		0.00	δ_{22}		0.5

Figure 1

An asymmetric positive fiscal shock in the monetary union

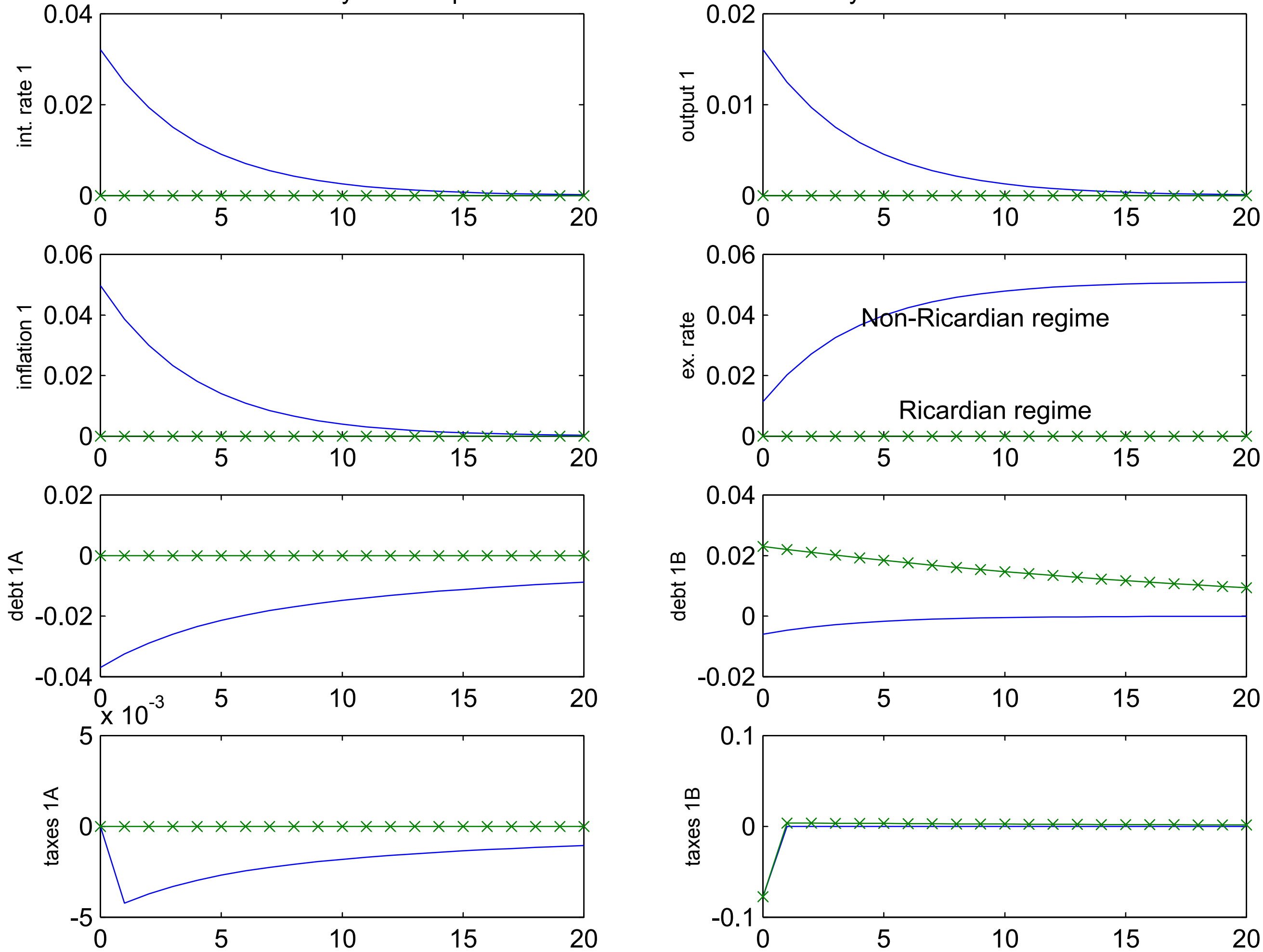

Figure 2
An asymmetric positive fiscal shock in the monetary union:
A non-Ricardian regime

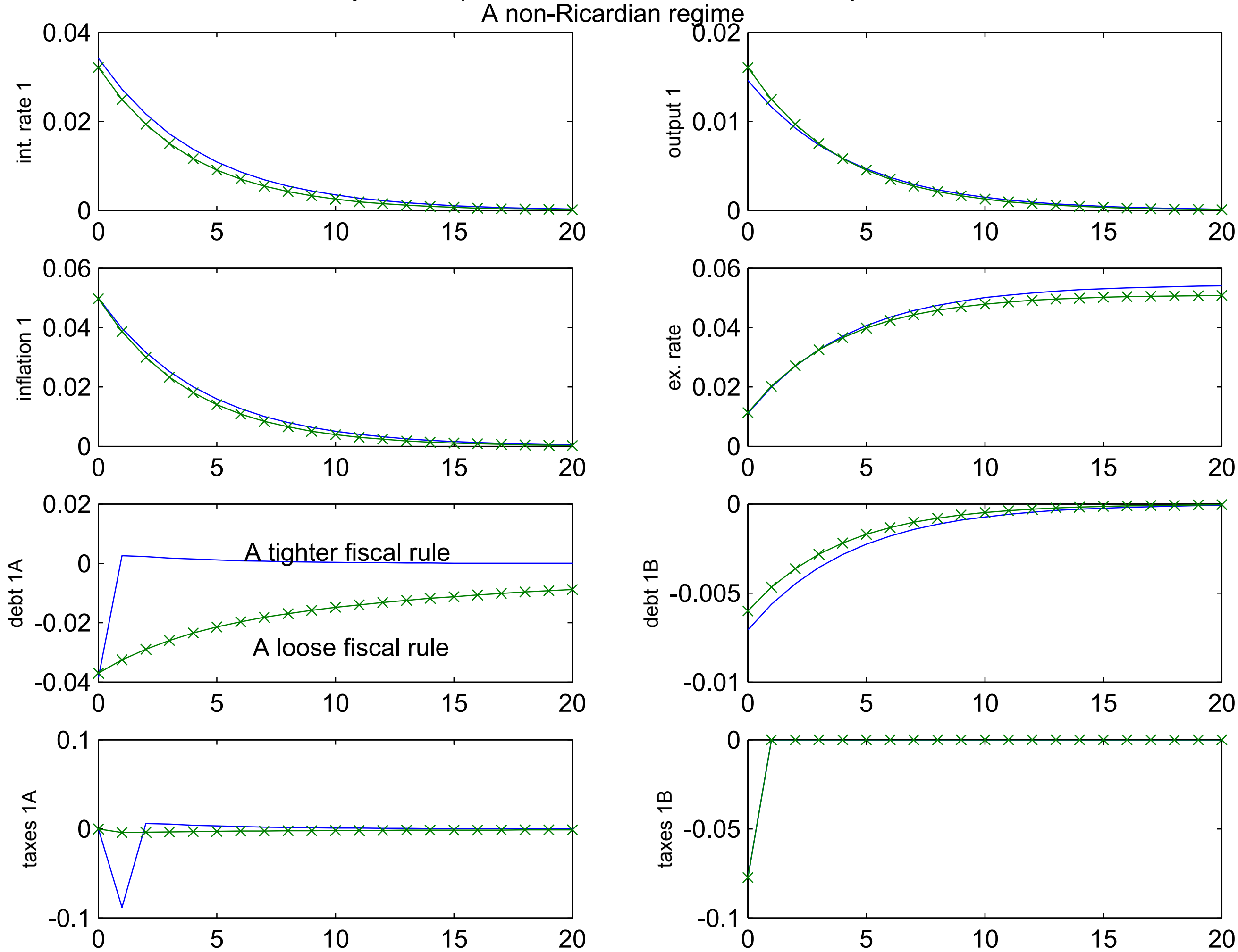

Figure 3
An asymmetric fiscal shock in the monetary union:
A non-Ricardian regime

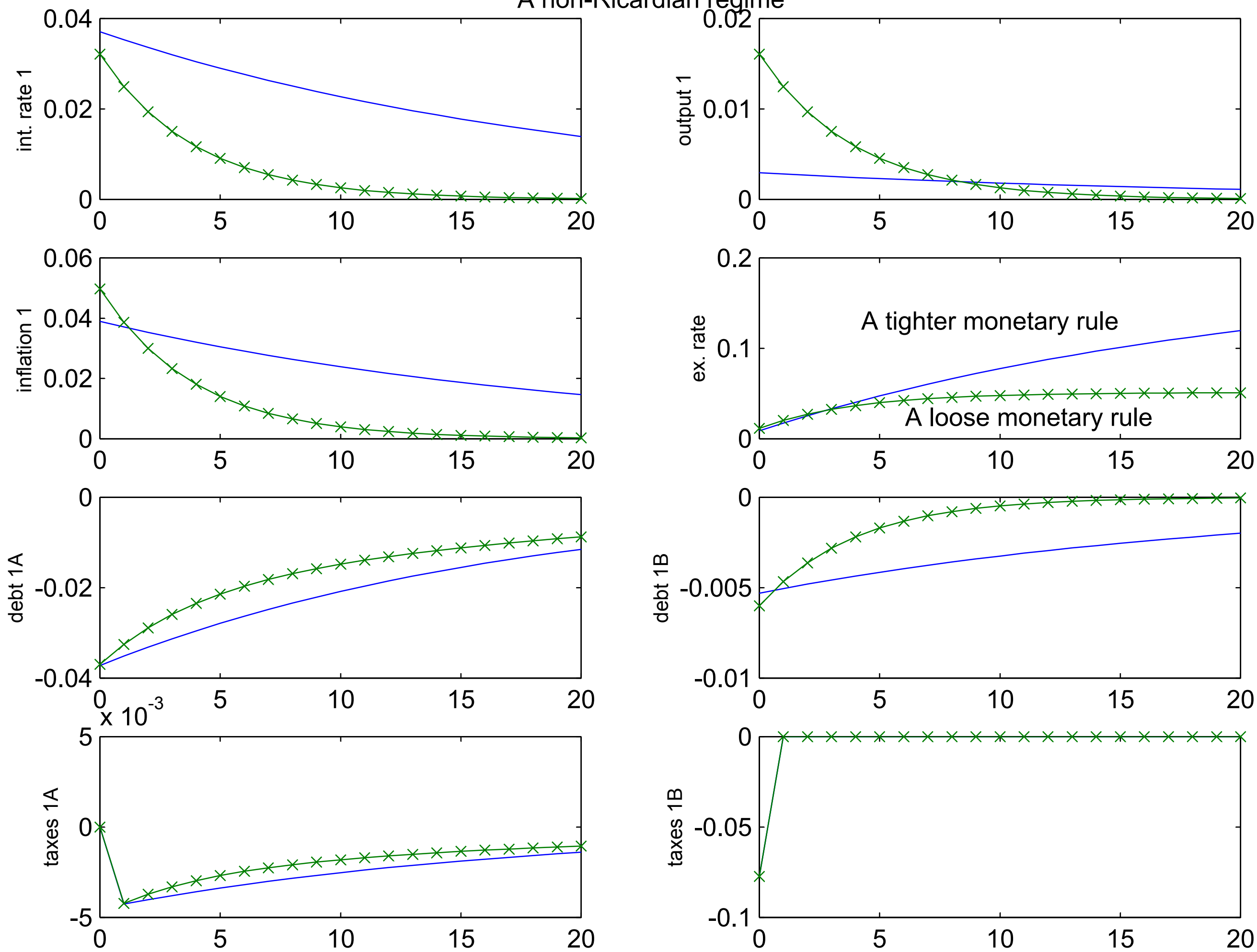

Figure 4
An Asymmetric positive fiscal shock in the monetary union:
Alternative non-Ricardian regimes