Forecasting corporate investment An indicator based on revisions in the French investment survey

Nicolas FERRARI[1]

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[1]Treasory and Economic Policy Directorate: nicolas.ferrari@dgtpe.fr. The author was in the Economic Forecast Department at Insee (France) at the time of this work. This document does not reflect the position of Insee and is issued on the sole responsibility of its author. The author's thanks go to Karine Berger, Olivier Biau, Matthieu Cornec, Michel Devilliers, Hélène Erkel-Rousse, Philippe Scherrer and Fabien Toutlemonde for their advice and attentive re-reading.

Abstract

The quarterly survey of investment in industry, conducted by the National Institute for Statistics and Economic Studies (Insee) is a prime source of information concerning short-term evolutions in productive investment, making it possible to estimate these evolutions at an early stage and with considerable precision.

However, the annual nature of the questions posed makes it is difficult to use the results for forecasting on a quarterly basis. This article proposes a quarterly indicator based on revisions in industrial firms' expectations regarding their investment. The proposed indicator measures the adjustments to investment figures made during the year in response to changes of a short-term nature. It turns out to be closely correlated with quarterly evolutions in firms' investment as measured in the national accounts. Moreover, it is available roughly three months before the publication of the initial quarterly national accounts figures.

As the distributions examined fail to verify the classic normality hypothesis (thick tails and heavy concentrations at zero) it is necessary to apply an estimation method that is robust to extreme revisions. Taking into account also the presence of heteroscedasticity, the method known as "Quasi-generalised M-estimators" was applied.

Keywords: productive investment, forecasts, business surveys, outliers, robust regressions, M-estimators, Quasi-generalised M-estimators, adaptative procedures.

Résumé

L'enquête trimestrielle sur les investissements dans l'industrie de l'Institut National de la Statistique et des Études Économiques (Insee) est une source d'information majeure concernant les évolutions conjoncturelles de l'investissement productif. Elle permet d'estimer de manière précoce et précise les évolutions de l'investissement industriel.

Toutefois, la nature annuelle des questions posées rend délicate son utilisation pour des prévisions selon un rythme trimestriel. Cet article propose un indicateur trimestriel des révisions d'anticipations d'investissement des industriels. L'indicateur proposé mesure les adaptations au cours de l'année des investissements en fonction des évolutions conjoncturelles. Il est très bien corrélé aux évolutions trimestrielles de l'investissement des entreprises mesurées par la comptabilité nationale. De plus, il est disponible environ trois mois avant la publication des premiers résultats des comptes trimestriels.

Les distributions étudiées ne vérifiant pas l'hypothèse classique de normalité (queues épaisses et fortes concentrations en zéro), il est nécessaire de mettre en œuvre une méthode d'estimation robuste aux révisions extrêmes. En prenant également en compte la présence d'hétéroscédasticité, il a été choisi d'utiliser la méthode dite des "M-estimateurs quasi-généralisés".

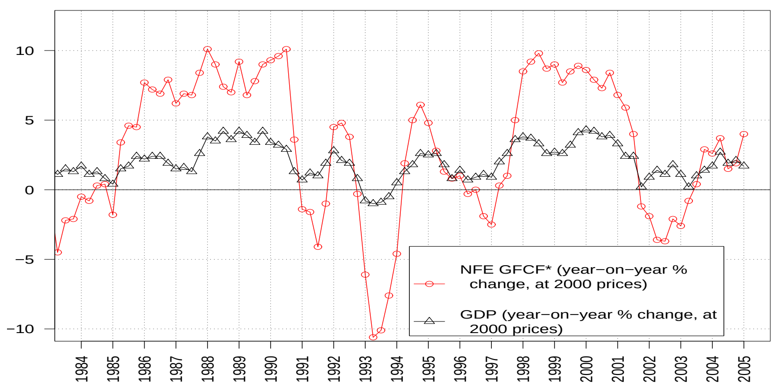
Mots-clés: Investissement productif, prévisions conjoncturelles, enquêtes de conjoncture, valeurs extrêmes, procédure adaptative, régression par les M-estimateurs, méthode des M-estimateurs quasi-generalisés.

Classification JEL: C14, C16, C42, C53, E22

1 Introduction

Corporate investment constitutes a very important variable in short-term economic analysis. While the size of the aggregate is fairly small in relation to GDP, between 10% and 12% depending on the particular year[2], it overreacts to variations in the level of activity (cf. figure 1). As a result, it makes a particular contribution to variations in GDP. Over a long period (from 1980 to 2003) NFE GFCF[3] contributed 32% to the year-on-year variations in GDP[4]. In addition to exerting a short-term influence on demand, investment makes it possible for firms to develop their productive resources. Current investment efforts have an impact on the future, with consequences in the medium term for corporate supply.

Figure 1: Comparative evolutions in GDP and corporate investment



Source: Insee, quarterly national accounts (base 2000).
* NFE GFCF: Gross Fixed Capital Formation by Non-Financial Entreprises.

Short-term indicators concerning investment are sparse. Because of the high degree of heterogeneity of individual behaviour, forecasting investment turns out to be a delicate matter. In fact, the survey of industrial investment (referred to here as the *investment survey*), carried out quarterly by Insee, is one of the rare sources of a short-term nature relating to capital spending by firms. By using a particular method known as that of the "Large Investors", the survey makes it possible to forecast industrial investment reliably and at an early stage. These estimates turn out to be very close to later evaluations carried out using exhaustive statistical sources[5]. As industrial investment turns out also to have a strong

[2] Since the beginning of the 1990s.

[3] Gross Fixed Capital Formation by Non-Financial Entreprises.

[4] The quarterly national accounts figures used in this article are the preliminary results for Q1 2005 on the 2000 base.

[5] Especially the EAE industry survey ("Enquête Annuelle d'Entreprise" in industry, the French annual business survey in this sector).

correlation with investment in other sectors of activity, the *investment survey* finally provides interesting information concerning productive investment for all sectors of activity taken together.

However, the annual nature of the questions makes it relatively difficult to use the survey for quarterly forecasting. Revisions between two successive surveys provide relevant information, but it is not easy to use the aggregate growth rates for the construction of a quarterly indicator, because of their annual nature.

On the other hand, examination of the revisions by individual firms between successive surveys of the amounts of investment they expect to make provides very interesting information. To be more precise, it is possible to construct a quarterly indicator of the revisions in own-firm investment expectations. This series turns out to be closely correlated with the quarterly evolutions in NFE GFCF, a figure that becomes available at a later date.

This indicator captures the within-year changes in firms' investment plans. Prior to the start of the year, firms forecast the level and growth rate for their investment in the light of their internal development plans. At this stage, short-term evolutions in the coming year are still highly uncertain. As the year goes on, the rate of investment is refined and adjusted to unforeseen short-term economic evolutions. This means that evolutions in expectations are then linked to the momentum of investment during the rest of the current year. The revisions indicator turns out as a result to be closely correlated with quarterly variations in the value of NFE GFCF, making it a very good indicator for forecasting variations in this variable.

The revisions in own-firm investment expectations do not show a random Gaussian distribution. The tails of the distributions, being particularly thick, require estimation methods that are robust to outliers. The revisions also show a high concentration at zero. In order to provide a response to the statistical difficulties, the estimators of the centre of distribution have been chosen from within the M-estimator class.

The revisions also show high heteroscedasticity, with the variance in the revisions expressed as a ratio of turnover declining significantly with size of firm. In order to improve the efficiency of the estimators, a two-stage approach was adopted. This is similar to the QGLS (Quasi-generalised Least Squares) method and is known as the Quasi-generalised M-estimators method.

In Part 2 we briefly describe the *investment survey*, together with the method of calculation used for the results published for the purposes of short-term analysis of investment. These results provide relevant annual information but are difficult to use for forecasting evolutions in investment on a quarterly basis. On the other hand, as the expectations are revised every quarter, these revisions can themselves be regarded as providing relevant quarter-to-quarter information. In

Part 3 the construction of the revisions indicator is described, with particular attention paid to the choice of the score function of the M-Estimators as well as to the correction of the heteroscedasticity. Part 4 comments on the series constructed in this way and proposes an example of how it might be used for forecasting corporate investment with the help of a VAR model. Part 5 contains conclusions.

2 Presentation of the *investment survey* and utilisation of its results

2.1 Presentation of the *investment survey* and French particularities

The *investment survey*, carried out in France by Insee[6], forms part of the harmonised European system of business and consumer surveys. In addition, however, this survey has a number of particularities. Four elements are in fact specific to the French survey and go beyond the norms set by the European Commission and as practised by other EU institutes:

1. The Commission requires biannual frequency for the *investment survey*, with a spring survey in March or April and an autumn survey in October or November. In France the *investment survey* is quarterly, being carried out in the months of January, April, July[7] and October.

2. Additional questions are asked in France compared with the European norm. These questions make it possible to evaluate half-yearly evolutions in investment, as well as annual evolutions in productive capacity and in the decommissioning of equipment. Since July 2003, new questions have been added concerning the nature of the investment and evolutions in IT spending, research and development and foreign direct investment.

3. As there is no harmonised European definition of investment, most economic institutes have chosen to leave it to firms to define for themselves what they mean by the term. Insee, for its part, has opted to give a precise definition of investment, which is clearly set out in the survey questionnaire. This definition is shown in Box 1 below. It is the result of a trade-off between two objectives: on the one hand, it must be capable of being easily adjusted to the headings used in firms' own accounts; second, it must be similar to that of Gross Fixed Capital Formation (GFCF), which is the aggregate in the national accounts that the short-term economic analyst is trying to forecast.

[6]Institut National de la Statistique et des Études Économiques.
[7]Only since July 2003.

4. Lastly, since there is no harmonised European method of aggregation, the individual heterogeneity of investment amounts, and in particular the presence of annual amounts that are nil or very small, require a method that is robust to extremely wide individual variations in own-firm investment. For this purpose, Insee has worked out a robust estimation method that meets these requirements. This method is described in detail in part 2.2.

Box 1: The definition of investment in the *investment survey*

The notion of investment in this survey covers...

1. **acquisitions of tangible assets excluding external capital contributions**
2. **the value of goods (movable or immovable) that have been the subject of a leasing contract**

   (The amount recorded is the value of the goods at the time of signature of the contract and not the sum of annual payments)
3. software (bought or developed in-house)

... and excludes lands and dwellings.

Investment is recorded after deduction of deductible taxes and before depreciation.

The results of the *investment survey* are published a fortnight after the end of the month in question. The survey covers a sample of roughly 4 000 firms representative of the whole of industry.

Two types of questions are included in each occurrence of the survey and make it possible to evaluate the outlook for the evolution in investment as seen by the firms questioned.

- first, firms report the annual amounts of investment carried out or planned for three consecutive calendar years;
- second, they give their opinion on past and planned evolutions in their half-yearly investment expenditure. These opinions are expressed in the form of a choice between “rising”, “stable” and “falling”. The results are aggregated and published in the form of balances of opinion (weighted differences between the number of those replying “rising” and those replying “falling”).

This article deals only with replies to the annual quantitative questions. More information on the *investment survey* can be found in the methodological fact sheet available on the Insee web site and in the book in the Insee-Méthode series relating to this survey (Rosenwald (1994)), currently being updated.

2.2 Calculating growth rates: the so-called "Large Investors" (LI) method

Questions relating to annual amounts of investment are of crucial importance in the survey. They make it possible to estimate expectations and outturns concerning annual changes in industrial investment. These results are the most informative and the most publicised of the survey. The quantitative nature of these questions and the heterogeneity of individual firms' investment behaviour nevertheless require the application of a special method.

Two methods have in fact been developed by Insee and are used for the French survey. The first, known as the "Large Investors" (LI) method, is the principal one, used in processing the results of the survey. This method is described in detail below. A second, more theoretical method has also been developed. This is based on the so-called "GM-estimators" (GM) method. It has been described in P. Ravalet (1996) but will not be set out in detail in the present document.

The principle underlying the "Large Investors" (LI) method

In order to estimate aggregate evolutions on the basis of individual sample survey data, the most natural estimator to use is the so-called "ratio" method, consisting of estimating the growth rate for the entire population from the growth rate for just the sample, restricted to firms that have reported their investment amounts for the two years between which the evolution is calculated.

However, the ratio estimator lacks robustness. A firm with atypical behaviour can have a very strong influence on the aggregate result. It is therefore essential to apply a method that is robust to atypical evolutions. The classic robust estimation methods are relatively complex and hence not easy to grasp. Because of this, a method that is more pragmatic and at the same time similar to the ratio method has been developed. This method, known as that of the "Large Investors" (LI), consists of underweighting certain firms that are judged to be non-representative of the entirety of firms in the sector being considered. This method has the robustness of complex estimators such as the GM-estimators, while at the same time permitting easy comprehension of the results.

Two types of firms are distinguished in this method:

- the firms known as "Large Investors" (LI) are regarded as non-representative of the total population of firms. They are therefore counted only in their own case within a given stratum.
- the other firms are described as "extrapolatable" and the growth rate of their investment is extrapolated to the whole of the stratum to which they belong, with the exclusion of firms regarded as "Large Investors". This extrapolation is carried out stratum by stratum.

The growth rate for an individual stratum is then estimated as the weighted sum of the growth rate of the "Large Investors" in the stratum in question and the growth rate of the investment of the extrapolatable firms in the same stratum. The growth rate for the "Large Investors" is weighted by the importance of these firms, while the growth rate of the extrapolatable firms is weighted by that of all other firms in the stratum (extrapolatable and non-sample). This means that the growth rate for the stratum between years 0 and 1 is estimated as follows:

$$t_{\text{stratum}} = p^{\text{LI}}_{\text{stratum}}\, t^{\text{LI}}_{\text{stratum}} + \left(1 - p^{\text{LI}}_{\text{stratum}}\right)\, t^{\text{extra}}_{\text{stratum}}$$

with:

- $t^{\text{LI}}_{\text{stratum}}$: growth rate between years 0 and 1 for the "Large Investors" in the stratum under consideration.
- $t^{\text{extra}}_{\text{stratum}}$: growth rate between years 0 and 1 for the extrapolatable firms in the stratum under consideration.
- $p^{\text{LI}}_{\text{stratum}}$: weighting attributed to the "Large Investors" in the stratum under consideration.
- $1 - p^{\text{LI}}_{\text{stratum}}$: weighting attributed to the extrapolatable firms in the stratum under consideration.

The two growth rates $t^{\text{LI}}_{\text{stratum}}$ and $t^{\text{extra}}_{\text{stratum}}$ are estimated using the ratio method. $i^{0,\text{LI}}_{\text{stratum}}$ and $i^{1,\text{LI}}_{\text{stratum}}$ ($i^{0,\text{extra}}_{\text{stratum}}$ and $i^{1,\text{extra}}_{\text{stratum}}$) denote the sum of the amounts of investment by firms selected as "Large Investors" and by extrapolatable firms, respectively, for years 0 and 1. The growth rates $t^{\text{LI}}_{\text{stratum}}$ and $t^{\text{extra}}_{\text{stratum}}$ are then given by the following equations.

$$\begin{cases} t^{\text{LI}}_{\text{stratum}} = \frac{i^{1,\text{LI}}_{\text{stratum}}}{i^{0,\text{LI}}_{\text{stratum}}} - 1 \\ \\ t^{\text{extra}}_{\text{stratum}} = \frac{i^{1,\text{extra}}_{\text{stratum}}}{i^{0,\text{extra}}_{\text{stratum}}} - 1 \end{cases}$$

The weight attributed to firms selected as "Large Investors" is given by equation (1) where I^{0}_{stratum} designates the total amount of investment of the population of firms in the stratum for the base year (year 0) and $i^{0,LI}_{\text{stratum}}$ that of the "Large Investors" in the stratum in the same year.

$$p^{\text{LI}}_{\text{stratum}} = \frac{i^{0,LI}_{\text{stratum}}}{I^{0}_{\text{stratum}}} \tag{1}$$

Starting with the survey for October $N-1$ and ending with that of July N, firms report their forecasts and outturns for investment expenditure in years $N-2$, $N-1$ and N. This means that at the time of each survey it is possible to calculate two year-to-year growth rates using the method described above. For the first growth rate, between years $N-2$ and $N-1$, the base year 0 is $N-2$.

For the second growth rate, between years $N-1$ and N, the base year 0 is $N-1$.

The EAE industry survey[8] provides as early as October $N-1$ figures for investment by the actual population of firms for the year $N-3$. Prior to this, the *investment survey* for July $N-1$ will have provided the definitive estimate for the growth rate in investment between years $N-3$ and $N-2$. By cross-comparison between the two sources of information it is then possible to estimate the investment amounts for the different strata of the actual population for year $N-2$. These amounts correspond to the terms I^0_{stratum} in the calculation of the first year-to-year growth rate (between years $N-2$ and $N-1$).

The investment amounts already estimated for year $N-2$ are then compared with the results of the latest survey for the first growth rate estimated in this way (between years $N-2$ and $N-1$). These two sources of information then make it possible to estimate the investment amounts for the actual population in year $N-1$. As previously, these amounts correspond to the terms I^0_{stratum} in the calculation of the second year-to-year growth rate (between years $N-1$ and N).

The selection of the "Large Investors"

A firm can be selected as a "Large Investor" if it is atypical or if it exerts a strong influence:

The atypical firm: A firm is considered to be atypical when the variations in its investment between two consecutive years show a very large amplitude. To be more precise, when a firm increases its investment between two consecutive years by a factor of 10 or reduces its investment between two consecutive years by a factor of 10, it is considered as atypical and is classified as a "Large Investor" for the calculation of the corresponding growth rate.

The influential firm: A firm is considered to be influential when it contributes significantly to the results published at the NES 16 level[9]. Influential firms are then classified manually as "Large Investors" for the calculation of both growth rates.

The choice of firms to be classified as "Large Investors" for being influential is guided by what are known as "indicators of change". These indicators specify the extent of the variation in the growth rate at the NES 16 level depending on which a firm is considered to be a "Large Investor" or not. When the change indicator shows an amplitude greater than a certain

[8] "Enquête Annuelle d'Entreprise" in industry, the French annual business survey in this sector.

[9] Nomenclature Économique de Synthèse (the French agregated economic classification for the activities), at the 16-heading level.

minimum, the firm is considered as influential and is then classified as a "Large Investor".

Depending on the individual sector, the minimum required may be more or less exacting. In manufacturing industry, this minimum is always less than one percentage point, meaning that to be extrapolatable a firm cannot contribute more than one percentage point of growth to one of the two growth rates at the NES 16 level.

The "Large Investors" method provides a precise estimate of evolutions concerning the total population of industrial firms

For any given year, the annual growth rate of investment estimated on the basis of the initial surveys is not comparable to any other statistical source, as the replies reflect firms' investment expectations and not the investment actually carried out. On the other hand, the latest annual growth rates of investment, estimated on the basis of the most recent surveys and going beyond the year under consideration, measure firms' investment outturns. These actual evolutions can be compared *a posteriori* with the exhaustive sources available at a later date. The statistical source that comes closest in terms of coverage is the EAE industry survey[10]. By way of illustration, the definitive results of the *investment survey* can be compared with the data provided by the EAE industry survey (cf. graph2). The correlation of the results in the case of manufacturing industry amounts to 97% for the period from 1990 to 2003.

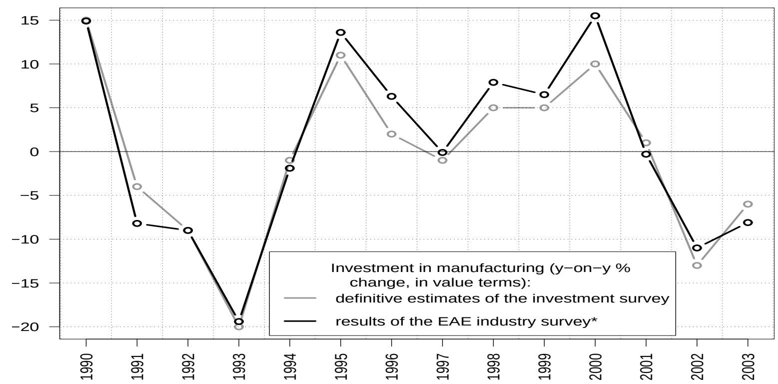
This strong correlation confirms the validity of the size of the sample (roughly 4 000 firms) and the robustness of the calculation method, which make it possible for the *investment survey* to provide a good reflection of the expectations, and later of the actual evolutions, in the investment carried out by the whole of the actual population of industrial firms.

2.3 Revisions in the views of firms questioned in the *investment survey* provide interesting information of a short-term nature

As indicated earlier, questions relating to the amounts of annual investment make it possible to evaluate firms' forecasts of their capital expenditure. For a given year, successive surveys provide several estimates of the evolutions – first forecast and then implemented – in industrial investment (cf. figure 3).

[10] "Enquête Annuelle d'Entreprise" in industry, the French Annual Business survey in this sector. This survey is carried out by the SESSI (Service des Études et des Statistiques Industrielles) and is exploited jointly by the SESSI and by Insee (Institut National de la Statistique et des Études Économiques).

Figure 2: Comparison for manufacturing industry of the definitive estimates of the *investment survey* and the results of the l'EAE industry survey*



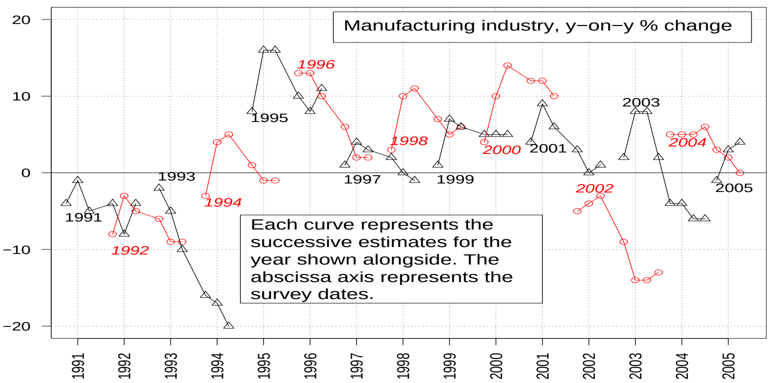
Source: *investment survey* (Insee) and EAE industry survey* (SESSI and Insee).
* EAE industry survey: "Enquête Annuelle d'Entreprise" in industry, the French annual business survey in this sector.

It can be seen from the graph that the estimated rates provide a good reflection of the pattern over time in capital expenditure. For example, the years 1998 to 2000 turn out to be a period when corporate investment was particularly strong. This period was then followed from 2001 on by a phase of economic slowdown and rationalisation of development projects on the part of industrial firms.

Figure 3 highlights the substantial scale of the revisions between successive estimates of the growth rate of industrial investment within a given year. As shown in the previous Part (2.2), with each successive estimate the results of the survey converge towards the actual evolutions in industrial investment as subsequently measured by the EAE industry survey. This convergence is a major criterion for assessing the quality of the *investment survey*: evolutions between successive estimates are not the result of statistical hazard but indeed reflect modifications over time in industrial firms' investment projects.

In other words, for year N, in April of year $N+1$ and roughly one year before the publication of the EAE industry survey and before that of the so-called "semi-definitive" annual national accounts, the *investment survey* gives a precise idea of GFCF by the industrial sector. However, despite the fact that this information is available well in advance of the EAE industry survey and the national accounts, it is still far too late from the point of view of utilisation for short-term economic forecasting. On the other hand, the previous estimates (those from October $N-1$ on) are published soon enough to be used in the forecasting exercise for the year N.

Figure 3: Successive estimates by industrial firms of the evolution in their investment



Source: *investment survey* (Insee).

The most natural manner in which to use this early information is to assume that firms' aggregate forecasts correspond to the evolutions that will in fact take place. It turns out that on average the growth rates, forecast and then achieved, are revised according to a relatively stable time-pattern (cf. figure 3). In other words, the result is a systematic revision for each estimate that is a function of the date of the survey in relation to the year under consideration. For the years 1990-2003, industrial firms' initial estimates turn out to have been overestimates in relation to the final estimate in the April survey of the following year. On average over this period, this overestimation amounts to 1.5 of a point for the first estimate at the time of the October survey of the previous year and then to 6.2, 5.8 and 2.3 points, respectively, in the January, April and October surveys of the year under consideration. Finally, the estimate made in January of the following year turns out to be very similar to that in the April survey of the same year, showing a slight downward difference of 0.2 of a point on average (cf. table 1). It is therefore essential to take the systematic nature of these revisions into account in any rigorous analysis of successive estimates of the investment growth rate.

Until 1994, this correction of the average revisions was incorporated directly into the published results. However, this correction turned out to be problematical for the year 1993: the economic situation deteriorated sharply and the initial expectation in October 1992 was revised downwards, not upwards, in the following surveys (for January and April 1993). Whereas the initial October 1992 estimate gave a relatively neutral item of information, later surveys converged towards a final figure showing a fall of 20% for industrial investment in 1993. It then became clear that it was not sufficient merely to correct by applying an average revision, as revisions between successive surveys are seen to depend

Table 1: Average revisions in industrial firms' estimates of annual growth in their investment, manufacturing industry, 1990-2003

Survey dates in relation to the year N being considered	Average investment rate as estimated by the survey	Average difference compared with estimates in the survey for April $N+1$
October $N-1$	1.1%	1.5 points
January N	5.8%	6.2 points
April N	5.4%	5.8 points
October N	1.9%	2.3 points
January $N+1$	-0.6%	-0.2 of a point
April $N+1$	-0.4%	0.0 of a point

Source: *Investment survey.* Author's calculations.

How to read the table: For example, from 1990 to 2003, at the time of the January surveys (year N) industrial firms estimated – on average over this period – that their investment would increase by 5.8% during the current year. Finally, at the time of the surveys for April of the following year (year N+1), they estimated on average that their investment had fallen by 0.4%, giving an average revision of 6.2 points between the survey for January of the current year and that of April the following year.

on the position in the economic cycle. For example, in the October 1999 survey, the forecast for growth in 2000 was 4%. When activity had been at the peak of the cycle, the final estimate, in the April 2001 survey, was 10%. If the average bias had been taken into account on its own, this would have led to a forecast of only 5% or 6% growth in 2000 at the time of the October 1999 survey.

On the other hand, the revisions between two successive surveys provide particularly interesting information. For example, for the year 1993, the decline between the October 1992 and January 1993 surveys in the forecast investment growth rate indicated a major change in firms' investment outlook (cf. figure 3). What is needed therefore is to consider at the same time both the level of firms' expectations (regarding growth in their capital expenditure) and the revisions of these expectations reported between successive estimates. For example, the downward revision of merely 2 points between the April and October 2004 surveys appears small by comparison with the average (3.5 points). This is therefore a positive piece of information, in line with the strength of corporate GFCF in Q4 2004. Using the data in this way permits a highly informative qualitative analysis but it hardly lends itself to quantitative analysis in the way a quarterly indicator would do.

3 A quarterly indicator of revisions in annual amounts

3.1 Revisions in individual expectations

Comparison of revisions in aggregate annual investment growth rates turns out to be interesting, but these revisions cannot properly be related, in one way or another, to quarterly evolutions in investment. For one thing, even if the aggregation method used (that of the “Large Investors”) provides a good estimate of forecasts of annual growth, it is not well-suited to the specific measurement of revisions in growth rates. With this method, the weighting attributed to a given firm can change between two successive surveys. Furthermore, certain revisions in growth rates are due more to modifications in the amounts relating to base years than to the years that are of interest. For example, take the case of a firm which in the April 2005 survey makes an upward revision in its estimate of investment spending for 2004 without modifying its forecast relating to 2005. In so doing, it has reduced the investment growth rate for 2005 but has not, for all that, modified the outlook for 2005.

Revisions in firms’ expectations turn out to be more informative, when consideration is given directly to the revisions between two successive surveys in the amounts of investment and not to the revisions in the estimates of the annual evolution.

At the time of each survey, a calculation is made of the average revision in investment in relation to the firm’s total turnover. The year considered is the most “advanced” possible: at the time of the October $N-1$ survey, firms are questioned for the first time regarding their expectations for the year N. In other words, at the time of the January N survey, it is then possible to calculate the difference between the amount declared in January N and that declared in October $N-1$.

At the time of each of the surveys carried out during year N (January, April, July and October), it is possible to calculate for each firm the evolution in replies compared with the previous survey (cf. table 2).

The July survey has only been in existence since 2003. For the period up to 2002, therefore, no calculation of the indicator can be made for July. Similarly, for the same period, the indicator for October can only be calculated as the revision between the April and October surveys.

Until such time as a sufficient number of July surveys are available and with the aim of achieving consistency over time in the series concerning October, the same method of calculation is used for the years 2003 and 2004. This means that there is no revisions indicator for the July surveys and that the replies to

Table 2: Theoretical timetable for the calculation of the indicator

Surveys	Figures requested in successive surveys	Indicators calculated on the difference...
January N	- Year $N-2$ - Year $N-1$ - Year N	between the survey of October $N-1$ and that of January N about the year N
April N	- Year $N-2$ - Year $N-1$ - Year N	between the survey of January N and that of April N about the year N
July N	- Year $N-2$ - Year $N-1$ - Year N	between the survey of April N and that of July N about the year N
October N	- Year $N-1$ - Year N - Year $N+1$	between the survey of July N and that of October N about the year N

Table 3: Provisional timetable for the calculation of the indicator

Surveys	Figures requested in successive surveys	Indicators calculated on the difference...
January N	- Year $N-2$ - Year $N-1$ - Year N	between the survey of October $N-1$ and that of January N about the year N
April N	- Year $N-2$ - Year $N-1$ - Year N	between the survey of January N and that of April N about the year N
October N	- Year $N-1$ - Year N - Year $N+1$	between the survey of April N and that of October N about the year N

the October surveys are all compared with those given six months earlier at the time of the April surveys. This is a shortcoming – for the time being only – of the indicator, meaning that it is available for only three quarters out of four (cf. table 3).

The individual revisions are related to the size of the firm, measured by its turnover. For any given survey t between the April N survey and that of January $N+1$, the figure used is the turnover for the year $N-1$ denoted by $TO_{i,t}$ for firm i[11]. Denoting by $I^a_{i,t}$ and $I^a_{i,t-1}$ the amounts of investment for the year a declared by firm i at the time of the t and $t-1$ surveys, the individual revisions indicator $d_{i,t}$ is provided by equation (2).

[11] In other words, $TO_{i,t}$ remains constant between the April N and January $N+1$ surveys.

$$d_{i,t} = \frac{I^a_{i,t} - I^a_{i,t-1}}{TO_{i,t}} \tag{2}$$

3.2 Stratification

These indicators of own-firm revisions are aggregated in such a way as to calculate for each survey date a position indicator m_t of the distribution of the own-firm revisions $d_{i,t}$. The sample is stratified by sector and by size of firm. It is therefore natural to calculate these parameters first by stratum and then to aggregate them with the help of adjustment coefficients. The adjustment coefficients used are the annual amounts of investment calculated on the basis of the EAE industry survey[12] for 2002. The aggregation method used for each stratum requires having available a sufficient number of observations. The level of stratification used must be at a fairly aggregated level: the sample is therefore divided up according to "NES 16"[13] and into three sizes of workforce (less than 100, from 100 to 499 employees, 500 employees or more). Until October 2003[14], coverage of the energy and food-processing sectors by the survey was relatively unsatisfactory. As a result, they were excluded from the estimate. This means that just the manufacturing sector is included, i.e. only four NES 16 industries: consumer goods, cars, capital goods and intermediate goods. Finally, the car sector, being too small and too concentrated to be divided into three groups according to size with the aggregation method used, has been grouped into a single stratum. In the end, therefore, the calculation was carried out for 10 strata.

3.3 The aggregation method used: the M-estimators

Within each stratum, the calculation of an aggregate revisions indicator poses technical difficulties. The distributions of the own-firm revisions indicators are very extensive, largely because some firms may make substantial revisions in their investment amounts between successive surveys. For example, a small firm may plan to purchase a building whose cost may be equivalent to several years' turnover. If the plan is not followed through, the resulting revision may itself be on a scale equivalent to several times the firm's total turnover. To take another example, at the time of the October survey a firm reports its investment forecast for the following year. This expected figure stems from strategic decisions taken by the firm and the forecast is not modified in successive survey responses until the company publishes its accounts, in other words some months after the end of the year in question, when a very substantial revision takes place, representing the gap between what had been forecast before the

[12] "Enquête Annuelle d'Entreprise" in industry, the French annual business survey in this sector.

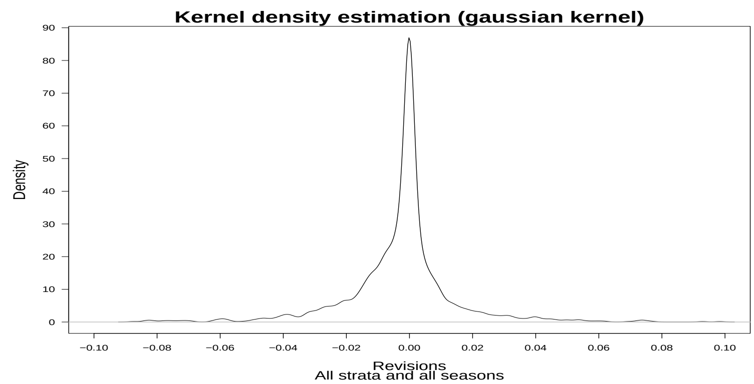
[13] Nomenclature Économique de Synthèse (the French agregated economic classification for the activities), at the 16-heading level.

[14] The *investment survey* became compulsory in 2004. This permitted a very significant rise in the response rate, especially in the food-processing and energy sectors.

beginning of the year and the amount recorded in the accounts a year and a half later.

In statistical terms, these large revisions are reflected in distributions with very thick tails (cf. figure 4). It then becomes necessary to carry out aggregation using robust estimators and not the usual arithmetic mean.

Figure 4: Empirical breakdown of revisions in own-firm investment



Source: Insee, *investment survey*. Author's calculations.

Of the possible robust estimators, the median is the most natural. However, it may not be suitable. Not only are the tails thick, but the distributions are highly concentrated around zero (cf. figure 4). Since numerous firms make no revisions to their investment expectations between two surveys, the median is almost systematically null.

The method chosen is that of the M-estimators, due to Huber (1964). This method is a generalisation of the so-called OLS (Ordinary Least Squares) method. In order to estimate the position of the centre of the distribution, instead of minimising the sum of the squares of the residuals as in the OLS method, in this case it is the sum of another objective function applied to the residuals that is minimised. This function is denoted by ρ.

To be more precise, for each survey season S (surveys for January, April and October) and for each stratum H, a robust regression is carried out for the revisions indicators on the time indicators explained by time-dummy variables. For all the surveys t in season S, this means calculating the $m_{H,t}$, i.e. the aggregated indicators of the revisions occurring in stratum H. $\mathbf{1}_{\{x\}}$ denotes the dummy function of x, which carries the value 1 in x and 0 otherwise. For each stratum H and each season S, the model is then written:

$$d_{i,t} = \sum_{\tau} m_\tau \mathbf{1}_{\{\tau\}}(t) + \epsilon_{i,t} \tag{3}$$

The disturbances $\epsilon_{i,t}$ show random distributions that we assume to be independent. In a first stage, we also assume that the disturbances $\epsilon_{i,t}$ are identically distributed (by stratum and by season).

The M-estimators are not linear. They are therefore not invariant to scale changes and it is then necessary to divide by a scale parameter $\sigma_{H,S}$ of the dispersion of the $d_{i,t}$ for the stratum and season in question:

$$\frac{d_{i,t}}{\sigma_{H,S}} = \sum_{\tau} m_\tau \frac{\mathbf{1}_{\{\tau\}}(t)}{\sigma_{H,S}} + \tilde{\epsilon}_{i,t} \tag{4}$$

Several statistics can be used as the scale parameters $\sigma_{H,S}$. The one chosen is the median of the absolute values of deviations. This statistic, known as the MAD (Median Absolute Deviation), is given by equation (5). This measure has the advantage of being robust to outliers. Applying a coefficient of 1.48 brings the MAD statistic into equality with the standard deviation in the case of a normal distribution.

$$\sigma\left(d_{H,S}\right) = MAD\left(d_{H,S}\right) = 1.48 \text{ Median}\left[d_{H,S} - \text{Median}\left(d_{H,S}\right)\right] \tag{5}$$

$n_{H,S}$ denotes the number of observations in stratum H for the surveys in season S. The minimisation problem associated with the estimate for stratum H and season S is then written:

$$\left(m_{H,t}\right)_{t\in S} = \text{Arg} \min_{(m_t)_{t\in S}} \frac{1}{n_{H,S}} \sum_{i\in H t\in S} \tilde{\rho}_{H,S}\left[t, d_{i,t}, \left(m_t\right)_{t\in S}\right] \tag{6}$$

$$\text{with} \qquad \tilde{\rho}_{H,S}\left[t, d_{i,t}, \left(m_t\right)_{t\in S}\right] = \rho\left(\frac{d_{i,t} - m_t}{MAD\left(d_{H,S}\right)}\right) \tag{7}$$

Under several theoretical conditions, $(m_{H,t})_{t\in S}$ converges to a finite limit $(m^0_{H,t})_{t\in S}$ when the sample size $n_{H,S}$ tends to infinite. These hypotheses and the theoretical background for M-estimators are described in detail in appendix A.

3.4 The choice of the objective function

The choice of the objective function ρ is the most delicate aspect of the M-estimator method. This function has to be suitably adapted to the general form of the distribution of the residuals. In addition, a derivable objective function facilitates the numerical solution of the minimisation problem. When the objective function is derivable, it can be defined by its derivative, known as the score function and denoted by ψ.

For the Gaussian distributions, it is natural to choose a quadratic function and the estimator of position of the distribution would then be the arithmetic mean.

More generally, the objective function needs to be chosen in such a way that the associated M-estimator performs satisfactorily in the family of distributions being considered. To be more precise, the estimator must be robust and close to efficiency. "Efficiency" means that the estimator is unbiased and of minimal variance. The robustness ensures that the removal of any observation makes little difference to the estimate. For a precise definition of robustness, the reader is referred to, for example, the book by Lecoutre and Tassi (1987).

The maximum likelihood theory indicates that, if f is the density of the actual distribution of the disturbances, the optimal objective function - in the sense of efficiency as defined above - is given by $\rho = -\log f'/f$. However, out of concern for robustness of the results at the cost of a loss of efficiency, it is possible to choose an objective function whose growth with the amplitude of the residuals is not as fast.

A large number of M-estimators can be found in the statistical literature. Those most frequently used are listed in box 2.

Box 2: The choice of the objective function ρ

The objective function ρ has to be chosen in such a way that the estimations are simultaneously robust and close to efficiency for the distributions being examined. A large number of families of functions is proposed in the statistical literature. We shall give here certain examples. The first of these (OLS, median) are mentioned for pedagogic purposes but do not have the properties suitable for the distributions examined here. However, the other families proposed are liable to be suitable.

In each case, the estimator is defined by the objective function ρ and/or by its derivative, i.e. the score function ψ.

OLS (Ordinary Least Squares):

$$\rho(x) = \frac{1}{2}x^2 \quad \text{and} \quad \psi(x) = x$$

The OLS estimator is a special case of an M-estimator. It is extremely widely used and has numerous properties. In particular, on the assumption of normality of distributions, it is the optimal estimator from the point of view of efficiency. (An estimator is said to be efficient when it is unbiased and its variance is minimal.) On the other hand, the quadratic nature of the objective function makes it highly sensitive to outliers. The empirical distributions envisaged here are not Gaussian. In particular, the very thick tails of the distributions make the OLS estimators too sensitive to outliers.

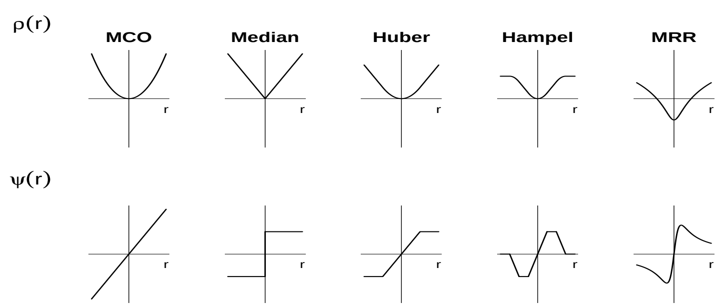
Median:

$$\rho(x) = |x| \quad \text{and} \quad \psi(x) = \text{sign}(x)$$

$$\text{with} \quad \begin{cases} \text{sign}(x) = -1 & \text{if } x < 0 \\ \text{sign}(x) = 0 & \text{if } x = 0 \\ \text{sign}(x) = 1 & \text{if } x > 0 \end{cases}$$

The median is another very classic indicator, having the advantage of being highly robust to outliers. However, it fails to capture any information away from the median point. In this case, the very large concentration at zero of the empirical distributions leads to medians that are in most cases nil and so provide absolutely no information.

Figure A: Examples of objective functions ρ and associated score functions ψ



Huber:

$$\psi(x) = x \quad \text{if} \quad |x| < c, \qquad \psi(x) = c \ \text{sign}(x) \quad \text{if} \quad |x| > c$$

The M-estimator proposed by Huber (1964) is equivalent to the OLS indicator as regards the centre of the distribution and equivalent to the median indicator as regards the tails. It has the additional advantage over the other M-estimators presented below of guaranteeing the existence and uniqueness of the solution of the associated minimisation problem. However, in the case of the distributions envisaged here (very thick tails and high concentration at zero) it turns out to be insufficiently robust. This leads to a preference for M-estimators with score functions that are said to be "redescending", meaning that they tend to zero at infinity.

Various "redescending" functions:

Tukey's Biweight: $\psi(x) = \frac{x}{c}\left(1 - \frac{x^2}{c^2}\right)^2$ if $|x| < c$, 0 otherwise

Andrew's Sine function: $\psi(x) = \frac{1}{\pi}\sin\left(\frac{\pi x}{c}\right)$ if $|x| < c$, 0 otherwise

Hampel:
$$\begin{cases} \psi(x) = x & \text{if} \quad |x| < a \\ \psi(x) = a \ \mathrm{sign}(x) & \text{if} \quad a < |x| < b \\ \psi(x) = a \frac{\mathrm{sign}(x) \ c-x}{c-b} & \text{if} \quad b < |x| < c \\ \psi(x) = 0 & \text{if} \quad c < |x| \end{cases}$$

All the three families of score functions given by the above formulae are "redescending". To be more precise, they tend to zero after a certain threshold. This means that points that are too distant from the centre of the distribution are completely rejected from the estimation. All are close to proportionality in 0. The estimators are therefore similar to the OLS indicator for the centre of the distribution. Use of the Biweight functions is due to Tukey, that of the Sine functions to Andrew and that of the piecewise score function to Hampel.

MRR functions (the family of score functions chosen):

$$\rho(x) = \frac{c}{2}\log\left[x^2 + c\right] \qquad \text{and} \qquad \psi(x) = \frac{cx}{x^2 + c}$$

For distributions of the kind seen here, Moberg, Ramberg and Randles (1980) propose the choice of score function in the family defined above. We name this family of functions after the authors (abbreviated to MRR). These functions have the advantage of being "redescending" but without totally rejecting certain estimation points - in contrast to the three previous families of functions (Biweight, Sinus and Hampel) - with the score functions not cancelling out beyond a certain point. They are still close to proportionality around zero, so that the corresponding M-estimators are still similar to the OLS indicators around the centre of the distribution. These estimators turn out to be very close to efficiency for distributions that are highly concentrated and have very thick tails. These are the estimators that were chosen for the rest of the analysis.

There is no *a priori* optimal choice of score function. The choice is made in the light of the family of distributions being examined. In this case, the appropriate score function is chosen by an adaptative method inspired by Ravalet (1996). This method is in several stages.

1. First, robust statistics are calculated on the empirical distributions in order to measure the thickness of the tails of the distributions and their concentration around zero.

2. Second, a theoretical distribution is constructed which, in the light of these statistics, has the best possible correspondence to the empirical distributions.

3. Next, samples are simulated using the theoretical distribution arrived at. The location parameters of the samples are estimated using the various families of score functions envisaged. The family that is adopted is the one with the smallest estimation errors.

4. Within this family, one particular score function is chosen on the basis of the same criterion (minimisation of a cost function of estimation errors of the simulations).

These various stages are described in detail in appendix B. We choose a so-called MRR function (cf. box 2) with a tuning constant $c = 0.10$:

$$\rho(x) = \frac{0.10}{2} \log\left[x^2 + 0.10\right] \tag{8}$$

The algorithm for the calculation of the minimisation problem adopts the method known as "iterative re-weighting of Ordinary Least Squares". This algorithm is described in appendix C.

3.5 Quasi-generalised M-estimators

However, the estimator described above can be brought even closer to efficiency. The residuals in fact turn out to have a high level of heteroscedasticity, with variance diminishing as the size of firm increases. In other words, in relation to turnover figures, the amplitude of the revisions decreases as the size of firm increases. There are two explanations. First, large firms are liable to be more rational in their investment planning and to have more efficient management control systems than small firms. Second, the multiplicity of activities carried out by a large firm and the diversity of its investment projects means that there are often partially compensating changes, with the reduction in certain investments possibly offset by the emergence of other investments.

In order to correct the heteroscedasticity, a two-stage procedure is used, on the same lines as the Quasi-generalised Least Squares (QGLS) method. First, an estimation is made using the M-estimators method. This makes it possible to extract the residuals, the amplitude of whose variance varies with the turnover for each observation. The dependence of the variance of the residuals $\Sigma^2_{H,S}(\cdot)$ on the size of firm is then estimated for each stratum H and each season S. In the second stage, the model is transformed by dividing by the square root of the variance $\hat{\Sigma}^2_{H,S}(\cdot)$ estimated in this way. Equation (4) is then replaced by equation (9), in which $\tilde{\epsilon}_{i,t}$ are then of the same variance, making it legitimate to accept the assumption of equality of the distributions of the pairs $(t, d_{i,t})$.

$$\frac{d_{i,t}}{\sigma_{H,S} \cdot \hat{\Sigma}_{H,S}\left(TO_{i,t}\right)} = \sum_{\tau} m_{\tau} \frac{\mathbf{1}_{\{\tau\}}(t)}{\sigma_{H,S} \cdot \hat{\Sigma}_{H,S}\left(TO_{i,t}\right)} + \tilde{\epsilon}_{i,t} \tag{9}$$

Equation (7) then becomes:

$$\tilde{\rho}_{H,S}\left[t, d_{i,t}, (m_t)_{t \in S}\right] = \rho\left(\frac{d_{i,t} - m_t}{MAD\left(d_{H,S}\right) \ \cdot \hat{\Sigma}_{H,S}\left(TO_{i,t}\right)}\right) \tag{10}$$

The estimation of the dependency of the dispersion of the residuals on the size of firm is carried out by stratum and by survey season (surveys for January, April and October). For this purpose, the residuals are re-ordered according to the turnover of the firms. The dispersion of the residuals is then estimated for a moving window of 100 observations, with the help of the median of the absolute values of deviations (the MAD or Median Absolute Deviation).

The logarithm of the dispersion of the residuals calculated in this way is then regressed on the logarithm of turnover. For each firm i in stratum H and each date t in season S, with the $\mu_{i,t}$ independent, identically distributed and with null mean, the chosen model is then written:

$$\log \Sigma_{H,S}\left(TO_{i,t}\right) = \alpha_{H,S} + \beta_{H,S} \log(TO_{i,t}) + \mu_{i,t} \tag{11}$$

In this way a theoretical dispersion as a function of the size of firm (measured by turnover) is estimated for each stratum H and each of the seasons S (surveys for January, April and October).

$$\log \hat{\Sigma}_{H,S}\left(TO_{i,t}\right) = \hat{\alpha}_{H,S} + \hat{\beta}_{H,S} \log(TO_{i,t}) \tag{12}$$

The regression is also carried out by the M-estimators. The score function used is then the Huber function.

After correction of the heteroscedasticity and for the second estimate, the procedure for the choice of score function (described in the section 3.4 and in more detail in appendix B) still ends in choosing a MMR function, but with a larger constant: $c = 0.12$.

3.6 Construction of the quarterly series by stratum and adjustment

For each stratum H, we therefore have three series $(\hat{m}_{H,t})_{t \in S}$, one for each season S (surveys for January, April and October). In order for these series to be comparable, they are centred at zero and reduced to unit variance.

In each stratum H, the revisions to the three annual series are then brought together to form a quarterly series. The revisions in the January survey N correspond to the first quarter of year N, those in the April survey to the second quarter and those in the October survey to the third and fourth quarters.

The quarterly series of revisions by stratum are then aggregated into a single series for the whole of manufacturing industry. For this purpose, constant adjustment coefficients are applied, calculated on the amounts of investment by stratum derived from the EAE industry survey for 2002.

4 The revisions indicator derived from the *investment survey* substantially improves the quality of the investment forecast

4.1 Description of the revisions series

The series constructed in this way turns out to be fairly well correlated with quarterly changes in NFE GFCF[15] in value terms[16] (cf. figure 5). Over the period from Q3 1991 to Q4 2003, the correlation comes out at 69%. The indicator for a given quarter is available in the middle of the same quarter, making it possible to forecast investment in the quarter for the purposes of the Insee publication "Conjoncture in France" which comes out at the end of the quarter. This is not yet possible for the third quarters of the year and hence for the briefer October issues in the series. In due course, however, the July survey will make it possible to construct an indicator available for end-August as well, making it possible to provide a forecast for Q3, particularly for use in the brief October "Conjoncture" report.

The series turns out to have a lead on the quarterly accounts. For example, the 1997 recovery appears in the survey series for January, but not before Q2 in the quarterly accounts.

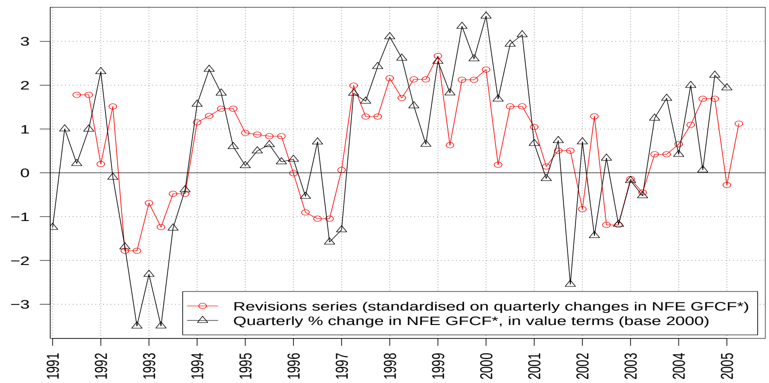
4.2 An example of a calibration model

In order to illustrate the possible use of the revisions indicator for forecasting investment on a quarterly basis, we give here an example of a calibration model with a Vector Autoregressive (VAR) process. This takes as unique explanatory variable the revisions indicator described in this article. In fact, other indicators forecasting investment provide little additional information compared with that provided by the indicator and its lags.

[15]Gross Fixed Capital Formation by Non-Financial Entreprises.

[16]The quarterly national accounts used in this article are the preliminary figures for the first quarter of 2005 on base 2000.

Figure 5: Comparison of the constructive series and the actual evolutions in firms' investment



Sources: Insee, *investment survey* and quarterly national accounts (base 2000).
Author's calculations.
* NFE GFCF: Gross Fixed Capital Formation by Non-Financial Entreprises.

The vector process X_t being modeled is formed by the quarterly evolutions in NFE GFCF in volume terms[17] (denoted by GFCF) and the revisions indicator (denoted by REV). GFCF_t is the evolution in GFCF in quarter t and REV_t is the revisions indicator attributed to quarter t. For example, in the case of Q2 2005, this is the indicator calculated on the basis of the *investment survey* for April 2005.

$$X_t = \begin{pmatrix} \text{GFCF}_t \\ \text{REV}_t \end{pmatrix}$$

The stationarity of the two series GFCF and REV is verified by means of unit root tests: ADF (Augmented Dickey-Fuller) tests reject at very significant level the non-stationarity of the two series. It is then natural to write X_t in the form of a VAR process. With L denoting the lag operator, X_t is modelled by the equation (13), where Φ is a polynomial, c_0 is a constant vector and ν_t is Gaussian white noise. The absence of autocorrelation of the disturbances ν_t and their normality were tested *a posteriori*.

$$X_t = c_0 + \Phi(L)X_{t-1} + \nu_t \tag{13}$$

The process is estimated for the period running from Q1 1992 to Q4 2003. Both the method of iterative tests of the maximum likelihood ratio method and the minimisation of the Akaike information criterion (AIC) point to introducing only one lag. The two dynamic equations are then estimated using OLS[18]:

[17] The quarterly national accounts used in this article are the provisional figures for the first quarter of 2005 on base 2000.
[18] The variations in GFCF are expressed as percentages.

$$\begin{cases} \text{GFCF}_t = \underset{(2.2)}{0.46} + \underset{(1.6)}{0.26}\ \text{GFCF}_{t-1} + \underset{(2.9)}{0.15}\ \text{REV}_{t-1} \\ \text{with} \quad R^2 = 0.45, \quad \overline{R}^2 = 0.42, \quad RMSE = 1.30 \quad \text{and} \quad \text{Durbin-Watson} = 1.97 \end{cases}$$

$$\begin{cases} \text{REV}_t = \underset{(-1.7)}{-1.10} + \underset{(2.9)}{1.41}\ \text{GFCF}_{t-1} + \underset{(1.5)}{0.24}\ \text{REV}_{t-1} \\ \text{with} \quad R^2 = 0.44, \quad \overline{R}^2 = 0.42, \quad RMSE = 3.89 \quad \text{and} \quad \text{Durbin-Watson} = 1.96 \end{cases}$$

It was verified *a posteriori* that the residuals do indeed take the form of white noise[19] and that they do indeed show a Gaussian bivariate distribution[20].

The REV indicator is available slightly more than three months before the publication of the corresponding quarterly accounts. In order to be able to use the latest value of the indicator (REV_t) at the time of forecasting investment in the current quarter t, it is useful to write the model in so-called "recursive block" form. This form can be obtained by transforming the system of the two above equations with the aid of a Cholesky decomposition of the variance-covariance matrix. However, a simpler and perfectly equivalent method is to carry out the regression of the GFCF series directly on the REV series, as well as on the two lagged series.

$$\begin{cases} \text{GFCF}_t = \underset{(3.4)}{0.65} + \underset{(0.1)}{0.01}\ \text{GFCF}_{t-1} + \underset{(4.0)}{0.17}\ \text{REV}_t + \underset{(2.4)}{0.11}\ \text{REV}_{t-1} \\ \text{with} \quad R^2 = 0.59, \quad \overline{R}^2 = 0.57, \quad RMSE = 1.12 \quad \text{and} \quad \text{Durbin-Watson} = 1.73 \end{cases}$$

The "recursive block" form then makes it possible to use all available information to forecast GFCF for the quarter concomitant with the indicator. For the following quarters, the canonic form is used to protract the process $(X_t)_t$.

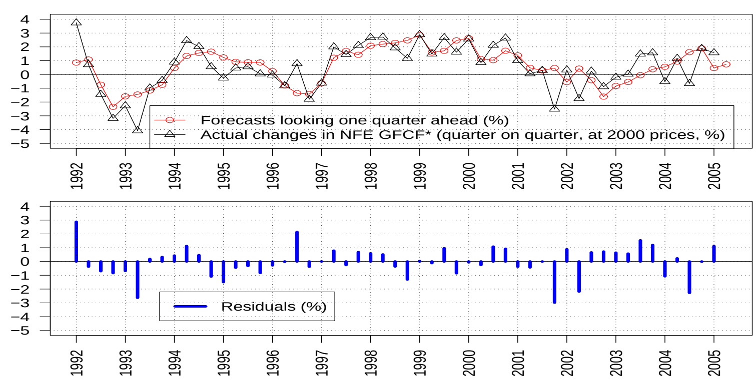
For example, in May 2005, the latest available quarterly national accounts were for Q1 2005. The REV indicator, for its part, was available for Q2 2005 (based on the April survey). For this quarter, the value of the indicator was 3.0. On the basis of the information available in May 2005, the "recursive block" form then gave a forecast for the evolution in NFE GFCF of 0.7% in Q2 2005. The canonic form makes it possible to protract the forecast for growth in NFE GFCF to Q3 2005 (1.1%) and Q4 2005 (0.8%).

[19] A multivariate portmanteau test was carried out. The null hypothesis was the absence of autocorrelation of the residuals. The test statistic came out at 50.5. This is to be compared with a χ^2 of 60 degrees of freedom, giving a p-value of 0.80. It is therefore not possible to reject the absence of autocorrelation of the residuals.

[20] The Skewness statistic and the Kurtosis statistic applied to the residuals come out at 2.64 and 0.40 respectively. The joint statistic for the Doornik and Hansen [2] test comes out at 3.04, giving a p-value of 0.21 for the hypothesis of normality of the residuals.

For the period from the beginning of 1992 to the beginning of 2005, the standard deviation of residuals for a one-quarter time horizon ("recursive block" form) is 1.10. Protracted using the canonic form over a two-quarter time horizon, the standard deviation comes out at 1.26 and for a three-quarter time horizon at 1.43. These values can be compared with the standard deviation of variations in GFCF over the same period, i.e. 1.66. As regards correlation, the one-quarter forecasts have a 75% correlation with the outturns. The corresponding figures for two and three quarters are 65% and 44%, respectively. These statistics show that the indicator is a good instrument for forecasting evolutions in investment over two quarters. However, like all short-term economic indicators, it becomes inefficient in forecasting evolutions over three quarters.

Figure 6: Forecast of the quarterly variations in NFE GFCF* in volume terms looking one quarter ahead



Sources: Insee, *investment survey* and quarterly national accounts (base 2000). Author's calculations.
* NFE GFCF: Gross Fixed Capital Formation by Non-Financial Entreprises.

Figure 6 represents forecasts over a single time horizon ("recursive block" form) as well as the outturns and the residuals.

In the past, forecasts provided by the VAR model correctly captured the turning points in the short-term investment cycle. For example, the upturn in GFCF in 1997 appears in the accounts starting in Q2 of that year. The model would have perfectly forecast this upturn on the basis of the results from the April survey. Similarly, the marked slowdown in investment in Q1 2001, marking the end of the "dotcom bubble", is well in line with the forecast by the model.

5 Conclusion

Given the sparsity of survey indicators for forecasting corporate GFCF, it turns out that the *investment survey* provides invaluable short-term information regarding the future evolution in corporate investment. The results published in the form of growth rates are very useful for a qualitative approach, but show certain limitations for the application of quantitative forecasting instruments on a quarterly basis. These limitations are avoided by the expectation revisions indicator, which provides relevant information for forecasting quarterly variations in investment.

This revisions indicator thus makes effective use of the quarterly nature of the *investment survey* carried out by Insee. Compared with surveys of industrial investment in other European countries, this special feature of the French survey represents a substantial advantage for the forecasting of evolutions in investment on a within-year basis.

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A The M-estimators theory

The "asymptotic problem" is defined as the minimisation problem for the M-estimators estimation when the sample size tends to infinite. The asymptotic problem of the optimisation program (6) is equivalent to that given by (14). In this equation, (T, D) is the list of the two random variables which are respectively the time of an observation and the value of the revision associated with this observation. The 2-dimension (T, D) variable has the distribution of the $(t, d_{i,t})$ for the stratum and the season in question. E_0 is the expected value given T, and E_T is the unconditional expectation.

$$\text{Arg} \min_{(m_t)_{t \in S}} \; E_T E_0 \; \tilde{\rho} \left[T, D, (m_t)_{t \in S}\right] \tag{14}$$

The convergence of estimators toward the solution of the asymptotic problem needs to be based on the following hypotheses, given in Gouriéroux-Monfort[6]:

1. The 2-dimension variables $(t, d_{i,t})$ are independent, identically distributed.
2. The subset of the optimisation is open ($\mathbb{R}^{\text{Card}(S)}$ can be used).
3. The function $\tilde{\rho}$ is continuous for $(m_t)_{t \in S}$ and is integrable under the true distribution of (T, D) for all $(m_t)_{t \in S}$. This is equivalent to the continuity of ρ and its integrability under the true distributions of stochastic disturbances.
4. $\frac{1}{n_{H,S}} \sum_{\substack{i \in H \\ t \in S}} \tilde{\rho}\left[t, d_{i,t}, (m_t)_{t \in S}\right]$ converges uniformly almost certainly to $E_T E_0 \; \tilde{\rho}\left[T, D, (m_t)_{t \in S}\right]$ when the sample size $n_{H,S}$ tends to infinite.
5. The asymptotic problem has one and only one solution $(m_t^0)_t$.

With these hypotheses, there is asymptoticly a solution to the first order conditions for the finite sample size minimisation program. This solution $(\hat{m}_t)_t$, named M-estimator, converges almost certainly to $(m_t^0)_t$ when the sample size $n_{H,S}$ tends to infinite.

With more hypotheses [21] , $\sqrt{n}\left((\hat{m}_t)_t - (m_t^0)_t\right)$ is asymptoticly distributed according to a Gaussian distribution, which mean is 0 and variance is $J^{-1} I J^{-1}$ with:

$$I = E_T E_0 \left(\frac{\partial \tilde{\rho}\left[T, D, (m_t^0)_{t \in S}\right]}{\partial (m_t)_t} \frac{\partial \tilde{\rho}\left[T, D, (m_t^0)_{t \in S}\right]}{\partial (m_t)_t'} \right)$$

$$J = E_T E_0 \left(- \frac{\partial^2 \tilde{\rho}\left[T, D, (m_t^0)_{t \in S}\right]}{\partial (m_t)_t \; \partial (m_t)_t'} \right)$$

[21] It is sufficient that $\tilde{\rho}$ is two time continuously derivable for $(m_t)_t$, that the matrix J (defined after) exists and that the matrix J is invertible.

The chosen objective function ρ must respect the conditions listed above. Practically, this function is chosen in the set of continuous and symmetric functions. It has to be minimum in 0 and to grow on $]0, +\infty[$. *Ex post*, integrability is checked, the finite sample size minimisation program has to have a solution under observed empirical distributions and the asymptotic problem has to have only one solution under a large set of theoretical distributions around these empirical distributions.

On the other hand, distributions have to be in accordance with hypothesis 1. There are no difficulties for independence. However the couples $(t, d_{i,t})$ are not identically distributed. The variance of $d_{i,t}$ grows when the size of the firm decreases. We have described in Part 3.5 how to correct the model in order to check this hypothesis 1.

B The choice of the M-estimator

An M-estimator is defined by its objective function ρ or by its derivative, i.e. the score function ψ. *Ex ante*, there is no optimal score function. The choice is made in the light of the family of observed distributions in order to ensure that the estimator is robust and close to efficiency with these distributions. Such a method to choose the score function is named “an adaptative procedure” because this function ψ is adapted for empirical distributions. Such methods are listed in the field of nonparametric techniques since it is not necessary to suppose hypotheses about the belonging of disturbances’ distribution in a parametric set of distributions.

B.1 Procedure for the choice of the score function

We describe here the adaptative procedure used to choose the score function. As it was said in the main part of the article, this procedure could be organized in four stages:

1. First, robust statistics are calculated on the empirical distributions in order to measure the thickness of the tails of the distributions and their concentration around zero.

2. Second, a theoretical distribution is constructed which, in the light of these statistics, has the best possible correspondence to the empirical distributions.

3. Next, samples are simulated using the theoretical distribution arrived at. The location parameters of the samples are estimated using the various families of score functions envisaged. The family that is adopted is the one with the smallest estimation errors.

4. Within this family, one particular score function is chosen on the basis of the same criterion (minimisation of a cost function of estimation errors of the simulations).

These various stages are described in detail here for the choice of the score function for the first stage estimation in the method of the "Quasi-genralised M-estimators" (cf. Part 3.5 in the main part of the article). The choice of the score function for the second stage estimation is done with the same procedure. Intermediate results are given in Part B.2 in this appendix.

1. Statistics used to measure the thickness of tails and concentration around zero need to be invariant by translation and by homothety of the distribution. Indicators used have been chosen by Ravalet [12].

   The thickness of tails' measure was proposed by Hogg [7]. Let us define $U(p)$ (respectively $L(p)$) the mean of the np biggest (respectively smallest) order statistics of a sample which size is n. We use a linear interpolation when np is not a whole number. Then, we compute τ, the thickness of tails' measure given with equation (15). The more tails of a distribution are thick, the more τ is important. For a Gaussian distribution, τ takes the value 2.59.

$$\tau = \frac{U(5\%) - L(5\%)}{U(50\%) - L(50\%)} \tag{15}$$

   We define $X(a, b)$ the mean of order statistics between the na^{th} and the nb^{th} order statistics (with interpolation if necessary). The measure of the concentration around the median is given by Hogg and *alii* [8] and denoted by P_k (formula given below). The more the distribution is concentrated around its median, the more P_k is important. For a Gaussian distribution, P_k takes the value 2,74.

$$P_k = \frac{X(85\%, 95\%) - X(5\%, 15\%)}{X(50\%, 85\%) - X(15\%, 50\%)} \tag{16}$$

   Computed statistics are quite constant over strata and seasons. We can retain the same values $\tau = 5.4$ and $P_k = 6.0$ for all strata and all seasons.

2. We want now to find a theoretical distribution close to empirical distributions of revisions. This closeness is measured with statistics defined above: they must have the same values on the theoretical distribution and on empirical distributions. Obviously, this criterion does not define an absolute closeness as does a distance in the distributions set (like the Kullback-Leibler distance for instance). Therefore, we also need to check this proximity with graphic representations of distributions, using Quantile-Quantile plots. Such an example is given by figure 7. We look

Table 4: τ et P_k statistics on several symmetric distributions

Theoretical distributions	τ	P_k
Gaussian distribution	2.6	2.7
Slash distribution	8.5	4.2
Double exponential distribution	3.3	3.4
"Gaussian distribution to the power of 2"	4.4	5.6
"Gaussian distribution to the power of 3"	6.1	10.0
"Gaussian distribution to the power of 6"	9.1	44.8

Author's calculations.

for a symmetric distribution on which τ and P_k take values 5.4 and 6.0. We test various distributions which statistics are given by table 4.

Slash distribution is defined by the ratio of the normalized Gaussian distribution and the uniform distribution on $[0, 1]$. Double exponential distribution is defined as the difference between two exponential distributions. "Gaussian distribution to the power of p" is here the distribution of $\text{sign}(N)|N|^p$ when N is a normalized Gaussian random variable. $\text{sign}(\cdot)$ is the function which takes values -1 on $]-\infty, 0[$, 1 on $]0, +\infty[$ and 0 on 0.

After these various examples, we look for a distribution given by equation (17):

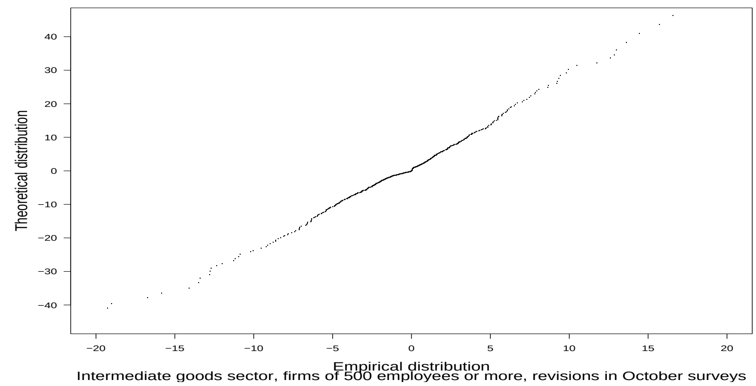
$$X = \alpha D + (1 - \alpha)\ \text{sign}(N) \left|\frac{N}{K}\right|^p \tag{17}$$

with D a double exponential random variable and N a normalized Gaussian random variable. Finally, with $\alpha = 0.60$, $K = 0.61$ et $p = 2.77$ (founded by Newton algorithm), this theoretical distribution has the same τ et P_k statistics than empirical distributions by strata.

Quantile-Quantile plots help to check closeness between residuals' empirical distributions and this theoretical distribution. For instance, figure 7 compares the chosen theoretical distribution and the residuals' empirical distribution for revisions in October surveys in the stratum of firms of 500 employees or more and in the intermediate goods sector. The graph is close to a straight line; consequently, we can assume that the two distributions are close (with an affine transformation).

3. Listed score functions (cf. box 2 page 19) are tested with several values of parameter c (or parameters a, b and c in the case of Hampel functions). With the chosen theoretical distribution, we simulate 1 000 samples of 200 observations. In each sample, the center of the distribution is measured by the M-estimator defined with the tested score function. The square root

Figure 7: Quantile-Quantile plot between the chosen theoretical distribution and residuals' empirical distribution



Source: *investment survey* (Insee). Author's calculations.

of the second empirical moment for the 1 000 estimated centers is used as a criterion of a bad efficiency of estimations. Results are given in table 5.

Regarding these estimations, the best M-estimators are M-estimators defined with MRR functions or Hampel functions. MRR functions have just one parameter and are therefore easier to use. Moreover, MRR functions do not totally reject extreme values outside the estimation: The score functions are not canceled out beyond a certain point. So, we choose to use MRR functions better than Hampel functions.

4. Finally, we have just to choose exactly the parameter c for a MRR function. We use the same criterion than in the previous stage (the minimisation of the square root of the second moment of estimated centers of simulated distributions). This criterion gives $c = 0.10$. In this case, the square root of the second moment of estimated centers of simulated distributions is 0.030.

This method has several limits. In particular, our definition of closeness is not a distance[22]. So, the choice of a theoretical distribution is quite arbitrary since it could be changed with other statistics for the thickness of the tails of the distributions and their concentration around zero. However, when these statistics are defined with other quantile windows (cf. equations (15) and (16)), the chosen theoretical distribution is very close.

[22] In fact, it's a semi-distance.

Table 5: Simulations results

M-estimator	**Square root of the second empirical moment**
OLS (i.e. arithmetic mean)	0.053
Median	0.037
Huber with $c = 1$	0.046
Huber with $c = 3$	0.081
Biweight with $c = 1$	0.038
Biweight with $c = 5$	0.044
Andrew's Sine function with $c = 1$	0.036
Andrew's Sine function with $c = 5$	0.047
Hampel with $a = 0,5$, $b = 1$, $c = 2$	0.033
Hampel with $a = 2$, $b = 4$, $c = 8$	0.054
MRR function with $c = 1$	0.032
MRR function with $c = 4$	0.043
MRR function with $c = 0.1$	0.030

Author's calculations.

Finally, the MRR score function with $c = 0.10$ seems to be a good choice for all strata and all seasons.

B.2 The choice of the score function for the second stage estimation with the "Quasi-generalised M-estimators" method

The choice of the score function for the second stage estimation in the "Quasi-generalised M-estimators" method uses the same procedure than that described before (cf. B.1 in this appendix).

After the heteroscedasticity correction, statistics for the thickness of the tails of the distributions and their concentration around zero are close to $\tau = 5.3$ and $P_k = 6.1$. The theoretical distribution is given by $\alpha = 0.60$, $K = 0.60$ and $p = 2.70$. So we choose a higher parameter for the second stage estimation: $c = 0.12$.

C The algorithm for the calculation of the M-estimator

M-estimators' estimations are not computed with the direct resolutions of associated minimisation programs but with an iterative method of re-weighting of OLS (Ordinary Least Square). This procedure converges faster. We explain below briefly the idea of this algorithm.

We define the general problem of the regression of a dependent variable Y on K independent variables $X^{(k)}$, k varying between 1 and K. These K variables can contain a unitary constant.

$$Y \approx X\beta$$

The dependent variable Y and independent vectors $X = (X^{(k)})_{k\in[1..K]}$ are observed N times and take values y_i and $x_i = (x_i^{(k)})_{k\in[1..K]}$ with i varying between 1 and N.

We want to estimate $\min_\beta \sum_i \rho(y_i - x_i\beta)$. ρ is derivable and its derivative is noted ψ. If a solution exists, then this solution checks necessary the first order conditions for the minimisation program. These conditions are given by $0 = \sum_{i,t} x_i^{(k)}\psi(y_i - x_i\beta)$ for all k between 1 and K.

Let us define $w(r) = \psi(r)/r$. We can write for all k between 1 and K:

$$0 = \sum_{i,t} x_i^{(k)}(y_i - x_i\beta)\ w(y_i - x_i\beta)$$

These K equations are exactly similar to that given by minimisation of the sum of residuals' square with individual weights $w_i = w(y_i - x_i\beta)$. So, we can resolve by iteration using weighted OLS: at the stage n, $\beta^{(n)}$ is the result of the OLS estimation weighted by individual weights:

$$w_i^{(n)} = w\left(y_i - x_i\beta^{(n-1)}\right)$$

If the series $\beta^{(n)}$ converges to a limit $\beta^{(\infty)}$, then $\beta^{(\infty)}$ checks the first order conditions of the initial minimisation program and $\beta^{(\infty)}$ is a local minimum.

According to the objective function, this algorithm shall not converge. The algorithm shall also converge to a local solution which is not a global minimum. The start point of the algorithm has to be correctly chosen and it is important to check after that the algorithm has converged. The start point is given by the non-weighted OLS method, i.e. with $w_i^{(0)} = 1$.