# Technical Report
# Effects of Mergers Involving Differentiated Products
**COMP/B1/2003/07**

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**October 7, 2004**

## INTRODUCTION AND SUMMARY

This Technical Report is offered in accordance with our contract to develop protocols and software for horizontal merger review under COMP/B1/2003/07. The report is divided into five sections. Section I is an overview of merger simulation methods, including estimation of the necessary elasticities to calibrate a simulation model. We discuss econometric estimation of a full AIDS demand system as well as use of logit and PCAIDS models that incorporate restrictions that allow one to perform simulation with much less data. Adding "nests" allows specification of more general logit and PCAIDS models with fewer restrictions on elasticities. In this Report we discuss nests within the PCAIDS framework and show how they may be parameterized exogenously or calibrated based on observed profit margins. We also compare the properties of AIDS, logit, and PCAIDS, since each of these models is likely to predict somewhat different price effects from mergers.

Section II discusses "Critical Loss Analysis" ("CLA"), which has become a widely used methodology for analyzing market definition and competitive effects. We describe the CLA calculation and we review recent literature that discusses interpretation and potential misuse of CLA. We also describe how CLA relates to merger simulation, since the two approaches share important similarities. For example, a recent reformulation of CLA in terms of "diversion ratios" is very similar to the use of cross-price elasticities to

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calibrate a merger simulation model. This section also describes how CLA and merger simulation can be applied to study coordinated effects.

Section III discusses geographic market definition. We review the leading articles on methodology and discuss the practical issues of implementing such methodologies. Experience has shown that the geographic market definition often has to be developed using facts that are highly specific to the case at hand, since suitable datasets for econometric estimation are often not available in a merger investigation. We discuss the role of limited "natural experiments" in interpreting the data on market definition and we compare the analysis used by the parties in Volvo Scania.

Section IV describes the econometric software for demand system estimation and unilateral effects simulation. Section V presents two case studies of estimation and simulation. It also includes commentary on geographic market definition and other aspects of unilateral effects analysis raised in the litigated *U.S. v. Oracle* merger case.

There are two appendices. Section VI is a detailed technical appendix that summarizes the analytical details associated with the econometric estimation of demand models using scanner data and with the simulation of the likely price effects of mergers. Section VII reproduces the full text of the 2002 *Antitrust Law Journal* article by Epstein and Rubinfeld that introduced PCAIDS. The article is included as a convenience for the reader since there are frequent references to it in this Report.

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# I. OVERVIEW OF MERGER SIMULATION

## A. Introduction

Merger simulation is a set of quantitative techniques to predict price effects of mergers with differentiated goods. Applied to unilateral effects analysis, it has been used to assess the magnitude of merger-specific efficiencies (reductions in marginal costs for the merging firms) required to offset predicted price increases and to evaluate the adequacy of proposed divestitures. Simulation can also help analyze the competitive effects of product repositioning and de novo entry. The development of simulation methods is continuing and the scope of their possible application is being broadened. For example, we will describe potential applications of simulation methods to the evaluation of coordinated effects and to Critical Loss Analysis.

In the history of merger analysis, merger simulation is a relatively new entrant. It has been used by the U.S. competition authorities since the early 1990s. While complex in its details, merger simulation is appealing because it allows one to generate quantitative predictions, and (within the framework of a well-specified model) to evaluate the robustness of those predictions. Put simply, merger simulation takes as a starting point a model of equilibrium pricing (typically Bertrand), calibrates that model to the available industry data (such as prices and shares), and uses the model to predict post-merger price changes.[1] Existing techniques focus on the relatively short-term price effects; a particular transaction may also raise concerns about longer-run issues such as the rate of product development and innovation that would require separate analysis.

Merger simulation analysis is carried out in two stages. In the first stage, the estimation of a demand model provides own and cross-price elasticities of demand for the goods in the pre-merger market. In the second stage, one solves the first-order conditions (FOCs) for post-transaction profit maximization by the new, post-merger entity. The post-transaction FOCs differ because they take account of both the cross-price elasticities

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[1] For recent discussions of the methodology of merger simulation see Epstein and Rubinfeld (2002); Werden, Froeb, and Scheffman (2004); and Harkrider and Rubinfeld (2004; forthcoming), chapter 11.

between the two merging firms and the merger-specific efficiencies. Moreover, the demand model implies new elasticities as prices change in the new equilibrium. The solution finds the new post-transaction prices that are consistent with all of these effects.

We discuss three alternative demand models for use in merger simulation. Because these models have been discussed extensively in the literature, we limit our overview to their most important features. The models are (i) the antitrust logit model[2] ("ALM"); (ii) the Almost Ideal Demand System[3] ("AIDS"), and (iii) the proportionality-calibrated AIDS known as "PCAIDS."[4] Each model should be viewed as an approximation to the "true" underlying demand structure. They differ in their data requirements, difficulty in calibration, flexibility in representing price elasticities, and bottom-line predictions of price changes.

The ALM requires only market shares, a measure of substitutability between products, and an estimate of the market demand elasticity. As a tradeoff for the relative simplicity of its inputs, the ALM relies upon a relatively strong assumption that the cross-elasticities are identical across products. This assumption will not always be appropriate in mergers involving differentiated products.

In contrast, AIDS is less restrictive but it requires detailed price and revenue information, generally supplied by scanner data. AIDS is structurally more complex than ALM and frequently requires estimation of dozens of coefficients. It can be a significant econometric challenge to obtain a complete set of coefficients with plausible algebraic signs, magnitudes, and statistical reliability.

The PCAIDS model offers a simplified version of AIDS that requires only market shares, an estimate of the market's demand elasticity, and an estimate of the price elasticity of demand for a single brand in the market. PCAIDS is similar to the ALM in assuming in its most basic form that cross-price elasticities between competing products are equal. PCAIDS and the ALM differ in their underlying mathematical structure, which leads to different predictions of unilateral effects. When the assumption of equal

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[2] Werden and Froeb (1994).

[3] Hausman, Leonard, and Zona (1994).

[4] Epstein and Rubinfeld (2002).

cross-price elasticities is not appropriate, both the ALM and PCAIDS can be generalized by introducing additional "nesting parameters" to make the demand model more flexible.

The models yield different estimates of price effects since elasticities change along the underlying demand curves as prices increase. Own-price elasticities increase and cross-price elasticities will also change. Even if all models are matched to the same set of pre-merger elasticities, the predicted post-merger prices will depend on the "curvature" of the mathematical relationships that define the demand system.

In this Report we spell out many of the technical details associated with the application of merger simulation methods. We also discuss factors that might be considered in deciding whether a particular merger simulation method is appropriate for the industry being studied. Simulation is an important tool but it should not be used uncritically or to the exclusion of other possible analyses of mergers involving differentiated goods.

## B. Parameter Reduction in Demand Models

### 1. Independence of Irrelevant Alternatives ("IIA")

The demand systems needed to carry out merger simulation are confronted by the practical problem of estimating a large number of parameters with limited amounts of data. In general, with N goods there are on the order of $N^2$ unknown own and cross-price elasticities. Even with scanner datasets, econometric estimation of an unrestricted system may be infeasible or the results may not be reliable.

The structure of the ALM and PCAIDS incorporates an assumption that greatly reduces the number of unknown parameters. Specifically, the cross-price elasticities for all goods with respect to the price of any one other good are the same. Formally, $\varepsilon_{ij} = \varepsilon_{kj}$, where the ε's are the cross-price and own-price elasticities. Economists refer to this assumption as the Independence of Irrelevant Alternatives ("IIA") property.

IIA implies that substitution from any good in the choice set (i.e., market) to all others in that set is proportional to their relative market shares. Suppose, for example, that the choice set consists of goods A, B, and C, with respective shares of 60 percent, 30 percent, and 10 percent. If the price of good C is increased, the IIA property says that the substitution to good A must be twice that to B because the share of A is twice that of B. IIA is a way to define what it means for goods to be equally close substitutes for each other. If the substitution away from C is proportional to the relative shares of A and B, then A and B are equally close substitutes for C. The intuition is that if sales of C were stopped altogether, the sales of A and B would increase by amounts that left their relative sizes unchanged.

IIA reduces the number of parameters in the system from $N^2$ to 2N, i.e., N own-price elasticities plus N cross-price elasticities. For markets with 5 brands, for example, it is only necessary to estimate 10 parameters instead of 25. The ALM, and PCAIDS with Slutsky-symmetry and homogeneity restrictions from economic theory, in fact have the property that each system has only two unknown parameters *regardless* of the number of brands in the system. This simplification is valuable because it significantly reduces the data requirements for merger simulation analysis.

IIA provides a convenient basis for inferring substitution patterns if they have not yet been, or cannot be, estimated. In the absence of reliable evidence to the contrary, a reasonable assumption is that the merging firms' products are neither especially close nor especially distant substitutes, which means that IIA applies, at least approximately.[5] IIA also ensures that all estimated elasticities make sense, i.e., that goods assumed to be substitutes have positive cross elasticities of demand.

If the IIA assumption does not describe the actual market, however, then the restriction can result in highly inaccurate patterns of substitution. For example, the implied cross-elasticity can force a high degree of substitution from a luxury car to a minivan if the minivan has a large share in a "market" defined as all passenger motor vehicles. Use of IIA is analogous to imposing a set of restrictions on an econometric model. Restrictions make it possible to obtain values for parameters that either could not be estimated with the available data, or would be estimated with very low precision. Imposing valid restrictions allows the remaining parameters to be estimated more precisely. But imposing invalid restrictions runs the risk of significant bias.

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[5] See Werden and Froeb (2002) and Willig (1991).

When IIA is not appropriate, the alternative is to turn to a more highly parameterized model of demand. The full AIDS is one possibility, although estimating this model is data intensive and involves estimating many more parameters. The other possibility is to specify either the logit or PCAIDS model with nests. In our view, PCAIDS with nests is often the most appealing alternative because it uses a relatively small number of additional parameters and is easy to use for sensitivity analyses.

## 2. Multi-Stage Budgeting Models and AIDS

AIDS is a less restricted system than either ALM or PCAIDS. With no structural restrictions, there are (N–1)(N+2) parameters. Imposing Slutsky-symmetry and homogeneity restrictions reduces the number of parameters to (N–1)(N/2+2), which is still of order $N^2$. Even with relatively large datasets, econometric estimation of so many coefficients can be problematic, with wrong algebraic signs, implausible magnitudes, and low statistical reliability for the estimated coefficients.

Restrictions to reduce the number of free parameters in AIDS may be imposed by assuming a multi-level decision-making process.[6] For example, suppose a dinner entrée in a restaurant could be beef, lamb, salmon, or trout. An unrestricted AIDS with the four goods would involve 18 parameters. But suppose customers first choose whether they want meat or fish, and then make a final choice from the sub-category. This might be represented as separate meat and fish "markets," each with two goods. There would be 8 parameters in all, a substantial reduction (in addition, there needs to be additional structure to determine the initial choice of meat vs. fish).[7]

Rubinfeld (2000) is a case study of a multi-level model in the context of a merger in the ready-to-eat ("RTE") cereal industry. In the RTE industry there are approximately 200 brands. An unrestricted AIDS model would involve nearly 40,000 (200 x 200) parameters. Simplification is necessary to make the model tractable; such simplification is usually achieved through a series of relatively strong assumptions about the

[6] The concept of multi-level budgeting is due primarily to Gorman (1995). For an exposition, see Maddala (1988), p. 66.

[7] See Hausman, Leonard, Zona (1994) for an application to "premium," "popular," and "lite" beer demand.

relationship between brands and the relationship between brands and/or attributes of brands.

With a multi-level decision-making model groups of individual brands are combined into sensible aggregates, and demands for brands in one "branch" or segment of a "tree structure" are assumed to be separable from the demands in other branches. As Rubinfeld (2000) suggests, "one might think of cereal choice as occurring at the third stage of a three-stage decision-making process. The top level determines the demand for RTE cereal, the second level divides the choice of the 200 cereal brands into three segments (Kid cereals, Family cereals, and Adult cereals), and the third stage determines the demand for brands within one of the three segments."[8]

This process greatly reduces the number of parameters to be estimated, but at a cost. The greater the number of restrictions built into the multi-level budgeting assumptions, the more the sensitivity of the resulting price predictions to those restrictions. Rubinfeld points out that by construction cross-price elasticities between products in different segments are likely to be small. The sensitivities of the merger simulation method to the specification of the multi-stage budgeting process have been discussed in a significant U.S. merger case involving the cereal industry.[9] Given these sensitivities, it is a good practice to evaluate the robustness of any simulated prices effects to the multi-stage model specification.

### C. ALM Structure

The ALM is a reformulation of the conventional logit model designed to make it more functional for antitrust practitioners.[10] Competitive interactions among goods are completely characterized by their shares and prices, which are observed, and values for two key parameters $\beta$ and $\varepsilon$, which must be estimated. The $\beta$ parameter measures the substitutability of the goods for each other (i.e., own and cross-price elasticities). The

[8] Rubinfeld (2000), pp. 173-174.

[9] *New York v. Kraft Gen. Foods, Inc*. 926 F. Supp. 321 (S.D.N.Y. 1995). Rubinfeld (2000), Appendix spells out how one should calculate the cross-price elasticities between brands that are in different segments.

[10] For a more detailed discussion of the application of the logit model, *see* Werden et. al. (1996).

value of $\beta$ can be estimated from aggregate data on prices and quantity of actual transactions, household level data on actual choices, or survey data. A natural experiment such as the entry and exit of a brand may also reveal patterns of sales diversion to develop an estimate. The $\varepsilon$ parameter is the price elasticity for the market as a whole and can be estimated using aggregate data.

The underlying demand model for the ALM is specified as follows.[11] Consumers make a discrete choice from a set of $n$ alternatives, where the choice provides the greatest utility. That is, the choice is a quantity (e.g., kilos of bread, liters of beer). The indirect utility function associated with choice of product j by consumer i is specified as

$$U_{ij} = \alpha_j - \beta p_j + e_{ij} .$$

The utility is a function of the own price $p_j$ and the fixed effect $\alpha_j$ summarizes perceived relative quality differences products. The price coefficient $\beta$ is assumed to be constant for all consumers and products. The $e_{ij}$ is an error term that measures the deviation of consumer i's utility from the mean utility for product j. The key probabilistic assumption in ALM and other logit-based choice models is that the error terms are independently and identically distributed according to the type I extreme value distribution (see Maddala, 1988). Given this distribution for the errors, the probability of choosing product j is given by

$$\pi_j = \exp(\alpha_j + \beta p_j) / \sum \exp(\alpha_k + \beta p_k), \ \ k=1\ldots n$$

This model describes choices over every good the consumer might purchase. In the context of merger simulation, the choice set is narrowed by categorizing the products/brands of interest as "inside" goods and aggregating all other goods as a single "outside" good. In particular, let product $n$ be the aggregate outside good and assume $p_n = 0$ to assign it a constant utility. Shares are then defined in the conventional sense by terming the inside goods the "market," so that each share $s_j$ equals the choice probability, conditional on the choice being an inside good. That is, $s_j = \pi_j / (1 - \pi_n)$. These quantity shares are observable (note that the logit theory is not developed in terms of revenue shares).

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[11] This discussion is based on Werden and Froeb (1994).

Let $\varepsilon_j$ be the own-price elasticity, $\varepsilon_{jk}$ be the cross-price elasticity with respect to the kth good, and let ε be the market elasticity. Finally, let *pbar* be the share-weighted average price. Then the elasticities for the system are[12]

$$\varepsilon_j = [\beta\, pbar(1 - s_j) + \varepsilon s_j]\, p_j \,/\, pbar$$
$$\varepsilon_{jk} = s_k(-\beta\, pbar + \varepsilon]\, p_k \,/\, pbar\ .$$

As Werden and Froeb emphasize, this formulation is appropriate only if the prices are appropriately normalized, e.g., price is measured per unit of volume or weight.[13] As a two parameter system, the ALM is generally calibrated with estimated values for ε and $\beta$. (Observe that is it possible to solve for $\beta$, given values for ε and a single own-price elasticity). Werden and Froeb (1994) also show that $\varepsilon = \beta\ pbar\ \pi_n$. So, the market elasticity also changes post-merger as *pbar* and $\pi_n$ change.

To complete calibration of the model, it is necessary to find the $\alpha$'s. As fixed effects, differences among the $\alpha$'s are all that matter so it is convenient to set $\alpha_n = 0$ as a normalization. Then, from the definition of the $\pi$'s, $\ln(\pi_j\ /\ \pi_n) = \alpha_j - \beta p_j$. From the definition of $s_j$, it follows that $\alpha_j = \ln((1 - \pi_n)/\pi_n) + \ln(s_j) + \beta p_j$, j=1…$n$–1. The outside good probability is updated dynamically in the simulation by setting $\pi_n = 1/\ (1+\sum\exp(\alpha_k - \beta p_k))$, k=1…$n$–1.

In the ALM, prices increase as a result of a merger, but the magnitudes of the price increases for different brands are different. All else equal, larger shares for the merging firms result in larger unilateral effects. If the merging brands have significantly different shares, the merger has asymmetric effects on the prices of those brands. The price of the smaller-share brand increases more than that of the larger-share brand. The explanation is that the larger brand serves as a larger "magnet" and is better able to capture sales diverted from the smaller brand. In addition, the prices of the merging brands typically increase much more than the prices of non-merging brands. Increased concentration

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12 Werden and Froeb (1994), p. 410.
13 Werden and Froeb, p. 409.

among the non-merging brands increases the price effects of a merger, but the effect is typically fairly weak.

Finally, we note that the study of logit choice models is an active area of research in economic theory. For example, one generalization is the “mixed” or “random-coefficients” logit model. These models incorporate customer heterogeneity by specifying that the observed demand is a mixture of distinct individual demands for consumers who have different characteristics. Suppose high-income consumers have relatively inelastic demands, while low-income consumers have relatively elastic demands. Observed demand then can be modeled as the weighted average or “mixture” of two logit demands, with the weights being the population proportions of high- and low-income consumers. Mixed logit models are more flexible than the ALM but the data requirements are intensive and estimation is considerably more complex.[14]

### D. AIDS Structure and Estimation

AIDS is a different model of demand. It is interpreted as a first-order approximation to any demand system, so it does not make the same structural assumptions as the ALM. AIDS explains the share of each good as a linear function of the logarithms of the prices of each of the N goods in the market and the “real” expenditure in the market. In contrast to logit, the shares are in terms of *revenue*, since the model was derived from an analysis of consumer expenditure (the details in terms of the underlying microeconomic theory are set forth in Deaton and Muellbauer, 1980). For example, the share equation for $s_i$ is

$$s_i = a_i + \sum b_{ij} \ln p_j + h_i \ln(x/P), \qquad j = 1 \text{ to } N$$

where $x$ is total market expenditure and $P$ is a price index. Each “own-coefficient” $b_{ii}$ specifies the effect of each brand’s own price on its share. These coefficients should have negative signs, since an increase in a brand’s price should (all other prices held constant) reduce its share. The “cross-effect” coefficients $b_{ij}$ should have positive signs (assuming that brands are substitutes), since these terms are related to the cross-price elasticities.

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[14].*See, e.g.*, Berry, Levinsohn, and Pakes (1995); Nevo (2000a).

AIDS has a number of desirable properties including flexibility in modeling elasticities, the ability to impose and test the properties of consumer demand, and the ability to aggregate. As already discussed, the downside to flexibility is the large number of parameters that need to be estimated.

Empirical implementation of AIDS may differ from the basic specification. The share equations can be modified to account for factors other than prices and expenditure. For example, consumer preferences for the product might grow over time or be seasonal. In addition, consumers in one geographic area might have a greater preference for the product than the consumers in other geographic areas. The expanded model may include fixed effects, trends, and seasonal variables. Retail scanning data on advertising and in-store promotional activity may also be available to augment further the estimated model.

AIDS estimation generally requires decisions about data aggregation. In particular, each brand may have different package sizes or minor differences in variety. Specifying a demand system to account for all of the individual products is not realistic. Instead, the products must in some way be aggregated and the demand system specified for the aggregates. The question then is the proper degree of aggregation and the appropriate aggregation method to use.

When disaggregated data are available, the degree of aggregation that should be undertaken is the outcome of practical considerations and the desire not to distort the econometric estimates. A good way to proceed is to test the effect of using different levels of aggregation within a range dictated by the practical considerations given the number of products in the category. In some cases the degree of aggregation does not significantly affect the results, while in others it does.

When high-frequency (e.g., weekly) data are used, a danger exists that the elasticity estimates obtained from a demand system represent short-run behavior rather than long-run behavior. Specifically, if consumers stock up on products when they go on sale, their short-run responsiveness to price changes (i.e., sales) might exceed their long-run responsiveness to price changes (i.e., permanent price changes). Consumer inventorying behavior could lead to incorrect conclusions when focusing on long-run elasticities.[15]

### E. PCAIDS

PCAIDS is an approximation to the AIDS model that requires much less data. It uses only revenue market shares and values for two elasticities—the price elasticity of industry demand and the price elasticity of any one product. This simplicity is achieved primarily by placing IIA restrictions on the structure of the AIDS model.

PCAIDS is similar to the ALM in that both rely on IIA to achieve reduction to a two-parameter system. There are differences, however. First, the predicted unilateral effects from PCAIDS differ from the ALM. The PCAIDS effects can be either higher or lower. These differences stem from different mathematical curvature of the functions underlying the demand systems and also from the possibility that revenue-based shares (for PCAIDS) differ from quantity-based shares (for the ALM). In addition, a constant industry elasticity is assumed in conventional implementations of PCAIDS (and AIDS), whereas industry demand becomes more elastic in the ALM. The two models might be viewed as providing approximate upper and lower bounds on the likely price effects of the transaction. Second, it appears easier to relax the IIA assumption for PCAIDS. Since proportionality may not be appropriate for many markets, it is important to be able to investigate the effect of not using it, and this analysis is easily manageable with PCAIDS. The third difference concerns aggregation. The revenue shares in PCAIDS are easy to calculate for "brands" defined as composites of underlying products. The logit model uses quantity shares, which can be difficult to define for aggregates of differentiated goods.

A fourth difference between PCAIDS and the ALM is the role of prices. As discussed earlier, the ALM requires suitably normalized prices, and the elasticities are functions of the prices. PCAIDS does not require price data and prices do not enter into the elasticity calculations. In this sense, PCAIDS has fewer data requirements. Because

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[15] For additional discussion of empirical estimation using scanner data, see Hosken, O'Brien, Scheffman, and Vita (2002) available at www.ftc.gov.

the two methods have different strengths, we think it is valuable in practice to compare results using both. The software that is provided along with this Report allows for these comparisons.

### 1. PCAIDS Calibration without Nests

Basic calibration of PCAIDS is achieved by assuming IIA (i.e., proportionality) and suppressing the expenditure term from the full AIDS.[16] The industry price elasticity $\varepsilon$ is assumed known. Assume also that $\varepsilon_1$ (the own-price elasticity for the first brand) is known. Then it can be shown that the entire set of elasticities for the market is given by[17]

$$\varepsilon_j = [(1 - s_j)\varepsilon_1 + (s_j - s_1)\varepsilon] / (1 - s_1)$$

$$\varepsilon_{jk} = s_k(\varepsilon - \varepsilon_1) / (1 - s_1)$$

PCAIDS is therefore a two-parameter system. As a two parameter system, PCAIDS can also be calibrated with values for any pair of price elasticities (e.g., $\varepsilon_1$ and $\varepsilon$ can be found given $\varepsilon_j$ and $\varepsilon_{jk}$).

### 2. Deviations from Proportionality — PCAIDS with Nests

Proportionality will not always characterize the diversion of lost sales accurately when products are highly differentiated. Fortunately, it is straightforward to modify PCAIDS to allow a more general analysis. The approach is to group brands in different "nests." IIA is assumed to hold *within* a nest, but not *across* nests. This allows a more flexible pattern of cross-price elasticities.

We illustrate the nest approach with a three-firm market, with shares of 20%, 30%, and 50% respectively. Firm 1 contemplates a price increase. Under proportionality, brand 2's market share of 30% and brand 3's share of 50% imply that 37.5% (30/80) of the share lost by brand 1 from a price increase would be diverted to brand 2 and 62.5%

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[16] PCAIDS suppresses the expenditure term under the assumption that data to measure this effect are not available. Though lacking the expenditure term, the PCAIDS brand price elasticities in general are different from AIDS price elasticities that impose homotheticity (compare equations (5) and (6) to the AIDS elasticities in Alston, Foster and Green, 1994). Consequently, the calculated brand elasticities from PCAIDS will, in general, differ from the unrestricted AIDS.

[17] See Epstein and Rubinfeld (2002), Appendix.

(50/80) would be diverted to brand 3. That is, under proportionality, brand 2 is only 60% (.375/.625) as likely to be chosen by consumers leaving brand 1 as brand 3.

Now suppose instead that brand 2 is relatively "farther" from brand 1 than would be predicted by proportionality in the sense that fewer consumers would choose brand 2 in response to an increase in the price of the first brand. For example, brand 2 may only be "half as desirable" a substitute as brand 3 so that it is only 30% as likely to be chosen instead of 60%. We describe this effect in terms of a "nesting parameter." In this case the nesting parameter equals 50% because the odds of choosing brand 2 are now only half the odds predicted by proportionality. It is straightforward to calculate that the implied share diversion to brand 2 becomes 23.1% and the diversion to brand 3 increases to 76.9% (implying odds of 30% =.231/.769). As expected, fewer consumers leaving brand 1 would choose brand 2.

The equations for PCAIDS elasticities with nests are more complex and have been presented elsewhere.[18] They now depend on the original parameters $\varepsilon$ and $\varepsilon_1$ and the set of nesting parameters. We summarize this example by comparing the implied elasticities with and without the nest for brand 2.

| | Non-Nested Demand Elasticity with Respect to: | | | Separate Brand 2 Nest, (Nesting Parameter = 0.5) Elasticity with Respect to: | | |
|---|---|---|---|---|---|---|
| Brand | $p_1$ | $p_2$ | $p_3$ | $p_1$ | $p_2$ | $p_3$ |
| 1 | –3.00 | 0.75 | 1.25 | –3.00 | 0.46 | 1.54 |
| 2 | 0.50 | –2.75 | 1.25 | 0.31 | –2.08 | 0.77 |
| 3 | 0.50 | 0.75 | –2.25 | 0.62 | 0.46 | –2.08 |

The nest has a variety of effects. The cross-price elasticities for brand 2 in the right-hand panel are scaled down by 50% relative to the other brands. That is, IIA no longer holds (the cross elasticities $\varepsilon_{12}$ and $\varepsilon_{32}$ remain equal because brands 1 and 3 are in the same nest). The nest implies diminished interbrand competition, as reflected by the

[18] Epstein and Rubinfeld (2002), Appendix.

smaller own-price elasticities for brands 2 and 3. However, brands 1 and 3 become relatively more substitutable on the basis of cross-price elasticity.[19]

The number of nesting parameters required in the model obviously depends on the number of nests. More specifically, the number of parameters equals the number of pairs of nests, because each parameter modifies the share diversion between the two associated nests. With 2 nests there is one nesting parameter; a 3-nest specification requires three parameters; and a 4-nest specification requires six parameters. Because the number of nesting parameters increases exponentially with the number of nests, a tractable simulation model probably should not have more than 3 or 4 nests.

What remains is the difficult question of when proportionality is inappropriate, making nests necessary for accurate merger simulations. To this point, there has been very little empirical testing of this question. Fortunately, PCAIDS makes it easy to detect whether nesting is likely have economically meaningful effects through a sensitivity analysis of the nesting parameters. A coarse grid (e.g., 0.75, 0.50, and 0.25) covering a range of nesting factors may be adequate to assess sensitivity.

### 3. Using Brand-Level Profit Margins to Calibrate PCAIDS with Nests

There is a potentially more useful, data based approach to the estimation of nesting parameters that relies on brand-level margin data.[20] Assume, for example, that firms 1 and 2 are the merger partners each of which produces a single brand pre-merger. Suppose further that the profit margins are available for these brands, since the merger authority may be able to obtain this information. PCAIDS yields the own-price elasticities, which in turn imply margins for each brand equal to $-1/\varepsilon_1$ and $-1/\varepsilon_2$, respectively. Since $\varepsilon_1$ was used to calibrate the model, the corresponding margin must be consistent with this value. If the implied brand 2 margin is also close to the actual, this indicates that a model without nests provides a good fit to the data.

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[19] The calculations assume an own-price elasticity of –3 for Brand 1 and an industry elasticity of –1. It would be incorrect to scale the non-nested elasticities in the left-hand panel directly. Nests affect the impact of adding-up, homogeneity, and symmetry, and the appropriate calculation takes account of these constraints to generate economically consistent elasticities.

[20] For more discussion, see Epstein and Rubinfeld (2004).

But the implied margin for brand 2 may not be close to the actual margin. This suggests that brand 2 belongs in a separate nest, since the result of imposing IIA is not consistent with the data. It is then possible to solve for the nesting parameter that yields an implied brand 2 margin that is equal to the actual. The nesting parameter would therefore be determined empirically.

A procedure to find the nesting parameters is as follows (for simplicity, the industry elasticity is assumed to equal –1). Let $E = \text{diag}(E_1, E_2)$ be the transposed block-diagonal matrix of pre-merger brand elasticities for the two merging firms. Let $B$ be the corresponding block-diagonal matrix of PCAIDS coefficients. It is straightforward to show that $E = BS^{-1} - I$, where $S$ is the diagonal matrix of corresponding brand shares and $I$ is a conformable identity matrix. Let $s$ be the vector of shares for the brands sold by the merging firms. As shown in Epstein and Rubinfeld (2002, Appendix), the vector of predicted pre-merger brand margins $\mu^{\text{pre}}$ for the merging firms is given by

$$\mu^{\text{pre}} = -S^{-1}\text{diag}(E_1, E_2, \ldots, E_n)^{-1}s.$$

As also shown in Epstein and Rubinfeld (2002, Appendix), the elements of the $B$ matrix are functions of the nesting parameters ω, as well as the shares and the exogenous $b_{11}$, i.e.,

$$b_{\text{ij}} = -\frac{s_{\text{i}}s_{\text{j}}}{s_1}\frac{\omega(\text{i},\text{j})}{\sum_{\text{k}=2}^{N} s_{\text{k}}\omega(\text{k},1)}b_{11}$$

for i≠j. The elasticities that determine the margins are therefore functions of the shares, $b_{11}$, and the nesting parameters.

The PCAIDS software has the capability to use manual iteration to find $b_{11}$ and the nesting parameters that yield the observed margins.[21] The recommended procedure is to specify an exogenous brand elasticity that results in the corresponding brand margin that matches the observed margin. Then, one should iteratively search over nesting parameters in the (0,1] interval until parameters are found that yield margins for the other brands that match the observed values. The software prints out the implied pre-merger margins for this purpose.

[21] It should be possible in future work to automate the solution process.

The "margin method" is potentially very useful, but it also raises its own set of questions.[22] For example, we expect that the incremental margins will be obtained through examination of a company's internal cost accounting reports. These reports may require adjustment to eliminate allocated fixed costs or to include additional incremental costs (such as selling expenses), depending on how the accounts in the system are defined. Moreover, deriving nesting parameters for multi-brand firms can lead to problems of overidentification (i.e., a range of nesting parameters would be consistent with the margin data) or underidentification (the data would not be sufficient to estimate all the nesting parameters). Nevertheless, the structure of PCAIDS still provides bounds for the nesting parameters that are consistent with the available information.

It is possible that reasonable nesting parameters to generate the exact actual margins will not exist in certain situations (see Epstein and Rubinfeld, 2004). This can happen for two main reasons. First, the system may be overidentified. In this case the number of equations (i.e., first-order conditions for brands with known margins) would exceed the number of unknown nesting parameters (which equals the number of unique pairs of nests). Second, one or more of the nesting parameters may require a value outside the (0,1] interval to generate the margins. "Fitting" the model can therefore give rise to either a deviation of an implied margin from an actual margin or a deviation of an implied nesting parameter from the feasible range.

The best way to proceed in this situation is case specific and requires judgment. For purposes of this discussion, we assume that actual margins have been determined as accurately as possible, so that measurement error is not a factor. The analysis should first consider whether the deviation is material. For example, if the actual and predicted margins differ by a small amount, it may be reasonable to ignore the difference by attributing it to the approximation error inherent in using any economic model. Similarly, an implied nesting parameter only slightly greater than 100% might be set to 100% if the resulting implied margins are only slightly affected (note that a nesting parameter of 100% means that the two nests should be combined into a single nest).

[22] For a detailed discussion see Epstein and Rubinfeld (2004).

Large deviations may indicate that brands have been not assigned to appropriate nests—regrouping the brands may result in a closer "fit" in terms of margins.

Ultimately, however, if no scenario yields implied margins that are reasonably close to the actual with acceptable values for the nesting parameters, it may be the case that one (or more) of the more fundamental assumptions underlying the demand model should be examined. In PCAIDS these assumptions include Bertrand equilibrium, homogeneity, Slutsky-symmetry, and symmetry of the matrix of nesting parameters. How one can best reconcile the model with the observed margin data, when they differ, should be decided on a case-by-case basis.

### F. General Solution of the Post-Merger FOCs

The same solution method for the post-merger price effects can be used when Bertrand pricing is assumed, regardless of the demand model. Recall that in the basic differentiated Bertrand model, firms compete over price.[23] Each firm maximizes its profit, conditional on its belief that the prices of competing firms remain unchanged. An equilibrium is reached when no firm wishes to change its pricing decision.

There is a FOC equation for each brand in the market. Under Bertrand price competition, a general expression for all of the FOCs is given by the matrix equation:

$$s + \text{diag}(E_1, E_2, \ldots, E_n)S\mu = 0.$$

where $E_i$ is the transposed matrix of own-price and cross-price elasticities for brand i. These pre-merger elasticities are provided by the demand model. The vector $s$ contains the brand revenue shares (even when using the ALM as the demand model), $S = \text{diag}(s)$, and $\mu$ is a vector of brand-level incremental profit margins. The models we consider treat incremental unit costs as independent of output pre- and post- merger but efficiencies are allowed to shift these costs.

The first stage of a simulation is used to calculate the brand-specific margins $\mu$. Assuming the pre-transaction shares and elasticities are known, the margins are given directly by:

[23] See, for example, Pindyck and Rubinfeld (2005), Chapter 11, or Carlton and Perloff (2000), p. 166-172.

$$\mu^{\text{pre}} = -S^{-1}\text{diag}(E_1, E_2, \ldots, E_n)^{-1}s.$$

The second stage analyzes the FOCs to predict price changes due to the transaction. In general, the post-transaction shares, elasticities, and margins are functions of the price changes. To simplify the notation, assume that the merger involves firms 1 and 2. There are $n$–1 firms in the post-transaction market, but the number of brands remains $N$. The merged firm requires a new cross-elasticity matrix $E^{+}$ for the $n_1$ plus $n_2$ brands it is now producing. The FOCs for the second stage are:

$$s + \text{diag}(E^{+}, E_3, \ldots, E_n)S\mu = 0, \qquad (1)$$

where all variables are understood to be taken at their post-transaction values.

To understand the solution of (1), consider the relation between $\mu^{\text{pre}}$ and $\mu^{\text{post}}$. For the jth brand,

$$c_j^{\text{pre}} = (1 - \mu_j^{\text{pre}})\, p_j^{\text{pre}}\,.$$

It follows from the definitions that $c_j^{\text{post}} = (1-\gamma_j)c_j^{\text{pre}}$ and that $p_j^{\text{post}} = \exp(\delta_j)p_j^{\text{pre}}$. In this notation, $\gamma_j$ is the merger-specific efficiency (i.e., percentage reduction in incremental cost) and $\delta_j$ is the unilateral price effect.[24] As a result,

$$\mu_j^{\text{post}} = 1 - c_j^{\text{post}}/p_j^{\text{post}}$$
$$= 1 - (1 - \mu_j^{\text{pre}})\,(1-\gamma_j)/\exp(\delta_j)\,.$$

This relationship can be expressed in matrix notation for all brands as

$$\mu^{\text{post}} = \mathbf{1} - \Gamma\Delta^{-1}(1 - \mu^{\text{pre}}),$$

where $\mathbf{1}$ is an $N$ vector of ones.

The second stage FOC can now be written as a function of the percentage price changes:

$$s + \text{diag}(E_1, E_2, \ldots, E^{*}_{n-1})S\,[\mathbf{1} - \Gamma\Delta^{-1}(1 - \mu^{\text{pre}})] = 0, \qquad (2)$$

where the price changes also generate post-transaction shares and elasticities through the demand model. That is, the solution to (2) is framed entirely in terms of finding the vector δ that solves the system of equations. Observe that the pre-transaction prices and costs $p^{\text{pre}}$ and $c^{\text{pre}}$ are not needed in the analysis.

Simulation of divestiture of a brand from the ith firm to the jth firm is accomplished by suitable definition of the price elasticity matrices. The rows and columns corresponding to the brands to be divested are deleted from $E_i$. When the jth firm is an

[24] See also section VI.D.2 below.

incumbent in the market, $E_j$ is augmented by a new row and a new column containing the own-price elasticity and the cross-price elasticities with the other brands for the firm. When the divestiture is to an entrant, the number of firms in the post-transaction market increases by one and an additional elasticity matrix is defined that consists of a single element equal to the own-price elasticity for the divested brand.

### G. Simulation and Product Market Definition

Unilateral effects merger simulation does not require that a relevant antitrust product market be defined. Instead, the procedure may be viewed as a way to model price effects for any given set of firms and brands, regardless of whether they constitute a formal relevant market. An example is the acquisition by Post of Nabisco's ready-to-eat cereal assets (see Rubinfeld, 2000). In the cereal merger, market definition played at best a minimal role in the evaluation of unilateral effects (but market definition was central to the analysis of coordinated effects). If the simulated effects are below the relevant threshold, then the merging brands do not constitute a relevant antitrust market. Sufficiently large predicted price effects suggest that the merging brands are a separate market. The key issues are the magnitude of the "market" elasticity and the own and cross price elasticities of the products explicitly included in the simulation.

To elaborate, the "market" elasticity in the model measures the "leakage" of demand, given the universe of brands that has been included in the analysis. For this reason, the market elasticity used in a merger simulation should depend on the number of different brands under consideration. All else equal, a small number of brands implies a higher market elasticity compared to a simulation with a larger number of brands. This "top level" adjustment is required for each of the models we have discussed: logit, PCAIDS (with or without nests), and AIDS. Indeed, merger simulation focuses market definition (at least for unilateral effects) as an analysis of the appropriate market elasticity, given the brands under consideration.

An advantage of merger simulation is that it provides a framework for estimating the market elasticity. Given the goods included in the analysis, one possibility is to construct an appropriate price index to estimate an aggregate demand elasticity econometrically.[25] The econometric model typically uses quantity or revenue as the dependent variable, and the right-hand side would include the price index as well as income and any other relevant control variables.

The relationship between merger simulation and market definition may also be approached from a different perspective. The correct relevant antitrust market is generally not known with certainty because it typically cannot be observed. The goods that actually compete with the merging firm's products are usually identified on the basis of prices and other data from the pre-merger market. The relevant antitrust market, at least a definition based on the "Small but Significant Non-transitory Increase in Price" ("SSNIP") test, goes beyond this analysis by asking which additional competitors would constrain the merging parties if the attempt were made to raise prices unilaterally post-merger. That is, the relevant market must also take account of entry and possible product repositioning. Merger simulation still captures these effects, however, to the extent that they contribute to the market elasticity used in the analysis.

Suppose a market definition is proposed which consists of at least the merging parties plus a set of existing competitors. The merger simulation can then test whether these competitors are sufficient to constrain unilateral price increases. If the simulated price effects are small, then one might infer that the merging firms do not constitute a market and that the merger does not pose a threat to competition. If the simulated effects are large, then one might investigate the role of entry, which is not modeled directly by the simulation.[26]

In sum, simulation is typically carried out using the firms identified in a candidate market definition that are already competitors of the merging firms. If the analysis shows that the unilateral effects are small, then one will have some assurance that the proper relevant market is broad and the merger is not anticompetitive. If the unilateral effects are large, then the simulation focuses attention on whether the universe of included firms is sufficiently large or whether entry on a sufficient scale is plausible. The simulation

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[25] See, e.g., Hausman and Leonard (2002), p. 248.
[26] For discussion of entry in the context of merger simulation see Epstein and Rubinfeld (2002).

does not presuppose a market definition. Instead, it provides a valuable empirical check on whatever market definition is under consideration.

### H. The Choice of Merger Simulation Model

Critiques have been made on a number of levels with respect to the use of merger simulation methods and the choice of demand model to be used as the basis for a simulation analysis. The most sweeping critique may be that simulation should not be used because it is too limited by the assumption of static, Bertrand competition. Moreover, even if Bertrand competition is accepted, the validity of IIA may be challenged in a given market when using the ALM or PCAIDS. Finally, an issue arises with respect to the interpretation and reconciliation of the simulation results, when one uses different underlying demand models.

#### 1. The Bertrand Assumption

The Bertrand pricing assumption is standard in existing simulation models for differentiated products both because it is intuitively plausible and analytically tractable. It treats a firm as having market power as a result of the product differentiation; however, the amount of market power can range from virtually none (i.e., perfectly competitive pricing) to a high level, depending on the particular circumstances. Moreover, the static Bertrand framework represents an advance over mere reliance on market shares for merger analysis, and (while the relevant literature is small) is supported by the empirical literature.[27]

Nonetheless, we do not claim that a static Bertrand model is universally applicable. It may not adequately describe prices if a large fraction of output is sold in a small number of "winner take all" auctions that might arise, for example in certain wholesale markets. Other examples of market structures where a static Bertrand assumption may not be appropriate include process industries (e.g., paper mills or steel mills) that operate continuously, industries with significant network effects (e.g., airlines), and industries with wasting assets (e.g., cruise ships, where lost revenue from unsold seats creates an

[27] See Berry, Levinsohn, and Pakes (1995) and Slade (2004).

incentive to operate at full capacity). If firms have excess capacity, product differentiation is limited, and/or incremental profit margins are high, the incentive to engage in rivalrous behavior may outweigh the unilateral effects predicted by the simulation. On the other hand, economic theory indicates that Bertrand behavior with homogeneous goods when there are capacity constraints can lead to Cournot outcomes, which clearly can be less than perfectly competitive.[28]

Even when the static Bertrand assumption is accepted as a working hypothesis, it is possible that a particular market will respond to price changes in a way that is not consistent with the simulation method. For example, the ALM, PCAIDS, and AIDS allow the demand elasticities to change with post-merger price increases, but these functional forms typically result in relatively limited variation in the elasticities. If price increases as small as 5% or 10% would affect the price elasticities very substantially, then the simulation will probably overstate the likely competitive effects. But in our view the burden should be on the party making such a claim to provide adequate factual support.

In sum, it would be a mistake either to dismiss merger simulation or to accept the results uncritically. Simulation is a valuable and uniquely informative tool that should be routinely available in a merger review. But in the end each transaction has to be judged using the totality of the economic evidence, of which simulation is only a part.

### 2. Testing IIA

The IIA assumption (like any assumption) may be criticized. It is clearly desirable to assess whether IIA is an appropriate assumption and to have a model specification strategy that allows for deviations from proportionality. We offer several different strategies for testing whether IIA characterizes demand in a given market.

First, if adequate data are available, estimation of the full AIDS is one possibility, since AIDS does not impose IIA. IIA could be tested by imposing an appropriate set of restrictions on the estimated coefficients. Recall that $b_{ij}$ in AIDS measures the share diversion from brand j to brand i in response to a price increase for brand j. IIA implies

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[28] Kreps and Scheinkman (1983).

that $b_{ij}/b_{kj} = s_i/s_k$, so the estimation procedure can impose the linear constraint $b_{ij} = (s_i/s_k)b_{kj}$. When imposing these restrictions, it is important not to be misled by statistical rejection of the null hypothesis of IIA. The demand system may be close to satisfying IIA, but this null hypothesis may still be rejected as an artifact of the large sample sizes generally available with scanner data. For example, in an econometric model the null hypothesis may assert that the true value of a given regression coefficient is zero when in actuality the coefficient differs from zero but by only a small amount. Because the null hypothesis is not *exactly* true in this case, standard hypothesis testing procedures that hold the probability of a Type 1 error fixed (e.g., at, 5%) are likely to reject the null hypothesis in a large sample. In the context of testing IIA, suppose that the ratio of the shares of two brands was 3 to 1 but the ratio of the respective estimated AIDS coefficients was 3.1 to 1. Economically, the AIDS results indicate good support for IIA, but the null hypothesis of IIA would likely be rejected in a large sample.

Second, marketing surveys or brand switching studies may exist that allow one to infer cross-price elasticities (see Baker and Rubinfeld, 1999 and Werden, 2000). Cross-price elasticities that are significantly different with respect to the same price change would be evidence that proportionality does not hold. A customer survey could also be designed to provide evidence on whether IIA is appropriate. For example, the survey could posit a 5% price increase for a given brand (which can be one of the merging brands or any other brand included in the simulation analysis) and ask the respondent to rank preferences for each of the other brands. At a minimum, evidence for IIA would require the preferences to be correlated with the shares of the other brands.

Third, if there is no independent source of information about cross-price elasticities, then it is easy to use PCAIDS with nests to test the sensitivity of results to IIA by specifying different nests and plausible nesting parameters. If the unilateral effects vary significantly, then the situation is no different from an ordinary econometric analysis when the estimation results are sensitive to model specification. In both situations, we think it is appropriate to rely on the judgment of experienced researchers to reach the most defensible outcome, or range of outcomes.

## 3. Comparing the ALM, PCAIDS, and AIDS

The ALM and PCAIDS have potentially testable differences in terms of the predicted pre-merger margins. If brand margin data are available, the actual margins may be compared to the predicted margins from the two models. Even if margins are only available for one of the firms, a comparison is still possible if the firm produces multiple brands. PCAIDS treats all the brands in the same nest for the same firm as a single aggregate brand in the sense that the pre-merger margins are identical. The ALM yields different margins when the brands have different prices. To be specific, suppose firm A produces brands 1 and 2. If the brands are placed in the same nest, PCAIDS will calculate identical margins and identical unilateral effects for them. However, the ALM will generate different margins and effects if they have different prices. If the empirical margins differ from the PCAIDS predictions, then nests should probably be used in PCAIDS to create a structure that more closely approximates the empirical margins. Similarly, empirical margins that differ from the ALM predictions suggest that a nested logit model could fit the data better than ALM.

If data are available to estimate the full AIDS and a logit demand system, non-nested hypothesis testing procedures can indicate which specification has a statistically better fit.[29] It is also possible, as discussed above, to impose constraints on the AIDS model to test IIA. If the hypothesis of IIA is not rejected, then one can restrict attention to the ALM and PCAIDS (we assume here that the expenditure term is not likely to be important).

In a given situation there is a choice between logit, PCAIDS, nested PCAIDS, and, data permitting, unrestricted AIDS. In practice, we think it is advisable to use each model, data permitting. Each model will yield different unilateral effects because they have somewhat different internal mathematical structures. The key question is whether the results yield conflicting indications as to the likely unilateral effects associated with the merger (especially in relation to price increase thresholds (e.g., 5%) used by the Commission). In an ideal world, different models will all lead to the same conclusion. But when they do not, expert judgment may help in deciding which analysis should

[29] See, for example, Mizon and Richard (1986).

receive the most weight. For example, the complete matrix of own and cross-price elasticities may appear more reasonable in one of the models. Alternatively, there may be strong a priori grounds for specifying particular nests.

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# II. CRITICAL LOSS ANALYSIS

## A. Introduction

Critical loss analysis ("CLA") has become widely accepted in merger analysis since it was introduced in 1989.[30] The method has applications to unilateral and coordinated effects analysis and market definition (both product market and geographic market). CLA asks a simple question: for a given price increase, what is the smallest loss of sales (in percentage terms) that would make the price increase unprofitable for a hypothetical monopolist? This loss is termed the Critical Loss ("CL"). The analyst then needs to determine the Actual Loss of sales in response to the price increase. If the Actual Loss would be greater than CL, then the price increase would not be profitable. Depending on the context, an Actual Loss greater than CL implies that a unilateral or coordinated price effect equal to the given price increase was not a concern, or that the goods involved did not form a separate antitrust market.

For example, suppose two adjacent aluminum firms each sell ingot at $100/ingot and that each firm has an incremental cost of $50/ingot. Suppose also that each firm has sales of 100 ingots. Each firm therefore has an incremental profit of $5,000 (100 units times $50 profit/unit). Are the products of the two firms in a relevant market? To answer the question, one might use the criterion that the products are in the same market if a simultaneous and permanent 5% price increase by both firms would be profitable. Such a price increase would be profitable if the combined firm's unit sales declined by no more than 9.1% (the CL). In other words, if the Actual Loss is predicted to be less than 9.1% then the two firms would form a relevant market. It can be shown in general that CL is equal to Y/(Y+CM), where Y is the hypothesized percentage price increase and CM is the actual contribution margin (i.e., percentage incremental profit margin, (P–c)/P). In this example CL = 5% / (5% + 50%) = 9.1%.

The original Harris and Simons article provides a simple application to geographic market definition. They analyze the acquisition by Occidental Petroleum of the polyvinyl

[30] Harris and Simons (1989).

chloride resin ("PVC") assets of Tenneco. The FTC alleged a US market for PVC, whereas the parties claimed a broader geographic market. The price of PVC was $0.25/lb, with a variable cost of $0.18/lb, implying a contribution margin of approximately 28%. For a 5% price threshold, the CL was approximately 15%. Assuming that all US producers increased price by 5%, the CL (in terms of total US consumption) was 875 million pounds. That is, US producers could restrict output collectively by as much as 875 million pounds and still find that a 5% price increase was profitable.

Harris and Simons then turn to the Actual Loss. Testimony in the case showed that foreign PVC was acceptable to US consumers. Moreover, foreign suppliers had available capacity to produce more than 875 million pounds of PVC that could profitably be used to defeat an output restriction in the US. The implication was that domestic consumers could defeat a 5% price increase in the US because they could divert a sufficient volume of purchases to foreign suppliers, implying that the relevant geographic market for PVC was larger than the US.

The price effects of a merger depend crucially on the magnitude of the loss sales that would be diverted to competitors as the result of a price increase, and the allocation of that diversion among the competing firms. The CL approach emphasizes the importance of aggregate sales diversion. As a quantity response to a price increase, CLA can be restated in terms of elasticities. In the PVC example, U.S. would not be a separate market if the magnitude of the elasticity of demand facing U.S. producers is 3 or higher (15% CL divided by 5% price increase). That is, if it were known that the elasticity was at least this large, then one would have enough information to infer a broader market. But if a direct measurement of the elasticity were not available, then the data on foreign production plus the assumption that foreign PVC was a close substitute would allow one to reach the same conclusion.

CLA can be controversial in more complicated settings.[31] One set of issues involves the analysis of Actual Loss with differentiated goods. Suppose, for example, that foreign

[31] Katz & Shapiro (2003); O'Brien & Wickelgren (2003); Scheffman and Simons (2003), *available at* www.antitrustsource.com.

PVC was a good but not perfect substitute for domestic PVC (a better analogy might be US beer compared to European beer). There would be diversion to the foreign product if the hypothetical US monopolist raised price. But would US consumers in this case purchase more than 875 million pounds after a 5% price increase? CLA does not answer this question. Scheffman and Simons state that it is a separate "factual matter [to determine] what the Actual Loss in volume is likely to be as a result of the hypothesized price increase" for which they suggest demand-elasticity analyses, customer-switching analyses, etc.[32] Determination of the Actual Loss conceptually stands on the same ground as determining the market elasticity used in merger simulation. In this respect the methods are quite similar; indeed, both require similar data to calibrate the analysis.

Cost accounting issues are central to CLA. Thus, proper use of the method entails addressing the definition and measurement of incremental cost. There can also be a question of the proper determination of CL when the merger partners (or the firms in the candidate market) have different costs, so that CM for the hypothetical monopolist is no longer uniquely defined. It should be noted, incidentally, that when the appropriate cost data have been collected for CLA, one essentially has all the information needed to implement PCAIDS with margin-based nests.

## B. CLA and Diversion Ratios

It may be useful to frame CLA in terms of elasticities. Suppose firms maximize profits by setting marginal revenue equal to marginal cost. Then the price elasticity for each firm equals the (negative) reciprocal of its Lerner index, implying $CM = -1/\varepsilon_i$. The critical loss for each firm is $CL_i = Y/(Y - 1/\varepsilon_i)$.[33] However, the industry elasticity facing the hypothetical monopolist (or the residual demand elasticity for the post-merger firm) should be smaller in magnitude than any $\varepsilon_i$ because it would face less competition than an individual firm. The monopolist may well have an Actual Loss that is smaller than the individual firm CL unless the industry elasticity is close to $\varepsilon_i$.

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[32] Scheffman and Simons (2003), pp. 3–4.

[33] For a general analysis of the relationship between demand elasticities, diversion, and critical loss, see Werden (1998).

This point has been debated. Scheffman and Simons argue that firms seldom set price to equate marginal revenue and marginal cost. They claim that kinks in the industry demand curve or in marginal costs make this assumption unrealistic. They conclude "…it is reasonable to assume that the Actual Loss for the hypothetical monopolist (or for a merged entity with a significant cross elasticity between the products of the two parties) will be lower than for an individual competitor. However, again, Actual Loss may also be substantially *greater* than Critical Loss."[34] [emphasis added] Because they deny a connection between margins and elasticity, the margins no longer constrain their inferences from the data. We agree that kinks complicate CLA (and merger simulation, as well). But in our view, the most reasonable course is to require significant factual evidence for the existence of a kink before abandoning the standard analysis.

Katz and Shapiro present a variant of CLA based on their concept of an "aggregate diversion ratio." In the context of defining a relevant market, they hypothesize a price increase for a single product Z. As with the logit choice model, the diversion ratio is framed in terms of units. Sales of Z decline, with some sales going to other products in the candidate market and other sales lost entirely. The aggregate diversion ratio is the number of lost sales diverted within the market divided by the total number of lost sales. For example, if sales of Z would fall by 200 units in response to a five percent price increase but 90 units would go to other products in the candidate market, the aggregate diversion ratio would be 45% (90/200). They conclude:

> Given the pre-merger gross margin M, calculate the critical loss associated with a ten-percent price increase. If and only if the aggregate diversion ratio associated with a group of products is at least as large as the critical loss, then this group of products forms a relevant market using a five-percent price-increase threshold.[35]

This analysis is for a price increase for a single good and where all products have the same profit margin (price minus incremental cost). The extension to a market-wide price increase with varying margins is considerably more complex.

[34] Scheffman and Simons (2003), p. 5.
[35] Katz and Shapiro (2003), p. 17.

The diversion ratio approach for CLA is similar to using merger simulation. Indeed, the diversion ratios are very closely related to price elasticities. Observe that for total unit sales of Z equal to $q_z$, the own-price elasticity in the example is $-200/q_z/.05$. Suppose for simplicity there was just one other good W in the candidate market, with sales $q_w$. The cross-price elasticity is then given by $90/q_w/.05$. Since the ALM and PCAIDS are two-parameter systems, this is sufficient information to calibrate the demand model and the price effects can be modeled directly.

When reliable data on diversion ratios are available we see some value in using CLA as a complement to simulation. We reiterate, however, that the usefulness of CLA may be limited to relatively simple cases. In more complex situations, the use of CLA may raise as yet unresolved analytical issues. For one thing, we are not aware of a general formulation of CLA for differentiated goods in the literature. For another, it is likely to be difficult to obtain the data for each of the competing products sold by the merging firms. Moreover, the CLA analysis becomes quite cumbersome for analyzing joint price changes for several goods, as is the case in merger analysis. Simulation has the advantage over CLA of automating the calculations. Moreover, the calibration of the simulation model accommodates heterogeneity in margins and provides values for all of the cross-price elasticities. Furthermore, the simulation model makes it easy to experiment with alternative values for the elasticities, and to investigate the interaction between the industry elasticity and the underlying cross-price elasticities.

### C. Coordination

The PVC example of geographic market definition may also be interpreted as an analysis of potential coordinated effects. Suppose that a merger was suspected to increase the probability of collusive pricing in an industry. The CLA identifies the minimum amount of sales the cartel would have to lose to make the Y% collusive price increase unprofitable. Additional analysis would be required to establish whether existing suppliers outside the suspected cartelizing firms could profitably expand output to make the required additional sales to defeat the price increase. This output expansion could come from increased production, diversion of sales from other markets, or repositioning of other products to compete more directly with the products of the cartel.

If alternative suppliers were not sufficient to provide the additional sales, then the analysis would have to turn to an evaluation of the likelihood of entry, taking into account timeliness and profitability (including recovery of sunk costs of entry).

A merger simulation framework can provide an alternative analysis of coordination. The simulation model can specify a hypothetical merger of *all* of the suspected cartelizing firms, even though the announced merger may formally involve only two firms. By treating the suspected cartel as a single entity after the merger, this approach yields a model of perfect coordination because all price effects relevant for maximizing the joint profits of the cartel are internalized. This analysis would also take the projected merger-specific efficiencies into account. The predicted price effects of this hypothetical merger may be useful not only to indicate the potential harm from collusion, but also to provide a measure of the incentive for firms to collude in the first place.

### D. Critical Loss Bibliography

This bibliography includes the most significant discussions of the underlying principles of CLA published to date. In our view, an important remaining area of analysis for this literature is to explore the relationship between merger simulation and CLA with differentiated goods more closely.


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# III. GEOGRAPHIC MARKET DEFINITION

## A. Overview

We treat geographic market definition analogously to product market definition: A geographic market is defined as a set of consumers and producers over which a hypothetical monopolist could profitably impose the SSNIP. Thus, if another product/producer constrains the prices of the producer in question, it is considered to be in the relevant market. While the SSNIP test remains conceptually appropriate for both geographic and product markets, empirical implementation of this test very much depends on the data available in a particular transaction.

The key issue in a unilateral effects analysis is to identify competitors that can expand output in the regions where the merging parties have the incentive to raise prices without collusion. Some general guidance is provided in the U.S. Horizontal Merger Guidelines, which describe four types of evidence for geographic market analysis:

(1) evidence that buyers have shifted or have considered shifting purchases between different geographic locations in response to relative changes in price or other competitive variables;

(2) evidence that sellers base business decisions on the prospect of buyer substitution between geographic locations in response to relative changes in price or other competitive variables;

(3) the influence of downstream competition faced by buyers in their output markets; and

(4) the timing and costs of switching suppliers.

As we describe below, a variety of econometric models have been developed to examine these effects but data requirements limit their applicability. When suitable datasets are not available for statistical analysis, the geographic market definition will often depend on interpretation of a few key facts and "natural experiments" that are sometimes provided by relevant supply/demand shocks, as well as the marketing and business planning documents obtained from the parties.

Assuming no price discrimination, a basic approach to geographic market definition would begin with a particular customer location and determine what other suppliers would find it profitable to ship product to the chosen location in the face of a hypothesized price increase, or alternatively how far customers would travel to a new supplier. Relevant markets can be very different, depending on the level of distribution. For example, wholesale markets for groceries might be national or even international given the ease of shipment, whereas retail markets are likely to be much narrower, given the limited distances retail customers are willing to travel to a store.

In general, there is not a single methodology for determining the relevant geographic market definition. At a minimum, one needs to identify the number, size, and locations of firms that compete with the merging parties (the economic market). The next step would then be to identify the additional competition in response to a hypothetical price increase (which yields the antitrust market). The availability of particular data for these tasks varies greatly from deal to deal, as do the institutional details of the market, including different national regulations (e.g., technical product standards), legal barriers to trade (such as licenses or import quotas), language differences, and other factors that impede entry or expansion of output. In what follows, we review a variety of approaches that have proven useful for one or both of these goals.

## B. Approaches to Geographic Market Definition

### 1. Elzinga-Hogarty Tests

A widely used analysis for identifying geographic markets based on shipments data was introduced by Elzinga and Hogarty (1973, 1978). The method relies on two simple threshold tests of concentration of imports and exports for a given product definition. The first test is called LIFO ("Little In From Outside") and the second is called LOFI ("Little Out From Inside").

For convenience, we refer to activities inside a region as "domestic," where the activities involve the relevant product. LIFO measures the importance of imports. Specifically, LIFO equals 1 minus the ratio of imports to domestic consumption. As imports fall, LIFO approaches unity. LOFI, on the other hand, involves exports and is defined as 1 minus the ratio of exports to domestic production. As exports fall, LOFI also approaches unity.

Elzinga and Hogarty consider a region to be a geographic market if *both* tests exceed a threshold. The authors originally proposed 75% (the "weak" market), but subsequently advocated 90% (the "strong" market). The intuition is that when imports and exports are relatively small, then prices are determined by domestic competition.

The Elzinga-Hogarty approach is implemented with historical data on trade flows. As many commentators have noted, the historical flows may not be completely informative in the context of a prospective merger analysis. In particular, they may indicate overly narrow markets.[36] Landes and Posner (1981) argue that even a de minimis level for LIFO can be consistent with a broader antitrust market. Their point is that if prices rose to anticompetitive levels due to the merger, imports could increase to defeat the price increase, regardless of the historical level of imports. Werden (1981) makes a similar point. The criticism has force, but additional analysis is generally necessary to determine empirically whether imports could actually expand to the necessary extent.

Elzinga-Hogarty is often a reasonable starting point for analysis. It is clearly important to understand historical product flow patterns before making judgments about future flows. But Elzinga-Hogarty does not offer a complete analysis. Indeed, the 25% and 10% thresholds are arbitrary. Values below these levels do not even necessarily prove a broad market. For example, imports may be constrained or exports may be sold at significantly different prices, suggesting a separate market.

[36] Problems involving the choice of a "region" when using other than national data (e.g., hospital admissions) are discussed in Frech et al. (2004). Capps et al. (2001) discusses complications for Elzinga-Hogarty that arise from the possibility of price discrimination.

### 2. Price Correlations

Stigler and Sherwin (1985) advocate using price correlations to define geographic markets.[37] They justify the use of correlations using Marshall's principle that the prices of similar goods in the same geographic market should tend towards equality after making allowance for transportation and other arbitrage costs. The correlation criterion is more general than Marshall's definition because products may be differentiated and therefore have different prices. Correlation is one way to take account of different units of measurement and quality differences in testing whether prices move together. Prices in a geographic market do not move in exact lock-step, however, so correlations will generally be less than 1.

The Stigler-Sherwin method does not provide a criterion for determining how high a correlation must be for two goods to be in the same market. Within a market, the deviation of the correlation from 1 will depend on the magnitude of the arbitrage costs and the time required for the arbitrage to be effective. One solution that is sometimes used is to identify a benchmark good that is known to be in the same market. For example, the products of the merging firms are presumably in the same market and could serve as benchmarks. The correlation with the benchmark might then serve as the minimum to identify other goods in the market. But even when such a benchmark is identified, a correlation analysis for market definition may be difficult to interpret. For example, Stigler and Sherwin compute a variety of correlations, using the data in levels, logs, and first differences of levels and logs. There is no guarantee that these different transformations yield consistent conclusions.

Other criticisms have been made of correlation analysis for market definition. For example, there is no statistical control for "supply side" shocks that cause prices of unrelated products to move together (see, e.g., Slade, 1986). A large change in the price of oil could change the prices of plastics and airline tickets but the correlation in this context is clearly specious. Werden and Froeb (1993) also point out that the usual correlation tests are limited to comparisons of pairs of potential markets. If, in fact, there

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[37] Stigler and Sherwin argue that correlations are superior to Elzinga-Hogarty tests and the regression approach presented by Horowitz (1981).

are many imperfect substitutes that exert, in the aggregate, a sufficient constraint on prices, the Stigler Sherwin approach may reach a false conclusion. That is, the correlations in a valid geographic market may be intrinsically "low." This is another manifestation of the general problem that the method does not identify a particular value of the correlation coefficient that delineates the boundaries of competition. A variety of other, more subtle criticisms are summarized in Kaserman and Zeisel (1996).

Despite these limitations, price correlations are still often informative for geographic market definition. Especially when it is reasonable to assume that markets are at least national in size, price correlations within a country can provide a benchmark for correlations with foreign goods. But to prove a single market the correlations need to be supplemented with additional evidence that arbitrage between two countries is feasible to constrain prices. This evidence could include, for example, data on historical trade flows, analysis of explicit entry barriers, and financial analysis of cost of entry.

### 3. Granger Causality and Cointegration

Some of the concerns with the use of price correlations can be remedied if one uses more sophisticated econometric methods. One approach is Granger causality tests (Granger (1969), Pindyck and Rubinfeld (2000, Chapter 9)). To test whether X "causes" Y (and is therefore in the same geographic market), one compares the $R^2$s of two regressions, the first in which Y is regressed only on its own lag values, and the second in which the regressors also include a series of lagged values for X. If the incremental increase in goodness of fit is statistically significant, then one concludes that X "causes" Y. Conversely, a similar procedure tests whether Y "causes" X. If the results of such tests (i) that there is immediate feedback between two variables X and Y (which might reflect two geographic locations), or if (ii) there is direct causality between X and Y and between Y and X; then X and Y would be deemed to be in the same antitrust market. By incorporating lags, this approach may be viewed as a "dynamic" version of price correlation analysis and is potentially superior. For empirical examples, see Uri and Rifkin (1985) and Benson and Faminow (1990). However, the approach does have some of the same drawbacks as Stigler-Sherwin since it relies ultimately on price correlations and not on underlying structural models of demand and supply. Further, reliable estimation of the lag structures often requires fairly long time series of relatively high-frequency (e.g., quarterly or higher).

More recently, cointegration (Engle and Granger (1987), Pindyck and Rubinfeld (2000, Chapter 15)) has also been proposed for geographic market definition This is a further refinement of analysis of correlations of time series data. Suppose that two price series behave as random walks, i.e., they are each nonstationary processes with a unit root. The simple correlation between them may be spuriously high because it is well-known that independent random walks can appear to move together even when there is no relationship between them. In essence, a cointegration test is a regression of Y on X (involving lags and differences of the variables), when both variables have been found to contain a unit root. If Y and X are truly unrelated (i.e., uncorrelated), then the residuals in that regression will also be a nonstationary, unit root process. But if the residuals are stationary, then Y and X are said to be cointegrated and the inference is that there is a statistically valid relationship between the two processes.

The concept of cointegration can be extended to a system of N variables. Consider a non-stationary N-vector of prices $P_t = \{p_{1t}, p_{2t}, \ldots, p_{Nt}\}$, where $p_{it}$ is the log-price of a commodity in period t. Suppose there are $s$ cointegrating relations, i.e., $s$ independent and stationary linear combinations of the elements of $P_t$. The Granger representation theorem (Engle and Granger, 1987) states that under fairly general conditions the process for $P_t$ can be written as

$$\Delta P_t = \mu + \Pi P_{t-1} + \Gamma_1 \Delta P_{t-1} + \Gamma_2 \Delta P_{t-2} + \ldots + \Gamma_{p-1} \Delta P_{t-p+1} + \varepsilon_t$$

where $\Gamma$ and $\Pi$ are n by n matrices and $\Pi$ has reduced rank $s$. The matrix $\Pi$ can be written as $\Pi = \alpha\beta'$, where $\alpha$ is an N by N–$s$ matrix of coefficients and $\beta$ is an N by N–$s$ matrix of cointegrating vectors. Estimation of the process for $P_t$ involves testing the rank of $\Pi$. A two-step estimation procedure for this model was given by Engle and Granger (1987). A one-step procedure that jointly estimates the cointegrating vectors with the $\Gamma$ and $\Pi$ parameters was given by Johansen (1988, 1991).[38]

[38] For additional discussion of cointegration in the context of geographic market definition see Gonzalez-Rivera and Helfand (2001).

Cointegration provides additional diagnostics and structure for the analysis. For example, if neither Y nor X has a unit root, then a conventional correlation using the levels of the prices may be calculated. If one variable has a unit root and the other does not, then it is unlikely that the variables have any meaningful relationship, regardless of the correlations that might be computed by using them.

Cointegration analysis for market definition raises two practical problems. First, statistically powerful tests using this framework require lengthy and high frequency time series. For many markets, such data may not be available. Second, cointegration measures the long run behavior of the processes. Even if the null hypothesis of no cointegration is rejected, it does not necessarily follow that prices in two different regions are linked tightly enough to be considered part of a single market for the merger review. For example, suppose that a price divergence in response to anticompetitive price shock lasted 10 years. This may result in the series being cointegrated in a purely statistical sense but for economic purposes one still may infer separate markets.

For example, consider a test for whether *x* and *y* are cointegrated. This amounts to testing whether there is a unit root in the residual $\underline{u}$ of the regression of *y* on *x*. Using the Dickey-Fuller procedure (see Hamilton, 2004), the residual may be analyzed with a distributed lag regression of the form $u_t = \sum \beta_i \Delta u_{t-i} + \gamma u_{t-1} + \varepsilon_t$ and testing $H_0$: $\gamma = 1$. The null hypothesis is that there is a unit root, i.e., no cointegration. Rejection of $H_0$ implies that *x* and *y* are cointegrated. But another way of framing the question is to rewrite the model for u as an autoregression $u_t = \sum \delta_i u_{t-i} + \varepsilon_t$. The impulse response function may be calculated for this autoregression (see Hamilton, 1994, p. 586), which measures the change in $u_{t+j}$ for a shock in period t. Rejection of $H_0$ is equivalent to the inference that a shock in period t has effects for all future j. That is, *x* and *y* could be cointegrated in a statistical sense provided only that the horizon for the impulse response function is less than infinity. But a shock (e.g., a price divergence between two regions) that lasted significantly more than one or two years may lead to the conclusion that the regions are economically separate even though they may be formally "cointegrated."

### 4. Other Methodologies

Shipment data and price correlations are simplified and potentially incomplete analyses because they do not specify a structural model of competition in the relevant market. Structural models are typically more difficult to estimate but may be more informative when suitable data are available. Early examples include the estimation of residual demand curves; see Spiller and Huang (1986), Scheffman and Spiller (1987), Baker and Bresnahan (1988). A variety of cautions and criticisms regarding residual demand estimation for market delineation are presented in Froeb and Werden (1991). At this point it appears that residual demand estimation is not widely used in merger analysis.

Even more sophisticated structural models for geographic market definition have been developed since residual demand curve estimation was introduced. Feenstra and Levinsohn (1995) model variations in product qualities, some of which have a spatial dimension.[39] Pinske et al. (2002) show how to derive firms' response functions semi-parametrically. They explain that the slope of a response functions is equivalent to a diversion ratio. These ratios in turn predict the likely price effects of mergers and to define markets. The approach is quite general, and it allows for a variety of distance measures and functional forms. The main disadvantage is massive data requirements, a general characteristic of semiparametric methods.

Critical Loss Analysis was originally developed as a tool for geographic market definition, as discussed earlier. The challenge, as always with Critical Loss Analysis is developing sufficient outside evidence on the Actual Loss, as well as to measure incremental cost.

Finally, there is a different approach based on creating stylized maps of competitive overlap described in Higgins (1999). These maps do not entail statistical estimation. Instead, in their most simple form, they describe the geographic market in terms of a distribution of representative customers on a closed section of a plane, with a uniform maximum demand price for exactly one unit of the product. A finite number of suppliers,

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[39] A similar approach was taken in Levinsohn and Feenstra (1990); for earlier studies, see, for example, Klein et. al. (1985) and Slade (1986).

each supplying a homogenous good, are also located in the plane. Suppliers have the same per km transportation costs (i.e., arbitrage costs) and total costs increase linearly in distance from a supplier. Suppliers sell on a delivered price basis, each has localized market power but eventually there is a distance where delivered prices for two suppliers are equal. Consider, for example, two suppliers and the perpendicular bisector of the segment that connects them. Each supplier has a delivered price advantage on its side of the bisector. The bisector defines the line of equal prices for that pair of suppliers. With three or more suppliers, the mutual bisectors divide the plane into regions of localized market power. A merger would increase market power by expanding the overlap regions where the merging firms have the first and second lowest costs. That is, the overlaps define the regions of concern. This methodology clearly makes highly restrictive assumptions, but it has potential to shed light on mergers at the retail level. An extension of the model with stochastic demand is given in Higgins et al. (2004).

Defining relevant geographic markets is conceptually straightforward but data limitations significantly constrain empirical work. Because it is possible for either demand or supply considerations to drive market definition, and because transportation and other arbitrage costs can be difficult to measure, there is no single best approach. As we have discussed, a variety of econometric methods is available when suitable datasets exist. The various studies of the gasoline markets we have cited are good examples of these approaches. Elzinga-Hogarty tests are less rigorous and have more modest data requirements but will often still provide useful information. Frequently, however, proof of the geographic market will depend on the ability to identify relevant but case-specific patterns in the data that can reveal substitution patterns across regions in response to price increases.

In this regard, we think the Commission made good use of the evidence on price and margin differences and country-specific vehicle requirements to define markets in the Volvo Scania investigation. This transaction was also interesting for raising the issue of the role of the dealer network as a barrier to new entry or expansion by an existing fringe supplier. It is not necessary here to review the detailed empirical analyses and criticisms of the work of the experts retained by the merging parties. Analytically, we would have been interested to see the results of merger simulation and CLA to the various markets identified in Volvo Scania, since it appears that margin data were available. We also would have liked more explicit financial analysis of the ability and incentive of fringe suppliers to expand their sales and dealer networks after a hypothetical price increase.

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# IV. ECONOMETRIC SOFTWARE

In this section we describe the use of the econometric software for demand system estimation. The software has two different components. First is a set of Stata modules to estimate AIDS and logit demand models econometrically using panel (i.e., scanner) data. Second is an Excel add-in (consisting of two .xla files) that carries out merger simulation in Excel. The Excel add-in will be referred to as the DG-COMP MSP, or Merger Simulation Program. The Stata output is easily transferred to Excel. DG-COMP MSP can perform simulation using the ALM, AIDS, PCAIDS, and PCAIDS with nests.

## A. Stata Programs for Econometric Demand System Estimation

There are four Stata programs. The user interacts only with the top-level program called demand_shell. This module is customized for each problem because it aggregates the scanner-level data to define the relevant brands. The demand_shell program invokes three other subroutines that prepare a final regression dataset and perform the actual AIDS and logit estimation. The discussion in this section assumes a working knowledge of Stata.

It is critical to configure Stata properly with the memory and matrix size commands. The case studies were carried out with the system parameters:

```
set memory 32m
set matsize 500
```

Larger problems may require higher limits. For simplicity, place the Stata programs and the input data in the same directory and use the Stata cd command to point to that directory.

The syntax to run the software from the Stata command line is

do demand_shell *rawinputfilename.csv* *outputlabel* [A | L]

As presently written, demand_shell assumes the input data is a comma-delimited file. The name of the actual datafile is used in place of rawdatafilename. The optional argument A means estimate only the AIDS and L means estimate only the logit system.

If the third argument is not given then both demand systems are estimated. The programs place the estimation results in log files with the following names: The AIDS results are appended to a log with the name AIDS_outputlabel_date and the logit results are appended to a log with the name LOGIT_outputlabel_date. For example, the toilet tissue case study was estimated using the command

do demand_shell subwtti.csv tissue

The AIDS results were placed in AIDS_tissue_23Jun04.log and the logit results were placed in LOGIT_tissue_23Jun04.log.

It is the user's responsibility to customize the input section of demand_shell to accommodate the actual input data for a particular problem. Each input observation must contain information sufficient to generate the following Stata variables:

brand (a string that contains the name of the brand)
rev (a float variable that measures sales revenue)
q (a float variable that measures units sold)
fixed (a string variable to define a fixed effect for each geographic location in the data, e.g., city1, city2…)
time_t (a float variable that codes the time period of the observation)

The user is also responsible for including Stata code in the input section to apply any filters to the data or check for data errors. The software aggregates the raw observations to the level of brand i, location j, time t. The "brands" in the analysis are typically aggregates of more detailed product offerings. For example, the SCOTT brand in the toilet tissue case study includes SCOTT WHITE, SCOTT YELLOW and other product line variants in the underlying data. The OTHER brand is an aggregate of various fringe suppliers.

The other Stata programs called by demand_shell are panel_data, AIDS_homogeneous, and logit_no_nest. These programs reorganize the data for Stata and should not require any user modification.

### B. Estimation and Endogeneity

The Stata programs AIDS-homogeneous and logit_no_nest use the REG3 procedure to estimate the demand systems. For AIDS, the program imposes Slutsky-symmetry and homogeneity constraints. The logit_no_nest program estimates the $\beta$ parameter for the ALM, imposing the cross-equation constraint that $\beta$ be the same in each brand share equation. The software in each case generates the necessary code to impose the constraints automatically. REG3 is used with the "mvreg" option to estimate the model as a system of seemingly unrelated regressions. The software treats prices as predetermined for purposes of estimation. REG3 allows the use of instrumental variables to control for possible endogeneity. It is straightforward to modify the Stata programs to include instruments, but we have not done so since the necessary code is likely to be case-specific and thereby not appropriate for a general program.

The most widely discussed method for controlling for endogeneity in the context of demand models for merger simulation is the instrumental variables technique presented in Hausman and Leonard (2002), which in turn is a further elaboration of the method in Hausman, Leonard, and Zona (1994). While the authors present the method for AIDS, the same procedure applies for the ALM. In either case, a basic problem is availability of sufficient data to allow one to utilize an adequate number of instruments in estimating the model. The approach of Hausman and his co-authors is to use prices from one city as instruments for other cities, on the assumption that stochastic city-specific factors are independent. Under this assumption prices in other cities can reasonably be characterized as predetermined, and they will serve as theoretically valid instruments. While formally correct, it is seldom possible to verify this assumption. The Hausman et al. method is discussed and criticized by Bresnahan (1996).

In our view, endogeneity is likely to be a second-order concern for estimation. The existing literature does not contain enough examples of instrumental variables estimation using scanner data panels to persuade us that the IV results are reliably and meaningfully different from ordinary least squares estimates. Some authors, notably LaFrance (1993), Berry (1994), and Dhar, Chavas, and Gould (2003), have argued, however, that endogeneity bias is a more significant issue. This is a worthwhile topic for additional research but, absent specific information to the contrary, we believe it is generally reasonable to treat prices in scanner data as predetermined. In the case studies discussed in Section V, for example, prices for individual products often remain unchanged for many weeks on end, while the corresponding shares display variation. This is evidence of a lack of simultaneous feedback from shares to prices.

### C. DG-COMP MSP Excel Menu Commands

The user interacts with three components of DG-COMP MSP. First is a set of commands that the system adds to the Excel menu bar. Second is a "data input sheet" that the user creates to specify the inputs to the simulation. Third is the output sheet that contains the results of the simulation. The software is intended to be "user friendly" and has a self-documenting internal help file.

The commands are placed onto the Excel menu bar automatically when DG-COMP MSP starts. They are available under the heading "Merger Simulation" and include commands to control data input options and to carry out simulation. The tree structure of the commands is summarized in the following table.

| **Data Input and Workbook Commands** | **Drop-down Menus** |
|---|---|
| Create New Data Input Sheet | |
| Select/Change Active DG-COMP MSP … | Workbook |
| | Data Input Sheet |
| Reset Active Data Input Sheet | |
| **Simulation Options** | |
| Divest Brands | |
| PCAIDS Merger Simulation | |
| PCAIDS Merger Simulation with Nests | |
| Non-Nested Logit | Parse Stata Log File |
| | Logit Simulation |
| Standard AIDS | Parse Stata Log File |
| | Define Input Area for AIDS Parameters |
| | Reorganize AIDS Parameters |
| | AIDS Simulation |
| Help | |

The user must create a "data input sheet" that contains the basic data to specify the simulation. This information always includes the industry elasticity of demand, the names of the firms, the names of the brands, and the brand shares. For PCAIDS it is also necessary to include the brand elasticity for a single brand. For nested PCAIDS it is necessary to specify the name of the nest for each of brands associated with it. For the ALM, it is necessary to specify the logit $\beta$ parameter (or, alternatively, the brand elasticity for a single brand) and brand unit prices. For the full AIDS it is necessary to include the matrix of estimated price and expenditure coefficients.

The following pages contain screenshots of the inputs for the different demand models using the beer case study describe in section V.

# Screenshot A: PCAIDS Data Input Sheet

## DG-COMP Merger Simulation Program 1.0



### Data Input Sheet

Industry Elasticity: -1.00

Logit Beta Parameter:
(Brand elasticity will override)

Enter the own-price elasticity for a single brand below.
Enter efficiencies as negative percentages.

Share Basis: 0
0=revenue, 1=quantity

### Parties in the Transaction

Merger Partner 1 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | BUD | 7.1% | -2.50 | | | |
| 2 | | | | | | |

Merger Partner 2 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OLD_STYLE | 13.7% | | | | |
| 2 | | | | | | |

### Other Firms in the Relevant Market

Competitor 1 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | MILLER | 25.1% | | | | |
| 2 | MILLER_LITE | 17.9% | | | | |

Competitor 2 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_LITE | 9.3% | | | | |
| 2 | | | | | | |

Competitor 3 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_REG | 26.9% | | | | |
| 2 | | | | | | |

Competitor 4 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |

# Screenshot B: PCAIDS with Nests Data Input Sheet

## DG-COMP Merger Simulation Program 1.0



### Data Input Sheet

Industry Elasticity: -1.00

Enter the own-price elasticity for a single brand below.
Enter efficiencies as negative percentages.

Logit Beta Parameter:
(Brand elasticity will override)

Share Basis: 0
0=revenue, 1=quantity

### Parties in the Transaction

Merger Partner 1 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | BUD | 7.1% | -2.50 | reg | | |
| 2 | | | | | | |

Merger Partner 2 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OLD_STYLE | 13.7% | | reg | | |
| 2 | | | | | | |

### Other Firms in the Relevant Market

Competitor 1 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | MILLER | 25.1% | | reg | | |
| 2 | MILLER_LITE | 17.9% | | lite | | |

Competitor 2 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_LITE | 9.3% | | lite | | |
| 2 | | | | | | |

Competitor 3 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_REG | 26.9% | | reg | | |
| 2 | | | | | | |

Competitor 4 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |

## Screenshot C: ALM Data Input Sheet

### DG-COMP Merger Simulation Program 1.0



### Data Input Sheet

Industry Elasticity: -1.00

Logit Beta Parameter: 61.70
(Brand elasticity will override)

Share Basis: 1
0=revenue, 1=quantity

Enter the own-price elasticity for a single brand below.
Enter efficiencies as negative percentages.

### Parties in the Transaction

Merger Partner 1 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | BUD | 6.6% | | | | 0.0441 |
| 2 | | | | | | |

Merger Partner 2 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OLD_STYLE | 17.2% | | | | 0.0328 |
| 2 | | | | | | |

### Other Firms in the Relevant Market

Competitor 1 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | MILLER | 25.3% | | | | 0.0409 |
| 2 | MILLER_LITE | 18.7% | | | | 0.0396 |

Competitor 2 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_LITE | 9.9% | | | | 0.0387 |
| 2 | | | | | | |

Competitor 3 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_REG | 22.3% | | | | 0.0497 |
| 2 | | | | | | |

Competitor 4 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |

## Screenshot D: AIDS Data Input Sheet

### DG-COMP Merger Simulation Program 1.0



#### Data Input Sheet

Industry Elasticity: -1.00

Logit Beta Parameter:
(Brand elasticity will override)

Enter the own-price elasticity for a single brand below.
Enter efficiencies as negative percentages.

Share Basis: 0
0=revenue, 1=quantity

#### Parties in the Transaction

Merger Partner 1 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | BUD | 7.1% | | | | |
| 2 | | | | | | |

Merger Partner 2 Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OLD_STYLE | 13.7% | | | | |
| 2 | | | | | | |

#### Other Firms in the Relevant Market

Competitor 1 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | MILLER | 25.1% | | | | |
| 2 | MILLER_LITE | 17.9% | | | | |

Competitor 2 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_LITE | 9.3% | | | | |
| 2 | | | | | | |

Competitor 3 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | OTHER_REG | 26.9% | | | | |
| 2 | | | | | | |

Competitor 4 Firm Name:

| Brand | Name | Share | Elasticity | Nest | Efficiencies (%) | Price |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |

## Screenshot D: AIDS Data Input Sheet (cont.)

### Parameters for AIDS Share Equations (if using Stata, paste from output log and select reorganize option)

| | Price Coefficients | | | | | | Expenditu | Mean |
|---|---|---|---|---|---|---|---|---|
| **Brand** | BUD | OLD_STYLE | MILLER | MILLER_LITE | OTHER_LITE | OTHER_REG | Term | Share |
| BUD | -0.197 | 0.019 | 0.040 | 0.039 | 0.027 | 0.072 | 0.0045 | 0.0879073 |
| OLD_STYLE | 0.019 | -0.139 | 0.056 | -0.003 | 0.029 | 0.038 | -0.008 | 0.1343332 |
| MILLER | 0.040 | 0.056 | -0.582 | 0.299 | 0.037 | 0.151 | 0.025 | 0.2270717 |
| MILLER_LITE | 0.039 | -0.003 | 0.299 | -0.500 | 0.045 | 0.119 | 0.0093 | 0.1880234 |
| OTHER_LITE | 0.027 | 0.029 | 0.037 | 0.045 | -0.147 | 0.009 | -0.001 | 0.0880335 |
| OTHER_REG | 0.072 | 0.038 | 0.151 | 0.119 | 0.009 | -0.389 | -0.029 | 0.2746309 |

The menu command "Select/Change Active DG-COMP MSP …" is used to switch among different input sources. Click on a cell in the desired workbook or data input sheet and then invoke the command to identify the choice to the software. The selected workbook or data input sheet is referred to as "active."

The "Reset Active Data Input Sheet" command clears the data from the active sheet and restores the default formatting.

The "Divest Brands" command is an option to specify hypothetical divestiture of one or more brands sold by the merging firms. Each brand can be divested either to an existing competitor or a hypothetical new entrant. A firm is not permitted to divest all of its brands. After the divestiture is specified, the actual simulation is carried out by using one of the following commands.

The "PCAIDS Merger Simulation" command performs PCAIDS simulation and appends the results to an output worksheet. The output worksheet has the name *datainputsheetname*_PCAIDS.

The "PCAIDS Merger Simulation with Nests" command performs PCAIDS simulation with nests and appends the results to an output worksheet. After this command is selected, the user is prompted to specify the lower triangle of the (symmetric) matrix of nesting parameters. The output worksheet has the name *datainputsheetname*_NESTED_PCAIDS.

The "Non-Nested Logit" command has submenus to process the Stata log file (when the logit $\beta$ parameter is estimated econometrically) and to perform simulation using the ALM. The simulation results are appended to an output worksheet with the name *datainputsheetname*_LOGIT. For further information on processing the Stata file see section IV.D.2 below.

The "Standard AIDS" command has submenus to process the Stata log file with AIDS estimation results and to perform simulation using the ALM. The simulation results are appended to an output worksheet with the name *datainputsheetname*_AIDS. For further information on processing the Stata file see section IV.D.1 below.

The "Help" command provides brief descriptions of these menu choices.

### D. Importing Stata Estimation Results into DG-COMP MSP

The following procedures are used to bring the Stata estimation results into DG-COMP MSP (it is assumed that MSP has already been installed). MSP has functions that largely automate the process.

#### 1. AIDS

The objective is to organize the raw Stata log into a matrix of AIDS parameters that can be recognized by MSP.

a) Open the Stata AIDS log file with Excel. In the Excel Text Import Wizard, choose the "Delimited" option and select the Space delimiter. Then click Finish.

b) On the Excel menu bar, select Merger Simulation — Standard AIDS — Parse Stata Log File. (The program may ask you to repeat the operation, which is sometimes necessary for Excel to convert Stata output to European decimal formats correctly.) This command will group the relevant coefficients and data for each estimated system in the log file into two matrices. The matrices are placed in the log next to the section that reports the R-sq for the equations in the associated system. The first matrix contains the brand names and the period T shares (averaged across all cross-sectional units). The second matrix contains the estimated AIDS coefficients for the price terms and the expenditure term, as well as the average brand shares over the sample period. (The program inserts an extra row and column to give effect to adding-up and homogeneity, respectively). Note that the Stata results are in alphabetical order by brand. For example, the matrices in the log file for the beer case study are:

| | Period T Share |
|---|---|
| BUD | 0.071 |
| MILLER | 0.251 |
| MILLER_LITE | 0.179 |
| OLD_STYLE | 0.137 |
| OTHER_LIGHT | 0.093 |
| OTHER_REG | 0.269 |

and (note the column without a name contains the price coefficients obtained by imposing homogeneity)

| lnp1 | lnp2 | lnp3 | lnp4 | lnp5 | | lnrexp | Mean Share |
|---|---|---|---|---|---|---|---|
| –0.197 | 0.040 | 0.039 | 0.019 | 0.027 | 0.072 | 0.004 | 0.088 |
| 0.040 | –0.582 | 0.299 | 0.056 | 0.037 | 0.151 | 0.025 | 0.227 |
| 0.039 | 0.299 | –0.500 | –0.003 | 0.045 | 0.119 | 0.009 | 0.188 |
| 0.019 | 0.056 | –0.003 | –0.139 | 0.029 | 0.038 | –0.008 | 0.134 |
| 0.027 | 0.037 | 0.045 | 0.029 | –0.147 | 0.009 | –0.001 | 0.088 |
| 0.072 | 0.151 | 0.119 | 0.038 | 0.009 | –0.389 | –0.029 | 0.275 |

c) Create an MSP input data sheet in the Excel file by selecting Merger Simulation — Create New Data Input Sheet. Alternatively, if a data input sheet already exists, activate it by selecting the appropriate Merger Simulation command. On the MSP input data sheet, enter the industry elasticity and the names and shares of the brands for each firm. The names of the brands **must** match the names used in Stata. If the brand shares are based on revenue, enter 0 in the yellow-highlighted cell M9. Usually, the easiest way to enter the brand names and shares is to copy the first matrix from the log file and paste it into a blank area of the input data sheet. Use the mouse to drag each brand and share to the appropriate area of the input data sheet.

d) Select Merger Simulation — Standard AIDS — Define Input Area for AIDS Parameters. This will create row and column labels for a blank matrix.

e) Copy the second matrix created in the AIDS log in step b) and paste special it into the area created in the input data sheet in step d). Copy only the numerical values, there should be N rows and N+2 columns, where N is the total number of brands in the analysis. Do not copy the header row with the column names.

f) Finally, select Merger Simulation — Standard AIDS — Reorganize AIDS Parameters. This orders the data in the matrix by brand by firm, as required for the simulation, instead of alphabetically.

If you choose to create the input data sheet in the log file after it was brought into Excel, be sure to save the log file as a standard Excel workbook without the ".log" extension in the filename.

The actual simulation is run by selecting Merger Simulation — Standard AIDS — AIDS Simulation. Note that the estimated fixed effects are not needed for the simulation.

### 2. Logit

The Stata log provides the brand names, period T average shares and T prices, and the estimated logit $\beta$ coefficient.

a) Open the Stata LOGIT log file with Excel. In the Excel Text Import Wizard, choose the "Delimited" option and select the Space delimiter. Then click Finish.

b) On the Excel menu bar, select Merger Simulation — Non-Nested Logit — Parse Stata Log File. (The program may ask you to repeat the operation, which is sometimes necessary for Excel to convert Stata output to European decimal formats correctly.) The command will group the brand names and period T shares and prices at the start of the log for each estimated system. The command also inserts the estimated logit $\beta$ coefficient from the regression output alongside this information. Note that the Stata results are in alphabetical order by brand. For example, the log for the beer case study after the parse command contained:

| Brand | q_shar~T | price_T | Beta |
|---|---|---|---|
| BUD | 0.066 | 0.044 | –61.697 |
| MILLER | 0.253 | 0.041 | |
| MILLER_LITE | 0.187 | 0.040 | |
| OLD_STYLE | 0.172 | 0.033 | |
| OTHER_LIGHT | 0.099 | 0.039 | |
| OTHER_REG | 0.223 | 0.050 | |

c) Create an MSP input data sheet in the Excel file by selecting Merger Simulation — Create New Data Input Sheet. Alternatively, if a data input sheet already exists, activate it by selecting the appropriate Merger Simulation command. On the MSP input data sheet, enter the industry elasticity and the names, shares, and prices of the brands for each firm. Usually, the easiest way to enter the brand names, shares, and prices is to copy the block of information from the log file and paste it into a blank area of the input data sheet (omitting the header row with the column names). Use the mouse to drag each brand/share/price combination to the appropriate area of the input data sheet. (Include the 3 blank cells between the share and the price when dragging to conform to the layout of the data input sheet).

d) Copy the value of the estimated $\beta$ coefficient and paste special it into the highlighted cell M7. Make sure that a brand elasticity is not entered elsewhere on the data input sheet. A brand elasticity, if specified, implies a value of $\beta$ that will override the value entered in M7.

e) If the brand shares are based on quantities, enter 1 in the highlighted cell M9, else enter 0. The shares in the logit log file should be based on quantity.

If you choose to create the input data sheet in the log file after it was brought into Excel, be sure to save the log file as a standard Excel workbook without the ".log" extension in the filename.

The actual simulation is run by selecting Merger Simulation — Non-Nested Logit — Logit Simulation. The estimated fixed effects are not needed for the simulation.

## V. CASE STUDIES

We now describe the results of two illustrative case studies. The first study involves beer and the second toilet tissue. The data were obtained from a public use database of supermarket scanner data maintained by the University of Chicago. The data are described in detail at http://www.gsb.uchicago.edu/kilts/research/db/dominicks/. The complete database contains approximately 9 years of weekly store-level data for the Dominick's supermarket chain in the Chicago area, covering sales of more than 3500 UPCs (Universal Product Codes) in several dozen stores. The case studies used a subset of the available data to simplify management of the input files.

### A. Beer

The beer data included weekly observations for approximately 2 years on 186 separate products in 5 stores. The UPCs track products at a very detailed level, e.g., sales of Beck's Bier are recorded separately for cans and bottles. Moreover, many of the UPC's have very small sales and some even have zero sales in a given week. For purposes of the case study, the UPCs were aggregated into 6 composite brands: Budweiser, Miller, Miller Lite, Old Style, Other Light Beer, and Other Regular Beer. The average shares across the sample stores at the end of the sample were as follows:

| Brand | Revenue Share (%) | Quantity Share (%) |
|---|---|---|
| BUD | 7.1 | 6.6 |
| MILLER | 25.1 | 25.3 |
| MILLER_LITE | 17.9 | 18.7 |
| OLD_STYLE | 13.7 | 17.2 |
| OTHER_LIGHT | 9.3 | 9.9 |
| OTHER_REG | 26.9 | 22.3 |
| Total | 100.0 | 100.0 |

The econometric estimation used these brands with fixed effects for each brand for each of the 5 stores.

### 1. AIDS Beer Estimation Results

The matrix of estimated AIDS coefficients (excluding the expenditure terms and fixed effects) is:

| | Ln(Price) of | | | | | |
|---|---|---|---|---|---|---|
| Share Eqn. | BUD | MILLER | MILLER_LITE | OLD_STYLE | OTHER_LIGHT | OTHER_REG |
| BUD | –0.197 | 0.040 | 0.039 | 0.019 | 0.027 | 0.072 |
| MILLER | 0.040 | –0.582 | 0.299 | 0.056 | 0.037 | 0.151 |
| MILLER_LITE | 0.039 | 0.299 | –0.500 | –0.003 | 0.045 | 0.119 |
| OLD_STYLE | 0.019 | 0.056 | –0.003 | –0.139 | 0.029 | 0.038 |
| OTHER_LIGHT | 0.027 | 0.037 | 0.045 | 0.029 | –0.147 | 0.009 |
| OTHER_REG | 0.072 | 0.151 | 0.119 | 0.038 | 0.009 | –0.389 |

These results are constrained to satisfy adding up (columns sum to zero), homogeneity (rows sum to zero), and symmetry ($b_{ij} = b_{ji}$). The coefficients on the diagonal are expected to have negative sign (generating the own-price elasticities) and the other coefficients are expected to have positive sign (generating the cross-price elasticities). The estimated Miller Lite-Old Style coefficient of –0.003 has the wrong sign but the associated t-statistic is only –0.26, which is consistent with a positive true coefficient that is obscured by sampling error.

### 2. Logit Beer Estimation Results

The purpose of estimating the logit demand system is to obtain the value of the price coefficient $\beta$. Prices were normalized on a per-ounce basis. Using the data described above, the estimated coefficient was 61.7.

### 3. Beer Simulation Results

Assume a proposed merger involving Budweiser and Old Style, both regular beers. The simulation results for AIDS, logit, and PCAIDS are based on the assumption that the industry elasticity is –1.0. The PCAIDS simulation assumed a brand elasticity of –2.5 for Budweiser, based on a reported 40% firm-wide gross profit margin (see Anheuser-Busch 2003 Form 10-K, p. 58),.

| Simulation Method | Brand | Predicted Unilateral Effect (%) |
|---|---|---|
| AIDS | BUD | 3.4 |
| | OLD_STYLE | 3.0 |
| Logit | BUD | 6.0 |
| | OLD_STYLE | 1.5 |
| PCAIDS | BUD | 4.5 |
| | OLD_STYLE | 2.5 |

Note that the logit simulation yields the highest single effect, as well as the lowest effect.

Alternatively, for the PCAIDS analysis one might expect that regular beer and light beer should be in separate nests. The following simulation assumes a nesting parameter equal to 0.25, implying that light beer is a poor substitute for regular beer.

| Simulation Method | Brand | Predicted Unilateral Effect (%) |
|---|---|---|
| NESTED PCAIDS | BUD | 6.1 |
| | OLD_STYLE | 3.5 |

The predicted effects are somewhat larger compared to the non-nested PCAIDS. Note that the nest structure effectively makes the pre-merger market more concentrated with respect to Budweiser and Old Style because they are both regular beers. The light beers nest has less ability to constrain the effect of the merger.

### B. Toilet Tissue

The toilet tissue data included weekly observations for approximately 2 years on 110 separate products in 4 stores. These products often have minimal differentiation, e.g., sales for a given manufacturer can be recorded separately depending on the color of the tissue. As with beer, some UPC's had very small and even zero sales in a given week. For the case study, the UPCs were aggregated into 5 composite brands: Charmin, Kleenex, Northern, Scott, and Other.

The average shares across the stores at the end of the estimation period were as follows:

| Brand | Revenue Share (%) | Quantity Share (%) |
|---|---|---|
| CHARMIN | 21.9 | 22.5 |
| KLEENEX | 24.1 | 19.8 |
| NORTHERN | 16.8.0 | 21.0 |
| OTHER | 11.7 | 17.8 |
| SCOTT | 25.5 | 18.9 |
| TOTAL | 100.0 | 100.0 |

The econometric estimation used these brands with fixed effects for each brand for each of the 4 stores. Note that the revenue share for Scott is much larger than its quantity share. A difference of this magnitude is likely to result in significantly larger unilateral effects in the AIDS framework. It should be noted, however, that the quantity share is given (as reported) in terms of rolls of paper. Casual inspection at the supermarket reveals that Scott has many more sheets per roll than other brands, so the Scott quantity share would be underestimated if number of sheets is the more appropriate unit of measurement.

### 1. AIDS Tissue Estimation Results

The matrix of estimated AIDS coefficients (excluding the expenditure terms and fixed effects) is:

| | | | Ln(Price) of | | |
|---|---|---|---|---|---|
| Share Eqn. | CHARMIN | KLEENEX | NORTHERN | OTHER | SCOTT |
| CHARMIN | –0.575 | 0.092 | 0.150 | 0.098 | 0.236 |
| KLEENEX | 0.092 | –0.500 | 0.177 | 0.086 | 0.145 |
| NORTHERN | 0.150 | 0.177 | –0.777 | 0.128 | 0.322 |
| OTHER | 0.098 | 0.086 | 0.128 | –0.360 | 0.048 |
| SCOTT | 0.236 | 0.145 | 0.322 | 0.048 | –0.751 |

The estimation imposes the same constraints of adding-up, homogeneity, and symmetry. All of the estimated coefficients have the expected sign.

### 2. Logit Tissue Estimation Results

The purpose of estimating the logit demand system is to obtain the value of the price coefficient $\beta$. Prices were normalized on a per roll basis. It is known that sheets per roll vary across brands, indicating it might be more appropriate to normalize on a per sheet basis. However, data on number of sheets per roll were not available. Using the data described above, the estimated coefficient was 9.12.

### 3. Tissue Simulation Results

Assume a proposed merger involving Charmin and Scott. The simulation results for AIDS, logit, and PCAIDS are as follows. An industry elasticity of –1.0 was assumed in each case. The PCAIDS simulation assumed a brand elasticity of –3.5 for Charmin.

| **Simulation Method** | **Brand** | **Predicted Unilateral Effect (%)** |
|---|---|---|
| AIDS | CHARMIN | 8.7 |
| | SCOTT | 8.0 |
| Logit | CHARMIN | 3.0 |
| | SCOTT | 5.3 |
| PCAIDS | CHARMIN | 9.2 |
| | SCOTT | 8.4 |

In contrast to the beer results, AIDS yields higher unilateral effects than logit. These effects are driven in large part because the revenue share for Scott in the AIDS simulations is much larger than the corresponding quantity share in the logit model.

Alternatively, for the PCAIDS analysis suppose one thought that Charmin was a "premium" tissue and that Scott was an "economy" tissue and that these products should be in separate nests. Further, suppose that Kleenex and Northern are premium and that Other was economy, and are also assigned to the appropriate nest. The following simulation assumes a nesting parameter equal to 0.5, implying that economy is a moderately good substitute for premium.

| **Simulation Method** | **Brand** | **Predicted Unilateral Effect (%)** |
|---|---|---|
| NESTED PCAIDS | CHARMIN | 6.5 |
| | SCOTT | 6.7 |

The predicted effects are somewhat smaller. Note that the nest structure effectively makes the pre-merger market less concentrated with respect to Charmin and Scott because they are in different nests.

### C. Business Software

In a recent case, the U.S. brought suit against Oracle Corporation, alleging that the proposed hostile acquisition of PeopleSoft by Oracle is anticompetitive and violates Section 7 of the Clayton Act. The District Court ruled against the U.S., but as this is written, it appears likely that the decision of the Court will be appealed. Whatever the ultimate outcome, the case raises interesting questions concerning unilateral effects analysis generally, and merger simulation and geographic market definition specifically. In what follows, we briefly highlight some of those questions.[40]

According to the Department of Justice, Oracle, PeopleSoft, and SAP are the only three firms in a $500 million U.S. market for high function business applications software (i.e., software sold to major businesses to manage their operations). DOJ argued that the merger is 3-to-2 in that market and will lessen competition. In particular, DOJ alleged the merging parties have a 48% share in "high function financial management software" ("FMS") and a 68% share in "high function human resource management software" ("HRM"). Oracle countered that these shares were overstated because the geographic market should include at least Europe and perhaps the rest of the world, and that the product market is too narrow. The parties also disagreed as to the extent, if any, of the unilateral effects that will be generated as the result of the merger.[41] DOJ claimed that software prices would increase. Oracle argued they would not, claiming the absence of significant barriers to entry into DOJ's alleged product market. In particular, Oracle pointed to a number of potential competitors, including Microsoft and Lawson, and argued that customers can protect against any exercise of market power by Oracle by outsourcing or by obtaining software from potential competitors that are able to reposition themselves to compete with Oracle and PeopleSoft.

The DOJ submitted economic testimony with respect to market definition (Prof. Kenneth Elzinga) and unilateral effects (Prof. Preston McAfee). Oracle offered counter economic testimony on each (by Profs. Jerry Hausman and Tom Campbell, respectively).

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[40] Redacted versions of the submissions by both parties are available on their respective websites: www.usdoj.gov/atr/cases, and www.oracle.com/peoplesoft. For a PeopleSoft point of view, see www.peoplesoft.com/corp/en/news_events/news/justice.jsp.

[41] Both parties appear to agree that a coordinated effects theory is not appropriate in this case.

Oracle's third economic expert, Prof. David Teece, testified that the market was dynamic and that the introduction of a new "infrastructure layer" of competitive products marked a paradigm shift that needed to be accounted for in defining a relevant market.

Judge Walker ruled for Oracle in his September 2004 decision. In Judge Walker's opinion, the U.S. did not meet its legal burden to prove by a preponderance of the evidence that (i) HRM and FMS (both forms of high function software) define separate relevant product markets; (ii) the relevant geographic market is the U.S. as opposed to a world market; (iii) there are any significant unilateral effects. The Court also ruled that Oracle had not proved any efficiencies, but that point will be moot unless the District Court's opinion is overturned and the case is remanded for further proceedings.

The Oracle case raises two issues that are relevant for this Report. The first one concerns which empirical methods for evaluating unilateral effects are relevant in an auction situation in which many contracts are put up for bid. The second involves application of the Elzinga-Hogarty test. We discuss each of these issues, in reverse order.

#### 1. Geographic Market

With respect to geographic market definition, DOJ (citing the *Horizontal Merger Guidelines*) argued that the scope of the market is the U.S. because "there was no evidence that [U.S. customers] could effectively turn outside of [the United States] for alternative sources of [the product.][42] The government argued further that customers are business units operating in the U.S., wherever their corporate headquarters might be located. Prof. Elzinga testified that licensing high function software involves on-going customer relationships that require on-site evaluations and demonstrations, as well as product support and continuing maintenance and upgrades. Such relationships, he argued, necessitate a vendor with significant domestic operations. He testified further that, because arbitrage is not possible, a hypothetical monopolist of U.S. sales of the product could raise prices in the U.S. (the price increase would not be defeated by importation). Ironically, Prof. Elzinga rejected the application of an Elzinga-Hogarty test in this case because the test "was designed for product markets that are literally shipped

[42] DOJ, Post-trial Brief, July, 2004. p.23.

from factory to customer, that have substantial transportation costs relative to the value of the product, and for which there are no legal impediments to shipment across geographic boundaries."[43] He also argued that the Elzinga-Hogarty test should not be applied to the case due to the existence of geographic price discrimination.

Oracle claimed the geographic market is worldwide and that if there were a U.S. market, Oracle does not have significant market power because market shares should "reflect global capabilities … because SAP's dominance globally, and in particular among global enterprises, would be enough to defeat plaintiffs' claims."[44] Oracle argued further for a worldwide market using four distinct "tests:"[45] (1) sellers such as SAP, Oracle, PeopleSoft, Siebel, and Microsoft market their software globally; (2) the Elzinga-Hogarty test indicates a worldwide market; (3) software transportation costs are minimal; and (4) prices in the U.S. tend to move uniformly with prices outside the U.S. (relying on the Stigler-Sherwin test and an empirical analysis of Prof. Hausman).

The Court rejected Prof. Elzinga's argument that the geographic marked is limited by the fact that products are marketed and supported in the U.S. The Court also concluded that an Elzinga-Hogarty test was appropriate in this case, from which it follows that the relevant geographic market is worldwide.[46] Nevertheless, the testimony by both sides confirms that there is no single best test to determine a relevant market, and that good economics involves a judicious evaluation of a range of documentary and empirical evidence as well as the choice of an appropriate methodology.

### 2. Unilateral Effects

DOJ argues that by eliminating PeopleSoft as a competitor, the proposed acquisition would cause Oracle to offer lower discounts to U.S. customers on its high function business software. The Government identifies four distinct unilateral effects theories, namely mergers that will raise price by:

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[43] *Id.*, p. 24.
[44] Oracle, Post-trial Brief, July, 2004, p. 22
[45] *Id.*, p. 23–24.
[46] Opinion of the District Court, pp. 132-136.

> "(a) creating a monopoly or dominant firms; (b) perpetuating a monopoly or dominant firm by eliminating a nascent rival; (c) giving one firm stronger control of its "niche" in a product-differentiated market; or (d) strengthening a firm's power to make noncompetitive bids that buyers will be unable to refuse."[47]

DOJ argued that points (c) and (d) apply to this case. In response to a challenge by Judge Walker, DOJ pointed in particular to the analysis of unilateral effects in *FTC v. Swedish Match* and to *FTC v. Staples, Inc.*[48] and cited legal commentator Herbert Hovenkamp:

> "Unilateral effects theories have proven to be among the most useable and robust contributions of the post-Chicago revolution in antitrust economics … Unilateral effects methodologies for analyzing mergers must be regarded as, if anything, more reliable than the methodologies used for evaluating mergers under the traditional concerns about increased concentration."[49]

In support of its unilateral effects case, DOJ cited documentary evidence and offered direct testimony from customers that Oracle and PeopleSoft are each others' closest competitors. Prof. McAfee emphasized that competition in this market involves a customer-specific bidding process. Prof. McAfee relied on three separate analyses to conclude that there will be significant price effects in the relevant market: (1) an analysis of 25 specific cases of competition between Oracle and PeopleSoft; (2) a regression analysis showing that when PeopleSoft is a competitor, Oracle offers higher discounts; and (3) a more general analysis of bidding behavior using auction theory. All three analyses led Prof. McAfee to conclude that the proposed merger will result in higher prices.

In response, Oracle argued that it will not be able to raise prices post-merger. Citing the *Guidelines* presumption that unilateral effects are not likely to be a problem if the post-merger market share of the firm is less than 35%, Oracle used product and geographic market definitions to support its view that the post-merger share will be

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[47] DOJ, Post-trial Brief, p. 28.

[48] 131 F. Supp. 2d 151 (D.D.C. 2000) and 970 F. Supp. 1066 (D.D.C. 1997), respectively. These were cases in which the Government as successful. The defendant was victorious in a number of other unilateral effects cases, including *U.S. v. Long Island Jewish Med. Ctr.* (983 F. Supp. 121 at 142–144) and *NewYork v. Kraft Gen. Foods* (926 F. Supp. 321 at 352–33).

[49] Herbert Hovenkamp, "Post-Chicago Antitrust: a Review and Critique," 2001 *Colum. Bus. L. Rev.* 257, 333.

significantly less than 35%. Oracle argued further that market shares are likely to give an inaccurate picture of this acquisition since (1) customers choose to limit the number of vendors, they do not care to have a large number of bidders; and (2) it is difficult to predict price effects in markets in which buyers as well as sellers have substantial market power.

Finally, Oracle argued that an appropriate unilateral effects analysis of the diversion of sales requires extensive econometric analysis and that DOJ failed to provide one. Interestingly, Oracle pointed explicitly to the potential of merger simulation as a valuable tool. Oracle's position, however, was that the relevant demand elasticities should be estimated econometrically and should not be based on qualitative assessments. Oracle also criticized Profs. Elzinga and McAfee for assuming closeness of products based on market shares, without developing appropriate evidentiary support.

In ruling for Oracle, Judge Walker placed a significant burden on the DOJ. According to Judge Walker, "to prevail on a differentiated products unilateral effects claim, a plaintiff must prove a relevant market in which the merging parties would have essentially a monopoly or dominant position."[50] In Judge Walker's view, DOJ's customer testimony was not sufficient to support the Government's market definition. As a result, he rejected the DOJ alleged market shares as inapplicable.[51]

Judge Walker also concluded that the Government failed to prove that Oracle and PeopleSoft were each others' closest competitor. He rejected the testimony of Prof. McAfee on unilateral effects because that testimony was based on the wrong market definition (a U.S. rather than global market). Judge Walker also criticized the Government (and its expert Prof. Elzinga) for not presenting market definition testimony based on econometric analyses of cross-price elasticities and/or diversion ratios.

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[50] District Court Opinion, p. 47.
[51] District Court Opinion, p. 140.

## VI. TECHNICAL APPENDIX

This appendix has two main purposes. First, it presents the specification of the ALM and AIDS econometric demand models used by the Stata programs. Second, it allows a user of the simulation software to follow the calculations and verify the solution. Because it is intended to be a complete, stand-alone explanation, it may repeat information presented elsewhere in this Report. We provide examples to illustrate the details of the calculations for each of the four simulation models.

### A. Notation

The following notation is used throughout the Appendix. There are n firms. Each firm produces $n_i$ brands, for a total of N brands. All brands are "inside" goods. There are I "cities" and T time periods in the data.

The kth brand has the following characteristics:

$q_{kit}$ — brand quantity sold in city i during period t.

$p_{kit}$ — brand price in city i during period t.

$s_{kit}$ — brand share in city i during period t. The share is based on quantity for ALM and is based on revenue for AIDS and PCAIDS.

$\varepsilon_{kk}$ — own-price elasticity

$\varepsilon_{jk}$ — cross-price elasticity of brand j with respect to change in price of brand k

$c_k$ — incremental cost (assumed constant)

$\mu_k$ = average brand profit margin $(p_k - c_k)/p_k$.

In addition, we require:

$E_i$ — transposed matrix of own-price and cross-price elasticities for the ith firm, with element (k, j) equal to $\varepsilon_{jk}$.

$\gamma_k$ — merger-specific efficiency for the kth brand, i.e., $c_k^{post} = (1 + \gamma_k)c_k$.

$\delta_{ki}$ — fixed effect for brand k, city i

$x_{it}$ — total nominal expenditure in city i during period t for the brands represented by $s_1 \ldots s_N$.

$P_{it}$ — a price index

$\varepsilon$ — industry elasticity of demand

## B. Econometric Estimation

### 1. ALM

The dependent variables for the regressions in the ALM are constructed by taking logarithmic transformations of the share data, which yields a convenient linear model. One brand is used as a numeraire good, so the estimated model contains only N–1 brand share equations. Without loss of generality, brand N is used as a numeraire. The logit model implies that the observed market shares satisfy

$$s_{kit} = \frac{\exp(\alpha_k + \beta p_{kit})}{\sum \exp(\alpha_j + \beta p_{jit})} \quad .$$

where $\beta$ and the $\alpha$'s are parameters to be determined. The model is linearized by using the transformation $\ln(s_{kit}/s_{Nit})$ as the dependent variable for the brand k share equation.

After the transformation, each brand share equation is of the form

$$\ln(s_{kit} / s_{Nijt}) = \alpha_k - \alpha_N + \beta(p_{kit} - p_{Nit}).$$

Because this is a panel model, it is appropriate to add fixed effects for each brand for each city. There are therefore I–1 fixed effects (to avoid perfect collinearity) per brand. The complete model can be written as

$$\ln(s_{1it} / s_{Nit}) = \gamma_1 + \beta(p_{1it} - p_{Nit}) + \sum_{m=1}^{I-1} \delta_{1m} + u_{1it}$$

$$\ldots$$

$$\ln(s_{N-1,it} / s_{Nit}) = \gamma_{N-1} + \beta(p_{N-1,it} - p_{Nit}) + \sum_{m=1}^{I-1} \delta_{N-1,m} + u_{N-1,it}$$

Because the underlying theory assumes that $\beta$ is the same in each equation, the model should be estimated with a systems estimation procedure to impose the appropriate cross-equation constraints. If data are available, the user may add additional variables to the system to control for factors other than price. For example, in the context of supermarket data, the system could include a dummy variable that takes on the value 1 if the brand was under promotion in a particular time period and 0 otherwise.

### 2. AIDS

The estimated share equations are of the form $s_{kit} = a_k + \sum b_{km} \ln p_{mit} + h_k \ln (x_{it} / P_{it}) + \sum \delta_{kj}$. That is, as a panel model, the brand share equations include I–1 city fixed effects in addition to the usual price terms and real expenditure term. Because the shares are defined to add to 100% for each time period in each city, it is appropriate to impose this "adding up" constraint on the estimation procedure. This is accomplished by estimating only N–1 share equations for each city and time period. The last equation is obtained as a residual where $s_{Nit}$ is calculated as 1 minus the sum of the other brand shares for the ith city in each period, which ensures that the constraint is satisfied. We deflate expenditure using a Stone price index, i.e., $\ln P_{ij} = \sum \omega_k \ln p_{kij}$.[52] Each weight $\omega_k$ equals the average brand k revenue share over the entire sample period. There are therefore I–1 fixed effects (again, to avoid perfect collinearity). The complete model can then be written as the N–1 equation system:

$$s_{1it} = a_1 + b_{11} \ln(p_{1it}) + b_{12} \ln(p_{2it}) + \ldots + b_{1N} \ln(p_{Nit}) + h_1 \ln(x_{it} / P_{it}) + \sum_{m=1}^{I-1} \delta_{1m} + u_{1it}$$

$$\ldots$$

$$s_{N-1,it} = a_{N-1} + b_{N-1,1} \ln(p_{1it}) + b_{N-1,2} \ln(p_{2it}) + \ldots + b_{N-1,N} \ln(p_{Nit}) + \\ h_{N-1} \ln(x_{it} / P_{it}) + \sum_{m=1}^{I-1} \delta_{N-1,m} + u_{N-1,it}$$

Slutsky-symmetry imposes the constraints $b_{ij} = b_{ji}$. In addition, the homogeneity constraint from economic theory is imposed by dropping the $\ln(p_N)$ term during estimation (this constraint is testable econometrically). The coefficient $b_{iN}$ in the ith equation is recovered as a residual defined as the negative of the sum of the other $b$'s in the equation. As with the ALM, if data are available the user may add additional

[52] The formal derivation AIDS requires a trans-log price index but Deaton and Muellbauer (1980, p. 316) suggested use of Stone's index as a convenient simplification to linearize the model. The resulting specification is sometimes referred to in the literature as LA/AIDS ("linear approximate AIDS").

variables to the system, such as a dummy variable for brand promotion, to control for factors other than price and expenditure.

### C. Shares and Elasticities for the Simulation Models

We summarize here the determination of shares and elasticities for the demand models analyzed in this Report.

#### 1. ALM

Elasticities in the ALM depend on two parameters. Typically, the parameters are the pre-merger industry elasticity $\varepsilon$ and the logit parameter $\beta$. Let *pbar* be the (quantity) share-weighted average price, $\sum s_i p_i$. The ALM elasticities are[53]

$$\varepsilon_j = [\beta\, pbar(1 - s_j) + \varepsilon s_j]\, p_j \,/\, pbar$$
$$\varepsilon_{jk} = s_k\,(-\beta\, pbar + \varepsilon]\, p_k \,/\, pbar\ .$$

The prices in these formulas are typically average prices for the entire sample in the period just prior to the merger. The IIA property holds because $\varepsilon_{jk} = \varepsilon_{ik}$. The sign convention is $\beta < 0$, $\varepsilon < 0$, $\varepsilon_j < 0$, $\varepsilon_{jk}$.>0.

Alternative calibrations are possible. For example, suppose exogenous information was available on $\varepsilon$ and one of the brand elasticities $\varepsilon_i$ (instead of $\beta$). Given the prices and shares, one can invert the elasticity formula for $\varepsilon_i$ to solve for $\beta$ and then calculate the remaining elasticities in the system.

Higher post-merger prices imply changes in the shares and elasticities. The calibration of the ALM to produce updated shares is achieved as follows (see Werden and Froeb, 1994). Define $z = \varepsilon / (\beta\, pbar)$ using pre-merger information. In addition, for the kth brand use the average pre-merger share to define $\alpha_k = \ln(s_k) - (\beta\, pbar) + \ln((1-z)/z)$, k=1…N. The $\alpha$'s calculated in this way take account of the averaged data used for the simulation. The shares are then given by

[53] Werden and Froeb (1994), p. 410.

$$s_k = \frac{\exp(\alpha_k + \beta p_k)}{\sum \exp(\alpha_j + \beta p_j)} \ .$$

Werden and Froeb (1994) also show that

$$\varepsilon = \beta \frac{pbar}{1 + \sum \exp(\alpha_k + \beta p_k)} \ .$$

The post-merger industry elasticity therefore also changes as *pbar* changes.

**2. AIDS**

The AIDS elasticities depend on the estimated coefficients from the AIDS model plus an exogenous value for the industry elasticity. The specific formulas are[54]

$$\varepsilon_j = (b_{jj} + \varepsilon\omega_j h_j)/s_j - 1 + \omega_j(1+\varepsilon)$$
$$\varepsilon_{jk} = (b_{jk} + \varepsilon\omega_k h_j)/s_j + \omega_k(1+\varepsilon)$$

The AIDS elasticities do not involve prices explicitly. Conventionally, the industry elasticity is assumed to remain constant. The weights $\omega$ are the average shares across cities for the entire sample period. The (revenue) shares $s$ are typically the average shares at the end of the sample period.

Post-merger shares to calculate post-merger elasticities are calculated as follows. The AIDS share equation for the ith brand is $s_i = a_i + \sum b_{ij}\ln p_j + h_i\ln(x/P)$. The post-merger share equals $s_i + ds_i$. Now, differentiate the share equation totally to set $ds_i = \sum b_{ij}dp_j/p_j + h_i\varepsilon\sum\omega_j dp_j/p_j$. The second term in this expression makes use of the equations $dx/x = (1+\varepsilon)dP/P$ and $dP/P = \sum\omega_j dp_j/p_j$.

**3. PCAIDS**

PCAIDS elasticities also depend on two parameters. Typically, the parameters are the pre-merger industry elasticity $\varepsilon$ and one of the brand elasticities. Assume that a value is available for $\varepsilon_1$. Then the entire set of elasticities for the market is given by[55]

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[54] Hausman and Leonard (2002), p. 250.

$$\varepsilon_j = [(1 - s_j)\varepsilon_1 + (s_j - s_1)\varepsilon] / (1 - s_1)$$

$$\varepsilon_{jk} = s_k(\varepsilon - \varepsilon_1) / (1 - s_1)$$

The IIA property holds because $\varepsilon_{jk} = \varepsilon_{ik}$. As a two parameter system, PCAIDS can also be calibrated with values for any pair of price elasticities (e.g., $\varepsilon_1$ and $\varepsilon$ can be found given $\varepsilon_j$ and $\varepsilon_{jk}$). The (revenue) shares $s$ are typically the average shares at the end of the sample period.

It is useful to express the elasticities in terms of the underlying demand system parameters. PCAIDS suppresses the AIDS expenditure terms to express shares as

$$s_1 = a_1 + b_{11} \ln(p_1) + b_{12} \ln(p_2) + \ldots + b_{1,N-1} \ln(p_{N-1})$$

$$\ldots$$

$$s_{N-1} = a_{N-1} + b_{N-1,1} \ln(p_1) + b_{N-1,2} \ln(p_2) + \ldots + b_{N-1,N-1} \ln(p_{N-1})$$

PCAIDS also imposes adding-up and homogeneity in a manner that is analogous to the full AIDS specification. Recall, however, that PCAIDS imposes IIA so that in general the $b$ coefficients in PCAIDS will be different from the $b$'s estimated econometrically for the full AIDS.

The PCAIDS elasticities can be expressed equivalently in terms of the $b$'s as

$$\varepsilon_j = b_{jj}/s_j - 1 + s_j(1+\varepsilon) \quad \text{(A1)}$$

$$\varepsilon_{jk} = b_{jk}/s_j + s_k(1+\varepsilon) \quad \text{(A2)}$$

The formulas resemble the AIDS formulas without the expenditure term component. (The shares $s$ are used in place of the weights $\omega$ on the assumption that time-series data to calculate $\omega$ are not available.)

The elasticities for PCAIDS with nests involve $\varepsilon$, $\varepsilon_1$, and the nesting parameters. Assume that there are w nests, $w \leq N$, with each brand assigned to a nest. Given a price increase for brand k in nest 1, the diversion of share to brand i in nest 2 deviates from proportionality by a nesting parameter $0 < \omega(\Im(k), \Im(i)) \leq 1$. $\Im()$ is an indicator function that maps a brand into its associated nest, i.e., $\Im(k) = 1$ in this example. We assume that $\omega(\Im(k), \Im(i)) = \omega(\Im(i), \Im(k))$. Similarly, the diversion from brand k to brand j in nest 3

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[55] See Epstein and Rubinfeld (2002), Appendix.

deviates from proportionality by $\omega(\Im(k), \Im(j))$. Proportionality is the special case where $\omega(\Im(k), \Im(i)) = 1$. In this case the diagonal and off-diagonal b coefficients in PCAIDS are given by

$$b_{jj} = \frac{s_j}{s_1} \frac{\sum_{m \neq j} s_m \omega(j, m)}{\sum_{m \neq 1} s_m \omega(1, m)} b_{11} \tag{A3}$$

and

$$b_{ij} = -\frac{s_i s_j}{s_1} \frac{\omega(i, j)}{\sum_{k=2}^{N} s_k \omega(k, 1)} b_{11}, \ i \neq j. \tag{A4}$$

The PCAIDS elasticities with nests are obtained by using these coefficients with formulas (A1) and (A2).

A general procedure to calibrate PCAIDS with nests is as follows. With values for $\varepsilon$ and $\varepsilon_1$, invert (A1) to find $b_{11}$. Use (A3) and (A4) to find the remaining $b$ coefficients in the demand system. Equations (A1) and (A2) then provide the rest of the elasticities.

When the brand margins are known, the DG-COMP MSP software can be used to calibrate PCAIDS with nests. Specify an exogenous brand elasticity that results in the corresponding brand margin that matches the observed margin. Then iteratively search over nesting parameters in the (0,1] interval until nesting parameters are found that yield margins for the other brands that match the observed values. The software prints out the implied pre-merger margins for this purpose. If no solution for nesting parameters can be found, the analysis must reconsider whether the brand shares and margins have been measured accurately and whether brands have been assigned to appropriate nests. If there is no scenario in which the model yields the observed margins using reasonable assumptions, this may be a signal that the assumption of Bertrand equilibrium is not consistent with the data.

Finally, post-merger shares to calculate post-merger elasticities are calculated by totally differentiating the PCAIDS share equation $s_i = a_i + \sum b_{ij} \ln p_j$. The post-merger share equals $s_i + ds_i$, which equals $s_i + \sum b_{ij} dp_j/p_j$.

## D. First-Order Conditions for Simulation

### 1. Pre-Merger

An example with 3 firms is sufficient to illustrate the structure of the first-order conditions ("FOC") for the simulation models treated in this Report. Assume that firm 1 produces two brands and that firms 2 and 3 each produce a single brand, for a total of four brands in the relevant market. We assume a pre-merger equilibrium under static Bertrand assumptions with constant incremental costs. A firm maximizes profits by varying its prices under the assumption that its competitors' prices remain fixed. That is, it solves $\partial[\sum(p_j - c_j)q_j]/\partial p_k = 0$ for each of the brands it produces.

For a single brand firm, the FOC is $1 - (p_k - c_k)\ \partial q_k/\partial p_k = 0$. Let $PQ$ represent total market revenue. It is convenient to multiply both sides of the equation by $s_k = p_k q_k$ / PQ to express the first-order condition as

$$s_k + s_k \varepsilon_{kk} \mu_k = 0 .$$

This formulation of the FOC is in terms of revenue shares (even if the demand model is the ALM). A multi-brand firm maximizes profits by taking account of the cross-price elasticity involving the brands it produces. The number of its first-order conditions equals the number of its brands. There is a separate first-order condition for each brand in the market. When the elasticities are known then the pre-merger margins can be inferred from the first-order conditions.

The set of pre-merger FOCs for the example is

(Firm 1) Brand 1: $s_1 + s_1\varepsilon_{11}\mu_1 + s_2\varepsilon_{21}\mu_2 = 0$ .

(Firm 1) Brand 2: $s_2 + s_1\varepsilon_{12}\mu_1 + s_2\varepsilon_{22}\mu_2 = 0$

(Firm 2) Brand 3: $s_3 + s_3\varepsilon_{33}\mu_3 = 0$

(Firm 3) Brand 4: $s_4 + s_4\varepsilon_{44}\mu_4 = 0$

A general matrix expression for all of the FOCs for all of the brands in the market is given by

$$s + \mathrm{diag}(E_1, E_2, \ldots, E_n)S\mu = 0. \qquad \text{(A5)}$$

Recall that $E_i$ is an $n_i$ by $n_i$ matrix of transposed price elasticities for the ith firm. In equation (A5), $s = (s_1, s_2, \ldots s_N)'$ is the vector of market shares (in terms of revenue) and $S = \mathrm{diag}(s)$. The corresponding vector of brand margins is $\mu = (\mu_1, \mu_2, \ldots, \mu_N)'$. Given the elasticities and shares, the brand margins $\mu$ in the pre-merger equilibrium are given by

$$\mu = -S^{-1}\mathrm{diag}(E_1, E_2, \ldots, E_n)^{-1}s.$$

### 2. Post-Merger

Suppose firms 1 and 2 merge to form NewCo. There are still four first-order conditions because the number of brands has not changed. However, post-merger there will be only two firms. The newly merged firm maximizes a new profit function involving all of the brands it now produces. The new first order conditions are:

(NewCo) Brand 1: $s_1 + s_1\varepsilon_{11}\mu_1 + s_2\varepsilon_{21}\mu_2 + s_3\varepsilon_{31}\mu_3 = 0$ .

(NewCo) Brand 2: $s_2 + s_1\varepsilon_{21}\mu_1 + s_2\mu_2\varepsilon_{22} + s_3\varepsilon_{32}\mu_3 = 0$

(NewCo) Brand 3: $s_3 + s_1\varepsilon_{31}\mu_1 + s_2\varepsilon_{32}\mu_2 + s_3\varepsilon_{33}\mu_3 = 0$

(Firm 3) Brand 4: $s_4 + s_4\varepsilon_{44}\mu_4 = 0$

NewCo has to take account of all of the cross-price elasticities between its products to maximize profits. The structure of the FOC for firm 3 does not change.

NewCo is characterized by an augmented elasticity matrix $E^*$ for the $n_1 + n_2$ brands it is now producing. The general matrix expression for all of the FOCs in the post-merger market is

$$s + \mathrm{diag}(E^*, E_3, \ldots, E_n)S\mu = 0,$$

where all variables are understood to be taken at their post-transaction values.

Provided the cross-price elasticities are positive so that the goods are substitutes, NewCo has the unilateral incentive to raise its prices. The new cross-price elasticity terms imply that its FOCs are positive at pre-merger prices (they had been equal to zero and the new cross-price term is greater than zero). By raising the brand price, the margin increases and gives greater weight to the negative own-price elasticity, thereby bringing the FOC back to zero.

Define the percentage changes in the brand prices (e.g., $dp_i/p_i$) as the unilateral effects from the merger. The new prices generated by the unilateral effects give rise to new margins and shares, which in turn generate new elasticities. The changes in the shares and elasticities are determined by the demand model, e.g., ALM, AIDS, or PCAIDS. The solution of the post-merger FOCs is obtained by optimizing with respect to the vector of unilateral effects.

We calculate the post-merger price for the ith brand as $p_i^{post} = p_i(1+ dp_i/p_i)$ in the ALM and $p_i^{post} = p_i\exp(dp_i/p_i)$ for AIDS and PCAIDS.[56] The post-merger margin $\mu_i^{post} = (p_i^{post} - c_i^{post})/p_i^{post}$ can be parameterized directly in terms of the unilateral effect and the exogenous efficiency. In particular, $\mu_i^{post} = 1 - (1 - \mu_i)(1+\gamma_i)/(1 + dp_1/p_1)$ for the ALM and $\mu_i^{post} = 1 - (1 - \mu_i)(1+\gamma_i)/\exp(dp_1/p_1)$ for AIDS and PCAIDS.

The calculation of post-merger shares and elasticities for use in the FOCs was discussed in Section C above.

### E. Solution Examples

We use the beer case study to illustrate the calculations underlying simulations with the alternative demand models used in this Report.

#### 1. ALM

The logit beer analysis is based on the following data:

| **Brand** | **Quantity Share (%)** | **Price ($)** |
|---|---|---|
| BUD | 6.6 | 0.0441 |
| MILLER | 25.3 | 0.0409 |
| MILLER_LITE | 18.7 | 0.0396 |
| OLD_STYLE | 17.2 | 0.0328 |
| OTHER_LIGHT | 9.9 | 0.0387 |
| OTHER_REG | 22.3 | 0.0497 |

[56] The difference reflects the fact that AIDS and PCAIDS are formulated in terms of the logarithm of prices.

The average price *pbar* is equal to 0.0412. The estimated $\beta$ is –61.7. The industry elasticity was set to –1.0.

DG-COMP MSP reports the following pre-merger matrix of ALM elasticities:

| | BUD | OLD_STYLE | MILLER | MILLER_LITE | OTHER_LITE | OTHER_REG |
|---|---|---|---|---|---|---|
| BUD | –2.61 | 0.21 | 0.39 | 0.28 | 0.14 | 0.42 |
| OLD_STYLE | 0.11 | –1.81 | 0.39 | 0.28 | 0.14 | 0.42 |
| MILLER | 0.11 | 0.21 | –2.14 | 0.28 | 0.14 | 0.42 |
| MILLER_LITE | 0.11 | 0.21 | 0.39 | –2.17 | 0.14 | 0.42 |
| OTHER_LITE | 0.11 | 0.21 | 0.39 | 0.28 | –2.24 | 0.42 |
| OTHER_REG | 0.11 | 0.21 | 0.39 | 0.28 | 0.14 | –2.65 |

These values match the results of using the formulas in section C.1.. For example, the own-price elasticity of Miller_Lite is –2.17 = (–61.7(.0412)(1–.187) + (–1)(.187)).0396/.0412. The software also reports pre-merger margins as:

| **Brand** | **Margin** |
|---|---|
| BUD | 0.3830 |
| OLD_STYLE | 0.5515 |
| MILLER | 0.5421 |
| MILLER_LITE | 0.5557 |
| OTHER_LITE | 0.4453 |
| OTHER_REG | 0.3769 |

To evaluate the FOCs as we have expressed them, the quantity shares in the ALM need to be converted to revenue shares. This is accomplished by rescaling $s_i' = p_i s_i / pbar$, where $s_i$ is the quantity share.

The pre-merger FOCs are satisfied with the calculated shares, elasticities, and margins. For example, the rescaled revenue shares for Miller and Miller Lite are 25.09% and 17.91%, respectively. The FOC for Miller is 0.0 = .2509 + .2509(–2.14)0.5421 + .1791(0.39)0.5557.

The post-merger FOCs are also satisfied. Consider NewCo. DG-COMP MSP reports post-merger (quantity) shares of 5.7% and 17.0% with unilateral effects of 6.0% and 1.5% for Bud and Old Style, respectively. The predicted post-merger prices are therefore 0.0467 (i.e., a 6% increase) and 0.0333. The program reports a weighted-average unilateral effect across all firms of 0.73%, implying a post-merger average price equal to 0.0415. Next, the post-merger margin is calculated for Bud as 0.418 = 1–(1–0.3830)/1.06 and for Old Style as 0.5581 = 1–(1–0.5515)/1.015. The post-merger industry elasticity becomes –1.017 (the DG-COMP MSP results include the logit $\alpha$ parameters to verify this calculation). The post-merger Bud own-price elasticity becomes –2.78 = (–61.7(.0415)(1–.057) + (–1.017) (.057)).0467/.0415 and the post-merger cross-price elasticity of Old Style with respect to Bud is 0.099 = .057(61.7(.0415)–1.017).0467/.0415. Finally, the implied post-merger revenue shares for Bud and Old Style are 6.41% and 13.64% by rescaling. The post-merger Bud FOC is 0.0= .0641 + .0641(–2.78)(.418) + .1364(.099).5581. The other FOCs may be verified similarly.

### 2. AIDS

The AIDS beer analysis is based on the following data:

| **Brand** | **Revenue Share (%)** | **Price Index Weights (%)** |
|---|---|---|
| BUD | 7.1 | 8.8 |
| MILLER | 25.1 | 22.7 |
| MILLER_LITE | 17.9 | 18.8 |
| OLD_STYLE | 13.7 | 13.4 |
| OTHER_LIGHT | 9.3 | 8.8 |
| OTHER_REG | 26.9 | 27.5 |
| Total | 100.0 | 100.0 |

The relevant matrix of estimated AIDS coefficients is:

| | Ln(Price) Of | | | | | |
|---|---|---|---|---|---|---|
| Share Eqn. | BUD | MILLER | MILLER_LITE | OLD_STYLE | OTHER_LIGHT | OTHER_REG |
| BUD | -0.197 | 0.040 | 0.039 | 0.019 | 0.027 | 0.072 |
| MILLER | 0.040 | -0.582 | 0.299 | 0.056 | 0.037 | 0.151 |
| MILLER_LITE | 0.039 | 0.299 | -0.500 | -0.003 | 0.045 | 0.119 |
| OLD_STYLE | 0.019 | 0.056 | -0.003 | -0.139 | 0.029 | 0.038 |
| OTHER_LIGHT | 0.027 | 0.037 | 0.045 | 0.029 | -0.147 | 0.009 |
| OTHER_REG | 0.072 | 0.151 | 0.119 | 0.038 | 0.009 | -0.389 |

The vector of coefficients for the expenditure term is (0.004, 0.025, 0.009, –0.008, –0.001, –0.029). The industry elasticity was set to –1.0.

DG-COMP MSP reports the following pre-merger matrix of AIDS elasticities:

| | BUD | OLD_STYLE | MILLER | MILLER_LITE | OTHER_LITE | OTHER_REG |
|---|---|---|---|---|---|---|
| BUD | –3.79 | 0.26 | 0.55 | 0.54 | 0.37 | 1.00 |
| OLD_STYLE | 0.15 | –2.01 | 0.42 | –0.01 | 0.22 | 0.29 |
| MILLER | 0.15 | 0.21 | –3.34 | 1.17 | 0.14 | 0.57 |
| MILLER_LITE | 0.21 | –0.02 | 1.66 | –3.80 | 0.25 | 0.65 |
| OTHER_LITE | 0.29 | 0.31 | 0.40 | 0.49 | –2.57 | 0.10 |
| OTHER_REG | 0.28 | 0.15 | 0.58 | 0.46 | 0.04 | –2.41 |

These values match the results of using the formulas. For example, the own-price elasticity of Miller_Lite according to the formula is –3.80 = (–.500 + .009(.188)(–1))/.179 –1 +0.188(1–1). The software also reports the pre-merger margins as:

| Brand | Margin |
|---|---|
| BUD | 0.2638 |
| OLD_STYLE | 0.4973 |
| MILLER | 0.4634 |
| MILLER_LITE | 0.4634 |
| OTHER_LITE | 0.3887 |
| OTHER_REG | 0.4145 |

The pre-merger FOCs are satisfied with the calculated shares, elasticities, and margins. For example, the FOC for Miller is 0.0 = .251 + .251(–3.34)0.4634 + .179(1.66)0.4634.

The post-merger FOCs are also satisfied. Consider NewCo. DG-COMP MSP reports post-merger (revenue) shares of 6.5% and 13.4% with unilateral effects of 3.4% and 3.0% for Bud and Old Style, respectively. Next, the post-merger margin is calculated for Bud as 0.2880 = 1–(1–0.2638)/1.034 and for Old Style as 0.5119 = 1–(1–0.4973)/1.03. The post-merger Bud own-price elasticity becomes –4.0 = (–.197 + .004(.088)(–1))/.065 –1 +0.088(1–1) and the post-merger cross-price elasticity of Old Style with respect to Bud is 0.15 = (.019 – .008(.088)(–1))/.134 +0.088(1–1). The post-merger Bud FOC is 0.0= .065 + .065(–4.0)(.2880) + .134(.15).5119. The other FOCs may be verified similarly.

### 3. PCAIDS

The PCAIDS beer analysis is based on the following data:

| Brand | Revenue Share (%) |
|---|---|
| BUD | 7.1 |
| MILLER | 25.1 |
| MILLER_LITE | 17.9 |
| OLD_STYLE | 13.7 |
| OTHER_LIGHT | 9.3 |
| OTHER_REG | 26.9 |
| Total | 100.0 |

The industry elasticity is set to –1.0 and an own-price elasticity of –2.5 is assumed for Budweiser.

DG-COMP MSP reports the following pre-merger matrix of PCAIDS elasticities:

| | BUD | OLD_STYLE | MILLER | MILLER_LITE | OTHER_LITE | OTHER_REG |
|---|---|---|---|---|---|---|
| BUD | –2.50 | 0.22 | 0.40 | 0.29 | 0.15 | 0.43 |
| OLD_STYLE | 0.11 | –2.39 | 0.40 | 0.29 | 0.15 | 0.43 |
| MILLER | 0.11 | 0.22 | –2.21 | 0.29 | 0.15 | 0.43 |
| MILLER_LITE | 0.11 | 0.22 | 0.40 | –2.32 | 0.15 | 0.43 |
| OTHER_LITE | 0.11 | 0.22 | 0.40 | 0.29 | –2.46 | 0.43 |
| OTHER_REG | 0.11 | 0.22 | 0.40 | 0.29 | 0.15 | –2.18 |

These values match the results of using the formulas. For example, the own-price elasticity of Miller_Lite according to the formula is –2.32 = ((1–.179)(–2.50)+(.179-.071)(-1))/(1–.071). The software also reports pre-merger margins as:

| **Brand** | **Margin** |
|---|---|
| BUD | 0.4000 |
| OLD_STYLE | 0.4179 |
| MILLER | 0.5208 |
| MILLER_LITE | 0.5208 |
| OTHER_LITE | 0.4059 |
| OTHER_REG | 0.4589 |

The pre-merger FOCs are satisfied with the calculated shares, elasticities, and margins. For example, the FOC for Miller is 0.0 = .251 + .251(–2.21)0.5208 + .179(.40)0.5208.

The post-merger FOCs are also satisfied. Consider NewCo. DG-COMP MSP reports post-merger (revenue) shares of 6.7% and 13.4% with unilateral effects of 4.5% and 2.5% for Bud and Old Style, respectively. Next, the post-merger margin is calculated for Bud as 0.4258 = 1–(1–0.4000)/1.045 and for Old Style as 0.4321 = 1–(1–0.4179)/1.025. To evaluate the post-merger elasticities we use the alternative formulas presented in Section C.3. Invert Equation (A1) to solve $b_{11}$ = –0.106 = (–2.5+1)(.071). The post-merger Bud own-price elasticity is –2.58 = –.106/.067 –1 and the post-merger cross-price elasticity of Old Style with respect to Bud (using Equation A4) is 0.11 = ((.134)/(1–.067))(.106)/.134. The post-merger Bud FOC is 0.0 = .067 + .067(–2.58)(.4258) + .134(.11).4321. The other FOCs may be verified similarly.

## 4. PCAIDS with Nests

The PCAIDS beer analysis with nests is based on the following data:

| Brand | Revenue Share (%) | Nest |
|---|---|---|
| BUD | 7.1 | Reg |
| OLD_STYLE | 13.7 | Reg |
| MILLER | 25.1 | Reg |
| MILLER_LITE | 17.9 | Lite |
| OTHER_LIGHT | 9.3 | Lite |
| OTHER_REG | 26.9 | Reg |
| Total | 100.0 | |

The industry elasticity is set to –1.0 and an own-price elasticity of –2.5 is assumed for Bud. Miller Lite and Other Lite are placed in a separate nest with a nesting parameter equal to 0.25.

DG-COMP MSP reports the following pre-merger matrix of nested PCAIDS elasticities:

| | BUD | OLD_STYLE | MILLER | MILLER_LITE | OTHER_LITE | OTHER_REG |
|---|---|---|---|---|---|---|
| BUD | –2.50 | 0.28 | 0.52 | 0.092 | 0.05 | 0.56 |
| OLD_STYLE | 0.15 | –2.36 | 0.52 | 0.09 | 0.05 | 0.56 |
| MILLER | 0.15 | 0.28 | –2.13 | 0.09 | 0.05 | 0.56 |
| MILLER_LITE | 0.04 | 0.07 | 0.13 | –1.57 | 0.19 | 0.14 |
| OTHER_LITE | 0.04 | 0.07 | 0.13 | 0.37 | –1.75 | 0.14 |
| OTHER_REG | 0.15 | 0.28 | 0.52 | 0.09 | 0.05 | –2.09 |

These values match the results of using the formulas. For example, use Equations (A1) and (A3) to find the own-price elasticity for Miller Lite. Invert Equation (A1) to solve $b_{11} = -0.106 = (-2.5+1)(.071)$. The calculation underlying Equation (A3) is summarized as follows:

| Brand | Share (%) | Numerator Nesting Parameters | Summation Term | Denominator Nesting Parameters | Summation Term |
|---|---|---|---|---|---|
| Bud | 7.1 | 0.25 | 0.018 | 1.00 | 0.0 |
| Old_Style | 13.7 | 0.25 | 0.034 | 1.00 | 0.137 |
| Miller | 25.1 | 0.25 | 0.063 | 1.00 | 0.251 |
| Miller_Lite | 17.9 | 1.00 | 0.0 | 0.25 | 0.045 |
| Other_Light | 9.3 | 1.00 | 0.093 | 0.25 | 0.023 |
| Other_Reg | 26.9 | 0.25 | 0.067 | 1.00 | 0.269 |
| Num. Sum | | | 0.275 | | |
| Denom. Sum | | | | | 0.725 |

The Miller Lite elasticity is calculated as –1.57 = (.179/.071)(.275/.725)(-.106)/.179 – 1. The software also reports pre-merger margins as:

| Brand | Margin |
|---|---|
| BUD | 0.4000 |
| OLD_STYLE | 0.4232 |
| MILLER | 0.4997 |
| MILLER_LITE | 0.6787 |
| OTHER_LITE | 0.5725 |
| OTHER_REG | 0.4787 |

The pre-merger FOCs are satisfied with the calculated shares, elasticities, and margins. For example, the FOC for Miller is 0.0 = .251 + .251(–2.13)0.4997 + .179(.13)0.6787.

The post-merger FOCs are also satisfied. Consider NewCo. DG-COMP MSP reports post-merger (revenue) shares of 6.6% and 13.3% with unilateral effects of 6.1% and 3.5% for Bud and Old Style, respectively. Next, the post-merger margin is calculated for Bud as 0.4345 = 1–(1–0.4000)/1.061 and for Old Style as 0.4373 = 1–(1–0.4179)/1.025. To evaluate the post-merger elasticities we use the alternative formulas presented in Section C.3. We already have $b_{11}$ = –0.106. The post-merger Bud own-price elasticity is –2.61 = –.106/.066 –1. A similar calculation to that used for the per-merger Miller Lite elasticity results in a post-merger cross-price elasticity of Old Style with respect to Bud equal to (using Equation A4) is 0.15. The post-merger Bud FOC is 0.0 = .066 + .066(–2.61) (.4345) + .133(.15).4373. The other FOCs may be verified similarly.

# VII. APPENDIX: MERGER SIMULATION: A SIMPLIFIED APPROACH WITH NEW APPLICATIONS

This appendix contains the complete article "Merger Simulation: A Simplified Approach with New Applications" by Roy Epstein and Daniel Rubinfeld that introduced PCAIDS. It originally appeared in 2002 in the *Antitrust Law Journal* (volume 69), published by the Antitrust Section of the American Bar Association.

The article has several minor printing errors. The last sentence in section 4.B of the appendix should read "the familiar $s_i/s_j$." The equation for $b_{ij}$ at the top of page 918 should have a leading minus sign. The line above Appendix Equation (A5) should replace the first two terms in the parentheses with the product $b_{ij}/p_j\ PQ/p_i$. On page 909, the predicted price increase for B should be 4.5%. The corresponding value of alpha is .061, implying a threshold share for the entrant of 0.26%. Finally, Table 3 (toilet paper shares) should read as follows:

| **Brand** | **Share (%)** |
|---|---|
| Scot Tissue | 16.7 |
| Cottonelle | 6.7 |
| Kleenex | 7.5 |
| Charmin | 30.9 |
| Northern | 12.4 |
| Angel | 8.8 |
| Private Label | 7.6 |
| Other | 9.4 |
| Total | 100.0 |

The discussion and simulation results based on Table 3 use the correct data.

# MERGER SIMULATION: A SIMPLIFIED APPROACH WITH NEW APPLICATIONS

Roy J. Epstein
Daniel L. Rubinfeld*

## I. INTRODUCTION

In recent years there have been significant developments in the use of empirical economic methods to study the likely competitive effects of mergers.[1] These developments have been shaped by the increased use of unilateral effects analyses by the competition authorities, as is expressed in part in the 1997 Horizontal Merger Guidelines. Such analyses evaluate the ability of the post-transaction firm to raise the prices of some or all of its (often differentiated) products through unilateral decisions and without resort to overtly collusive activities.[2]

Unilateral effects analyses encompass a broad set of issues that arise when the differentiated brands produced by the merging firms constitute the first and second choices for some group of customers. Absent de novo entry or product repositioning, a unilateral price increase may become profitable as the result of a merger if a substantial number of customers who previously would have been lost to competitors can now be retained because the merged firm also offers the customers' second choice. If, however, this "1–2" customer group is relatively small, then

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* Director, LECG Inc., Cambridge, MA, and Robert L. Bridges Professor of Law and Professor of Economics at the University of California, Berkeley, respectively. Professor Rubinfeld served as Deputy Assistant Attorney General in the Antitrust Division of the Department of Justice from June 1997 through December 1998. They wish to thank Jonathan Baker, Steven Brenner, Luke Froeb, Richard Gilbert, Jerry Hausman, Gregory Werden, and the referees for helpful comments; all errors remain their own.

[1] For a recent survey, see Jonathan B. Baker & Daniel L. Rubinfeld, *Empirical Methods Used in Antitrust Litigation: A Review and Critique,* 1 J. Am. L. & Econ. Rev. 386 (1999).

[2] The 1997 U.S. Department of Justice and Federal Trade Commission Horizontal Merger Guidelines and the 1995 U.S. Department of Justice and Federal Trade Commission Intellectual Property Guidelines have also emphasized the potential effects of a transaction on innovation, in general, and on the intensity of research and development efforts, in particular. For a general discussion and further references, see Daniel L. Rubinfeld & John Hoven, *Innovation and Antitrust Enforcement, in* Dynamic Competition and Public Policy ch. 3 (Jerome Ellig ed., 2001). To our knowledge, merger simulation has yet to be applied to evaluate competitive issues that involve innovation markets explicitly.

at best only a minimal price increase will be profitable.[3] In essence, the forgone profits from the lost sales to diverted customers would be roughly comparable to the incremental profits from price increases to customers that do not switch.

The technique known as "merger simulation" has emerged as a promising framework for this analysis.[4] Simulation uses economic models grounded in the theory of industrial organization to predict the effect of mergers on prices in relevant markets. There is a common theoretical core to all simulation approaches in use today, although the details of a given simulation will depend on data availability and on the mathematical characterization of the market or markets at issue.

While merger simulation is not a panacea for all of the economic issues that arise in a difficult transaction, it nonetheless can offer assessments of competitive effects and remedies that are beyond the reach of other methods of inquiry. For example, simulation has been used to evaluate the likelihood that potential merger-specific efficiencies (associated with reductions in the marginal cost of production) are sufficiently great to offset predicted price increases. Simulation can also be used to analyze the competitive effects of product repositioning and de novo entry. Finally, simulation can help one to evaluate the adequacy of proposed divestitures.[5] With time, we believe that simulation techniques will be better understood and more widely used by antitrust lawyers and economists.[6]

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[3] Unilateral effects simulation can predict price increases or decreases for a merger involving firms in the same market, depending on efficiencies and changes in market structure, such as repositioning and divestitures.

[4] *See, e.g.,* Jerry Hausman, Gregory Leonard & J. Douglas Zona, *Competitive Analysis with Differentiated Products,* ANNALES D'ÉCONOMIE ET DE STATISTIQUE 34 (1994); Gregory J. Werden, *Simulating the Effects of Differentiated Products Mergers: A Practical Alternative to Structural Merger Policy,* 5 GEO. MASON L. REV. 363 (1997); Carl Shapiro, *Mergers with Differentiated Products,* ANTITRUST, Spring 1996, at 23; Gregory J. Werden, *Simulating Unilateral Competitive Effects from Differentiated Products Mergers,* ANTITRUST, Spring 1997, at 27; Jerry A. Hausman & Gregory K. Leonard, *Economic Analysis of Differentiated Products Mergers Using Real World Data,* 5 GEO. MASON L. REV. 321 (1997); *see also* Gregory J. Werden *The Effects of Differentiated Products Mergers: A Practitioners' Guide, in* STRATEGY AND POLICY IN THE FOOD SYSTEM: EMERGING ISSUES 95 (Julie A. Caswell & Ronald W. Cotterill eds., 1997).

[5] In recent years the agencies have begun to look critically at remedies involving restructuring. *See, e.g.,* Robert Pitofsky, Chairman, FTC, The Nature and Limits of Restructuring in Merger Review, Remarks at Cutting Edge Antitrust Conference (Feb. 17, 2000), *available at* http://www.ftc.gov; and Richard G. Parker & David A. Balto, *The Evolving Approach to Merger Remedies,* ANTITRUST REP., May 2000.

[6] For a lawyer's assessment of merger simulation, see James F. Rill, *Practicing What They Preach: One Lawyer's View of Econometric Models in Differentiated Products Mergers,* 5 GEO. MASON L. REV. 393 (1997).

A variety of different economic models can be utilized as the basis for a simulation analysis.[7] When sufficient data are available, demand models can be estimated econometrically. When these *estimated-demand simulation models* are not feasible, models requiring less data can be valuable if one is willing to make additional assumptions about the nature of demand. The logit demand model and "PCAIDS"—a new model to be introduced in this article—both fit into this *calibrated-demand simulation model* category. We will suggest that PCAIDS offers advantages over a number of other calibrated-demand models.

We have undertaken this review and update of work on merger simulation with a number of goals in mind. First, we offer a relatively nontechnical description of the principles of merger simulation—principles that are consistent with the methodologies currently in use by the competition authorities. Second, we describe PCAIDS, the new calibrated-demand merger simulation methodology. Third, we present examples that apply PCAIDS, including some applications that to our knowledge have not previously appeared in the literature on merger simulation. Fourth, we suggest how simulation analyses might be used to evaluate the safe harbors of the Merger Guidelines.

Calibrated-demand models are relatively easy to implement and make detailed simulation feasible for nearly any transaction because they require neither scanner nor transaction-level data. The PCAIDS model, in particular, requires only information on market shares and reasonable estimates of two elasticities. Estimates of these elasticities often can be obtained from marketing information or, when appropriate, through demand estimation. As with any calibrated-demand simulation model, one can test the sensitivity of the PCAIDS results to changes in the values of the estimated elasticities and to other simulation parameters.

We believe that calibrated-demand simulation models can offer valuable screening devices for "quick looks" by enforcement agencies and by merging firms. The models can be used to review the potential antitrust exposure resulting when unilateral effects issues are raised but sufficient information is not available to estimate reliably a full set of cross-price elasticities. The models also can offer a useful means of working out the implications of the range of qualitative judgments an analyst might make based on documentary and interview evidence, and to test the sensitivity of competitive effects predictions to plausible variations in those assumptions. The analyses may be particularly useful for weighing opposing forces, as when comparing the potential anticompetitive loss of localized

[7] For an overview of publicly available merger simulation tools, see http://www.antitrust.org/economics/mergers/simulation.html.

competition to the procompetitive gain relating to merger-specific efficiencies and product repositioning.

The balance of this article is organized as follows. Part II discusses the economic fundamentals of merger simulation. Because the pros and cons of merger simulation have been extensively debated elsewhere, we do not undertake such a treatment here. In Part III we introduce the PCAIDS approach to modeling demand. We explain how a key assumption about the relationship between market shares and the diversion of lost sales from price increases can be used to calibrate the PCAIDS model. Part IV offers some examples of merger simulation with PCAIDS that includes comparisons with other simulation models. In Part V we show how PCAIDS can be applied to the analysis of product repositioning and entry. Part VI presents an analysis of the Merger Guidelines's safe harbors using PCAIDS simulation, and Part VII contains some brief concluding remarks. We have relegated the more technical mathematical details to the Appendix.

## II. THE BASICS OF MERGER SIMULATION

Merger simulation models predict post-merger prices based on information about a set of premerger market conditions and certain assumptions about the behavior of the firms in the relevant market. Simulation models typically assume that firms' behavior is consistent with the Bertrand model of pricing, both pre- and post-merger. According to this theory, each firm sets the prices of its brands to maximize its profit, while accounting for possible strategic, noncollusive interactions with competitors. An equilibrium results when no firm can increase its profit by unilaterally changing the prices of its brands. This equilibrium can be interpreted as the outcome of the interactions between each firm's pricing decisions and its expectations of the price reactions of its competitors.[8]

Merger simulation requires a "demand model" that specifies the relationships between prices charged and quantities sold in the relevant market. A reasonable demand model must satisfy a number of conditions. The most basic is that the own-price elasticities (i.e., the percentage change in quantity for a given percentage change in its own price) should be negative. Increases in a product's own price should reduce the quantity demanded of that brand. Cross-price elasticities would normally be

[8] For a basic introduction to the "Nash-Bertrand" equilibrium, see ROBERT S. PINDYCK & DANIEL L. RUBINFELD, MICROECONOMICS ch. 12 (5th ed. 2000); a more advanced presentation appears in JEAN TIROLE, THE THEORY OF INDUSTRIAL ORGANIZATION (1988).

expected to be positive; a price increase for one brand normally leads to an increase in the quantity demanded of each of the remaining brands in the market (so long as the brands are economic substitutes for each other).[9] Implementation of the demand model requires particular values for these own- and cross-price elasticities.

In addition, simulation models require assumptions about supply or, more specifically, about how total cost responds to incremental changes in post-merger output. Most simulation analyses assume that incremental costs do not vary with output. The effects of any merger efficiencies are analyzed by changing the level of incremental costs (keeping the assumption that the level of incremental cost does not change as output changes).

A merger simulation analysis typically proceeds in two stages. First, one assumes that the market shares and own-price and cross-price elasticities for each brand in the pre-transaction market are known. The assumption of profit maximization then generates a set of mathematical "first-order conditions" (FOCs) that can be used to calculate pre-transaction gross profit margins for each brand.[10] Second, one takes into account the fact that the merged firm in general will set different prices than the premerger firms, to the extent that the merger removes some competition or there are potential efficiencies. The merged firm recognizes that, when it raises price on one of its brands, it keeps the profits from customers whose purchases are diverted to a brand of its merger partner. The demand model translates these price changes into corresponding changes in margins, elasticities, and shares. This second step, in essence, involves solving for the price changes that generate post-transaction margins, elasticities, and shares that are consistent with the merged firm maximizing the sum of its profits from all of the brands it now produces.[11]

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[9] In a general demand model there is no requirement that own-price elasticities be equal for the different brands or that cross-price elasticities take on particular values.

[10] See Appendix equation (A1). Using the first-order conditions to estimate margins avoids the distortions associated with the inclusion and allocation of fixed costs in accounting data, a particular problem for multibrand firms. Moreover, relevant accounting data are likely only to be available for the brands sold by the merging parties. As a result, the FOC approach is particularly useful if one is to perform the simulation when there are more than two firms in the market and data sources are limited. We note, however, that the FOCs may yield negative margins, which are generally not consistent with the assumption that goods are substitutes. Because estimated margins depend on the price elasticities in the model, negative estimated margins could signal that the model is relying on inappropriate elasticities.

[11] See Appendix equations (A2) and (A3) for the solution to the relevant optimization problem.

## III. THE PCAIDS MODEL

### A. Background: Almost Ideal Demand Systems

Economists have explored a variety of demand models for merger simulation with a range of virtues: every model must strike a balance between theoretical rigor, tractability, and success in explaining the actual data. As might be expected, the simulated price effects of a merger will depend on the particular demand model chosen.[12] A demand model that we find particularly appealing is the Almost Ideal Demand System, or "AIDS."[13] AIDS is a widely accepted and intuitively reasonable model in economics that allows a flexible representation of own-price and cross-price elasticities. Moreover, its economic properties are arguably superior to alternatives that have often been used in merger simulation, including linear, constant-elasticity (log-linear), and logit demand models.

The major problem with AIDS is a practical one. AIDS typically requires econometric estimation of a large number of parameters, and it is not unusual for the estimated cross-price elasticities to have low precision and algebraic signs that are inconsistent with economic theory. We explain below how it is possible to implement a variant of the AIDS model in a manner that ensures the correct signs, without the use of complex econometric methods. This simplicity is not costless, however, because PCAIDS requires additional structural assumptions beyond the AIDS model.[14] We believe that these costs are often reasonable in comparison to the benefits associated with both the variety of applications that can be handled with PCAIDS or other calibrated-demand simulation models.

A simple example with three independent firms, each owning a single brand, will help explain the logic of AIDS (and PCAIDS). The AIDS model specifies that the share of each brand depends on the prices of all brands. More formally, the share of the ith brand, $s_i$, as a percent of total market revenues is a function of the natural logarithms of the prices, $p_i$, of all of the brands in the relevant market:

$$\begin{aligned} s_1 &= a_1 + b_{11}\ln(p_1) + b_{12}\ln(p_2) + b_{13}\ln(p_3) \\ s_2 &= a_2 + b_{21}\ln(p_1) + b_{22}\ln(p_2) + b_{23}\ln(p_3) \\ s_3 &= a_3 + b_{31}\ln(p_1) + b_{32}\ln(p_2) + b_{33}\ln(p_3) \end{aligned}$$

[12] *See* Philip Crooke, Luke Froeb, Steven Tschantz & Gregory Werden, *The Effects of Assumed Demand Form on Simulated Post Merger Equilibria,* 15 Rev. Indus. Org. 205 (1999).

[13] For the original presentation of AIDS, see Angus Deaton & John Muellbauer, *An Almost Ideal Demand System,* 70 Am. Econ. Rev. 312 (1980).

[14] Calibrated-demand models based on other types of demand systems also require comparably strong structural assumptions.

The coefficients $b_{ij}$ (for i, j = 1,2,3) must be determined to use this system to simulate the effects of a merger.[15] As shown in the Appendix (Section 3, Equations (A4) and (A5)), the $b$'s underlie the own-price and cross-price elasticities. The three "own-coefficients" $b_{11}$, $b_{22}$, and $b_{33}$ specify the effect of each brand's own price on its share. These coefficients should have negative signs, since an increase in a brand's price should (all other prices held constant) reduce its share; indeed, these coefficients are closely related to and have the same signs as the own-price elasticities. The six other $b_{ij}$'s specify the effects of the prices of other brands on each brand's share. For example, $b_{12}$ specifies the effect of an increase in the price of brand 2 on share 1, while $b_{13}$ describes the effect of an increased price of brand 3 on brand 1's share. These "cross-effect" coefficients are expected to be positive (assuming the three brands are substitutes), since these terms are related to and have the same signs as the cross-price elasticities.[16]

When we use this AIDS (or PCAIDS) model to simulate a merger, we wish to predict changes in the share of each brand resulting from the transaction. These changes (obtained formally by differentiating each equation totally) are given by the following:

$$\begin{aligned} ds_1 &= b_{11}(dp_1/p_1) + b_{12}(dp_2/p_2) + b_{13}(dp_3/p_3) \\ ds_2 &= b_{21}(dp_1/p_1) + b_{22}(dp_2/p_2) + b_{23}(dp_3/p_3) \\ ds_3 &= b_{31}(dp_1/p_1) + b_{32}(dp_2/p_2) + b_{33}(dp_3/p_3)\,. \end{aligned} \tag{1}$$

We can see from (1) that there is a linear relationship between the change in each brand's market share ($ds$) and the percentage changes in the three prices ($dp/p$), where the $b$'s provide the weights.[17] Note, for example, that an increase in $p_1$ leads to a decrease in $s_1$ (since $dp_1/p_1$ is positive and the weight $b_{11}$ is negative), while an increase in $p_2$ leads to an increase in $s_1$ (since $b_{12}$ is positive).

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[15] In this presentation we have suppressed the aggregate expenditure terms from the original Deaton and Muellbauer specification. This "homotheticity" assumption is reasonable to the extent that changes in industry expenditure have no significant effects on share. Since we are concerned only with changes created by the merger, the $a_i$ intercepts drop out in the analysis that follows.

[16] The market shares predicted by AIDS are required to sum to 100%—the *adding-up* property. We also impose *homogeneity*, the assumption that equal proportional changes in all prices have no effect on market share (e.g., if all prices went up by 10 percent, the market shares for the various brands should not change). As explained in the Appendix, adding-up and homogeneity effectively reduce the number of brands to be analyzed in the AIDS model from $N$ to $N$–1.

[17] The price changes will in general also affect the total size of the market (see the Appendix, section 1).

## B. ECONOMETRIC ESTIMATION OF DEMAND FOR SIMULATION MODELS

The simple 3-brand example also allows us to illustrate the difficulty in estimating elasticities. In the example, a model with 3 brands has 9 *b* parameters: 3 own coefficients and 6 cross-effect coefficients, which correspond to 3 own elasticities and 6 cross- elasticities. More generally, a market with "$n$" brands gives rise to a total of $n^2$ elasticities: $n$ own-price elasticities and $n(n-1)$ cross-price elasticities. In the AIDS context, $n^2$ $b_{ij}$ coefficients generate these elasticities.[18] While 9 coefficients ($n =$ 3) may be easily tractable in this simple example, merger analysis can involve many more brands and parameters. In the ready-to-eat cereal industry, for example, there are approximately 200 brands. As a result, a complete cereal model could involve 40,000 elasticities. To estimate the parameters of a demand model with many brands, it is necessary either to have a large data set, or to impose assumptions that reduce the number of independent parameters to be estimated.[19]

Econometric estimation using supermarket scanner data is sometimes thought to be the only practical way to determine demand parameters for large simulation models (AIDS-based or otherwise). When available, these data can indeed be quite valuable. For example, they often track detailed price variations across many cities or market areas on a weekly or monthly basis, and provide important information concerning trade promotion, couponing, and other marketing practices. Nevertheless, there are important limitations that can handicap many applications.

First, scanner data are typically available only for brands sold in supermarkets and the largest drug stores and mass merchandisers. Unless supplemented by separate audits, retail sales data in smaller outlets are typically not available. Moreover, sales of many consumer goods, and nearly all intermediate goods, are not tracked by scanner data. Second, the scanner data describe the retail prices of consumer goods, whereas many mergers occur at the production or wholesale level. To use scanner data in such cases, one must incorporate a set of assumptions about mark-ups and margins that link wholesale and retail prices. Third, scanner data generally must be analyzed with complex econometric procedures that can sometimes be open to criticism. For example, econometric issues

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[18] Other demand models will also require a similar number of estimated coefficients.

[19] In addition to imposing adding-up and homogeneity, the number of parameters can also be reduced significantly by specifying a demand model that results from a multilevel decision-making process. For an evaluation of this approach, see Daniel L. Rubinfeld, *Market Definition with Differentiated Products: The Post/Nabisco Cereal Merger,* 68 ANTITRUST L.J. 163, 173–76 (2000).

involving model identification and estimation must be overcome before demand effects can be distinguished from supply effects. Finally, despite one's best efforts, econometric estimation may yield results at odds with common sense and intuition. With many parameters to be estimated, it is frequently the case that at least some of the empirically estimated elasticities suffer from low levels of statistical significance, implausible magnitudes, and/or wrong algebraic signs.

### C. PCAIDS: Proportionality-Calibrated AIDS

Calibrated-demand simulation models offer an alternative to models that rely on econometric estimation of demand. Because they reduce the number of required demand parameters, these models are especially valuable when there are data limitations or estimation problems, or when a rapid and less costly analysis is required.[20] We offer Proportionality-Calibrated AIDS (PCAIDS) as a calibrated-demand model that provides analytical flexibility while retaining many of the desirable properties of AIDS.

PCAIDS requires neither scanner data nor data on premerger prices. It requires information only on market shares, the industry price elasticity, and the price elasticity for one brand in the market. The logic of PCAIDS is simple. The share lost as a result of a price increase is allocated to the other firms in the relevant market in proportion to their respective shares. In effect, the market shares define probabilities of making incremental sales for each of the competitors.[21]

We believe that the proportionality assumption is practical and often reasonable when data are limited.[22] With proportionality and PCAIDS, one can take a "quick look" at the likely price effects of a merger; these results are likely to be reliable when applied to markets with limited product differentiation, or when the merger brands are not unusually

[20] See Baker & Rubinfeld, *supra* note 1, for a survey of a variety of approaches to the calibration of demand systems, including auction models and conjoint survey methods.

[21] This approach has long been used in other settings involving economics and law when data are limited. For example, in *State Industries Inc. v. Mor-Flo Industries, Inc.,* 883 F.2d 1573 (Fed. Cir. 1989), one of the leading decisions in the patent damages area, the assumption is that the patent holder suffers lost sales equal to its market share applied to the infringer's sales (the remaining infringing sales would have been made by the other firms in the market in proportion to their respective shares). For a recent analysis of this decision see Roy J. Epstein, *State Industries and Economics: Rethinking Patent Infringement Damages,* 9 FED. CIR. B.J. 367 (2000).

[22] Earlier discussions of proportionality in the context of merger analysis include Robert D. Willig, *Merger Analysis, Industrial Organization Theory, and Merger Guidelines, in* BROOKINGS PAPERS ON ECONOMIC ACTIVITY: MICROECONOMICS 299 (M. Baily & C. Winston eds., 1991); Shapiro, *supra* note 4.

close (or distant) in terms of their attributes and substitutability. In this sense, proportionality reflects the analytical framework in the Merger Guidelines, which suggest that market share sometimes may be used to measure the relative appeal of the merging firms' products as first and second choices for consumers.[23] Moreover, as we discuss below, PCAIDS can be extended to situations where extensive product differentiation makes proportionality suspect. Indeed, PCAIDS can be used as an approximation of the AIDS model, with a structure that ensures proper signs and consistent magnitudes for the elasticities.[24] Another potential advantage compared to other simulation methods is that PCAIDS can be implemented on a conventional spreadsheet without additional specialized software. In summary, PCAIDS is a general method for calibrating AIDS demand with minimal data, and for which proportionality is a useful starting point.

The simplifications that flow from the proportionality assumption of PCAIDS can be illustrated in a simple example. The three equations in Equation (1) show that a change in the price of the first brand, $p_1$, affects the market shares of all three brands. Recall that the own-effect of the price of brand 1 on the share of brand 1 is $b_{11}$. The cross-effects of $p_1$ on the shares of brands 2 and 3 are given by $b_{21}$ and $b_{31}$. With proportionality, sales are diverted to brands 2 and 3 in proportion to the market shares of the two brands. For example, if brand 2 has a share of 40 percent and brand 3 a share of 20 percent, an increase in the price of brand 1 will increase the share of brand 2 by twice as much as it increases the share of brand 3. Formally, the proportionality assumption implies that the cross-effects associated with $p_1$ can be expressed in terms of $b_{11}$ and the observed shares; $b_{21}$ is equal to $-s_2/(s_2+s_3)\,b_{11}$ and $b_{31}$ equals $-s_3/(s_2+s_3)\,b_{11}$.[25] The same relationships between own and cross effects hold for other prices; for example, $b_{12}$ equals $-s_1/(s_1+s_3)\,b_{22}$.

The proportionality assumption reduces the number of unknown $b$'s in (1) from 9 to 3. We only need to know the 3 own-effect coefficients (and market shares) to calculate the remaining 6 cross-effect coefficients. More generally, the proportionality assumption posits a direct relation-

[23] *See* Horizontal Merger Guidelines ¶ 2.211.

[24] Our discussion of PCAIDS focuses on implementation with aggregate market share information. However, the method is also applicable as a set of restrictions that could be imposed when estimating standard AIDS with scanner data. We show in the Appendix that PCAIDS and its extensions to non-proportionality satisfy Slutsky symmetry, an important theoretical property for demand systems.

[25] The minus sign is necessary because $b_{11}$ is negative (it is associated with the own-effect). It is easy to verify that the sum of the cross-effects in this case equals $-b_{11}$, which confirms that adding-up is satisfied.

ship between all cross-effects associated with a particular price change and the corresponding own-effect.[26] The implication is that the only unknowns in the model are the $n$ own-effect coefficients. The assumption that the predicted market shares sum to 100 percent eliminates one additional unknown, so the number of unknown parameters is then reduced from $n^2$ to $n - 1$, or from 40,000 to 199 in our cereal example.

In fact, the proportionality assumption reduces the information requirement of PCAIDS even further. It is not necessary to know all $n$ (or even $n - 1$) own-price effects or elasticities. The PCAIDS model can be calibrated with only two independent pieces of information (in addition to the shares): the elasticity of demand for a single brand and the elasticity for the industry as a whole. For example, only the industry elasticity and the own-price elasticity for brand 1 are needed as inputs in the calculation of the own-effect coefficient for brand 1, $b_{11}$:[27]

$$b_{11} = s_1(\varepsilon_{11} + 1 - s_1(\varepsilon + 1)). \tag{2}$$

In Equation (2), $\varepsilon_{11}$ is the own-price elasticity for brand 1 and $\varepsilon$ is the industry elasticity. Then, as shown in Section 4.A. of the Appendix, proportionality implies that all remaining unknown own-effect coefficients can be determined as simple multiples of $b_{11}$, as Equation (3) illustrates:

$$b_{ii} = \frac{s_i}{1 - s_1} \frac{1 - s_i}{s_1} b_{11} . \tag{3}$$

We have already seen that once the $b_{ii}$ own-effects have been calculated, the cross-price effects can then be calculated from the own-price effects and market shares. This means that knowledge of the own-price elasticity of any one brand and the overall industry price elasticity is sufficient to obtain estimates of all relevant demand parameters of the PCAIDS model from the market share data. This is true whether there are 3 or 200 brands.

Elasticities can be calculated directly from the values for the $b$ parameters, the market shares ($s_i$), and the industry elasticity ($\varepsilon$), as follows (see Appendix equations (A4) and (A5) for details):

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[26] Note that elasticities derived using the assumption of proportionality may be sensitive to the market definition. If additional brands are thought to be in the market, and are therefore included in the model, the estimated price effects of the merger could change.

[27] More generally, the own-effect coefficient for any one brand can be determined from the industry elasticity and the own-price elasticity for that brand; the result is proven in the Appendix.

Own-price elasticity for the ith brand:

$$\varepsilon_{ii} = -1 + \frac{b_{ii}}{s_i} + s_i(\varepsilon + 1)\,. \tag{4}$$

Cross-price elasticity of the ith brand with respect to the price of the jth brand:

$$\varepsilon_{ij} = \frac{b_{ij}}{s_i} + s_j(\varepsilon + 1)\ . \tag{5}$$

Under the assumption that the magnitude of the industry elasticity $\varepsilon$ is smaller in magnitude than any brand own-price elasticity, PCAIDS implies that the cross-elasticities will be positive. Moreover, it can be shown that all pre-transaction cross-elasticities corresponding to a given price change are equal, i.e., $\varepsilon_{ij} = \varepsilon_{kj}$ for all brands i, j, and k. This equality is a consequence of the assumption of proportionality.[28]

All the information required to calibrate PCAIDS should be available. Market shares typically are known with reasonable accuracy. It should be feasible to infer the own-price elasticity for at least one brand sold by the merging parties from marketing studies in the party's documents (including surveys and focus groups), from econometric analyses, or from accounting data.[29] The industry elasticity typically is considerably smaller than the price elasticity of any one brand, because brand substitution is easier than industry substitution.[30] Absent independent information about the magnitude of that elasticity, we suggest an industry elasticity of −1 as a good starting point for a preliminary merger simulation. If the market under study is a relevant antitrust market, the industry elasticity will be equal to or greater than 1 in magnitude. As a result, this assumption will be conservative in its tendency to overpredict the price effects of mergers.[31]

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[28] The assumption of proportionality is equivalent to the assumption of "Irrelevance of Independent Alternatives" (IIA) that underlies the logit model. Unlike the logit model, however, the PCAIDS post-merger elasticities are not constrained by IIA.

[29] For an extensive discussion of the range of empirical methods that can be used to obtain estimates of demand elasticities, see Baker & Rubinfeld, *supra* note 1, Section 3.

[30] Suppose the prices of all cereals rose by 10%. Because many consumers, particularly children, are likely to continue eating the same quantities of cereal for breakfast (some, of course, will not and consumption of cereal for other purposes, such as snacks, may fall), ready-to-eat demand is not likely to be highly price-sensitive. On the other hand, a 10% increase for a single brand, such as corn flakes, with no change in competitors' prices, will be more price-sensitive, because it will likely result in substantial switching to other products within the cereal category.

[31] This follows from the rule of thumb for pricing by a monopolist. *See, e.g.,* PINDYCK & RUBINFELD, *supra* note 8, ch. 11.

To illustrate PCAIDS, reconsider the demand system in (1). Assume that the shares for the 3 brands (each sold by a different firm) are 20%, 30%, and 50%, respectively. Now, assume that there is a proposed merger between firms 1 and 2, the industry elasticity is −1, and the own-price elasticity for the first brand is −3. The formulas for PCAIDS given above and in the Appendix allow calculation of all parameters of the demand system (1) and all elasticities as shown in Table 1 below.

**Table 1**
**PCAIDS Coefficients and Elasticities**

| *PCAIDS Coefficient with Respect to:* | | | | *Elasticity with Respect to:* | | | |
|---|---|---|---|---|---|---|---|
| *Brand* | $p_1$ | $p_2$ | $p_3$ | *Brand* | $p_1$ | $p_2$ | $p_3$ |
| 1 | −0.400 | 0.150 | 0.250 | 1 | −3.00 | 0.75 | 1.25 |
| 2 | 0.150 | −0.525 | 0.375 | 2 | 0.50 | −2.75 | 1.25 |
| 3 | 0.250 | 0.375 | −0.625 | 3 | 0.50 | 0.75 | −2.25 |

The calculated own-elasticities—the negative values on the diagonal of the right panel of the table—can be either larger or smaller than the elasticity for the brand used to calibrate the system.[32] Reading down each column of elasticities, the cross-elasticities corresponding to a given price are equal, as expected given proportionality. PCAIDS simulation with these parameters predicts a unilateral post-merger price increase (absent efficiencies) of 13.8% for Brand 1 and 10.8% for Brand 2.

### D. Deviations from Proportionality—PCAIDS with Nests

Proportionality will not always characterize the diversion of lost sales accurately when products are highly differentiated.[33] Fortunately, it is straightforward to modify PCAIDS to allow a more general analysis. Products that are closer substitutes for each other than proportionality suggests may be placed together in "nests." The approach is analogous to using nests in a logit context, but we believe it is easier and more flexible to calibrate PCAIDS with a nest structure.

[32] The PCAIDS coefficients satisfy adding-up and homogeneity and are symmetric, as required.

[33] *Cf.* Horizontal Merger Guidelines ¶ 2.211: "The market shares of the merging firms' products may understate the competitive effect of concern, when, for example, the products of the merging firms are relatively more similar in their various attributes to one another than to other products in the relevant market. On the other hand, the market shares alone may overstate the competitive effects of concern when, for example, the relevant products are less similar in their attributes to one another than to other products in the relevant market."

To illustrate, return to the three-brand example discussed in the previous section. In that example, brand 2's market share of 30% and brand 3's share of 50% implied that 37.5% (30/80) of the share lost by brand 1 when its price increased would be diverted to brand 2 and 62.5% (50/80) would be diverted to brand 3. This effect can be characterized using an *odds ratio*. Here, the odds ratio between brand 2 and brand 3 is 0.6 (0.375/0.625). That is, under proportionality, brand 2 is only 60% as likely to be chosen by consumers leaving brand 1 as brand 3. Now suppose instead that brand 2 is relatively "farther" from brand 1 in the sense that that fewer consumers would choose brand 2 in response to an increase in $p_1$ than would be predicted by proportionality. For example, brand 2 may only be "half as desirable" a substitute as brand 3 and the appropriate odds ratio is really only 0.3. It is straightforward to calculate in this case that the share diversion to brand 2 becomes 23.1% and the diversion to brand 3 increases to 76.9% (an odds ratio of 0.3 = 0.231/0.769). As expected, fewer consumers leaving brand 1 would choose brand 2.

We generalize PCAIDS to cover such situations by constructing separate "nests" of brands. Diversion among brands within each nest is characterized by proportionality. Share diverted to a brand in a different nest deviates from proportionality in the following sense: the odds ratio is equal to the odds ratio under proportionality, multiplied by an appropriate scaling factor ranging from 0 to 1. The result is that brands within a nest are closer substitutes than brands outside the nest. PCAIDS with nests allows a more flexible pattern of cross-elasticities, as the model is no longer fully constrained by the proportionality assumption.

Continuing with the example, we capture the effect of brand 2 being a less-close substitute for brand 1 than indicated by market shares by placing brand 2 in a separate nest with a scaling or odds ratio factor of 0.5. We then use formulas in the Appendix to recalculate the *b* coefficients and resulting elasticities with this nesting assumption.[34] Table 2 reports the calculated elasticities for both the nested model and the original model.[35]

The nest parameter rescales the cross-elasticities in the right-hand panel; the cross-elasticities measuring the responses of brands 2 and 3 to the price of brand 1, and those measuring the responses of brands 1 and 2 to the price of brand 3 are no longer equal. (The cross-elasticities

[34] It would be incorrect to scale the non-nested elasticities in the left-hand panel directly. Nests affect adding-up, homogeneity, and symmetry and the appropriate calculation takes account of these constraints to generate economically consistent elasticities.

[35] The calculations continue to assume an own-price elasticity of −3 for Brand 1 and an industry elasticity of −1.

**Table 2**
**PCAIDS Elasticities with Nests**

| | Non-Nested Demand | | | | Separate Brand 2 Nest, (Odds Ratio Factor = 0.5) | | |
|---|---|---|---|---|---|---|---|
| | *Elasticity with Respect to:* | | | | *Elasticity with Respect to:* | | |
| *Brand* | $p_1$ | $p_2$ | $p_3$ | *Brand* | $p_1$ | $p_2$ | $p_3$ |
| 1 | −3.00 | 0.75 | 1.25 | 1 | −3.00 | 0.46 | 1.54 |
| 2 | 0.50 | −2.75 | 1.25 | 2 | 0.31 | −2.08 | 0.77 |
| 3 | 0.50 | 0.75 | −2.25 | 3 | 0.62 | 0.46 | −2.08 |

measuring the responses of brand 1 and brand 3 to the price of brand 2 remain equal, but at lower values, because brands 1 and 3 are in the same nest while brand 2 is outside.) With nesting, brand 2 becomes a poorer substitute for brands 1 and 3 (as indicated by the smaller cross-elasticities of brand 2 to the prices of brands 1 and 3 and of brands 1 and 3 to the price of brand 2), while brands 1 and 3 become better substitutes for each other (as indicated by the larger cross-elasticities of brands 1 to the price of brand 3 and of brand 1 to changes in the price of brand 3).

Simulation of a merger of brand 1 and brand 2 using this nested PCAIDS model predicts a unilateral price increase (without efficiencies) of 10.1% for both brand 1 and brand 2, compared to the original increases of 13.8% and 10.8% without nests. The unilateral effects are smaller because the merging brands are less-close substitutes for each other.

What remains is the difficult question of when proportionality is inappropriate, making nests necessary for accurate merger simulations. To our knowledge, there has been very little empirical testing of this question.[36] We note, however, that if PCAIDS introduces the possibility of biased values for the *b* coefficients, it may still provide an economically useful approximation.[37] Fortunately, PCAIDS makes it easy to detect

[36] A statistical test procedure is described in Jerry A. Hausman & Daniel McFadden, *Specification Tests for the Multinomial Logit Model,* 52 ECONOMETRICA 1219 (1984). One recent AIDS analysis of a grocery item using scanner data indicates that proportionality is reasonable but it does not formally test the hypothesis. *See* David A. Weiskopf, Assessment of the Relationship Between Various Types of Estimation Bias and the Simulated Economic Impact of Certain Anti-Competitive Scenarios at 55 and Table B2 (unpublished Ph.D. dissertation, Vanderbilt University 1999).

[37] In econometric terms, coefficients estimated with the PCAIDS restrictions could have lower mean square error, i.e., the reduced variance of the estimates may more than balance any bias that is introduced. *See, e.g.,* PINDYCK & RUBINFELD, *supra* note 8, at 29–32.

whether nesting is likely have economically meaningful effects through a sensitivity analysis of the odds ratio factors. We suspect that most simulations will justify very few nests, because simulation results appear to be robust to modest departures from proportionality. We also believe that a coarse grid (e.g., 0.75, 0.50, and 0.25) covering a range of odds ratio factors is adequate to assess sensitivity.

### E. PCAIDS and Other Calibrated-Demand Simulation Models

The PCAIDS model shares some characteristics with models based on logit demand structures that have been used to simulate mergers. Both assume proportionality (the logit model makes a comparable assumption of "independence of irrelevant alternatives"), yield positive cross-elasticities, and can be calibrated with only two parameters. We prefer PCAIDS to logit, however, for several reasons. First, PCAIDS does not require premerger price data. There will doubtless be occasions where prices are either not available for all firms in the market or are not measured accurately. Second, one can depart from proportionality in the PCAIDS framework using nested demands. Logit models can be generalized with nests as well, but we believe that logit is more difficult to calibrate econometrically and the additional nesting parameters are less intuitive.[38] Third, we prefer PCAIDS because it has mathematical "curvature" that approximates that of the standard AIDS model.[39] We suggest that the "curvature" of AIDS models is likely to fit data better than that of logit demand, although we recognize that this opinion invites further empirical research.[40] In essence, we view PCAIDS as a desirable mix of the best features of both logit (few parameters, correct signs) and AIDS (ability to fit the data, curvature).[41]

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[38] For a discussion of estimation problems with nested logits, see Gregory J. Werden & Luke M. Froeb, *The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy,* 10 J.L. Econ. & Org. 407, 420 (1994).

[39] For an analysis of curvature of alternative demand models, see Crooke et al., *supra* note 12.

[40] We are aware of very few studies that directly compare AIDS and logit using real-world data. A recent article that uses grocery scanner data on white pan bread sales indicates AIDS fit the data significantly better than logit. *See* Atanu Saha & Peter Simon, *Predicting the Price Effect of Mergers with Polynomial Logit Demand,* 7 Int'l J. Econ. Bus. 149, 154 (2000).

[41] The informative discussion at http://www.antitrust.org/mergers/economics/simulation.html concludes that "much progress has been made using the linear and nested logit demand specifications. . . . However, more progress can be made, by simulating the effects of mergers within the context of more flexible functional forms, like the AIDS model."

Our approach is similar in spirit to one suggested by Carl Shapiro.[42] Shapiro offers a rule-of-thumb formula for calculating the predicted prices of the post-merger firm, assuming that the merger involves two firms and two symmetric merging brands. As inputs, he requires markups (or equivalently gross margins) and diversion ratios. Shapiro's diversion ratio-symmetry assumptions in his two-brand example are similar to our proportionality assumption. However, his approach differs from ours in a number of ways. First, in much of the paper Shapiro assumes that demand elasticities are constant, an assumption that can create simulation difficulties because (a) such models sometime fail to converge; (b) the price increases resulting from a merger tend to be overstated; (c) non-merging firms do not raise prices in response to unilateral increases by the merged entity. Second, his approach does not readily generalize to multibrand firms. Finally, Shapiro does not discuss possible extensions when the proportionality assumption does not appear to be reasonable.

## IV. USING PCAIDS

This section offers a number of examples of applications of PCAIDS that are intended to make some of the principles discussed above more concrete. Our goal is to demonstrate that PCAIDS can provide reasonable estimates of the simulated effects of mergers at relatively low cost and with some transparency. The examples demonstrate the calibration of the PCAIDS demand model using shares and elasticities, the incorporation of efficiencies, sensitivity analyses using nests, and divestiture. The examples utilize available data on toilet paper, baby food, and white pan bread.

### A. The Kimberly-Clark/Scott Merger Revisited

We first use PCAIDS to re-examine the acquisition of Scott by Kimberly-Clark. A PCAIDS analysis of this 1992 merger may be compared to an earlier published simulation analysis by Hausman and Leonard that used supermarket scanner data to estimate econometrically a standard AIDS model.[43]

There were eight toilet paper brands premerger with national shares as shown in Table 3:

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[42] Shapiro, *supra* note 4; Carl Shapiro, Mergers with Differentiated Products, Address Before the ABA and IBA (Nov. 9, 1995), *available at* http://www.usdoj/atr/public/speeches/shapiro.spc.txt.

[43] Jerry A. Hausman & Gregory K. Leonard, *Economic Analysis of Differentiated Products Mergers Using Real World Data,* 5 GEO. MASON L. REV. 321 (1997).

**Table 3**
**Toilet Paper Market Shares**

| *Brand* | *Share (%)* |
|---|---|
| ScotTissue | 30.9 |
| Cottonelle | 7.5 |
| Kleenex | 6.7 |
| Charmin | 12.4 |
| Northern | 8.8 |
| Angel | 16.7 |
| Private Label | 7.6 |
| Other | 9.4 |
| Total | 100.0 |

Scott produced both ScotTissue and Cottonelle. Kimberly-Clark produced only Kleenex. We calibrate PCAIDS using a price elasticity for Scott of −2.94, reported by Hausman and Leonard, and an estimate of −1.17 for the industry elasticity inferred from their article.

Table 4 compares PCAIDS price elasticities calculated using these parameters to the elasticities estimated econometrically by Hausman-Leonard.

**Table 4**
**PCAIDS and Hausman-Leonard Elasticities**

| | *Own-Price Elasticity* | | *Cross-Price Elasticity* | |
|---|---|---|---|---|
| | *PCAIDS* | *Hausman-Leonard* | *PCAIDS* | *Hausman-Leonard* |
| ScotTissue | −2.9 | −2.9 | 0.36 | 0.24 |
| Cottonelle | −3.2 | −4.5 | 0.14 | 0.22 |
| Kleenex | −3.1 | −3.4 | 0.16 | 0.13 |
| Charmin | −2.6 | −2.7 | 0.66 | 0.35 |
| Northern | −3.0 | −4.2 | 0.26 | 0.41 |
| Angel | −3.1 | −4.1 | 0.19 | 0.26 |
| Private Label | −3.1 | −2.0 | 0.16 | 0.09 |
| Other | −3.1 | −2.0 | 0.20 | 0.27 |
| Average | −3.0 | −3.2 | 0.27 | 0.24 |

The two methods yield similar results brand by brand, and on average there appears to be relatively little difference.[44] We take this as evidence

[44] Each Hausman-Leonard cross-price elasticity in the table is calculated as the average of the cross-price elasticities with respect to the price of the brand given in the left-most column. The Hausman-Leonard study reported several negative cross elasticities (for non-merging goods) that we found difficult to interpret. The average values reported in the table exclude any negative cross-price elasticities.

that the proportionality assumption of PCAIDS is reasonably consistent with the toilet paper data. Moreover, differences between the elasticities yielded by the two methods may not be statistically significant. Hausman-Leonard report low precision for many of the estimated cross-price elasticities between the merging products in their model. For example, they report a Kleenex/Scott cross-price elasticity of 0.061 with a standard error of 0.066; this means that their estimated cross-elasticity is within two standard errors of our calibrated PCAIDS value of 0.16. Uncertainty about the true value of this cross-elasticity is particularly crucial to the merger simulation analysis because the magnitude of this cross-elasticity has a large effect on the price increases predicted from the merger.

The two simulation methods (taking into account the efficiencies assumed by Hausman-Leonard) yield predicted price changes for the merging firms as shown in Table 5:

**Table 5**
**Simulated Unilateral Effects—Toilet Paper**

| | *Price Change (%)* | |
|---|---|---|
| | *PCAIDS* | *Hausman-Leonard* |
| ScotTissue | –0.3 | –1.1 |
| Cottonelle | 0.7 | 0.5 |
| Kleenex | 4.3 | 0.2 |

The two models predict similar price changes for ScotTissue and Cottonelle. There is a greater difference between the price changes predicted by the two models for Kleenex, although this difference may not be statistically significant. As a sensitivity test, we introduced a nest structure that lowered the PCAIDS Kleenex/Scott cross-elasticity to 0.061 and left the other cross-elasticities in the model essentially unchanged. The price increase for Kleenex predicted by this nested PCAIDS model fell to 1.7 percent. This experiment suggests that increasing the same cross-price elasticity by two standard errors in the Hausman-Leonard simulation would produce a Kleenex price change much closer to the PCAIDS result.

## B. Efficiencies in a Baby Food Acquisition

The recently terminated effort by Heinz to acquire the Beech-Nut baby food assets raises many interesting questions about the role of efficiencies in merger analysis. We were not involved in that transaction, but it is our understanding that the litigation centered on coordinated effects. Indeed, we cannot ascertain from the published opinion whether either side presented testimony that relied on a merger simulation analysis of unilateral effects.[45] Nevertheless, we will use this proposed merger as an example of how PCAIDS might be applied to evaluate unilateral effects issues.

According to the court, there is a national relevant market for baby food in jars. The industry is concentrated, with three major firms and a small fringe (which we represent as a composite "private label" firm[46]). The market shares are given in Table 6:

**Table 6**
**Baby-Food Market Shares**

| *Brand* | *Share (%)* |
|---|---|
| Heinz | 17.4 |
| Beech-Nut | 15.4 |
| Gerber | 65.0 |
| Private Label | 2.2 |
| Total | 100.0 |

The pre-transaction HHI was 4,770, with a delta of 536, well above the safe harbor limits in the Merger Guidelines. Market shares and the HHI alone, however, do not provide sufficient information to analyze the potential magnitudes of a unilateral price increase or the mitigating effect of efficiencies.

We do not analyze individual brands, but instead treat each firm as if it produced a single aggregate. We also do not distinguish competition at the retail level (for customers) from competition at the wholesale level (for shelf space). Because the written opinion does not offer specific price elasticities, we have assumed an industry elasticity of −1.0 and we have estimated a price elasticity for Heinz of −2.60 from financial information.[47]

We consider three alternative simulations. First, we model the four firms as belonging in a single nest. Proportionality implies that most of the share lost by Heinz due to a price increase would be diverted to

[45] *See* FTC v. H.J. Heinz Co., 246 F.3d 708 (D.C. Cir. 2001).

[46] The use of composite goods or firms is common in merger simulation because, when appropriate, it greatly diminishes the number of parameters in the model and simplifies the analysis.

[47] The elasticity was calculated as the negative of the ratio of sales ($9,407,949) to gross profit ($3,619,424). At the profit-maximizing price for a firm, the negative of its markup of price over cost as a proportion of price equals the inverse of its elasticity. *See* H.J. Heinz

Gerber instead of Beech-Nut. The ratio of the Gerber to the Beech-Nut market share equals 65/15.4. This yields an odds ratio of 4.22, which indicates that consumers leaving Heinz would be more than four times as likely to shift to Gerber as to Beech-Nut. For the second simulation, we put Heinz and Beech-Nut in a separate nest from Gerber and private label, with an odds ratio factor of 0.5. This nest structure represents the hypothesis that one group of consumers strongly prefers Gerber to Heinz and Beech-Nut. In this scenario the Gerber Beech-Nut odds ratio falls by half to 2.11, indicating that Gerber becomes a poorer substitute (now only about twice as many consumers would choose Gerber). For the third simulation, we put Heinz and private label in a separate nest from Gerber and Beech-Nut, also with an odds ratio factor of 0.5. This scenario tests the implication of treating Gerber and Beech-Nut as closer substitutes because they are both premium-priced brands. Since proportionality holds within a nest, the odds ratio would revert to 4.22 (the ratio of their market shares).

The simulated unilateral effects for each of these scenarios, in the absence of any efficiencies, are given in Table 7:

**Table 7**
**Simulated Unilateral Effects—Baby Food**

| | *Simulated Price Change* | | |
|---|---|---|---|
| *Firm* | *No Nests* | *Heinz Beech-Nut Nest* | *Beech-Nut Gerber Nest* |
| Heinz | 6.2% | 12.3% | 3.9% |
| Beech-Nut | 6.8% | 13.3% | 3.4% |

These results illustrate the importance of the nesting assumption for the magnitude of the price increases. Predicted price increases are largest when the merging firms are in the same nest (which implies consumers view them as closer substitutes for each other than market shares alone suggest), and smallest when the merging firms are in separate nests (which implies consumers view then as less-close substitutes for each other than market shares alone suggest).

PCAIDS can also be used to provide estimates of the efficiencies that would fully offset the predicted price effects. For the no-nest case, we calculate that reductions in marginal costs of approximately 8% for both Heinz and Beech-Nut would be required. If Heinz and Beech-Nut are closer substitutes and in the same nest, reductions in marginal costs of

Company Form 10-K for fiscal year ended May 3, 2000, Consolidated Statements of Income, *available at* http://www.edgar-online.com.

approximately 16% for each firm are necessary to offset the predicted price increase.

The Court notes that the merging parties claimed expected efficiencies of 22.3% for Beech-Nut.[48] It is not clear to what extent the claimed cost-reductions for Beech-Nut would translate into merger-specific efficiencies for the merged entity.[49] However, our analysis in this hypothetical suggests that evidence on efficiencies would have been crucial to any argument that unilateral effects of the merger on price were not likely to be significant.

### C. Merger with Divestiture

Some proposed transactions raise concerns about unilateral price effects that cannot be overcome by expected efficiencies or repositioning. Divestiture may be an option to "fix" such a deal, and simulation analysis can help evaluate whether and which divestitures would eliminate competitive concerns. We illustrate an analysis of divestiture using data from a recent study of a merger between two large white pan bread bakeries.[50] The pre-transaction market contained six firms with market shares as shown in Table 8:

**Table 8**
**Market Shares—White Pan Bread**

| *Firm-Brand* | | *Share (%)* |
|---|---|---|
| A-1 | | 14.2 |
| A-2 | | 8.1 |
| A-3 | | 7.6 |
| B-1 | | 8.8 |
| C-1 | | 7.0 |
| D-1 | | 7.6 |
| Grocery | | 31.5 |
| Other | | 15.2 |
| | Total | 100.0 |

[48] *Heinz Co.,* 246 F.3d at 721.

[49] We understand (from personal communication) that Jonathan Baker testified (on behalf of Beech-Nut and Heinz) to an expected 15% reduction in marginal cost for the gains passed through to the Beech-Nut brand. According to Baker, these gains would come from a price reduction; the gains to Heinz buyers would come from getting a brand that is 15% higher in quality (at the same price as their old brand, according to the merging parties).

[50] *See* Saha & Simon, *supra* note 40.

Firms A and B are the merging parties. "Grocery" and "Other" are composites of smaller suppliers. The pre-transaction HHI was 2,317 with a change of 524, values that could trigger detailed agency review.

According to the study, the industry elasticity was −1.0. We set the elasticity for B-1 to the study's estimate of −1.34 to complete the PCAIDS calibration of the demand model. Initially we assume proportionality. Table 9 shows the unilateral price increases for the merged firm predicted by PCAIDS in the absence of efficiencies.

**Table 9**
**Simulated Unilateral Effects—**
**White Pan Bread**

| *Brand* | *Price Increase* |
|---|---|
| A-1 | 10.0 |
| A-2 | 10.0 |
| A-3 | 10.0 |
| B-1 | 28.7 |

The share-weighted average price increase for the brands in the merger is 14.3%. Further analysis shows that even if the merger yielded efficiencies that reduced the marginal costs of each brand by 10%, the PCAIDS simulation would predict a price increase of approximately 18% for B-1. The share-weighted average price increase for the merged firm with these efficiencies is 4.4%, which may still raise concerns. We also experimented with nests, since A-3 was a premium-priced brand and perhaps was less of a substitute for the lower-priced B-1. However, we did not find that plausible nest structures significantly affected the results.[51] Without the prospect of timely entry or of efficiencies greater than 10%, the transaction would certainly raise anticompetitive concerns.

Divestiture by Firm A of one or more of its three brands is one possible strategy to restructure the deal. The effect of divestiture on unilateral pricing behavior will depend both on what brand or brands are divested and what firm acquires those brands. Simulation models can help analyze the effects on prices of specific divestitures. We first simulated the merger assuming a sale of A-3 to the smallest firm, C. For this merger and divestiture, assuming no efficiencies, the predicted share-weighted average price increase for the four brands originally sold by the merging

[51] We even tried an extreme case of putting A-3 in a separate nest from all of the other brands in the market and setting the odds ratio factor to 0.01 to minimize the competitive overlap with B-1.

firms is only 2.8%. Even modest merger-related efficiencies would eliminate this average price increase. Alternatively, we simulated the merger with divestiture of A-3 to a hypothetical new entrant and found a share-weighted average price increase of only 1.8% before efficiencies.

The evaluation of these simulated post-divestiture price effects also raises the issue of appropriate measurement of prices. In our example, the range of price changes for the various brands is quite wide. For example, if A-3 is divested to firm C, its price is predicted to *decrease* by 11.0%, while A-1 and A-2 have predicted price increases of 1.3% and B-1 has a predicted price increase of 18.6%. Divestiture reduces considerably the predicted price increases for brands the merged firm retains and results in a predicted price decrease rather than increase for A-3. An important issue facing the merger authorities in this situation is whether a transaction should be judged by its effect on average prices in the relevant market, or by its separate effects on the prices for individual brands.

## V. ANALYZING PRODUCT REPOSITIONING AND ENTRY WITH PCAIDS

The Horizontal Merger Guidelines acknowledge entry and product repositioning as competitive responses to a merger with unilateral price increases.[52] The Guidelines distinguish between "committed" entry, which requires significant sunk costs of entry and exit, and "uncommitted" entry, which does not.[53] Uncommitted entrants are capable of increasing output sufficiently quickly (e.g., by redeploying existing assets) that they are able to constrain the market pre-transaction. For this reason, the Guidelines focus on committed entry as truly new competition that may be generated by unilateral price increases. For committed entry to be an effective competitive check according to the Merger Guidelines, it must occur within two years (timeliness), must be profitable at *pre-transaction* prices (likelihood), and "must be responsive to the localized sales opportunities that include the output reduction associated with the competitive effect of concern" (sufficiency).

Merger simulation (which could be based on PCAIDS or other demand models) provides a prediction of the unilateral price increases that would occur absent entry or repositioning. Associated with any such price increase will be a reduction in output. The central question is whether

[52] The Horizontal Merger Guidelines ¶ 2.12 n.23 indicates that the same analysis applies to both cases.

[53] *See id.* ¶¶ 1.0 & 3.0.

repositioning or entry can increase output sufficiently to defeat the price increase.

A complete analysis of entry and repositioning raises difficult modeling issues that go beyond the scope of this article. It would require an assessment of sunk costs and minimum viable scale (the smallest scale at which its average cost is equal to the pre-transaction price) for committed entry, as well as a financial-accounting analysis to determine whether pre-transaction prices are adequate for long-run profitability. Nonetheless, we believe that PCAIDS can provide a useful framework in which to analyze under the conditions under which committed and uncommitted responses might be expected to constrain unilateral price increases.

We use the following procedure to identify the amount of entry that should be sufficient to eliminate unilateral price increases. For any brand sold by the merged firm, the post-merger revenue can be defined in terms of the pre-merger revenue and the unilateral percent change in price ($\delta^*$) and percent change in quantity (denoted $\alpha$) for the brand. Since the shares and industry elasticity are known, and the merger simulation yields the unilateral price changes, it is possible to solve for the percentage reduction in output $\alpha$. Using the expression $p^{post}q^{post} = (1+\delta^*)p^{pre}q^{pre}(1-\alpha)$, it can be shown that (see Section 4.D. of the Appendix for details)

$$\alpha = 1 - \frac{s^{post}}{s^{pre}} \frac{(1 + (\varepsilon + 1)\mathrm{d}P/P)}{(1 + \delta^*)}\ . \qquad (6)$$

The predicted output reduction therefore depends on two price effects: the unilateral brand price increase and the average price change ($\mathrm{d}P/P$) for the market as a whole.

The magnitude of the reduction in output in terms of the pre-transaction revenue market share for the brand is $\alpha s^{pre}$. If the entrant's sales were a close substitute for the restricted output, then we could expect sales at this share level for the new brand to be sufficient to constrain the merged firm at pre-transaction prices.[54] The rationale is that the sales opportunities of the entrant would effectively restore the restricted output to the market, implying a return to the pre-transaction prices.[55] This analysis can be applied to solve for the value of $\alpha$ for each

[54] Normally, we would expect the entrant to offer a close substitute, because entry is intended to take advantage of the sales opportunities resulting from unilateral price increases.

[55] We implicitly assume that the combined sales of the entrant and the brand produced by the merged firm equal the pre-transaction level; that is, the entrant does not merely "cannibalize" sales from the incumbent.

brand sold by the merged firm for which unilateral price increases are a concern. The total required entry would then be the sum of the shares from the individual $\alpha$ factors.

The merger simulation may also indicate that other firms in the market would raise price and restrict output, generating additional sales opportunities. It may be appropriate to require additional entry to constrain these price increases as well, in order to make sure that the entrant is not diverted from pursuing the opportunities from the merged firm's output restrictions.

This analysis can, in principle, be applied to both uncommitted and committed repositioning. In the uncommitted case, sunk repositioning costs are assumed to be zero. For committed repositioning, it is necessary to carry out additional analyses to determine required sunk costs and minimum viable scale. As the Merger Guidelines point out, entry is unlikely if the minimum viable scale is larger than the sales opportunities available to entrants. In addition, the profits on the sales opportunities at pre-transaction prices must be sufficient to justify the sunk costs.

To illustrate some of the issues involved in an analysis of entry, we consider a hypothetical transaction involving ready-to-eat (RTE) cereals.[56] RTE cereal products are highly differentiated along several dimensions (e.g., sweetness, texture, grains, vitamin and fiber content, color and packaging). Because this example uses aggregated data and relies on other simplifying assumptions for purposes of illustration, we do not identify individual companies or their product lines. In our example there are six firms: firms A, B, C, and D are "majors," firm E is a private label composite, and firm F is another composite firm that represents an aggregation of other, smaller brands. Firms C and D each sell two brands. We use PCAIDS to analyze a hypothetical merger between firms A and B.

We account for the fact that the characteristics of firms' brands affect consumers' substitution patterns by placing the brands of the six firms in two nests, based on whether each firm's brands appeal primarily to adults or to children. (Each nest in the example could contain multiple brands.) The premerger shares and nests are given in Table 10.[57]

Proportionality holds within each nest. We assume a scaling factor of 50% for share diversion across nests. That is, the share diverted from a

[56] We wish to thank Kraft Foods for providing us with the breakfast cereal data.

[57] We use the notions of Kids and Adult nests for illustrative purposes only. We believe, nevertheless, that the relevant market for antitrust purposes is all ready-to-eat cereals. *See* New York v. Kraft Gen. Foods, Inc., 926 F. Supp. 321, 356 (S.D.N.Y. 1995).

**Table 10**
**Pre-Merger Market Shares**

| *Firm-Brand* | *Share (%)* | *Nest* |
|---|---|---|
| A-1 | 13.0 | Kids |
| B-1 | 4.2 | Adult |
| C-1 | 26.5 | Kids |
| C-2 | 8.8 | Adult |
| D-1 | 21.8 | Kids |
| D-2 | 5.4 | Adult |
| Private Label | 6.0 | Kids |
| Other | 14.2 | Kids |
| Total | 100.0 | |

Kids brand to an Adult brand (and vice versa) is only half as large as predicted by their market shares. This structure introduces a simple, but flexible alternative to strict proportionality (with a factor of 100%).

To complete the data requirements for the simulation, we assume an industry price elasticity of −1.0 and an own-price elasticity of −1.60 for A.[58] We also assume that a merger between A and B will generate efficiencies that lower incremental costs for each firm by 2%.

Taking into account the efficiencies (but not repositioning or entry), the PCAIDS simulation predicts that the merger will result in no change in A's prices. However, the predicted price increase for B is 4.9% and its share falls to 4.1%. This post-merger price increase could raise competitive concerns, but it might also induce other firms to enter de novo or to redesign and reposition their products to compete more directly with B.

We calculate the required entry to constrain B as follows. The value for $\alpha$ obtained from Equation (6) is 0.065. As a result, the value of the restricted output is 0.27 percentage points of market share (0.065 multiplied by the pre-transaction share of 4.2%). If an entrant could achieve this share with a new brand that is a close substitute for B then the unilateral price increase can be prevented.

The small amount of required entry in the example is not surprising, since B is a relatively small firm. (The amount of restricted output must be less than the size of B.) This highlights the potential importance of the analysis of minimum viable scale because entry on such a limited

[58] The own-price elasticity for the example is calculated as the ratio of gross profits to sales from aggregate financial statements for A. A more refined estimate would require information on sales and costs by product line.

basis may not be economic. In the RTE cereal industry, one possibility for low-cost entry might be repositioning of existing brands (or capacity) from the Kids segment to the Adult segment.

Ultimately it is a matter of judgment as to whether an entrant would be capable of achieving the requisite share to make raising prices unprofitable for the merging firm. Additional analysis would also be necessary to determine whether the entrant would achieve minimum viable scale and be profitable at pre-merger prices. Nevertheless, we are optimistic that the approaches just described can provide a feasible and useful framework to evaluate the range of issues raised when entry and repositioning are discussed.

## VI. PCAIDS AND THE MERGER GUIDELINES SAFE HARBORS

In this section we briefly discuss some applications of our simulation analysis to the evaluation of safe harbor rules for unilateral effects. A safe harbor offers a boundary below which transactions are not likely to be challenged, thereby reducing transactions costs and conserving enforcement resources. The Merger Guidelines suggest two alternative safe harbors with respect to unilateral effects. The first applies when the combined market share of the merging firms is less than 35%; the other is available when the change in the HHI is less than 50 (with a pre-transaction HHI over 1,800) or less than 100 (with a pre-transaction HHI between 1,000 and 1,800).[59]

If taken literally, the 35% safe harbor would shelter transactions from review for unilateral effects when the merging firms have shares as large as 17.5% each, magnitudes that might not be uncommon. To evaluate this safe harbor, we used PCAIDS (and reasonable elasticity assumptions) to investigate potential unilateral effects when the merging firms have a combined share of 35%.[60] The results indicated price increases of 6% or more for at least one of the merging firms, irrespective of firm size. The simulations suggest that a 35% safe harbor runs too great a risk of sheltering anticompetitive transactions.

Moreover, we note that the 35% standard, if enforced, would make the HHI safe harbor virtually irrelevant for analyzing unilateral effects.

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[59] The Horizontal Merger Guidelines ¶¶ 2.211 and 2.22 leave open the possibility of finding significant unilateral effects when the merging firms have combined market shares of less than 35%, indicating that this criterion is not equal in importance to the HHI safe harbor. For simplicity, however, we will refer to the 35% standard as a safe harbor and investigate its properties.

[60] The simulations used an industry elasticity of −1, a brand elasticity of −3 for the first merger partner, and a third firm with a 65% share. There were no efficiencies or nests.

The only mergers not already protected by the 35% rule that would be sheltered by the change in the HHI would be of minimal interest. Indeed, in these circumstances the smaller merging firm could have at most a 1.5% share (pre-transaction HHI between 1,000 and 1,800) or a 0.7% share (pre-transaction HHI over 1,800). These constraints are inherent in the mathematics associated with the existing safe harbors (see Section 5 of the Appendix for details), and are not dependent on our merger simulation analysis.

We have separate concerns about the HHI safe harbor in cases involving unilateral effects. The HHI safe harbor by itself shelters relatively few mergers because it is only satisfied when the smaller merging firm has at most a 7% share (pre-transaction HHI between 1,000 and 1,800) or a 5% share (pre-transaction HHI over 1,800). Again, as shown in the Appendix, these limits follow directly from the definition of the safe harbor in the Merger Guidelines. By "protecting" only mergers involving relatively low market shares, the HHI safe harbors pose a low risk of unilateral effects. This was confirmed by PCAIDS simulations that yielded maximum price increases under 5%.[61]

At the same time, it is natural to ask whether there is a basis for an alternative safe harbor (perhaps tied to the HHI or the sum of market shares) that could expedite a greater number of merger reviews while providing similar protection against anticompetitive transactions.[62] For example, our preliminary investigation suggests that a 25% safe harbor would typically generate unilateral effects below 5%, using similar assumptions as before. Moreover, the weighted average price increase for the merged firm will be even smaller when the merger partners are different sizes. We realize, of course, that the choice of an alternative safe harbor is a complex question that will involve substantial further study. However, the benefits in the form of reduced enforcement and transaction costs could make this a worthwhile effort.

## VII. CONCLUSION

Merger simulation can be used to evaluate many transactions that raise competitive concerns. It adds to the information provided by methods that rely on econometrically estimated demand systems, surveys of con-

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[61] The HHI simulations used an industry elasticity of −1, a brand elasticity of −3 for the first merger partner, and merging parties ranging from equal 5% shares to 24% and 1% shares, and a third firm with the residual share.

[62] Other researchers who advocate simulation have found little support for the 35% rule and have concluded that the existing HHI criterion "makes sense only if one believes either that mergers are likely to generate no efficiencies or that only consumer welfare should be considered in merger cases." *See* Gregory J. Werden & Luke M. Froeb, *Simulation*

sumer preferences, and the analytical strategies described in the Horizontal Merger Guidelines. The PCAIDS simulation approach presented in this article represents a simplification over existing techniques that we believe offers advantages in many applications. It requires only aggregate market shares, the industry price elasticity, and the own-price elasticity for a single brand in the relevant market. We have also shown that this approach can be easily extended to accommodate additional information on substitution and diversion patterns by constructing product nests. It allows a range of sophisticated analyses at relatively low cost. We have provided examples that evaluate efficiencies, nesting, brand divestiture, and entry/repositioning.

Our work is also relevant to recent criticisms of the use of market shares, especially in the form of HHIs, for merger analysis. PCAIDS shows that market shares can be highly informative when combined with well-grounded economic principles. In our view, the PCAIDS model justifies renewed reliance on market shares as a pragmatic benchmark to assess competition. We note that the Merger Guidelines themselves spell out the option of using market shares in an analysis of unilateral effects when market shares are reliable indicators of the closeness of substitutes and demand (which are essentially the conditions under which the proportionality assumption is appropriate).

Merger simulation is evolving and its techniques are improving. We expect that PCAIDS can help establish simulation as a standard tool to analyze potential unilateral effects. We hope that the methods introduced in this article will provide a basis to evaluate options and possibilities that might otherwise be quite difficult to subject to quantitative analysis.

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*as an Alternative to Structural Merger Policy in Differentiated Products Industries, in* THE ECONOMICS OF THE ANTITRUST PROCESS 77 (Malcolm B. Coate & Andrew N. Kleit eds., 1996).

## APPENDIX

Merger simulation builds on a demand-supply model that specifies a set of equations that relate three types of information for the brands in the relevant market: (i) own and cross-price elasticities, (ii) market shares, and (iii) gross profit margins. The demand model implies a "first-order condition" (FOC) for each brand, which specifies necessary mathematical relationships among these variables under the assumption that the firms in the market are maximizing profits without engaging in overt collusion. Each FOC involves the elasticities, shares, and margins both for that brand and for all of the other brands in the relevant market owned by the same firm. In this way the FOCs take into account possible trade-offs in pricing that are the primary source of unilateral effects.

**1. Notation and Assumptions**

A. There are $n$ firms in the relevant market, each producing $n_i$ brands. There are $N$ brands in total.

B. The jth brand has the following characteristics:

1. Average price $p_j$
2. Quantity $q_j$
3. Share $s_j$ of revenues in the relevant market
4. Own-price elasticity $\varepsilon_{jj}$ and cross-price elasticities $\varepsilon_{jk}$
5. Incremental cost $c_j$ and profit margin $\mu_j = (p_j - c_j)/p_j$.

C. The average industry price is $P$, calculated as $\ln P = \sum s_i \ln p_i$, for i = 1 to $N$. Also, $\Delta P/P = \sum s_i(\Delta p_i/p_i)$.

D. The $n$ firms face an aggregate industry demand curve with a (pre-merger) price elasticity of $\varepsilon$. An estimate of the percentage change in industry revenue due to industry-wide price changes is $\Delta(\sum p_i q_i)/\sum p_i q_i = \Delta P/P(\varepsilon+1)$.

E. There is at least one known own-price elasticity $\varepsilon_{jj}$. Each known own-price elasticity is larger in magnitude than the industry elasticity $\varepsilon$, $\text{abs}(\varepsilon_{jj}) > \text{abs}(\varepsilon)$, where abs(.) is the absolute value function.

F. Define the brand-specific vectors $s = (s_1, s_2, \ldots, s_N)'$ for market shares, $p = (p_1, p_2, \ldots, p_N)'$ for prices, $c = (c_1, c_2, \ldots, c_N)'$ for incremental costs, and $\mu = (\mu_1, \mu_2, \ldots, \mu_N)'$ for margins.

G. Define the brand-specific vector $\delta = (\delta_1, \delta_2, \ldots, \delta_N)'$ of exponential rates of price changes due to the transaction. Each $\delta_j = \ln(p_j^{\text{post}}) - \ln(p_j^{\text{pre}})$. Define the brand-specific vector $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_N)'$ of percentage changes in incremental costs due to the transaction. Each $\gamma_j = c_j^{post} / c_j^{pre} - 1$.

H. Define the matrices $S = \text{diag}(s)$, $\Gamma = \text{diag}(1+\gamma)$, and $\Delta = \text{diag}(\exp(\delta))$.

I. For the brands produced by the ith firm, define the $n_i$ by $n_i$ matrix $E_i$ with element (k, j) equal to $\varepsilon_{jk}$. That is, $E_i$ is the transposed matrix of own-price and cross-price elasticities.

J. Define the solution vector $\delta^*$ of price changes measured at compound rates as $\exp(\delta)-1$. The FOCs are solved using the $\delta$ vector and the conversion to $\delta^*$ expresses the solution in more convenient units.

## 2. General First-Order Conditions for Merger Simulation

There is a FOC equation for each brand in the market. A general expression for all of the FOCs is given by the matrix equation:

$$s + \text{diag}(E_1, E_2, \ldots, E_n) S\mu = 0.$$

The first stage of a simulation is used to calculate the brand-specific margins $\mu$. Assuming the pre-transaction shares and elasticities are known, the margins are given by:

$$\mu^{pre} = -S^{-1}\text{diag}(E_1, E_2, \ldots, E_n)^{-1}s. \tag{A1}$$

The second stage analyzes the FOCs to predict price changes due to the transaction. In general, the post-transaction shares, elasticities, and margins are functions of the price changes. To simplify the notation, assume that the merger involves firms $n-1$ and $n$. There are $n-1$ firms in the post-transaction market, but the number of brands remains $N$. The merged firm requires a new cross-elasticity matrix $E^*_{n-1}$ for the $n_{n-1}$ plus $n_n$ brands it is now producing. The FOCs for the second stage are:

$$s + \text{diag}(E_1, E_2, \ldots, E^*_{n-1}) S\mu = 0, \tag{A2}$$

where all variables are understood to be taken at their post-transaction values.

To understand the solution of (A2), consider the relation between $\mu^{pre}$ and $\mu^{post}$. For the jth brand,

$$c_j^{pre} = (1-\mu_j^{pre})\ p_j^{pre}.$$

It follows from the definitions that $c_j^{post} = (1+\gamma_j)\, c_j^{pre}$ and that $p_j^{post} = \exp(\delta_j) p_j^{pre}$. As a result,

$$\begin{aligned}\mu_j^{post} &= 1-c_j^{post}/p_j^{post}\\ &= 1-(1-\mu_j^{pre})\ (1+\gamma_j)/\exp(\delta_j).\end{aligned}$$

This relationship can be expressed in matrix notation for all brands as

$$\mu^{post} = \mathbf{1} - \Gamma\Delta^{-1}(1 - \mu^{pre}),$$

where $\mathbf{1}$ is an $N$ vector of ones.

The second stage FOC can now be written as a function of the percentage price changes:

$$s + \mathrm{diag}(E_1, E_2, \ldots, E^*_{n-1})\, S\, [\mathbf{1} - \Gamma\Delta^{-1}(1 - \mu^{pre})] = 0, \quad \text{(A3)}$$

where the price changes also generate post-transaction shares and elasticities through the demand model. That is, the solution to (A3) is framed entirely in terms of finding the vector $\delta$ that solves the system of equations. Observe that the pre-transaction prices and costs $p^{pre}$ and $c^{pre}$ are not needed in the analysis.

Simulation of divestiture of a brand from the ith firm to the jth firm is accomplished by suitable definition of the price elasticity matrices. The rows and columns corresponding to the brands to be divested are deleted from $E_i$. When the jth firm is an incumbent in the market, $E_j$ is augmented by a new row and a new column containing the own-price elasticity and the cross-price elasticities with the other brands for the firm. For divestiture to an entrant, the number of firms in the post-transaction market increases by one and an additional elasticity matrix is defined that consists of a single element equal to the own-price elasticity for the divested brand.

## 3. Properties of AIDS

### A. Share Equations

Associated with the ith firm are $n_i$ equations that model changes in brand-specific shares. They take the form $ds_{ik} = \sum b_{ij} dp_j/p_j$, where j = 1, . . ., $N$ and k = 1, . . ., $n_i$. We omit the AIDS expenditure terms in our analysis as a convenient simplification. The system can be written in matrix notation as $ds = B\delta$, where $B$ is the $N$ by $N$ matrix of $b$'s. The vector of pre-transaction shares $s^{pre}$ is assumed known. The post-transaction shares are $s^{post} = s^{pre} + B\delta$.

The "adding-up" property requires the shares of all the brands in the market to always sum to one. Since this identity holds for any set of price changes, it implies for any j that $\sum b_{ij} = 0$, i = 1, . . ., $N$. Adding-up makes one of the equations redundant because its coefficients can be completely expressed in terms of the coefficients from the other equations.

The homogeneity property requires shares to be unaffected by a uniform percentage change in all prices in the model. It implies for any i that $\sum b_{ij} = 0$, j = 1, . . ., $N$. Homogeneity makes one of the prices in the model redundant because its coefficients can be completely expressed in terms of the coefficients for the other prices in the same equation.

### B. AIDS Own-Price Elasticities

$$\varepsilon_{ii} = \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i} = \left(\frac{-q_i}{p_i} + \frac{b_{ii}}{p_i}\frac{PQ}{p_i} + \frac{s_i}{p_i}\frac{\partial PQ}{\partial p_i}\right)\frac{p_i}{q_i}$$

$$= -1 + \frac{b_{ii}}{s_i} + \frac{p_i}{PQ}\frac{\partial PQ}{\partial p_i} = -1 + \frac{b_{ii}}{s_i} + \frac{p_i}{\partial p_i}\frac{\partial P}{P}(\varepsilon + 1) \tag{A4}$$

$$= -1 + \frac{b_{ii}}{s_i} + s_i\,(\varepsilon + 1)\,.$$

### C. AIDS Cross-Price Elasticities

$$\varepsilon_{ij} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i} = \left(\frac{-b_{ij}}{p_j} + \frac{PQ}{p_i} + \frac{s_i}{p_i}\frac{\partial PQ}{\partial p_j}\right)\frac{p_j}{q_i}$$

$$= \frac{b_{ij}}{s_i} + s_j(\varepsilon + 1)\,. \tag{A5}$$

## 4. Properties of PCAIDS

### A. PCAIDS Calibration of the Demand System

We now show that PCAIDS can be fully calibrated regardless of the number of brands in the market, using only information on the own-price elasticity of demand for a single brand, the industry price elasticity of demand, and the market share data. The same result holds for the extension of the method using nests.

Each element of $B$ can be written as $b_{ik} = \theta_{ik} b_{kk}$, where the $\theta$'s are known but the diagonal elements $b_{kk}$ are unknown. The relative share diversion between brand i and brand j for a price change in brand k is given by the odds ratio $\theta_{ik}/\theta_{jk}$. For example, under strict proportionality $\theta_{ik} = -s_i/(1-s_k)$ and the odds ratio equals $s_i/s_j$. Impose adding-up and homogeneity. The constraints imply a system of $N$–1 independent equations in the $N$ unknown own-coefficients. Without loss of generality, assume that $\varepsilon_{11}$ is known. We normalize with respect to the first brand and define a vector $\beta$ with $N$–1 elements equal to $b_{jj}/b_{11} = \beta_j$, j > 1. The equation system is then non-singular and can be written in matrix form as

$$\begin{pmatrix} \theta_{12} & \theta_{13} & \dots & \theta_{1N} \\ 1 & \theta_{23} & \dots & \theta_{2N} \\ \vdots & & & \\ \theta_{N-1,2} & \dots & 1 & \theta_{N-1,N} \end{pmatrix} \begin{pmatrix} \beta_2 \\ \vdots \\ \beta_N \end{pmatrix} = \begin{pmatrix} -1 \\ -\theta_{21} \\ \vdots \\ -\theta_{N-1,1} \end{pmatrix} \tag{A6}$$

(A6) can be inverted to solve for the $\beta$ vector, which will be a function of the market shares. It can be shown that each $\beta_i$ equals $[(1-s_i)/(1-s_1)](s_i/s_1)$.

Since $\varepsilon_{11}$ and $\varepsilon$ are known, we can invert the formula for own-price elasticity to find $b_{11} = s_1(\varepsilon_{11} + 1 - s_1(\varepsilon + 1))$. The PCAIDS system can therefore be calibrated completely using market shares and the two elasticities.

We now prove that each PCAIDS own-price elasticity is larger in magnitude than the industry elasticity. By assumption, $\mathrm{abs}(\varepsilon_{11}) > \mathrm{abs}(\varepsilon)$. Assume that $\mathrm{abs}(\varepsilon_{ii}) < \mathrm{abs}(\varepsilon)$ for some $i > 1$. Substituting $b_{ii}=[(1-s_i)/(1-s_j)](s_i/s_j)\, b_{11}$ in the expression for the own price elasticity for $\varepsilon_{ii}$ yields the contradiction that $\mathrm{abs}(\varepsilon_{11}) < \mathrm{abs}(\varepsilon)$.

Finally, we prove that all PCAIDS cross-price elasticities are greater than zero. Suppose $\varepsilon_{ik} < 0$ for some i, k. By substitution, this implies $-b_{kk}/(1-s_k)+s_k(\varepsilon+1)<0$. Substitute for $b_{kk}$ in terms of $\varepsilon_{kk}$, and rearrange yielding the implication $((\varepsilon_{kk}+1)-s_k(\varepsilon+1))s_k > (1-s_k)s_k(\varepsilon+1)$. However, since $\varepsilon_{kk} < \varepsilon$, this is a contradiction.

### B. PCAIDS Nests

Assume that there are $w$ nests, $w \leq N$, with each brand assigned to a nest. Given a price increase for brand k in nest $f_1$, the diversion of share to brand i in nest $f_2$ deviates from proportionality by a multiplicative factor $\omega(k, i) > 0$. We assume that $\omega(k, i) = \omega(i, k)$. Similarly, the diversion from brand k to brand j in nest $f_3$ deviates from proportionality by $\omega(k, j)$. Proportionality is the special case where $\omega(k, i) = 1$. It can be shown in this general setting that:

$$\theta_{ik} = -s_i \frac{\omega(k,i)}{\sum_{m \neq k} s_m \omega(k,m)}.$$

The odds ratio under nesting is $\theta_{ik}/\theta_{jk} = (s_i/s_j)[\omega(k,i)/\omega(k,j)]$. In the case of proportionality for all nests, this reduces to the familiar $s_i/s_k$.

### C. Slutsky Symmetry of *B* with PCAIDS

We now show that the matrix $B$ of PCAIDS coefficients is symmetric both under strict proportionality and with nests as we have defined them. The discussion in Section 4.A implies that, under adding up and homogeneity, $\beta_j = \theta_{j1}/\theta_{1j}$. It follows that

$$\beta_j = \frac{s_j}{s_1} \frac{\sum_{m \neq j} s_m \omega(j,m)}{\sum_{m \neq 1} s_m \omega(1,m)}$$

and from before, $b_{jj} = \beta_j b_{11}$.

By the definition of $B$ and substitution for $\beta_i$ and $\beta_j$,

$$b_{ij} = \frac{s_i s_j}{s_1} \frac{\omega(i,j)}{\sum_{k=2}^{N} s_k \omega(k,1)} b_{11} .$$

for i≠j. Symmetry of $B$ follows directly.

### D. Required Market Share for Entry to Defeat Unilateral Effects

Let $\alpha$ represent the unilateral output reduction. For any brand produced by the merged firm, post-transaction revenue $p^{post}q^{post}$ is related to pre-transaction revenue $p^{pre}q^{pre}$ as follows:

$$p^{post}q^{post} = (1+\delta^*)p^{pre}q^{pre}(1-\alpha),$$

where $\delta^*$ is the unilateral percentage price increase. Total post-transaction market revenue equals pre-transaction market revenue $PQ$ multiplied by $1+(\varepsilon+1)\mathrm{d}P/P$, where $P$ is the average market price change (see 1.D.). Dividing both sides of the equation by post-transaction market revenue yields

$$\frac{p^{post}q^{post}}{PQ(1 + (\varepsilon + 1)\mathrm{d}P/P)} = (1 + \delta^*) \frac{p^{pre}q^{pre}}{PQ(1 + (\varepsilon + 1)\mathrm{d}P/P)} (1 - \alpha) .$$

Rewrite in terms of shares as

$$s^{post} = (1 + \delta^*) \frac{s^{pre}}{(1 + (\varepsilon + 1)\mathrm{d}P/P)} (1 - \alpha)$$

and solve for $\alpha$ as

$$\alpha = 1 - \frac{s^{post}}{s^{pre}} \frac{(1 + (\varepsilon + 1)\mathrm{d}P/P)}{(1 + \delta^*)} .$$

## 5. Proof of Maximum Firm Sizes Under Merger Guidelines Safe Harbors

If the 35% safe-harbor rule were enforced, then the HHI safe harbor would have independent relevance only for transactions where one of the firms is very small. By the algebra of the HHI (see Merger Guidelines at note 18), the safe harbor for merging firms 1 and 2 can be expressed as:

$$2s_1s_2 < \delta,$$

where $\delta$, the maximum safe harbor change in the HHI, is either 100 (pre-HHI less than 1,800) or 50 (pre-HHI greater than 1,800). It follows that $s_2 < \delta/(2s_1)$.

By assumption, $s_1 + s_2 > 35\%$, so that $s_2 > 35\% - s_1$. Putting these two conditions together implies

$$35\% - s_1 < \delta/(2s_1),$$

or, equivalently,

$$s_1^2 - 35s_1 + \delta/2 > 0.$$

Apply the quadratic formula, assuming the expression is equal to zero, and solve for the two possible values for $s_1$. The inequality is then satisfied when $s_1$ is either smaller than the lower value (and $s_2 > 35\% - s_1$) or greater than the higher value (and $s_2 < \delta/(2s_1)$. By substituting for $\delta$, it can be seen that the HHI safe harbor limits the smaller merging firm to at most a 1.5% share (pre-transaction HHI between 1,000 and 1,800) and a 0.7% share (pre-transaction HHI over 1,800)

It also follows that when the maximum safe harbor change in the HHI is 50, and the 35% standard is not enforced, then the smaller firm can be no larger than 5% (and must be below this level when the share of the larger firm is above 5%). When the maximum safe harbor change is 100, then the smaller firm can be no larger than 7.1% and must be below 5% when the share of the larger firm is above 10%.