Uncertainty bounds for CABs

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1. Introduction

This paper addresses issues concerning the uncertainty surrounding estimates of the output gap and the cyclically adjusted budget deficit. Uncertainty about the scale of the output gap is likely to be reflected in uncertainty about the size of the cyclical adjustment to be applied to the budget deficit. Our main objective in this paper is to evaluate the uncertainty associated with a commonly used estimate of the output gap and to demonstrate how this should be reflected in uncertainty over the cyclically adjusted budget deficit. The degree of uncertainty associated with the estimate of the output gap for a period in the past is noticeably less than that associated with the estimate of the current (or real-time) gap, and hence we also evaluate quasi-real time estimates that use data up to the point where we record the gap, with the next period's gap updating this estimate using the next period's outturn. Uncertainty of the cyclically adjusted budget deficit is compounded by uncertainty over the link between the output gap and the budget deficit. If the gap were to be caused by tax rich elements of demand such as consumption then the impact of a given gap on the deficit would be larger than when the gap is caused by changes in a tax poor component of demand such as exports. We therefore undertake a further analysis of the sources of shocks and their impact on the uncertainty surrounding the budget deficit.

In the second section of the paper we discuss methods of estimating the output gap and the associated uncertainty. We focus on the Harvey-Trimbur (2003) estimator, which approximates a parametric version of the ideal band pass filter discussed in Baxter and King (1999). In the third section we present full sample and real time estimates of the output gap and the associated uncertainty for the UK, Germany, France, Italy, Spain, the Netherlands and Denmark, and we note that over much of the sample (although not at the end) the real time estimate is much more variable (and more uncertain) than the full sample estimate, reflecting the difficulty we have in separating fluctuations in the output gap and in trend output in real time. In the fourth section we calculate the cyclically adjusted budget deficits from real time and full sample output gaps and compare them. We also compare the uncertainty in the cyclically adjusted deficit given the uncertainty associated with each measure of the gap. We discuss the differences in the cyclically adjusted budget deficits for the period 1989 to 1994 using real time and full sample output gap estimates. This allows us to stress the differences in these two estimates that we might see in a period of slow growth, and we also note that it is possible for the real time estimate to fall outside the 95% confidence bounds for the full sample estimate. The fourth section looks at the sensitivity of our results, first by using a different parameterisation of the Harvey-Trimbur filter for the UK, and then by applying different coefficients to the output gap in the calculation of the cyclically adjusted deficit. These coefficients come from a model based analysis of multipliers set out in an annex. The final section draws conclusions about the difficulties associated with making reliable estimates of the cyclically adjusted budget deficit.

2. Estimating the output gap and its uncertainty

2.1 Model uncertainty

The output gap, the difference between actual and potential output, has an importance in the popular debate which can tend to run ahead of the problems in measuring it; the output gap is not observable. The choice of what measure, or estimator, of the output gap to use is more than a dry academic issue. As the analysis of Canova (1998), for example, showed inference can be sensitive to measurement. In this paper we largely abstract from issues concerning model uncertainty and consider just one leading output gap estimator – the Harvey-Trimbur cycle. This is sufficient to make our point that use of the output gap in real-time can be problematic since it is often measured imprecisely. The Harvey-Trimbur cycle is a model-based estimator based on unobserved components (UC). It is a generalisation of the class of Butterworth filters that have the attractive property of allowing smooth cycles to be extracted from economic time series - indeed ideal band pass filters emerge as a limiting case; see Harvey and Trimbur (2003).

2.2 Statistical and parameter uncertainty: estimation in real-time

An additional source of uncertainty associated with output gap estimates derives from the fact that policy makers do not have the luxury of being able to wait before deciding whether the economy is currently lying above or below its trend level. They have to decide, without the benefit of hindsight, whether a given change to output in the current period is temporary or permanent, that is whether it is a cyclical or trend movement. As discussed by Mitchell (2003) their problem can be interpreted as a forecasting one, since these real-time output gap estimates are forecasts, in the sense that they are expectations of the output gap conditional on incomplete information. Only with the arrival of additional information, such as revised historical data and data not available at the time, do the output gap estimates eventually settle down at their 'final' values.[1] These real-time or end-of-sample output gap (point) estimates have been found to be unreliable, in the sense that there is a large and significant revision or forecasting error; see Orphanides and van Norden's (2002) application to the US economy and Mitchell's (2003) to the Euro-area.

Since in the absence of data revisions, revisions to real-time output gap estimates are explained by forecasting errors, it is important when de-trending in real-time to produce good forecasts of the (log) level of the underlying series. Application of an UC model can be seen implicitly to forecast future values optimally. This follows from the fact that for a correctly specified model, application of the one-sided Kalman filter is equivalent to application of the two-sided filter (smoother) to the underlying series extended infinitely into the future with optimal forecasts. These optimal, minimum mean square error, forecasts are derived via the Kalman filter, exploiting the state-space representation for a given output gap estimator. There is therefore no need for forecast extensions.

We note that when computing the cyclically adjusted budget deficit the European Commission in fact rely on a production function approach to measuring the output

[1] Values are never truly final because data revisions and the arrival of new data are a continuous process.

gap; see Denis et al. (2006). But since TFP is de-trended with a Hodrick-Prescott filter there remains an end-of-sample problem. Future values of TFP are therefore forecast prior to application of the de-trending filter in an attempt to improve the reliability of the real-time estimates. But as mentioned above reliability will only be improved if 'good' forecasts are produced. Indeed, since the Hodrick-Prescott filter implicitly assumes the data are generated by an IMA(2,2) model, if future values are forecast using an alternative model, this then raises concerns about why the Hodrick-Prescott filter is itself used if an IMA(2,2) model is not believed to be the 'true' model. In any case, given that the European Commission still expect revisions to their real-time output gap estimates they should, as we consider below, produce a measure of uncertainty associated with it and in turn the cyclically adjusted budget deficit.

2.3 Real-time and 'final' estimates of the output gap in the Euro-area

To investigate the unreliability of real-time output gap point estimates for Euro-area economies the following experiment is undertaken. Full sample or final estimates of the output gap are derived using data available over the (full) sample-period using data up to 2006q1. Real-time output gap estimates are computed recursively from 1980q1. This involves using data from the 1960s to 1980q1, to provide an initial estimation period of for most countries 20 years, to compute the real time estimate for 1980q1.[2] Then data up to 1980q2 are used to re-estimate the output gap (that involves re-estimation of the parameters of the models used to measure the output gap) and obtain real-time estimates for 1980q2. [3] This recursive exercise, designed to mimic real-time measurement of the output gap, is carried on until data for the period up to 2006q1 are used to estimate the real-time output gap for 2006q1. These last estimates are the 'final' values.

Throughout we use only the latest vintage of GDP data. What we call the real-time estimate is strictly the quasi-real estimate of Orphanides and van Norden (2002). If we considered real-time data we should expect to find an even greater degree of unreliability associated with the real-time estimates. Nevertheless revisions to published GDP data were found by Orphanides and van Norden (2002) to be less important than so-called statistical revisions. Statistical revisions are explained by the arrival of new data helping macroeconomists, with the advantage of hindsight, better understand the position of the business cycle, and also perhaps revising what model they use to identify and estimate it.

Figures 1-7 give an indication of what the real-time and final output gap estimates look like. Consistent with previous research, these indicate how unreliable real-time output gap estimates can be. They often paint a different picture to the final estimate and clearly illustrate how one can misjudge the position of the business cycle in real-time. In general the full sample estimates are more 'long period volatile than are the real time estimates, which clearly tend to track actual output more closely, suggesting that perhaps too much signal is read into the current data.

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[2] We ignore the fact that GDP data are published with, at least, a one quarter lag.

[3] In the notation of Harvey and Trimbur (2003) we set m=n=2 in each recursive sample, although for the UK we consider also increasing n to n=3. The models are not subjected to standard goodness-of-fit tests to help identify potential weaknesses/model mis-specification. Future work should consider this.

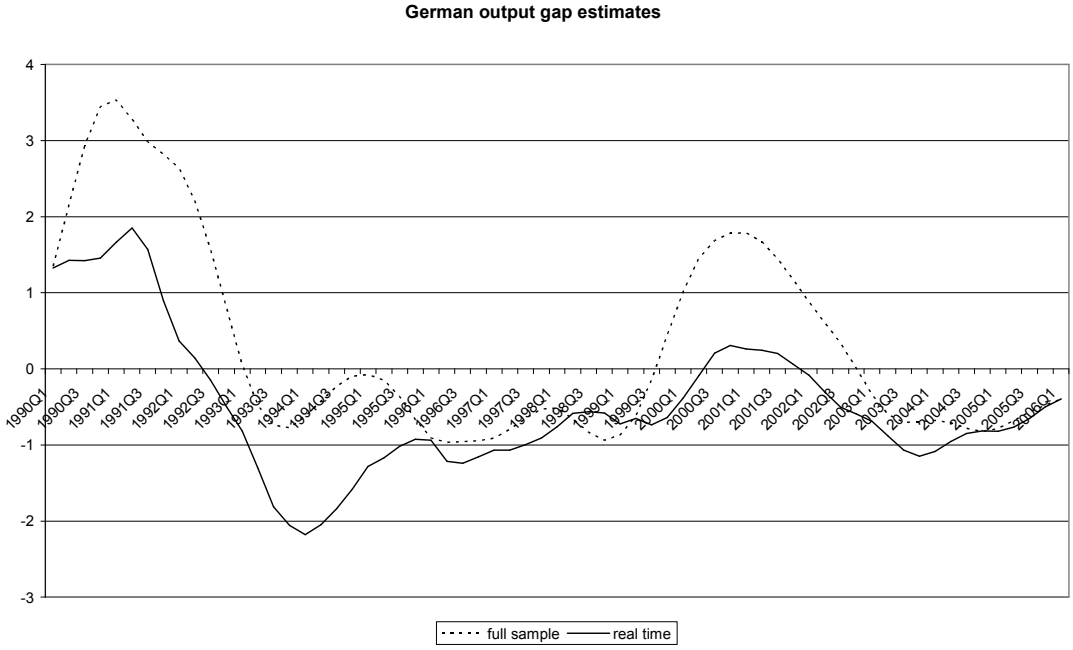


Figure 1

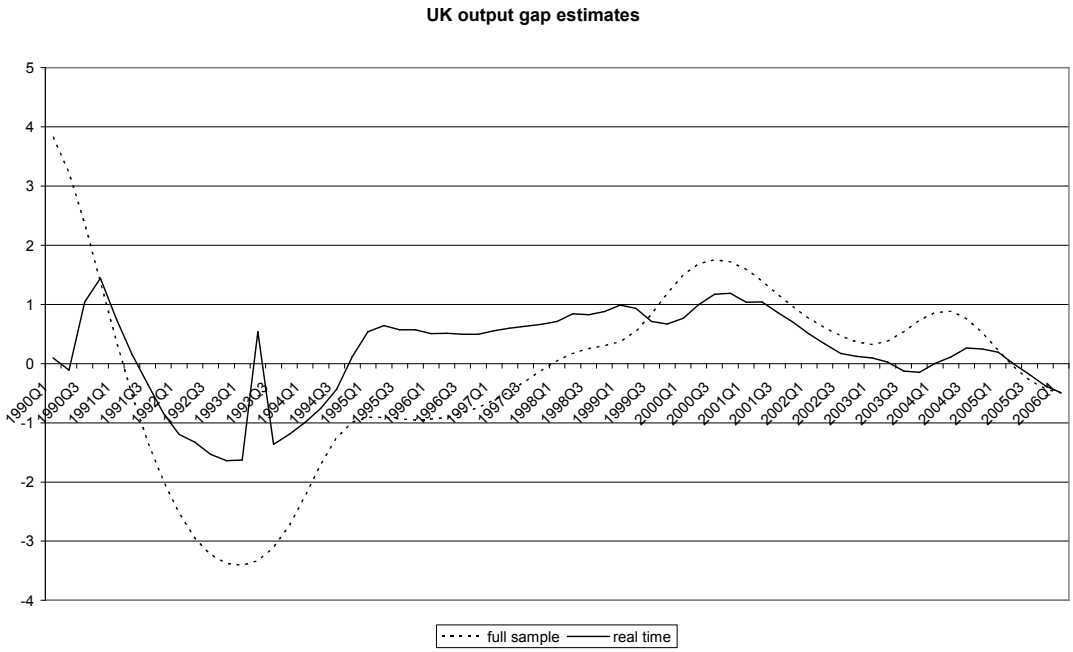


Figure 2

Output gap estimates using this technique suggest that it was difficult to estimate the degree of excess capacity use in the period after German unification and that data at the time may have been interpreted as suggesting trend growth had risen when it probably had not. The same could be said of the mini-boom around 2000, when perceptions of the subsequent period of slow growth were absent. The estimates of UK output gaps using this technique allow a lot of noise into the cycle and hence there is a tendency for the estimate to spike.

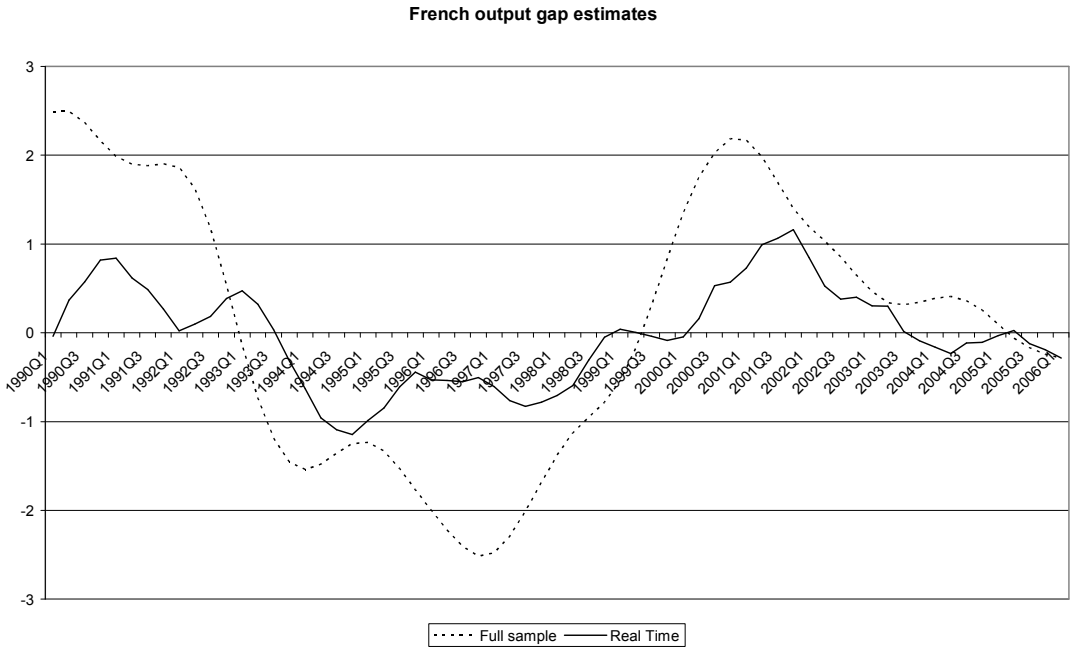


Figure 3

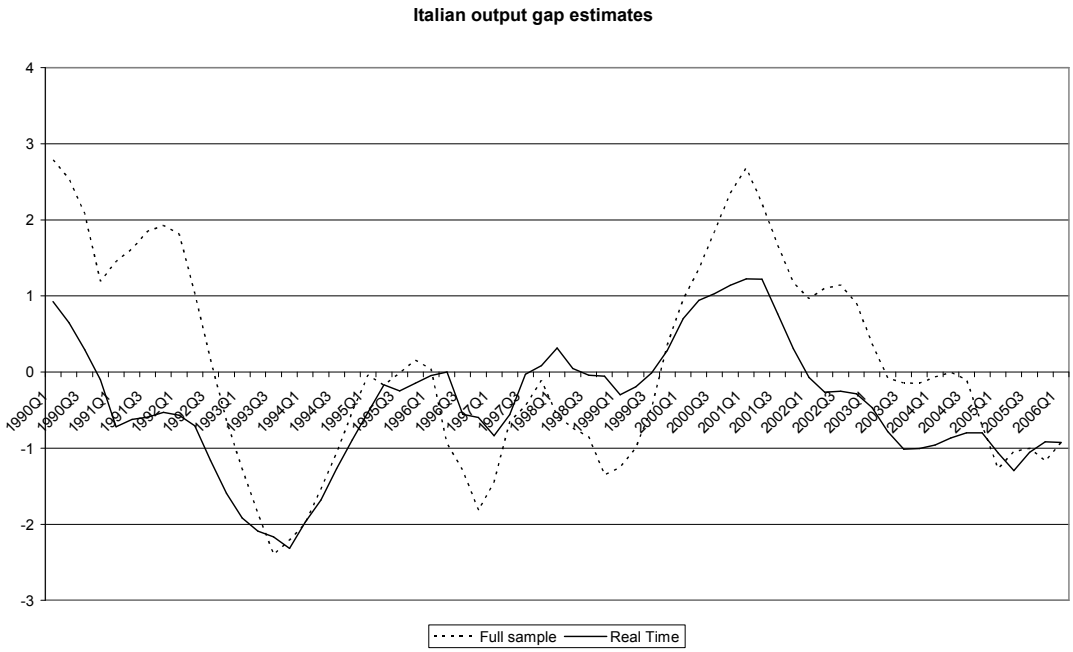


Figure 4

The lack of cyclicality in the real time output gap estimator is also apparent in the outturns for France and Italy, but in both cases the 'short period' volatility of the real time estimator is apparent. The strength of the early 2000s boom in France is only really apparent with hindsight, and the same is true in Italy, although subsequent below capacity growth may be apparent there. The correlation of the real time and final estimates is stronger in Italy than in the other estimates presented here.

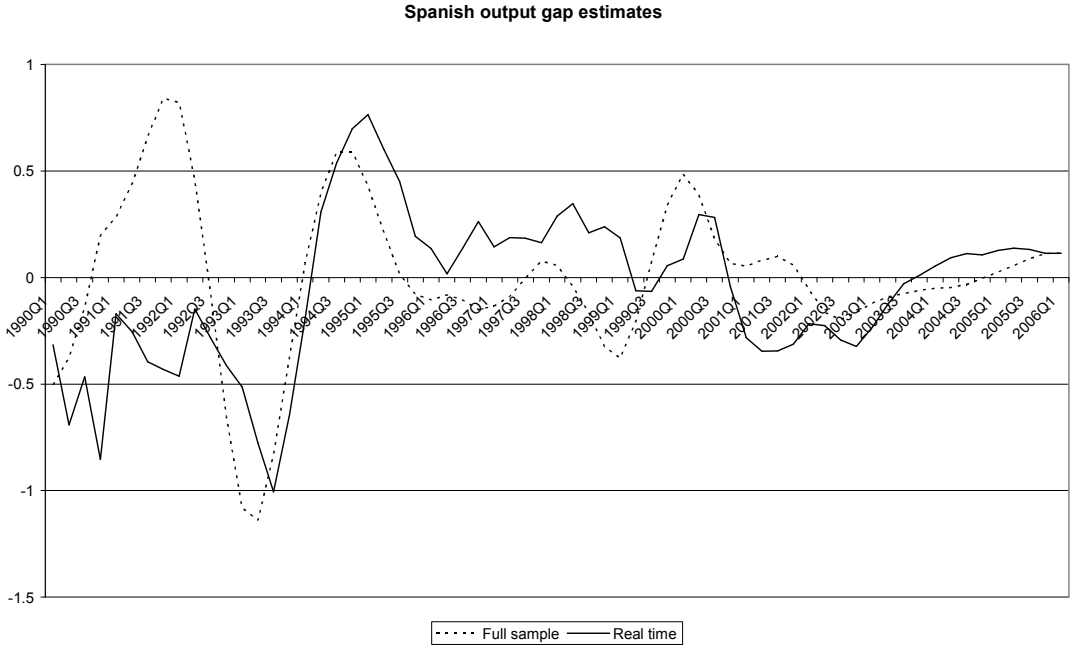


Figure 5

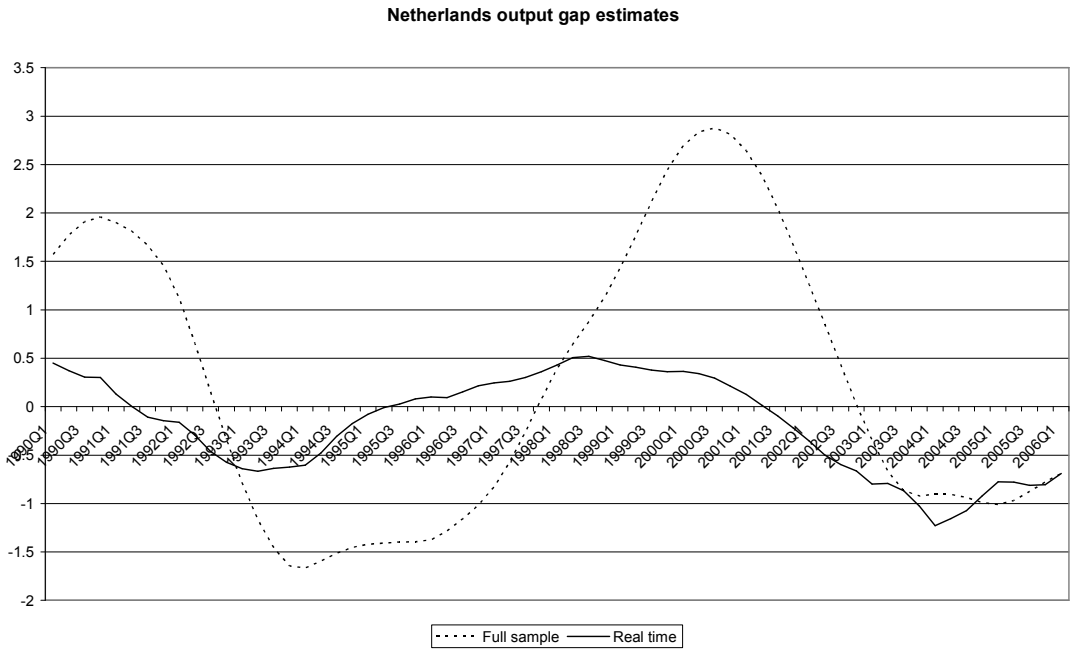


Figure 6

The Spanish output gap, measured either in real time or over the full sample appears to be much less ‘long term’ volatile than the other estimates presented here. This may reflect data based excess quarterly noise in the data, or the generally smooth evolution of actual output. Conversely the Netherlands shows long and large cycles which evolve in a smooth way whichever technique is used, whilst Belgian data show much more ‘short period’ volatility in the estimates of the cycle.

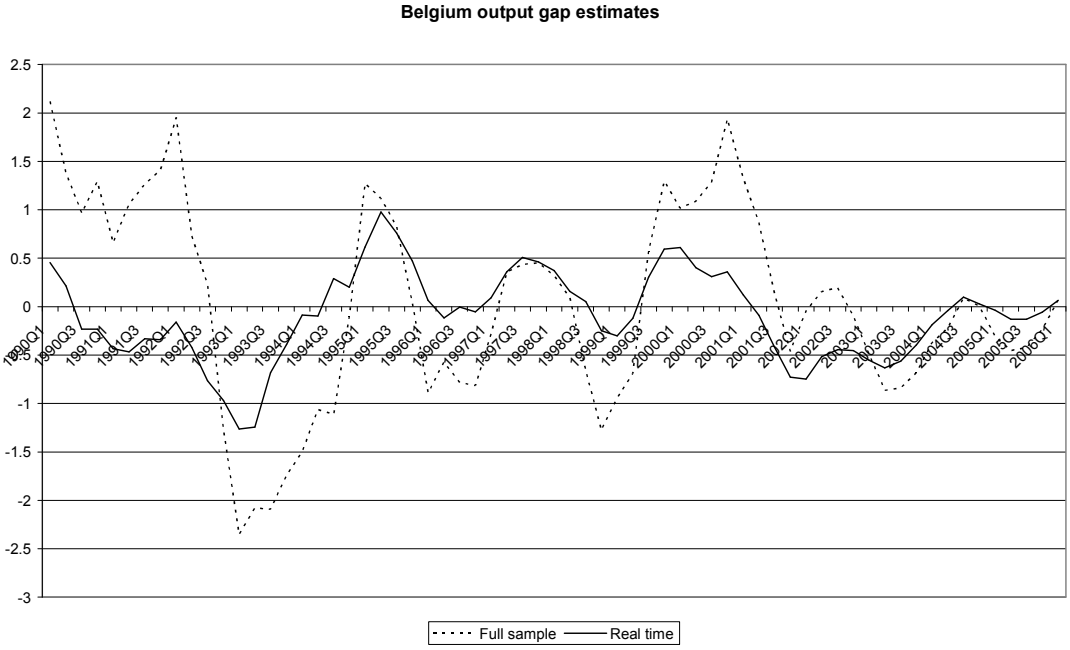


Figure 7

The real-time unreliability of output gap (point) estimates is summarised more fully in Table1. Reflecting the fact that the output gap estimator uses the data in two ways, as well as considering the real-time estimate, Table 1 also examines the filtered estimates or the quasi-final estimates in the parlance of Orphanides and van Norden (2002). UC models use the data in two ways in the sense that first they estimate the parameters of the model, denoted by the vector Θ, and secondly they use these estimates to obtain the filtered and smoothed estimates of the output gap, namely the quasi-final and final estimates of the output gap, respectively. The filtered and smoothed estimates are the expected value of the output gap conditional on information available at time t (t=1,2,...,T) and T, where $T \geq t$, respectively. Let us denote the real-time estimates, based on recursively updated estimates of the parameters of a given UC model Θ, by $y_{t|t}$; let $y_{t|T}$ denote the final estimates of the output gap at time t, that use full-sample information T to estimate Θ, and then let $y_{t|t}$ ($\hat{\Theta}_T$) denote the filtered estimates of the output gap at time t using these full-sample based parameter estimates.

Table 1 indicates that as the future becomes the present output gap estimates are revised. Real-time point estimates of the output gap in the Euro area are unreliable, in the sense that there is a large and important revision error. This is reflected by the correlation coefficients being on average only 0.44. Reflecting the importance of ex post information in re-defining the parameter values the filtered estimates are more reliable than the real-time estimates: correlation is higher averaging 0.66. Parameter uncertainty appears to be a dominant source of the unreliability of real-time estimates. We note that while one might consider an output gap estimator for which there is no parameter uncertainty (such as the Hodrick-Prescott filter), so that the filtered and real-time estimates are equivalent, our experience suggests that there remains a significant difference between the real-time and 'final' output gap estimates; e.g. see Mitchell (2003).

Table 1The unreliability of real-time output gap point estimates

		mean	min	max	s.d.	correlation vs. 'final'	RMSE
Belgium	*'final'*	-0.042	-2.353	2.117	0.958	1	0
	filtered	-0.025	-1.526	1.029	0.555	0.51	0.827
	real time	-0.016	-1.265	0.975	0.492	0.391	0.89
France	*'final'*	-0.15	-2.516	2.496	1.44	1	0
	filtered	-0.405	-1.541	1.165	0.691	0.773	1.038
	real time	-0.231	-1.768	1.161	0.555	0.59	1.203
Germany	*'final'*	-0.083	-2.399	3.533	1.423	1	0
	filtered	-0.212	-2.187	2.432	1.033	0.644	1.103
	real time	-0.299	-2.402	1.852	1.064	0.485	1.317
Italy	*'final'*	-0.048	-3.732	3.659	1.597	1	0
	filtered	-0.388	-2.661	1.455	1.029	0.726	1.156
	real time	-0.316	-2.499	1.67	1.064	0.614	1.291
Netherlands	*'final'*	-0.158	-3.275	2.877	1.491	1	0
	filtered	-0.124	-1.718	0.954	0.647	0.687	1.148
	real time	-0.079	-1.227	0.716	0.536	0.232	1.465
Spain	*'final'*	0.001	-1.139	0.842	0.337	1	0
	filtered	0.021	-0.86	0.743	0.285	0.467	0.325
	real time	0.039	-1.005	1.228	0.371	0.385	0.396
UK spec 1	*'final'*	-1.263	-6.515	3.116	3.084	1	0
	filtered	0.908	-3.649	3.716	2.138	0.974	2.438
	real time	1.197	-3.187	3.501	1.715	0.756	3.241
UK spec 2	*'final'*	-0.438	-4.998	4.868	2.461	1	0
	filtered	0.481	-2.181	3.136	1.207	0.84	1.835
	real time	0.317	-2.9	2.474	1.071	0.405	2.375

Notes: s.d. is the standard deviation of the output gap; correlation vs. 'final' is the correlation of the filtered or real-time output gap estimate against the full-sample estimate; RMSE is the root mean squared error of the filtered or real-time estimate against the 'final' estimate

2.4 Uncertainty associated with output gap estimates

It is not simply a question of this output gap forecast proving to be right and another forecast proving to be wrong. Point forecasts are better seen as the central points of ranges of uncertainty. A forecast of 2% must mean that people should not be surprised if the output gap turns out to be a little larger than that. Moreover perhaps they should

not be very surprised if it turns out to be much larger or indeed nothing at all. Therefore, consistent with recent developments in the forecasting literature, it is important to provide a description of the uncertainty associated with real-time output gap estimates via interval or density forecasts. Indeed, the 'optimal' real-time estimate of the output gap need not equal the mean or conditional expectation. It can be 'rational' to use biased real-time estimates. Furthermore, measures of uncertainty are useful in their own right if interested in analysing and communicating, for example, risk and volatility, or the probability of a downturn. Our concern in this paper is with producing measures of uncertainty associated with the cyclically adjusted budget deficit estimate, as measured in quasi real-time

2.5 Measuring the uncertainty associated with the output gap

We consider a simple approach to measure uncertainty that relies on a state-space representation for the output gap estimator. Conditional on Gaussianity (of the disturbances driving the components of the state vector) and knowledge of the covariance matrix of the estimated state vector confidence intervals around the output gap can be presented, and density estimates derived.[4] Other distributions could be considered although we confine attention to the Gaussian case since this both has the advantage of simplicity/familiarity and is sufficient to illustrate the role of forecasting uncertainty.

Let $P_{t|t}$ denote the Kalman filter based variance of the output gap at time t, $y_{t|t}$, using information available up to time t; the output gap is one of the elements of the state vector in the UC model. Then conditional on Gaussianity, the density is $N(y_{t|t}, P_{t|t})$, from which the 95% confidence interval around the point estimate, for example, can be extracted. Note that we are considering filter uncertainty only and ignoring parameter uncertainty. Accounting for parameter uncertainty should be expected to increase the width of the confidence bands around the point estimates.

Uncertainty is greater around the real-time than 'final' estimate. This is seen as follows. Denote the revision between the real-time and final estimates by $R_{t|T} = y_{t|T} - y_{t|t}$. With known and constant parameters (i.e. for the filtered rather than real-time estimates of Table 1), this variance $P_{t|t'}$ decreases as t′ increases; $P_{t|t} = P_{t|T} + Var(R^*_{t|T})$. The variance of the filtered (or one-sided) estimate is therefore greater than that of the smoothed (or two-sided) estimate, $P_{t|T}$, by a positive scalar equal to the variance of the revisions between the filtered and smoothed estimates $R^*_{t|T}$ where $R^*_{t|T} = y_{t|T} - y_{t|t}(\hat{\Theta}_T))$. This is the familiar result that the variance of the filtered (forecasted) estimate is equal to the variance of the outturn, $P_{t|T}$, plus the square of the bias, $Var(R^*_{t|T})$. This assumes the revision process has mean zero so that $Var(R^*_{t|T})=E(R^*_{t|T})^2$.

Just as point estimates of the output gap forecast in real-time can be evaluated ex post against the final estimates (as in Table 1), the accuracy of real-time measures of uncertainty can and should be evaluated ex post; see Mitchell (2003) for further

[4] The Kalman filter recursions automatically return estimates of the covariance matrix of the state vector. The diagonal elements of these matrices then can be used to construct the confidence intervals and density estimates. This approach has also been followed, for example, by Orphanides and van Norden (2002).

details. This is possible since a "good" interval forecast, as defined by Christoffersen (1998), should both have correct *unconditional* coverage (in the sense that on average observations fall in the interval to the predicted degree) and secondly be such that observations fall inside the interval in a random manner which is not clustered. Below we focus on how the uncertainty estimates associated with the output gap can be used to produce a measure of uncertainty associated with the cyclically adjusted budget deficit.

We detail the uncertainty concerning output gap estimates in Table 2 below for selected years. Although recursive, real time, estimates give less 'long term' volatility, they are in general noticeably less certain than full sample estimates. Hence even if we think, in real time, that the economy has an output gap of around zero, we cannot be at all certain about that. In 1990 six out of seven countries have real time estimates that were 30 to 100 per cent less certain than the full sample estimate. A similar pattern is clear in 1995 s well, with only the Netherlands having a real time estimate that is more certain than the full sample. Full sample uncertainty rises over the time period and approaches that of the real time estimate. Only in Spain are both estimates of the uncertainty in the gap low and stable.

Table 2 Standard Deviations of output gap estimates in selected years – full sample and recursive compared

		1990	1995	2000	2005
Belgium	*Full-sample*	0.881	0.881	0.887	1.069
	Recursive	1.145	1.380	1.283	1.261
France	*Full-sample*	2.169	2.184	2.192	2.686
	Recursive	3.956	2.996	3.057	2.822
Germany	*Full-sample*	1.018	1.021	1.043	1.448
	Recursive	1.309	1.605	1.976	1.557
Italy	*Full-sample*	1.067	1.071	1.089	1.472
	Recursive	1.645	1.800	1.691	1.680
Netherlands	*Full-sample*	1.718	1.722	1.738	2.043
	Recursive	1.164	1.373	1.422	2.078
Spain	*Full-sample*	0.430	0.430	0.431	0.522
	Recursive	0.799	0.503	0.649	0.625
UK	*Full-sample*	1.393	1.431	1.470	2.226
	Recursive	2.526	2.740	2.521	2.363

3. Cyclically adjusting the budget deficit

The patterns of evolving uncertainty detailed above should be reflected in our uncertainty about the scale of the cyclically adjusted budget deficit. The relationship between the deficit as a percent of GDP (BUD) and the cyclically adjusted deficit as a per cent of GDP (CABUD) depends of the sensitivity of revenues and expenditure to the cycle. The links between output and the budget deficit are discussed, for instance in Mellitz (2000), Blanchard and Perrotti (2002) and Wyplosz (2003) and it is common to assume that an increase in the output gap of 1.0 per cent of GDP (more use of capacity) reduces the deficit by 0.5 per cent of GDP, We nay write the cyclically adjusted budget deficit (CABUD) for the UK as dependent on the observed deficit (BUD and the output gap (OG)

UKCABUD= UKBUD -0.5*UKOG

Given the uncertainty in the output gap, and on the assumption that the budget deficit and the cyclical coefficient are known then we may write the standard deviation SD() of the CABUD as a function of the standard deviation of the output gap:

SD(UKCABUD) = 0.5*SD(UKOG)

We apply this formula across seven European countries, first estimating the real time and full sample cyclically adjusted deficits for each country, and then applying the time varying estimates of the standard deviation of the output gap to produce bounds of uncertainty around budget deficits. The SD of the full sample estimate has generally settled down by the start of our chart period, but the real time estimates vary noticeably over time, and may not be symmetrical. In each chart we plot the outturn for the deficit along with the cyclically adjusted deficit for that period, and in the real time chart we also include the full sample cyclically adjusted deficit.

Our budget deficit data has not been adjusted for any of the myriad of one off measures we have seen in the last decade in Europe, and hence we include mobile phone receipts in the deficit, which explains why most countries go into surplus around 2000. After 2002, when these receipts fade away, Germany and Italy show cyclically adjusted deficits that are more than 2 standard deviations away from zero, suggesting it is very unlikely that they could ever be judged to have been 'in balance or surplus'. France on the other hand has a cyclically adjusted deficit that just about include zero within its 95% confidence bounds, but this reflects the noticeably greater uncertainty about the French output gap than that in the other countries. The Netherlands, Belgium and Spain are all close to balance or in surplus in a cyclically adjusted way in the last few years. The UK has been in a statistically significant deficit position for some years, but this judgement is both marginal, and less pressing than for France, for instance, because the UK is not a member of EMU.

In general the two CABS tell a similar story to the actual deficit, but this is not always the case. When evaluated in real time the German deficit of the early 1990s was within the 95 per cent confidence bands around the cyclically adjusted deficit, whilst the full sample suggest it was consistently outside these bands. This reflects the large difference between the small real time estimate of the gap as compared to the larger full sample estimate.

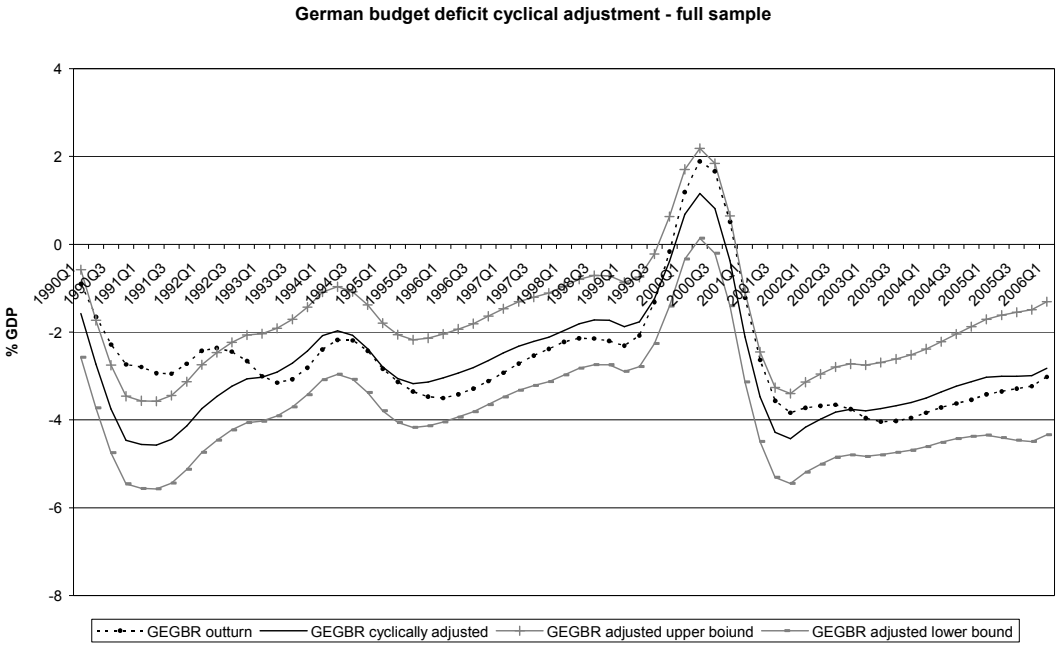


Figure 8

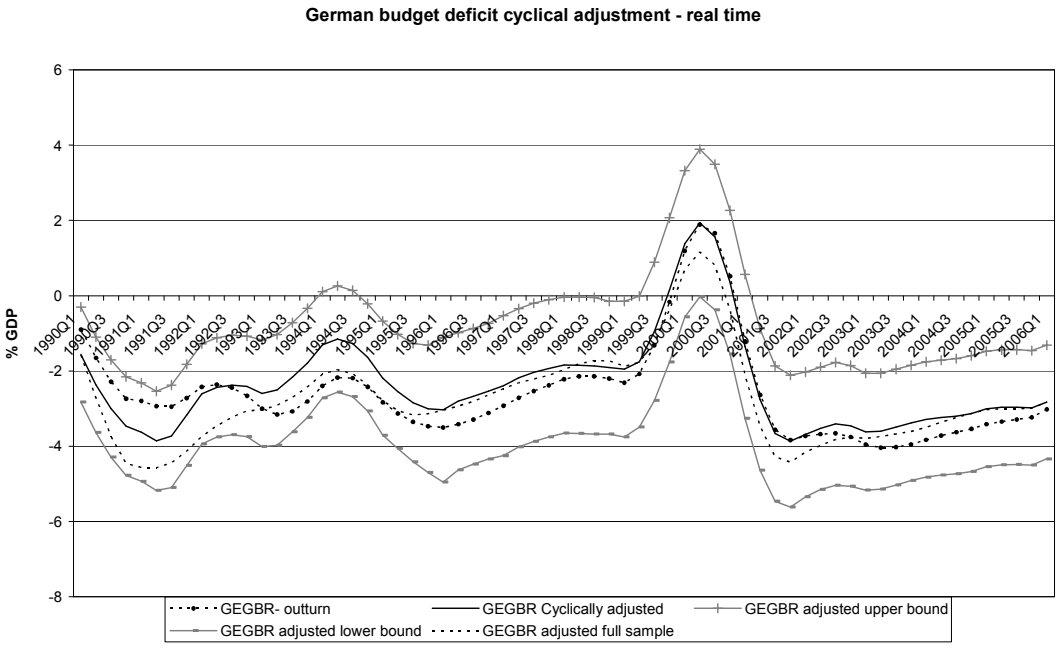


Figure 9

The real time deficit in the UK is almost twice as uncertain as the full sample, although they are the same at the present time. In the early 1990s the outturn was outside the 95 per cent bounds for the full sample estimate. The real time and full sample estimates differ by 2 percentage points of GDP at this time. The real time estimates are similar to outturns for most of the sample, as the real time gap is small.

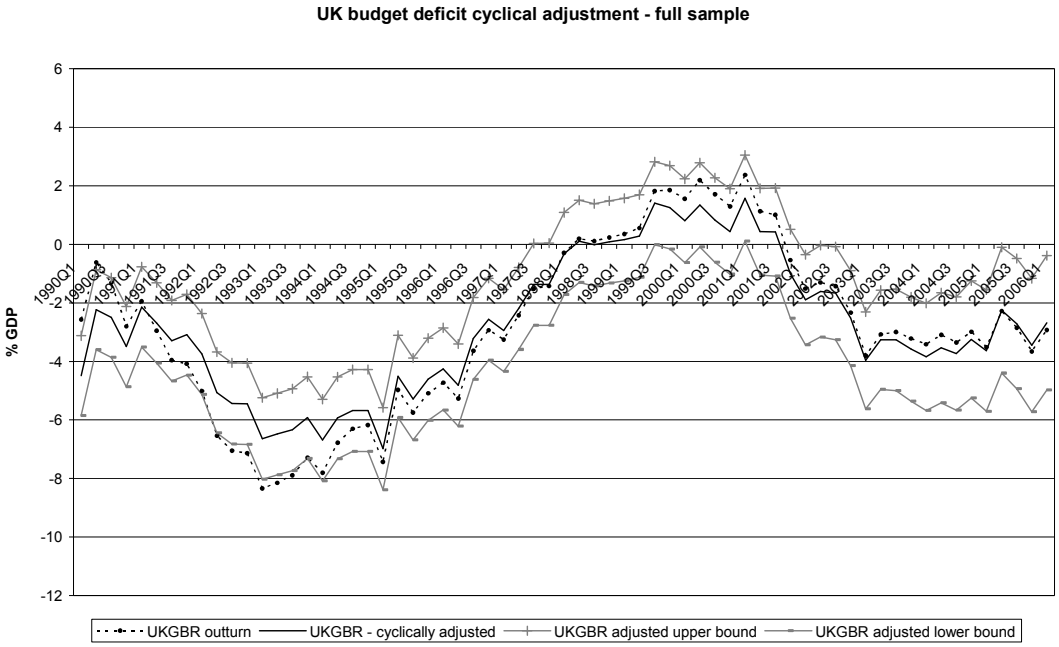


Figure 10

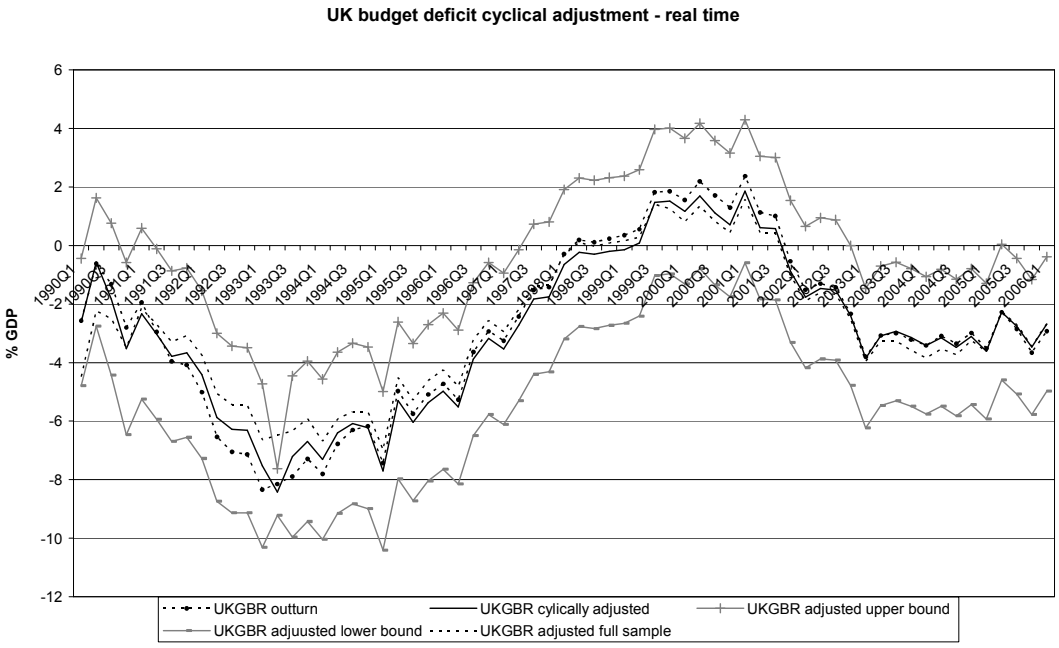


Figure 11

There is little conflict between the real time and full sample estimates of the CAB for France, and in both cases the estimates suggest that the country was in sustained deficit with little chance of achieving its commitments, However, there was a sustained improvement in the run up to EMU, with zero being well inside the uncertainty bounds for some years. After the formation of EMYU fiscal discipline appears to have been reduced, and zero drops out of the bounds.

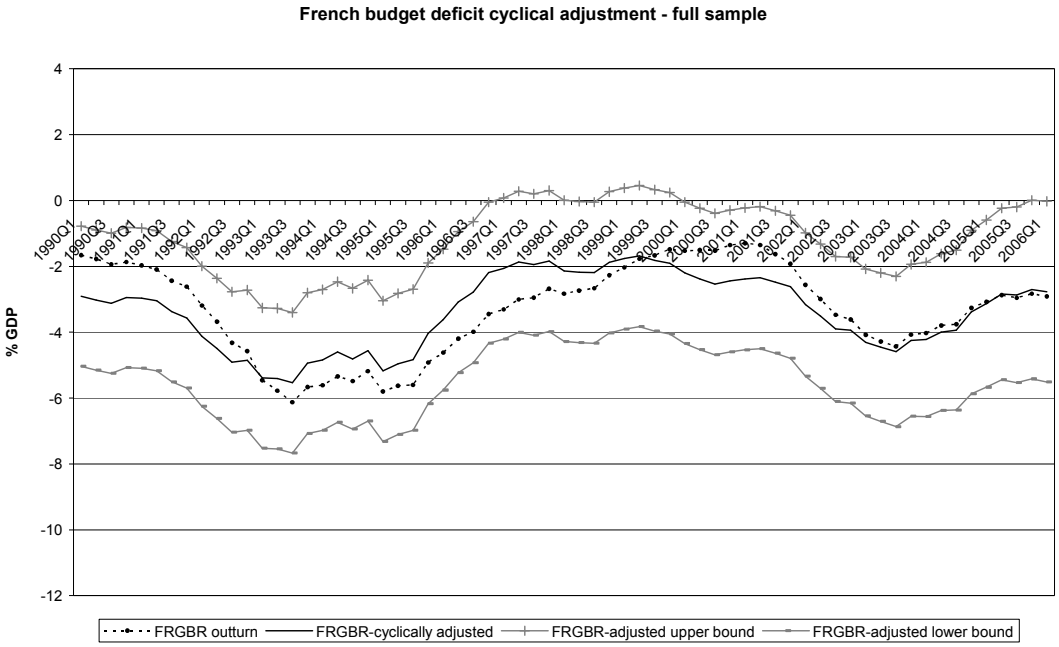


Figure 12

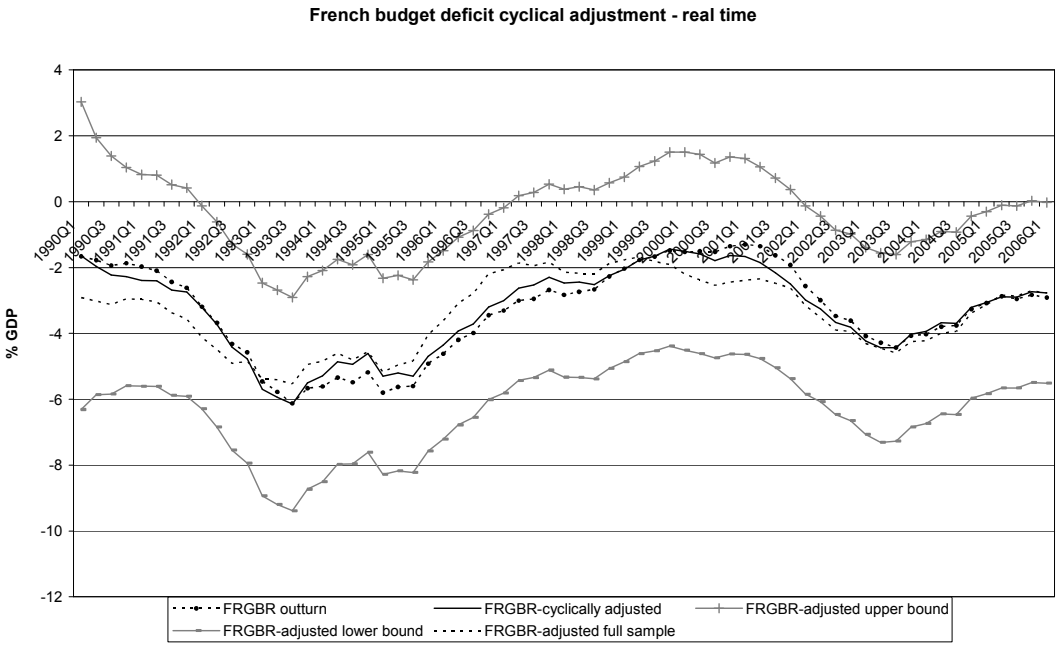


Figure 13

The Italian budget position improved markedly between 1990 and 1996, and by 1997 the target of zero appears to be temporarily at least within the bounds of probability. Special measures between 1997 and 2001 gave a plausible looking outturn, but the central estimates and the bounds have recently moved away from zero. In the early 1990s there was a noticeable difference between the real time and full sample estimates of the CAB.

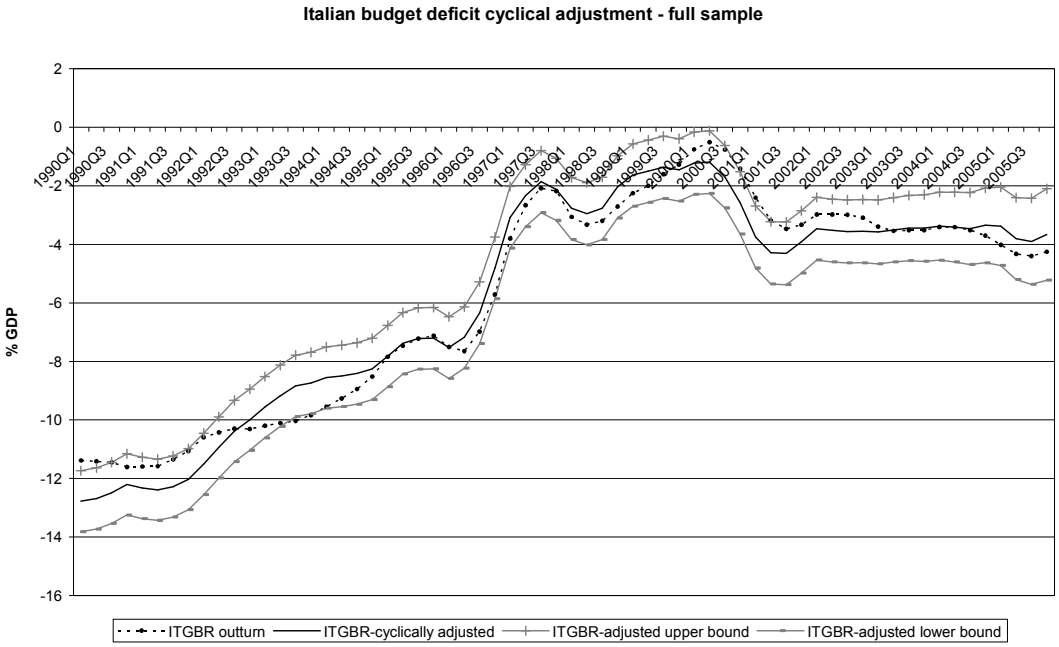


Figure 14

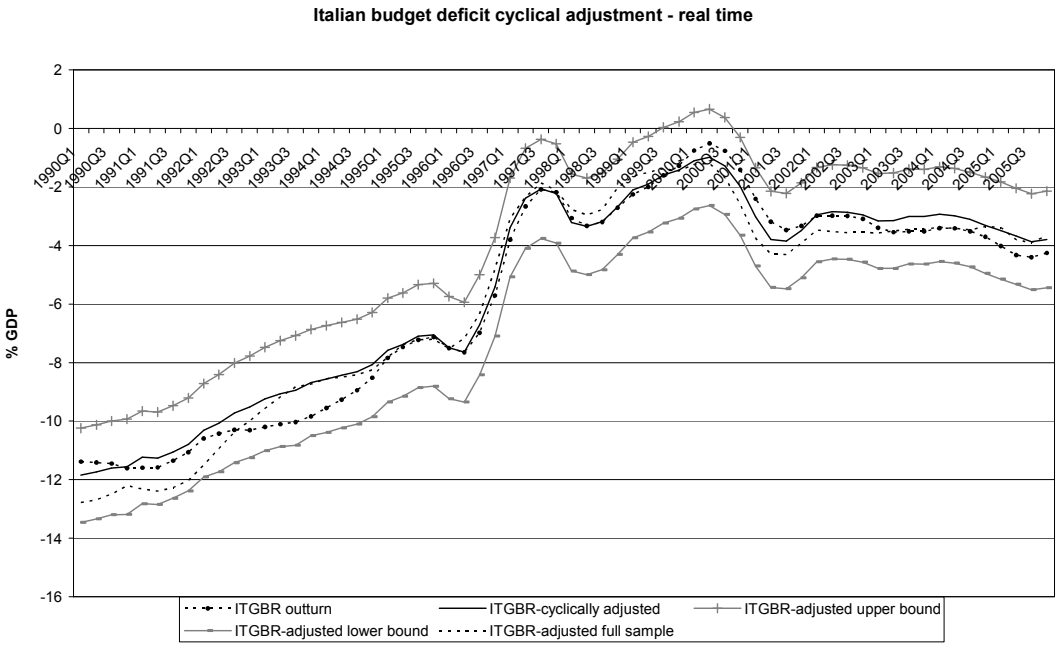


Figure 15

The low level of uncertainty around the Spanish budget deficit reflects the low uncertainty around the output gap estimates using this technique. However, these low bands leave the budget well within the bounds of close to zero over the last 3 years. The bounds using this estimate still suggest that the underlying position is strong, but the estimator is not the only one available, and an alternative estimator would help shed light on the position.

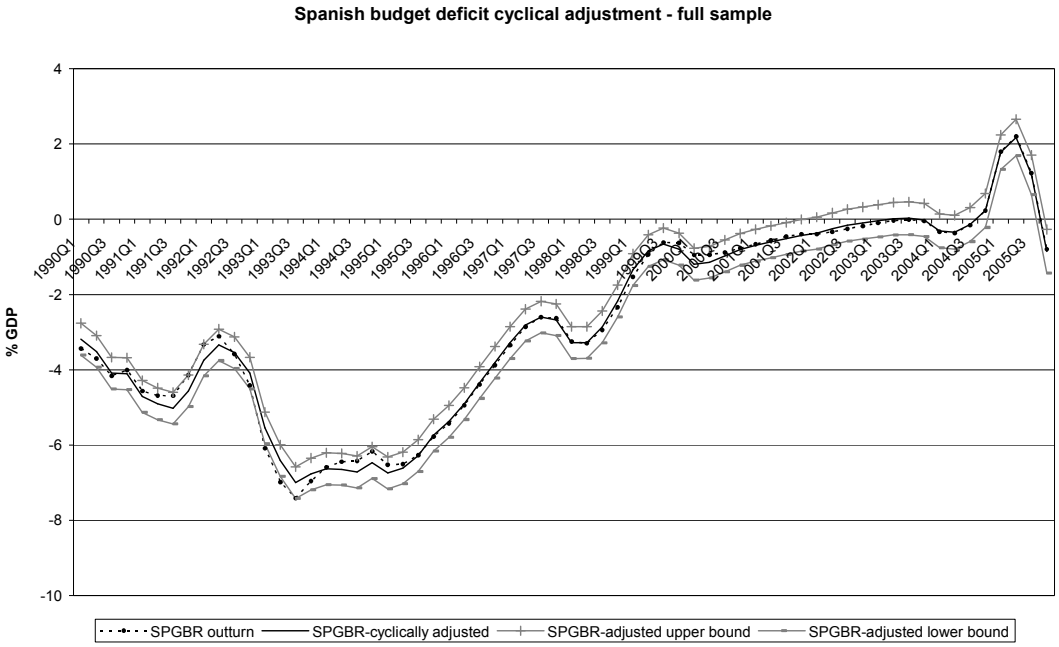


Figure 16

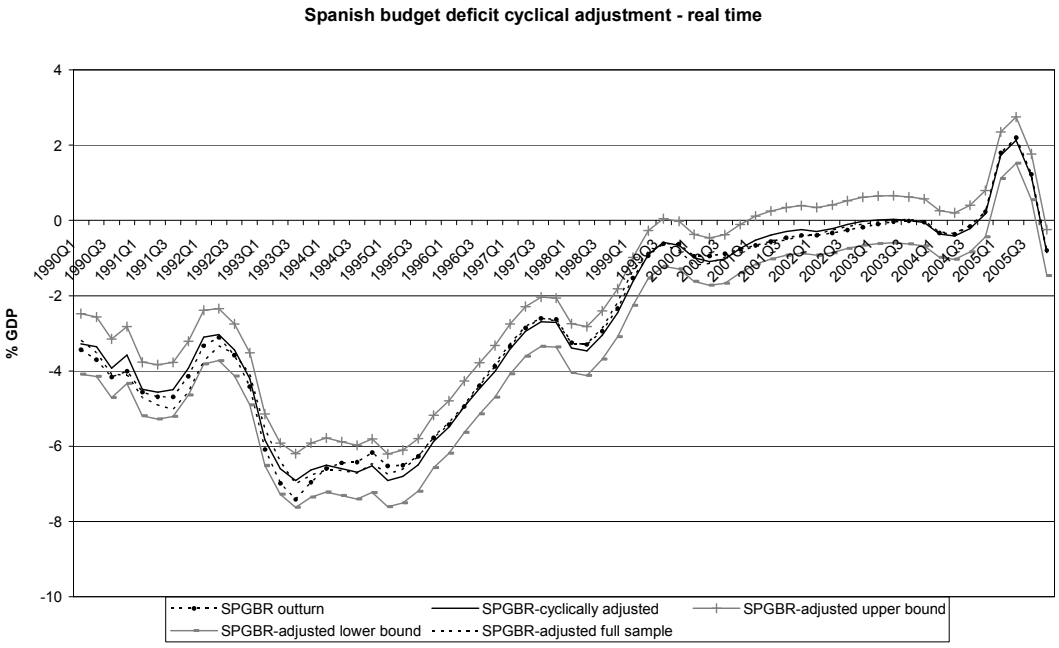


Figure 17

The Netherlands budget deficit is noticeably more uncertain over the full sample than that of most other countries considered here. The real time estimate is has a smaller standard deviation, except of course at the end of the sample. These differences are reflected in the budget deficit bounds, which are relatively wide, and indeed it is the only country to include both zero and 3 per cent within the full sample and real time 95 per cent confidence interval in the last 5 years of our sample. It is hence the only country where we are statistically unable to see whether it is likely to meet one or both of its treaty and Council commitments.

However, even given the uncertainty about the current cyclical position, which is greater here than in most other countries (except perhaps France) we can be reasonably sure that the surplus we currently see has some structural component and is unlikely, at the 95 per cent confidence level, to involve an underlying deficit position in breach of the 3 per cent floor. However, in 2003 for a period the budget deficit was sufficiently large that it is unlikely that it could be explained by cyclical factors, in that the bounds did not include zero.

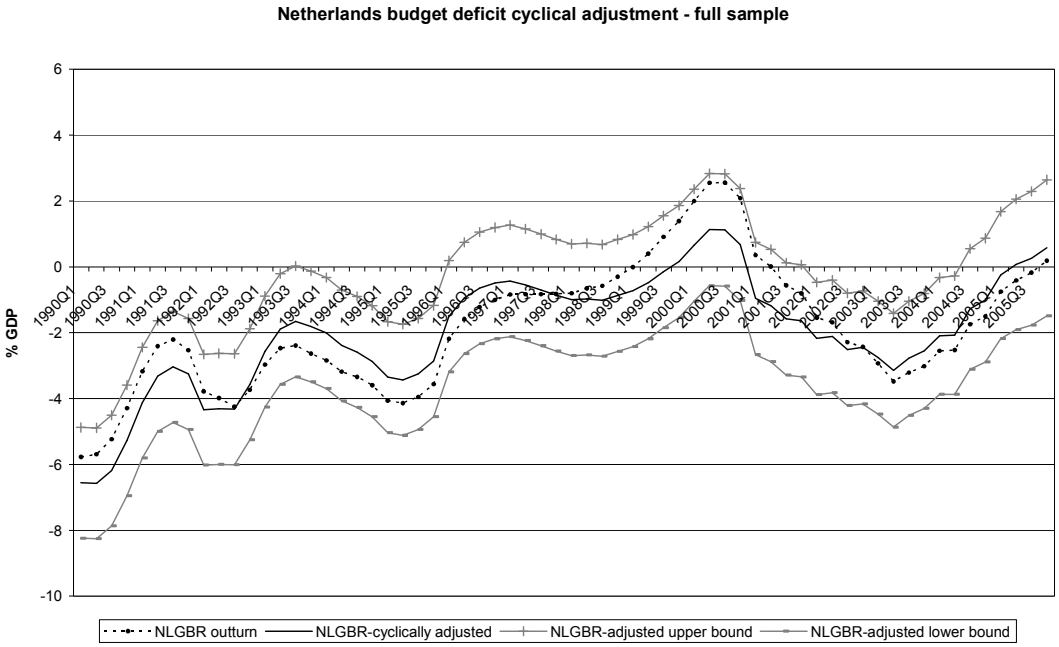


Figure 18

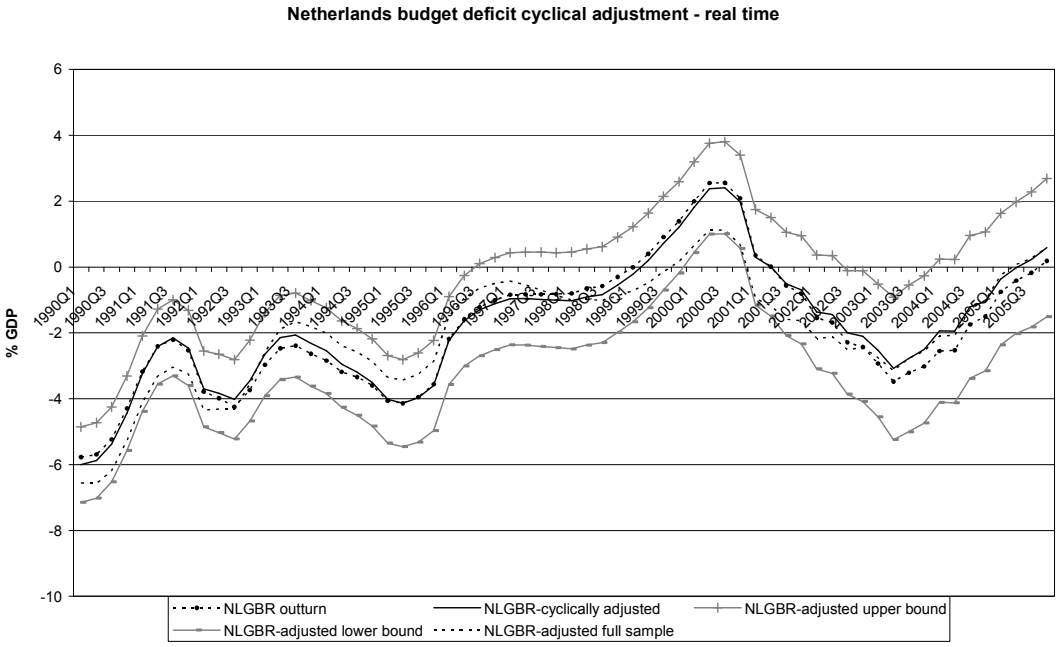


Figure 19

The uncertainty surrounding the output gap estimate in Belgium is noticeably lower than in the Netherlands, and we can be relatively sure, at the 95 per cent level that the current position represents a structurally adjusted budget deficit of somewhere between -1.5 and +1.5 per cent of GDP.

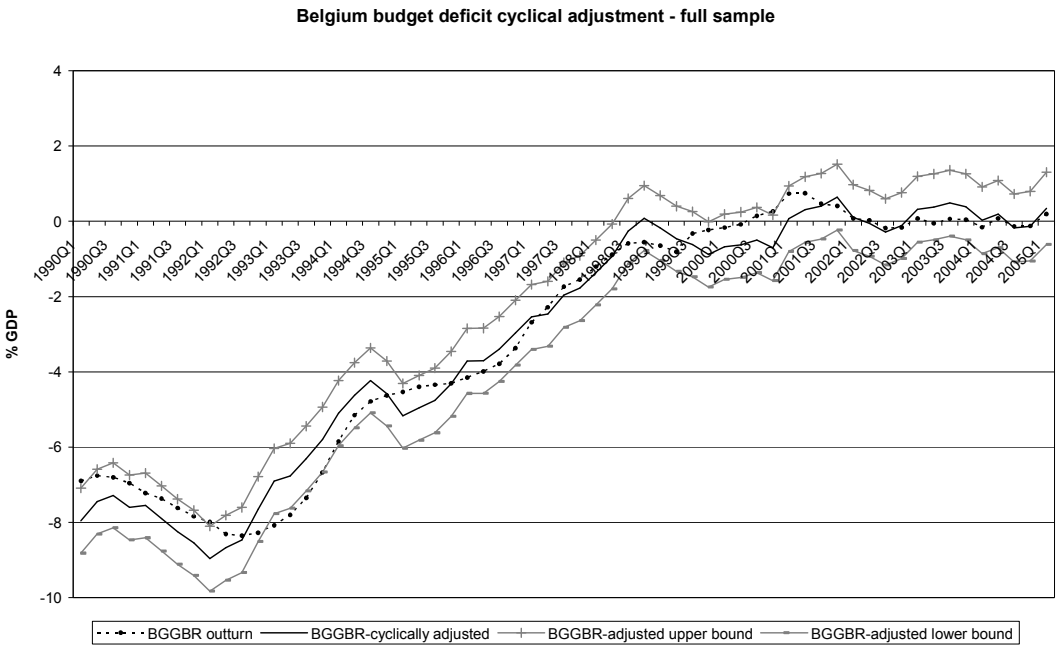


Figure 20

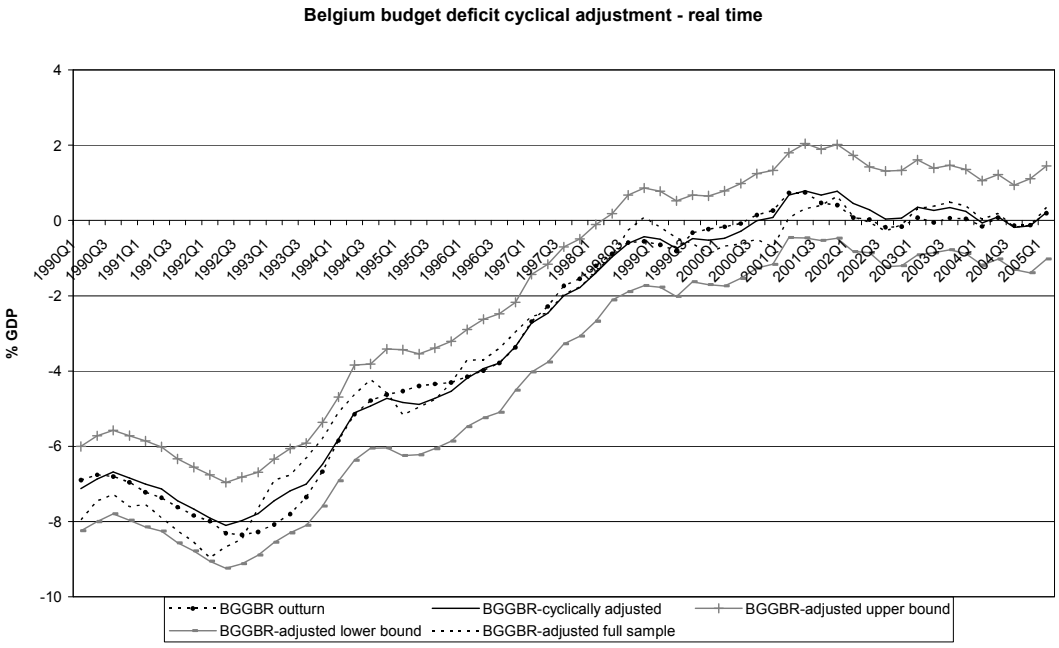


Figure 21

4. Uncertainty in the adjustment

The cyclical adjustment of the budget deficit is not just subject to uncertainty over the output gap, but also to uncertainty over the impact of the output gap on revenues and on spending. Tax revenues tend to fall with the cycle, and at least some items of expenditure can be regarded as cyclically sensitive[5]. Transfers to individuals in relation to their employment state are clearly cyclically sensitive, and this sensitivity varies across countries. In our sample it is low in Italy and high in Germany for instance, and hence we would expect the German budget deficit to be more cyclically sensitive than that of Italy. Expenditure taxes generally move less than one for one with the level of spending, although in some countries, such as Germany, tax rates are higher on cyclically sensitive variables. Hence a slow down in activity in Germany driven by weak consumption would reduce tax revenues rather more than it would in the UK, for instance. Income taxes also fall less than in proportion to the output gap because of the existence of thresholds and higher rates. Hence the details of individual countries benefit and tax system should affect the relationship between budgets and cycles.

If receipts have different sensitivities to the output gap, then the nature of the shock facing the economy will determine whether the downturn is budget rich or not. Appendix I sets out a model based analysis of the relationship between output effects on budgets, and given the frequency of different types of shocks in the 1990s to the five economies we look at, we can say that the average adjustment for demand shocks should be 0.52 for Spain, 0,51 for Germany, 0.48 for Italy, 0.45 for France and 0.42 for the small government UK. We can see that if shocks are from exports or from investment then the impacts on the deficit are noticeably smaller than the multipliers associated with consumption shocks, with an average impact of 0.33 for investment shocks in the large Euro Area countries and 0.32 for export shocks in the same group. Consumption shocks have more impact on tax revenues, and the average impact is 0.66 for France and Germany, and 0.75 for Italy and Spain. Hence we need to look at the source of demand shocks in order to evaluate the cyclical adjustment that would be appropriate in a given situation. This parameter uncertainty increases the uncertainty bounds involved in evaluating deficits, but it may not be an insurmountable problem to decide whether a downturn in demand is driven by domestic or by foreign factors.

In Table 3 we set out the upper and lower bounds for budget deficits in both real time and over the full sample for 1992 and for 2004 for the UK, France and Germany using here estimates of the impacts of the gap on the deficit. We use 0.33 to represent the impacts of external shocks on deficits, 0.66 to estimate the impacts of domestic demand shocks. We also include estimates based on a sensitivity of 0.45 which is the shocks weighted average from Barrell and Hurst (2003). Charts of these deficits over the period from 1990 are included in Appendix II.

We are particularly interested in whether or not a set of bounds around a cyclically adjusted deficit might for instance include a possibility that the country was close to balance or in surplus in a cyclically adjusted sense. We can see from the table that

[5] Although we may be able to find a statistical relation between other items of expenditure and the cycle, we regard such movements as discretionary, and we would not include then in our adjustments

only France saw itself in this position, and then only in 2004 if shocks were predominantly from consumption and the real time estimate was to be used. In this table no country and coefficient pair, either in real time or over the full sample could be said to be clearly in line with a Treaty commitment not to breach a deficit floor of 3 per cent of GDP, even in a probabilistic sense of being 95% certain that this is not the case. These impressions are confirmed by the charts in appendix two, which suggest that even if we include all the one off measures in the last 5 years it is very unlikely that any of these three countries have been running a structural surplus, and it is very likely that they have breached a reasonable level of borrowing of 3 per cent of GDP[6].

Table 3 Cyclically Adjusted budget deficits – changing the cyclical effect

			full sample			*real time*		
		GBR	Cyclically adjusted	Upper bound	Lower bound	Cyclically adjusted	Upper bound	Lower bound
UK b=0.5	1992	-6.433	-4.923	-3.538	-6.308	-5.720	-2.871	-8.570
	2004	-3.211	-3.590	-1.680	-5.501	-3.289	-0.956	-5.623
UK b=0.33	1992	-6.433	-5.436	-4.522	-6.351	-5.963	-4.082	-7.843
	2004	-3.211	-3.462	-2.201	-4.723	-3.263	-1.723	-4.803
UK b=0.45	1992	-6.433	-5.074	-3.828	-6.321	-5.792	-3.227	-8.356
	2004	-3.211	-3.552	-1.833	-5.272	-3.282	-1.181	-5.382
UK b=0.66	1992	-6.433	-4.440	-2.612	-6.268	-5.492	-1.731	-9.254
	2004	-3.211	-3.712	-1.190	-6.234	-3.314	-0.234	-6.395
Germany b=0.5	1992	-2.474	-3.376	-2.378	-4.374	-2.457	-1.131	-3.783
	2004	-3.679	-3.305	-2.130	-4.480	-3.216	-1.684	-4.747
Germany b=0.33	1992	-2.474	-3.069	-2.411	-3.728	-2.463	-1.587	-3.338
	2004	-3.679	-3.432	-2.657	-4.208	-3.373	-2.363	-4.384
Germany b=0.45	1992	-2.474	-3.286	-2.388	-4.184	-2.459	-1.265	-3.652
	2004	-3.679	-3.343	-2.285	-4.400	-3.262	-1.884	-4.640
Germany b=0.66	1992	-2.474	-3.665	-2.348	-4.982	-2.452	-0.701	-4.202
	2004	-3.679	-3.186	-1.635	-4.737	-3.068	-1.046	-5.089
France b=0.5	1992	-3.945	-4.595	-2.465	-6.726	-4.032	-0.908	-7.155
	2004	-3.710	-3.887	-1.480	-6.295	-3.633	-0.859	-6.407
France b=0.33	1992	-3.945	-4.374	-2.968	-5.780	-4.002	-1.941	-6.064
	2004	-3.710	-3.827	-2.238	-5.416	-3.659	-1.829	-5.490
France b=0.45	1992	-3.945	-4.530	-2.613	-6.448	-4.023	-1.212	-6.834
	2004	-3.710	-3.870	-1.703	-6.036	-3.641	-1.144	-6.137
France b=0.66	1992	-3.945	-4.803	-1.991	-7.616	-4.059	0.064	-8.183
	2004	-3.710	-3.944	-0.766	-7.122	-3.608	0.053	-7.270

[6] The UK uses a different set of guidelines, and sets its borrowing to be no greater than the level of public investment over a cycle. Given this has not exceeded 2 per cent of GDP we could make a simple adjustment to out figures of this magnitude, and hence we can say that the balance of probabilities is that it has not been meeting its target over the last four years.

5. Implications and Conclusions

Monetary Union requires some form of fiscal pact, and although it is in the interests of all to follow that pact, it is in the interests of each country to find special reasons why the pact constraints may not apply to it in a particular year. The pact was designed to set limits to borrowing, with penalties available for sustained breaches of the targets. These penalties are supposed to take account of the impacts of the cycle on the deficit, but these impacts are uncertain. We have investigated one representative estimator of the output gap and derived its distribution, giving us a simple way of evaluating its uncertainty. This in turn allows us to put uncertainty bounds around the budget deficit given an estimate of the output gap on the deficit. When then discuss the uncertainties involved in the relationship between the output gap and the deficit, showing that the source of shocks impacts on the uncertainty surrounding a deficit projection.

All estimates of the output gap are uncertain, and ours is probably no worse than others. Even those that are not revised are not known with certainty at the point where they are made. There is a strong case for using production function based estimates of the output gap, as in Denis et al (2006) but these are more difficult to calculate than our measure, and it is more complex, but not impossible, to evaluate the associated uncertainty. Even if we know with certainty the parameters of the production function we still have to estimate trend hours, trend productivity and the equilibrium level of employment. If these are estimated using a filter, which is common, then it is possible to directly evaluate their uncertainty and feed this into the gap. If they are the result of reduced form calculations, then a more complicated process of looking at the error variance on the reduced form has to be gone through.

It is our contention that the error bounds around any estimate of the output gap are wide. Given these uncertainties it will always be possible to put bounds around the cyclically adjusted deficit, and we strongly advocate that this should be done by all official bodies involved with the evaluation of deficits in Europe as well as by the scientific community. It would be more productive to accept that we are ignorant rather than argue about gaps, and hence evaluate budget deficits in probabilistic terms. This would involve asking whether a budget target is within the 95 per cent (or 64 per cent) bounds around the cyclically adjusted deficit. In undertaking this analysis it is also useful to discuss the source of weak demand that has caused the output gap and hence use an appropriate coefficient in the adjustment. In all cases humility over our ignorance and strength about the obvious, such as the scale of recent deficits in France and Germany, are important characteristics of sensible debate.

Appendix I Cyclical, Discretionary and Random Components in the Budget

Introduction

Seemingly random fluctuations in the government budgetary position are common. Tax revenues change for many reasons, and probably the least significant are those associated with the economic cycle. Tax revenues tend to fall with the cycle, whilst expenditure rises with it, and hence when evaluating the budgetary stance it is important to understand the cyclical position and it is important to know what has driven a particular cycle before one can asses its implications for the budget position.

There has been considerable debate about the scale of changes in revenues and spending in relation to the cycle, and it is widely thought that a 1% deviation in GDP from its trend might cause a 0.5 percent of GDP change in the budget deficit. This figure is perhaps a little high and it should differ noticeably between countries and will also depend on the causes of the cyclical movement. A slowdown in activity driven by falling export demand is likely to have noticeably less impact on revenues than one driven by weak consumer spending, as the latter is more tax rich than the former. It is clear from this that producing a single gross measure of the sensitivity of the budget to the cycle is a best misleading. Although a single measure is simple and transparent, the trade off with information about the causes of the shift in the budget may be too great.

Discerning discretionary fiscal policy moves is difficult, as it involves the detailed analysis of budgets and their impacts. Changes in tax rates and the definition of tax bases must be seen as discretionary movements in the government position, as must changes in expenditure on goods services and transfers. In general an improvement in the budgetary position made by a conscious discretionary policy might be expected to have a contractionary impact on the economy in the short term, although it might lead to lower long term real interest rates, and these might offset the contractionary impact of the policy. However, it is important not to assume that any non-cyclical movement in the budget represents discretionary policy.

Random variations in tax receipts or in spending may push the budget outturn well away from its predicted level without there being a cyclical explanation or a discretionary policy driving the change. Economic models cannot encompass all of the factors driving the economy, and models explaining tax receipts are bound to be incomplete descriptions. Tax receipts may change even when both the tax base and the tax rate do not move. A change in tax paying behaviour after the introduction of self assessment in the UK in the 1990s could not have been picked up by any model. A shift in the pattern of consumption away from taxed to untaxed goods may not be picked up in equations for indirect tax receipts unless we have very detailed models. Both of these events will look like random (but explicable) elements in our analysis of taxes.

Modelling the economy

This appendix uses the National Institute Global Model, NiGEM to scale the relationship between output gaps and the budget deficit. It is an estimated world model, which uses a 'New-Keynesian' framework in that agents are presumed to be forward-looking but nominal rigidities slow the process of adjustment to external

events[7]. Economies are linked through the effects of trade and competitiveness and are fully simultaneous. There are also links between countries in their financial markets as we model the structure and composition of wealth, emphasising the role and origin of foreign assets and liabilities. We have forward-looking wages, forward looking consumption, forward-looking exchange rates and long-term interest rates are the forward convolution of short-term interest rates.

Each country has a description of its domestic economy that can be broken up into sectors: the government, the labour market, consumption behaviour, the supply side of the economy and financial markets. We need to ensure that interest rates are set to stabilise the economy. We use a policy of nominal aggregate targeting and inflation rate targeting, or two pillar strategy advocated by the European Central Bank

$$r_t = \gamma_1 (P_t Y_t - P^*_t Y_t^*) + \gamma_2 (\Delta P_t - \Delta P_t^*) \qquad (1)$$

All variables are in logs, PY is (the log of) nominal GDP, P is (the log of) the Consumer Price Index (CPI) inflation rate, and a * denotes a target.

We have models of direct and indirect taxes, and of government spending. We consider the financing of the government deficit (BUD), and we allow either money (M) or bond finance (DEBT).

$$\text{BUD} = \Delta\text{M} + \Delta\text{DEBT} \qquad (2)$$

Current fiscal revenues can be disaggregated. Personal taxes (TAX, which includes both personal income tax and social security contributions) depend on personal incomes. Corporate taxes (CTAX) depend on longer term profitability. Indirect taxes (MTAX) depend on consumer expenditure. Transfers to individuals (TRAN) depend upon prices and on unemployment, and hence these vary with the economic cycle. Government consumption and investment (GC and GI) which are assumed to be on plan except for random fluctuations, and they are not influenced by the cycle. As GC and GI are in constant prices, we convert them to nominal terms using the private consumption deflator CED deflator. Government interest payments (GIP) are modelled as the income on a perpetual inventory, the change in the debt stock each period paying the long interest rate in the issue period until it is replaced[8]. The budget balance thus reads:

$$\text{BUD} = \text{TAX} + \text{MTAX} + \text{CTAX} - \text{TRAN} - \text{GIP} - \text{GC*CED} - \text{GI*CED} \qquad (3)$$

We normally assume budget deficits are kept within bounds in the longer term, and taxes rise to do this. We can describe the simple fiscal rule as

$$\text{Tax}_t = \text{Tax}_{t-1} + \phi\,[\text{GBRT} - \text{GBR}] \qquad (4)$$

Where Tax is the direct tax rate, GBR is the government surplus target and actual surplus. The feedback parameter ϕ is designed to remove an excess deficit in less than five years. If fiscal solvency is 'off', it is turned back on again after our experiment.

Evaluating Shocks

Economies are subject to many shocks, and they are the source of uncertainty in the economy. We define a shock as the part of the level or change in an economic variable, such as consumption, that we cannot explain using other economic variables.

---

[7] The structure and the simulation properties of NiGEM are described in, Barrell et al (2003)

[8] The perpetual inventory attempts to take account of countries like Italy and Belgium where there are large proportions of short-term public debt.

Take the equation for employment, for instance, where the number of people employed (EE) depends on the level of output (Y), the rate of technical progress (TP) and the real wage (W/P). Suppressing dynamics for simplicity we may write this as

$$Log(EE) = a + b*Log(Y) + c* Log (W/P) + d*TP + residual \qquad (5)$$

The residual, or shock, is the part of the evolution of employment that cannot be explained by economic factors. As such it is exogenous to the economic process we are considering. We describe it here as the residual on an estimated structural equation for investment, consumption, equity prices or employment, for instance and we discuss the nature of these residuals below. Where we assume rational expectations and the next period's outcome is taken as the expected value in the current period[9].

We can decompose shocks into demand shocks, supply shocks and fiscal shocks as in Barrell and Hurst (2003). Demand shocks are those that affect the level of spending in the economy, and we include consumption, investment, equity prices and exports. Supply shocks are those affecting the labour market, such as employment, self employment, hours and real wage (compensation equation) shocks. Tax shocks are those that directly affect the budget deficit independently of economic events elsewhere and we include transfers to individuals as well as direct, indirect and corporate taxes.

Barrell and Hurst (2003) take the weighted average of these shocks by country to see who has the largest shocks and which countries faced more supply shocks, demand and fiscal shocks. Table I.1 gives these indicators for the five largest European economies using France as a base. They weight together demand shocks using shares in GDP multiplied by the standard deviation of the individual shock over the period 1991q1 to 1999q4. Hence if exports are twice as important in GDP as investment but the shocks have half the standard deviation then export shocks take the same weight as investment shocks. Buti, Brunila and in't Veld (2002) use the Commission model QUEST to quantify the impacts of output on the budget deficit, and we can evaluate the properties of our model to do the same. In general output effects on the budget increase with the size of the government sector and the share of cyclically sensitive components of taxation and spending, and hence we would expect them to vary across countries. Country specific factors such as the degree of openness and the flexibility of the labour market will also matter. Blanchard (2000) and Barrell and Hurst (2003) suggest that supply shocks are different from demand shocks in their impacts, and hence we only analyse the relation between output and the deficit in response to shocks to demand. In order to do this we evaluate the impact of demand changes on the economy and on tax revenues, and then look at the effects of tax changes on output.

Evaluating Shock Multipliers

We may write the shock multiplier as (where tax is direct taxes, itax is indirect taxes, ctax is corporation, tran is transfers, C is consumption, I is investment, X is exports and Y is GDP):

[9] This is the 'industry standard' and is discussed at length in Barrell, Dury and Hurst (2003). Expectations in these equations could come from a variety of sources. We could use a learning rule, or we could assume that risk premia vary from tome period to time period to remove the unexplained component in our equation. We do not regard either as satisfactory, and we use the residuals from our estimated equations under the assumption of rational expectations.

$$Dy/DS = dy/dtax * dtax/dS + dy/ditax * ditax/dS + dy/dctax * dctax/S$$

$$+ dy/dtran * dtran/dS \qquad (6)$$

$$+ dy/dC * dC/dS + dy/dI * dI/dS + dy/dX * dX/dS$$

The left hand side of this expression is the shock multiplier, which we evaluate for consumption, investment and exports, and the last three terms represent the shock multipliers if there were no automatic stabilisers. If we have a consumption shock then dI/dS and dX/dS are set to zero by definition, and similarly for investment and export shocks. In order to evaluate the impacts of shocks on the deficit we need to evaluate the first four terms of the right hand side of this expression. This requires that we calculate the impact of each shock on tax revenues and on transfer spending, and that we calibrate the effect of an unanticipated change in tax revenues on output.

We first look a the impact of 1% of GDP changes in consumption, investment and export volumes sustained for 1 year, when they return to baseline for one quarter and subsequently the dynamics of the model are allowed to work. We assume that there is no interest rate response in this year, but after the year the monetary authorities respond. The fiscal authorities are assumed to leave tax rates unchanged for the year and then adjust direct taxes to achieve their budget target. The model is run with forward looking financial markets, and exchange rates and long rates jump in the first period.[10] These multipliers are generally below those for discretionary changes in government spending on goods and services, but these multipliers are also likely to be below 1.0. Barrell et al (2003) discuss standard fiscal simulations on NiGEM and suggest reasons why Germany has the largest multipliers on this table.

Table I.1: Multiplier Effects of a 1% of GDP impulse for one year

	Consumption	Investment	Exports	Average
France	0.697	0.615	0.667	0.682
Germany	0.885	0.765	0.89	0.886
Italy	0.599	0.529	0.582	0.586
UK	0.635	0.6	0.642	0.639
Spain	0.796	0.732	0.852	0.825
Euro Area	0.723	0.647	0.725	0.724

UK in EMU. Interest rates fixed for the first year, No fiscal feedbacks for the first year' The ECB uses a two pillar strategy, the exchange rate and the long rate are forward looking

The impact of the shocks on the public finances will depend upon the importance of the three types of tax we model (direct, indirect and corporate) as well as the significance of transfers in the economy. We would expect shocks to consumption to have a much more significant impact on the budget as consumers expenditure is a significant part of the indirect tax base. However, the significance of indirect taxes varies between countries, and hence the impacts of the consumption shock also vary. Investment and export shocks are likely to be less tax rich.

Shocks and the Budget

The impact of the shock on the economy also affects the Government budget. If the output effect of the shock is small then the impact on income and corporate tax revenues will be smaller, as will the impact on transfer payments. In addition the

[10] Multipliers are always less than one in these models, and are generally less than those given in unstructured VAR analyses such as that of Blanchard and Perotti (2002).

generosity of transfer payments differs significantly between countries, and is probably least important in Italy for instance. Hence small open economies with low multipliers will have lesser effects on the budget, and large countries with generous social security systems will have large effects. We can see that the impacts of consumption shocks on the budget is markedly higher than investment or export shocks, and in general export shocks have slightly more budgetary impacts than do investment shocks

Table I.2: Effects on Budget as a % of GDP of a 1% of GDP impulse for one year

	Consumption	Investment	Exports	Average
France	0.46	0.14	0.15	0.31
Germany	0.58	0.27	0.31	0.45
Italy	0.47	0.18	0.20	0.28
UK	0.36	0.18	0.20	0.27
Spain	0.59	0.24	0.28	0.42
Euro Area	0.51	0.19	0.22	0.34

UK in EMU. Interest rates fixed for the first year, No fiscal feedbacks for the first year' The ECB uses a two pillar strategy, the exchange rate and the long rate are forward looking

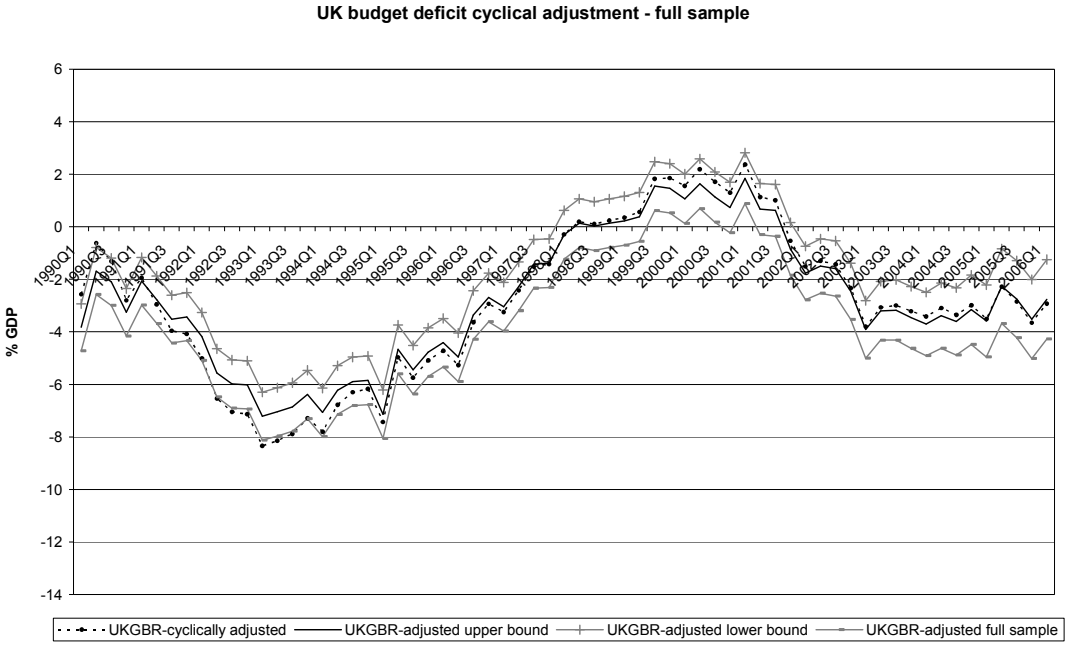
Table I.3: Effects on Budget as a % of GDP when GDP changes by 1% as a result of shocks

	Consumption	Investment	Exports	Average
France	0.67	0.23	0.22	0.45
Germany	0.66	0.35	0.35	0.51
Italy	0.79	0.35	0.34	0.48
UK	0.57	0.30	0.30	0.42
Spain	0.74	0.33	0.32	0.52
Euro Area	0.73	0.29	0.29	0.46

UK in EMU. Interest rates fixed for the first year, No fiscal feedbacks for the first year. The ECB uses a two pillar strategy, the exchange rate and the long rate are forward looking

Table I.3 records the rescaled impact of the shocks, so that we can seethe tax richness of each shock when GDP changes by 1% as a result. We utilise these estimates in the analysis of uncertainty over the budget deficit, and we attribute the probabilities form Barrell and Hurst (2003) to each source of shocks in order to produce an average effect. This should perhaps be closer to 0.45 on average, rather than the standard 0.5, but if we are clear on the source of the shocks that have driven the output gap then we can be more accurate in our assessment of the cyclically adjusted budget position.

Appendix II Deficit bounds with differing cyclical impacts on the deficit



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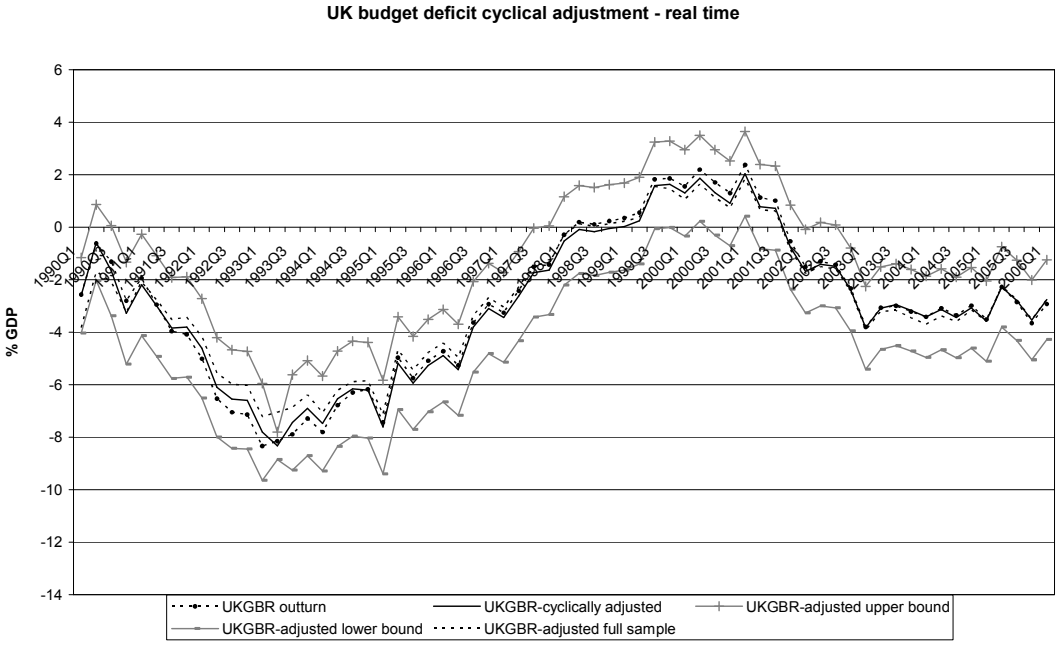

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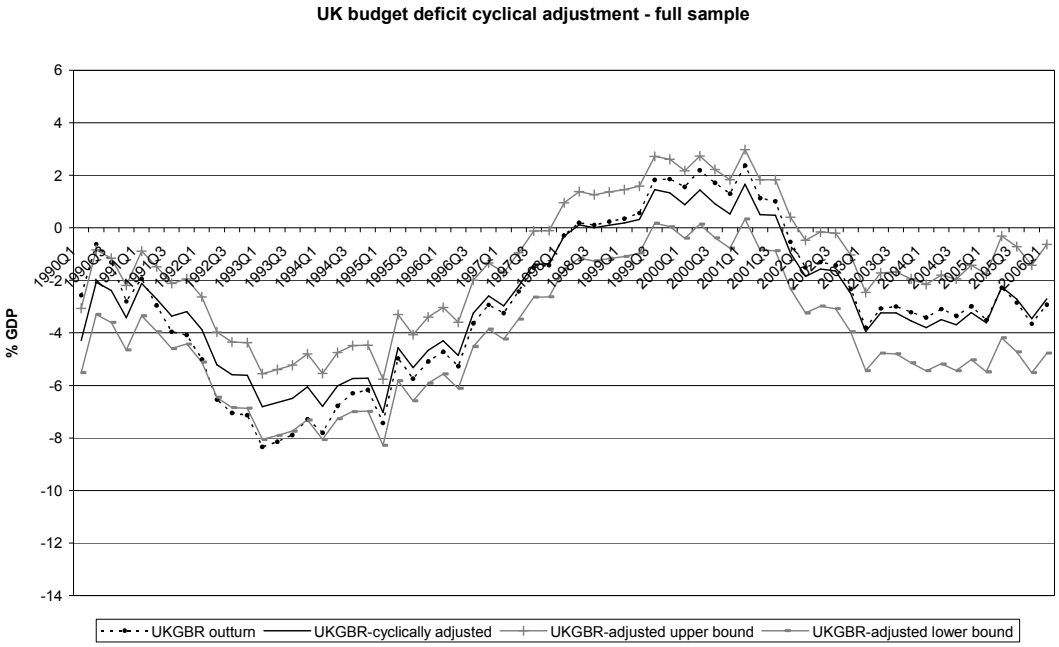

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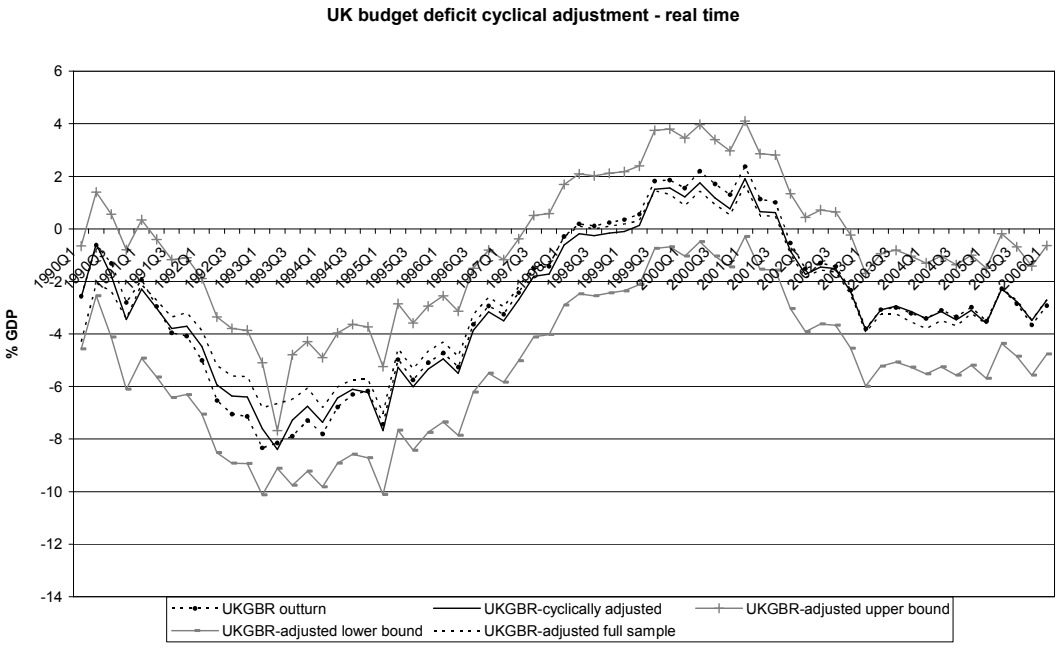

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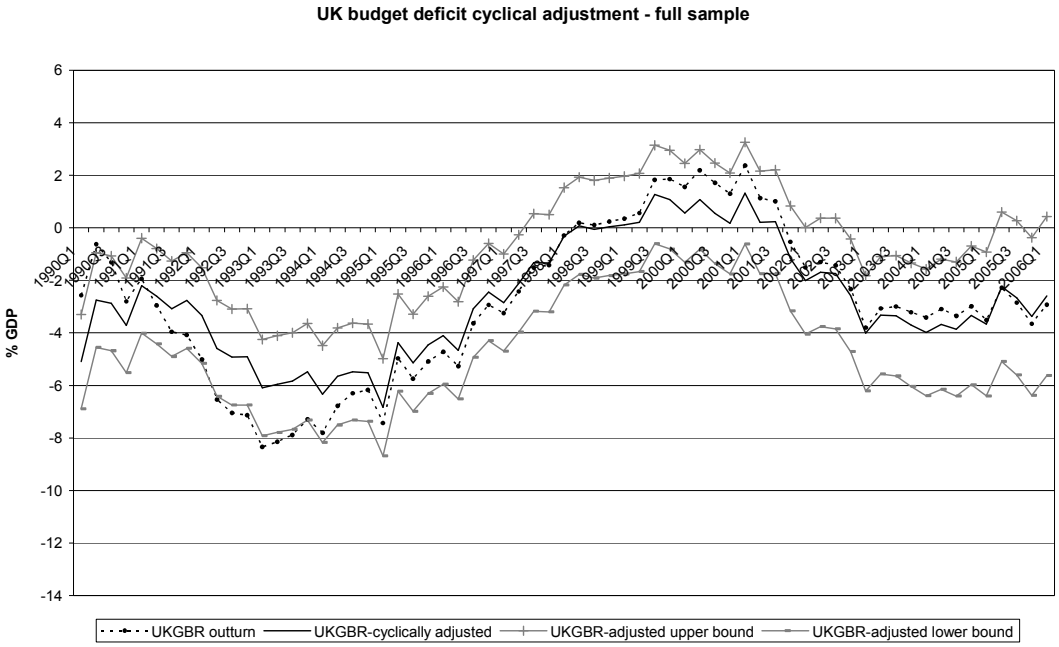

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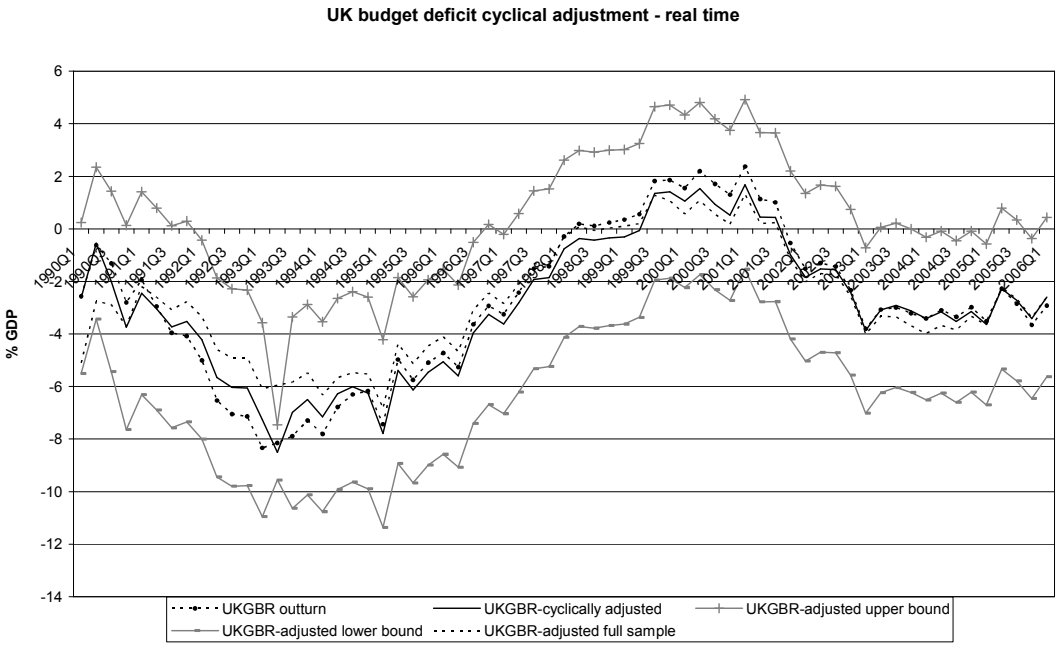

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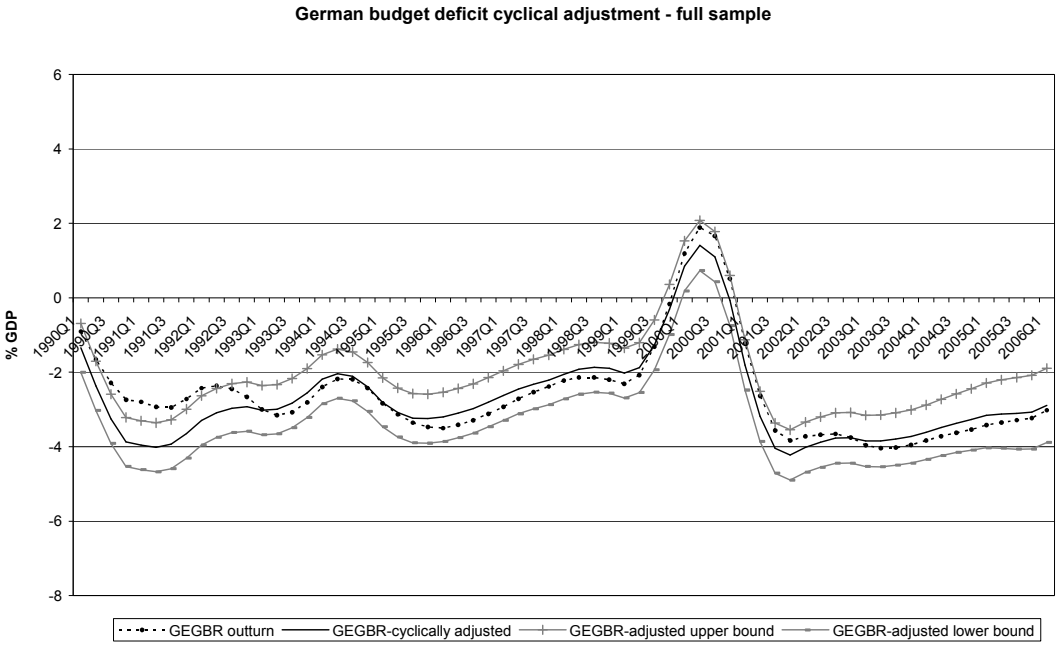

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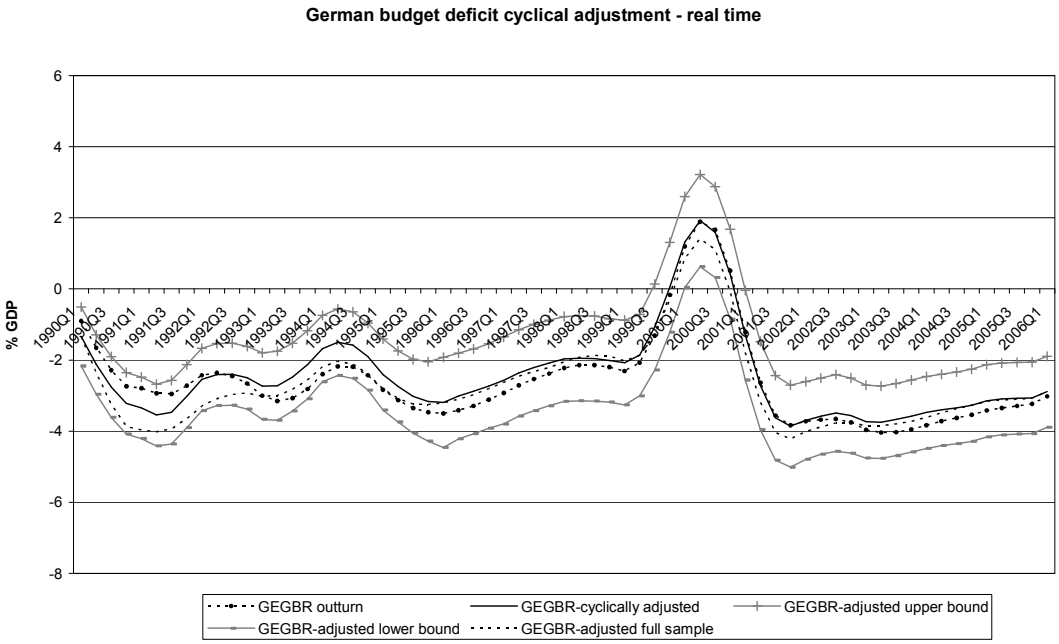

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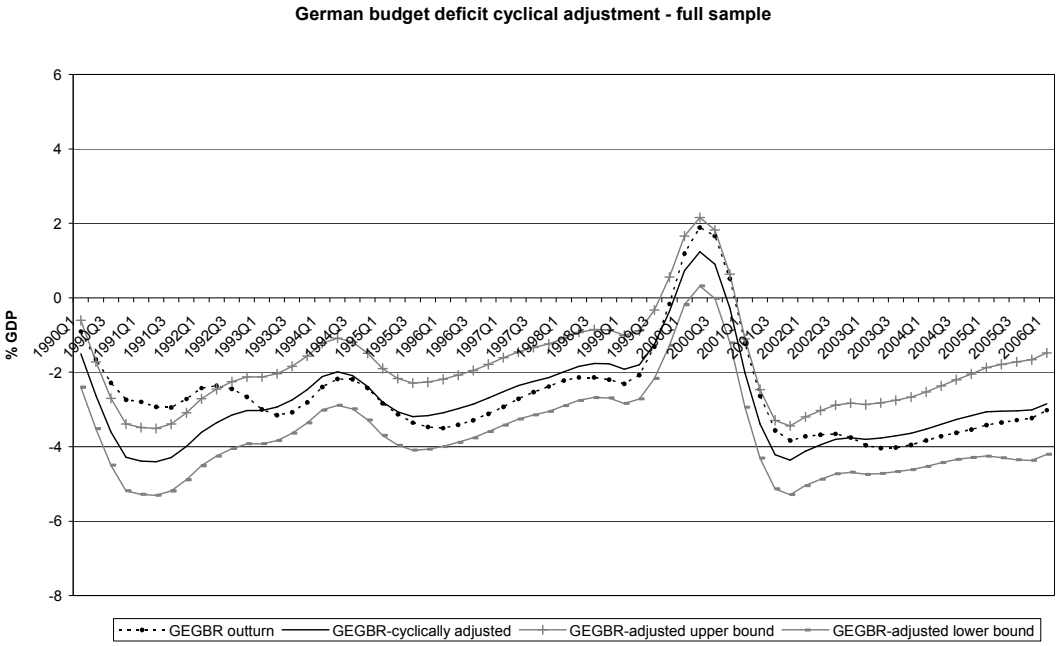

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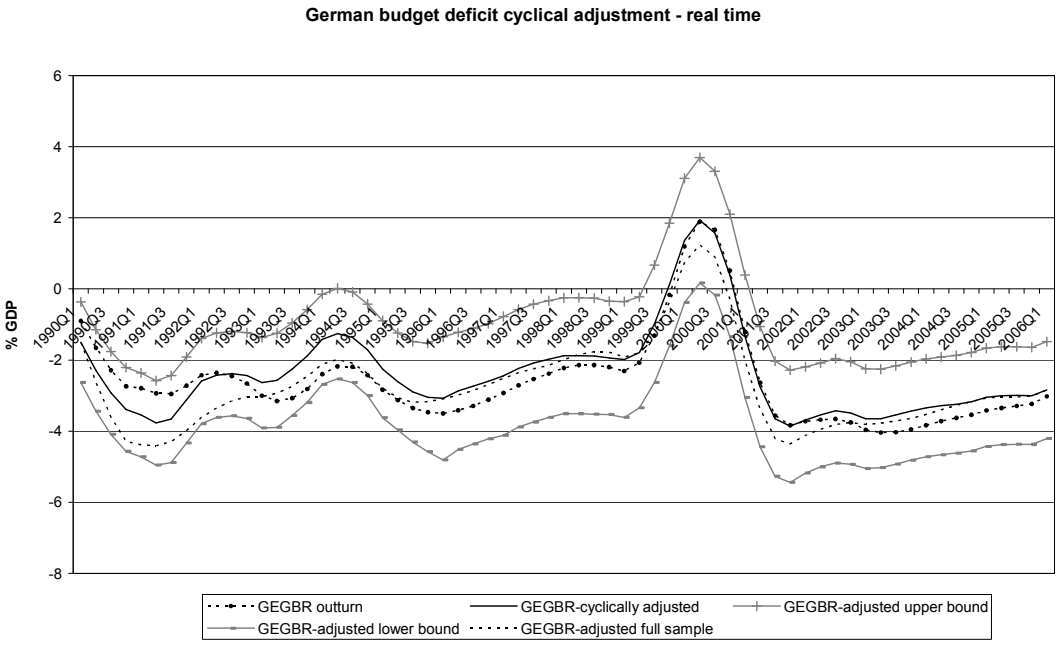

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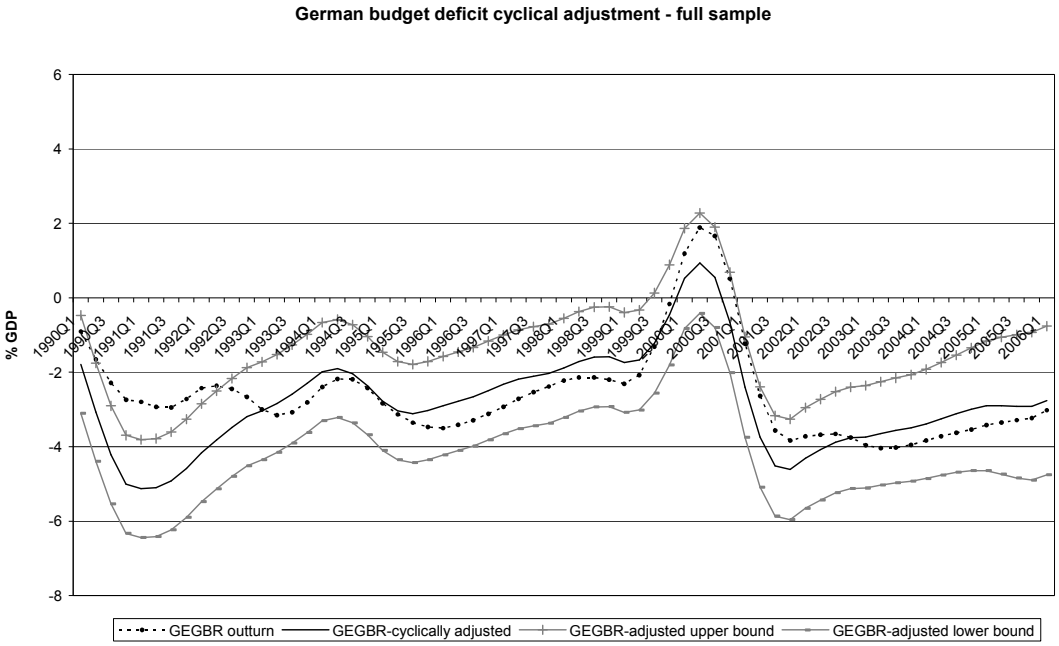

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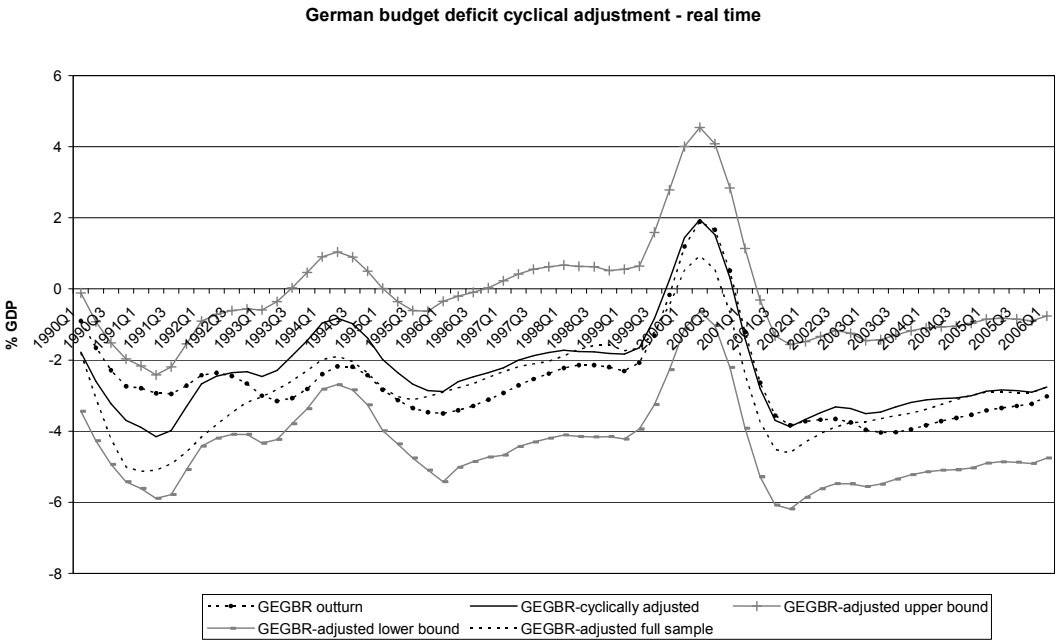

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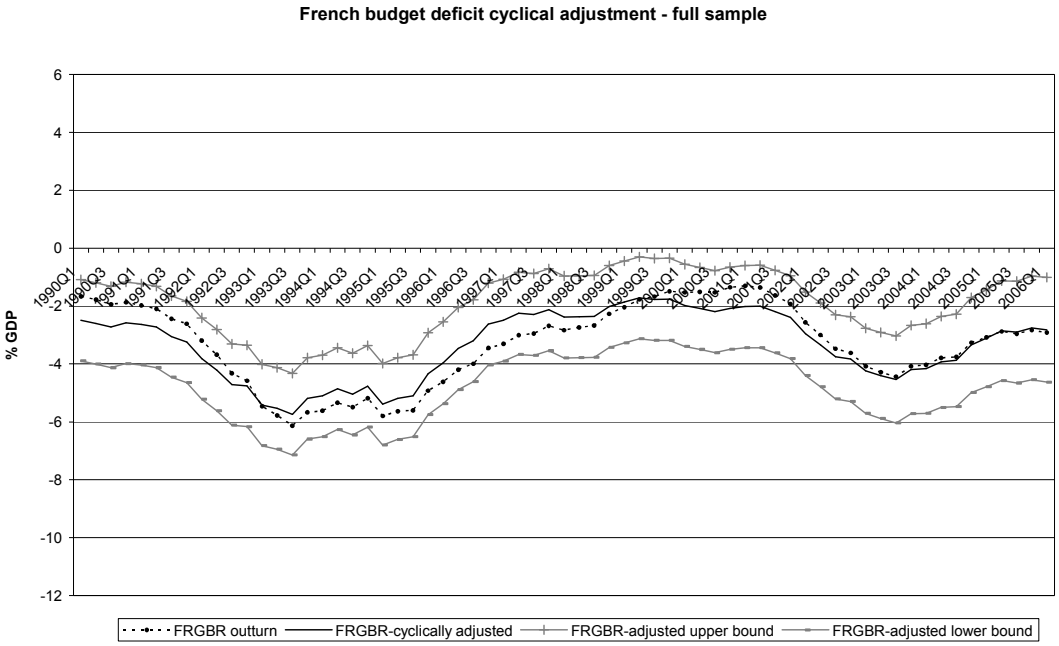

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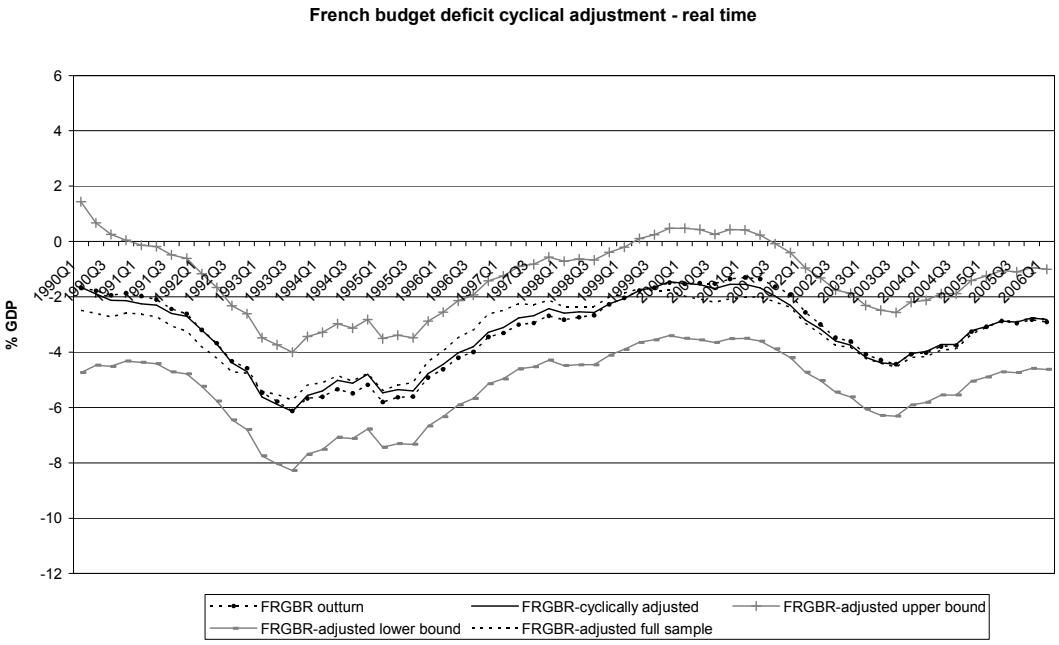

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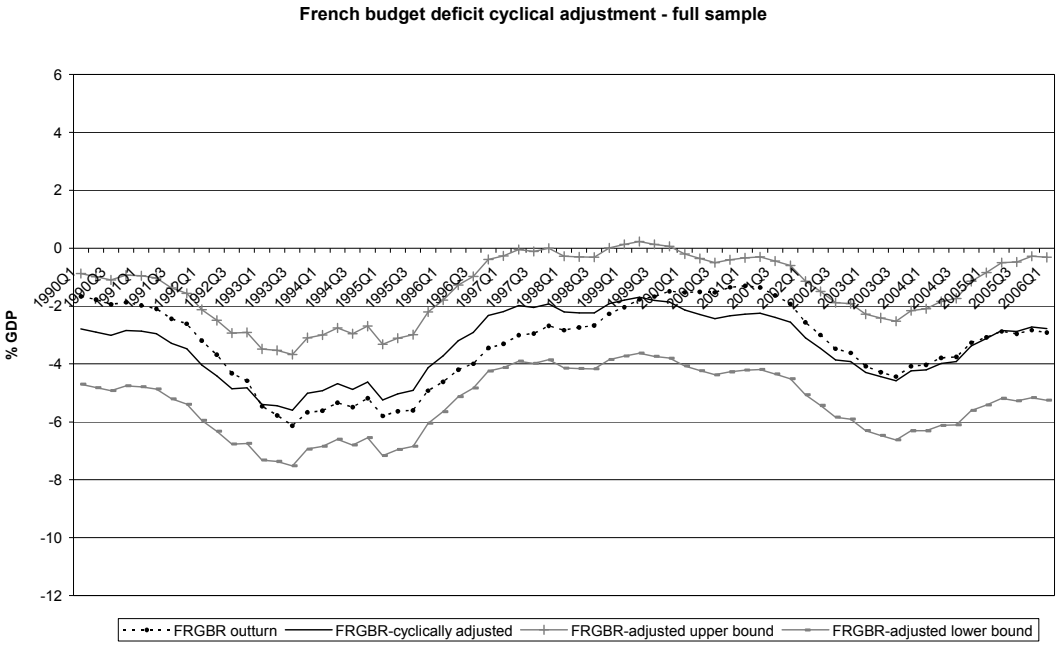

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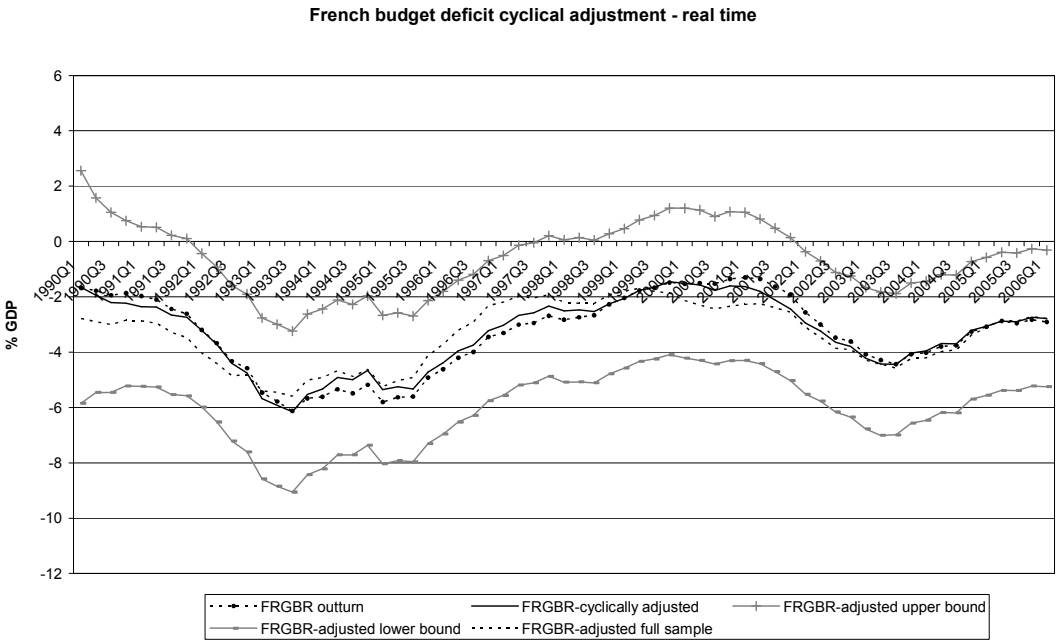

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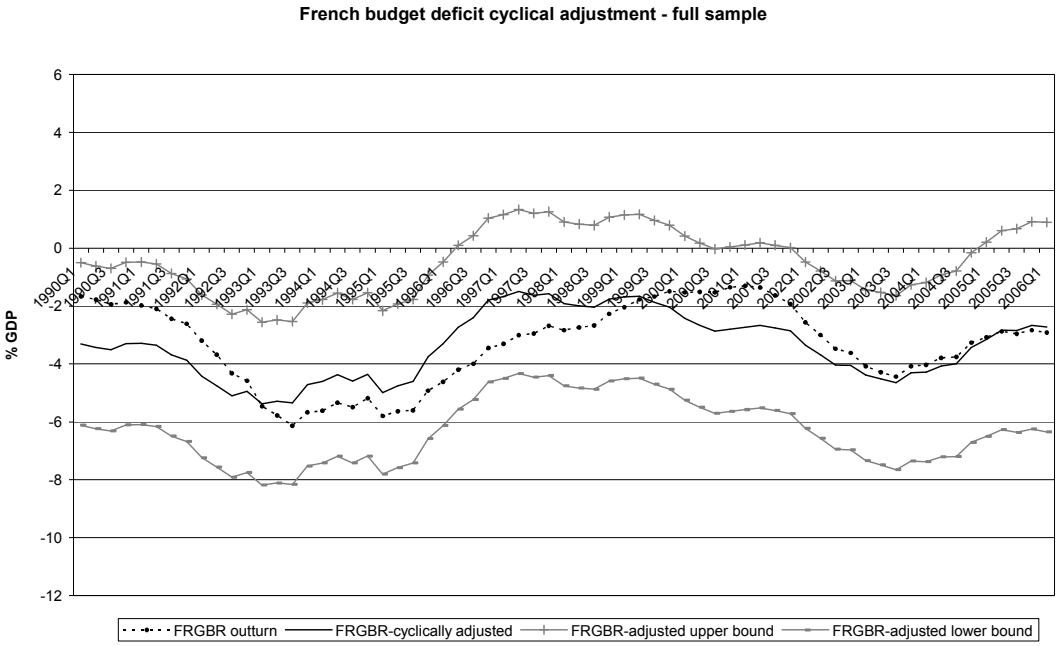

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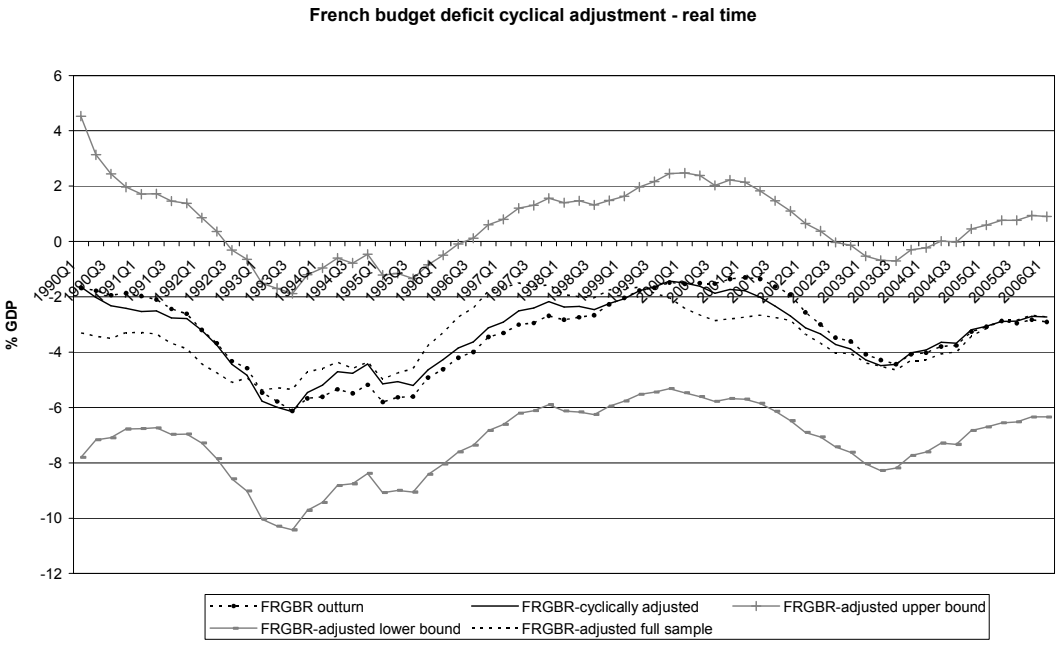

Figure 39 b = 0.66

References

Barrell, R., and Hurst, A. I., (2003) 'Benchmarks and Targets under the SGP' *National Institute Economic Review*, July 2003

Barrell, R., Becker, B., Byrne, J., Gottschalk. S., Hurst, A.I., and van Welsum, D., (2003) 'Macroeconomic Policy in Europe: Experiments with monetary responses and fiscal impulses' *Economic Modelling*

Baxter, M. and King, R.G. (1999), "Measuring business cycles: Approximate band-pass filters for economic time series", *Review of Economics and Statistics*, 81, 575-93.

Blanchard, O (2000) 'Commentary' *Federal Reserve Bank of New York Economic Policy Review'* 6 (1) 69-74

Blanchard, O., and Perotti, R., (2002) 'An Empirical Characterisation of the Dynamic Effects of Changes in Government Spending and Taxes on Output' *Quarterly Journal of Economics*

Brunila, A., Buti, M., and J.W., in't Veld (2002) 'Fiscal Policy in Europe: How Effective are the Automatic Stabilisers?' *European Commission Economic Paper* No 177

Canova, F. (1998), "Detrending and business cycle facts, *Journal of Monetary Economics*, 41, 475-512.

Christoffersen, P.F. (1998) 'Evaluating Interval Forecasts', *International Economic Review*, Vol. 39, pp. 841-862.

Denis, C, Grenouilleau, D., Mc Morrow, K. and Roger, W. (2006), "Calculating potential growth rates and output gaps – a revised production function approach", European Commission Economic Papers No. 247.

H.M. Treasury (2003) '*Fiscal Stabilisation and EMU'*

Harvey, A.C. and Trimbur, T. (2003), "General model based filters for extracting trends and cycles in economic time series", *Review of Economics and Statistics*, 85, 244-255.

Melitz, J., (2000) 'Some Cross Country Evidence about Fiscal Policy Behaviour and the Consequences for EMU' *European Economy* 2: 3-21

Mitchell, J., (2003), "Should we be surprised by the unreliability of real-time output gap estimates? Density estimates for the Eurozone", National Institute of Economic and Social Research Discussion Paper No. 225.

Orphanides, A. and van Norden, S. (2002), "The unreliability of output-gap estimates in real time", *Review of Economics and Statistics*, 84, 569-583.