Detecting Economic Regimes in France: a Turning Point Index using Mixed Frequency Data

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Abstract

This paper proposes an indicator for detecting business cycle turning points involving mixed frequency business survey unbalanced data. It is based on a hidden Markov-switching model and allows for the detection of regime changes in a given economy where information is displayed monthly and/or quarterly. Starting from Gregoir and Lenglart (2000) we propose an adaptable framework which can be applied to many situations involving monthly, bimonthly and quarterly data.

The proposed methodology is applied to the French economy. Using balances from business survey, this indicator measures the probability of being in an accelerating or a decelerating phase referring to the output growth rate cycle. The index is confronted over the past with a reference dating based on the growth cycle of the French GDP estimated through a Cristiano-Fitzgerald filter. By extracting information from business survey, our index exhibits quite clearly and timely regime changes in France. Moreover, the message delivered by the indicator is mainly unrevised and available many quarters before the ex-post dating. Considering this adequacy with the reference dating over the past, the turning point index therefore provides an accurate signal on the current outlook.

Key Words: Business Cycle; Business Survey; Turning points; Markov Switching Indicator; Multifrequency Data

JEL Classification: E32, E37, E66

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Introduction

Knowing if an economy has already entered a slowing down phase or, on the contrary, if a recovery has started is a key issue for economists. Standard forecasting methods, based upon linear regressions, have generally good forecasting properties, but fail to detect turning points in time. Specific methods have therefore been developed to detect regime changes.

Since Burns and Mitchell (1946) underlined comovement among different series as a main feature of business cycles, the basic idea of recent models has been to represent regime structural changes through discrete Markov chains. Since Hamilton (1989), Markov chains combined with AR or VAR processes have been widely used for instance in Kim (1994), Lahiri and Wang (1994), Diebold and Rudebusch (1996), Chauvet (1998) or Krolzig (1997). Computational improvements have also allowed Gibbs sampling estimates instead of E-M algorithm as in Kim and Nelson (1998 and 1999) for example. All these approaches are based on Markov-state conditional distributions of the observed variable and a stochastic indicator which is driven from the hidden Markov states probabilities. Otherwise dynamic factor models are also largely developed to extract business cycle indicators. These methods aim to provide a quantitative message on the cycle.

Much fewer are the models signaling only a qualitative message upon the phase of the cycle as Neftçi (1982) who built an indicator based on the probability of the next potential recession. In recent years, detecting turning points through likelihood based surveillance has been in focus; an example may be found in Andersson *et al.* (2004). Alternatively using an original pattern developed in seismographic detection, Keilis-Borok *et al.* (2000) presented an alarm model detecting recession occurrences.

Because they range among the few early, informative and unrevised economic data, business surveys should be among the most fruitful areas to apply such methods. They provide helpful information to determine if a turning point has yet occurred or not. However, several different questions are asked to the firms in this survey and they may yield separately to divergent messages about the cycle, preventing the forecaster from having a clear picture of the economic situation. Hence, there is a need of a synthetic indicator focusing on the detection of the currently expanding or contracting phase of the economy.

In particular, Gregoir and Lenglart (2000) approach is used here as the groundwork upon which an indicator mixing different frequency data is elaborated. This method generates a simple two-regime indicator which can be estimated over the whole period where survey data are available. But it is necessary to adapt the original method to deal with mixed frequency data.

The work presented here describes a method to build turning point indexes coping with mixed frequency unbalanced data. Indeed, in a lot of cases, turning point indexes are grounded on business survey data. Sometimes the survey frequency have changed, and different frequencies may coexist at the same time. Moreover, some survey can begin later than other. Adapting existing methods for turning point index, we present in the first section a framework allowing for mixing monthly, bimonthly and quarterly data. Section 2 gives an application of this method to the French economy. Finally, out-of-sample simulations are led in section 3 on the retained turning point index to assess its accuracy in real-time, the index is also compared with sector indicators.

1 Building a Markov-switching Index from unbalanced dataset

A turning point index aims at determining for a given reference series the probability of being in a phase of acceleration or deceleration. It pinpoints the phase shifts over the last months. This section details the model mixing monthly and quarterly data and starts with a general presentation of the estimating process of the turning point index. Then, in subsections 1.2 an 1.3 the detailed method is exposed. This method is applied to French business survey data in the section 2.

1.1 Main features of the qualitative turning point index

The aim is to synthesize the information given by the business survey (Y_t) for a discrete unobserved variable Z_t revealing the phase the economic cycle is in. Its value should be close to $+1$ (resp. -1) if the sector output witnesses an accelerating (resp. a decelerating) phase. Z_t has the dynamics of a homogeneous first order Markov chain whose transition matrix parameters constitutes the first set of parameters to be estimated (probabilities $\boldsymbol{\eta} = P(Z_t|Z_{t-1})$).

In addition, the data are observations (missing data are allowed) of a set of Q quantitative series, $Y = (Y_t^q)_{(t,\,q)\in\{1,\ldots,T\}\times\{1,\ldots,Q\}}$, where t is the month and q the series number. These observed series are coded into qualitative variables (X_t^q) which can take only two possible values ($+1$ or -1) at each date t and for each series q, depending on the 3-month difference of the series compared to its long run trend. This is the coding step.

Having a set of coded monthly observations (or missing data for some series) X_t^q, it is necessary to determine at each date the probabilities of being in each hidden state of the economy, given information available up to this date: these are the filtered probabilities $P(Z_t|I_t)$, where $I_t = (X_1, \ldots, X_t)$. Smoothed probabilities $P(Z_t|I_T)$ are also defined given the whole information $I_t = (X_1, \ldots, X_T)$.

The probabilities of X_t^q depend on the hidden variables Z_t, and this dependence is conveyed by constant conditional probabilities $P(X_t^q|Z_t)$, constituting the second set of parameters to be estimated. For instance, if the hidden state is acceleration ($Z_t = +1$), the probability of obtaining an "above the trend" ($X_t^q = +1$) is larger than if the hidden state is deceleration ($Z_t = -1$).

The state-space model is estimated by an E-M algorithm (Expectation-Maximization). By iterative likelihood maximization, it enables to obtain the estimate of the whole set of parameters as well as filtered and smoothed hidden states probabilities. The procedure uses Kitagawa-Hamilton filters. Lastly, the filtered probabilities index is coded as follows: $Index_t = P(Z_t = +1|I_t) - P(Z_t = -1|I_t)$.

Before further descriptions, one shall note that the filtering step involved here is almost directly adapted from the original framework of Gregoir and Lenglart (2000). On the contrary, the coding step is very different from their approach (indeed, they use the innovation sign of an univariate AR estimates on each input quantitative time series Y_t^q). Our choice for the coding step is closer to Baron and Baron (2002) one's. Both steps are described more precisely in the following subsection. Above all the main innovation of this work comes from the ability to mix monthly, bimonthly and quarterly data whereas the previous methods only deal with monthly data.

Other specifications have been tried for our multifrequency turning point index. Particularly, an alternative quantitative model based on Hamilton's (1989) methodology has been used. This method can be called quantitative because there is no coding step and the observations Y_t^q have conditional normal density functions. Instead our choice is a qualitative index because the observations are coded into discrete values for each series. The results given by the two approaches are quite close with slightly better results for the qualitative turning point index retained here.

1.2 Coding step

To transform quantitative information (y_t^q) into (x_t^q) we choose to differentiate the quantitative series with a 3-month lag and the resulting coding depends on the position of the 3-month variation towards the median of all observed 3-month variations:

$$x_t^q = \begin{cases} +1 & \text{if } \left(y_t^q - y_{t-3}^q\right) > m^q = median\{y_r^q - y_{r-3}^q,\ r = 4,\ \ldots,\ T\} \\ -1 & \text{otherwise} \end{cases}$$

In the case of bimonthly series, the coding step is quite similar. Given the fact that the 3-month earlier value is unobserved, the proxy of the mean between 2-month and 4-month earlier values is involved :

$$x_t^q = \begin{cases} +1 & \text{if } \left(y_t^q - \frac{1}{2}(y_{t-2}^q + y_{t-4}^q)\right) > m^q = median\{y_r^q - \frac{1}{2}(y_{r-2}^q + y_{r-4}^q),\ r = 5,\ \ldots,\ T\} \\ -1 & \text{otherwise} \end{cases}$$

This coding method presents the advantage to fit well the quarterly-monthly frequency change. Actually, the coded information is recorded every three months before the frequency change as the survey is carried out at each quarter's first month, then, as the survey becomes monthly, the coded information switches to a monthly rate. Moreover this coding also allows easily to introduce bimonthly or semestrial series.

1.3 Filtering step

Having a set of observations (x_t^q), it is necessary to determine for each date the probabilities of being in each possible state of the economy, given the information available up to this date: these are the filtered probabilities $\boldsymbol{\xi}_{t/t} = P(Z_t|I_t)$, where $I_t = (X_1,\ \ldots,\ X_t)$ stands for all the information gathered until the date t. Although parameters are estimated over the whole period, filtered probabilities give a message on the actual economic outlook involving the information available until the current date t.

Following Gregoir and Lenglart (2000), the hidden Markov state Z_t is in fact built as the product of two independent Markov chains $\tilde{Z}_t(= +1 \text{ or } -1)$ and W_t $(= 0 \text{ or } 1)$. $\tilde{Z}_t$ represents the economic phase and W_t means a strong (value 1) or dubious (value 0) economic trend. There are actually 2×2 hidden Markov states $Z_t = (\tilde{Z}_t, W_t) \in \mathcal{Z} = \{(+1, 1); (-1, 1); (+1, 0); (-1, 0)\}$.

The probabilities of X_t^q depend on the hidden variables Z_t, and this dependence is conveyed by constant conditional probabilities $\pi_z^q(x) = P(X^q = x|Z = z)$, constituting the second set of parameters to be estimated.

In the case $W_t = 0$, the effect of the $\tilde{Z}_t$ state on X_t^q is cancelled in accordance with an assumption of conditional equiprobability, so for each q:

$$P(X_t^q|Z_t = (+1,\,0)) = P(X_t^q|Z_t = (-1,\,0)) = \frac{1}{2}$$

1.4 Estimation

The state-space model is estimated by an Expectation-Maximization (E-M) algorithm. Using iterative likelihood maximization, it enables to calculate both the estimate of the whole set of parameters as well as the filtered hidden states probabilities as presented in appendix 1. The procedure uses Hamilton filters which main recursive equations are recalled here:

$$\boldsymbol{\xi}_{t/t-1} = \boldsymbol{\eta}\boldsymbol{\xi}_{t-1/t-1} \quad (1)$$

$$\xi_{t/t}(z) = \frac{\xi_{t/t-1}(z)\prod_{q=1}^{Q}\pi_z^q(x_t^q)^{1(x_t^q \text{ is not missing})}}{\sum_{s\in\mathcal{Z}}\xi_{t/t-1}(s)\prod_{q=1}^{Q}\pi_s^q(x_t^q)^{1(x_t^q \text{ is not missing})}} \quad (2)$$

where:

- $X = (x_t^q)_{(t,\,q)\in\{1,\dots,T\}\times\{1,\dots,Q\}}$ is the matrix of the coding of 3-months difference observations,
- $\boldsymbol{\xi}_{t/t'}$ is the column vector of $\xi_{t/t'}(z) = P(Z_t = z|I_{t'})$ for $z \in \mathcal{Z}$ the probability that the hidden Markov variable takes the value z given information until date t',

and the $4 + 2 \times Q$ parameters of the model to be estimated:

- the transition matrix $\boldsymbol{\eta} = (\eta_{ij})_{(i,\,j)\in\mathcal{Z}^2}$ where $\eta_{ij} = P(Z_t = i|Z_{t-1} = j)$.
- the conditional probabilities $\boldsymbol{\pi}^q = (\pi_z^q(x))_{(x,\,z)\in\{-1,+1\}\times\mathcal{Z}}$ where $\pi_z^q(x) = P(X_t^q = x|Z_t = z)$.

Finally, the likelihood can be written and estimated through E-M algorithm in order to estimate the parameters

$$L(x_2,\dots,x_T) = \prod_{t=2}^{T} l(x_t|I_{t-1})$$

The filtered turning point index (TPI) is estimated as the probability difference between high and low regimes:

$$TPI_t = P(Z_t \in \{(+1,\ 1),\ (+1,\ 0)\}|I_t) - P(Z_t \in \{(-1,\ 1),\ (-1,\ 0)\}|I_t)$$

This index is filtered as the probabilities of the hidden states are conditional only to past information. This message is useful because at each date, it is close to the real time message given by the TPI. The filtered index therefore shows the accuracy of the indicator for detecting regime changes.

On the contrary if one wants to assess the dating power of the index the smoothed TPI can be more appropriate :

$$TPI_t^{smoothed} = P(Z_t \in \{(+1,\ 1),\ (+1,\ 0)\}|I_T) - P(Z_t \in \{(-1,\ 1),\ (-1,\ 0)\}|I_T)$$

The hidden state probabilities are conditional to all the information set - past and future - and the message given by this smoothed index is less volatile. See appendix 1 for the backward recursion to estimate the smoothed probabilities.

2 Application to the French business survey

In our case, applied to French business survey data, this index provides a reading of alternative accelerations and decelerations in the French output, and at the end of the time span it gives an evaluation of the economic situation in progress.

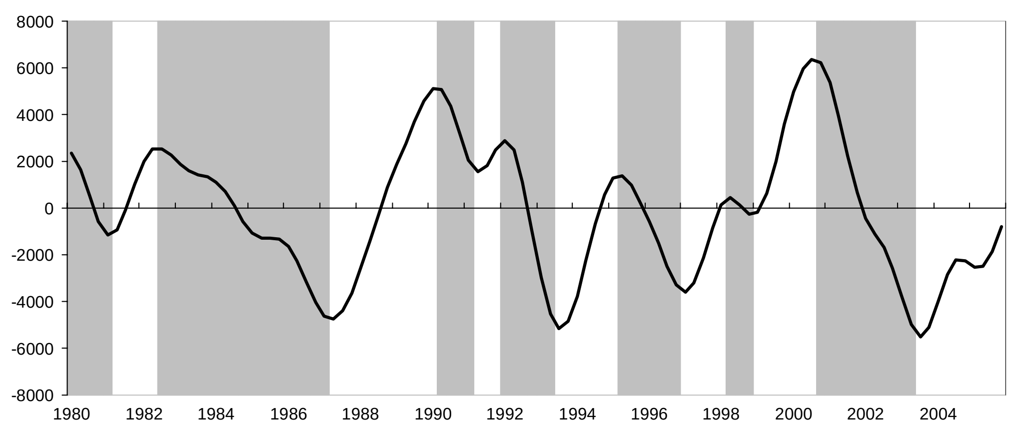
2.1 A reference dating based on the GDP deviation to trend

To assess the relevance of the turning point index, its confrontation with a reference dating on French economy is necessary. As no dating committee exists in France such as the National Bureau of Economic Research (NBER) for the United States, we have to build a reference dating as described thereafter.

First, it is essential to define the cycle we want to monitor. We retain the definition given by Burns and Mitchell's 1946 treatise, *Measuring Business Cycles.* We are not interested in detecting the business cycle (alternation of increase and decrease in the level of the output) because, there are very few GDP decreasing phases in France over the last decades.

But, like King and Plosser (1994), our implementation of the Burns and Mitchell procedures differs in the manner in which the reference cycle dates are selected. We determine the turning points (peaks and troughs) in the reference cycle using a measure of aggregate economy activity such as real GDP. Moreover, the turning points are determined by the deviation of the GDP series to its trend. The trend is estimated through a Cristiano-Fitzgerald (2003) filter which extract the business cycle frequencies components of the observed time series. Precisely, the filter is a band pass-filter which cuts off short-term fluctuations (whose cycle lenghts are shorter than 18 months in our case) and long-term trends (cycle lenghts longer than 120 months here). The filtered GDP series therefore represent the medium-term movements corresponding to the business cycle. Applying this filter to the French GDP (which is taken as a series whith drift and unit root in the filter) gives the deviation presented on Graph 1 between 1980 and 2005.

Graph 1 - *French reference dating and GDP deviation to trend*



Notes: the full line represents the GDP deviation to trend extracted through a Cristiano-Fitzgerald filter, the grey (white) zones show the deviation-to-trend decreasing (increasing) phase of the GDP growth rate cycle.

2.2 The business survey data

The model of the TPI is applied to 26 seasonally adjusted balances of opinions given by five business survey carried out by Insee: business survey in the retail trade (prefixed by RT), in the wholesale trade (prefixed by WT), in the industry (prefixed by I), in the building industry (prefixed by BI) and in the services (prefixed by S).

For each survey, between 4 and 6 main balances of opinion are involved to estimate the turning point index. These balances are relative to the past and expected activity and workforce, order books, bottlenecks and general business outlook for example. More precisely, table 1 reports the main features for each balances of opinion involved in the turning point index:

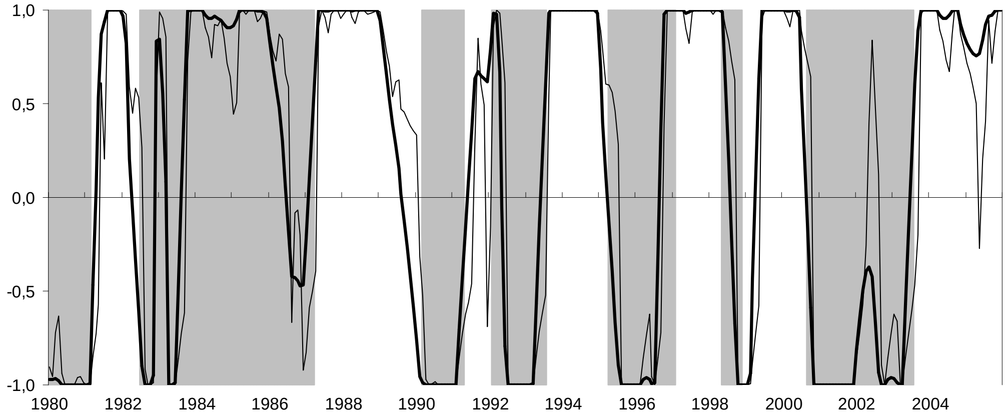
TABLE 1 - *Balances of opinion in input of the TPI*

Balances of opinion	**availability and periodicity**
in Industry (6 balances):	
past and expected changes in production (I_{PCP} and I_{ECP})	monthly from 1976m3
general business outlook in industry as a whole (I_{GBO})	monthly from 1976m3
total and foreign order books (I_{TOB} and I_{FOB})	monthly from 1976m3
inventories level (I_{INV})	monthly from 1976m3
in the Building Industry (5 balances):	
past and expected changes in activity (BI_{PCA} and BI_{ECA})	quarterly from 1976q1 to 1993q3 and monthly from 1993m9
total order books (BI_{TOB})	quarterly from 1976q1 to 1993q3 and monthly from 1993m9
past change in workforce (BI_{PCW})	quarterly from 1976q1 to 1993q3 and monthly from 1993m9
utilization of production capacity (BI_{UPC})	quarterly from 1976q1 to 1993q3 and monthly from 1993m9
in the Retail Trade (4 balances):	
total order books (RT_{TOB})	monthly from 1991m1
general business outlook in retail trade as a whole (RT_{GBO})	monthly from 1991m1
past sales (RT_{PS})	monthly from 1991m1
expected changes in workforce (RT_{ECW})	monthly from 2000m3
in the Wholesale Trade (5 balances):	
past sales (WT_{PS})	twice-monthly from 1979m7
foreign past sales (WT_{FPS})	twice-monthly from 1979m7
ordering intentions (WT_{OI})	twice-monthly from 1979m7
general business outlook in wholesale trade as a whole (WT_{GBO})	twice-monthly from 1979m7
delivery received from abroad (WT_{DRA})	twice-monthly from 1979m7
in the Services (6 balances):	
past and expected changes in turnover (S_{PCT} and S_{ECT})	quarterly from 1988q1 to 2000q2 and monthly from 2000m6
general business outlook in Services as a whole (S_{GBO})	monthly from 2000m6
expected demand (S_D)	quarterly from 1988q1
past and expected changes in profit and loss account (S_{PCPLA} and S_{ECPLA})	quarterly from 1988q1 to 2000q2 and monthly from 2000m6

The data set constituted by theses 26 series allows to estimate our turning point index (see Appendix 2 for the parameters estimates). The filtered and smoothed indexes are presented on graph 2 in comparison to the reference dating based on the deviation to trend of the French GDP (growth cycle).

As the smoothed probabilities are calculated using all the information set, they naturally shows less false signals than the filtered probabilities. The adequation to the dating of the French GDP growth rate cycle is very good although the input information of the TPI is completely different from the one used in the GDP dating.

Graph 2 - *Turning point index and French GDP accelerating and decelerating phases*



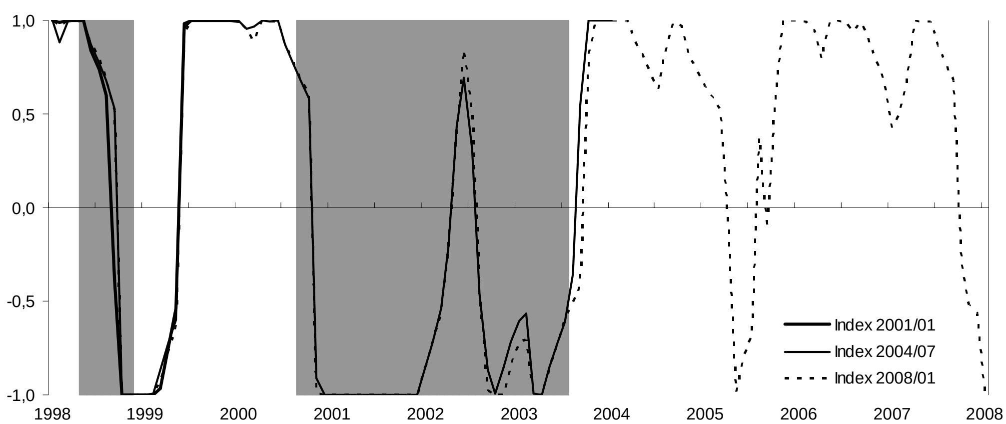
Note: the thin line represents the estimated filtered TPI, the thick line represents the corresponding smoothed TPI, the grey (white) zones show the decelerating (accelerating) phase of the GDP growth cycle.

3 The French turning point index gives reliable outlook

3.1 A good indicator in real-time

Whereas GDP growth can be sensibly revised from one quarter to the other, the TPI can help detecting the turning point with very low revisions. Because its estimation is based on almost unrevised survey data and because the parameters estimates are rather precise (see appendix 2 for standard errors), the TPI nearly does not change from one month to the other. Indeed, we performed an out-of-sample simulation of our TPI from 2001 to 2008, re-estimating the parameters and the TPI adding each monthly data after the other. From one month to the following month the filtered TPI changes its message on the regime for at the most a few months over the entire period of estimation (1977-2008). Most of the time there is no change in the signal and the mean revision only concerns the regime for one month over the entire period (see Graph 3). For the smoothed indicator, the picture is almost the same, the more frequent revisions around the last months of available data are compensated by less frequent revisions over the past.

Graph 3 - *Real time TPIs: very little revisions*

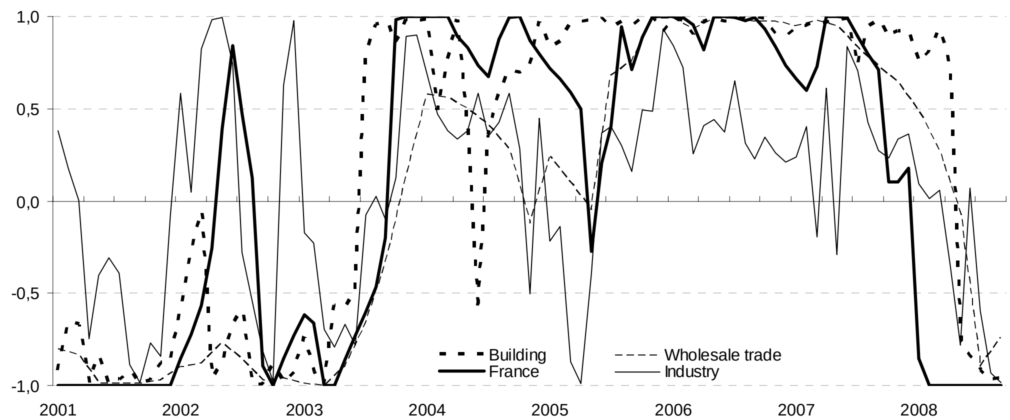


On the long run, when we compare the message given by the TPI estimated with data up to 2001 and the most recent one, there are only two months differing in the messages on the regimes.

3.2 A better message on the cycle than sectoral information

A great improvement of our index is that it is much clearer and accurate than similar sectors TPIs. Indeed, Insee (the French National Institute for Statistics and Economics Studies) already publish TPIs on sector surveys. The industry TPI has been introduced by Gregoir and Lenglart (2000) work. The recently TPI for building sector and wholesale trade are based upon the same methodology and give also relevant outlooks of these sector cycles. Even if activity changes can be different from one sector to another due to structural differences and idiosyncratic shocks, all sectors are driven by a common cycle imposing general economic movements (following for instance Burns and Mitchell (1946) definition). In that sense, Graph 4 shows that our TPI on the whole economy is clearer and presents less back and forth movements than each of the sector TPIs.

Graph 4 - *Sectors and French economy TPIs*



Conclusion

In order to detect and forecast the regimes of an economy, it is useful to assess its inflexions in real-time. A turning point index (TPI) meets entirely this requirement: it gives a qualitative signal about the growth, signaling accelerating and decelerating phases.

Among a lot of existing different hidden Markov models that can be used to build a switching regime indicator, we propose a new method inspired by Gregoir and Lenglart (2000) but adapted in order to handle unbalanced dataset. This model has been applied to the French economy using business survey data for different sectors. As such frequency changes are not unusual, our method could be applied with profit to many other cases.

After having elaborated a reference dating for the French economy through Cristiano and Fitzgerald filter applied on the GDP series, the signal given by the TPI has been systematically compared to this reference over the past. Most of the time the index pinpoints the right economic regime as well as the right turning points. This TPI has then been simulated in out-of-sample estimation to assess its real-time performance. When the index for the whole French economy and the different sectoral existing indexes are compared, the message delivered by the general index is much clearer. It is less volatile and it delivers less false signals. Such a method allowing quarterly, bimonthly and monthly data could be applied with great success to others countries business survey or hard data as well.

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Contents

Appendix

Appendix 1: the filter method

The likelihood expression
Thanks to an iterative formula, we can write the likelihood as follows:

$$\begin{aligned} L &= P(x_1, x_2, \dots, x_T) \\ &= \prod_{t=1}^{T} P(x_t | I_{t-1}) \end{aligned}$$

where t is the month. Knowing the expression of the probabilities $P(x_t|I_{t-1})$ given the parameters -$\boldsymbol{\eta}$ and $\boldsymbol{\pi}$- and the observations (x_t) will allow to maximise the likelihood by iterations.

The recursive filter
The recursive equations presented before can be estimated given the parameters $\boldsymbol{\eta}$ and $\boldsymbol{\pi}$. The procedure uses Hamilton filters whose main recursive equations are in the monthly case:

$$\boldsymbol{\xi}_{t/t-1} = \boldsymbol{\eta}\boldsymbol{\xi}_{t-1/t-1} \quad (1)$$

$$\xi_{t/t}(z) = \frac{\xi_{t/t-1}(z) \prod_{q=1}^{Q} \pi_z^q (x_t^q)^{1(x_t^q \text{ is not missing})}}{\sum_{s \in \mathcal{Z}} \xi_{t/t-1}(s) \prod_{q=1}^{Q} \pi_s^q (x_t^q)^{1(x_t^q \text{ is not missing})}} \quad (2)$$

where notations are the same as in section 1.

The first equation is the forecasting equation of the state-space model while the second is the updating equation. From Bayes formula, they are established as follows:

$$\begin{aligned} \xi_{t/t}(z) &= P(Z_t = z | I_t) \\ &= P(Z_t = z | X_t = x_t, I_{t-1}) \text{ where some terms } x_t \text{ containing the observations } x_t^p \text{ can be missing} \\ &= \frac{P(Z_t = z, X_t = x_t | I_{t-1})}{P(X_t = x_t | I_{t-1})} \\ &= \frac{P(X_t = x_t | Z_t = z, I_{t-1}) P(Z_t = z | I_{t-1})}{P(X_t = x_t | I_{t-1})} \\ &= \frac{\xi_{t/t-1}(z) P(X_t = x_t | Z_t = z, I_{t-1})}{P(X_t = x_t | I_{t-1})} \end{aligned}$$

As observations at date t depend on I_{t-1} only through Z_t and that observed series are independent between each other knowing Z_t, the former formula becomes:

$$\begin{aligned} \xi_{t/t}(z) &= \frac{\xi_{t/t-1}(z) P(X_t = x_t | Z_t = z)}{P(X_t = x_t | I_{t-1})} \text{ where some terms } x_t \text{ containing the observations } x_t^p \text{ can be missing} \\ &= \frac{\xi_{t/t-1}(z) \prod_{q=1}^{Q} P(X_t^q = x_t^q | Z_t = z)}{P(X_t = x_t | I_{t-1})} \\ &= \frac{\xi_{t/t-1}(z) \prod_{q=1}^{Q} \pi_{Z_t=z}^q (x_t^q)^{1(x_t^q \text{ is not missing})}}{P(X_t = x_t | I_{t-1})} \end{aligned}$$

Indeed, when the observation x_t^q is missing, $P(X_t^q = x_t^q | Z_t = z) = 1$. And adding on the

$card(\mathcal{Z})$ possible hidden state, the denominator can be expressed as well:

$$
\begin{aligned}
P(X_t = x_t | I_{t-1}) &= \sum_{s \in \mathcal{Z}} P(X_t = x_t, Z_t = s | I_{t-1}) \text{ where some terms } x_t \text{ containing the observations } x_t^p \text{ can be missing} \\
&= \sum_{s \in \mathcal{Z}} P(Z_t = s | I_{t-1}) P(X_t = x_t | Z_t = s, I_{t-1}) \\
&= \sum_{s \in \mathcal{Z}} P(Z_t = s | I_{t-1}) P(X_t = x_t | Z_t = s) \\
&= \sum_{s \in \mathcal{Z}} P(Z_t = s | I_{t-1}) \prod_{q=1}^{Q} P(X_t^q = x_t^q | Z_t = s) \\
&= \sum_{s \in \mathcal{Z}} \xi_{t/t-1}(s) \prod_{q=1}^{Q} \pi_s^q (x_t^q)^{1(x_t^q \text{ is not missing})}
\end{aligned}
$$

Finally, at each iteration of an Expectation-Maximization algorithm, the calculated log-likelihood function can be maximized for the $\boldsymbol{\eta}$ and $\boldsymbol{\pi}$ parameters. After the convergence of the algorithm, the filtered probabilities of the hidden states are recovered as well as the $\boldsymbol{\eta}$ and $\boldsymbol{\pi}$ parameters.

The smoothed probabilities

The smoothed probabilities of the hidden state are calculated thanks to this backward recursive equation:

$$P(z_t | I_T) = \sum_{z_{t+1} \in \mathcal{Z}} P(Z_{t+1} = z_{t+1} | Z_t = z_t) \frac{P(z_t | I_t)}{P(z_{t+1} | I_t)} P(z_{t+1} | I_T)$$

This comes from :

$$P(z_t | I_T) = \sum_{z_{t+1} \in \mathcal{Z}} P(Z_{t+1} = z_{t+1}, Z_t = z_t | I_T) = \sum_{z_{t+1} \in \mathcal{Z}} P(Z_t = z_t | Z_{t+1} = z_{t+1}, I_T) P(Z_{t+1} = z_{t+1} | I_T)$$

As, conditionally on Z_{t+1}, the observed $x_{t+1}, \ldots,\ x_T$ are independant of I_t :

$$
\begin{aligned}
P(Z_t = z_t | Z_{t+1} = z_{t+1}, I_T) &= P(Z_t = z_t | Z_{t+1} = z_{t+1}, x_{t+1}, \ldots,\ x_T, I_t) \\
&= \frac{P(x_{t+1}, \ldots,\ x_T | Z_{t+1} = z_{t+1}, Z_t = z_t, I_t)}{P(x_{t+1}, \ldots,\ x_T | Z_{t+1} = z_{t+1}, I_t)} P(Z_t = z_t | Z_{t+1} = z_{t+1}, I_t) \\
&= P(Z_t = z_t | Z_{t+1} = z_{t+1}, I_t) \\
&= \frac{P(Z_{t+1} = z_{t+1} | Z_t = z_t, I_t) P(Z_t = z_t | I_t)}{P(Z_{t+1} = z_{t+1} | I_t)}
\end{aligned}
$$

Therefore, the expression presented above is obtained.

Appendix 2: parameters estimates and standard errors

The following tables show the estimates (and their standard errors) of the parameters (transition matrix and conditional matrix) for the TPI model. The 26 series available in the business survey are taken as observed series. The matrix $P(X \mid Z)$ shows the probabilities of the signal X_t (high) of an observed series given the hidden state $Z_t = (\tilde{Z}_t, W_t)$ (4 possible states: high certain regime, high uncertain regime, low certain regime and low uncertain regime). The matrix $P(\tilde{Z}_t \mid \tilde{Z}_{t-1})$ gives the transition probabilities for the hidden state of the economy and the matrix $P(W_t \mid W_{t-1})$ is the transition matrix for the certainty Markov state(standard errors are in parentheses).

$$\boldsymbol{\eta}_{\tilde{Z}} = P(\tilde{Z}_t \mid \tilde{Z}_{t-1}) = \begin{pmatrix} 0.93 & 0.05 \\ (0.011) & (0.006) \\ 0.07 & 0.95 \\ (0.011) & (0.006) \end{pmatrix}$$

$$\boldsymbol{\eta}_{W} = P(W_t \mid W_{t-1}) = \begin{pmatrix} 0.84 & 0.25 \\ (0.026) & (0.036) \\ 0.16 & 0.75 \\ (0.026) & (0.036) \end{pmatrix}$$

$$\boldsymbol{\pi} = P(X = +1 \mid Z) = P\left(\begin{pmatrix} I_{PCP} \\ I_{ECP} \\ I_{GBO} \\ I_{TOB} \\ I_{FOB} \\ I_{INV} \\ BI_{PCA} \\ BI_{ECA} \\ BI_{TOB} \\ BI_{PCW} \\ BI_{UPC} \\ RT_{TOB} \\ RT_{GBO} \\ RT_{PS} \\ RT_{ECW} \\ WT_{PS} \\ WT_{FPS} \\ WT_{OI} \\ WT_{GBO} \\ WT_{DRA} \\ S_{PCT} \\ S_{ECT} \\ S_{GBO} \\ S_{D} \\ S_{PCPLA} \\ S_{ECPLA} \end{pmatrix} = \begin{pmatrix} +1 \\ +1 \end{pmatrix} | Z_t \right) = \begin{pmatrix} 0.79 & 0.5 & 0.33 & 0.5 \\ (0.03) & & (0.03) & \\ 0.87 & 0.5 & 0.36 & 0.5 \\ (0.02) & & (0.05) & \\ 1.00 & 0.5 & 0.00 & 0.5 \\ (0.00) & & (0.00) & \\ 0.88 & 0.5 & 0.23 & 0.5 \\ (0.03) & & (0.02) & \\ 0.79 & 0.5 & 0.37 & 0.5 \\ (0.04) & & (0.05) & \\ 0.88 & 0.5 & 0.21 & 0.5 \\ (0.04) & & (0.03) & \\ 1.00 & 0.5 & 0.11 & 0.5 \\ (0.00) & & (0.01) & \\ 0.92 & 0.5 & 0.13 & 0.5 \\ (0.03) & & (0.01) & \\ 0.84 & 0.5 & 0.19 & 0.5 \\ (0.02) & & (0.03) & \\ 0.12 & 0.5 & 0.78 & 0.5 \\ (0.03) & & (0.04) & \\ 0.89 & 0.5 & 0.15 & 0.5 \\ (0.01) & & (0.03) & \\ 0.96 & 0.5 & 0.07 & 0.5 \\ (0.03) & & (0.02) & \\ 0.91 & 0.5 & 0.16 & 0.5 \\ (0.01) & & (0.03) & \\ 0.85 & 0.5 & 0.18 & 0.5 \\ (0.01) & & (0.03) & \\ 0.85 & 0.5 & 0.25 & 0.5 \\ (0.02) & & (0.03) & \\ 0.83 & 0.5 & 0.21 & 0.5 \\ (0.02) & & (0.02) & \\ 0.74 & 0.5 & 0.27 & 0.5 \\ (0.02) & & (0.03) & \\ 0.76 & 0.5 & 0.33 & 0.5 \\ (0.01) & & (0.03) & \\ 0.95 & 0.5 & 0.13 & 0.5 \\ (0.03) & & (0.02) & \\ 0.68 & 0.5 & 0.38 & 0.5 \\ (0.02) & & (0.03) & \\ 0.61 & 0.5 & 0.34 & 0.5 \\ (0.02) & & (0.04) & \\ 0.79 & 0.5 & 0.17 & 0.5 \\ (0.04) & & (0.02) & \\ 0.75 & 0.5 & 0.32 & 0.5 \\ (0.01) & & (0.04) & \\ 0.89 & 0.5 & 0.18 & 0.5 \\ (0.03) & & (0.03) & \\ 0.96 & 0.5 & 0.07 & 0.5 \\ (0.01) & & (0.01) & \\ 0.84 & 0.5 & 0.21 & 0.5 \\ (0.01) & & (0.03) & \end{pmatrix}$$