Constructing a conditional GDP fan chart with an application to French business survey data

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Abstract
Among economic forecasters, it has become a more common practice to provide point projection with a density forecast. This realistic view acknowledges that nobody can predict future evolution of the economic outlook with absolute certainty. Interval confidence and density forecasts have thus become useful tools to describe in probability terms the uncertainty inherent to any point forecast (for a review see Tay and Wallis 2000). Since 1996, the Central Bank of England (CBE) has published a density forecast of inflation in its quarterly Inflation Report, so called “fan chart”. More recently, INSEE has also published a fan chart of its Gross Domestic Production (GDP) prediction in the Note de Conjoncture. Both methodologies estimate parameters of exponential families on the sample of past errors. They thus suffer from some drawbacks. First, INSEE fan chart is unconditional which means that whatever the economic outlook is, the magnitude of the displayed uncertainty is the same. On the contrary, it is common belief among practitioners that the forecasting exercise highly depends on the state of the economy, especially during crisis. A second limitation is that CBE fan chart is not reproducible as it introduces subjectivity. Eventually, another inadequacy is the parametric shape of the distribution. In this paper, we tackle those issues to provide a reproducible conditional and semi-parametric fan chart. For this, following Taylor 1999, we combine quantile regression approach together with regularization techniques to display a density forecast conditional on the available information. In the same time, we build a Forecasting Risk Index associated to this fan chart to measure the intrinsic difficulty of the forecasting exercise. The proposed methodology is applied to the French economy. Using balances of different business surveys, the GDP fan chart captures efficiently the growth stall during the crisis on an real-time basis. Moreover, our Forecasting Risk Index increased substantially in this period of turbulence, showing signs of growing uncertainty.

Key Words: density forecast, quantile regression, business tendency surveys, fan chart.

JEL Classification: E32, E37, E66, C22
1. Introduction

Usually, institutions display economic point forecasts. However, the forecast is not free of uncertainty: assuming that forecasts are not biased, they may be considered as the expected figure given the available information. Indeed, many shocks can affect the forecast in regard to oil prices, exchange rates, interest rates, and other variables. Another reason is that the behaviors of economic agents can only be estimated imprecisely over the past (when they do not change). The potential scenarios are therefore numerous: forecasters thus condense their forecast into a single, "baseline" scenario.

In the end, readers may lose sight of the uncertainty inherent in this type of exercise. Among economic forecasters, it has therefore become a more common practice to provide point projection with a density forecast. The longest running series of macroeconomic density forecasts dates back to 1968, when the ASA and the NBER initiated a survey of forecasters. For a detailed historical review, we refer to Tay and Wallis (2000). In particular, to dispel their risk, the Central Bank of England (cf. Britton et al 98) and INSEE display a “fan chart” to provide a concise illustration of the uncertainty affecting point forecasts. This realistic view recognizes that future evolution of the economic outlook cannot be predicted with absolute certainty. Confidence intervals and density forecasts have appeared as useful tools to describe in probability terms the uncertainty inherent to any point forecast (for a review, see Tay and Wallis 2000).

Both methodologies suffer from some drawbacks. First, the INSEE fan chart is unconditional which means that the magnitude of the displayed uncertainty is the same, whatever the economic outlook is. On the contrary, it is a common belief among practitioners that the forecasting exercise highly depends on the state of the economy, especially during crisis. Secondly, CBE’s fan chart is conditional but this conditionality comes from the subjective assessments by the members of the Monetary policy committee, and is therefore not reproducible. Eventually, another limitation is the parametric shape of the distribution, which is usually assumed to be exponential, that is without fat tails. In this paper, we tackle these issues to provide a reproducible methodology to build a non parametric conditional density forecasts.

This paper is organized as follows: in section 1, we review the main methods to describe uncertainty used by both INSEE and CBE, namely confidence intervals and density. We introduce our main notations and we give a brief description of French business surveys in section 2. In section 3, we introduce our methodology to derive conditional density forecasts and to construct our Forecasting Risk Index. Eventually, the proposed methodology is applied to the French economy in section 4.

Uncertainty description

A common way to describe uncertainty is to consider the future evolution of the economy (i.e. GDP growth rate) as the realization of a continuous random variable. The uncertainty is then fully characterized by the random variable density. A peaky density means a small uncertainty. On the contrary, a large uncertainty is translated into a loose distribution.
Confidence intervals

Confidence intervals may be the first simple method to describe the point forecast uncertainty (for a review see Tay and Wallis 2000). The principle is the following: the economic forecaster provides an interval together with his point forecast. The future observed value is then supposed to lie within this interval with a specified probability. For example, if the forecaster provides 95%-confidence intervals, the observed value is supposed to lie around 95% of the time into these intervals. It is also common to provide confidence intervals at different probability levels, i.e. 20%, 50%, 99% confidence intervals.

In the case of confidence intervals, the length of the interval measures the level of uncertainty. For example, we usually expect uncertainty to grow as the forecasting horizon increases. This is displayed by an increasing length of confidence intervals for a specified level of probability (cf. figures 1 and 2).

However, if their simplicity makes confidence intervals very attractive, they do not completely describe uncertainty.

Density forecasts

Density forecasts have thus become more and more appealing since they fully describe uncertainty. Notice that they can also be used to easily derive confidence intervals based on the appropriate distribution quantiles. In the case of density, the level of uncertainty is measured by the inverse of the sharpness of the distribution. Many indicators have been proposed to characterize the level of uncertainty, among them variance and entropy (see. Lugosi).

Since 1996, the Central Bank of England has a published a density forecast of inflation in its quarterly Inflation Report, so called “fan chart” (cf. figure 1). More recently, INSEE has also published a fan chart of its GDP prediction in the Note de Conjoncture (cf. figure 2).
Figure 1  CBE inflation fan chart

An example of an RPIX fan chart

Percentage changes on a year earlier

Figure 2  INSEE GDP fan chart

Source: INSEE
INSEE and CBE density estimation

Both methodologies assume that the error density belongs to exponential families. Estimations of the parameters are based on the sample of past forecast errors.

INSEE fan chart displays a normal distribution:

\[ f(y) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(y - \mu)^2}{2\sigma^2}\right). \]

Thus INSEE perceives the possible GDP outcomes symmetrically dispersed around the central most probable value. This is the famous bell-shaped curve (cf. figure 3 for the density of a standard normal).

Figure 3   density of a standard normal distribution

Central Bank of England perception is that in some circumstances the forecast error is more likely to be in one direction than the other. In statistical terms, their fan chart distribution is skewed. That is why they chose a particular form of statistical distribution called “two-piece” normal (cf. figure 4):
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\[ f(y) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2} \left\{ (x - \mu)^2 + \gamma \left( \frac{x - \mu}{|x - \mu|} (x - \mu)^2 \right) \right\} \right). \]

Eventually, CBE’s methodology allows the injection of subjectivity by deforming the density shape.

**Figure 4: example of a skewed distribution by CBE**

Even if both institutes display density forecasts, a fundamental philosophical difference remains between both methodologies. INSEE methodology attempts to capture an intrinsic uncertainty.

“assuming no change in the volatility of growth figures and the methodology used by INSEE forecasters during the period, the distribution of forecasting errors calculated from past exercises is a reliable indicator of the distribution of future errors” (cf. Note de conjoncture)

“we have chosen not to depart from the historic variance of forecasting errors, as the injection of a dose of subjectivity seems hard to justify ”( cf. Note de conjoncture)
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On the contrary, CBE’s fan chart somehow measures the CBE subjective uncertainty about: *The aim of the fan chart has been to convey to the reader a more accurate representation of the Bank’s subjective assessment of medium-term inflationary pressures.* It is therefore a forward-looking view of the risks to the forecast, not an extrapolation of past uncertainty. For more insights about this debate, we refer to B. de Finetti (1975) and references therein.

**Drawbacks**

**Conditional versus unconditional**

The first practical and important consequence is that the INSEE fan chart is unconditional. Thus, whatever the economic outlook is, the magnitude of the displayed uncertainty is the same. It means that on a long-term basis it is supposed to be correct on average. In other words, during a recession, the usual unconditional central confident interval can be significantly wrong. Let’s take a simple example by comparing forecasting with flying a plane: when crossing a turbulence area, flying gets more difficult. In the same way, during a crisis period, the intrinsic uncertainty of the forecasting exercise increases.

Instead of unconditional forecasting, conditional forecasting which can change from one period to another should be favoured to describe uncertainty. We illustrate the interest of conditional forecasting versus unconditional forecasting by a simple toy model.

**Toy Model**

Let $Z_t$ be a random variable that describes the state of the economy at time $t$ and that can only take two values: $A$ for acceleration with probability $p$, $D$ for deceleration $1 - p$.

Let $Y_t$ be a random variable describing the GDP growth rate.

To make it simple, we suppose the conditional distributions of $Y_t$ to be such that:

$E_{Y_t}(y|Z_t) = \mu_{Z_t}$, $Var_{Z_t}(y|Z_t) = \sigma^2_{Z_t}$ with $\mu_D < \mu_A$ (the growth rate is smaller during recession) $\sigma^2_A < \sigma^2_D$ (but the volatility is higher).

Then the description of uncertainty by an unconditional forecaster is $E(Var(y|I_t))$ which is equal to $p\sigma^2_D + (1 - p)\sigma^2_A$.

On the contrary, for a conditional forecaster, uncertainty as measured by $Var(y|I_t)$ would be equal to $\sigma^2_D$ during recession and to $\sigma^2_A$ during acceleration.

We can deduce that on a long-term average the unconditional description is correct. However, at each date $t$, knowing the information $Z_t$, it is either too small or too big.
Indeed, since $\sigma^2_A < p\sigma^2_D + (1 - p)\sigma^2_A < \sigma^2_D$, the unconditional description of uncertainty is either too loose (during acceleration period) or too sharp (during crisis). The unconditional forecast error neglects the information embedded in it.

On the contrary, since CBE’s methodology allows the injection of subjectivity from one period to another, their fan chart is thus conditional. However, their method is not reproducible as it is a measure of CBE’s subjectivity.

**Exponential distribution versus nonparametric**

To check a density forecast, we can use Talagrand’s diagram. The principle is the following: $\hat{F}_i(Y_i)$ is supposed to be a sequence of independent identically distributed uniform variables on $[0,1]$ with $\hat{F}_i$ the forecasted cumulative distribution function. The histogram of $\hat{F}_i(Y_i)$ is called Talagrand’s diagram and is supposed to be a straight line if our forecast is correct (cf. Dowd, K., 2004). Figure 5 displays Talagrand’s diagram for the INSEE fan chart. Instead of a straight line, it exhibits a concave profile indicating that the probabilities of right extreme risks are overestimated.
The aim of this paper is to suggest a reproducible methodology to build conditional density forecasts. In the same time, we will release the constraint of the parametric distribution shape.

Notations and business surveys description
Recall that $Z_t$ describes the state of the economy in our toy model. The role of $Z_t$ will be played by business surveys. Indeed, business surveys are a useful source of information when forecasting, for they present three types of advantages: (1) they provide reliable information coming directly from the economic decision makers, (2) they are rapidly available (about a month after the questionnaires are sent), on a monthly, bimonthly or quarterly basis, and (3) they are subject to small revisions (each publication presents a generally negligible revision, only on the preceding point). The disseminated statistics compiled from these surveys are usually balances of opinions. Since there are a large number of available survey variables (cf. table 1), composite indexes (CI) have been developed over the years to provide suitable summaries by extracting the common trend, and suppressing the undesirable "noise" of numerous data. In particular, the French composite indicator (cf. figure 6) gives an assessment of the global climate of the whole French economy.

**Figure 6: French composite indicator and year-on-year GDP growth rate**
Table 1: balances of opinions used in the French CI (table from Bardaji 09)

<table>
<thead>
<tr>
<th>Balances of opinion</th>
<th>availability and periodicity</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>in Industry</em> (6 balances):</td>
<td></td>
</tr>
<tr>
<td>past and expected changes in production (<em>I</em><em>{PCP} and <em>I</em></em>{ECP})</td>
<td>monthly from 1976m3</td>
</tr>
<tr>
<td>general business outlook in industry as a whole (<em>I</em>_{GBO})</td>
<td>monthly from 1976m3</td>
</tr>
<tr>
<td>total and foreign order books (<em>I</em><em>{TOB} and <em>I</em></em>{FOR})</td>
<td>monthly from 1976m3</td>
</tr>
<tr>
<td>inventories level (<em>I</em>_{INV})</td>
<td>monthly from 1976m3</td>
</tr>
<tr>
<td><em>in the Building Industry</em> (5 balances):</td>
<td></td>
</tr>
<tr>
<td>past and expected changes in activity (<em>B</em><em>{PCA} and <em>B</em></em>{ECA})</td>
<td>quarterly from 1976q1 to 1993q3 and monthly from 1993m9</td>
</tr>
<tr>
<td>total order books (<em>B</em>_{TOB})</td>
<td>quarterly from 1976q1 to 1993q3 and monthly from 1993m9</td>
</tr>
<tr>
<td>past change in workforce (<em>B</em>_{PCW})</td>
<td>quarterly from 1976q1 to 1993q3 and monthly from 1993m9</td>
</tr>
<tr>
<td>utilization of production capacity (<em>B</em>_{UPC})</td>
<td>quarterly from 1976q1 to 1993q3 and monthly from 1993m9</td>
</tr>
<tr>
<td><em>in the Services</em> (6 balances):</td>
<td></td>
</tr>
<tr>
<td>past and expected changes in turnover (<em>S</em><em>{PCT} and <em>S</em></em>{ECT})</td>
<td>quarterly from 1988q1 to 2000q2 and monthly from 2000m6</td>
</tr>
<tr>
<td>general business outlook in Services as a whole (<em>S</em>_{GBO})</td>
<td>monthly from 2000m6</td>
</tr>
<tr>
<td>expected demand (<em>S</em>_{D})</td>
<td>quarterly from 1988q1</td>
</tr>
<tr>
<td>past and expected changes in profit and loss account (<em>S</em><em>{PCPLA} and <em>S</em></em>{ECPLA})</td>
<td>quarterly from 1988q1 to 2000q2 and monthly from 2000m6</td>
</tr>
<tr>
<td><em>in the Wholesale Trade</em> (5 balances):</td>
<td></td>
</tr>
<tr>
<td>past sales (<em>W</em>_{P})</td>
<td>twice-monthly from 1979m7</td>
</tr>
<tr>
<td>foreign past sales (<em>W</em>_{PFPS})</td>
<td>twice-monthly from 1979m7</td>
</tr>
<tr>
<td>ordering intentions (<em>W</em>_{OI})</td>
<td>twice-monthly from 1979m7</td>
</tr>
<tr>
<td>general business outlook in wholesale trade as a whole (<em>W</em>_{GBO})</td>
<td>twice-monthly from 1979m7</td>
</tr>
<tr>
<td>delivery received from abroad (<em>W</em>_{DRA})</td>
<td>twice-monthly from 1979m7</td>
</tr>
<tr>
<td><em>in the Retail Trade</em> (4 balances):</td>
<td></td>
</tr>
<tr>
<td>total order books (<em>R</em>_{TOB})</td>
<td>monthly from 1991m1</td>
</tr>
<tr>
<td>general business outlook in retail trade as a whole (<em>R</em>_{GBO})</td>
<td>monthly from 1991m1</td>
</tr>
<tr>
<td>past sales (<em>R</em>_{PS})</td>
<td>monthly from 1991m1</td>
</tr>
<tr>
<td>expected changes in workforce (<em>R</em>_{ECW})</td>
<td>monthly from 2000m3</td>
</tr>
</tbody>
</table>

Notes: all business survey data are seasonally adjusted.

Denote the French CI by \( (i_t) \). Recall that our goal is to forecast the first release of the quarterly French GDP growth rate, denoted by \( y_t \). The first release will be published only 45 days after the end of the current quarter. Usually, economists also forecast the next quarters. For the sake of simplicity, we will restrict ourselves to the forecast of the current quarter. Finally, we define \( y_t := (y_t, \ldots, y_t) \). Our quarterly historical data of French GDP first release starts from 1988 Q1.

\[ 1 \text{ To be more precise, we define } i_t \text{ as the mean of the three last known monthly releases when forecasting takes place.} \]
Methodology

Adapting the sequential framework in Biau and Patra (2009): at each quarter $t = 1 \ldots T$, we observe business surveys data $z_t$ but the first release of the q-o-q GDP growth rate $y_t$ is unknown. To model uncertainty, we consider $(z_t)$ and $(y_t)$ as the outcome of random variables $(Z_t)$ and $(Y_t)$, such that the process $(Z_t, Y_t)_{t \in \mathbb{Z}}$ is jointly continuous stationary ergodic.

In a sequential version of the density prediction problem, the forecaster is asked to guess the next conditional density of $Y_t$ of a sequence of random variables $Y_1, \ldots, Y_{t-1}$ with knowledge of the past observations $y_{t-1} = y_1, \ldots, y_{t-1}$ and $z_t = z_1, \ldots, z_t$. In other words, the observations $y_1, y_2 \ldots$ and $z_1, z_2 \ldots$ are revealed one at a time, beginning with $(y_0, z_0), (y_1, z_1), \ldots$.

Quantile regression techniques provide a suitable semi-parametric tool to achieve a proper density forecast. We introduce here the main outlines of the quantile regression methodology.

Brief introduction to quantile regression

(Koenker and Bassett 78) developed a theory for the estimation of the quantiles of a variable $Y$ based on past observations.

The starting point is to notice that many probability quantities can be characterized by a minimization problem. For example, the expectation of a $L^2$ random variable satisfies

$$E(Y) = \arg \min_{m \in \mathbb{R}} E(Y - m)^2.$$ 

In the same way, the median corresponds to

$$med(Y) = \arg \min_{m \in \mathbb{R}} E|Y - m|.$$ 

The natural question is then: can the quantile function $Q_{\theta}(\theta) := \inf \{t \in \mathbb{R} : F_Y(t) \geq \theta\}$ be described by a minimization problem? (Koenker and Bassett 78) generalizes the observation that minimizing the $L_1$ loss yields to the median. To that end, they transform the $L^1$ loss into a suitable loss function called pinball loss function or also check loss (cf. figure 7).

$$\rho_{\theta}(y) := \theta y^+ + (1 - \theta) y^- \text{ with } y^+ := \max(0, y), y^- := \max(0, -y).$$
Then they obtain (for a proof see Biau and Patra (2009)):

$$Q_Y(\theta) = \arg \min_{m \in \mathbb{R}} E \rho_\theta (Y - m)$$

These ideas extend readily to conditional quantiles

$$Q_Y(\theta \mid z) := \inf \{ t \in \mathbb{R} : F_Y(t \mid z) \geq \theta \}.$$ 

We can then verify that

$$Q_Y(\theta \mid z) = \arg \min_{m(z)} E \rho_\theta (Y - m(z)) \mid X = z$$

(Koenker and Bassett 78) considered $m(z)$ of a linear form $z' \beta$ and the coefficients are estimated by minimizing $\sum_{i=1}^n \rho_\theta (y_i - z_i' \beta)$ on past sample observations. Their estimated quantile curve is then defined by

$$\hat{Q}_Y(\theta \mid z) := z_i' \hat{\beta}_\theta.$$ 

To justify quantile regression, assume the data to be generated by the following model with heteroscedasticity:

$$Y_i = \mu_i(Z_i) + \sigma_i(Z_i) \varepsilon_i,$$

with $\mu_i(Z_i) := E(Y_i \mid Z_i)$ and $\sigma_i(Z_i) := V(Y_i \mid Z_i)$, and $\varepsilon_i$ an error term independent of $Z_i$. 

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**Figure 7: pinball loss**

![Pinball Loss Diagram]
Based on the aforementioned model, the conditional quantile of $Y_t$ given $Z_t$ has the form

$$Q(Y_t|Z_t) = \mu_t(Z_t) + \sigma_t(Z_t) Q_{\alpha_t}(\theta).$$

(Koenker and Bassett 82) showed that if $\mu_t(Z_t)$ and $\sigma_t(Z_t)$ are linear functions of $Z_t$ then quantile regression estimates are asymptotically consistent. The appeal of quantile regression is that past observations of the quantiles are not required.

**Reference Model**

The first step is to find a proper set of explicative variables to both:

- reduce the uncertainty since

$$\text{Var}(Y_t) = E(\text{Var}(Y_t|Z_t)) + \text{Var}(E(Y_t|Z_t)) \geq E(\text{Var}(Y_t|Z_t))$$

In other words, the square error of an uninformed forecaster is equal to the error of an informed forecaster plus a residual term (knowledge term).

- and explain correctly the remaining uncertainty (i.e. $\text{Var}(\varepsilon_t)/\text{Var}(\sigma_t(Z_t))$ small).

As mentioned above, there is a huge number of balances of opinions (hundreds). Thus, we face the curse of dimension. The French composite indicator (FCI) is a way to reduce the dimension by assessing the global climate of the whole French economy.

In a simplified framework, FCI would be roughly equal to GDP growth rate $Y_t \equiv \alpha + \beta I_t$.

However, in these surveys, entrepreneurs are asked to give a qualitative appreciation on the occurred or expected changes of some variables of interest (output, order book, foreign order book, inventories, ...) through the three following categories: increase, no change, decrease. There is thus a difference between qualitative answers from microeconomic surveys and quantitative macroeconomic measures. To fill the gap, we suggest a model of the following form:

$$Y_t = Z_t \alpha + \sigma(Z_t) u_t$$

with $Z_t := (1, Y_{t-1}, I_t, \Delta I_t, \Delta I_{t-1})$, $\alpha := (\alpha_0, \alpha_1, \alpha_2, \alpha_3)$ and $u_t$ a white noise independent of the available information.
**Point forecast**

Once the model set, the forecaster needs first a point forecast. It is usual (though not optimal for prediction as we do not need the estimator to be unbiased) to estimate $\alpha$ by Ordinary Least Squares (OLS) and we get:

$$\hat{Y}_t = \hat{\alpha} Z_t$$

with $\hat{\alpha} := (Z_t' Z_t)^{-1} Z_t' Y_t$.

The unconditional variance $E(Var(Y_t|Z_t))$ of the error term can then be estimated by

$$\hat{E}(Var(Y_t|I_t)) := \frac{1}{T} \sum_t \|\hat{Y}_t - Y_t\|^2.$$  

**Conditional confidence intervals**

A simple way to derive conditional confidence intervals for $Y_t$ is to use quantile regressions. In order to construct a confidence interval with probability at least 90%, it is sufficient to estimate the quantile curve at levels 5% and 95% by quantile regression on the past available observation $(y_{t-1}, z_{t-1}), \ldots, (y_1, z_1)$: $\hat{Q}_{y_t}(0.05|z_t)$ and $\hat{Q}_{y_t}(0.95|z_t)$. Our confidence interval at quarter $t$ can be written as

$$CI_{90\%}^t := [\hat{Q}_{y_t}(0.05|z_t), \hat{Q}_{y_t}(0.95|z_t)]$$

**Conditional Density forecasts**

Recall that the quantile curve $\theta \rightarrow Q_{y_t}(\theta|z_t)$ completely describes the distribution. Thus, it would be sufficient to give the estimated quantile curve $\theta \rightarrow \hat{Q}_{y_t}(\theta|z_t)$. Instead, it is a common practice among forecasters to display a density forecast $y \rightarrow \hat{f}_t(y|z_t)$.

We give here a simple heuristic to derive a density forecast from the estimated quantile curve $\theta \rightarrow \hat{Q}_{y_t}(\theta|z_t).$
Recall that $Q_t(U|z_t)$ with $U$ a uniform distribution is distributed as $Y_t|z_t$. Moreover, 

$$
\hat{f}_t(y|z_t) := \frac{1}{Th} \sum_{i} K\left(\frac{y - Y_t}{h}\right) \text{ (with } K \text{ a kernel and } Y_t \approx Y_t|z_t \text{) is a consistent estimator of } f_y(y|z) \text{ under suitable conditions.}
$$

Box 1

Heuristic path to estimate $f_y(y|z)$

Set $N$ a large number (a practical choice is 100).

Compute at each $u_i := \frac{i}{N}; 1 \leq i \leq N$, $y_i := \hat{Q}(u_i|z)$

Eventually, compute 

$$
\hat{f}_t(y|z) := \frac{1}{Th} \sum_{i} K\left(\frac{y - y_i}{h}\right) \text{ with } K \text{ an Epanechnikov kernel and } h \text{ the bandwidth}
$$

**Forecasting Risk Index: FRI**

In this section, we aim at building a Forecasting Risk Index for each quarter $t$, associated to this fan chart to measure the intrinsic difficulty of the forecasting exercise. At quarter $t$, the difficulty of forecasting is linked to the sharpness of the distribution: the sharper the distribution, the easier forecasting is. We must thus set a quantitative indicator to measure how much the distribution is spread. In the literature, classical measures are variance or entropy. As far as the root mean square error is concerned, the $L^2$-norm (root of variance) suits our needs. The definition of conditional variance is given by:

$$
\sqrt{\text{Var}(Y|z)}
$$

We estimate $\sqrt{\text{Var}(Y|z)}$ by $\sqrt{\text{Var}(Y|z)} := \sqrt{\text{Var}_{f_y}(Y)}$

We are now in position to define our Forecasting Risk Index for each quarter $t$ as

$$
FRI_t := \sqrt{\hat{\text{Var}}(Y|z_t)}
$$
When the point forecast is based on our reference model, the expected forecasting error is approximately equal to the value of the Forecasting Risk Index.

**Out-of-sample Validation**

Recall the forecaster is asked to guess the next conditional density of $Y_t$ of a sequence of random variables $Y_1, \ldots, Y_{t-1}$ with knowledge of the past observations $y_{t-1} = y_1, \ldots, y_{t-1}$ and $z_t = z_1, \ldots, z_t$.

Thus, all previous quantities must be computed on a real time basis: $\hat{Y}_t = \hat{\alpha}_t Z_t$, with $\hat{\alpha}_t := (Z_t' Z_t)^{-1} Z_t' Y_t$, $CI^{90\%}$, $\hat{f}_Y(y|z_t)$

**Interval forecast**

If the confidence intervals for $Y_t$ are correct, $y_t$ should lie in $CI^{90\%}$ around 90% of the time.

**Density forecast**

To check our assumptions on real time basis, we can notice that $Y_t$ is supposed to follow $\hat{f}_Y(y|z_t)$. Thus the cumulative function $\hat{F}_Y (Y_t|x_t) := \int_{-\infty}^{Y_t} \hat{f}_Y(y|z_t) \, dy$ should be independent random variables distributed as uniform variable on $[0,1]$. The histogram of $(\hat{F}_Y(Y_t|z_t))$, can be plotted and compared with a uniform density: this is the classical Talagrand’s diagram. Other more sophisticated tests might be considered (see Berkowitz 2003, Clement 2004, Wallis 2003 and references therein).
Numerical results

Point forecasts

$Z_t := (1, Y_{t-1}, I_t, \Delta I_t, \Delta I_t)()$

Parameters estimation by OLS (from 1988 Q1 to 2010Q1) leads to the following chart:

**Table 1 Parameters estimation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>T value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.61</td>
<td>0.05</td>
<td>11.84</td>
<td>***</td>
</tr>
<tr>
<td>$Y_{t-1}$</td>
<td>-0.41</td>
<td>0.09</td>
<td>-4.42</td>
<td>***</td>
</tr>
<tr>
<td>$I_t$</td>
<td>0.09</td>
<td>0.01</td>
<td>6.89</td>
<td>***</td>
</tr>
<tr>
<td>$\Delta I_t, \Delta I_t$</td>
<td>0.10</td>
<td>0.01</td>
<td>8.28</td>
<td>***</td>
</tr>
</tbody>
</table>

Figure 8 displays the out of sample forecasts together with the first GDP release. The residual standard error is 0.31.

Recall that we have $Var(Y_t) = E(Var(Y_t|Z_t)) + Var(E(Y_t|Z_t))$. In other words, the square error of an uninformed forecaster is equal to the error of an informed forecaster plus a residual term (knowledge term). We estimate $Var(Y_t) \approx 0.24$ and $E(Var(Y_t|Z_t)) \approx 0.12$. Thus, our reference model gives us a 50% gain of accuracy for the $L^2$ loss.
Figure 8: out of sample point forecasts
Conditional confidence intervals

In table 2, we obtain the estimates (from 1988 Q1 to 2010Q1) for the quantile regression coefficients. Interestingly, the coefficient of the acceleration term is almost zero for the 95% quantile whereas it is significant for the 5% quantile. In other words, troughs are much worse than peaks are great.

Table 2 Parameters estimation

<table>
<thead>
<tr>
<th></th>
<th>theta = 0.05</th>
<th>Theta = 0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.17</td>
<td>1.23</td>
</tr>
<tr>
<td>$Y_{t-1}$</td>
<td>-0.35</td>
<td>-0.46</td>
</tr>
<tr>
<td>$I_t$</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>$\Delta I_t</td>
<td>\Delta I_t$</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Figure 9 displays GDP first releases together with 90% confidence intervals of its last forecast. Notice that the length of the interval depends on the forecasting date. The percentage of GDP first releases outside our 90%-confidence intervals is 13% estimated on our historical data.
Figure 9: Out of sample 90% confidence intervals
Conditional Fan chart

Figures 10 show density forecasts (methodology from box1) together with GDP first release (vertical line) at different quarters (from 2008 Q2 to 2010Q2).

Figure 10: density forecasts and GDP first release (vertical line)

As mentioned above, we draw Talagrand’s diagram for our new fan chart (cf. figure 11). In comparison with INSEE diagram (cf. figure 11), we can see that our new method leads to a better estimate of probability tails. The bumpy fan charts come from the semi-parametric methodology as no unimodal distribution has been imposed. This raised the philosophical question if the error distribution should be unimodal, parametric or nonparametric. If necessary, it is always possible to display a unimodal density but with a conditional variance. For this, it is sufficient to plug the forecasting risk index, as the conditional standard error of a gaussian density forecast.
Forecasting Risk Index

Figure 12 displays the Forecasting Risk Index. On an out-of-sample basis, the FRI exhibits clear signs of turbulence during the previous crisis. This gives an early signal of growing uncertainty. For example, when FRI is equal to 0.6 in 2008Q4, it can be interpreted as an expected error of 0.6 for a point forecast based on our reference model. It is worth recalling that our reference model is only based on surveys data. Thus, any uncertainty that may be not reflected by business surveys such as uncertainty on oil prices or exchange rates will not be displayed by our forecasting risk index. However, Talagrand's diagram shows that our fan charts enjoy nice empirical properties. Moreover, in a bayesian manner, an economic forecaster could always start from the uncertainty as measured by the surveys data and add his own subjective assessment of uncertainty.
Focus on the crisis period

Our new GDP density forecast captures efficiently the growth stall during the crisis on a real time basis. Indeed, the first release of 2008 Q4 (-1.3% in volume) lies inside the confidence interval (cf. figure 13). On the contrary, it was almost considered as an outlier by the INSEE fan chart (cf. figure 15). It is also interesting to compare results during the rebound of 2009 Q2. The first release (+0.3%) was in our confidence interval but it was far outside the INSEE fan chart (cf. figure 16). It could be surprising that our confidence interval captures efficiently the growth stall during the crisis on a real-time basis. Indeed, before 2008 Q4, the minimum q-o-q GDP growth rate of our historical data was -0.6%, far above the -1.3% of 2008 Q4. This feature is made possible both by the linear form of every quantile regression and by new extreme business surveys values during the last crisis.
Figure 13: Out-of sample confidence intervals during the last crisis
Figure 14: density forecasts during crisis

Chart on 2008 Q2

Chart on 2009 Q1

Chart on 2008 Q3

Chart on 2009 Q2

Chart on 2008 Q4

Chart on 2009 Q3
Figure 15: INSEE fan chart on 2008 Q3

Source: INSEE

Figure 16: INSEE fan chart on 2009 Q1

Source: INSEE
Conclusion

In this paper, we developed a reproducible methodology to build conditional density forecasts based on business surveys data. Even though results are not fully comparable since the forecasting device is not exactly the same, this methodology seems able to better catch the uncertainty pattern associated with forecasts of French GDP. In particular, we derive a Forecasting Risk Index from the conditional density forecasts, which aims at quantifying the intrinsic uncertainty of a point forecast. Interestingly, our index methodology leads to uncertainty that increases on recessions, a characteristic that Insee forecasts have witnessed in the recent crisis.

References


INSEE Note de Conjoncture for June 2008, pages 15 to 18


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